

The effective theory of time reparameterizations $f(\tau)$ broken down to $\text{SL}(2, \mathbb{R})$, and gauge transformations $g(\tau) = e^{i\phi(\tau)}$ is

$$S_{\text{eff}}[f, \phi] = \frac{K}{2} \int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi \mathcal{E} T) \partial_\tau f)^2 - \frac{\gamma}{4\pi^2} \int_0^{1/T} d\tau \{ \tan(\pi T f(\tau)), \tau \},$$

where $f(\tau)$ is a monotonic map from $[0, 1/T]$ to $[0, 1/T]$, the couplings K , γ , and \mathcal{E} can be related to thermodynamic derivatives and we have used the Schwarzian:

$$\{g, \tau\} \equiv \frac{g'''}{g'} - \frac{3}{2} \left(\frac{g''}{g'} \right)^2.$$

Specifically, an argument constraining the effective action at $T = 0$ is

$$S_{\text{eff}} \left[f(\tau) = \frac{a\tau + b}{c\tau + d}, \phi(\tau) = 0 \right] = 0,$$

and this is origin of the Schwarzian.