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Topologically-protected Edge States, Topological order, and Entanglement in Quantum Condensed Matter.

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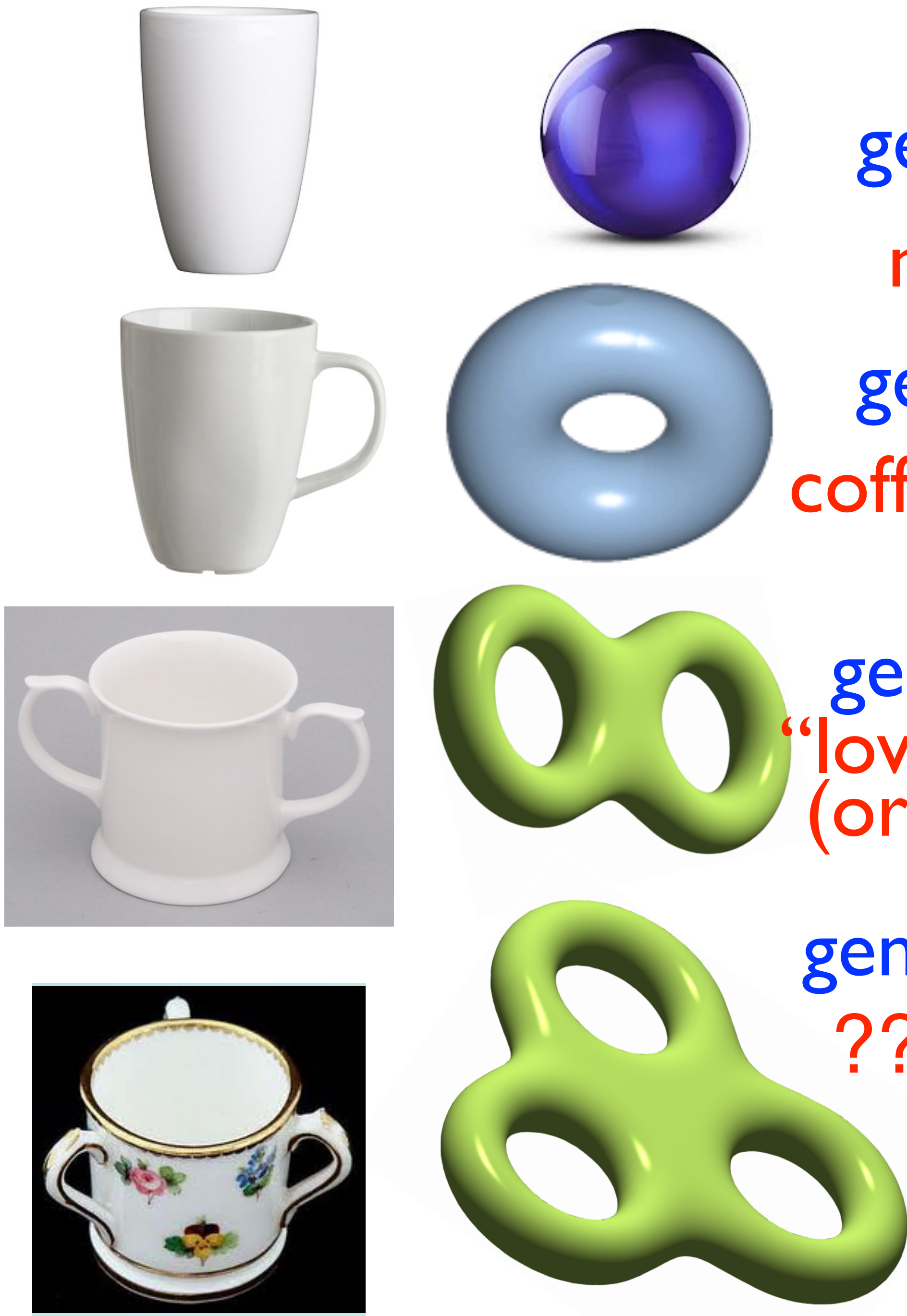
- Review of Berry Phase, Chern number
- 1D edges, 2D edges, Z2 invariant
- Fractional QHE and conformal field theory
- FQHE Entanglement, Geometry.

Topological equivalence



genus 0

mug

genus 1

coffee cup

genus 2

“loving cup”

(or soup?)

genus 3

???

- inequivalent objects cannot be continuously transformed into each other
- these 2D surfaces are classified by an integer, their *genus* (number of handles)

topology (genus) is a **global** property

- **geometric** properties (such as curvature) are ***local*** properties
- but integrals over local geometric properties may characterize global topology!

Gauss-Bonnet (for a closed surface)

$$\int d^2\boldsymbol{r}(\text{Gaussian curvature}) = 4\pi(1 - \text{genus})$$

$$\frac{1}{R_1 R_2} \qquad\qquad = 2\pi(\text{Euler characteristic})$$

$$4\pi r^2 \times \frac{1}{r^2} = \qquad\qquad 4\pi(1-0)$$

- trivially true for a sphere, but non-trivially true for any compact 2D manifold

- A more abstract generalization of the Gauss-Bonnet formula due to Chern found its way into quantum condensed-matter physics in the 1980's

- Quantum states are **ambiguous** up to a phase:
- Physical properties are defined by expectation values $\langle\Psi|\hat{O}|\Psi\rangle$ that are left unchanged by

$$|\Psi\rangle \mapsto e^{i\varphi}|\Psi\rangle$$

- As noticed by Berry, this has profound consequences for a family of quantum states parametrized by a continuous d-dimensional coordinate x *in a parameter space.*

- $|\Psi(\boldsymbol{x})\rangle$ can be expanded in a fixed orthonormal basis

$$|\Psi(\boldsymbol{x})\rangle = \sum_i u_i(\boldsymbol{x})|i\rangle \qquad \langle i|j\rangle = \delta_{ij}$$

$$|\partial_\mu \Psi(\boldsymbol{x})\rangle \equiv \sum_i \frac{\partial u_i(\boldsymbol{x})}{\partial x^\mu}|i\rangle$$

- we need a "gauge-covariant" derivative

$$\boxed{|D_\mu \Psi(\boldsymbol{x})\rangle = |\partial_\mu \Psi(\boldsymbol{x})\rangle - |\Psi(\boldsymbol{x})\rangle\langle\Psi(\boldsymbol{x})|\partial_\mu \Psi(\boldsymbol{x})\rangle}$$

$$\langle\Psi(\boldsymbol{x})|D_\mu \Psi(\boldsymbol{x})\rangle = 0$$

parallel transport

projects out parts of $|\partial_\mu \Psi(\boldsymbol{x})\rangle$ not orthogonal to $|\Psi(\boldsymbol{x})\rangle$

- The gauge-covariant derivative can also be written

$$|D_\mu \Psi(\boldsymbol{x})\rangle = |\partial_\mu \Psi(x)\rangle - i\mathcal{A}_\mu(\boldsymbol{x})|\Psi(\boldsymbol{x})\rangle$$

Lots of analogies with electromagnetic gauge fields in Euclidean space!

an analog of the electromagnetic vector potential in the parameter space x

- Berry's phase factor for a closed path Γ in parameter space is the analog of a Bohm-Aharonov phase

$$e^{i\phi_\Gamma} = \exp i \oint_\Gamma dx^\mu \mathcal{A}_\mu(\boldsymbol{x})$$

- The key gauge-invariant quantity is

$$\langle D_\mu \Psi(\boldsymbol{x}) | D_\nu \Psi(\boldsymbol{x})\rangle = \frac{1}{2}\left(\mathcal{G}_{\mu\nu}(x) + i\mathcal{F}_{\mu\nu}(\boldsymbol{x})\right)$$

Real symmetric positive
Fubini-study metric
(defines "quantum geometry")

Real antisymmetric
Berry curvature
$\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$

Chern's generalization of Gauss-Bonnet

$$\int_{\mathcal{M}_2} dx^\mu \wedge dx^\nu \mathcal{F}_{\mu\nu}(\boldsymbol{x}) = 2\pi \mathbb{C}_1$$

"Chern number"

first Chern class (an integer) replaces Euler's characteristic

integral over a closed orientable 2-manifold

- In quantum mechanics, “geometry” relates to energy, “local deformations” become adiabatic changes of the Hamiltonian, and “smoothness” (short-distance regularization) of the manifold derives from an energy gap

- the topology of quantum states is conserved so long as energy gaps do not close.

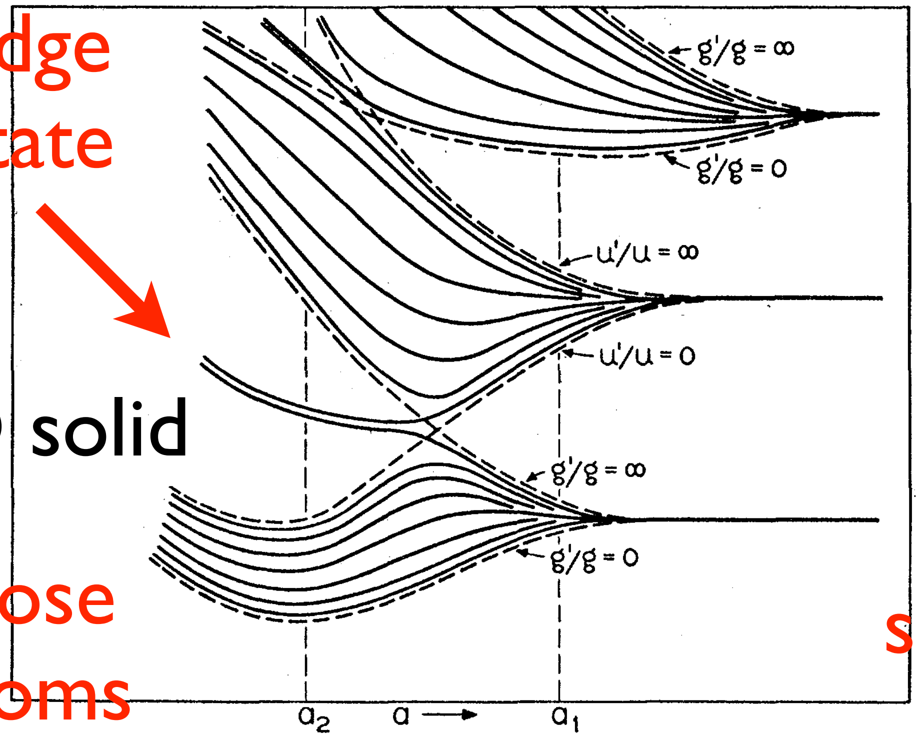
Now we know to look for topology, one can see that it the past its effects were noticed on an ad hoc basis as “oddities” !

- Tamm (1932), Shockley (1939) 1D edge states

edge state

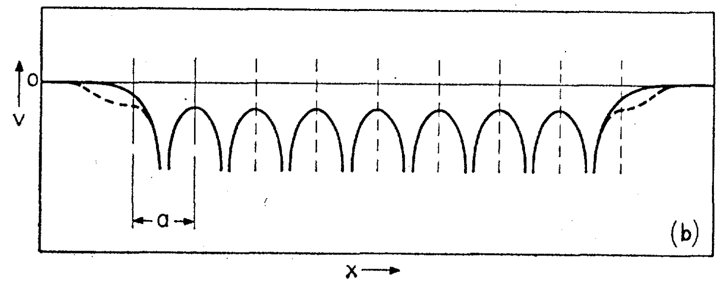
1D solid

close atoms



widely separated atoms

FIG. 2. Energy spectrum for a one-dimensional lattice with eight atoms.

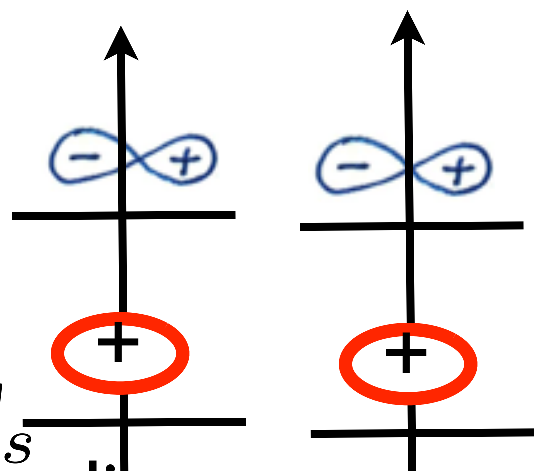


Fermi level pinned to edge state if neutral charge +1/2 electron if full , -1/2 if empty, per edge

fractionalization is typical in topological states

odd parity E_p

even parity about site E_s

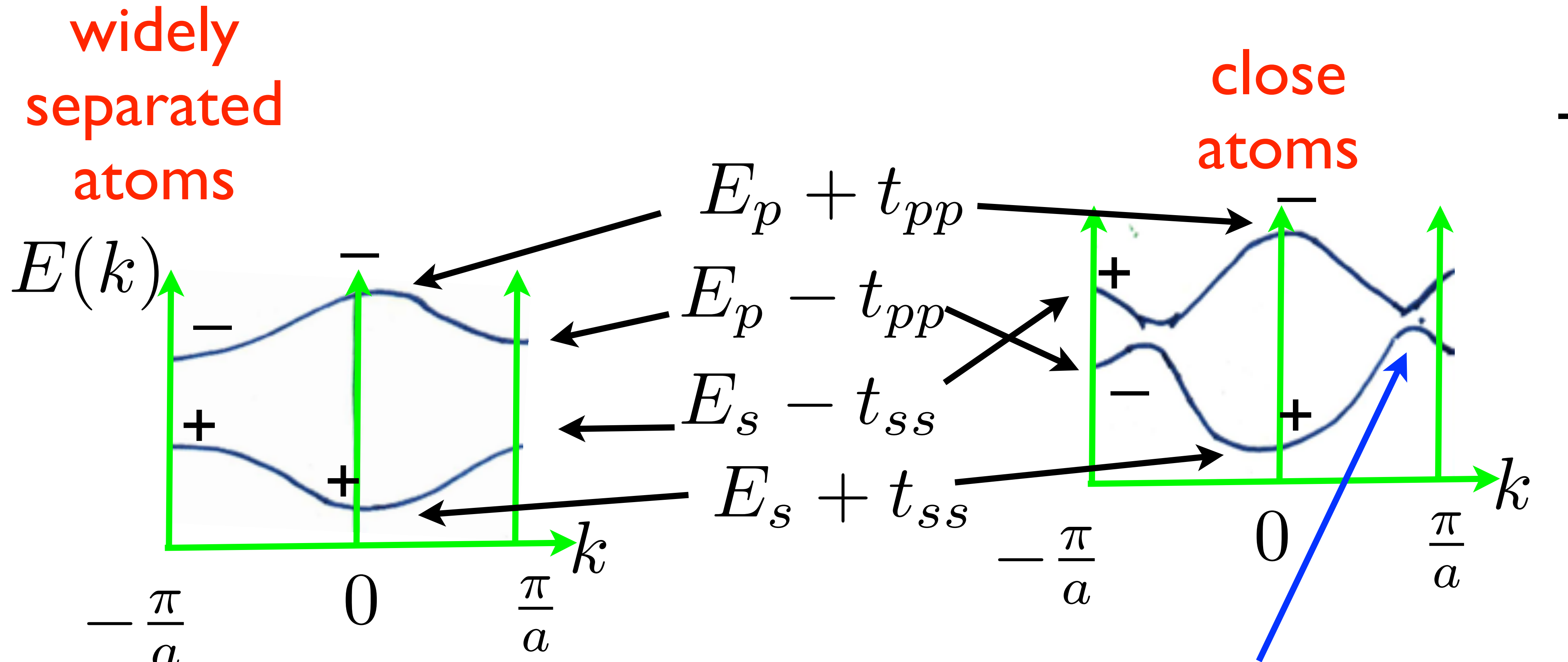


adjacent atoms

$t_{pp} > 0$

"hopping"

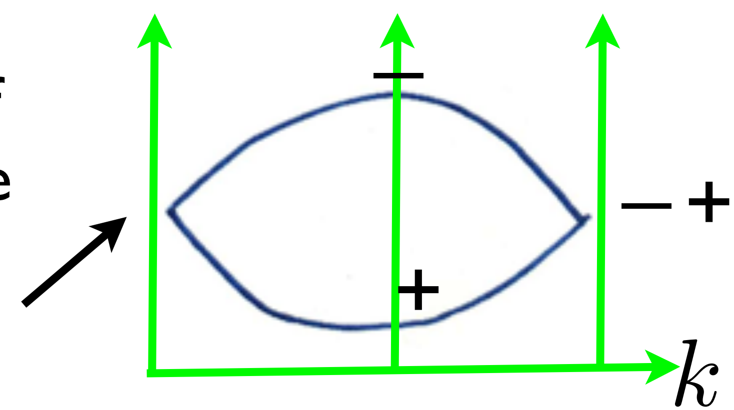
$t_{ss} < 0$



effect of t_{sp} (vanishes at k=0,π)

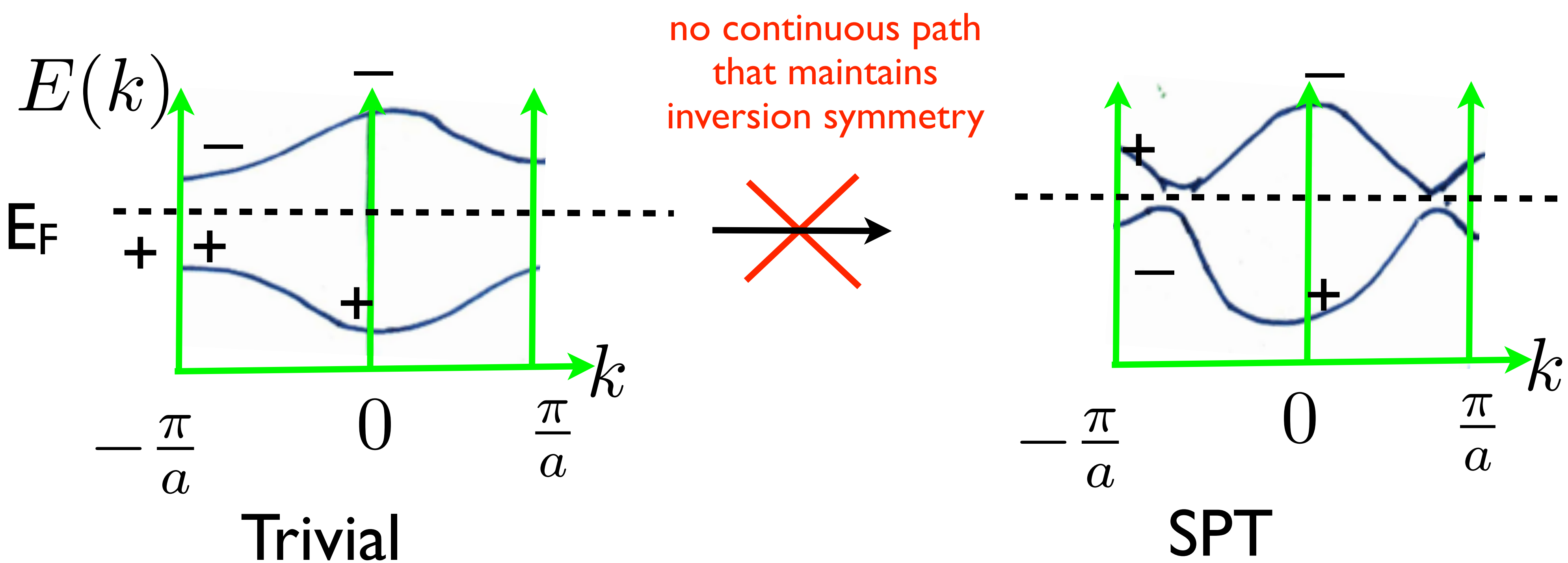
- If the bulk solid has inversion centers, bands have parity quantum numbers at ka = 0 and π, cannot mix there.

- As the atoms get closer, the magnitude of the hopping amplitude increase. At some point, the gap at ka = π will close, as the opposite-parity levels pass through each other without mixing!



- The Tamm-Shockley edge state is a boundary between the trivial vacuum and a <u>**Symmetry-Protected Topological (**</u>SPT) state

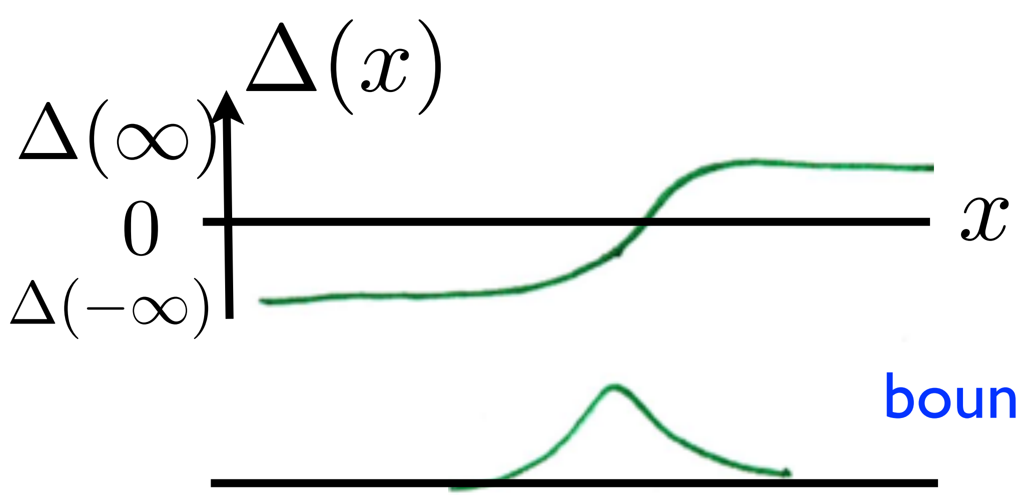
- The symmetry is **inversion symmetry** in the bulk solid: without this there would be no qualitative difference between the two limits, and no closing of the gap



- Inversion symmetry is semi-fragile (electric fields break it). The Tamm-Shockley edge state remained an obscure oddity for years. Only now can we see it as a simple example of a general principle.

- The edge-state is modeled quite generally by a Dirac-like equation where the “mass: changes sign (Jackiw-Rebbi)

$$H = \begin{pmatrix} \Delta(x) & vp_x \\ vp(x) & -\Delta(x) \end{pmatrix} = \Delta(x)\boldsymbol{\sigma}^3 - i\hbar v \boldsymbol{\sigma}^1 \frac{d}{dx}$$



bound state amplitude

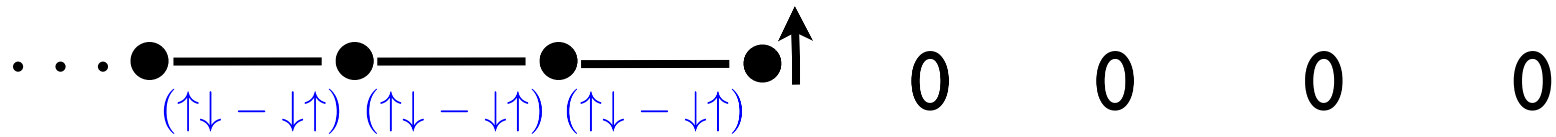
- 2D and 3D Time-reversal-Invariant topological insulators are SPT protected by fermionic time-reversal symmetry (Kramers degeneracy)
- The Z2 invariant was obscure until Kane and Fu considered <u>inversion symmetry</u> as an extra feature: The Z2 classification then is simple: just examine inversion symmetry of occupied bands at the 2^d inversion-symmetric points in k-space, just like for Tamm-Shockley effect!

Product = +1 or -1

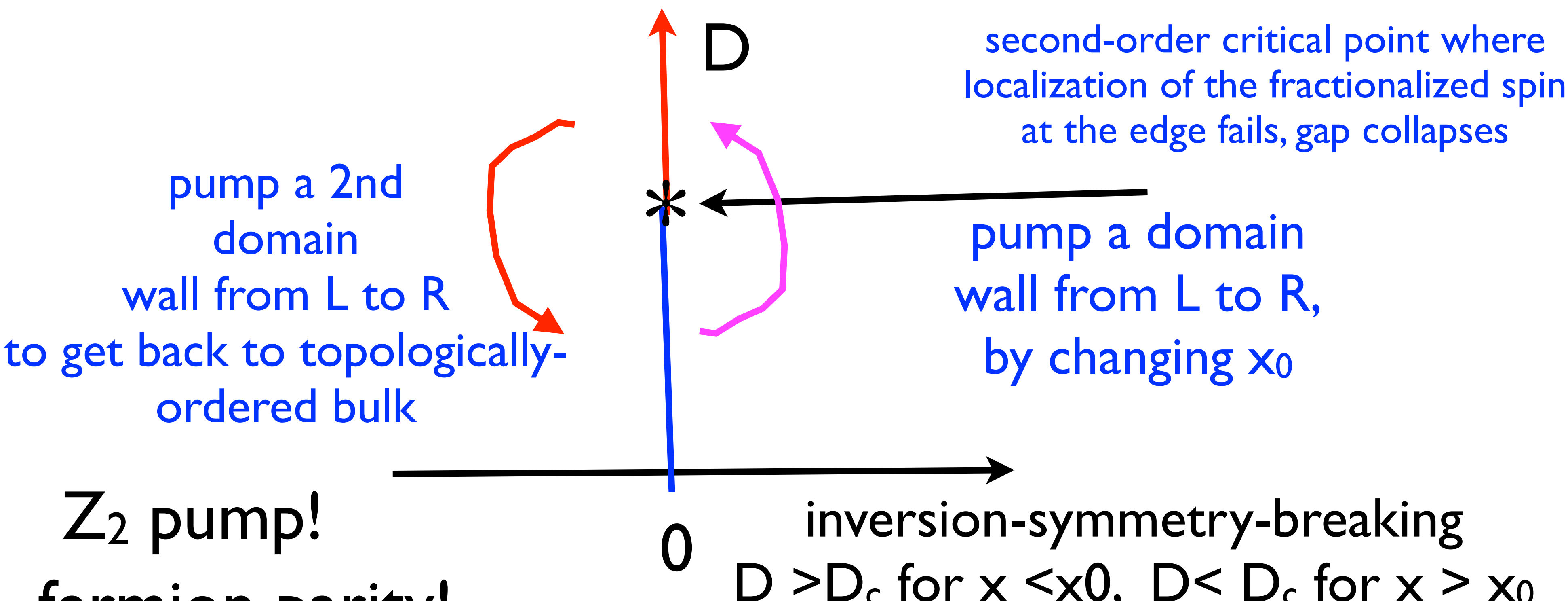
- Another instructive example of an SPT state is the spin-1 chain "Haldane gap" state,
- This exhibits fractionalization, topological order and entanglement, characterized by the entanglement spectrum (Li and FDMH 2008) which has become an impotant tool for investigating Topological Order.
- Wen clarified that is is an SPT with respect to Inversion and Time-reversal, and a key prototype for SPT's.

A spin-1 degree of freedom can be represented as **two** spin-$1/2$ degrees of freedom, projected into a symmetric state.

half-integer spin = fermion number!

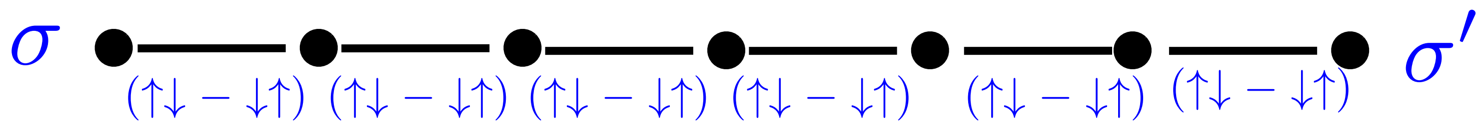


- The stability of topologically ordered states generally arises because no local modification can cause a change between topologically-distinct states



Z_2 pump!

fermion parity!



valence bond picture (AKLT) spin

2x2 Matrix product state)

$$\sum_{\sigma\sigma'} \psi_\sigma^{L*} (\boldsymbol{M}^{(1)} \ldots \boldsymbol{M}^{(N)})_{\sigma\sigma'} \psi_{\sigma'}^R$$

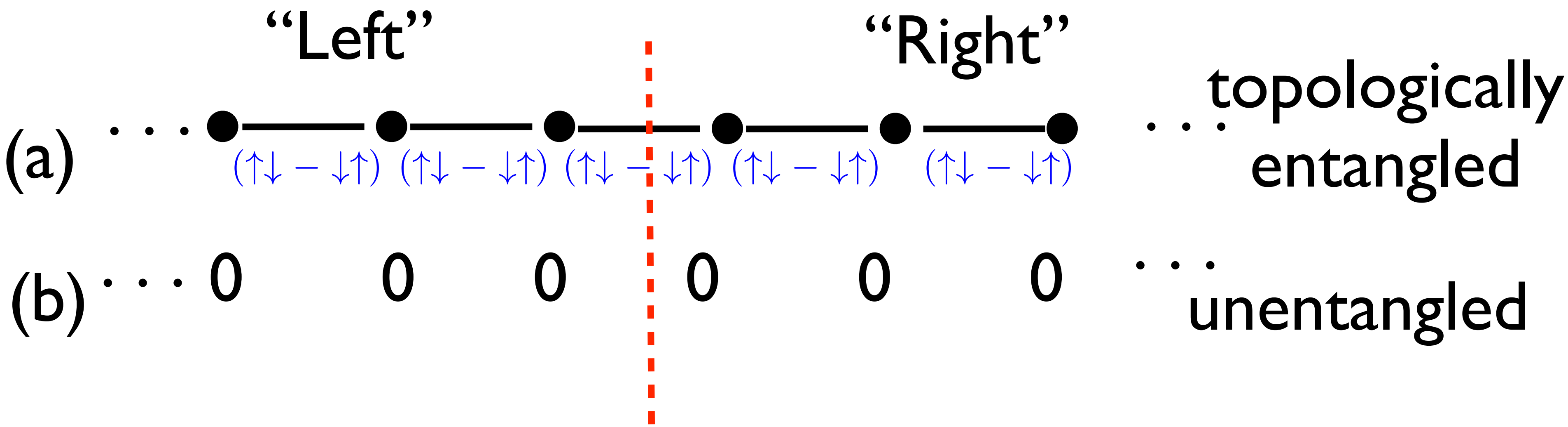
gapped (incompressible) state,unbroken symmetry

free spin-(1/2) states at free ends!

$$H = \sum_i J \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + D(S_i^z)^2$$

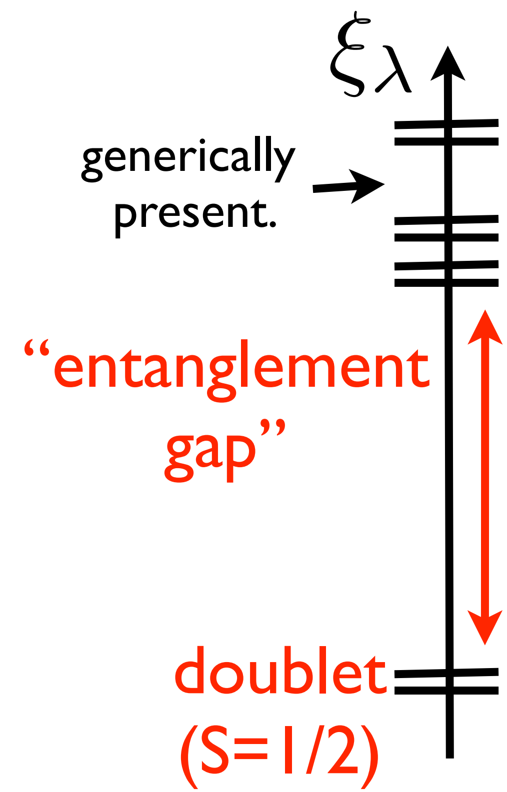
- Large D favours a state with $S^z_i = 0$, all i.

- topological order = long-range entanglement



$$|\Psi\rangle = \sum_{\lambda} e^{-\xi_\lambda/2} |\Psi^L_\lambda\rangle \otimes |\Psi^R_\lambda\rangle$$

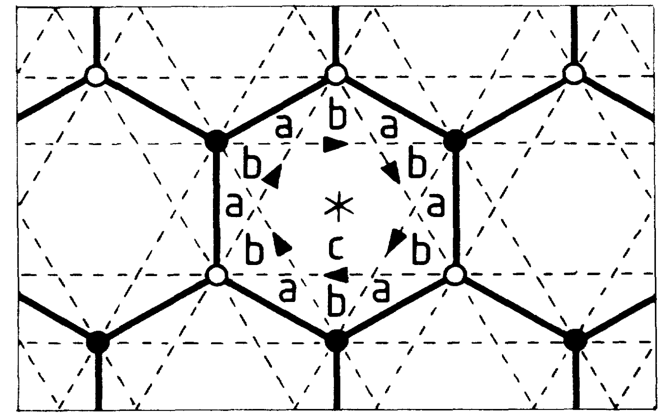
Bipartite Schmidt-decomposition of ground state reveals entanglement



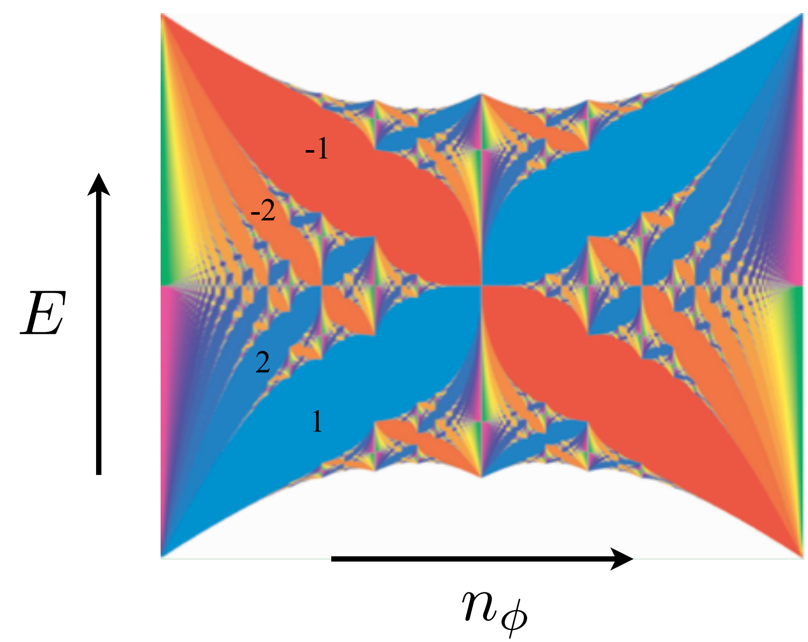
- a gapless "topological entanglement spectrum" separated from other Schmidt eigenvalues by an "entanglement gap" is characteristic of long-range topological order (Li + FDMH, PRL 2008)

The 2D Chern insulator

- This was a model for a "quantum Hall effect without Landau levels" (FDMH 1988), now variously known as the "quantum anomalous Hall effect" or "Chern insulator".

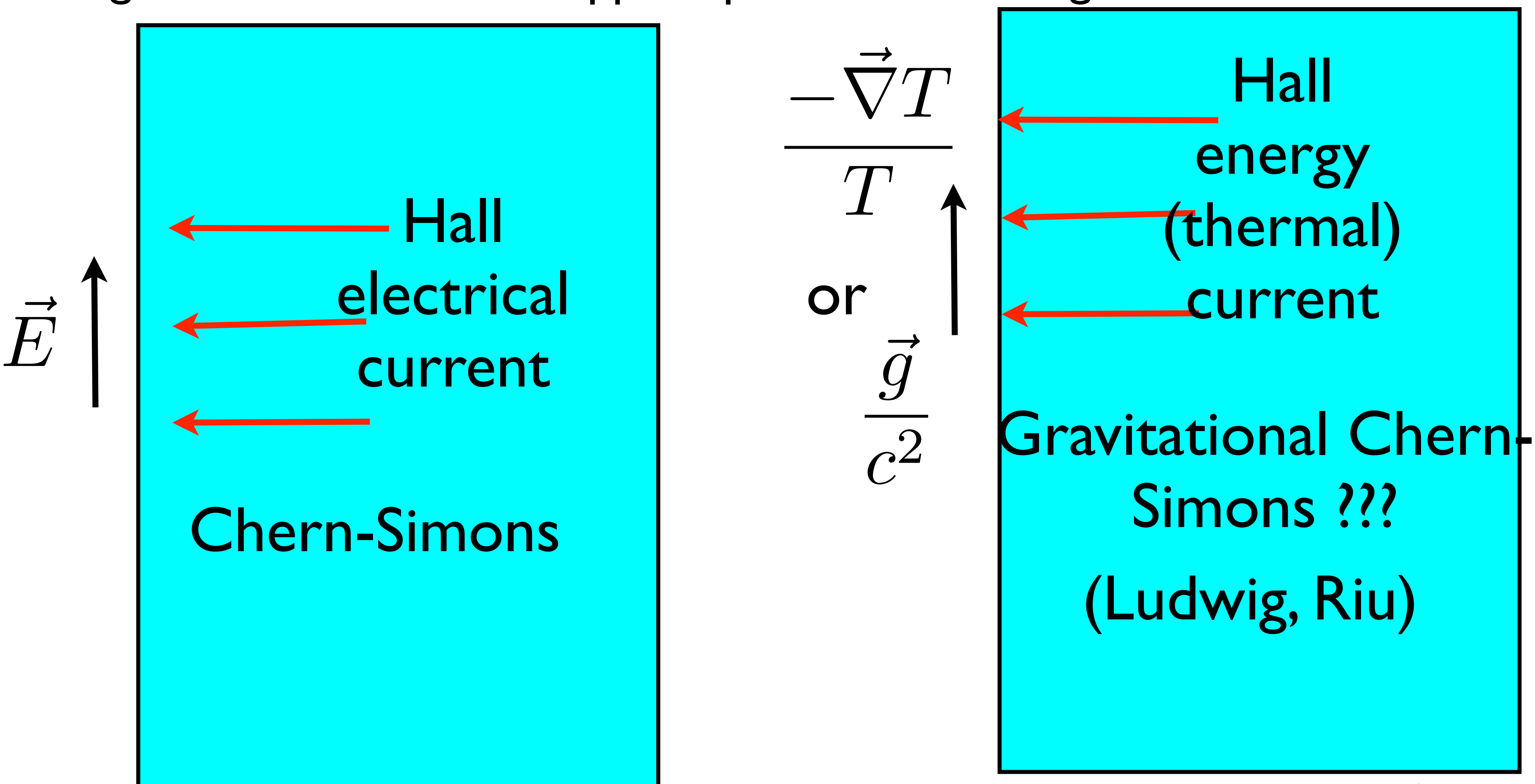


- Previously, Thouless, Kohmoto, Nightingale and den Nijs (TKNN) had analysed the QHE in the Hofstadter model, and found the invariant subsequently identified by Simon as the Chern number.



colored by Avron et al.

- quantum Hall state must have chiral edge states to absorb discontinuities in Hall currents if electric or gravitational fields are applied parallel to the edge



Luttinger showed that thermal transport coefficients could be formally obtained as a linear response to gravity, using

$$\frac{\vec{g}}{c^2} = -\frac{\vec{\nabla}T}{T}$$

Hall effects as anomalies

Electric current

$$J^a = \nu \frac{e^2}{2\pi\hbar}\epsilon^{ab}E_b$$

Electrical force on unit charge

Virasoro anomaly

$$J^a_E = \frac{\tilde{c}}{12}\frac{(2\pi k_B T)^2}{2\pi\hbar}\epsilon^{ab}\frac{g_b}{c^2}$$

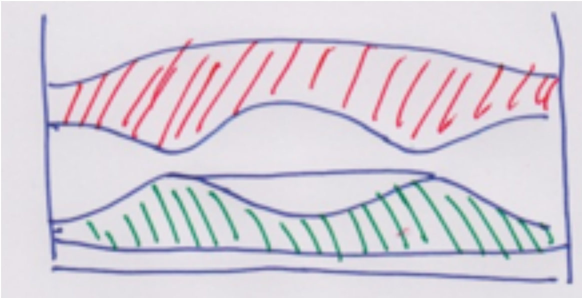
Energy (heat)current

gravitational force on unit energy!!!

$$\frac{\vec{g}}{c^2} = -\frac{\vec{\nabla}T}{T}$$

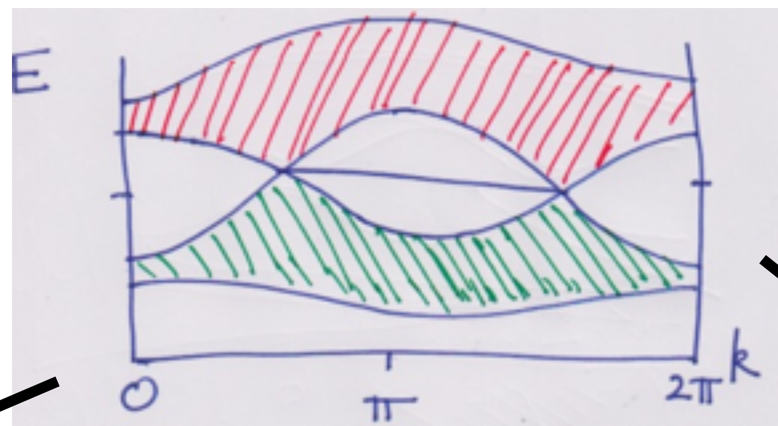
For 2D band electrons,
$\nu = \tilde{c}$
= sum of Chern numbers of occupied bands

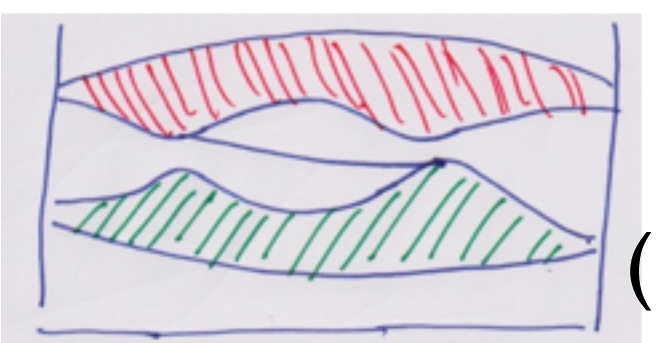
TKNN



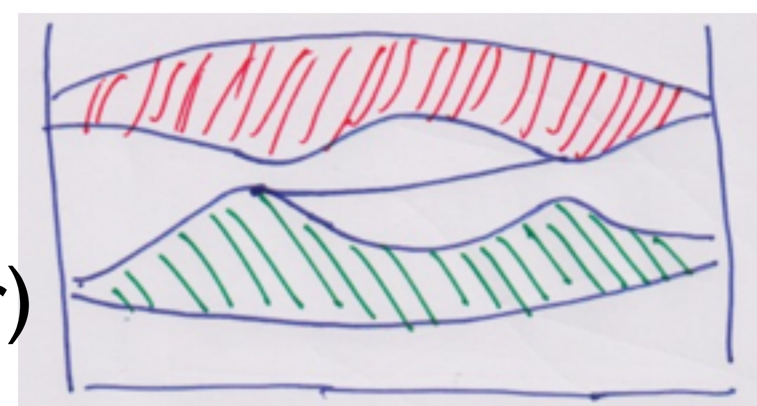
Broken
inversion

- gapless graphene "zig-zag" edge modes





Broken
time-reversal
(Chern insulator)

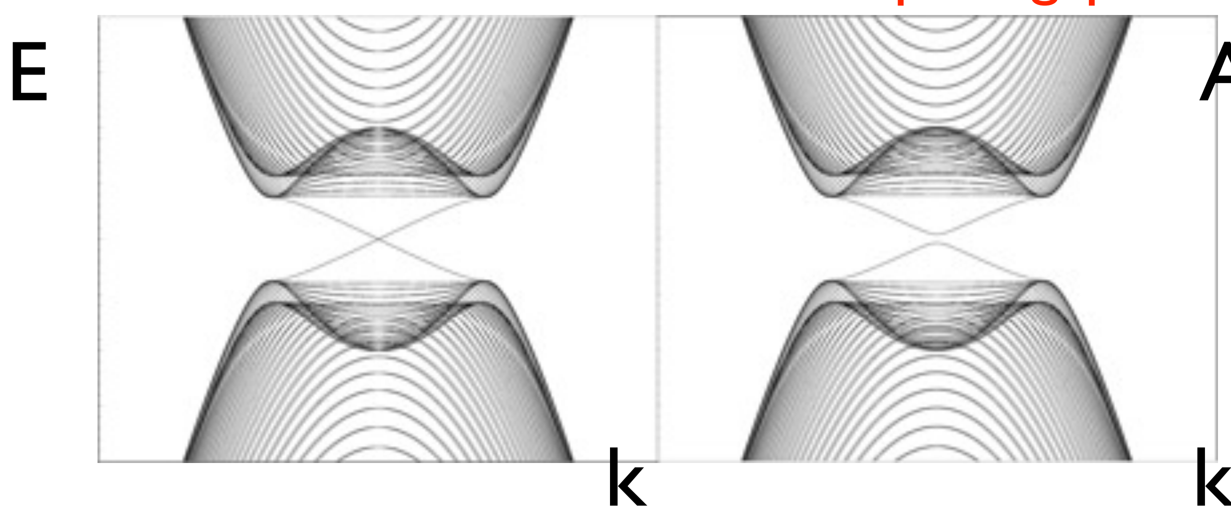


Kane and Mele 2005

- Two conjugate copies of the 1988 spinless graphene model, one for spin-up, other for spin-down

Zeeman coupling opens gap

B=0



At edge, spin-up moves one way, spin-down the other way

If the 2D plane is a plane of mirror symmetry, spin-orbit coupling preserves the two kind of spin. Occupied spin-up band has chern number +1, occupied spin-down band has chern-number -1.

- This looks "trivial", but Kane and Mele found that the gapless "helical" edge states were still there when Rashba spin-orbit coupling that mixed spin-up and spin-down was added.

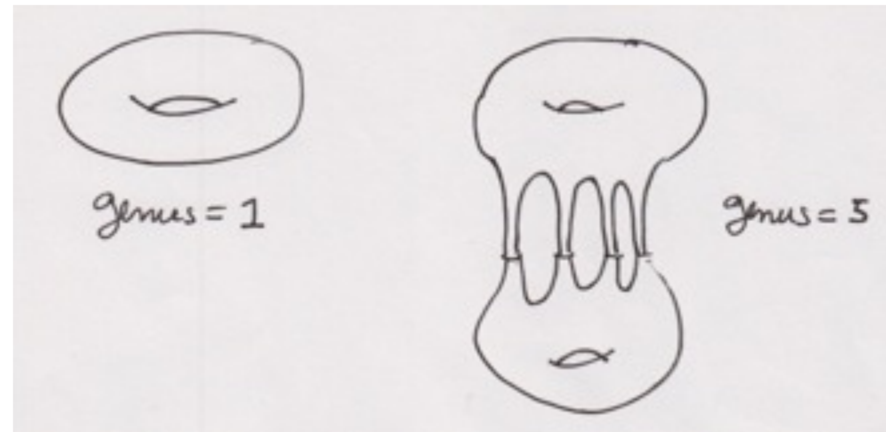
- They found a new "Z2" topological invariant of 2D bands with time-reversal symmetry that takes two values, +1 or -1. The invariant derives from **<u>Kramers degeneracy</u>** of fermions with time-reversal symmetry.

- This launched the new "topological insulator" revolution when an experimental realization was demonstrated.

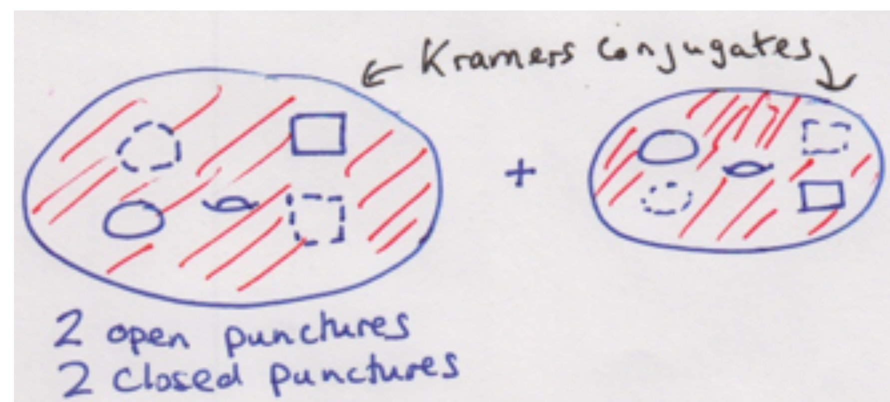
An explicitly gauge-invariant rederivation of the Z2 invariant

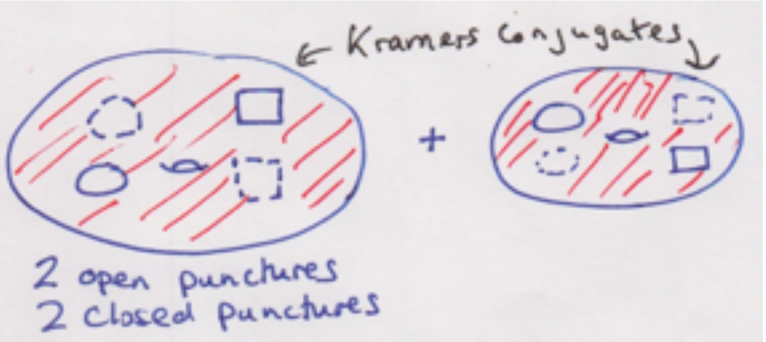
FDMH unpub.

- If inversion symmetry is absent, 2D bands with SOC split except at the four points where the Bloch vector is 1/2 x a reciprocal vector. The generic single genus-1 band becomes a pair of bands joined to form a genus-5 manifold



- This manifold can be cut into two Kramers conjugate parts, each is a torus with two pairs of matched punctures. In each pair, one puncture boundary is open one is closed.





- on a punctured 2-manifold

$$\exp i \int d^2\boldsymbol{k}\, \mathcal{F}^{12}(\boldsymbol{k}) = \prod_i e^{i\phi_i}$$

product of Berry phase-factors of puncture boundaries

- without punctures,

$$\int d^2\boldsymbol{k}\, \mathcal{F}^{12}(\boldsymbol{k}) = 2\pi C$$

- punctures come in Kramers pairs: $\prod_{i=1}^{2n} e^{i\phi_i} = \left(\prod_{i=1}^{n} e^{i\phi_i}\right)^2$

a perfect square, so we can take a square root!

$$\boxed{\left(\exp i\frac{1}{2}\int d^2\boldsymbol{k}\, \mathcal{F}^{12}(\boldsymbol{k})\right) \prod_{i=1}^{n} e^{-i\phi_i} = \pm 1}$$

- If inversion symmetry is present, the bands are unsplit and doubly-degenerate at all points in k-space, so the Berry curvature is undefined.

- Fu and Kane found a beautiful formula

$$\prod_{n} \prod_{k^*} I_{n,k^*} = \pm 1$$ = the Z_2 invariant

occupied bands

T+I-invariant k-points

Inversion quantum number ± 1 (about any inversion center)

Fractional QHE

- There is a mysterious connection to conformal field theory, even though there is no conformal invariance or scale invariance
- In critical phenomena, the conformal metric (that defines the conserved angles) is defined at large distance scales. In the FQHE, it seems to be defined at short-distance scales
- The Virasoro algebra seems to be the common feature of cft and fqhe

some related “mysteries”

- Why are model wavefunctions related to (Euclidean) 2+0 d cft good models for the FQHE?
- If the Laughlin state is a “lowest Landau level Schroedinger wavefunction” why does it occur in the second Landau level?
- Why is it “holomorphic”?
- What aspects of 1+1d cft apply to edge states?

- The conformal group is the group of coordinate transformations that preserve the unimodular part of a metric

$$ds^2 = e^{-2\varphi(x,t)} \left(v^{-1} dx^2 - v\, dt^2 \right) \qquad \text{(1+1)d}$$

"universal speed of massless particles"

$$ds^2 = e^{-2\varphi(\boldsymbol{x})} \left(g_{ab} dx^a dx^b \right) \qquad \text{(2+0)d}$$

"Euclidean metric, det g = 1"

- model FQH "wavefunctions" (Laughlin, Moore-Read, Read-Rezayi,...) are related to Euclidean 2D conformal theories characterized by a **unimodular 2D Euclidean metric** $g_{ab}, \det g = 1,$ that determines the shape of their guiding-center correlation functions

- The metric defines dimensionless complex coordinates $z, \ z^*$

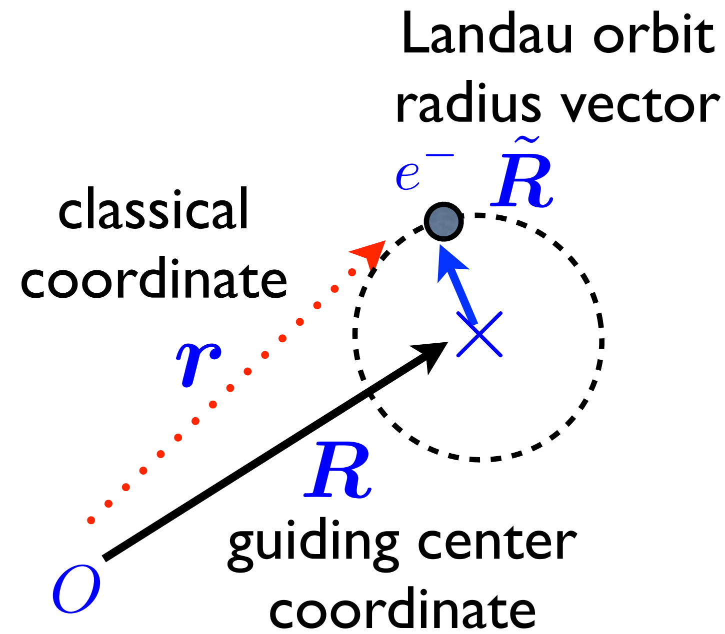
$$\frac{1}{2\ell_B^2} g_{ab} r^a r^b = z^* z \qquad \ell_B = \left(\frac{\hbar}{|eB|}\right)^{\frac{1}{2}}$$

- The metric is a continuously-variable "hidden" variational parameter determined by minimizing the correlation energy of the FQH state

- after Landau quantization, residual guiding center degrees of freedom are non-commutative

$$\boldsymbol{r} = \boldsymbol{R} + \cancel{\tilde{\boldsymbol{R}}} \longleftarrow$$

eliminated by Landau quantization



antisymmetric symbol

$$[R^a, R^b] = -i\ell_B^2 \epsilon^{ab}$$

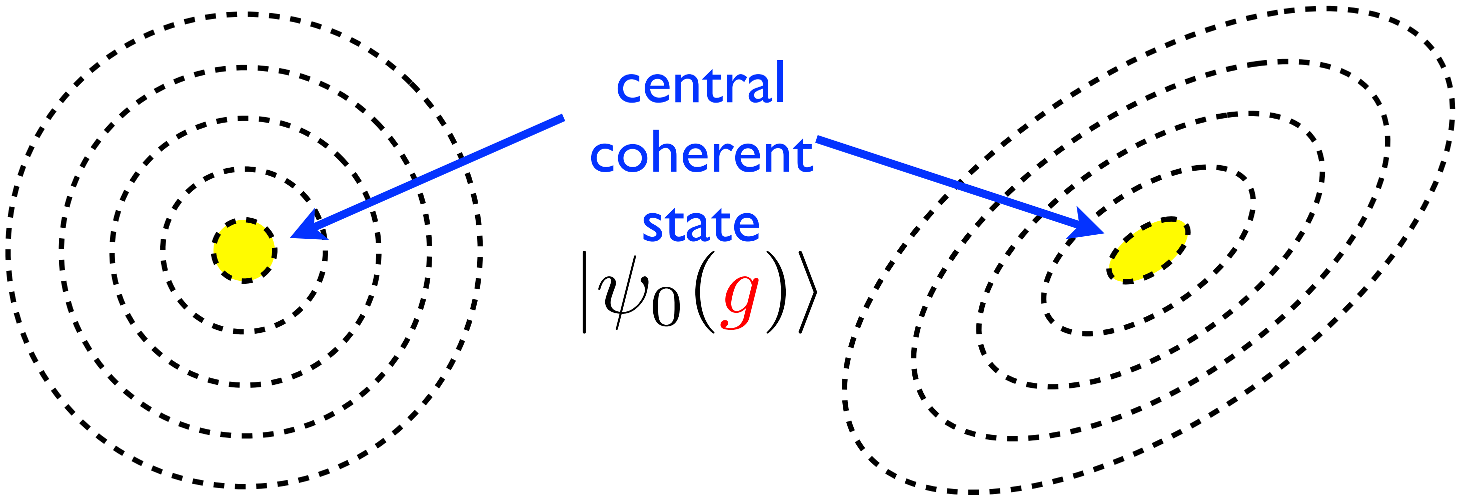
- isomorphic to phase space, they obey an uncertainty principle

$\boxed{\text{guiding centers cannot be localized within an area less than } 2\pi\ell_B^2}$

- The metric defines the shape of the coherent state at the center of the "symmetric gauge" basis of guiding-center states

different choices of metric:

("squeezed" relative to each other)



- Guiding-center "spin" (rotation operator) is defined by the metric

$$L(g) = \frac{g_{ab}}{2\ell_B^2} R^a R^b$$

$$[L(g), a^\dagger(g)] = a^\dagger(g)$$

$$a(g)|\psi_0(g)\rangle = 0$$

$$|\psi_m(g)\rangle = \frac{(a^\dagger(g))^m}{\sqrt{m!}}|\psi_0(g)\rangle$$

- Model cft-based states such as the Laughlin state have a **constant** (flat, rigidly-fixed) metric
- In real FQH states of electrons contained in a non-uniform background potential, the metric **varies locally and dynamically** to allow the incompressible fluid to adjust to non-uniform flow induced by the background.
- The metric $g_{ab}(\boldsymbol{r}, t)$ then becomes an emergent dynamical collective degree of freedom of the FQH state.

FDMH, Phys. Rev. Lett. **107**, 116801 (2011)

holomorphicity:

- coherent state basis

$$\bar{a}|\bar{z}\rangle = \bar{z}|\bar{z}\rangle \quad |\bar{z}\rangle = e^{\bar{z}\bar{a}^\dagger - \bar{z}^*\bar{a}}|0\rangle$$

$$S(\bar{z}, \bar{z}^*; \bar{z}', \bar{z}^*) = \langle\bar{z}|\bar{z}'\rangle = e^{\bar{z}^*\bar{z}' - \frac{1}{2}(\bar{z}'^*\bar{z}' + \bar{z}^*\bar{z})}$$

- non-null eigenstates of the overlap define an orthonormal basis

$$\int \frac{d\bar{z}' d\bar{z}'^*}{2\pi} S(\bar{z}, \bar{z}^*; \bar{z}', \bar{z}'^*)\psi(\bar{z}', \bar{z}'^*) = \lambda\psi(\bar{z}, \bar{z}^*)$$

- non-null eigenstates are degenerate with $\lambda = 1$

$$\psi(\bar{z}, \bar{z}^*) = f(\bar{z}^*)e^{-\frac{1}{2}\bar{z}^*\bar{z}}$$

holomorphic!

“accidentally” coincide with lowest-Landau level wavefunctions if $\bar{z} = z^*$!!!

- This is the true origin of holomorphic functions in the theory of the FQHE
- NOTHING to do with lowest Landau level states, derives from overlaps between states in a non-orthogonal overcomplete basis!
- Has obvious parallels in theory of flat-band Chern insulators, where the projected lattice-site basis is non-orthogonal and overcomplete

$$|\Psi_L^q\rangle = \prod_i \int \frac{d\bar{z}_i^* d\bar{z}_i}{2\pi} \boxed{\prod_{i<j} (\bar{z}_i^* - \bar{z}_j^*)^q \prod_i e^{-\frac{1}{2}\bar{z}_i^* \bar{z}_i}} |\bar{z}_1, \bar{z}_2, \ldots, \bar{z}_N\rangle$$

"Laughlin wavefunction"

many-particle coherent state

$$\bar{a}_i|\bar{z}_1, \bar{z}_2, \ldots, \bar{z}_N\rangle = \bar{z}_i|\bar{z}_1, \bar{z}_2, l \ldots, \bar{z}_N\rangle$$

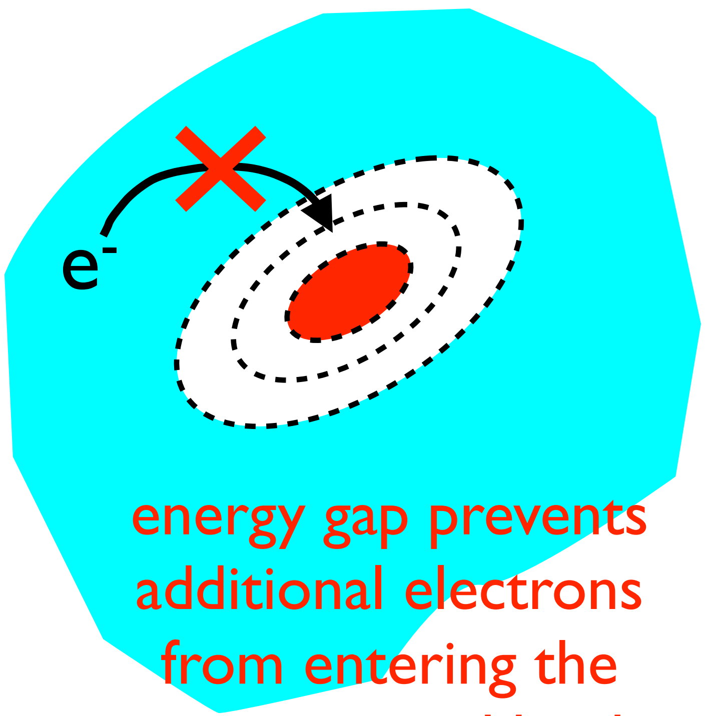
- It is a common misconception that the Laughlin state is fundamentally "a lowest Landau-level wavefunction" of Galileian-invariant Landau levels

- The similarity to a lowest-LL wavefunction is entirely accidental, as should have been clear when it was also found in the second LL. The recent discovery that Laughlin-like states occur in "flat band" Chern insulators now makes this entirely clear!

- The holomorphic character of the Laughlin state is entirely a property of the "quantum geometry" of the flat band (Landau level) encoded in $s(\boldsymbol{r}_1, \boldsymbol{r}_2)$, which in turn depends on the choice of metric g_{ab}.

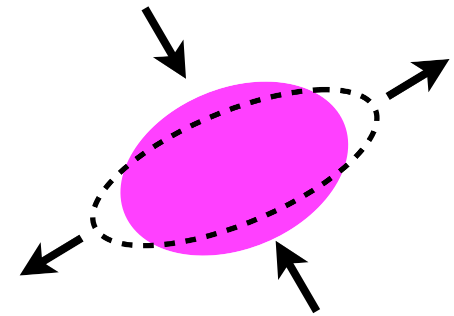
- Origin of FQHE incompressibility is analogous to origin of **Mott-Hubbard gap** in lattice systems.
- There is an energy gap for putting an **extra particle** in a quantized region that is **already occupied**

- **On the lattice** the "quantized region" is an atomic orbital with a fixed shape
- **In the FQHE** only the **<u>area</u>** of the "quantized region" is fixed. The **<u>shape</u>** must adjust to minimize the correlation energy.



energy gap prevents additional electrons from entering the region covered by the composite boson

- The metric (shape of the composite boson) has a preferred shape that minimizes the correlation energy, but fluctuates around that shape



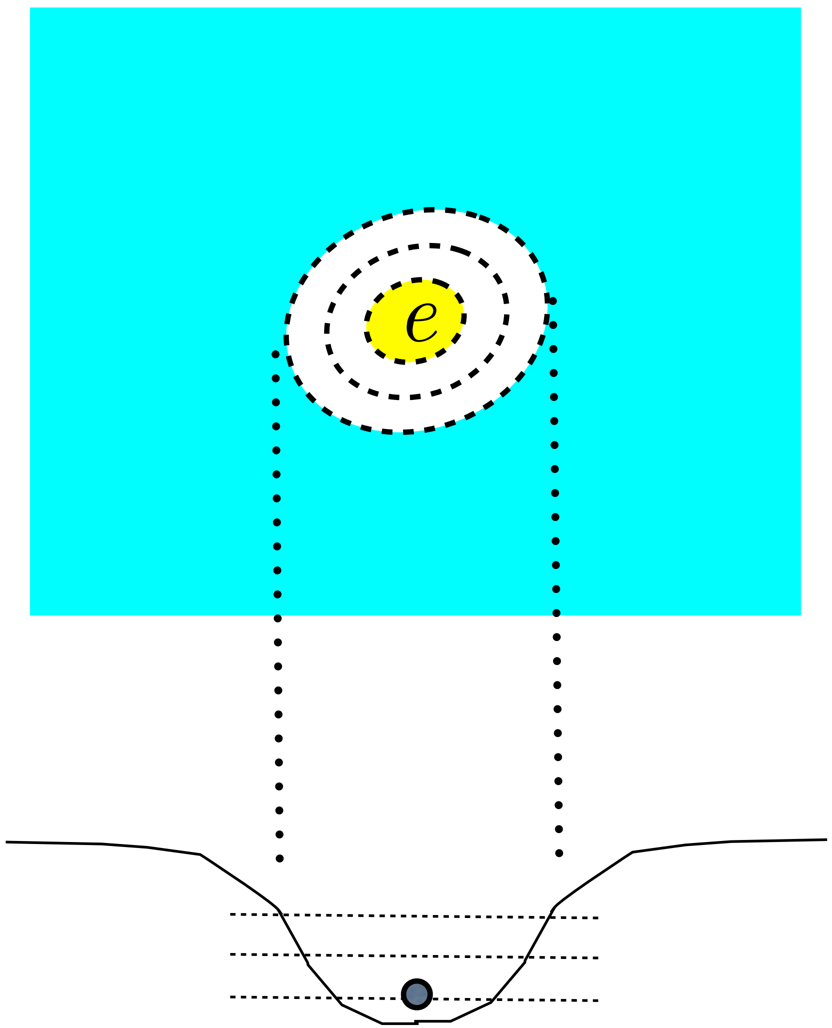
$\delta E \propto (\text{distortion})^2$

- The zero-point fluctuations of the metric are seen as the O(q^4) behavior of the "guiding-center structure factor" (Girvin et al, (GMP), 1985)

- long-wavelength limit of GMP collective mode is fluctuations of (spatial) metric (analog of "graviton")

FDMH, Phys. Rev. Lett. **107**, 116801 (2011)

$1/3$ Laughlin state



If the central orbital is filled, the next two are empty

The composite boson has inversion symmetry about its center

It has a "spin"

$$
\begin{array}{ccccccc}
 & \frac{1}{2} & \frac{3}{2} & \frac{5}{2} & & & \\
 & \boxed{1} & \boxed{0} & \boxed{0} & \cdots\cdots & & L = \frac{1}{2} \\
- & \boxed{\frac{1}{3}} & \boxed{\frac{1}{3}} & \boxed{\frac{1}{3}} & \cdots\cdots & - & L = \frac{3}{2} \\
\hline
 & & & & & & s = -1
\end{array}
$$

the electron excludes other particles from a region containing 3 flux quanta, creating a potential well in which it is bound

- crucial new physics:

composite bosons couple to the combination

$$\boxed{peB(\boldsymbol{r}) - \hbar s K(r)}$$

Gaussian curvature of metric

charge of composite boson

guiding-center "spin" of boson

* gauge field is

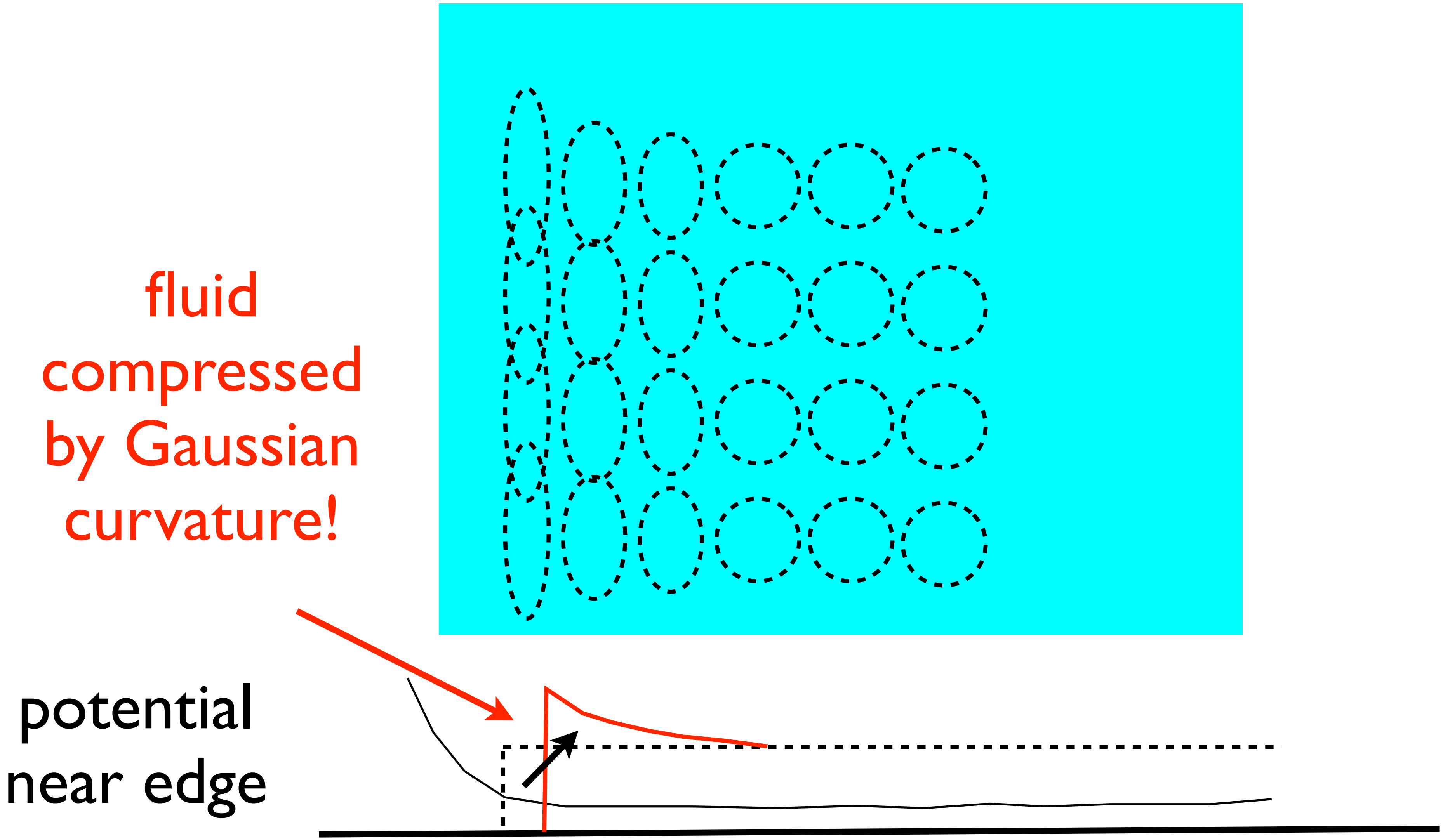
$$peA_\mu(\boldsymbol{r}) - \hbar s \Omega_\mu(r)$$

analog of spin-connection

related to Wen and Zee 1992

+ Chern-Simons

- metric deforms (preserving $\det\ g = 1$)in presence of non-uniform electric field



produces a dipole momemt

- multicomponent quantum Hall edge states do not have a universal speed, so are **not** Lorentz and conformally invariant.

- components of the cft energy momentum tensor:

Momentum density is independent of v:

$$T^0_x = T - \bar{T}$$

Energy density and stress are proportional to v

$$T^0_0 = -T^x_x = v(T + \bar{T})$$

Tracelessness (in flat space-time) is independent of v

$$T^0_0 + T^x_x = 0$$

Energy current density is proportional to v^2

$$T^x_0 = v^2(T - \bar{T})$$

The only speed-independent properties are

- The (signed) Virasoro algebra of the Fourier components of the momentum density (with the topologically-conserved chiral central charge

$$\tilde{c} = c - \bar{c}$$

This is a fundamental quantity that has nothing to do with conformal invariance (and in fact must vanish in a "true" (modular-invariant) 1+1d cft)

It controls a "Casimir momentum" $\frac{1}{24}\hbar\tilde{c}/L$

- Tracelessness of the energy-momentum tensor ($1\mathrm{d}$ pressure = energy density), which is true for linearly-dispersing modes, independent of their speed.

- Tracelessness of the 2D stress tensor in the Euclidean field theory is the absence of hydrostatic 2D pressure

$$P = -\frac{1}{2}\left(T^x_x + T^y_y\right) = 0$$

- Incompressible 2D quantum fluids at T = 0 do not transmit pressure through their bulk because of their energy gap (no gapless sound modes) - they only transmit pressure around their edges via gapless edge excitations.

- The traceless stress tensor of Euclidean 2d conformal field theory (and its dependence on a metric) may explain its applicability to FQHE

One final result

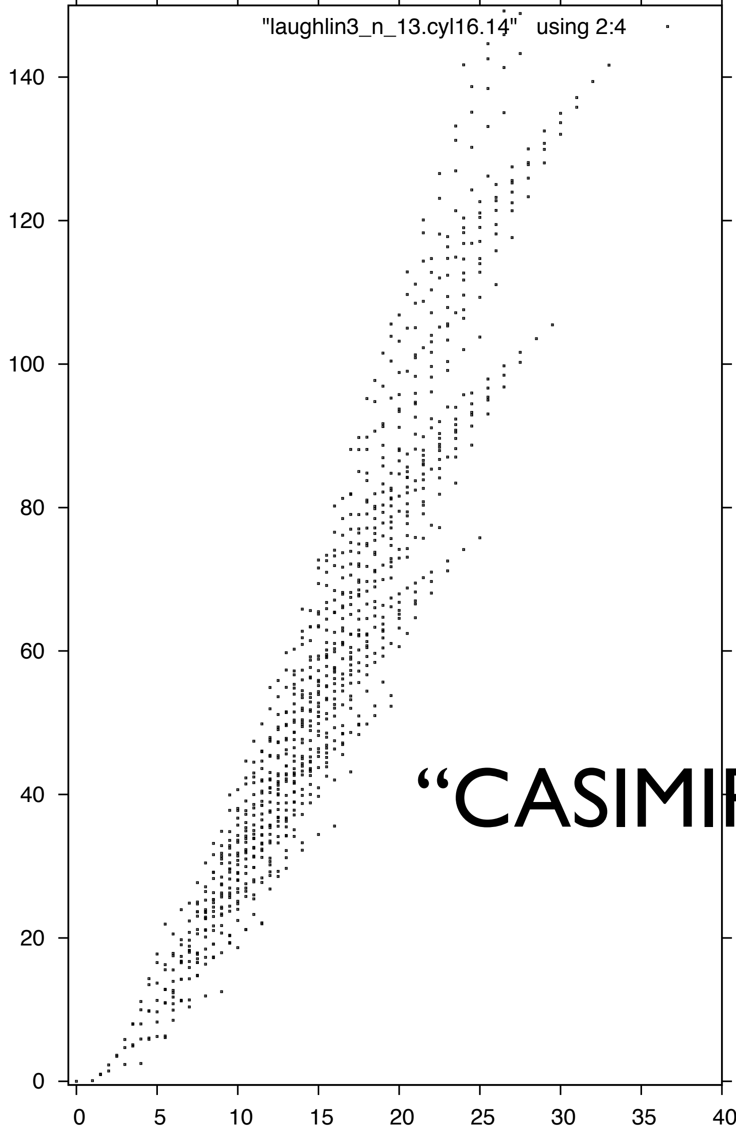
- In the "trivial" non-topologically-ordered integer QHE (due to the Pauli principle)

$$\tilde{c} = \nu = \text{Chern number}$$

$$\tilde{c} - \nu = 0$$

- the (guiding-center) "orbital entanglement spectrum" of Li and Haldane is insensitive to filled (or empty) Landau levels or bands, and allows direct determination of non-zero $\tilde{c} - \nu$

previous methods used the onerous calculation of the "real-space" entanglement spectrum to find $\tilde{c}$



ξ

m

ORBITAL CUT

$$\frac{P_a L^a}{2\pi} = \frac{\sum_\alpha m_\alpha e^{-\xi_\alpha}}{\sum_\alpha e^{-\xi_\alpha}} = \eta_H^{cd}\epsilon_{ac}\epsilon_{bd}\frac{L^a L^b}{2\pi\ell_B^2}$$

$$+\frac{1}{24}(\tilde{c}-\nu) - h$$

signed conformal anomaly (chiral stress-energy anomaly)

chiral anomaly

virasoro level of sector

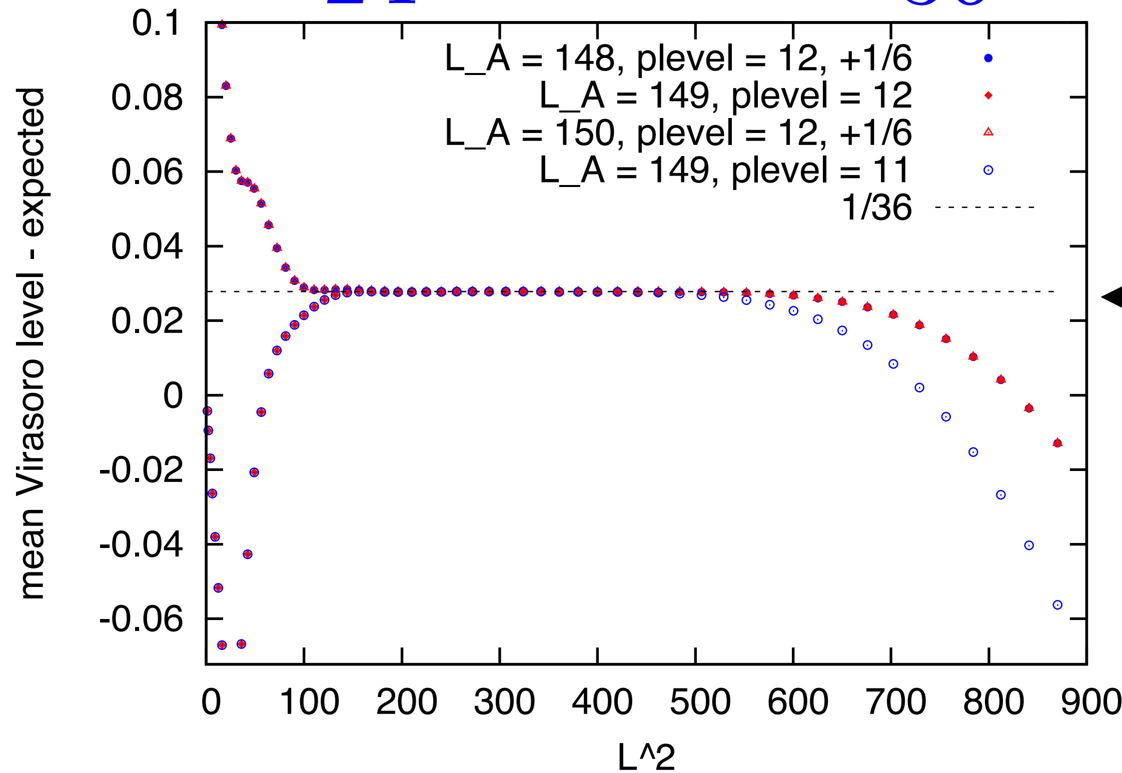
"CASIMIR MOMENTUM" term

(NOT "real-space cut" which requires
the Landau orbit degrees of freedom and their
form factor to be included

- Hall viscosity gives "thermally excited" momentum density on entanglement cut, relative to "vacuum", at von Neumann temperature T = 1

Yeje Park, Z Papic, N Regnault

$$\frac{1}{24}\left(\tilde{c}-\nu\right)=\frac{1}{24}\left(1-\tfrac{1}{3}\right)=\frac{1}{36}$$



$$\frac{1}{36}$$

Matrix-product state calculation on cylinder with circumference L
("plevel" is Virasoro level at which the auxialliary space is truncated)

- How universal is the Thermal Hall effect formula? It also depends on
- If Lorentz invariance is present, its $\tilde{c}$ essentially the same calculation as Casimir momentum
- When Lorentz invariance is broken by different speeds for different modes, but the remain independent, the result still stands
- How much information about the Hamiltonian ($T^0{}_0$) is needed? Is there a clean "gravitational" derivation just based on the momentum $T^0{}_x$ Virasoro anomaly?

$$J_E^a = \frac{\tilde{c}}{12} \frac{(2\pi k_B T)^2}{2\pi\hbar} \epsilon^{ab} \frac{g_b}{c^2} \qquad \frac{\vec{g}}{c^2} = -\frac{\vec{\nabla} T}{T}$$

Momentum density is universal:

$$T_x^0 = \frac{1}{L}\sum_m T_m \exp(2\pi i x/L)$$

$$[T_m, T_n] = (m-n)T_{m+n} + \tfrac{1}{12}\tilde{c}m(m^2-1)\delta_{m+n,0}$$

chiral central charge

Can we obtain the Thermal Hall effect just from this plus "gravity"?