

There are  $2^N$  many body levels with energy  $E$ . Shown are all values of  $E$  for a single cluster of size  $N = 12$ . The  $T \rightarrow 0$  state has an entropy  $S_{GPS} = Ns_0$ , where  $s_0 < \ln 2$  is determined by integrating

$$\frac{ds_0}{dQ} = 2\pi\mathcal{E}.$$

At  $Q = 1/2$ ,

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\dots$$

where  $G$  is Catalan's constant.