Making Room for Perturbation Theory in the Nonperturbative Strong Interactions

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1. **Seeing the Unseen at Accelerators:**
   Scaling, Jets and the Birth of Quantum Chromodynnamics
2. **Infrared Safety:**
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3. **Factorizations and Evolutions:**
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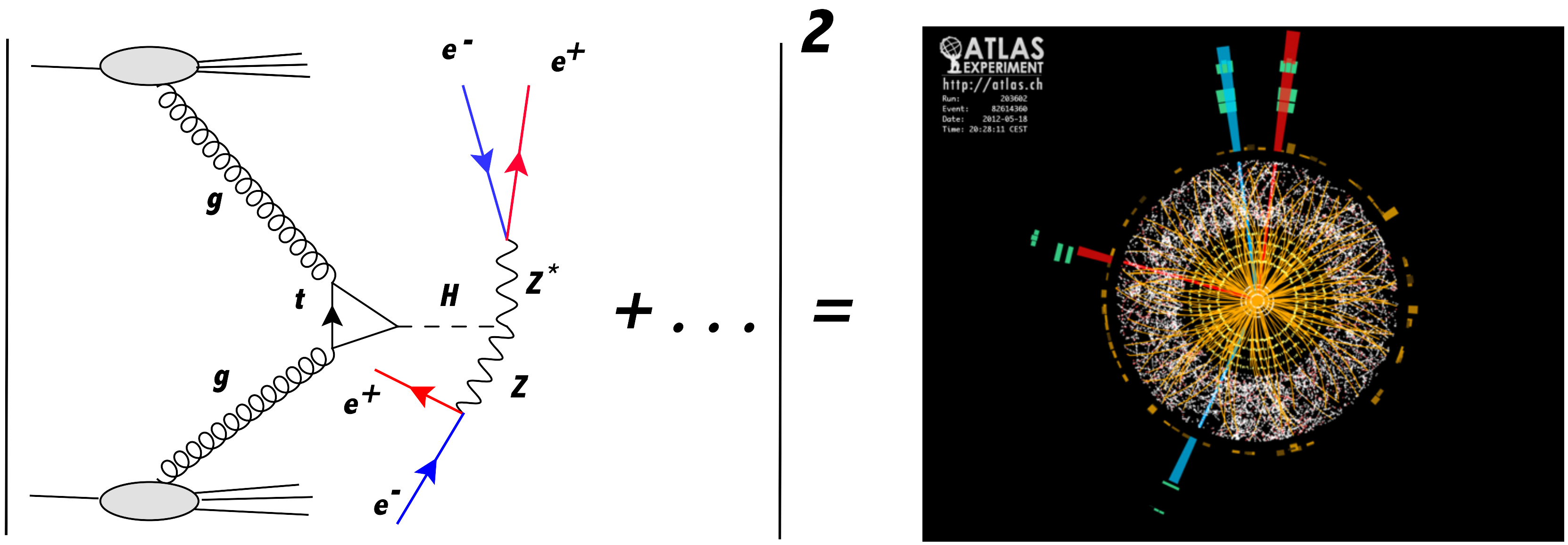
Starting from the beginning . . .

1. Seeing the Unseen at Colliders: *Scaling, Jets and the Birth of Quantum Chromodynnamics*

We can sum it up with a picture worth a thousand words:



From $SU(3)$ color through the Higgs into $SU(2)_L \times U(1)$.

Every observed final state is the result of a quantum-mechanical set of stories, and so far the stories supplied by the Standard Model, built on an unbroken $SU(3)$ color gauge theory (very much like the original Yang-Mills Lagrangian) and a spontaneously-broken $SU(2)_L \times U(1)$, account for essentially all observations at accelerators.

- **The Standard Model developed through the latter half of the Twentieth Century in parallel with modern field-theoretic ideas of flow: couplings within theories (renormalization group) and between theories (Wilsonian).**

- **A primary theme of Twenty-first Century physics is strongly coupled theories with emergent degrees of freedom. This is part and parcel of contemporary challenges.**

- **The mid-20th century picture of strong interactions: nucleons, nuclei bound by meson exchange, with multiple excitations evolved into:**

- **THE QUARK MODEL, with (mostly) qqq' baryons and $q\bar{q}'$ mesons.**

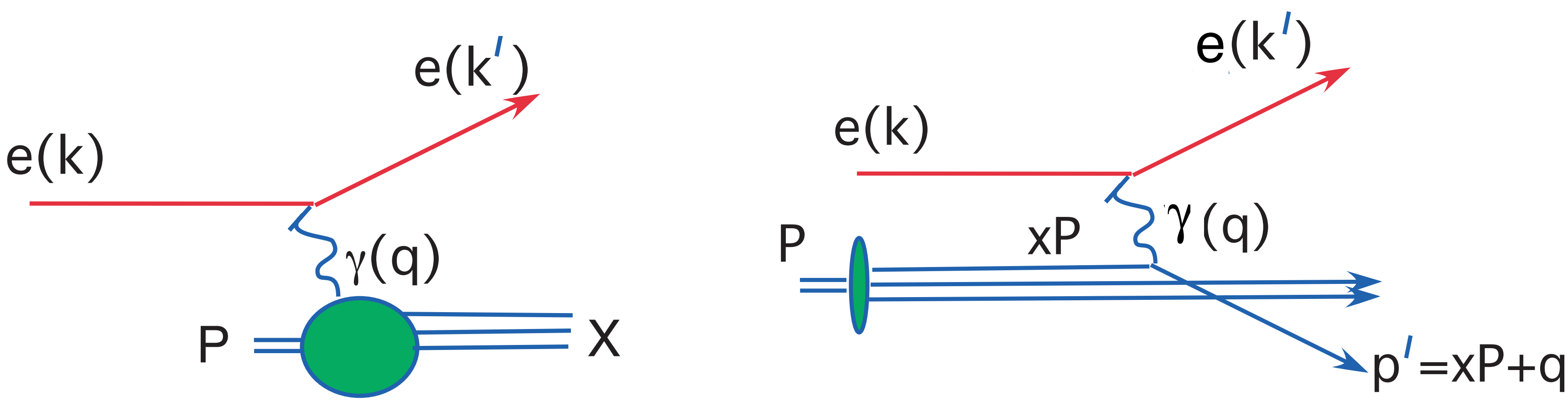
- **QUANTUM CHROMODYNAMICS, a part of the Standard Model, is in some ways the exemplary QFT, still not fully understood, but illustrating the fundamental realization that quantum field theories are protean: manifesting themselves differently on different length scales, yet experimentally accessible at all scales.**

- **To make a long story short: Quantum Chromodynamics (QCD) reconciled the irreconcilable.** **Here was the problem . . .**

1. Quarks and gluons explain spectroscopy, but aren't seen directly – confinement.

2. In highly ("deep") inelastic, electron-proton scattering, the inclusive cross section was found to be well-approximated by lowest-order elastic scattering of point-like (spin-1/2) particles (="partons" = quarks here) a result called "scaling":

$$\frac{d\sigma_{e+p}(Q, p\cdot q)}{dQ^2}\Big|_{\text{inclusive}} \simeq F\left(x = \frac{Q^2}{2p\cdot q}\right) \frac{d\sigma^{\text{free}}_{e+\text{spin}\,\frac{1}{2}}}{dQ^2}\Big|_{\text{elastic}}$$



- **If the "spin-$\frac{1}{2}$" is a quark, a paradox: how can a confined quark scatter freely?**

- **This paradoxical combination of confined bound states at long distances and nearly free behavior at short distances was explained by asymptotic freedom: In QCD, the force between quarks behaves at short distances like**

$$\mathrm{force}(r) \sim \frac{\alpha_s(r)}{r^2}, \qquad \alpha_s(r^2) = \frac{4\pi}{\ln\left(\frac{1}{r^2\Lambda^2}\right)}$$

where $\Lambda \sim 0.2$ GeV. For distances much less than $1/(0.2GeV) \sim 10^{-8}$cm the force weakens. These are distances that began to be probed in deep inelastic scattering experiments at SLAC in the 1970s.

- **The short explanation of DIS scaling: Over the times $t \ll \hbar/0.2GeV$ it takes the electron to scatter from a quark-parton, the quark really does seem free. Later, the quark is eventually confined, but by then it's too late to change the probability for an event that has already happened. This is how inclusive can be given by elastic.**

- **The function $F(x)$ is interpreted as the probability to find quark of momentum xP in a target of total momentum P – a parton distribution.**

- **Asymptotic freedom is a big deal:**

$$\frac{\text{Scaling}}{\text{QCD}} = \frac{\text{Elliptical Orbits}}{\text{Newtonian Gravity}}$$

- **A beginning, not an end.**

- **For Newtonian gravity, the three-body problem.**

- **For QCD . . . the challenge**

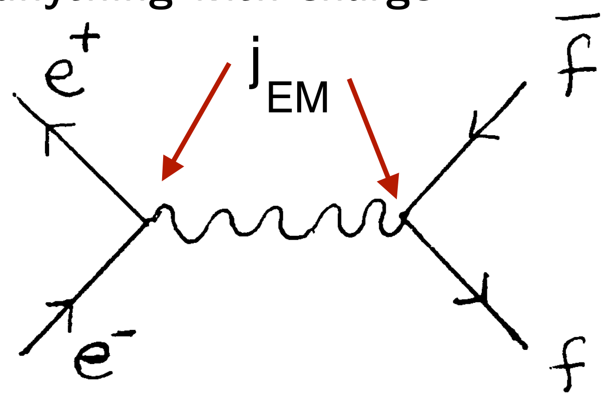
$$\frac{\text{Nuclear Physics}}{\text{QCD}} = \frac{\text{Chemistry}}{\text{QED}}$$

- **But can we**
  - **Study the particles that give the currents (quarks)?**
  - **Study the particles that give the forces (gluons)?**
  - **Expand in number of gluons? Perturbation Theory**

- To explore further, SLAC used the quantum mechanical credo:

  anything that can happen, will happen.

- Quarks have electric charge, so if they are there to be produced, they will be. This can happen when colliding electron-positron pairs annihilate to a virtual photon, which ungratefully decays to just anything with charge



- Of course, because of confinement, it's not really that simple. But more generally, we believe that a virtual photon decays through a local operator: $j_{\rm em}(x)$.

- This enables translating measurements into correlation functions . . . In fact, the cross section for electron-positron annihilation probes the vacuum with an electromagnetic current.

- **On the one hand, all final states are familiar hadrons, with nothing special about them to tell the tale of QCD, $|N\rangle = |\text{pions, protons}\ldots\rangle$,**

$$\sigma_{\mathrm{e^+e^-\to\ hadrons}}(Q) \;\propto\; \sum_N |\langle 0|j^\mu_{\mathrm{em}}(0)|N\rangle|^2\,\delta^4(Q-p_N)$$

- **On the other hand, $\sum_N |N\rangle\langle N| = 1$, and using translation invariance this gives**

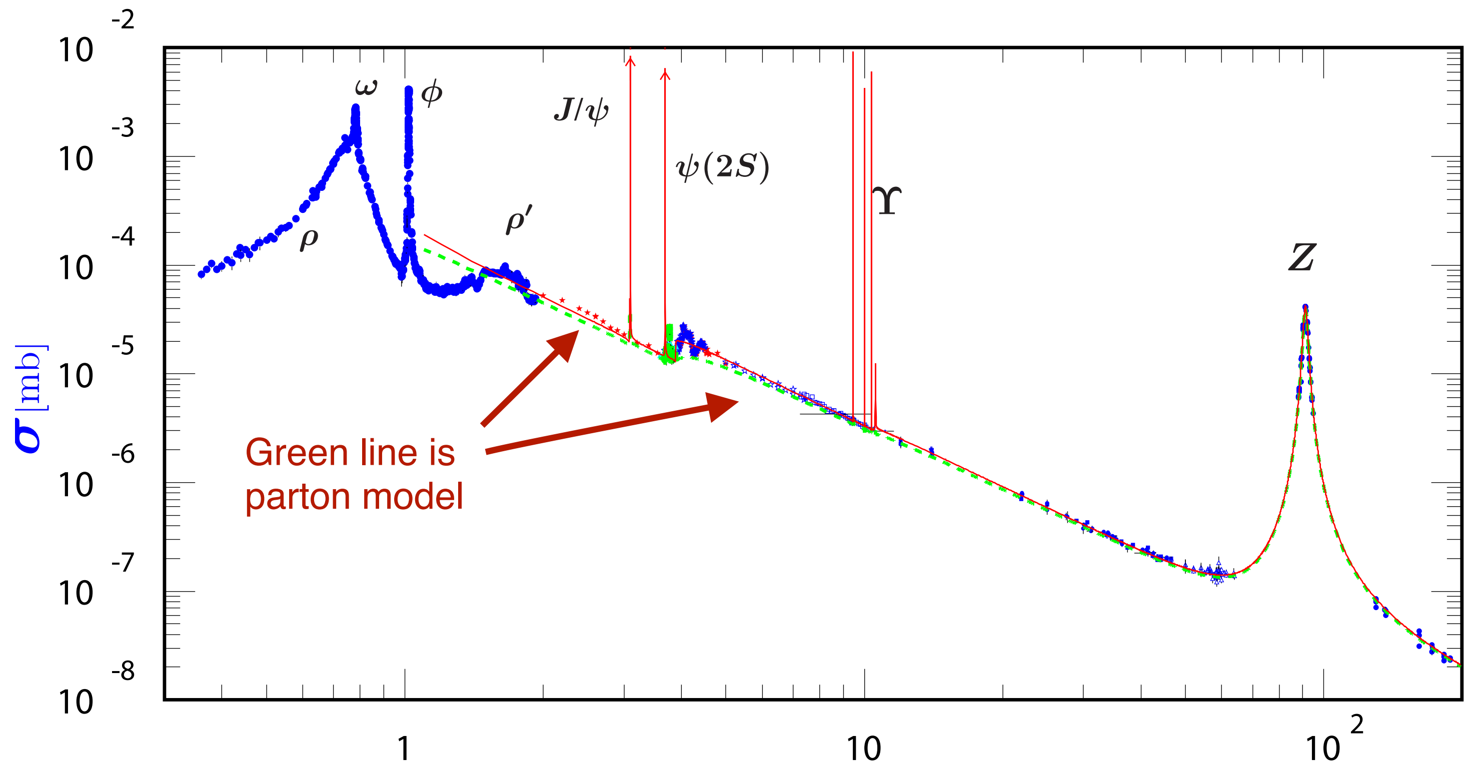
$$\sigma_{\mathrm{e^+e^-\to\ hadrons}}(Q) \;\propto\; \int d^4x\, e^{-iQ\cdot x}\ \langle 0|j^\mu_{\mathrm{em}}(0)\ j^\mu_{\mathrm{em}}(x)|0\rangle$$

- **We are probing the vacuum at short distances, imposed by the Fourier transform as $Q \to \infty$. The currents are only a distance $1/Q$ apart.**

- **Asymptotic freedom suggests a "free" result: QCD at lowest order ("quark-parton model") at cm. energy Q**

$$\sigma^{tot}_{e^+e^-\to\mathrm{hadrons}} \;=\; \frac{4\pi\alpha^2_{\mathrm{EM}}}{3Q^2}\sum_q e_q^2$$

• **This works for σ_{tot} to quite a good approximation!** **(with calculable corrections)**



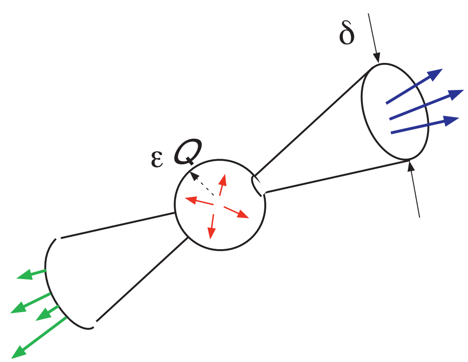
• **So the "free" theory again describes the inclusive sum over confined (nonperturbative) bound states – another "paradox".**

- **Is there an imprint on these states of their origin? Yes. What to look for? The spin of the quarks is imprinted in their angular distribution:**

$$\frac{d\sigma(Q)}{d\cos\theta} = \frac{\pi\alpha_{\rm EM}^2}{2Q^2}\left(1+\cos^2\theta\right)$$

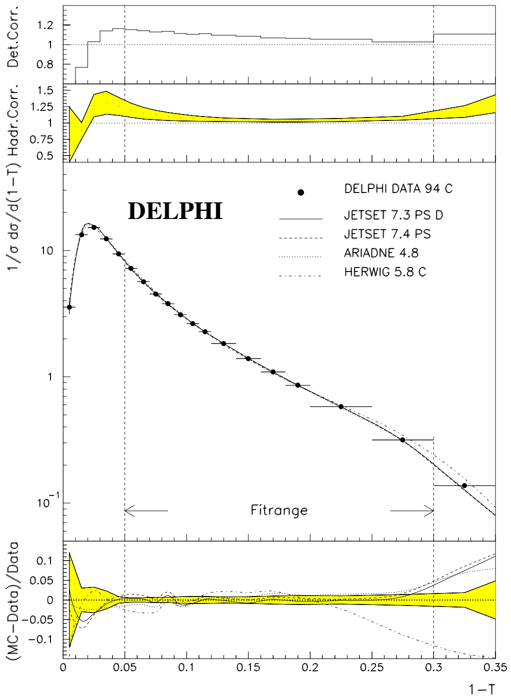
- **It's not quarks, but we can look for a back to back flow of energy by finding an axis that maximizes the projection of particle momenta ("thrust")**

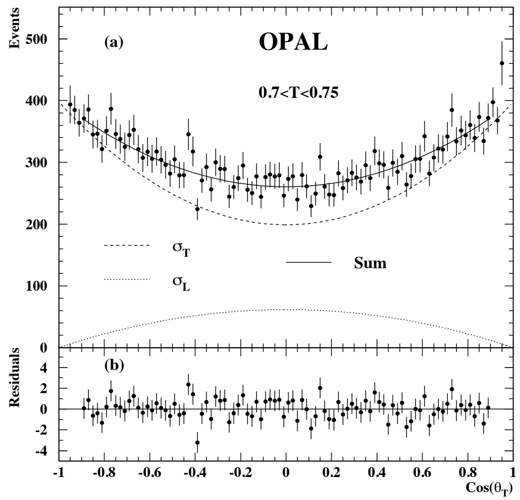
$$\frac{d\sigma_{\rm e^+e^-\to\ hadrons}(Q)}{dT} \propto \sum_N |\langle 0|j^\mu_{\rm em}(0)|N\rangle|^2\,\delta^4(Q-p_N)\ \delta\left(T-\frac{1}{Q}\max_{\hat{\rm n}}\sum_{i\in N}|\vec{p}_i\cdot\hat{\rm n}|\right)$$

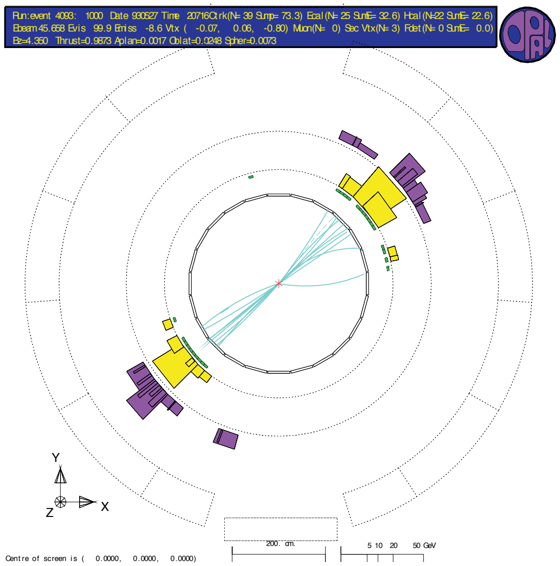


- **When the particles all line up, $T\to 1$ (neglecting masses). So what really happens?**

- **Here's what was found (from a little later, at LEP):**







- **Thrust is peaked near unity and follow the $1 + \cos^2\theta$ distribution – reflecting the production of spin $\frac{1}{2}$ particles – back-to-back. All this despite confinement. Quarks have been replaced by "jets" of hadrons. What could be better?**

- **But what's going on? How can we understand persistence of short-distance structure into the final state, evolving over many many orders of magnitude in time? The particles seen in the final states are pions and protons, not quarks and gluons.**

- **We are required to describe a theory with different degrees of freedom at different momenta and length scale. Nature transitions between the two effortlessly, but we can't yet.**

- **Setting this aside, what can we do with the tools at hand, and how can we seek to improve them?**

- **More specifically, how can we use perturbative QCD to measure, if not understand, the nonperturbative content of QCD?**

- **Can we characterize breakdowns of perturbation theory and use it to organize nonperturbative corrections?**

2. Infrared Safety: *Finding Something We Can Calculate, Better and Better*

Pre-gauge theory lessons for all perturbation theories: Even if a "final" theory isn't known, provisional or model Lagrangians can act as a valuable guide. And, if you know the Lagrangian, so much the better.

- **Landau Equations – singularities in external momenta, p_j are determined by linear equations in loop momenta. They occur when gradients with respect to loops l_c^μ of a set of propagator denominators $k_i^2(l_m, p_j)$ become linearly dependent**

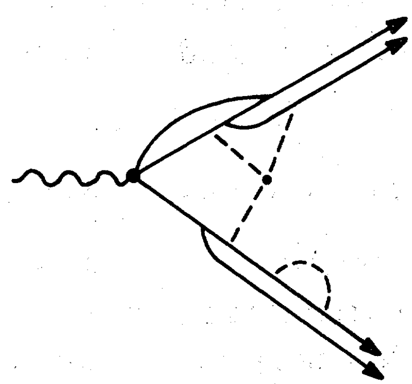
$$\sum_{\text{lines } i \text{ in loop } c} \alpha_i \, k_i^\mu(l_c) \; = \; 0 \, .$$

- **Coleman and Norton: the momenta of the on shell lines at Landau equations describe physical processes.**

- **Singularities (= enhancements) in amplitudes "tell a story".**

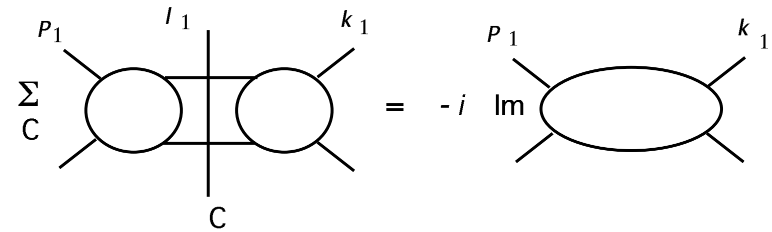
- **A tool for analyzing arbitrary diagrams in arbitrary theories.**

- **Infrared safety: From analyticity and unitarity to jets and event shapes.**
- **For an arbitrary diagram, what is the source of long-distance behavior (infrared divergences)? Consult the Coleman-Norton interpretation of Landau equations. For e^+e^- annihilation to hadrons, the only phyiscal pictures are like these (illustrated for two "jets" of collinear particles).**

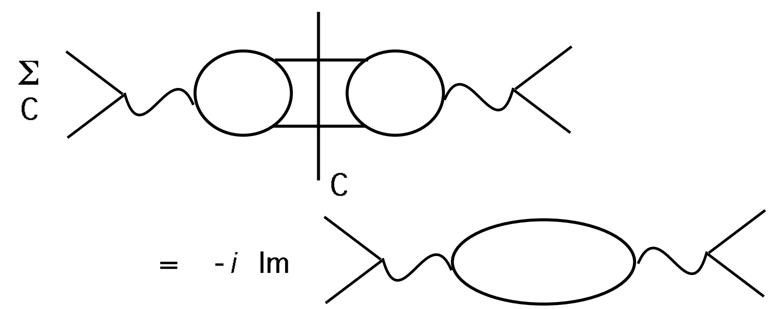


- **all intermediate states have the same flow of momentum – it's just redistributed between collinear particles, with additional soft radiation. No momentum can flow from one jet to another!**
- **Analytic calculations at one loop and beyond confirm this structure. Away from the singularities, numerical evaluation is possible, and only recently being systematically explored.**

- **For cross sections, cut diagrams and generalized unitarity**
- **Basic expression of unitarity at the level of diagrams:**



- **Or for** $\mathrm{e}^{+}\mathrm{e}^{-}$**,**

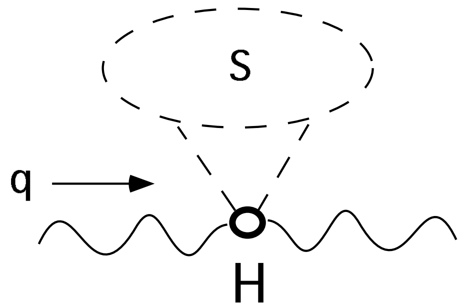


$$\pi(q^2)(q_\mu q_\nu - q^2 g_{\mu\nu}) = i \int d^4x\, e^{iqx} < 0|\; T\; J_\mu(x) J_\nu(0)\; |0 >$$

$$\sigma^{(\mathrm{tot})}_{e^+e^-}(q^2) = \frac{e^2}{q^2} \mathrm{Im}\, \pi(q^2)\,,$$

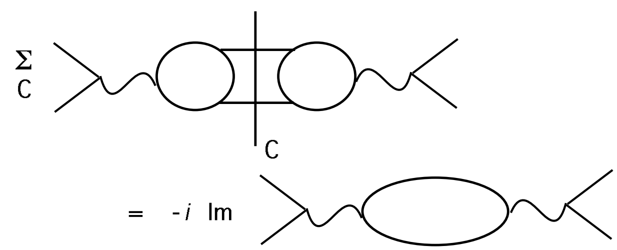
The function π is defined in terms of the two-point correlation function of the relevant electroweak currents J_μ (with their couplings included) as above.

- **Singularities? The only physical pictures for $\langle JJ \rangle$ and hence for $\sigma^{(\mathrm{tot})}_{e^+e^-}(q^2)$:**



- **Power counting confirms finiteness.**

- **But the method is much more general – unitarity holds point-by-point in *spatial* loop momenta $\vec{l}$ in the diagrams:**



$$\sum_{\mathrm{all}\ C} G_C(p_i, k_j, \vec{l}) = 2\ \mathrm{Im}\ \left(-iG(p_i, k_j, \vec{l})\right)\ .$$

- **Proof (and the origin of jet analysis): Do the time integrals for a general amplitude in part I, and get time-ordered perturbation theory (TOPT). This is equivalent to the sum over Feynman diagrams. The amplitude and its complex conjugate are given by a sum over virtual states:**

$$\sum_m \Gamma^*_m \Gamma_m = \sum_{m=1}^{A} \prod_{j=m+1}^{A} \frac{1}{E_j - S_j - i\epsilon}\,(2\pi)\delta(E_m - S_m) \prod_{i=1}^{m-1} \frac{1}{E_i - S_i + i\epsilon}$$
$$= -i\left[-\prod_{j=1}^{A} \frac{1}{E_j - S_j + i\epsilon} + \prod_{j=1}^{A} \frac{1}{E_j - S_j - i\epsilon}\right]$$

- **From**

$$i\left(\frac{1}{x+i\epsilon} - \frac{1}{x-i\epsilon}\right) = 2\pi\delta(x)$$

At the level of the loop integrands of TOPT.

For any cut, there are divergences only when virtual particles are collinear to final state particles, but then the virtual particles appear in another final state and cancel. A cross section that doesn't distinguish between different collinear particle configurations will be finite in perturbation theory even with massless particles: "infrared safety".

- **General condition for IR safety: treat states with the same flow of energy the same way.**

- **Weight functions: $e_n(\{p_i\})$:**

$$\frac{d\sigma}{de} = \sum_n \int_{PS(n)} |M(\{p_i\})|^2 \delta\left(e_n(\{p_1 \ldots p_n\}) - e\right)$$

(We're suppressing the initial state here.)

e is infrared safe if it satisfies

$$\begin{aligned} e_n(\ldots p_i \ldots p_{j-1}, \alpha p_i + \delta p, p_{j+1} \ldots) &= \\ &e_{n-1}(\ldots (1+\alpha) p_i \ldots p_{j-1}, p_{j+1} \ldots) + \mathcal{O}\left(\left[\frac{\delta p}{E_{\rm tot}}\right]^t\right) \end{aligned}$$

for some $t > 0$.

In the spirit of yesterday's and today's talks, let's define a final-state density matrix,

$$\rho \;=\; \sum_{\{p_i\}} |M(\{p_i\})|^2 \; |\{p_i\}\rangle \, \langle\{p_i\}| \; .$$

(D. Neil, W. Waalewijn, 1811.01021). Then define another operator $\hat{E}(e)$, designed to measure some weight for an arbitrary final state,

$$\hat{E}(e) | \, |\{p_i\}\rangle \;=\; \delta\left[e \;-\; e\left(\{p_i\}\right)\right] \, |\{p_i\}\rangle \qquad (1)$$

In these terms,

$$\frac{d\sigma}{de} \;=\; {\rm Tr}\left[\, \rho \; \hat{E}(e) \,\right]$$

- **We can avoid long times dynamics in the initial state by looking at $\mathrm{e}^+\mathrm{e}^-$ annihilation: event shapes and jet cross sections.**

- **Weight functions e_n can pick out jets and/or fix their properties.**

- **Some event shape and jet cross sections are known up to two and even three loops. Generally, however, the full power of unitarity is not built into our calculations of cross sections yet. We calculate infrared singularities and then cancel them. We know it's going to work and it does, but it's a lot of work.**
  (Yao Ma and GS: try to formulate calculations so that they are manifestly finite at every step.)

- **Anyway, whenever the event shape is extensive in phase space, we can define all sorts of information correlations between observed (region Ω) and unobserved ($\overline{\Omega}$) regions of radiation**

$$\hat{E}_\Omega(e)|\,|\{p_i\}\rangle \;=\; \delta\left[e_\Omega \;-\; e\left(\{p_i\in\Omega\}\right)\right]\,|\{p_j\}\rangle \tag{2}$$

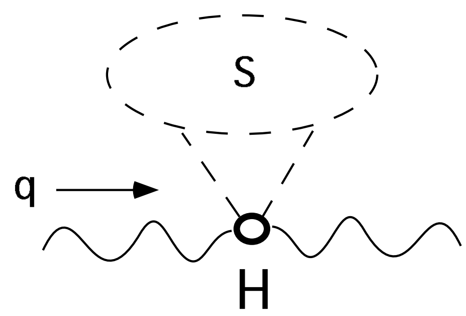
so that in these terms,

$$\frac{d\sigma}{de_\Omega} \;=\; \mathrm{Tr}\left[\,\rho\;\hat{E}_\Omega(e)\,\right]$$

- **There is entropy and mutual entropy to be defined and computed perturbatively and beyond.**

3. Factorizations and Evolutions: *Perturbative tools for Nonperturbative physics*

- **Infrared safety order-by-order is great, but we know its not the whole story.**
- **A great example is in the total e^+e^- cross section above: the only Landau pinches: a cloud of soft gluons attached at a point:**

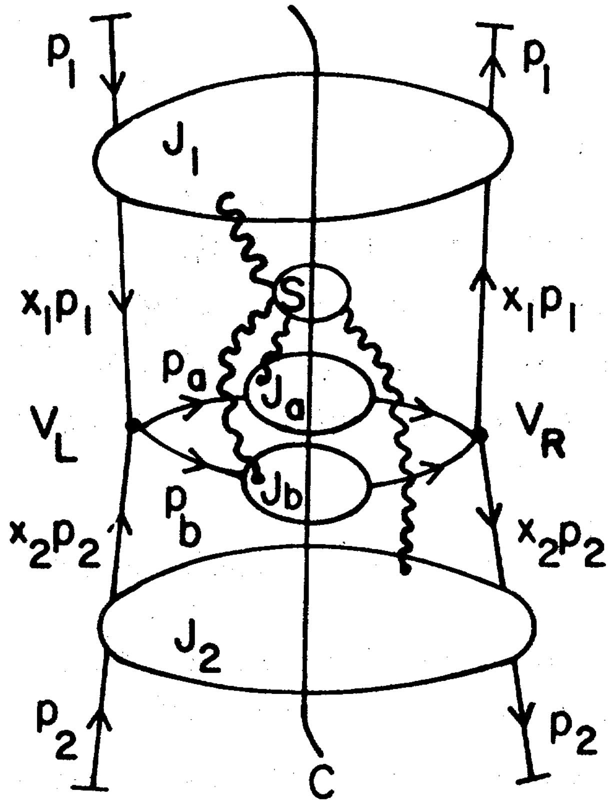


- **(Mueller, 1985): Summed to all orders, this diagram is proportional to ("renormalon"):**

$$\frac{\int d^4k\ \alpha_s(k^2)}{Q^4}\ \sigma_0(Q)\ \leftrightarrow\ \frac{\langle 0|F^2(0)|0\rangle}{Q^4}\ \sigma_0(Q)\ \text{(OPE)}$$

- **The LHS is not defined because the perturbative running coupling diverges at $k^2 = \Lambda^2_{\rm QCD}$.**
- **Perturbation theory signals the necessity for nonperturbative corrections. The perturbative door to vacuum dynamics . . . instantons, for example.**
- **Something analogous occurs in relation to the structure of hadrons . . .**

• **Here's the general Coleman-Norton picture for a large momentum-transfer process in hadron-hadron scattering:**



- As an example, a factorized jet cross sections looks like this:

$$d\sigma(a+b \to \{p_i\}) \;=\; \int dx_a dx_b \; f_{a/A}(x_a p_A)\; f_{b'/B}(x_b p_B)$$

$$\times \;\; C(x_a p_a, x_b p_b, Q)_{ab \to c_1 \ldots c_{N_{\rm jets}} + X}$$

$$\times d \left[\prod_{i=1}^{N_{\rm jets}} \; J_{c_i}(p_i) \right]$$

(Amati, Petronzio, Veneziano; Ellis, Machachek, Efremov, Radyushkin; Politzer, Ross: Libby, GS (1979); Bodwin; Collins Soper, GS (1985,1988), GS & Aybat (2009), Collins (2015))

- Parton distributions, short distance "coefficients" and functions of the jet momenta tell a story.

- In short, the essence of factorization proofs:

  - For an IR-safe sum over final states, the effects of final state interactions cancel, including their interference with initial state interactions (so-called "Glauber" or "Coulomb" exchanges).
  - Remaining initial state interactions reproduce the same, factorized, parton distributions as in deep-inelastic scattering, as imposed by causality.

Factorization follows new stories into the final state: Before the collision, there are lots of stories inside the proton, but the probability for each is the same in every proton!

The essence of predictions for the Standard Model and proposed theories:

$$Q^2\sigma_{\rm phys}(Q,m,f) \;=\; \hat\sigma(Q/\mu,\alpha_s(\mu),f) \otimes f_{\rm LD}(\mu,m) \;+\; \mathcal{O}\left(\frac{1}{Q^p}\right)$$

μ = factorization scale; m= IR scale **(**m *may* **be perturbative)**

- **"First this and then that" multiplication of probabilities – the essence of factorization. It requires a "sufficiently" inclusive cross section, much as in the calculation of jets in e^+e^- annihilation.**

- **Newly-minted jets and possible "new physics" are in $\hat\sigma$; $f_{\rm LD}$ "universal"**

- Again, the factorized cross section:

$$Q^2 \sigma_{\text{phys}}(Q, m, f) = \hat{\sigma}(Q/\mu, \alpha_s(\mu), f) \otimes f_{\text{LD}}(\mu, m) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$

- **What we do:**
  - Compute σ_{phys} and f_{LD} in an IR-regulated variant of QCD, where we can prove the factorization explicitly, then extract $\hat{\sigma}$, assuming it is the same in true QCD as in its IR-regulated version.

  - We compare the formula with unknown physical parton distributions to a suite of data and do a "global fit" for the $f(x, \mu)$ for different quarks and the gluon.

- **What we get (1): absolute predictions for the creation of jets and heavy particles from QCD, and for new degrees of freedom in BSM hypotheses.**

- **What we get (2) : a measurement of how partons share the proton's momentum.**

- The process is a "bootstrap", resulting in feedback between parton distributions, predictions and measurements.

The range of these predictions is greatly extended by Evolution & Resummation: If we have factorization, we can automatically extrapolate from one energy scale to another.

– Whenever there is factorization, there is evolution

$$0 = \mu \frac{d}{d\mu} \ln \sigma_{\text{phys}}(Q, m)$$

$$\mu \frac{d \ln f}{d\mu} = -P(\alpha_s(\mu)) = -\mu \frac{d \ln \hat{\sigma}}{d\mu}$$

– **We can calculate P because we can calculate $\hat{\sigma}$.**

(Dokshitzer, Gribov, LIpatov, Altarelli, Parisi)

– Wherever there is evolution there is resummation,

$$\sigma_{\text{phys}}(Q, m) = \sigma_{\text{phys}}(q, m) \;\otimes\; \exp\left\{\int_q^Q \frac{d\mu'}{\mu'} P\left(\alpha_s(\mu')\right)\right\}$$

– In effective theories SCET (soft-collinear), these evolution equations typically appear through renormalization group. A very efficient and flexible approach. (Bauer, Fleming, Pirjol, Rothstein, Stewart (2002) Becher, Neubert (2006))

– Multiscale problems can be dealt with by extended factorization analysis ("kT" and "threshold" resummations, for example: Dokshitzer, Diakonov, Troian; Parisi, Petronzio, Chiapetta, Greco; Catani, Trentedue, Grazzini; Collins, Soper, GS, ...)

– The same factorization $\rightarrow$ evolution step applies to our jets, and they "evolve"

$$J(\text{scale } \mu_2) \ \sim \ J(\text{scale } \mu_1) \exp\left[\int_{\mu_1}^{\mu_2} \frac{d\mu'}{\mu'} \ \int dx \ P(x, \alpha_s(\mu'))\right]$$

– Each term in the exponent corresponds to the potential emission of a new "sub-jet", which factors from the remaining jet and evolves nearly autonomously into the final state, branching further sub-jets along the way. (This is what event generators do.)

- **Other measurements of the nonperturbative structure of nucleons using factorization:**

- **1) Toward a more detailed picture of how quarks and gluons are distributed in the proton: measure transverse momentum and/or position of partons in nuclei. Factorization into measured spin and transverse-momentum distributions. "Generalized" parton distributions, involving elastic scattering of protons can probe orbital angular momentum.**

- **2) Correlations between partons: multiparton distributions in the proton.**

4. Dialing into the Nonperturbative Regime.

- **A sample of where all this leads . . . Much recent work has concentrated on jet substructure systematizing effects of hadronization.**
- **The thrust, with "averaged" nonperturbative input and best available perturbative calculations: (From R. Abbate *et al.* 1006.3080.)**

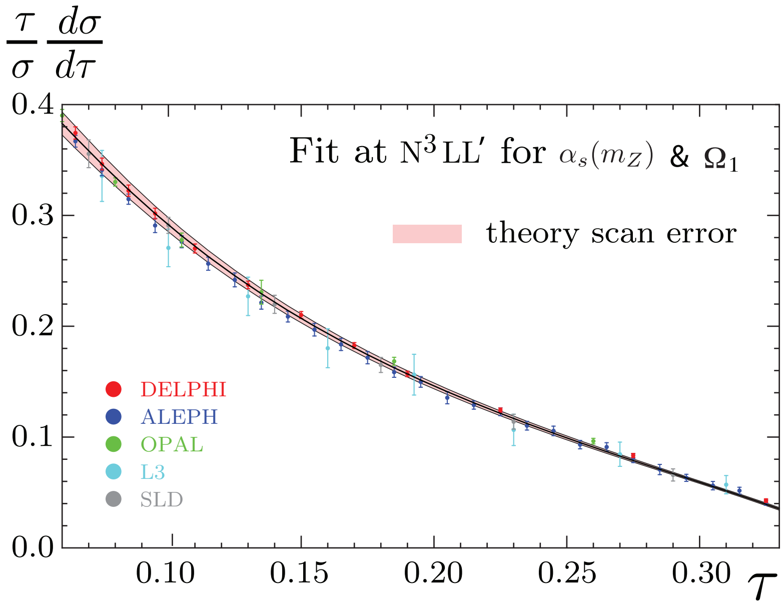


FIG. 13: Thrust distribution at N^{3}LL$'$ order and $Q = m_Z$ including QED and m_b corrections using the best fit values for $\alpha_s(m_Z)$ and Ω_1 in the R-gap scheme given in Eq. (68). The pink band represents the perturbative error determined from the scan method described in Sec. VI. Data from DELPHI, ALEPH, OPAL, L3, and SLD are also shown.

- **Event shapes, generalizing thrust, e.g., "angularities": (G. Bell *et al* 1808.07867.)**

$$\tau_a = \frac{1}{Q}\sum_i |\mathbf{p}_\perp^i|\, e^{-|\eta_i|(1-a)}$$

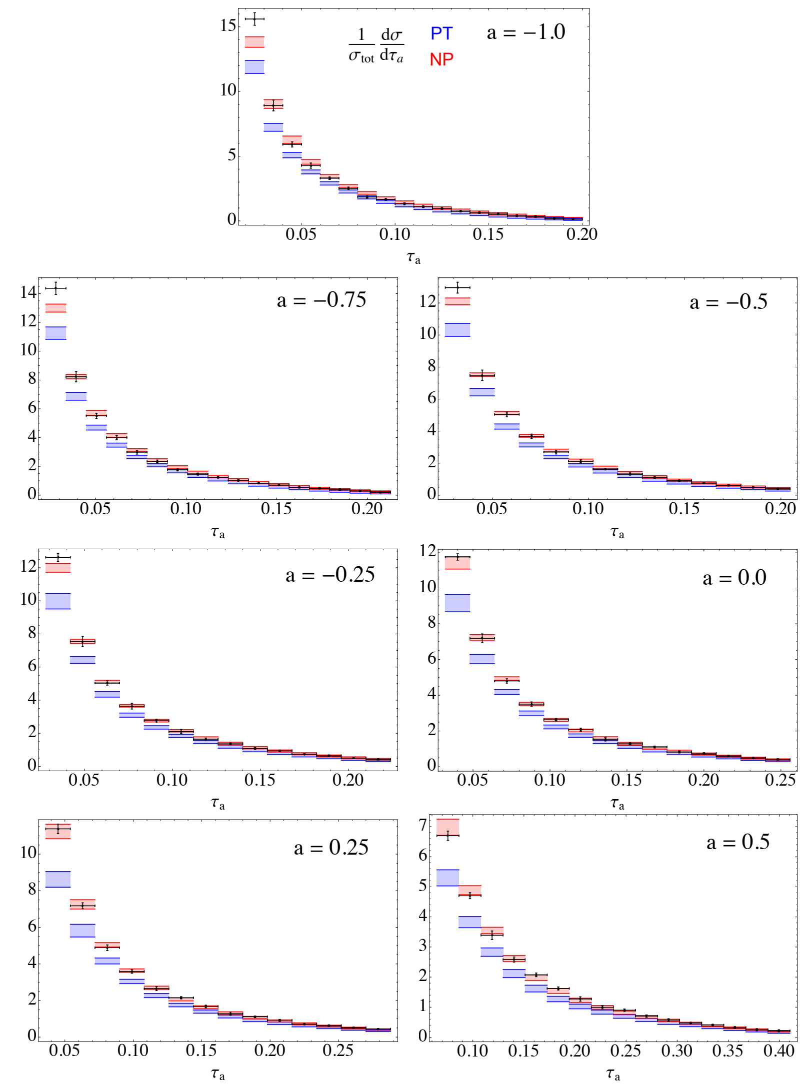


Figure 15. NNLL′ resummed and $\mathcal{O}(\alpha_s^2)$ matched angularity distributions for all values of a considered in this study, $a \in \{-1.0, -0.75, -0.5, -0.25, 0.0, 0.25, 0.5\}$, at $Q = m_Z$, with $\alpha_s(m_Z) = 0.11$. The blue bins represent the purely perturbative prediction and the red bins include a convolution with a gapped and renormalon-subtracted shape function, with a first moment set to $\Omega_1(R_\Delta, R_\Delta) = 0.4$ GeV. Overlaid is the experimental data from [48].

- **Distinguishing the stories of quark and gluon jets.**
  (Larkoski, Moult, Nachman, 1709.04464.)

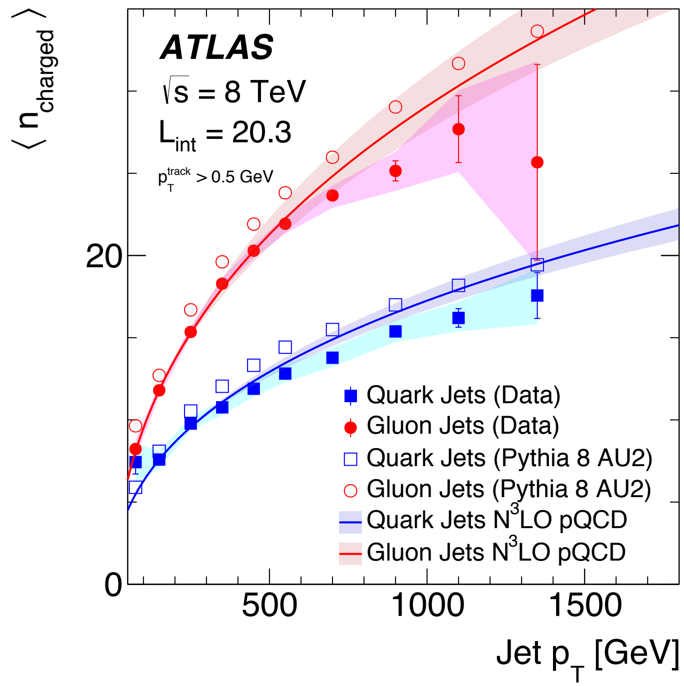


FIG. 12. Plot comparing the NNNLO prediction of Refs. [232, 233] (solid line) of quark (lower) and gluon (upper) jet mean charged particle multiplicities as a function of jet p_T to the ATLAS measurement. Taken from Ref. [247].

- **There is a special interest in recognizing signs of new particles within jets ("boosted decays"). Machine learning . . . (L. Olivera *et al.* 1511.05190.)**

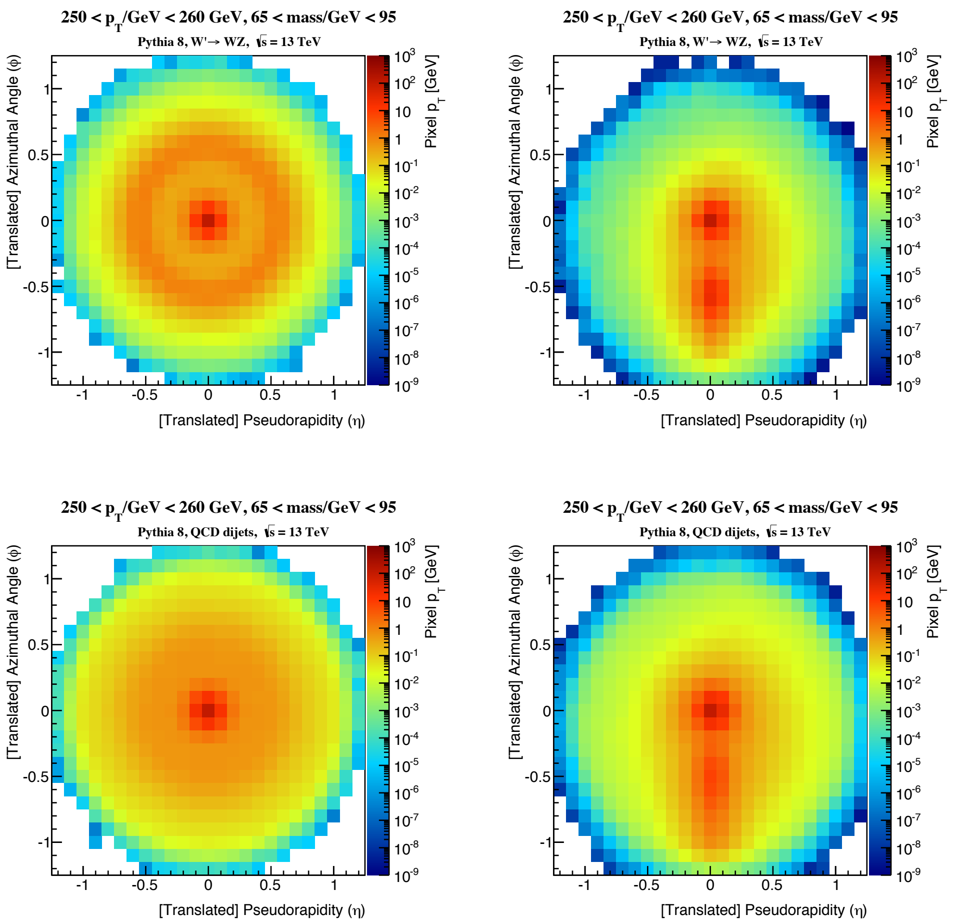


Figure 2: The average jet image for signal W jets (top) and background QCD jets (bottom) before (left) and after (right) applying the rotation, re-pixelation, and inversion steps of the pre-processing. The average is taken over images of jets with 240 GeV $< p_T <$ 260 GeV and 65 GeV $<$ mass $<$ 95 GeV.

Using resummation to push PT to the limit with available data.

- **Get started: First – find a jet.**

- **Then assign an axis $\hat{n}_J$: by minimizing $\sum_i E_i \cos\theta_{(i,\hat{n}_J)}$ for particles i in jet J.**

- **Thats the thrust again:**

$$\tau \ \equiv \ (1 - T) \ \equiv \ \frac{1}{Q_J} \sum_{i\,\mathrm{in}\,N} p_{Ti}\, e^{-|\eta_i|}$$

- **p_{Ti}, η_i measured relative to jet axis (minimizes $1 - T$) (can be chosen jet-by-jet).**

- **For multijet final states, define η_i relative to closest jet.**

- **Classify real and virtual quanta as in the jet, short distance, or part of the soft radiation. These subprocesses factorize (effective theories for each).**

- **Three-way factorization $\Rightarrow$ CO/IR ("Sudakov") resummation.**

  Two logarithmic integrals exponentiate:

$$\sigma(\nu) \;=\; \int_0 d\tau_a \, \mathrm{e}^{-\nu\tau_{Ji}} \, \frac{d\sigma}{d\tau} = \mathrm{e}^{\frac{1}{2}E(\nu,Q)}$$

$$E(\nu,Q) \;=\; 2\int_0^1 \frac{du}{u} \int_{u^2Q^2}^{uQ^2} \frac{dp_T^2}{p_T^2} A\left(\alpha_s(p_T)\right)\left(\mathrm{e}^{-u\nu(p_T/Q)} - 1\right) + \ldots$$

- **Expansion in $\alpha_s(Q)$ finite at all orders. The "cusp" anomalous dimension $A(\alpha_s)$ depends on color representation of the parent parton, only.**

- **For $u \to 0$ find the same sort of "renormalon" singularity that gave the operator product expansion, this time as**

$$\nu \, \frac{\int dp_T \; \alpha_s(p_T)}{Q}$$

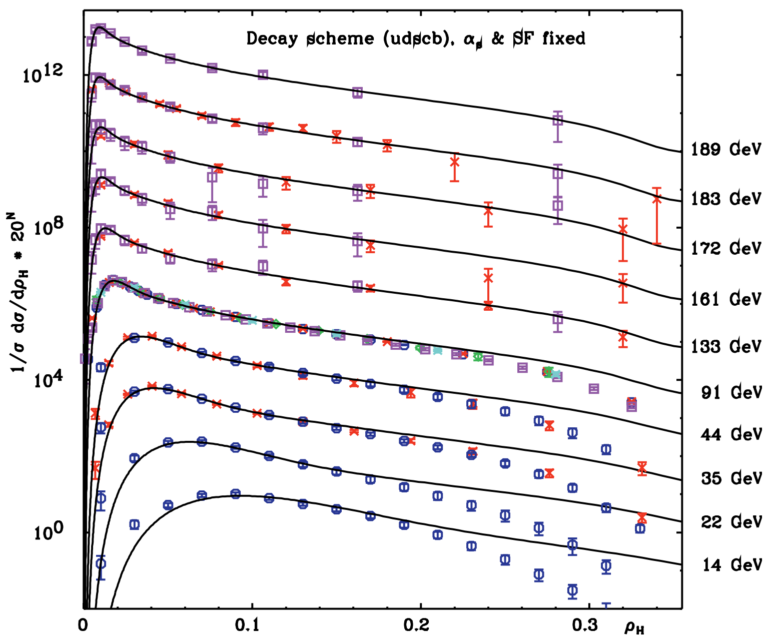
  Dial ν and again the breakdown of perturbation theory points to the structure of non-perturbative corrections - additive in the exponent in transform space $\to$ a convolution in event shapes, here with (just) $1/Q$ suppression.

- Convolution with non-perturbative but universal “shape function:, f_{NP}”, generically

$$\frac{d\sigma}{de} = \int d\xi \; f_{\mathrm{NP}}(e') \, \frac{d\sigma_{\mathrm{PT}}(e-e')}{de'}$$

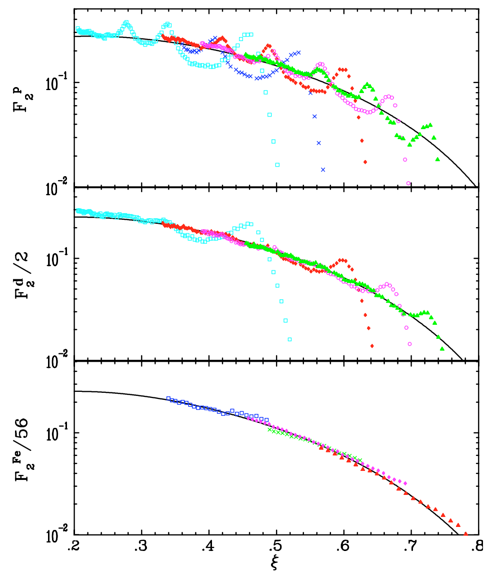
- Dial the moment variable for infrared (in)sensitivity in single or multiple events.

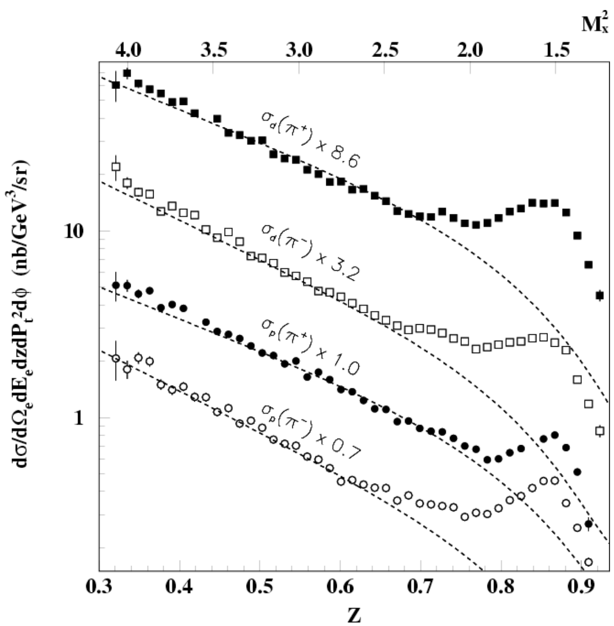
- Shape function phenomenology for thrust at LEP. (Korchemsky, Gardi . . .)



- All these stories (like the power corrections) are additive in $E(\nu, Q)$.

- On the horizon, the role of individual particles in jet structure.
- In principle, an analysis of shapes in ep, pp, eA and pA for thrust or other cleverly-chosen event shapes could provide the transition between the vacuum cusp function A and the quantum history of fast partons in a nuclear medium.
- The additive nature of the shape function, and its kinematic linkage with fragmentation functions for $z \to 1$ suggest a duality-based analysis, given sufficient data.
- Bloom-Gilman duality: Nonperturbative contributions act to 'redistribute probability around a smooth extrapolation of perturbation theory. How does this information flow?



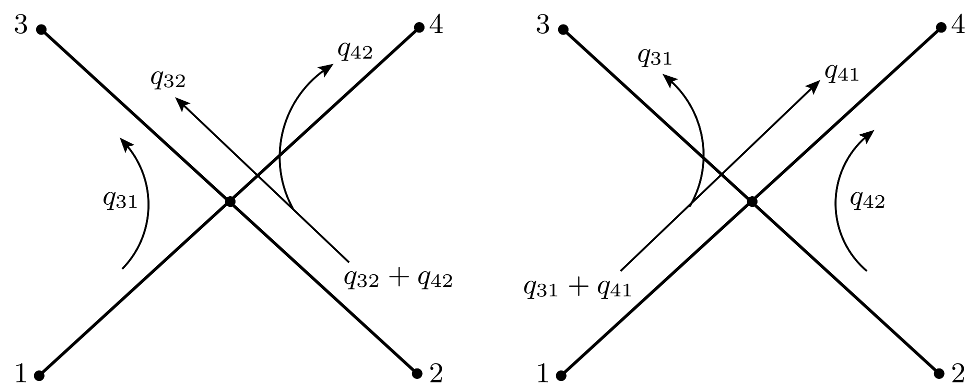


And last – is there a "geometric hint" in pQFT?

- **Could we take the messy sum of Feynman diagrams and resolve them into contributions that link external points? As in**

$$G\left(\{x_d\}_{\rm out},\{x_c\}_{\rm in}\right) \;=\; \langle 0|T(\prod_d \phi(x_d)\prod_c \phi(x_c))|0\rangle \;?$$

- **At tree level it's pretty clear, just imagine possible momentum flows:**



- **In fact, it applies to any and all orders, in terms of specific sets of maps $\pi_I : \{x_c\}_{\rm in} \to \{x_d\}_{\rm out}$,**

$$A\left(\{x_d\}_{\rm out},\{x_c\}_{\rm in}\right) \;=\; \sum_{\text{maps}\,\pi_I} A^{(\pi_I)}\left(\{x_d\}_{\rm out},\{x_c\}_{\rm in}\right)$$

- **Here, each map is associated with an infinite series in PT,**

$$A^{(\pi_I)}\left(\{x_d\}_{\rm out},\{x_c\}_{\rm in}\right) \;=\; \sum_{\text{graphs } G}\;\sum_{\text{vtx orderings } \mathcal{P}} G_{\mathcal{P}}^{(\pi_I)}\left(\{x_d\}_{\rm out},\{x_c\}_{\rm in}\right)$$

with terms given by

$$\begin{aligned} G_{\mathcal{P}}^{(\pi_I)}\left(\{x_d\}_{\rm out},\{x_c\}_{\rm in}\right) \;&=\; (2\pi)^N\,\frac{(-g)^N}{(4\pi^2)^L}\int\left(\prod_{i\in V} d^3y_i\right)\prod_{\text{all } j}\frac{\theta(z_j^+)}{2z_j^+} \\ &\times \prod_{\{P_{(ba)}^{(\pi_I)}\}}\frac{-1}{x_b^- \;-\; x_a^- \;-\; D_{(ba)}^{(\pi_I)} - i\epsilon}\,. \end{aligned}$$

$P_{(ba)}^{(\pi_I)}$ **is a path through internal vertices and for each such path**

$$D_{(ba)}^{(\pi_I)} \;=\; \sum_{i\in P_{(ba)}^{(\pi_I)}}\frac{(z_{i\perp} \;-\; z_{i-1\perp})^2}{2(z_i^+ - z_{i-1}^+)}\,. \tag{3}$$

- **Paths in coordinate space play the role of virtual states in momentum space, and energy deficits are replaced by deficits in light-cone "distance". (O. Erdoğan, GS, 2017)**

- **It's still very complicated, but the concept of paths can perhaps be lifted from a specific choice of degrees of freedom, and might even transcend the perturbative/nonperturbative dicotomy. Well, with that thought, maybe I should wrap it up . . .**

Conclusions

- The methods of perturbative QCD are powerful both in their flexibility and in their self-consistent limitations. We are capable of improving its independent predictions, and also in honing it as a tool to connect experiment to knowledge of the nonperturbative structure of hadrons
- The way forward will be in a range of energy with luminosity to match, as at an EIC. New observables that are sensitive to nonperturbative effects in a controllable fashion, combined with more sophisticated control over perturbation theory will lead to new insights,
- And perhaps to breakthroughs in understanding.