- In the near-horizon region, we change co-ordinates from r to ζ so that

$$r - R_h = \frac{R_2^2}{\zeta} \quad , \quad R_2 = \frac{LR_h}{\sqrt{6R_h^2 + L^2}}.$$

Then the near-horizon metric becomes $\text{AdS}_2 \times \text{S}_2$, with

$$ds^2 = R_2^2 \left[\frac{-dt^2 + d\zeta^2}{\zeta^2} \right] + R_h^2 d\Omega_2^2 \quad , \quad A = \frac{\mathcal{E}}{\zeta} dt.$$

where the dimensionless electric field $\mathcal{E}$ is

$$\mathcal{E} = \frac{g_F R_h \sqrt{3R_h^2 + L^2}}{6R_h^2 + L^2}.$$