Asymptotic Symmetries and Charges in General Relativity

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New Directions in Theoretical Physics
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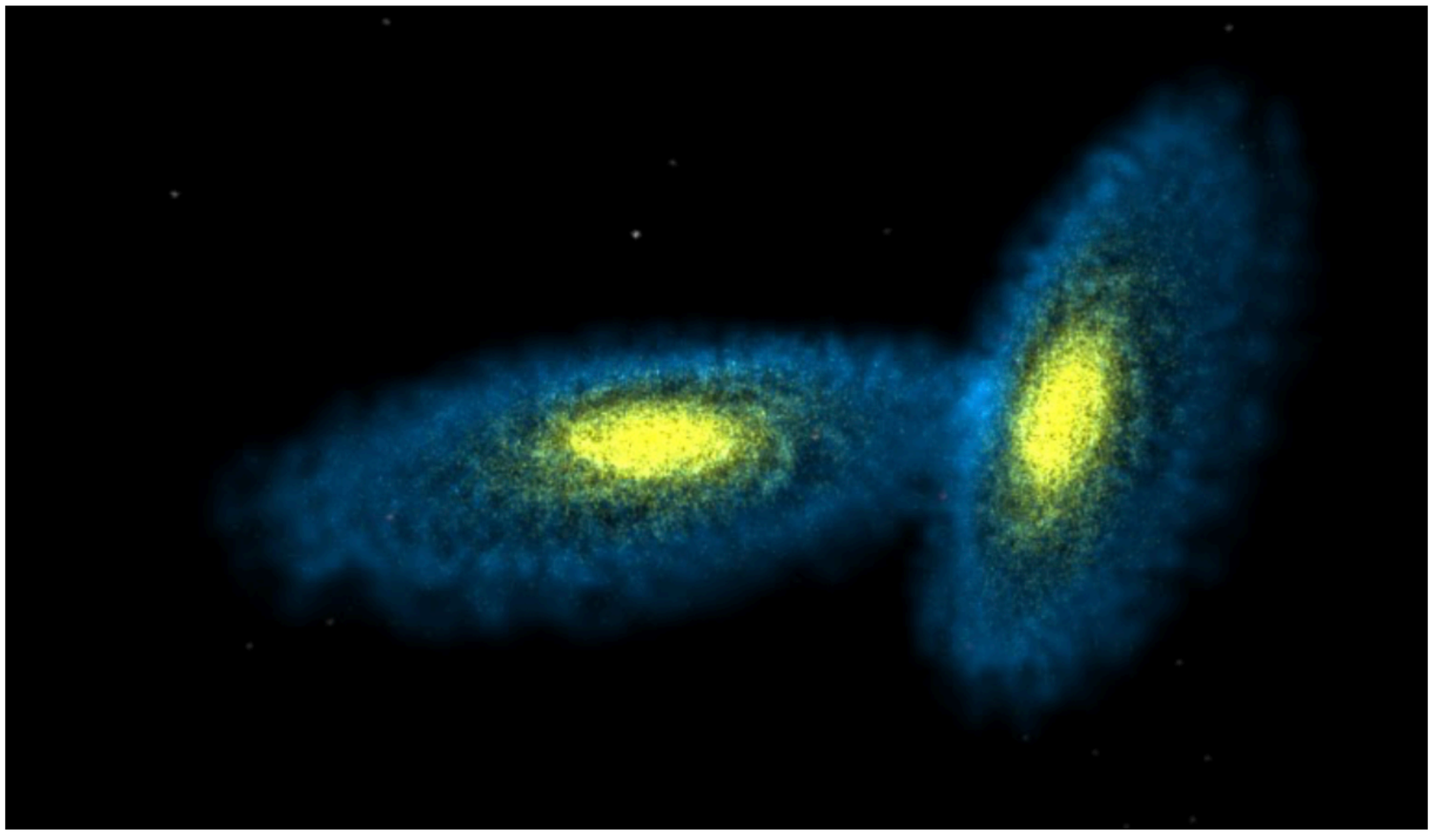
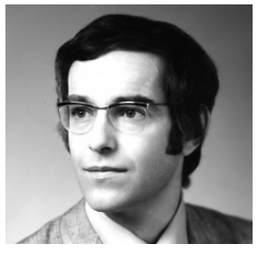
Preamble : Colliding Galaxies



Credits

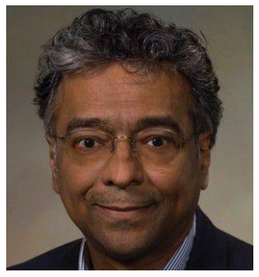
Hermann Bondi



Kenneth Metzner

Rainer Sachs

Bob Wald



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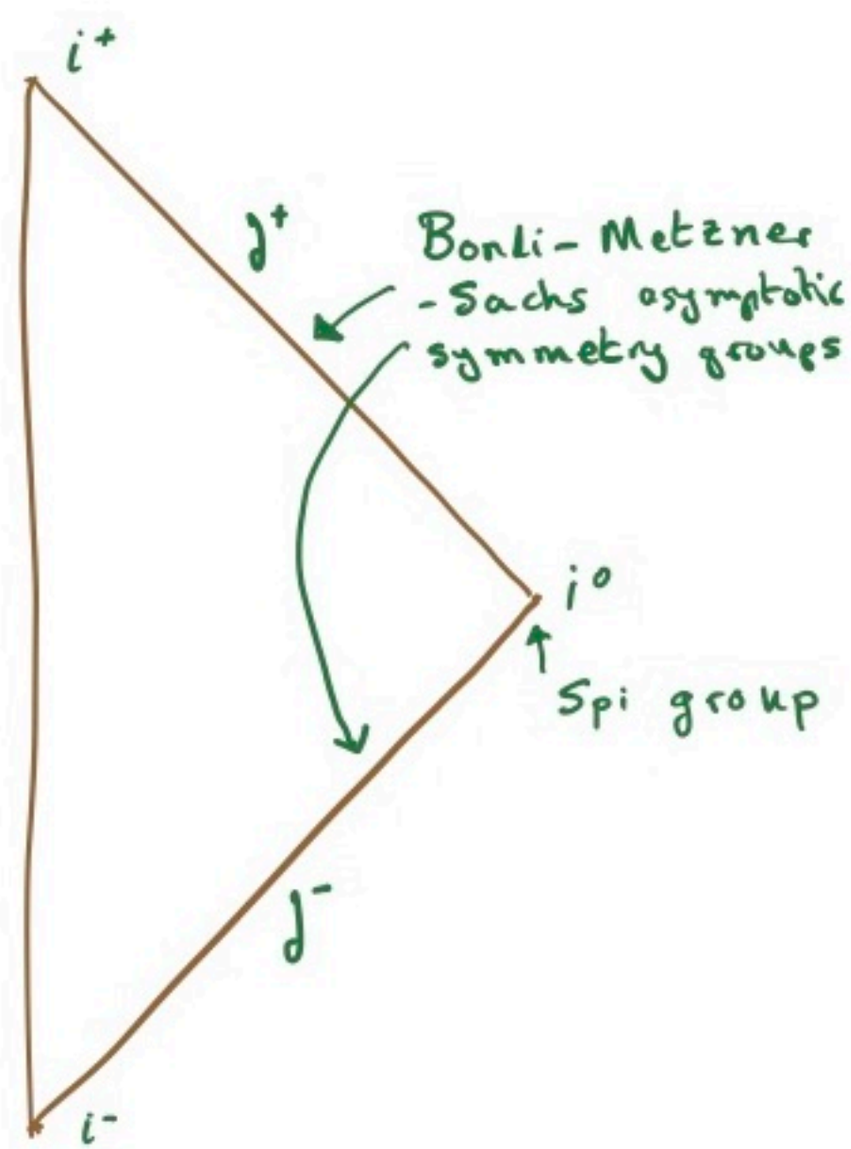
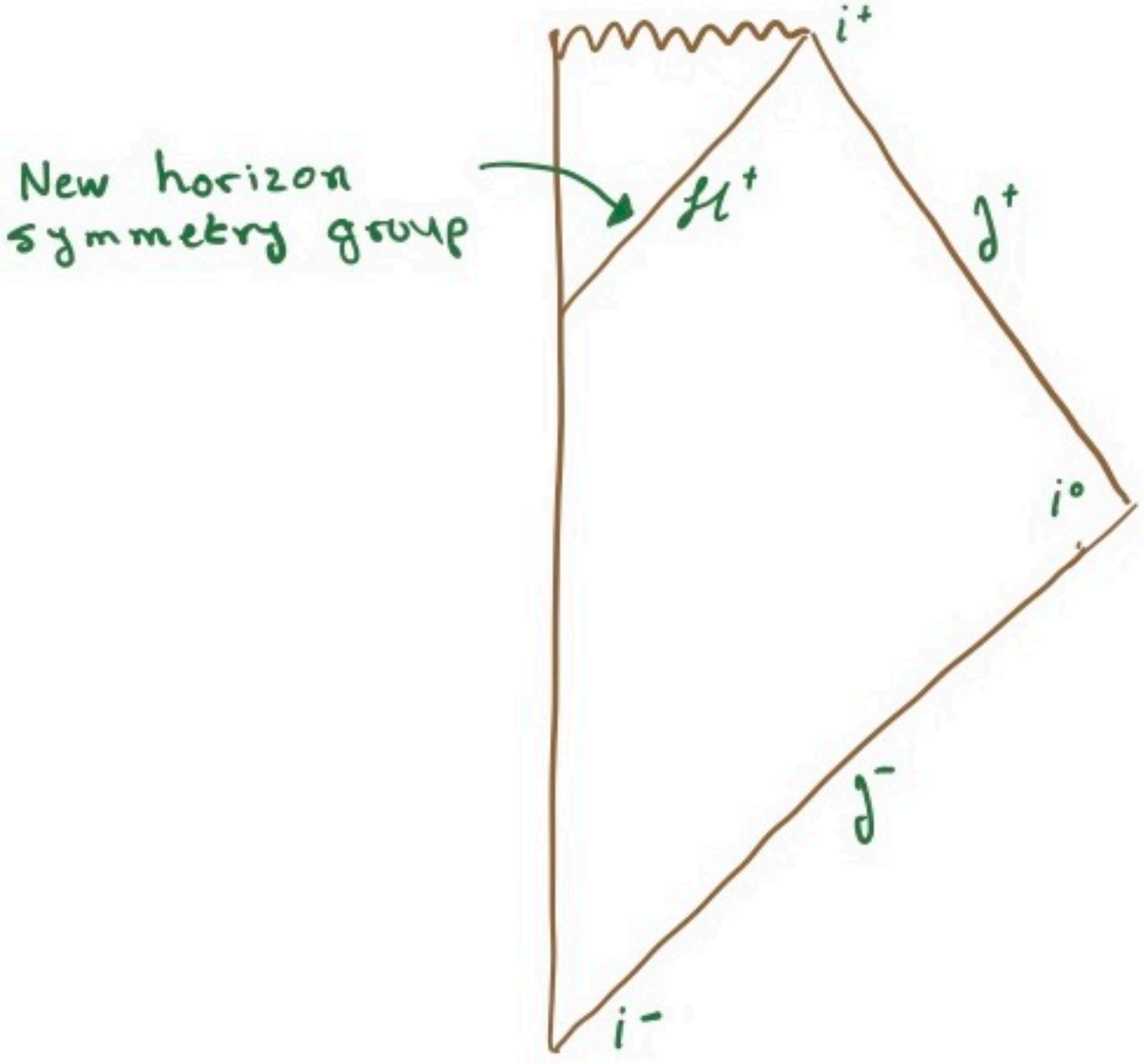
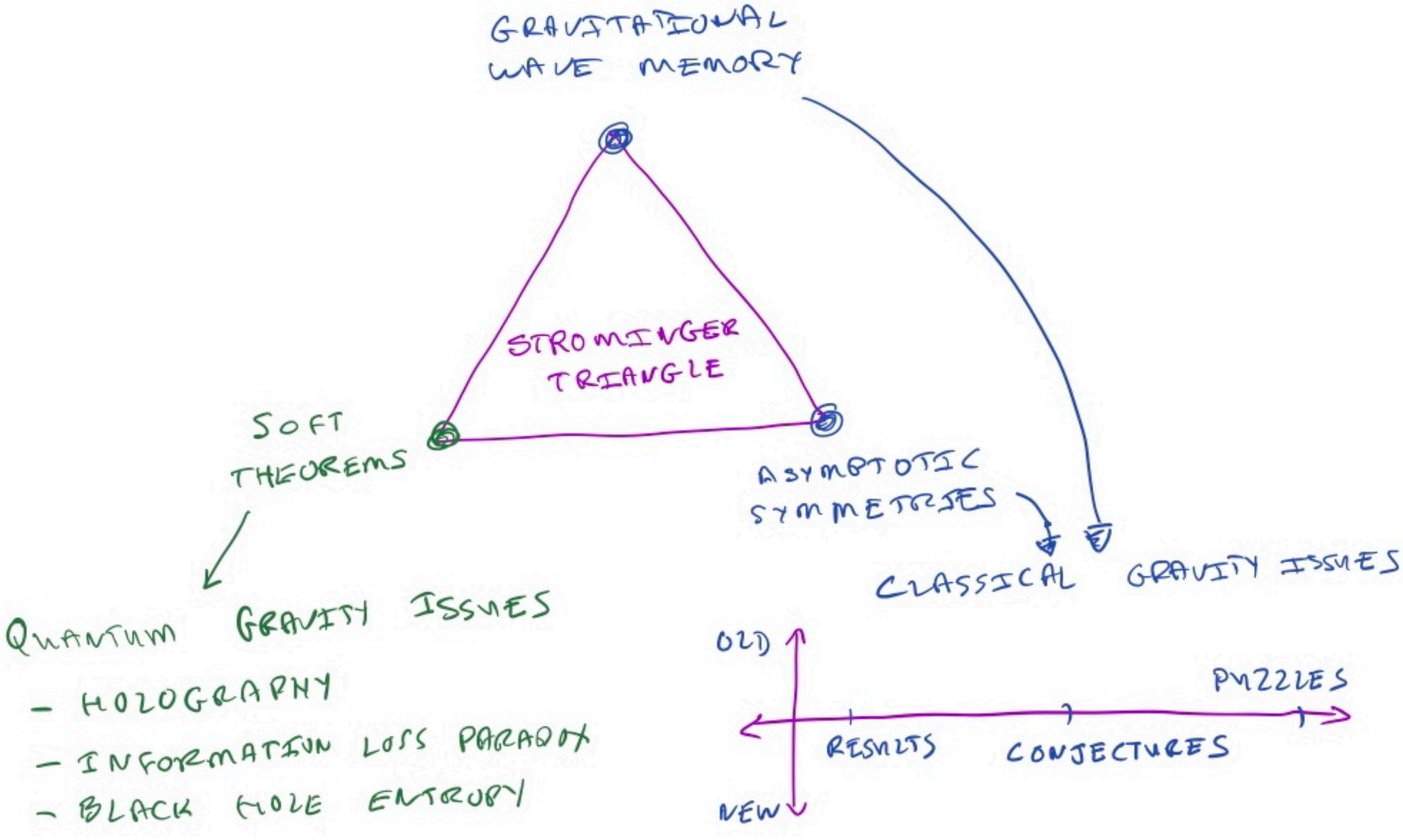
Context

★ General relativity in 3+1 dimensions, $\Lambda = 0$



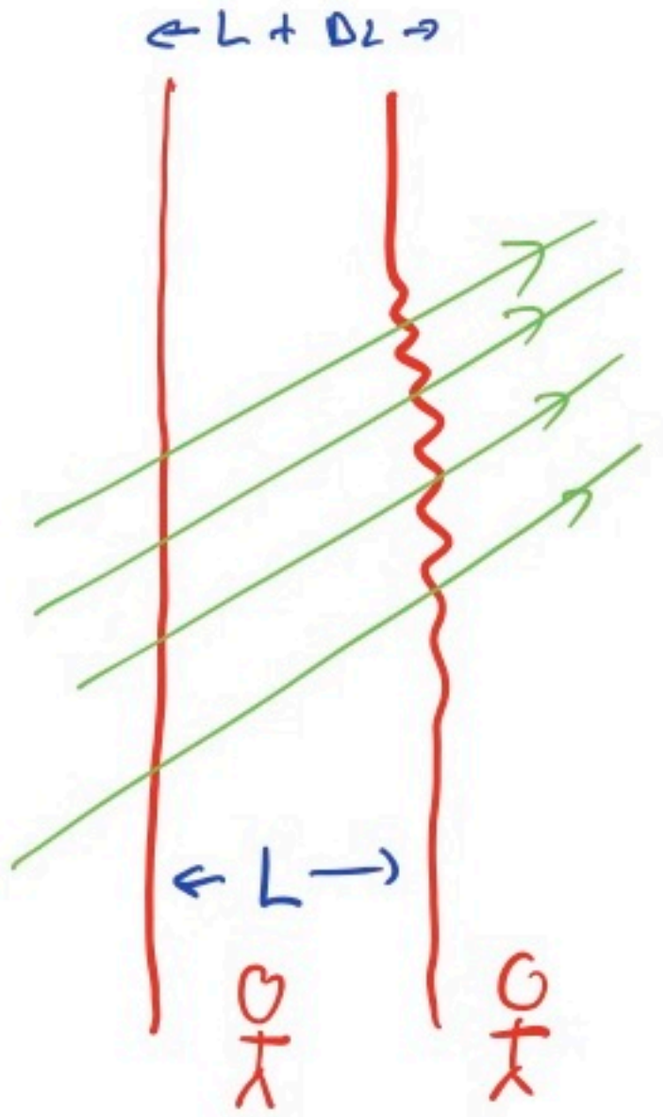


Overview


GRAVITATIONAL WAVE MEMORY
STROMINGER TRIANGLE
SOFT THEOREMS
ASYMPTOTIC SYMMETRIES
CLASSICAL GRAVITY ISSUES
QUANTUM GRAVITY ISSUES
- HOLOGRAPHY
- INFORMATION LOSS PARADOX
- BLACK HOLE ENTROPY
OLD
NEW
RESULTS
CONJECTURES
PUZZLES

Gravitational Wave Memory : Review

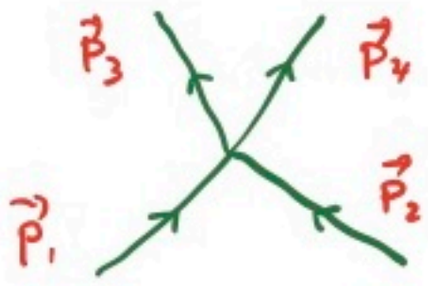
★ The permanent relative displacement of a pair of freely falling test masses (Zel'dovich and Polnarev 1974)



★ Geodesic deviation : $\xi^i \to \xi^i + h^{ij}\xi_j$ with $h \sim \int dt \int dt'$ (Riemann)

★ Intuition : $h \sim \frac{\ddot{Q}(t-r,\theta,\varphi)}{r} \to \frac{\ddot{Q}(\theta,\varphi)}{r}$

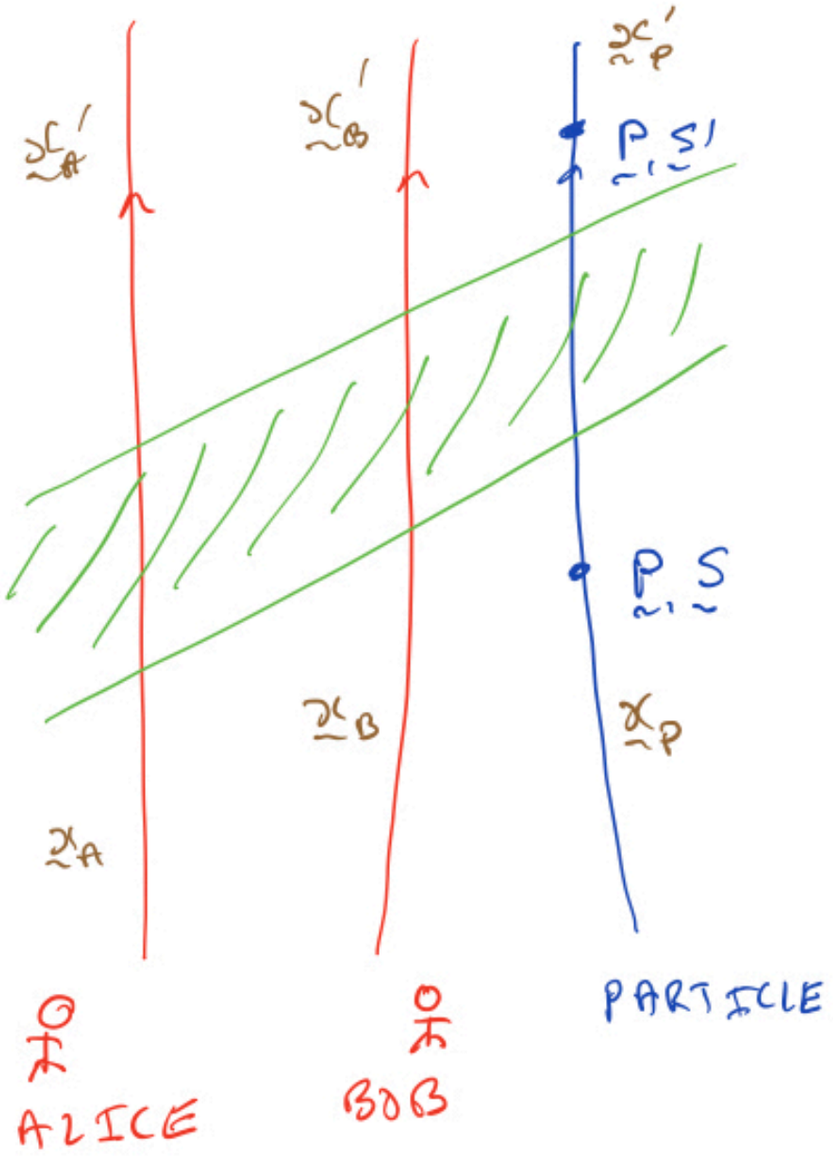
(non-radiative, non-stationary)



★ Detectable soon with LIGO/VIRGO/GEO

Memory Effect Hints at New Physics

★ Memory → Angular momentum becomes observer-dependent



$$\underset{\sim}{J}_A = \underset{\sim}{S} + (\underset{\sim}{x}_P - \underset{\sim}{x}_A) \times \underset{\sim}{P}$$

$$\underset{\sim}{J}'_A = \underset{\sim}{S}' + (\underset{\sim}{x}'_P - \underset{\sim}{x}'_A) \times \underset{\sim}{P}$$

$$\delta \underset{\sim}{J}_A = \underset{\sim}{J}'_A - \underset{\sim}{J}_A + \delta \underset{\sim}{x}_A \times \underset{\sim}{P}$$

Displacement measurement

$$\delta \underset{\sim}{J}_A - \delta \underset{\sim}{J}_B = \Big[(\underset{\sim}{x}_A - \underset{\sim}{x}_B) - (\underset{\sim}{x}'_A - \underset{\sim}{x}'_B) - (\delta \underset{\sim}{x}_A - \delta \underset{\sim}{x}_B) \Big] \times \underset{\sim}{P}$$

accelerometer measurement

$$\boxed{\delta \underset{\sim}{J}_A - \delta \underset{\sim}{J}_B = \frac{1}{2} h \cdot (\underset{\sim}{x}_B - \underset{\sim}{x}_A) \times \underset{\sim}{P}}$$

New Symmetry Group : Two Approaches

Alice
Theorist

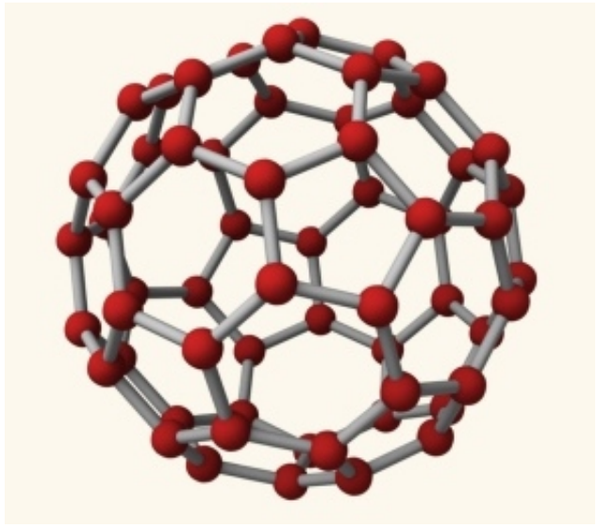
Top Down

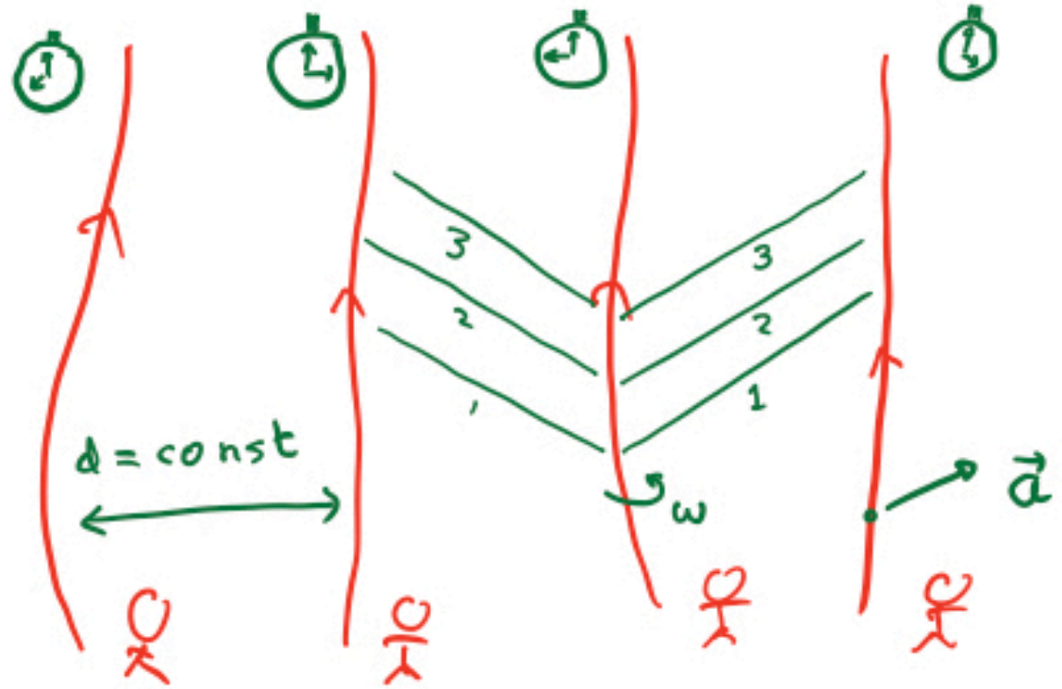
Bottom Up

Bob
Experimentalist

	General Relativity at $\mathcal{I}^+$	Special Relativity
Derive invariant structure	Geometric structure on $\mathcal{I}^+$	Minkowski metric
Symmetry group	BMS (or extensions)	Poincaré
Transformation law for observables		
Identify property of observer, construct reference frame	Asymptotic Bondi frame	Velocity ; Lorentz frame
Observable becomes observer-dependent	$(P^\nu, J^{\alpha\beta})$	Energy E of particle
	Special Relativity (or stationary GR)	Newtonian Physics

Construction of Asymptotic Bondi Frame





- Spatial geometry time independent (Epp, Mann, McGrath 2008) and round (corrections $\propto 1/r^2$)
- Construct (t, r, θ, φ) coordinates in vicinity of shell as in Special Relativity
- Clock synchronization: $\Delta t_{\text{inconsistency}} \sim \sqrt{E_{GW}\, \tau_{GW}}$
- Pick an arbitrary convention for zeros of clocks. A change in this convention $\Delta t(\theta, \varphi)$ is a supertranslation.

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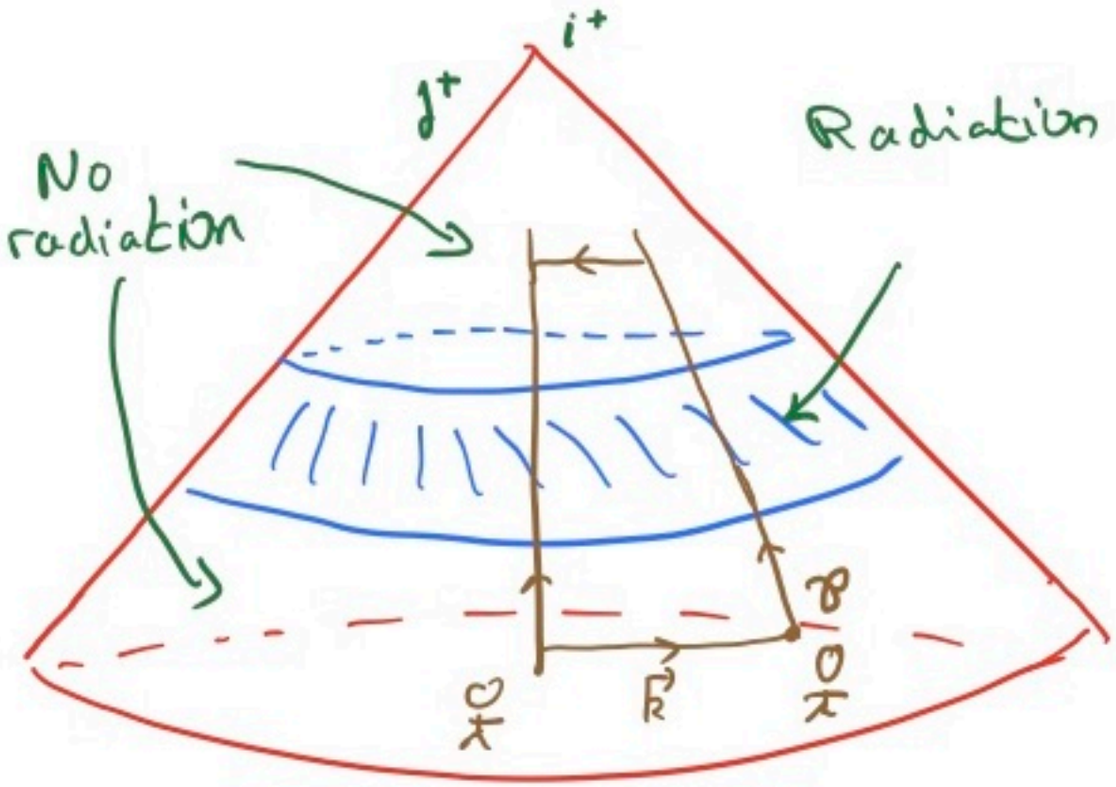
Observer Dependence of Angular Momentum Revisited



- Alice measures at $\mathcal{P}$, near stationary region of $\mathcal{J}^+$,

  $R_{abcd}, \nabla_a R_{bcde}, \nabla_a \nabla_b R_{cdef} \rightarrow P^a(\mathcal{P}), J^{ab}(\mathcal{P})$

- Compares her measurement to Bob's via transport law

  $k^a \nabla_a P^b = \frac{1}{4} R^b{}_{acd} J^{cd} k^a, \quad k^a \nabla_a J^{bc} = 2 P^{[b} k^{c]}$

- Asymptotic consistency

  $P^a = \quad O(1) + O(\frac{1}{r})$

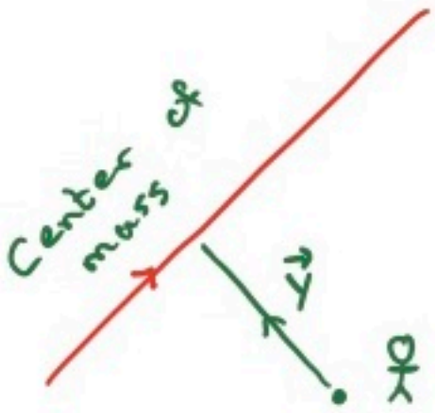
  $J^{ab} = \underbrace{O(r) + O(1)}_{\text{consistent}} + \underbrace{O(\frac{1}{r})}_{\text{small errors}}$

- Wilson-loop-like holonomy observable around closed loop

  $\begin{pmatrix} P^a \\ J^{ab} \end{pmatrix} \rightarrow \begin{pmatrix} \Lambda^a{}_c & \Lambda^a{}_{cd} \\ \Xi^{ab}{}_c & \Xi^{ab}{}_{cd} \end{pmatrix} \begin{pmatrix} P^c \\ J^{cd} \end{pmatrix}$. Identity unless encloses radiation

- Must enlarge set of observables to $(P^a, J^{ab}, \cdots)$

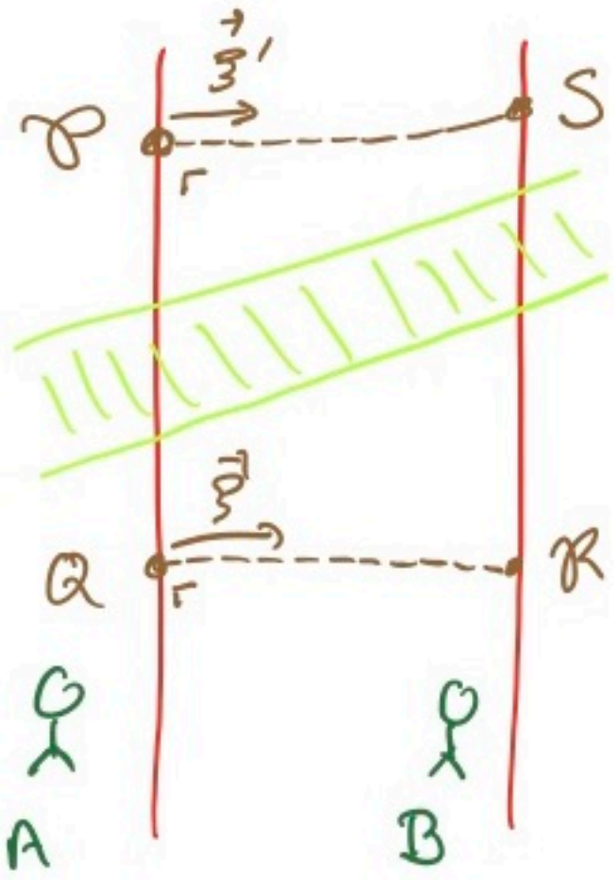
Relation to Gravitational wave Memories



★ Decompose J^{ab} as $J^{ab} = \frac{1}{M}\epsilon^{abcd} P_c S_d + 2\, y^{[a} P^{b]}$

★ Transport law with Riemann term dropped reduces to $k^a \nabla_a y^b = -k^b$, $k^a \nabla_a P^b = 0$, $k^a \nabla_a S^b = 0$ whose solution is the Poincaré transformation

$P^a \to \Lambda^a{}_b P^b$, $S^a \to \Lambda^a{}_b S^b$, $y^a \to \Lambda^a{}_b y^b + \Delta y^a$



★ Poincore transformation $(\Lambda^a{}_b, \Delta y^a)$ encodes four types of memory

- Displacement memory $\vec{\xi}' - \vec{\xi}$ at $\mathscr{I}$ $\left\{ \alpha \frac{1}{r} \right.$ near $\mathscr{I}^+$
- Proper time memory $\Delta\tau_A - \Delta\tau_B$ (Strominger and Zhiboedov 2014)
- Velocity memory $\vec{u}_A(P) - \vec{u}_B(S)$ (Grishchuk & Polnarev 1989) $\left. \alpha \frac{1}{r^2} \right\}$
- Frame dragging memory, net relative rotation of gyroscopes

Four Flavors of Displacement Memory

★ General solution of geodesic deviation equation $\ddot{\xi}^i = R_{0i0j}\xi^j$ is

$$\xi^i(t) = \xi^i_0 + v^i_0 t + \Delta h_{ij}\xi^j_0 + \left(\Delta\tilde{h}_{ij} - t\,\Delta h_{ij}\right)v^j_0$$

Leading Memory

$$\Delta h_{ij} = \int_{-\infty}^{\infty} dt \int^{t} dt' \, R_{0i0j}$$

Subleading Memory

$$\Delta\tilde{h}_{ij} = \int_{-\infty}^{\infty} dt \left[t \int^{t} dt' \, R_{0i0j} \right]$$

★ Decompose into E modes and B modes (even/odd parity). Two measurement methods.

E mode:

$$\Delta h \sim \frac{1}{r}\left[\underbrace{\text{Change in Coulomb Field}}_{\text{Conserved Charge}} + \underbrace{\int_{g+} \text{Energy Flux}}_{\text{Hard Piece of charge}}\right]$$

Ordinary Memory — Null Memory

Soft piece of charge

B mode: No null term. Likely vanishes in vacuum. Non vanishing for general matter sources (Wald 2019)

E mode: Discovered recently (Nichols 2017), "Center-of-mass memory". Sourced by flux of C.O.M. piece of angular momentum

B mode: "Spin Memory" (Pasterski, Strominger and Zhiboedov, 2015). Sourced by flux of intrinsic spin piece of angular momentum.

Top Down Approach to Symmetry Group

★ Metric near $\mathcal{I}^+$ in Bondi-Sachs coordinates $(u, r, \theta^1, \theta^2)$ is

$$ds^2 = -du^2\left[1 - 2\underbrace{\frac{m(u,\theta^A)}{r}}_{\text{Bondi mass aspect}}\right] + r^2\left[h_{AB}(\theta^C) + \frac{1}{r}\underbrace{C_{AB}(u,\theta^C)}_{\text{Shear tensor}}\right]$$

$$+ \left[D^B C_{AB} + \frac{4}{3r}\underbrace{N_B(u,\theta^A)}_{\substack{\text{Angular momentum} \\ \text{aspect}}} + \frac{\text{"}C^2\text{"}}{r}\right] du\, d\theta^A + \cdots$$

★ BMS symmetries

Supertranslation

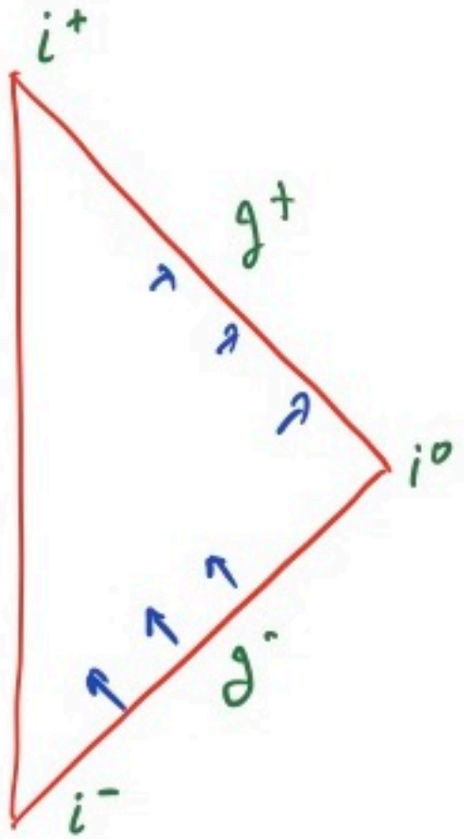
$$u \to u + \overbrace{\alpha(\theta^B)} + \frac{1}{2}u D_A Y^A + O\left(\frac{1}{r}\right)$$

$$\theta^A \to \theta^A + \underbrace{Y^A(\theta^B)}_{\substack{\text{Conformal Killing Vector} \\ \text{- 6 parameters}}} + O\left(\frac{1}{r}\right)$$

★ Extended BMS Algebra ???

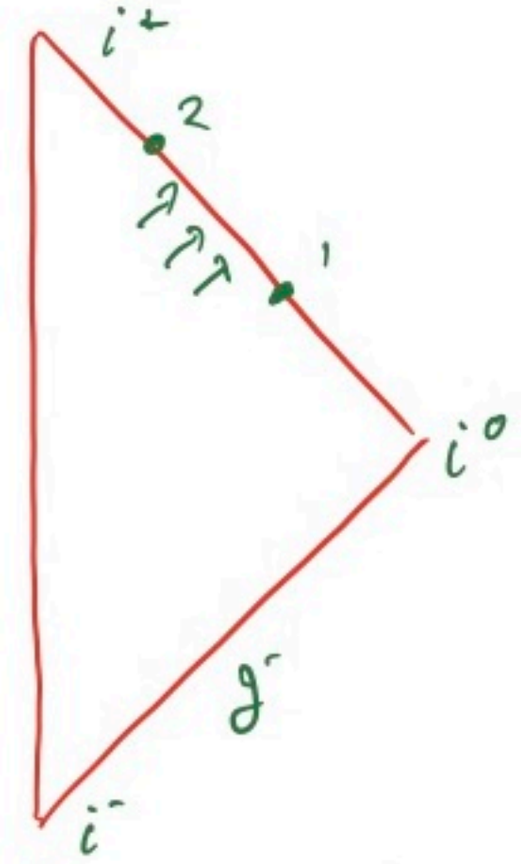
- Allow Y^A to be a singular conformal Killing vector, called superrotation (Barnich & Troessaert 2010)
- Allow $Y^A(\theta^B)$ to be any smooth diffeomorphism of the two-sphere (Campaglia and Laddha 2014)

Two Types of Conservation Laws



★ Global conservation law which constrains scattering

$$\int_{g^-} (\text{flux}) = \int_{g^+} (\text{flux})$$



★ Localized conservation law

$$Q_2 - Q_1 = \int_{1 \to 2} (\text{flux})$$

Status of Charges and Conservation Laws

	Supermomentum	Supersspin	Super Center-of-mass (soft hair)
Metric Function	m	N_A (B mode)	N_A (E mode)
Symmetry	Supertranslation	Superrotation (B mode)	Superrot. (B mode)
Interpretation of cons. law	Energy cons. at every angle	Spin cons. at every angle	C.O.M. cons at every angle
Localized cons. law	Ashtekar 1981	Pasterski et. al. 2015	Flanagan & Nichols 2016
Global cons. law	Conjectured (special case Strominger 2015)	Conjectured (special case Pasterski et. al. 2015)	Conjectured
Relation to memory	Change in charge is ordinary piece of leading order, E mode memory	Change in charge is ordinary piece of spin memory	Change in charge is ordinary piece of C.O.M. memory

Connection to Noether's Theorem?

Conclusions

- Our understanding of asymptotic charges and symmetries in general relativity has deepened over the past several years.
- Even within the realm of classical physics, there are open issues and puzzles to be resolved.