

The large N saddle point is

$$G_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-1/2}.$$

The saddle point will be invariant under a reparamaterization $f(\tau)$ when choosing $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$ leads to a transformed $\tilde{G}(\sigma_1, \sigma_2) = G_s(\sigma_1 - \sigma_2)$ (and similarly for Σ). It turns out this is true only for the $\text{SL}(2, \mathbb{R})$ transformations under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d}, \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken down to $\text{SL}(2, \mathbb{R})$ by the saddle point.