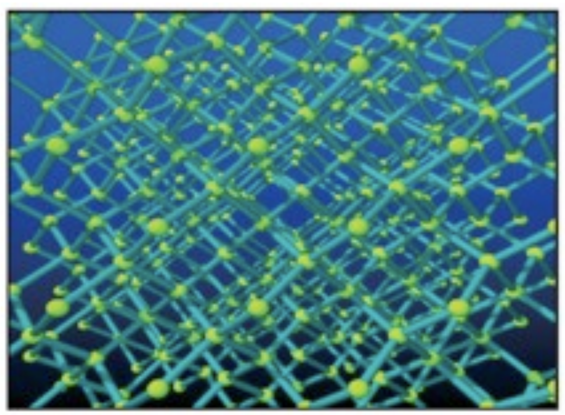
Detecting topological edge and surface states + Non-equilibrium steady states in 1D

Edinburgh, September 2013



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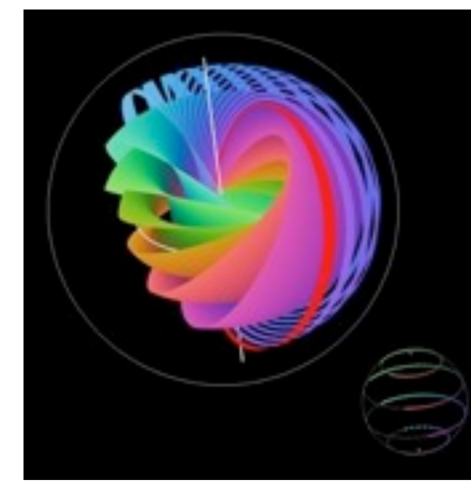


Outline

1. Quick introduction to some topological phases

2. How is the TI surface or edge fundamentally different in transport measurements from a normal metal *or graphene*?
Unconventional magnetotransport in 3D TI nanowires
(J. Bardarson, P. Brouwer, JEM, PRL 2010)

Toward superconducting state transport: understanding perfect transmission and Majoranas in SC/3DTI/SC junctions.
(R. Ilan, J. Bardarson, H.-S. Sim, JEM, arXiv 2013)
Impurity effects in *nonequilibrium* QSH edge state transport via integrability
(R. Ilan, J. Bardarson, J. Cayssol, JEM, PRL 2012)

Two old problems now approachable with new methods:
3. Strong interactions: the fractional quantum Hall edge
(J. Kjäll, JEM, PRB 2011; D. Varjas, M. Zaletel, JEM, arXiv 2013).
4. A basic problem of non-equilibrium in many-electron systems
(C. Karrasch, R. Ilan, JEM, arXiv 2013).

“Integer” topological phases in 2D: IQHE and 2D topological insulator

The integer quantum Hall effect is induced by a strong magnetic field. It is recently understood that there are similar one-electron phases from spin-orbit coupling.

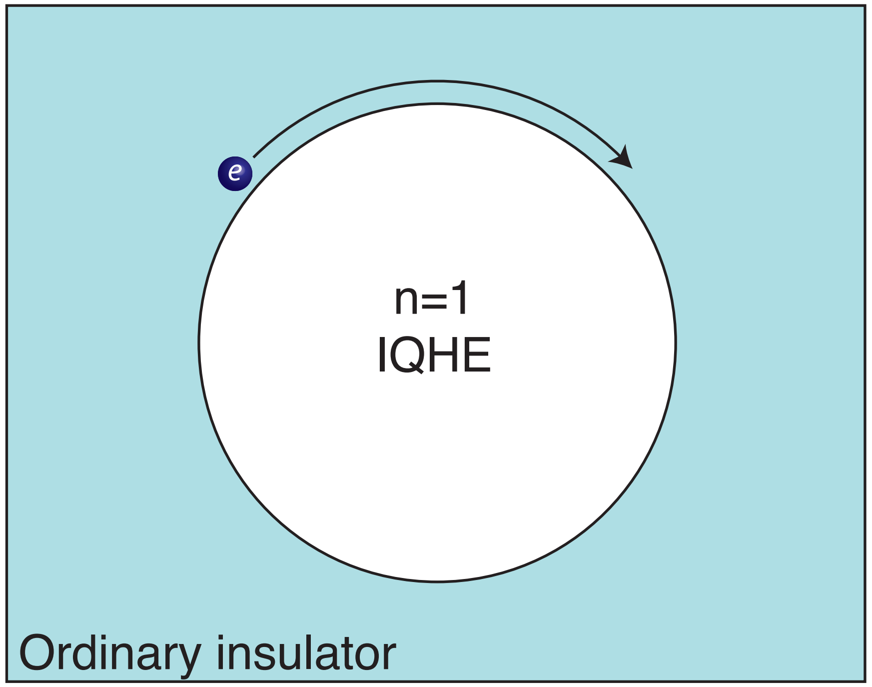
Spin-orbit coupling appears in nearly every atom and solid. Consider the standard atomic expression

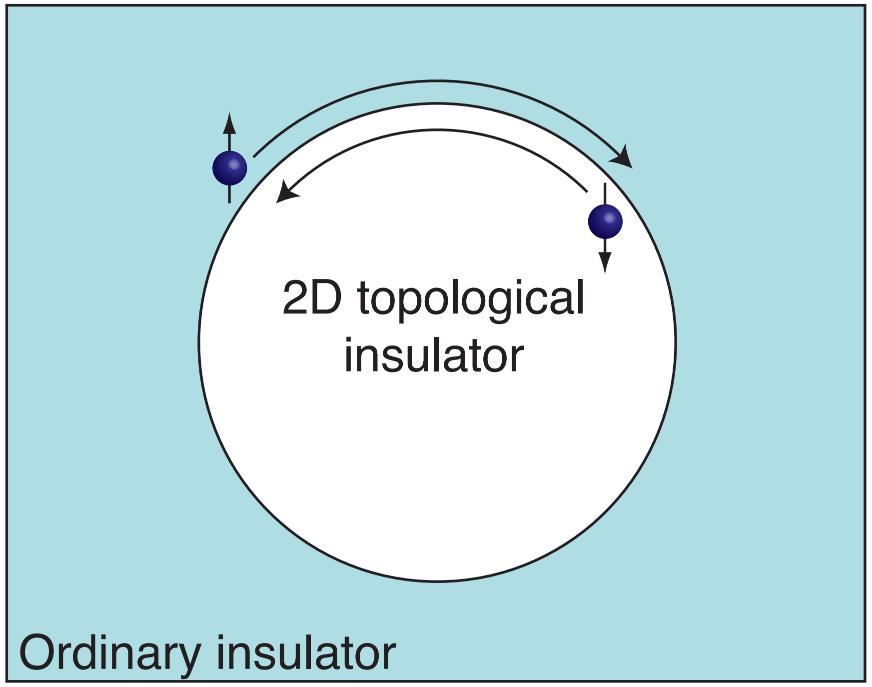
$$H_{SO} = \lambda \mathbf{L} \cdot \mathbf{S}$$

For a given spin, this term leads to a momentum-dependent force on the electron, somewhat like a magnetic field.

The spin-dependence means that the *time-reversal symmetry* of SO coupling (even) is different from a real magnetic field (odd).

spin-up and spin-down electrons are in IQHE states, with opposite “effective magnetic fields”.



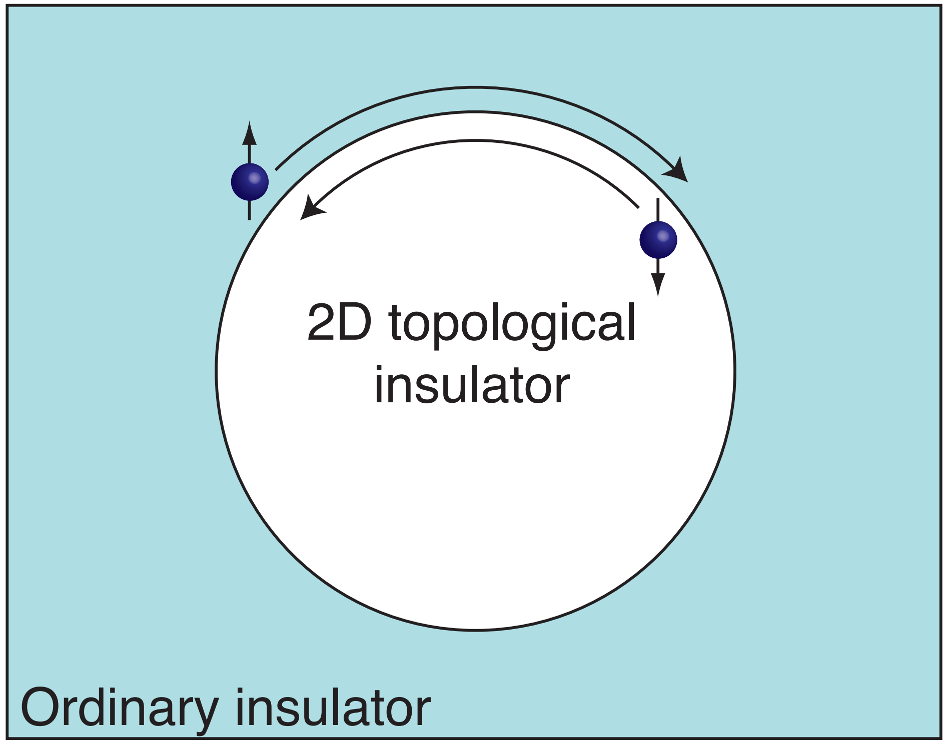


The 2D topological insulator

People were somewhat skeptical until it was shown in 2005 (Kane and Mele) that, in real solids with all spins mixed and no “spin current”, edge physics survives.

Kane and Mele found a new topological invariant in time-reversal-invariant systems of fermions.

It isn’t isn’t an integer. It is a Chern *parity* (“odd” or “even”), or a “Z2 invariant”.



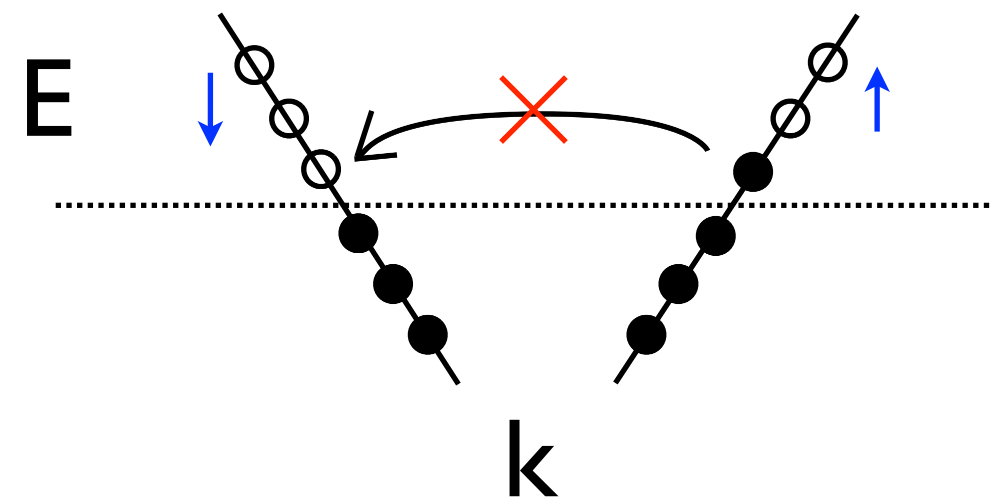
Systems in the “odd” class are “2D topological insulators”

1. Where does this “odd-even” effect come from?
2. How can this edge be seen?

The 2D topological insulator

Where does this “odd-even” effect come from?

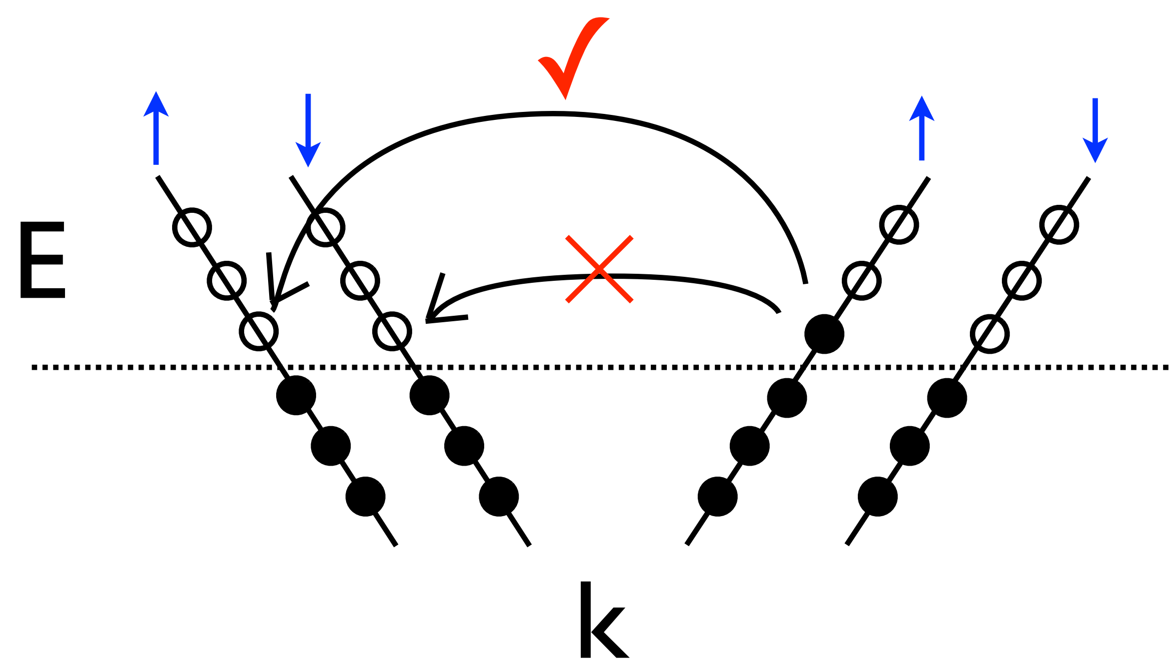
In a time-reversal-invariant system of electrons, all energy eigenstates come in degenerate pairs.



The two states in a pair cannot be mixed by any T-invariant perturbation. (disorder)

So an edge with a single Kramers pair of modes is perturbatively stable (C. Xu-JEM, C. Wu et al., 2006).

But this rule does not protect an ordinary quantum wire with 2 Kramers pairs:



The topological vs. ordinary distinction depends on time-reversal symmetry.

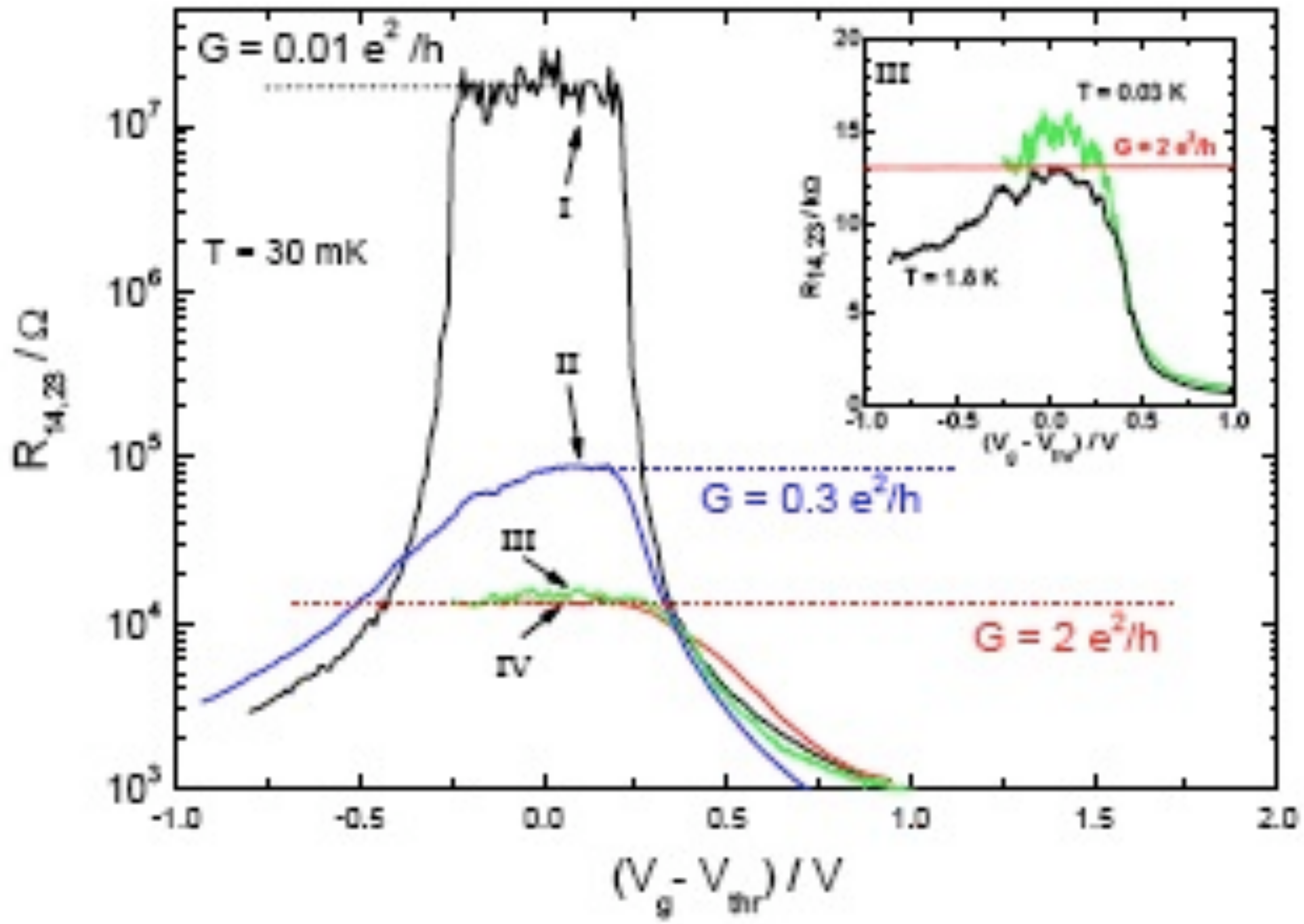
The 2D topological insulator

Key: the topological invariant predicts the existence of "quantum wires".

While the wires are not one-way, so the Hall conductance is zero, they still contribute to the *ordinary* (two-terminal) conductance.

There should be a low-temperature *edge* conductance from one spin channel at each edge:

$$G = \frac{2e^2}{h}$$



König et al., *Science* (2007)

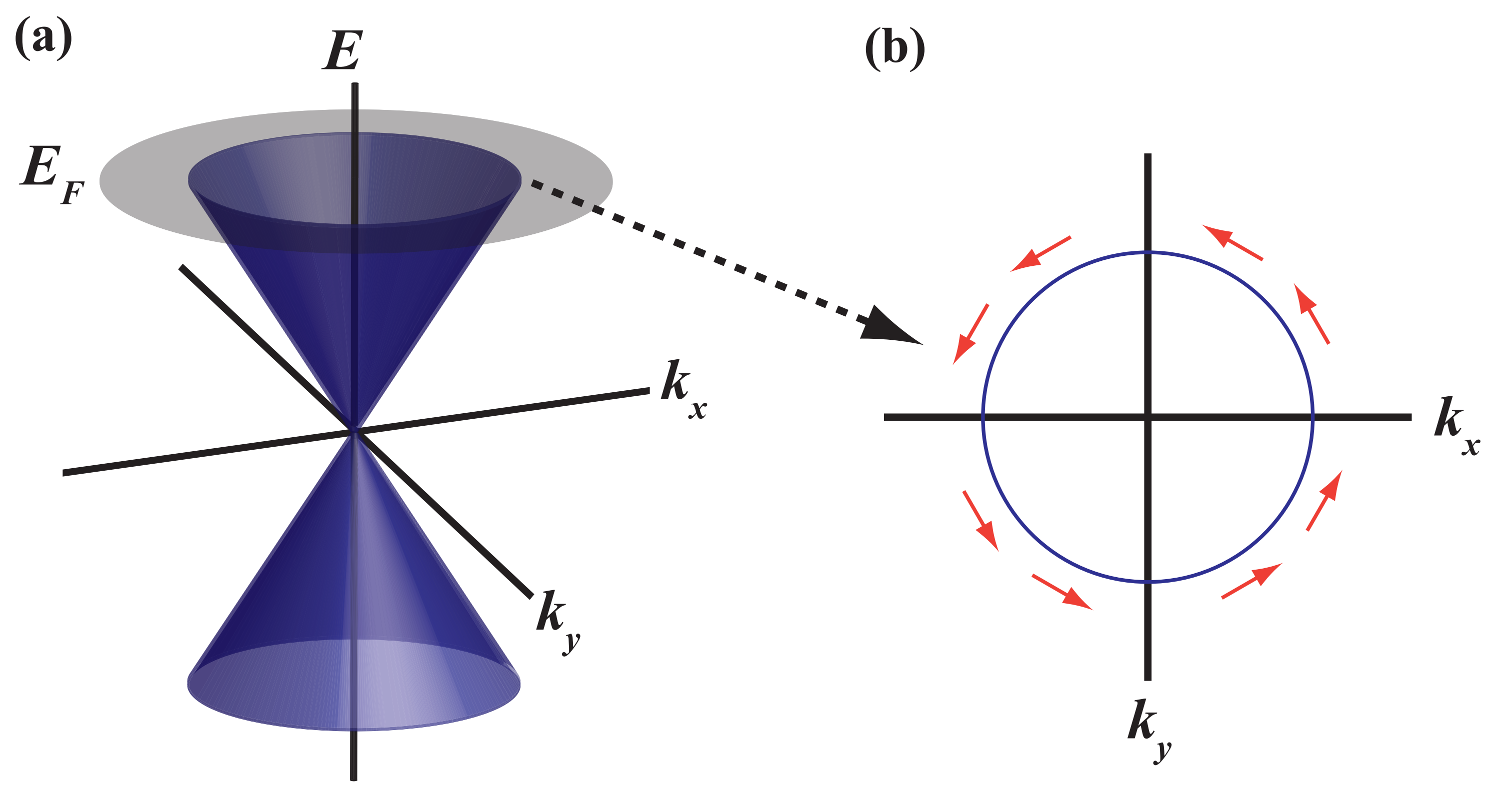
Laurens Molenkamp

This appears in (Hg,Cd)Te quantum wells as a quantum Hall-like plateau *in zero magnetic field.*

Topological insulators in 3D

In 3D, there are 4 band structure invariants: 3 "weak" invariants and 1 "strong".
(JEM-Balents, Roy, Fu-Kane-Mele, Fu-Kane, summer 2006)

The fourth gives a robust 3D phase whose metallic surface state in the simplest case is a single massless "Dirac fermion"



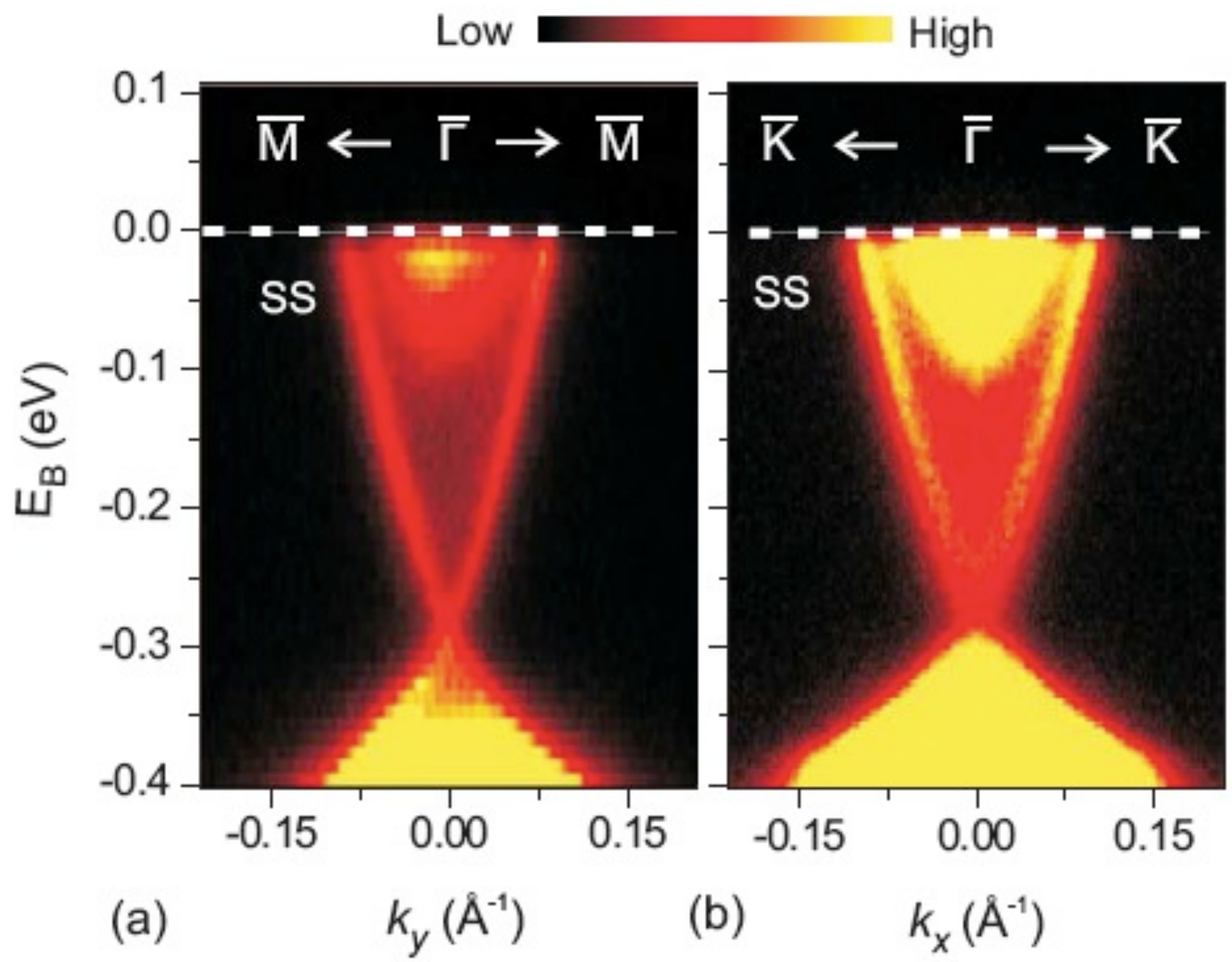
Surface state = "1/4 of graphene": no spin or valley degeneracy

2. Some fairly standard 3D materials turn out to be topological insulators!
Claim:
Certain insulators will *always* have metallic surfaces with strongly spin-dependent structure

ARPES of topological insulators

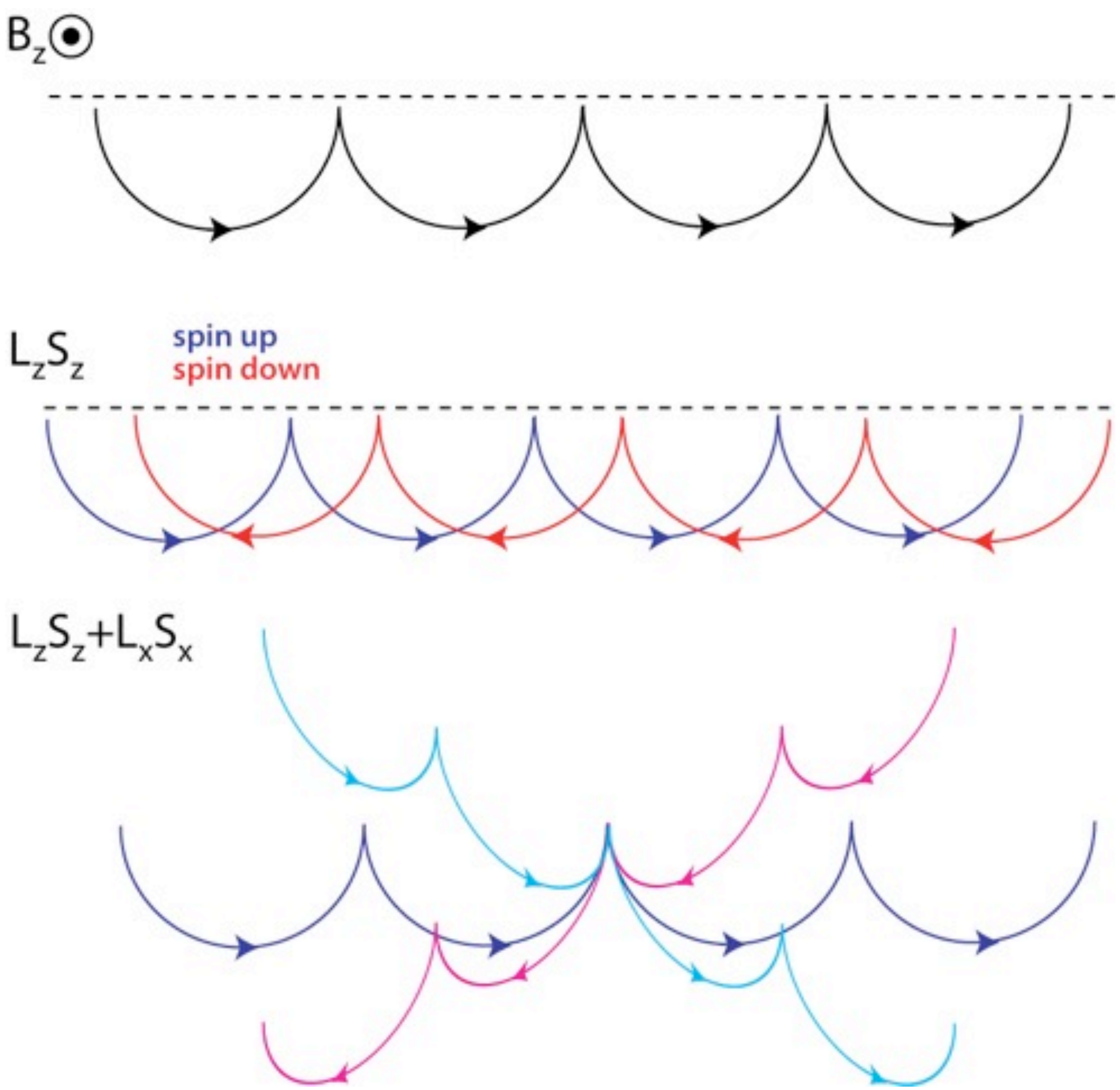
First observation by D. Hsieh et al. (Z. Hasan group), Princeton/LBL, 2008.

This is later data on Bi_2Se_3 from the same group in 2009:



The states shown are in the "energy gap" of the bulk material--in general no states would be expected, and especially not the Dirac-conical shape. Supported by STM measurements, optics, magnetotransport in best materials.

Topological Insulators from Spin-orbit Coupling
Semiclassical picture



1D edge of Quantum Hall Effect

1D edge of "Quantum Spin Hall Effect" (discovered 2007)

2D surface of 3D Topological Insulator (discovered 2008)

3D topological insulators have a special metallic 2DEG at any surface

How do these topological states connect to other physical properties?

Theme: When is the factor of 2 between an ordinary metal and the 2D and 3D edge/surface states more than just a factor of 2?

How can we tell in normal-state transport that the 3D TI surface is different from both graphene and conventional 2DEGs?

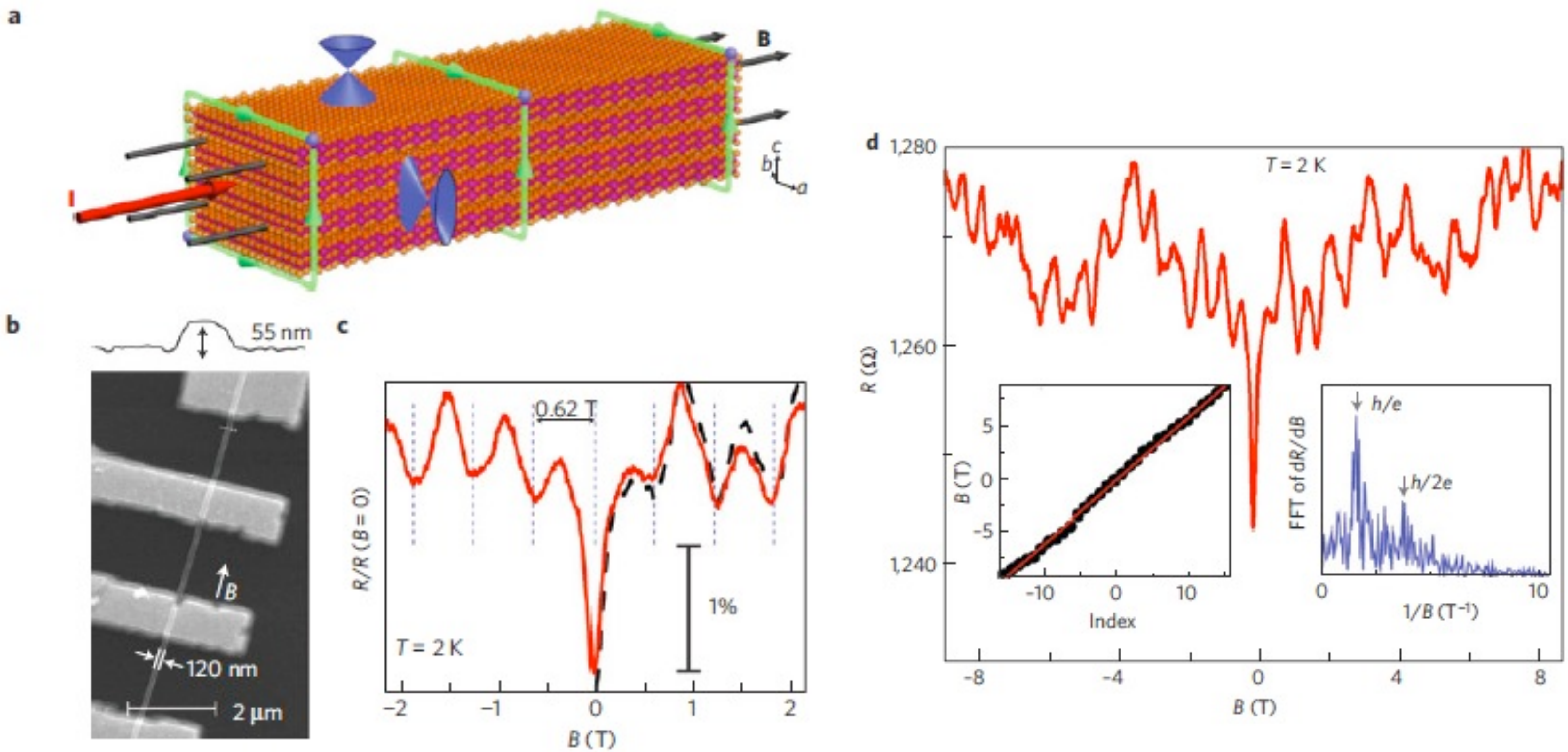
$$H = v(\sigma^x \pi_y - \sigma^y \pi_x) + \tfrac{g\mu_B}{2}\mathbf{B}\cdot\boldsymbol{\sigma}$$

$$(\boldsymbol{\pi} = \mathbf{p} + e\mathbf{A})$$

Beyond just having Dirac fermions, we would like a way to count them that does not require achieving quantized Hall conditions.

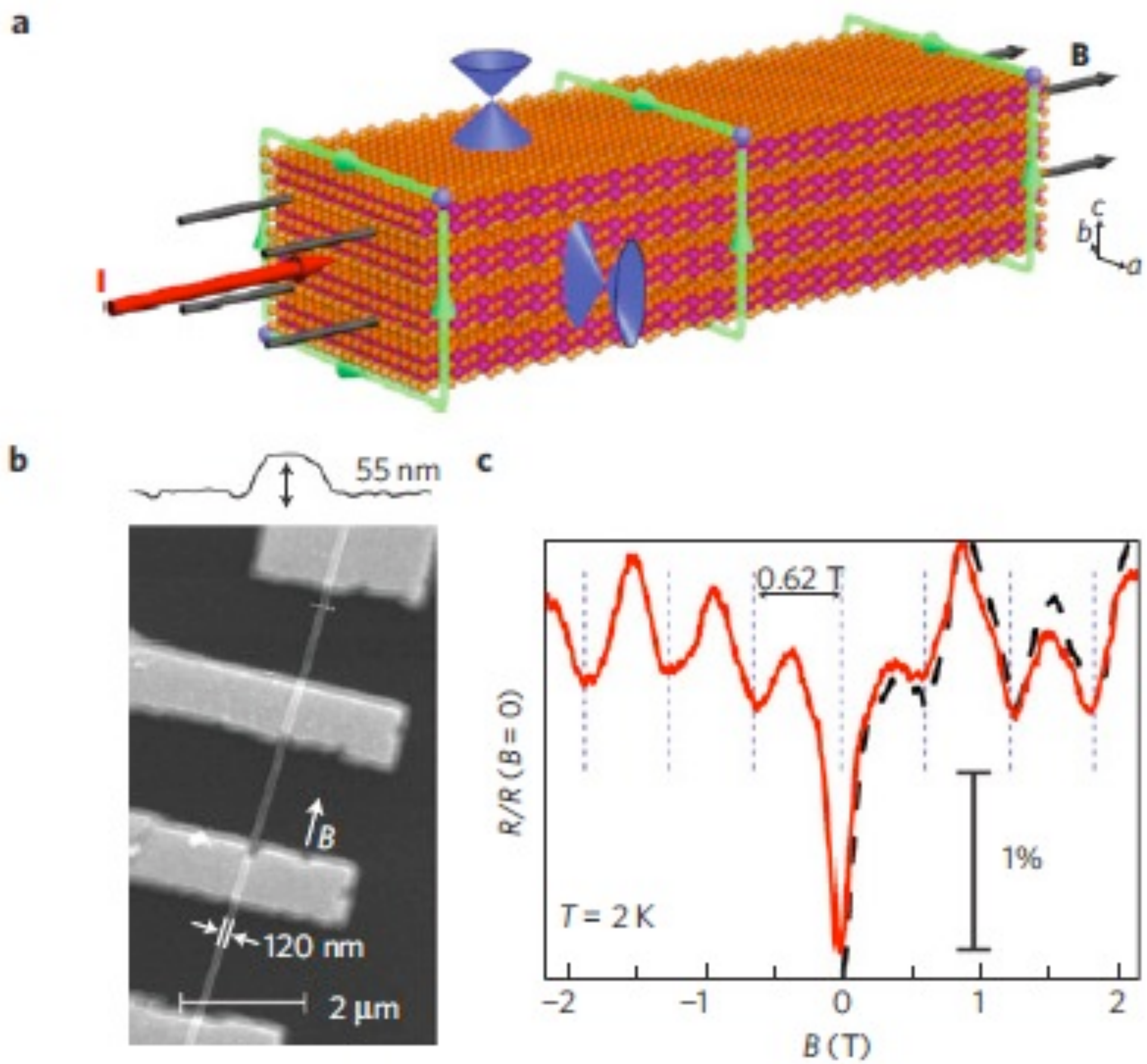
Conductance along a Bi2Se3 nanoribbon pierced by magnetic flux

Use geometry to isolate surface even when bulk is conducting.



H. Peng et al. (Y. Cui group), Nature Materials 9, 225 (2010)

Conductance along a Bi2Se3 nanoribbon pierced by magnetic flux



Aside from the original experimental motivation, piercing the TI surface by a flux is an interesting system:

putting π flux through a hole in a 3D TI, or through a TI nanowire, leads to a protected mode analogous to the "helical edge" of the 2D QSHE.

Berry phases in transport

Puzzle: Stanford nanowire experiment (Yi Cui et al., Nature Materials)

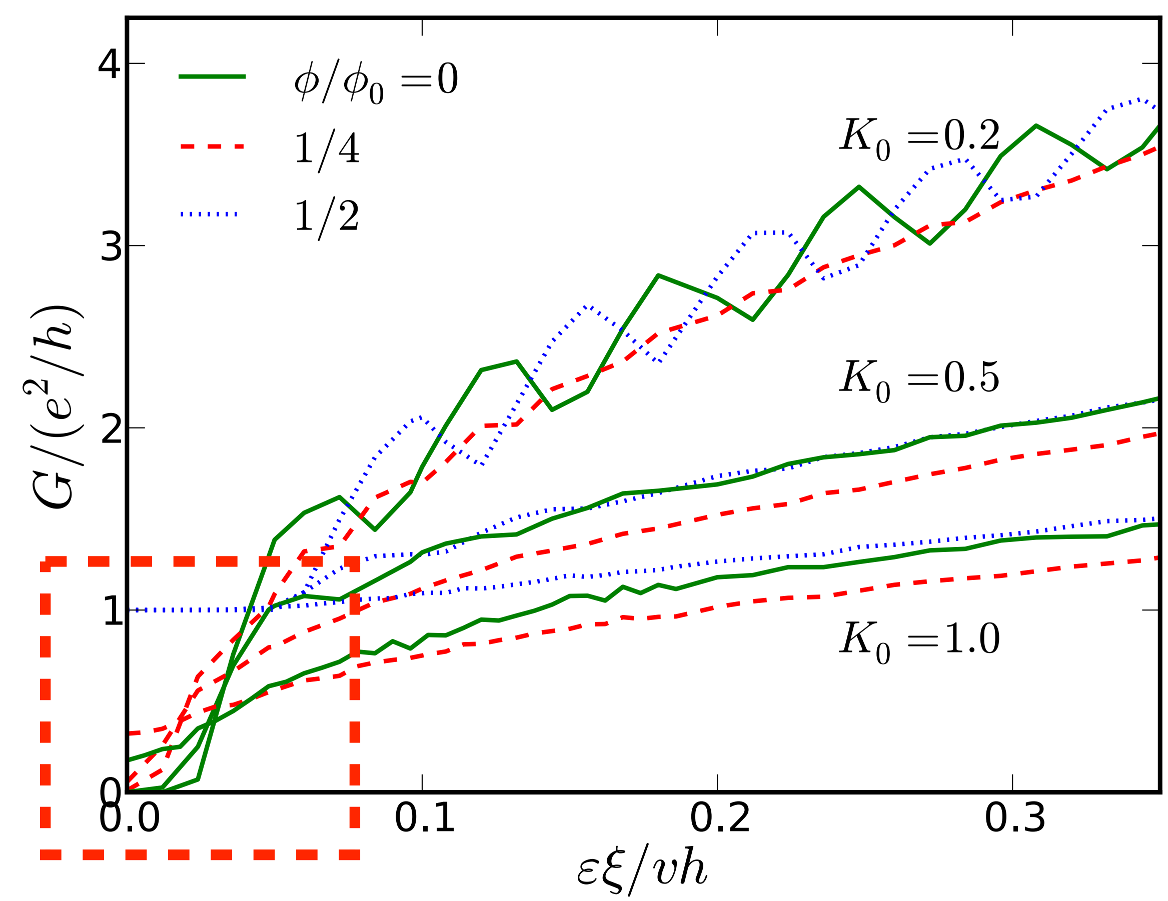
sees Aharonov-Bohm (h/e) oscillations, as expected for a clean system, rather than Sharvin & Sharvin (h/2e), as expected for a diffusive metallic cylinder.

The *sign* is also not what is expected in the strong-disorder limit: the Berry phase protects a mode at pi flux, rather than 0 flux as in a nanotube.

Intuition: spin-momentum locking means tha spin direction rotates through 2π as electro circles the cylinder. This gives a - sign that is compensated by the π flux.

(Bardarson, Brouwer, JEM, PRL 2010; Zhang and Vishwanath, PRL 2010)

K_0 = disorder strength



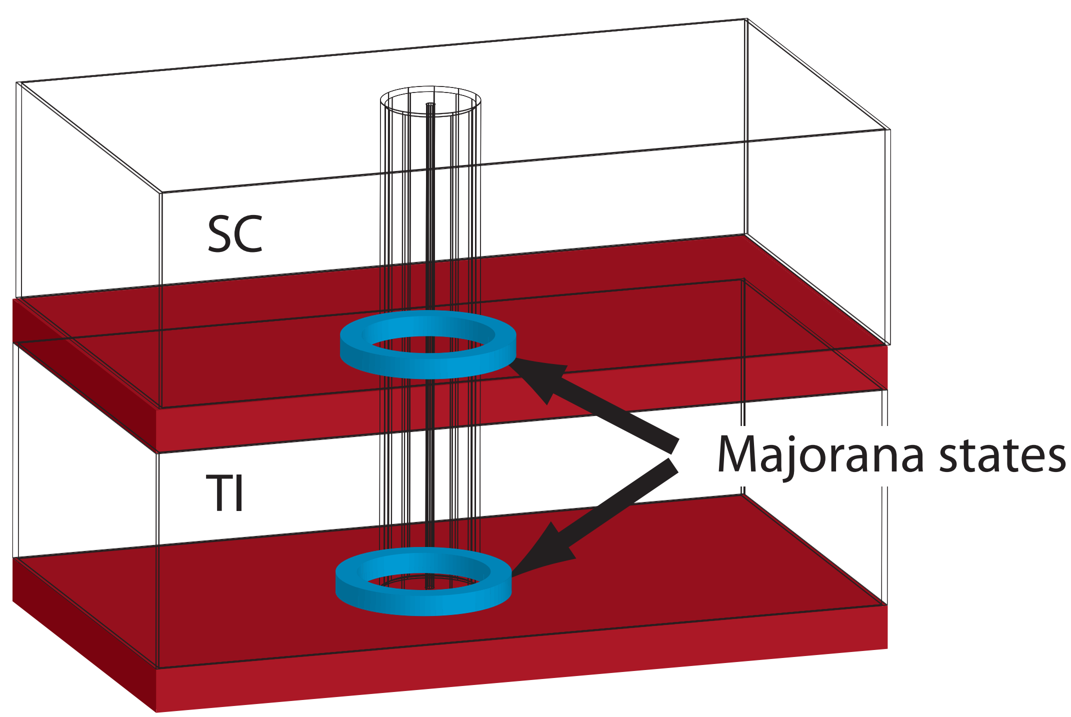
Scaled chemical potential relative to Dirac point

SC/3DTI/SC junctions

At certain fields and phase differences, there are bound "Majorana fermions" at the SC/3DTI interface. (Fu-Kane, Sato, ...)

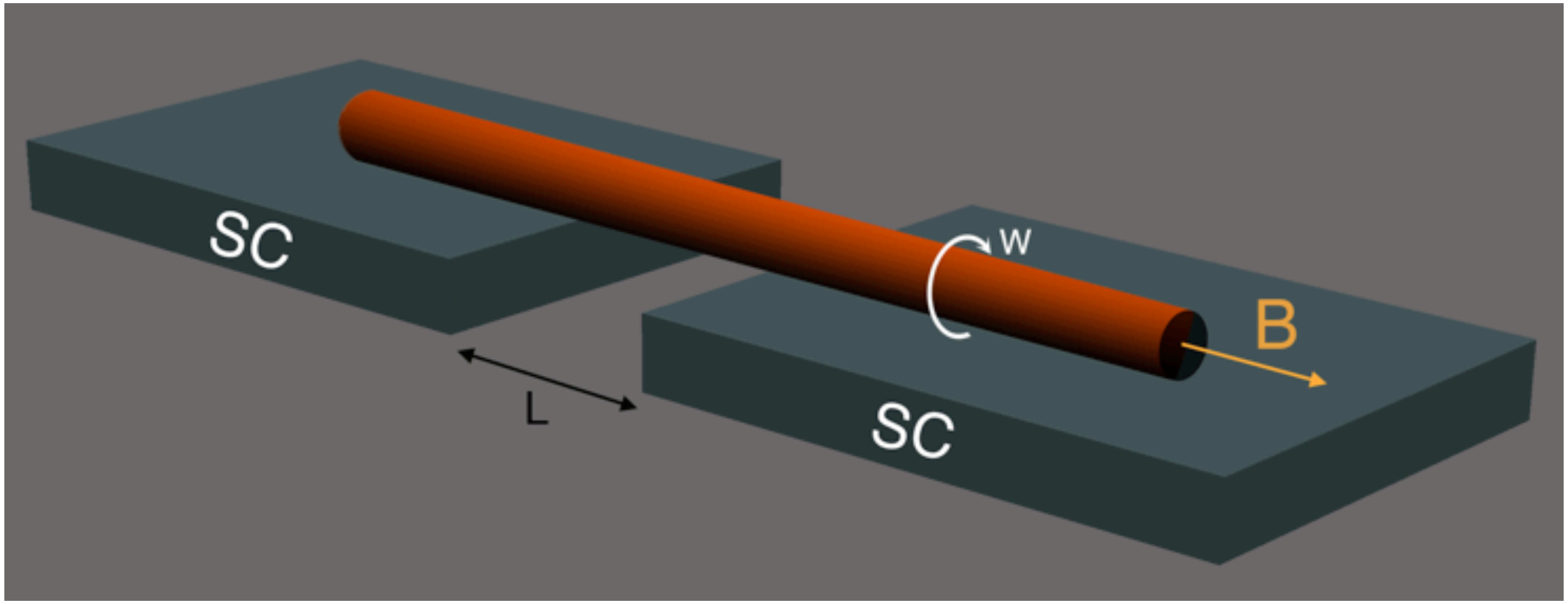
Majoranas=the most interesting factor of 2; "half" of a two-level system

Majoranas in vortex cores:

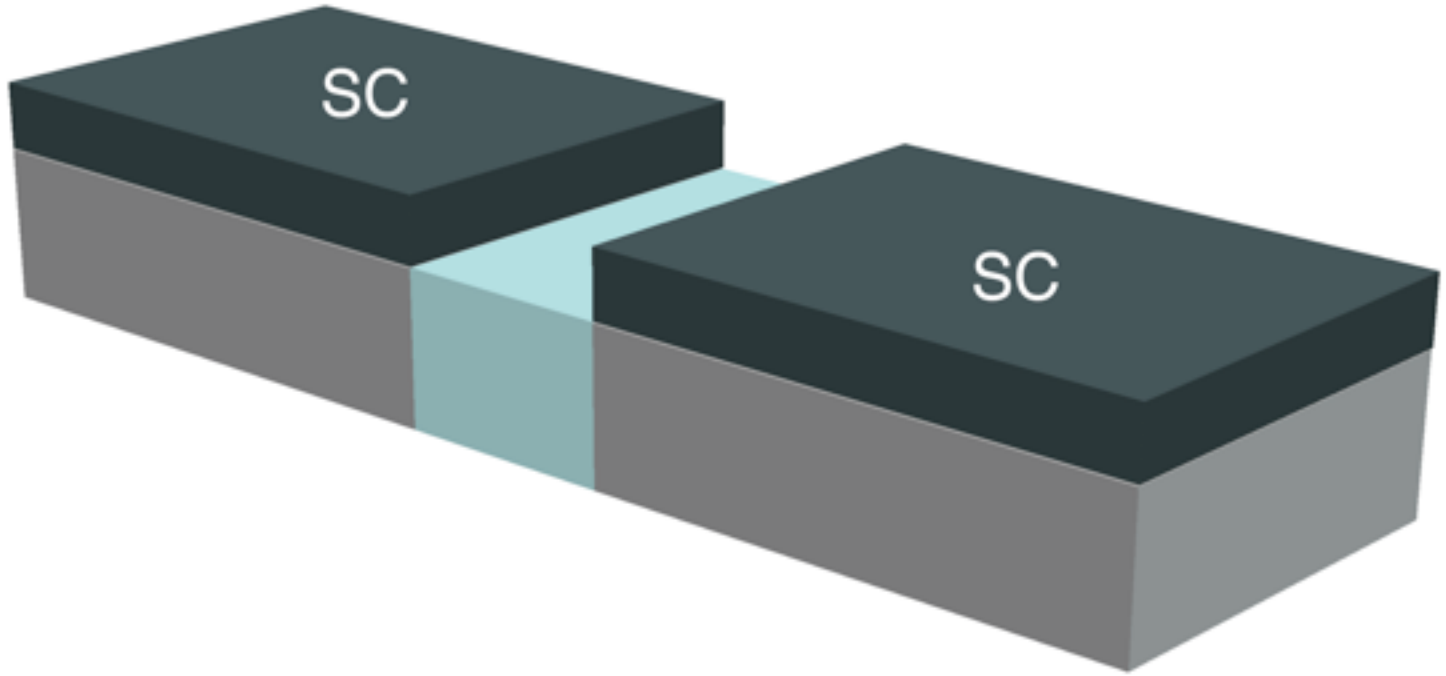


How can this be probed experimentally?

At π flux, the 3D TI nanowire mimics the perfect helical edge. The latter has been proposed as a platform for Majoranas, so...



Alternately, view this nanowire as a vortex, except that the empty region is outside rather than inside.



"Whereas bulk transport tends to obscure
the observation of surface state transport in the normal state, the supercurrent is found to be carried mainly by the surface states"

[Veldhorst et al. Nature Mater. 2012]

Majorana states $\Delta\phi = \pi$

[Fu and Kane, PRL 2008]

[Alicea Rep Prog Phys (2012), Beenakker, Annu. Rev. Cond. Mat. Phys. 2013, Grosfeld and Stern 2011, Potter and Fu 2013, Weider et al 2013]

3DTI and Majorana fermions

Short JJ $L \ll \xi$

[Beenakker 2006,
Titov and Beenakker 2006]

$$E_n(\phi) = \Delta\sqrt{1 - \tau_n \sin^2(\phi/2)}$$

$$\phi = \pi$$
$$+$$
$$\tau = 1$$

$$E = 0$$

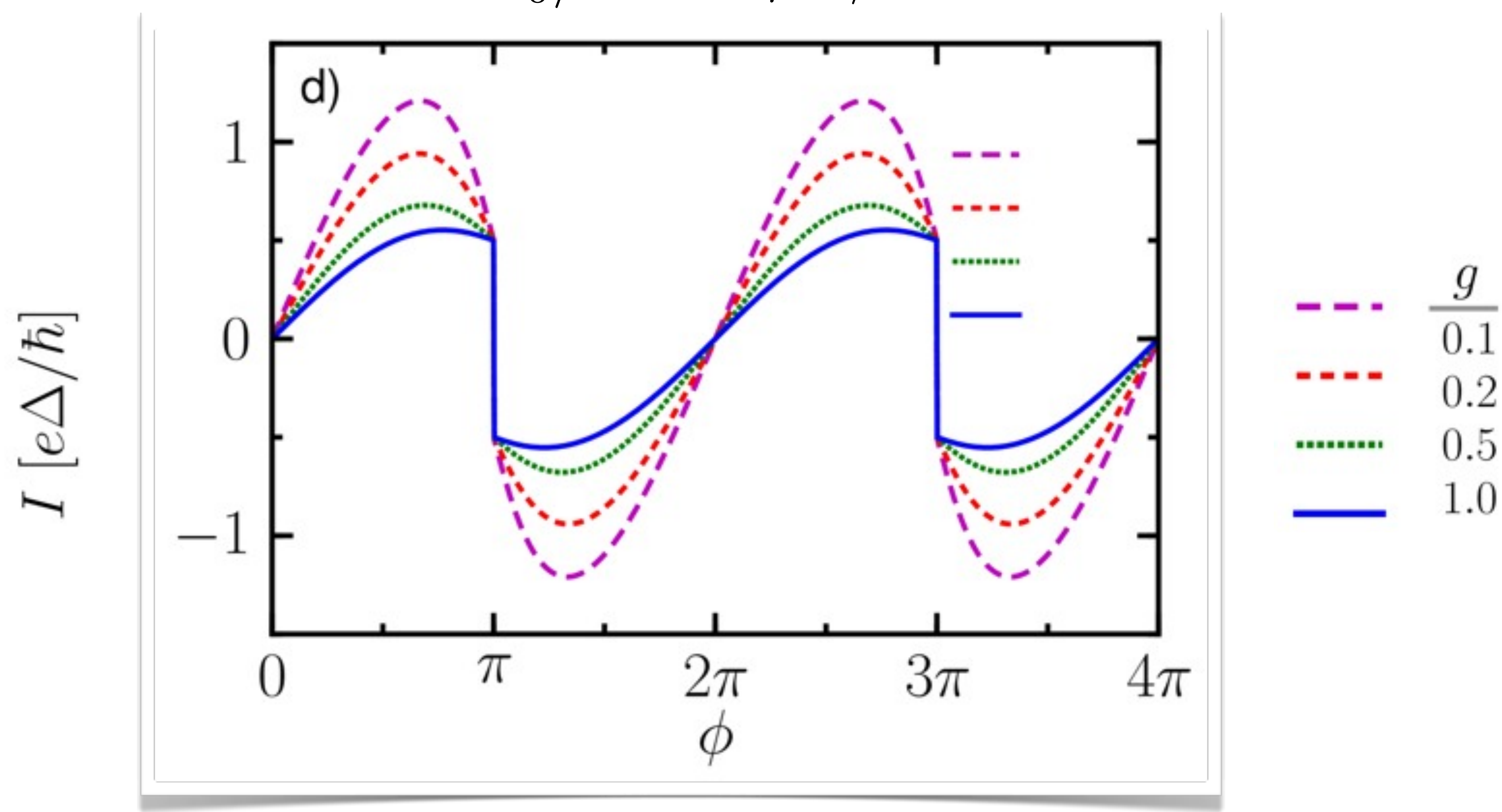
Zero energy Majorana state requires a *perfectly transmitted mode*

Current Phase relation - disorder

$$\langle V(r)V(r')\rangle = g\frac{\hbar v}{2\pi\xi_D^2}e^{|r-r'|/2\xi_D^2}$$

[Bardarson et al. PRL 2007]

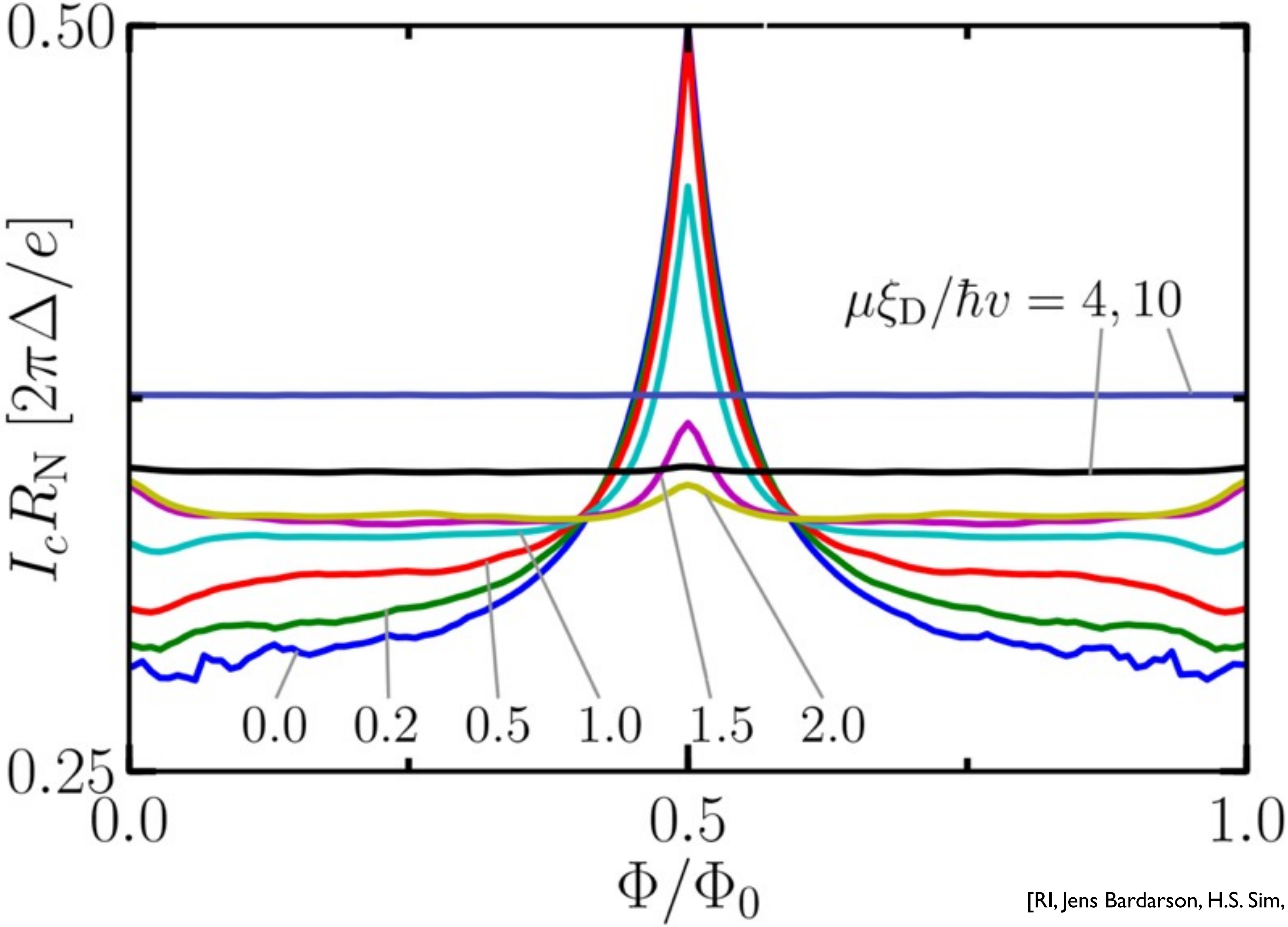
$\Phi = \Phi_0/2 \qquad \mu W/\hbar v = 20$



[RI, Jens Bardarson, H.S. Sim, Joel Moore,2013]

$I_c R_N$ - disorder

$g = 2$



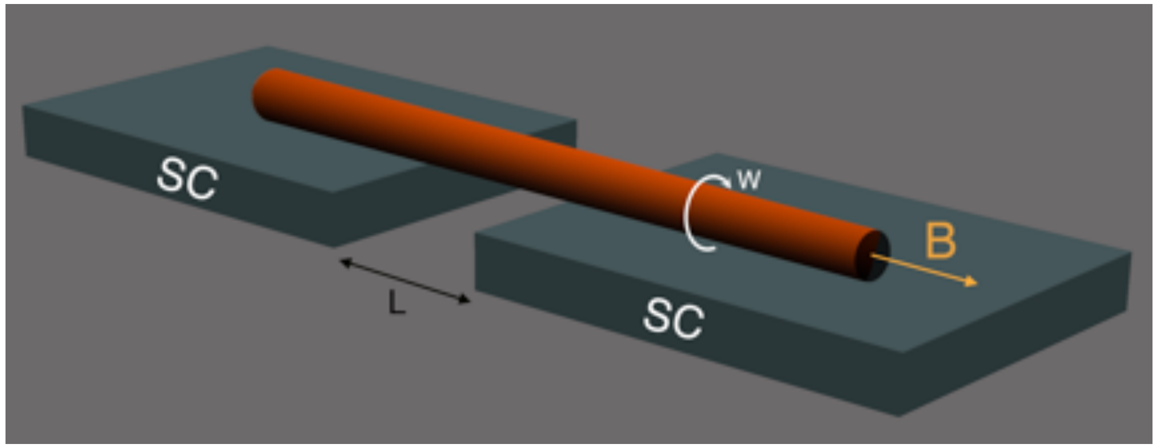
[RI, Jens Bardarson, H.S. Sim, Joel Moore,2013]

Experimental feasibility

Short junction limit	$L \ll \xi$
Mode suppression	$L/W \sim 1$
Flux through wire	$\Phi > \Phi_0/2$
Distance from Dirac point	$\mu\xi_D/\hbar v$
Disorder strength	g

$W \sim 400nm$

$B \approx 0.2T$



$g \approx 1 \quad \xi_D \sim 10nm \quad n \sim 8 \times 10^{10}\ cm^{-2}$

[Sacepe et al. Nature Commun 2011, Kong et al. Nano Letters 2010, Beidenkophf et al. 2011, ,Williams et al. PRL 2012, Tian et al. Sci. Rep. 2012, Veldhorst et al Nature Mater. 2012, Kim et al Nature Phys. 2012, Cho et al. Nature Commun 2013...]

The perfectly transmitted mode at π flux is a Majorana signature that:

1. has fewer competing explanations than the zero-bias anomaly;

2. appears under conditions when the Fraunhofer pattern is ordinary;

3. can be realized with existing large-bandgap materials.

So far, nearly free electrons at equilibrium.
What can we say more generally?

1. Analytical: effects of a magnetic perturbation at QSHE edge.
2. Numerical: FQHE edge & steady-states.

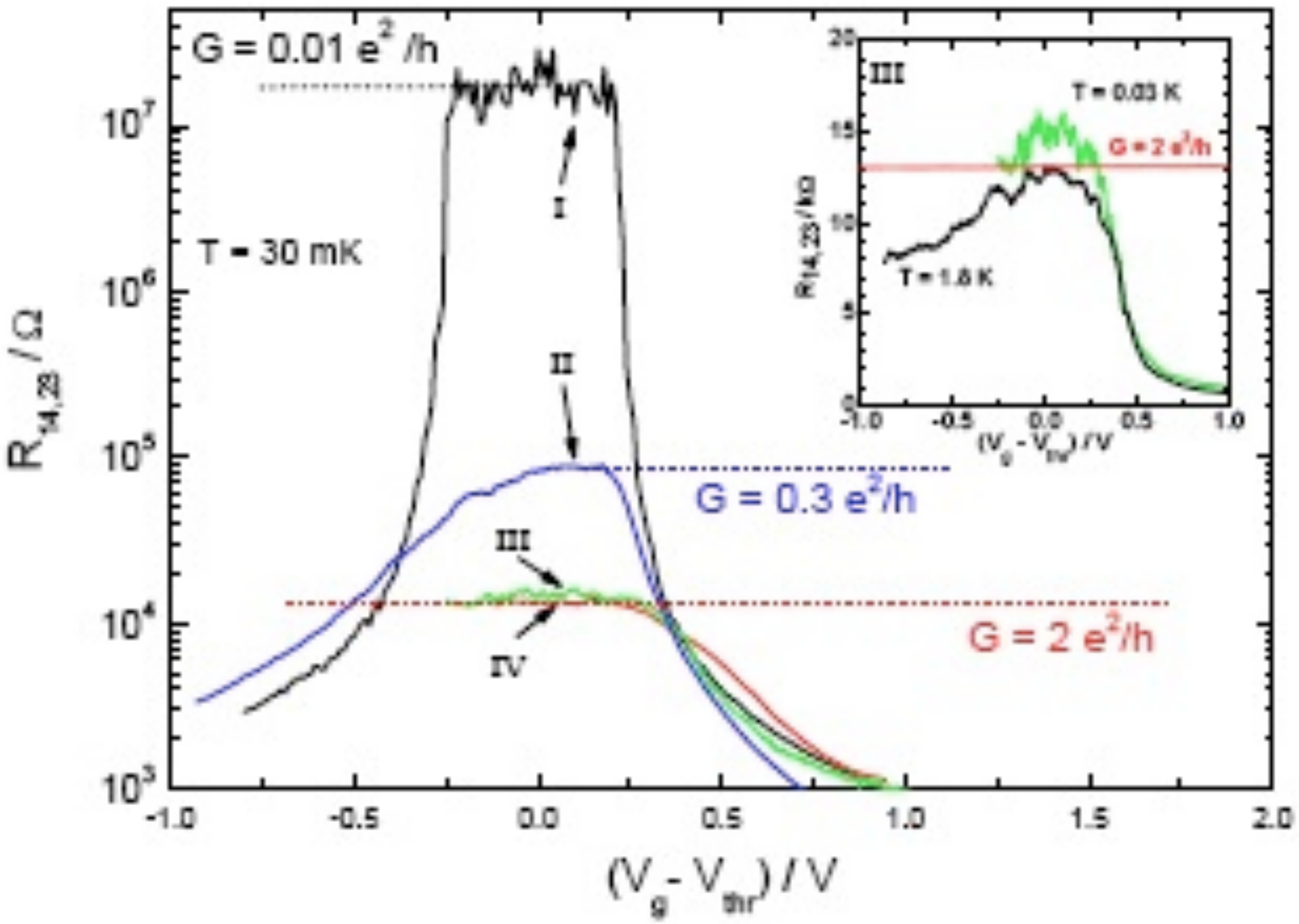
The 2D topological insulator

Key: the topological invariant predicts the existence of "quantum wires".

While the wires are not one-way, so the Hall conductance is zero, they still contribute to the *ordinary* (two-terminal) conductance.

There should be a low-temperature *edge* conductance from one spin channel at each edge:

$$G = \frac{2e^2}{h}$$



König et al., *Science* (2007)

Laurens Molenkamp

This appears in (Hg,Cd)Te quantum wells as a quantum Hall-like plateau *in zero magnetic field.*

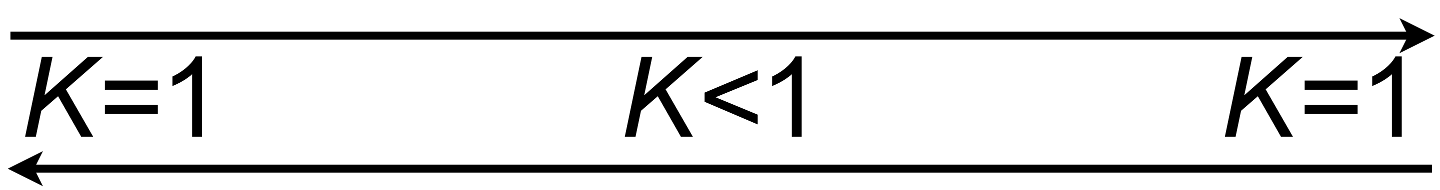
What about QSHE edge transport?
When is the factor of 2 interesting?

Question: if the measured two-terminal conductance is

$$G = \frac{2e^2}{h}$$

with no factor of K (Luttinger parameter),
then is the edge non-interacting?

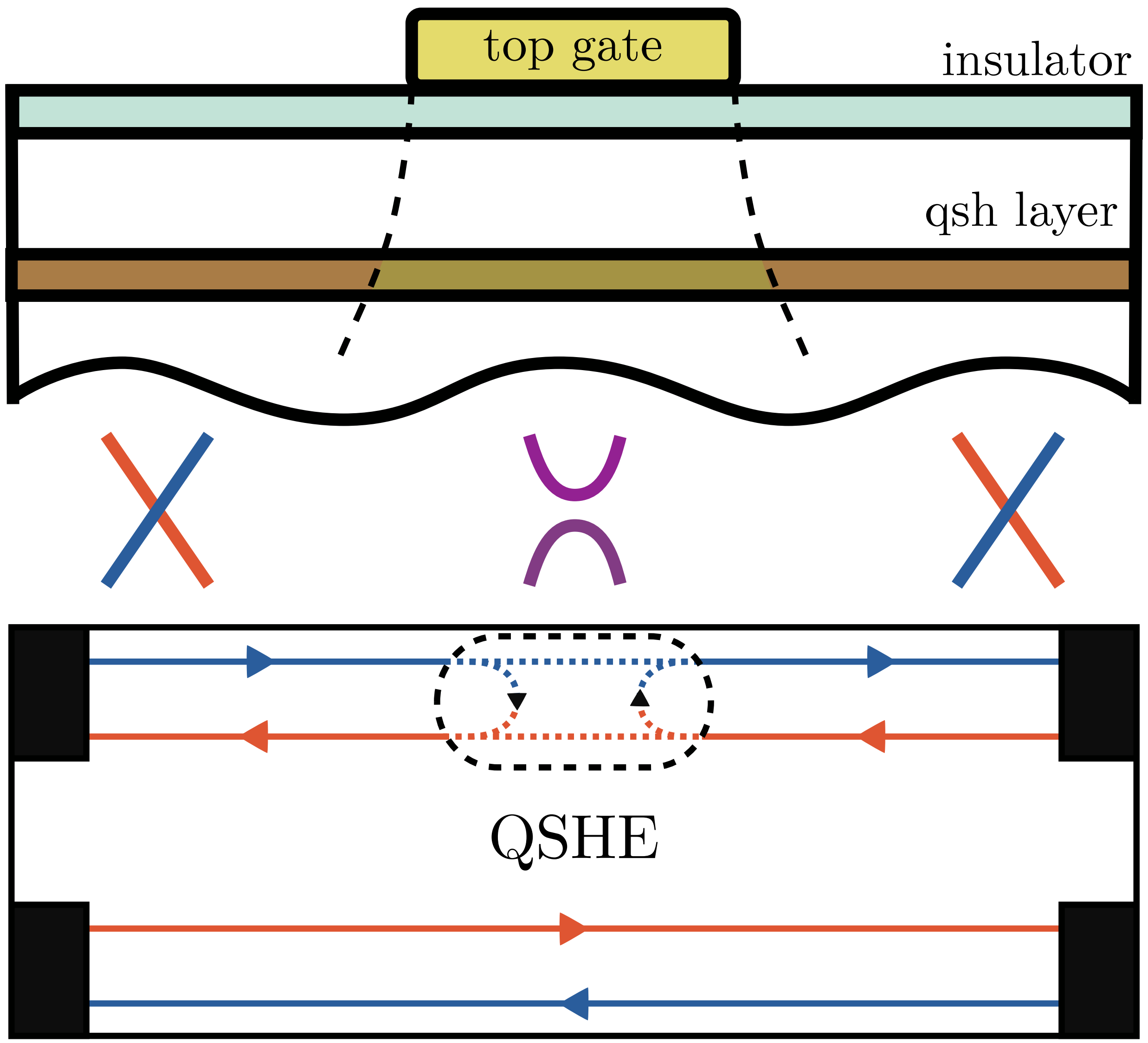
Not necessarily (Maslov-Stone, Safi-Schulz, 1995): Fermi-liquid contacts mean that the conductance is not the naive value Ke^2/h, but just e^2/h.



1. What are effects of interactions at the QSHE edge?
2. How can we verify time-reversal protection?

We want to include back-scattering at a single point, generated by a T-breaking impurity potential (not a Kondo impurity with a degree of freedom).

A remarkable non-equilibrium transport solution exists for one impurity in a *spinless* Luttinger liquid, with conductance interpolating from Ke^2/h to 0 as temperature is lowered, for repulsive interactions. (Fendley-Ludwig-Saleur, Fendley-Lesage-Saleur, 1995)



Note that this is not a point contact between edges but point backscattering at a single edge (as can happen in non-chiral FQHE states).

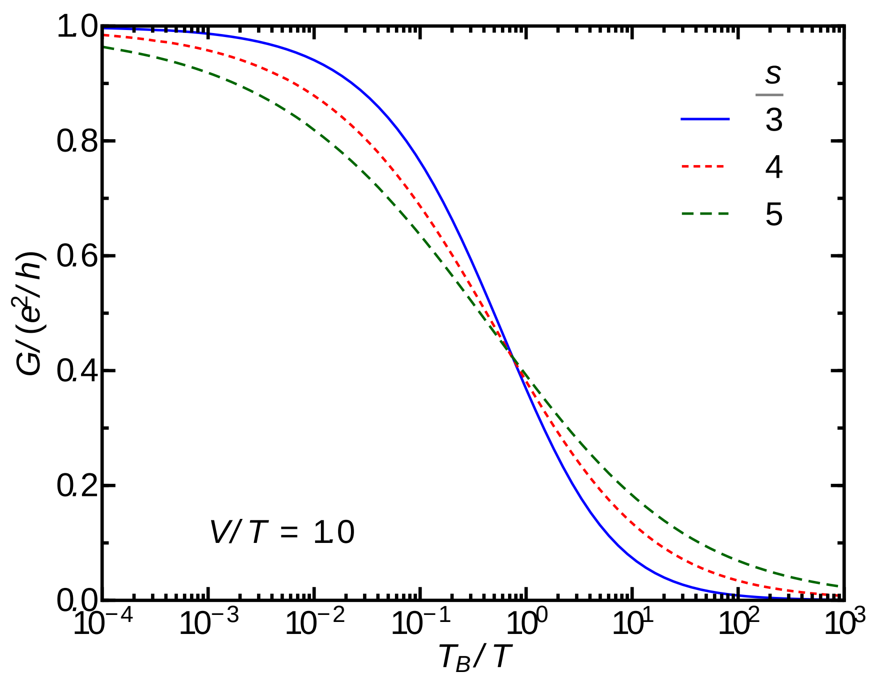
Impurity in a QSHE edge via integrability

So we need to do the following (Ilan, Cayssol, Bardarson, JEM, arxiv:1206.5211):

1. Do a mesoscopic calculation of the non-interacting edge to find the backscattering induced by a tunable T-breaking impurity.

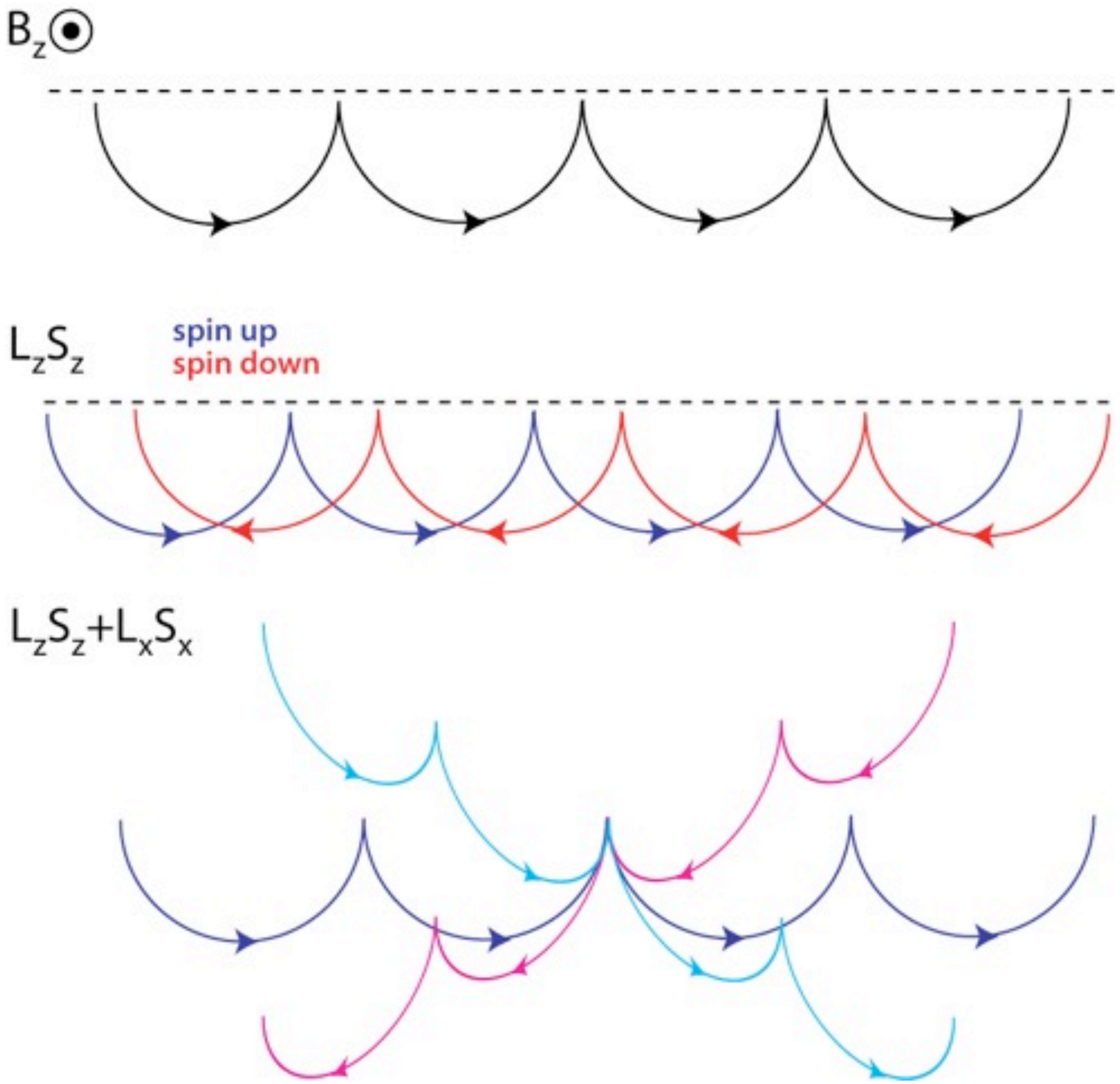
2. Use this as input to the Fendley-Lesage-Saleur solution for K = 1-1/m.

3. Solve self-consistently the combined set of TBA equations and the "contact correction" (Egger-Grabert, 1998) to obtain the current I(T,V).



Why do all this work? We believe that this experiment would prove "Z2-ness", measure *K* at the edge, and provide the first test of the contact correction and the FLS solution for non-quantized *K*.

Semiclassical picture of some *one-particle* phases



1D edge of Quantum Hall Effect

1D edge of “Quantum Spin Hall Effect” (discovered 2007)

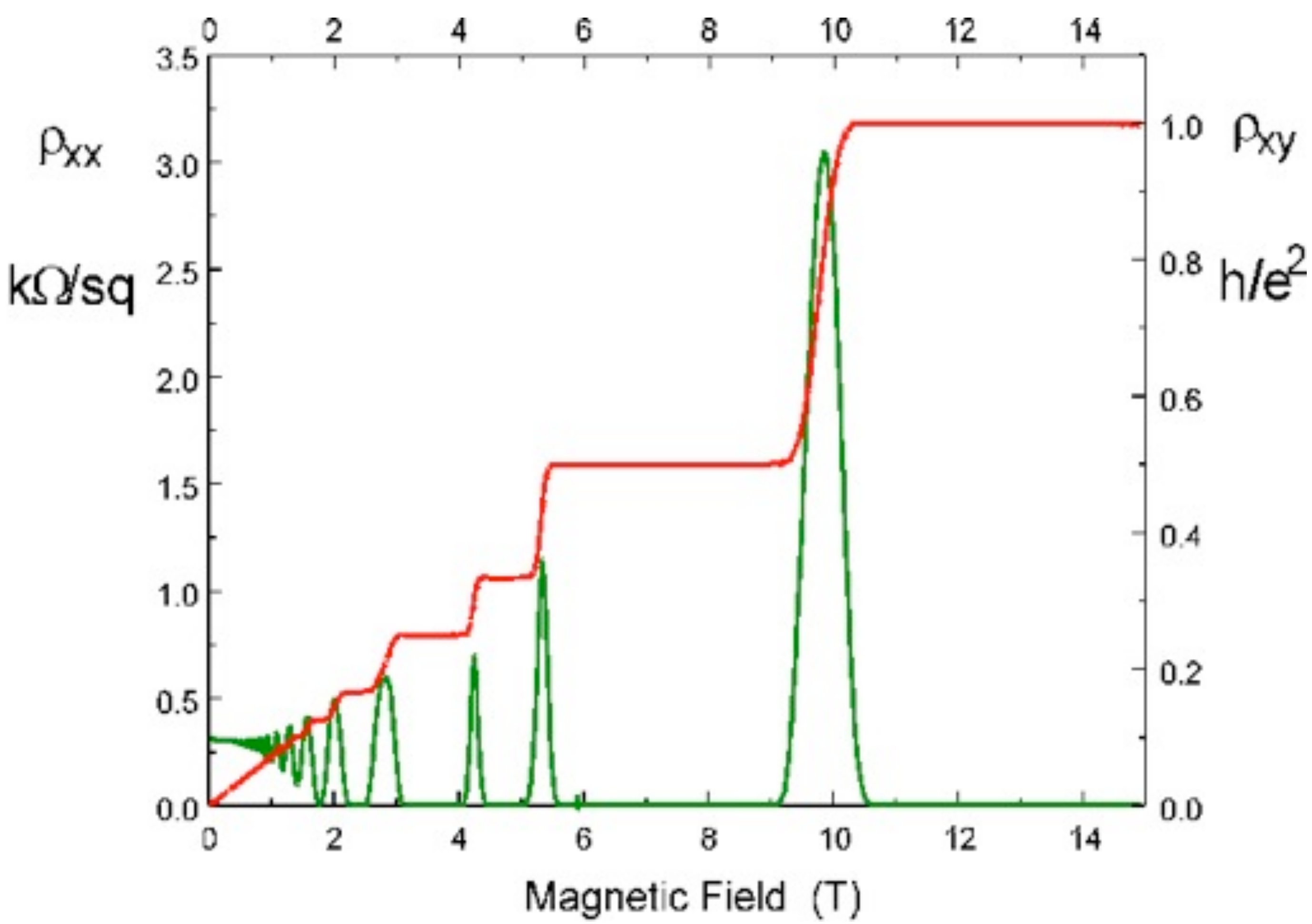
2D surface of 3D Topological Insulator (discovered 2008)

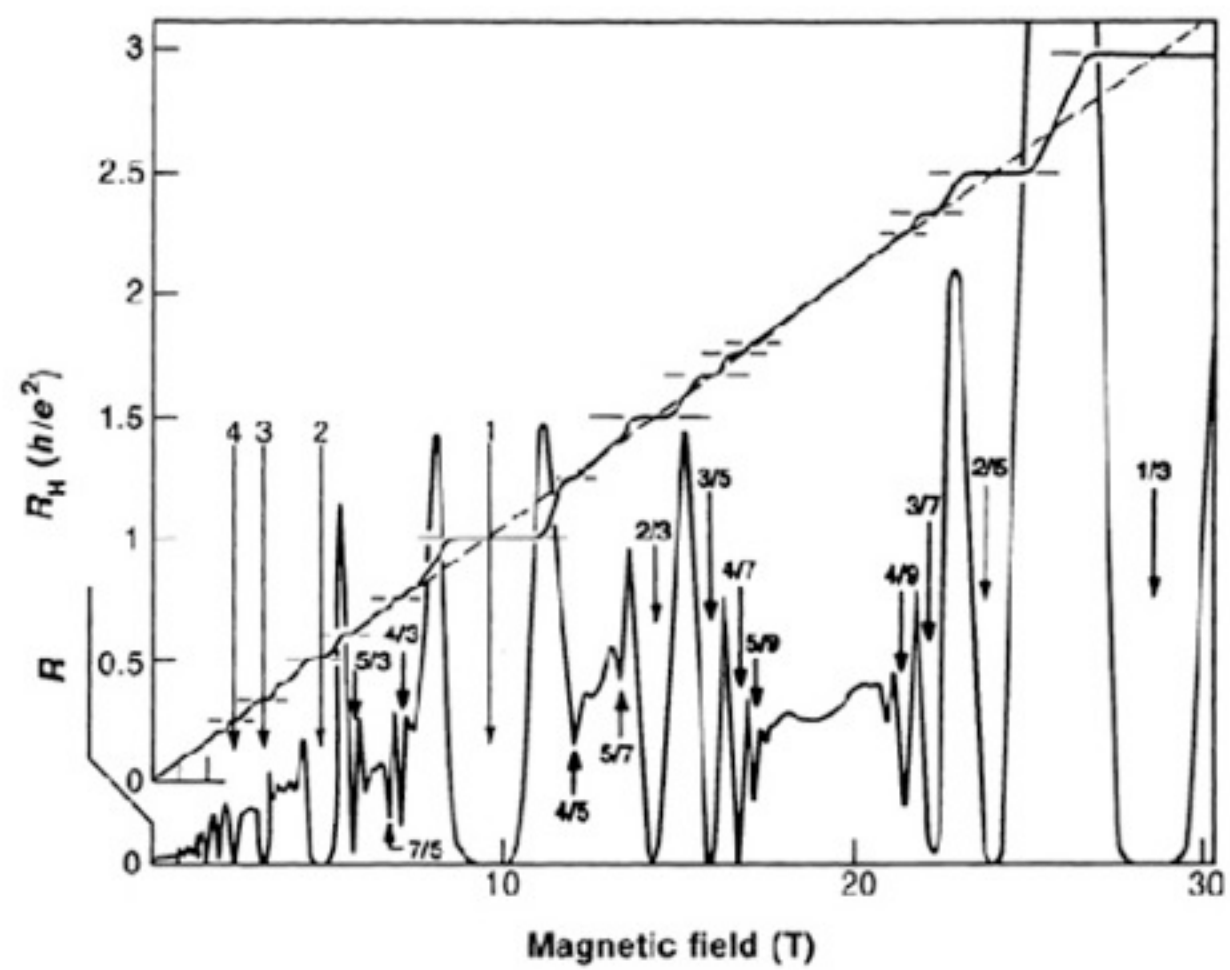
3D topological insulators have a special metallic 2DEG at any surface

What can happen as a result of strong Coulomb interactions?

The fractional quantum Hall effect and "anyons"

Experiment: in good samples, there are quantum Hall plateaus at "fractional" values that cannot be understood in a non-interacting model.





Early samples: few plateaus

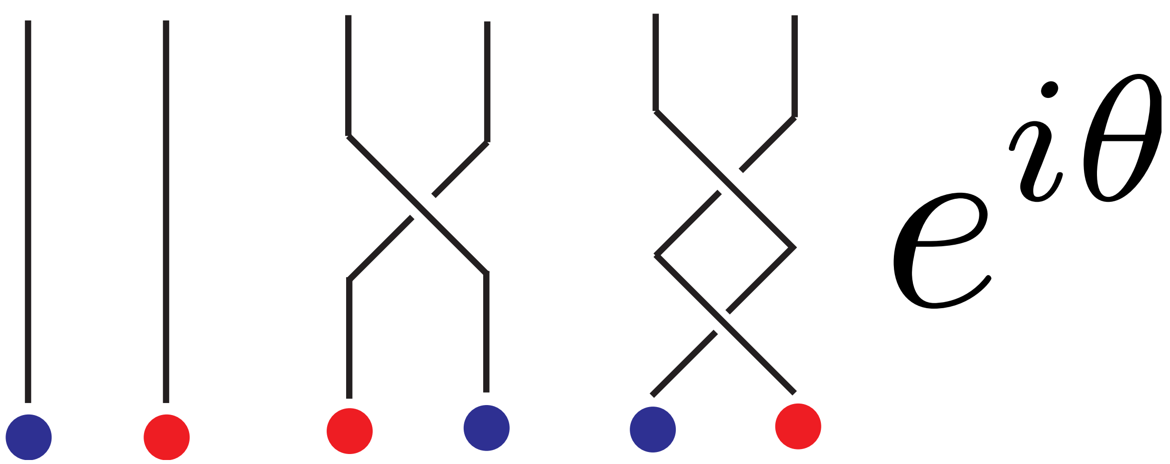
Modern samples: lots!

The fractional quantum Hall effect and “anyons”

Why is there so much effort going into making these complicated fractional states?

Theory says they have new kinds of quasiparticles with fractional statistics
“anyons” = neither bosons nor fermions

Key idea: proper statistics in 2D is about “braiding”, not just permutations->very complicated mathematical structure.



There are even “non-Abelian” states, with a ground-state degeneracy: representation of braiding is a matrix, not just a scalar, and can implement quantum gates.

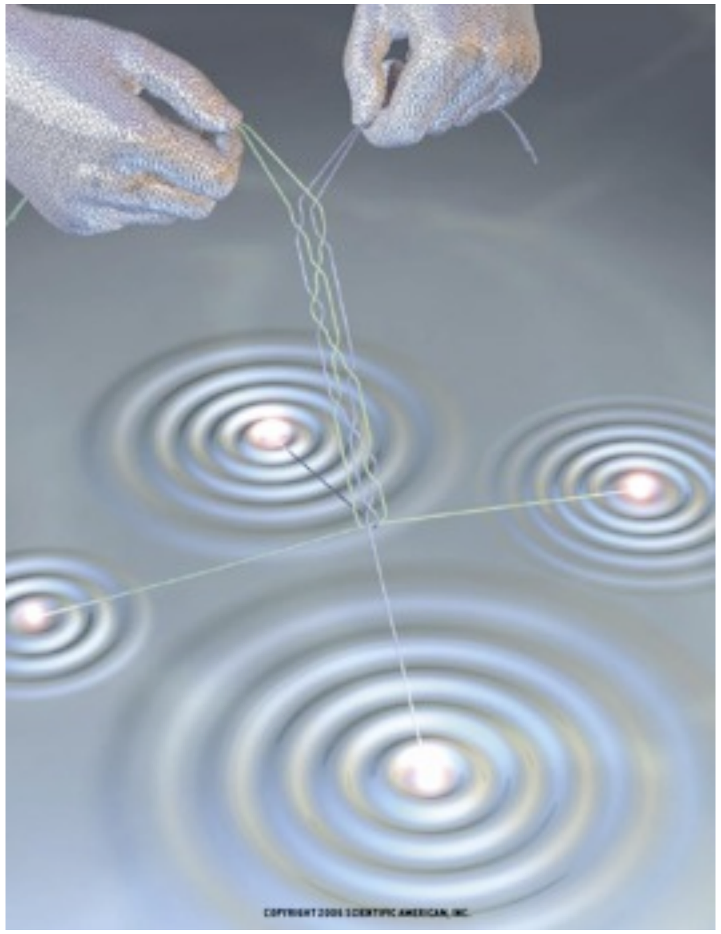
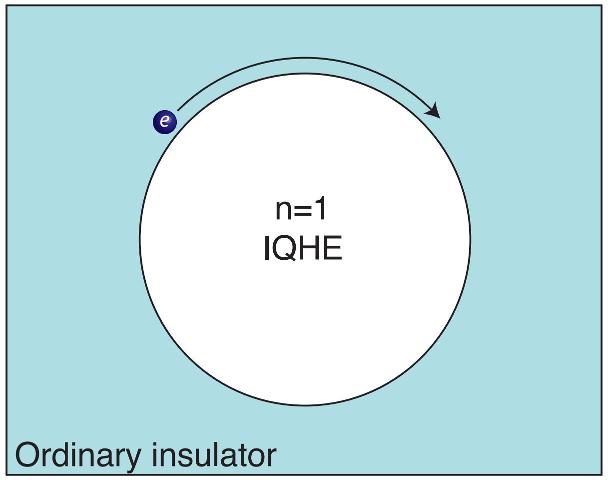


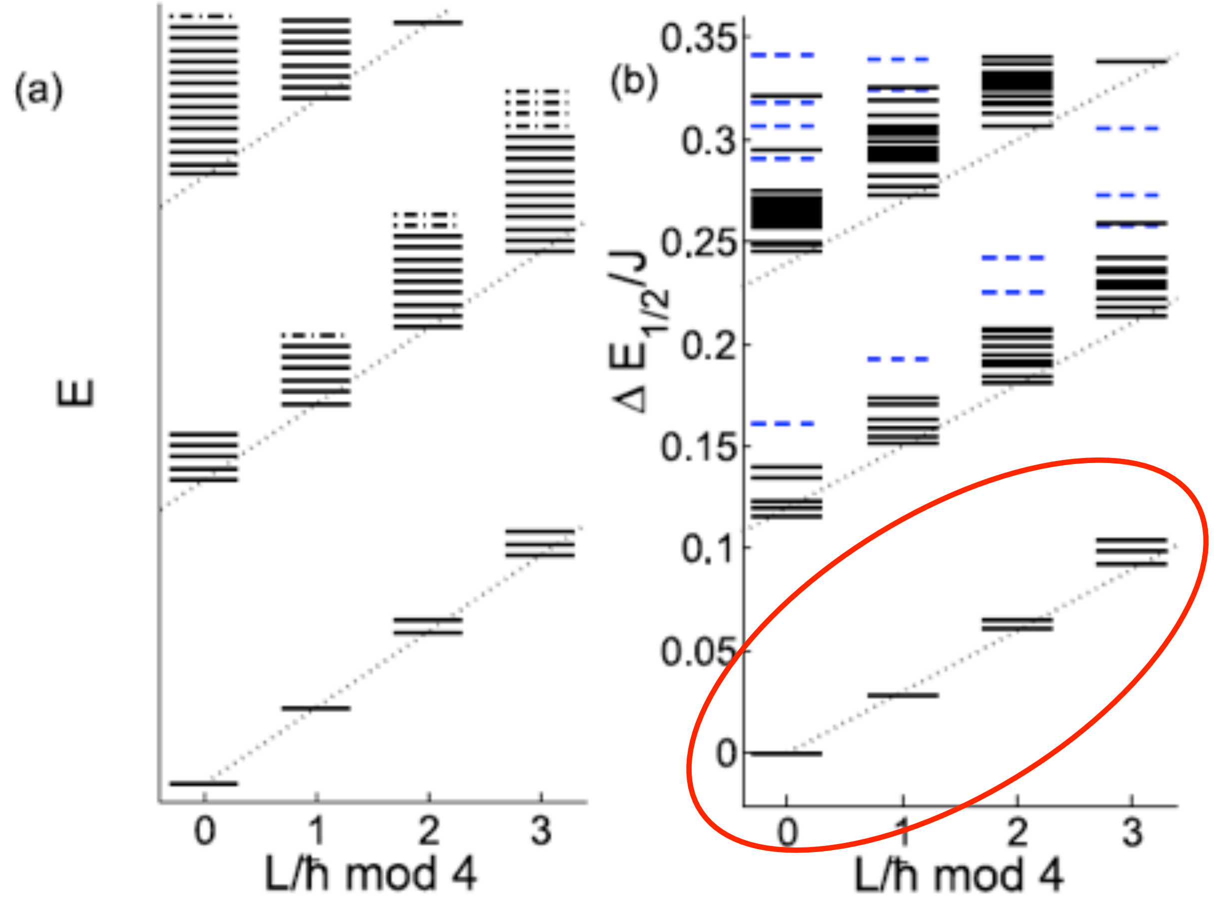
Figure from “Quantum Computing with Knots”, Sci. Am.

Topological edges via exact diagonalization

Ex: the square lattice with constant flux per plaquette.
Can do band projection if
(maximum interaction energy) < (interband splitting)
and then have essentially the standard FQHE problem.



Put on a trap and look for edge states.
(Laughlin 1/2 state of bosons)



Lesson:

Small systems are enough to see *incompressibility*, but not *fractionalization* (at least in the neutral sector).

Incompressibility is already a surprise for bosons--

For fermions, edge spectrum looks exactly the same for Laughlin 1/3 state as for IQHE n=1 state.

Point: one can see the edge state, even for a small number of particles, but the spectrum doesn't really tell us anything about fractional charge or statistics.

Need something better: can we approach thermodynamic limit for a gapless system?

Summary of FQHE experiments

1. There is indeed a gapless edge state.
2. It is certainly not an ordinary metal ("Fermi liquid").

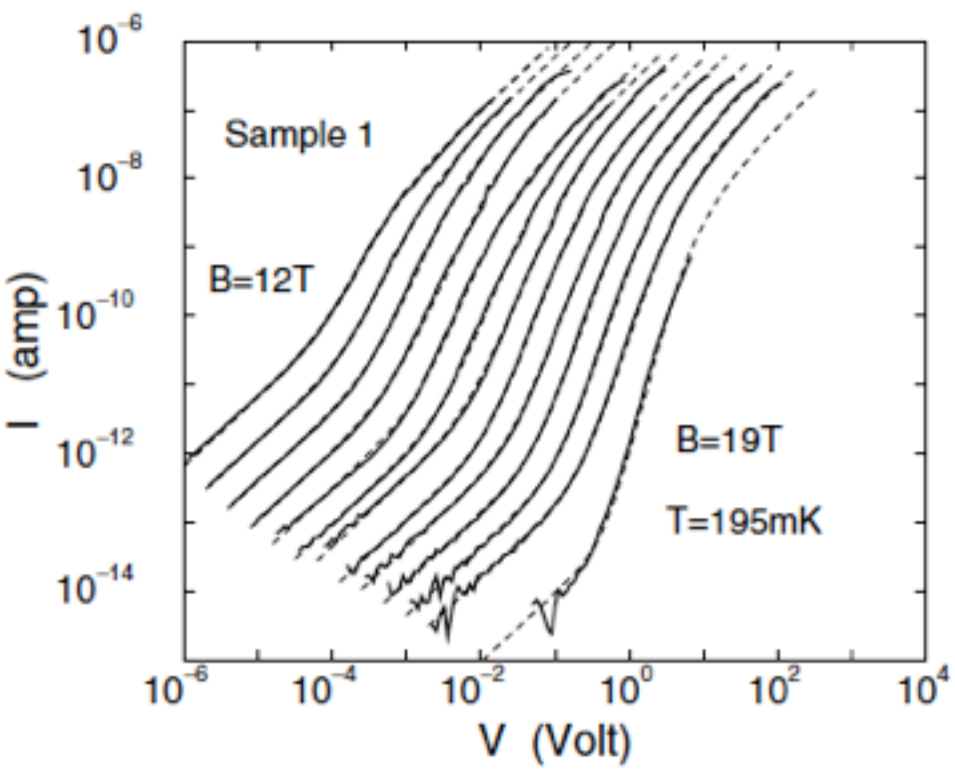


FIG. 2. Log-log plot of the I-V characteristics (solid lines) for electron tunneling from the FQH edge into the bulk doped $n+$ GaAs in sample 1 at various magnetic fields from 12 to 19 T in steps of 0.5, 18, and 18.5 T excluded. Corresponding filling factors vary from 2.69 to 4.26. Dashed lines represent best fits to Eq. (1). Successive curves are shifted by 0.3 units (a factor of 2) in the x direction for clarity.

3. However, it isn't very close to what standard theory predicts either.

Model: field theory of QHE

How can we describe the topological order in the quantum Hall effect, in the way that Landau-Ginzburg theory describes the order in a superconductor?

Standard answer: Chern-Simons Landau-Ginzburg theory

$$L_{CS} = -\frac{k}{4\pi}\varepsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda + j^\mu a_\mu, \qquad j^\mu = \frac{1}{2\pi}\varepsilon^{\mu\nu\lambda}\partial_\nu A_\lambda$$

There is an "internal gauge field" *a* that couples to electromagnetic *A*.

Integrating out the internal gauge field *a* gives a Chern-Simons term for *A*, which just describes a quantum Hall effect:

$$L_{QHE} = -\frac{1}{4k\pi}\varepsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$

There is a difference in principle between the topological field theory and the topological term generated for electromagnetism; they are both Chern-Simons terms.

Topological field theory of QHE

What good is the Chern-Simons theory? (Wen)

$$L_{CS} = -\frac{k}{4\pi}\varepsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + j^\mu a_\mu, \qquad j^\mu = \frac{1}{2\pi}\varepsilon^{\mu\nu\lambda}\partial_\nu A_\lambda$$

The bulk Chern-Simons term is not gauge-invariant on a manifold with boundary.

It predicts that a quantum Hall droplet must have a chiral boson theory at the edge:

$$S = \frac{k}{4\pi}\int \partial_x\phi(\partial_t\phi - v\partial_x\phi)\,dx\,dt$$

For fractional quantum Hall states, the chiral boson is a "Luttinger liquid" with strongly non-Ohmic tunneling behavior.

Experimentally this is seen qualitatively--perhaps not quantitatively.

Problems in FQHE theory vs. experiment

(1990-present)

1. Edge tunneling I-V exponent (Chang et al.)

Fitting required even to see plateaus clearly, and the exponents in some samples are displaced relative to theory; other samples seem roughly to agree with theory.

2. Fractional charge measured via shot noise at 2/3 (Heiblum et al.)

3. Conductance through constrictions at 2/3

4. "Upstream" (non-chiral) heat flow at 1/3 ?!

How can we study theoretically a gapless edge in the thermodynamic limit to understand what's going on?

Studying quantum correlations with classical algorithms: applied entanglement entropy

Basic (hazy) concept: "Entanglement entropy determines how much classical information is required to describe a quantum state."

Example:
how many classical real numbers are required to describe a *product* (not entangled) state of N spins?

simple product $$|\psi\rangle = A_{s_1} A_{s_2} A_{s_3} A_{s_4} |s_1 s_2 s_3 s_4\rangle$$

Answer: ~ N (versus exponentially many for a general state)

How do we efficiently manipulate/represent moderately entangled states?

Applied entanglement entropy

The remarkable success of the density-matrix renormalization group algorithm in one dimension (White, 1992; Ostlund and Rommer, 1995) can be understood as follows:

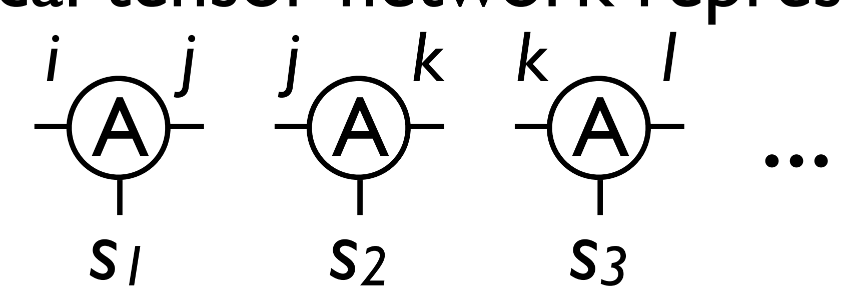
DMRG constructs "matrix product states" that retain local entanglement but throw away long-ranged entanglement.

Example states for four spins:

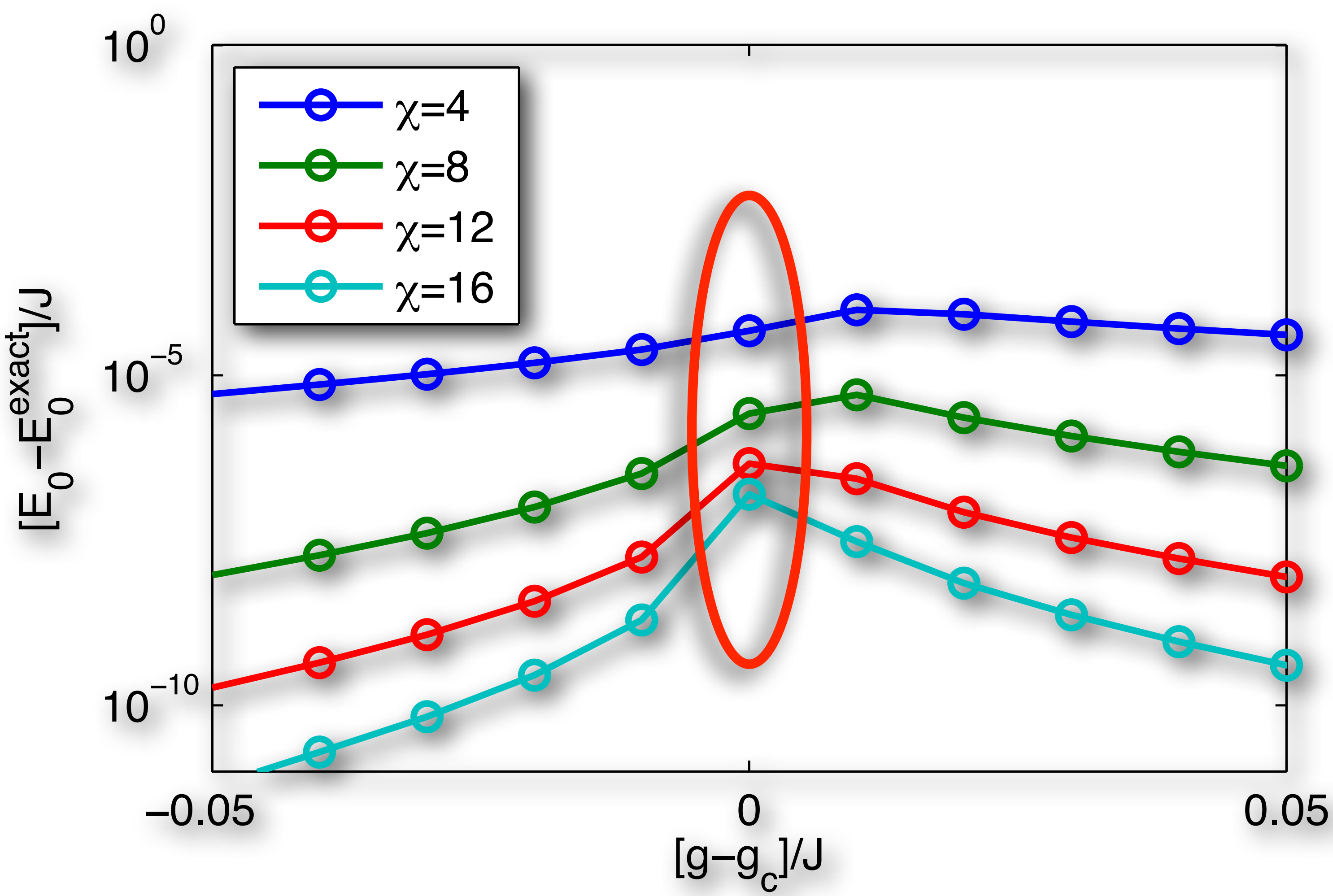
simple product $$|\psi\rangle = A_{s_1} A_{s_2} A_{s_3} A_{s_4} |s_1 s_2 s_3 s_4\rangle$$

matrix product $$|\psi\rangle = A^{ij}_{s_1} A^{jk}_{s_2} A^{kl}_{s_3} A^{li}_{s_4} |s_1 s_2 s_3 s_4\rangle$$

Graphical tensor network representation:



- find the ground state of a system by using **imaginary time evolution** (almost unitary for small time steps)
- parallel updates for **infinite/translational invariant** systems: **iTEBD** [Vidal '07]
- **example**, transverse Ising model: $H = \sum_i \left(J\sigma_i^z \sigma_{i+1}^z + g\sigma_i^x \right)$

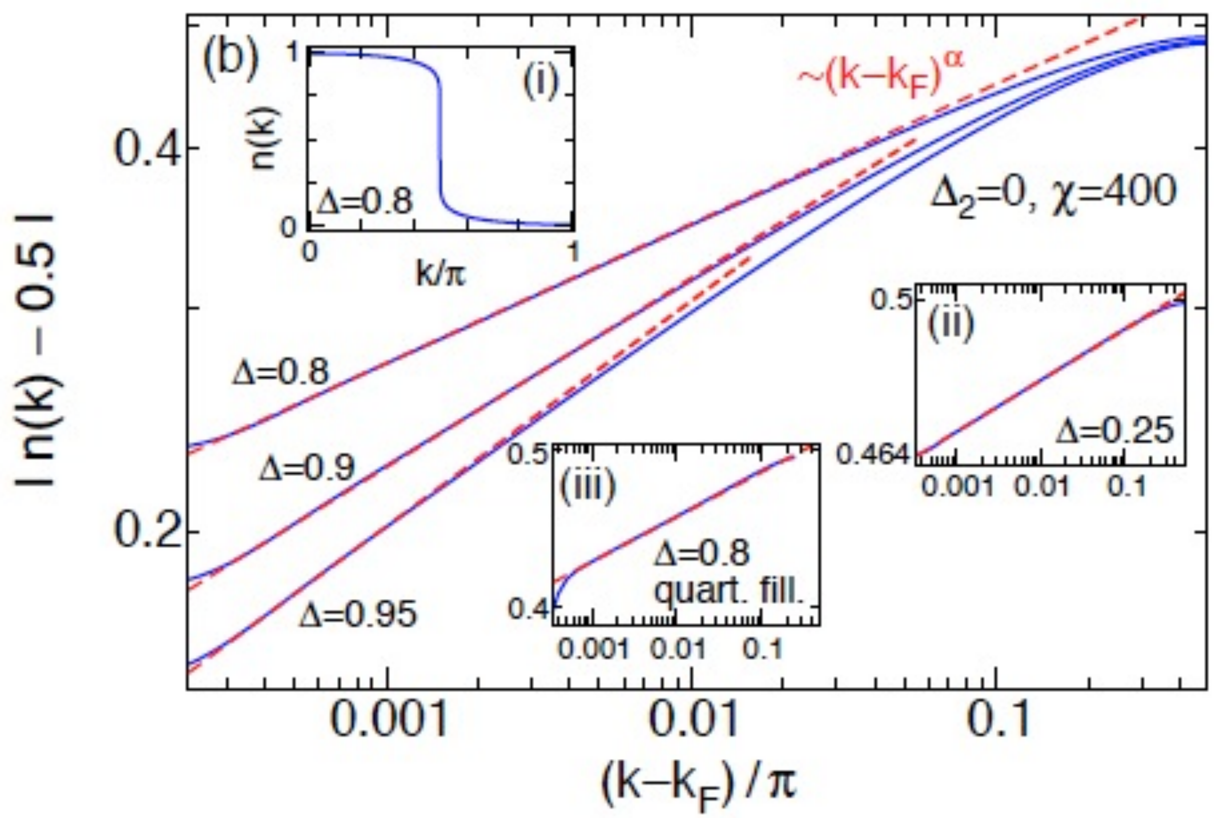


➡convergence of wave function is worst at the **critical point**

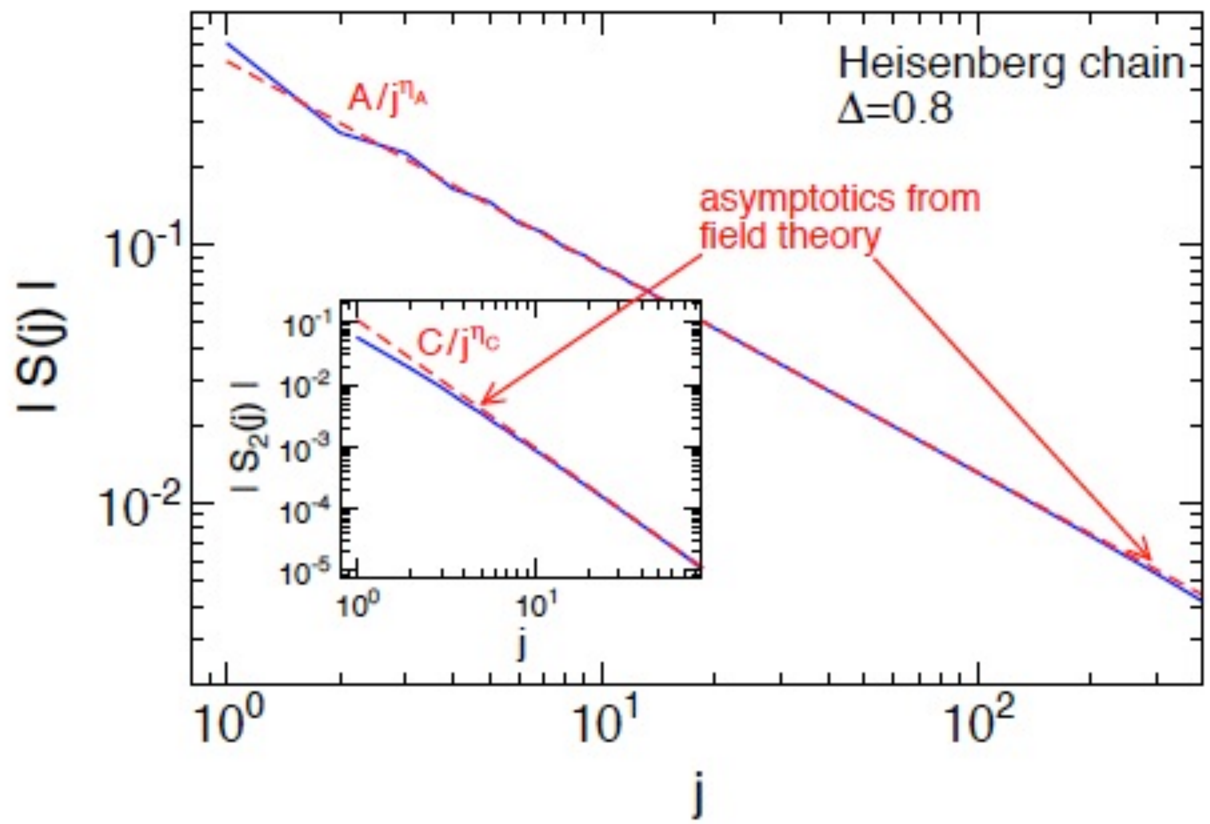
➡**conformal invariance**

Good news: can make "finite-entanglement" theory of convergence at critical points; they are hardly inaccessible.

Tests of Luttinger liquid behavior in the XXZ model



Momentum distribution n(k)



Check of leading staggered and uniform correlators against Lukyanov and Terras

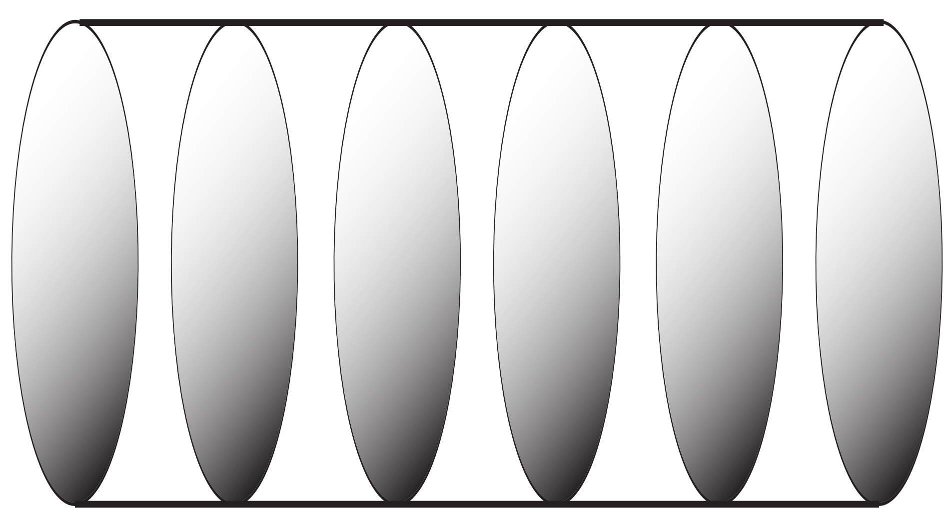
(C. Karrasch and JEM, PRB)

Later: try to solve open problems of dynamical properties at finite temperature.

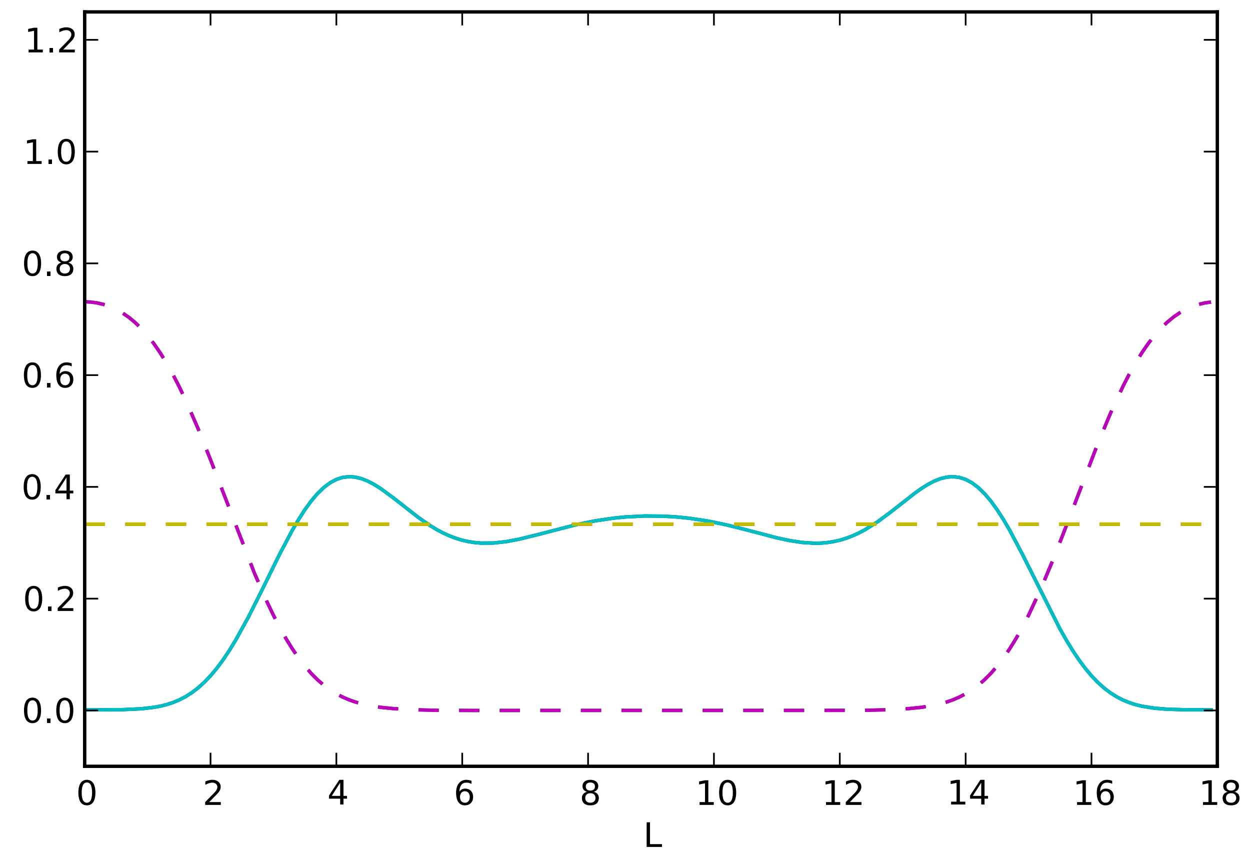
DMRG for FQHE edges: 2D -> 1D

Put LL on a cylinder: get an unusual spinless fermion model with interaction range determined by the cylinder size.

Add a confining potential around the cylinder to create a strip, with *edges: now a 1D CFT.*



electron density nonuniform



1/3 state: density profile

DMRG for FQHE edges

Wen's theory predicts that the edge electrons should have strongly non-Fermi liquid correlations.

With ED, one gets a discrete spectrum; hard to confirm expt.

Now the edges run along the infinite cylinder: continuous spectrum

1/3 edge:

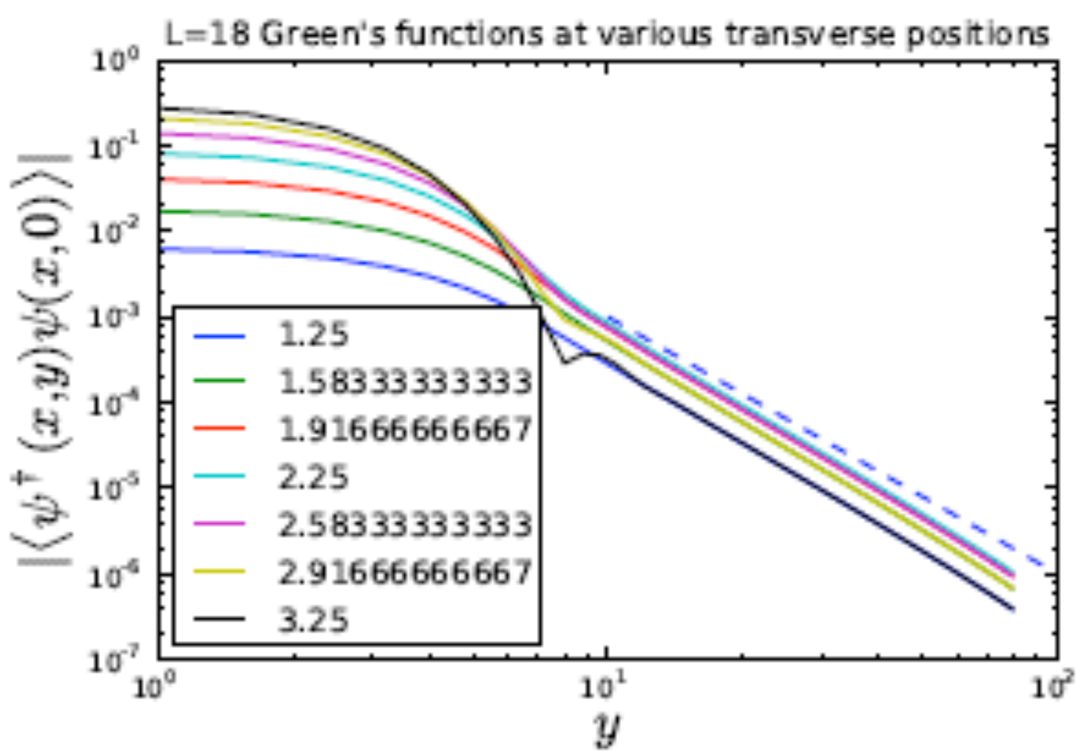
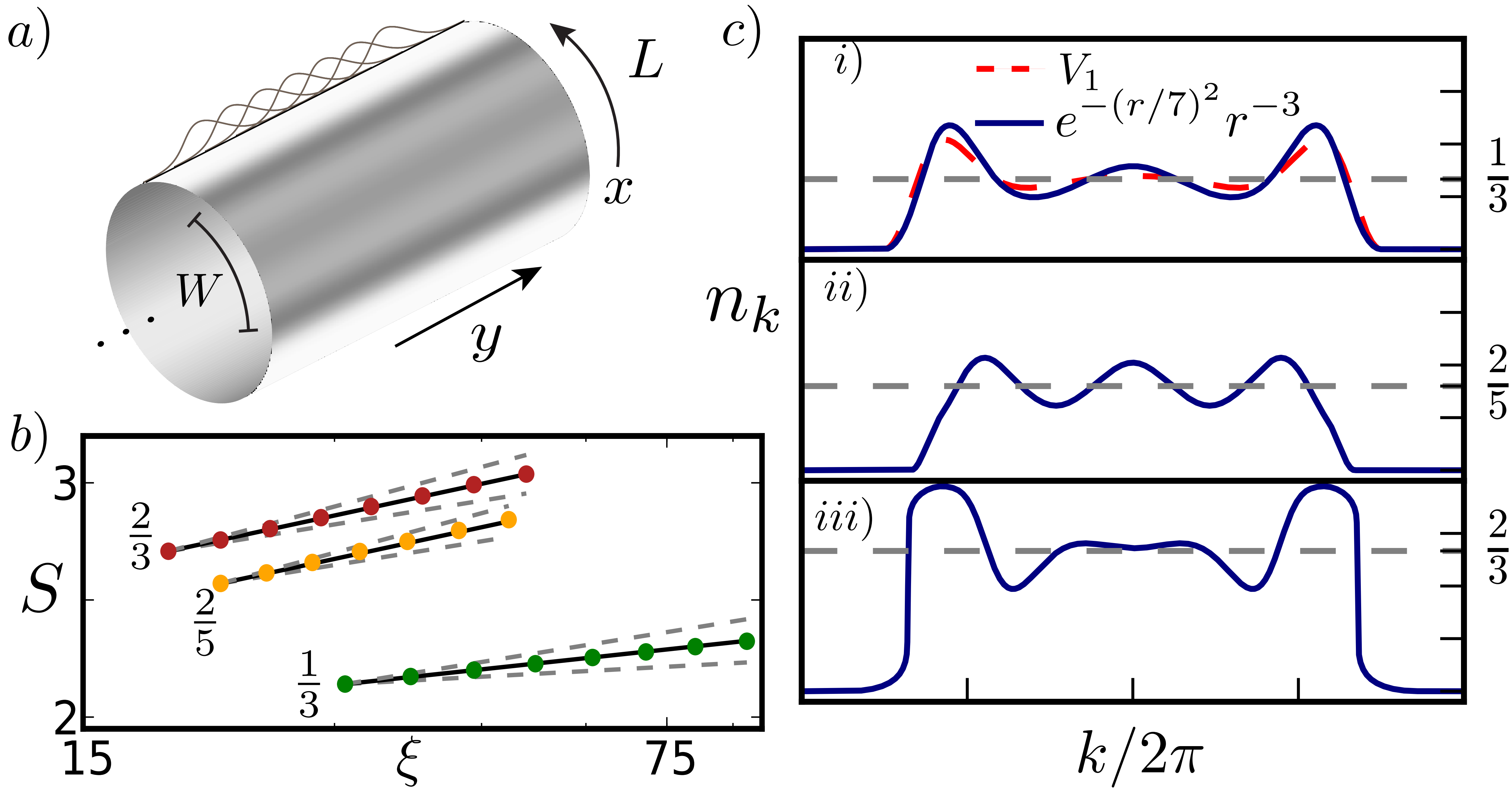


FIGURE 3. $G(y;x)$ at $L = 18$. $\eta = 3$ given by dashed line.

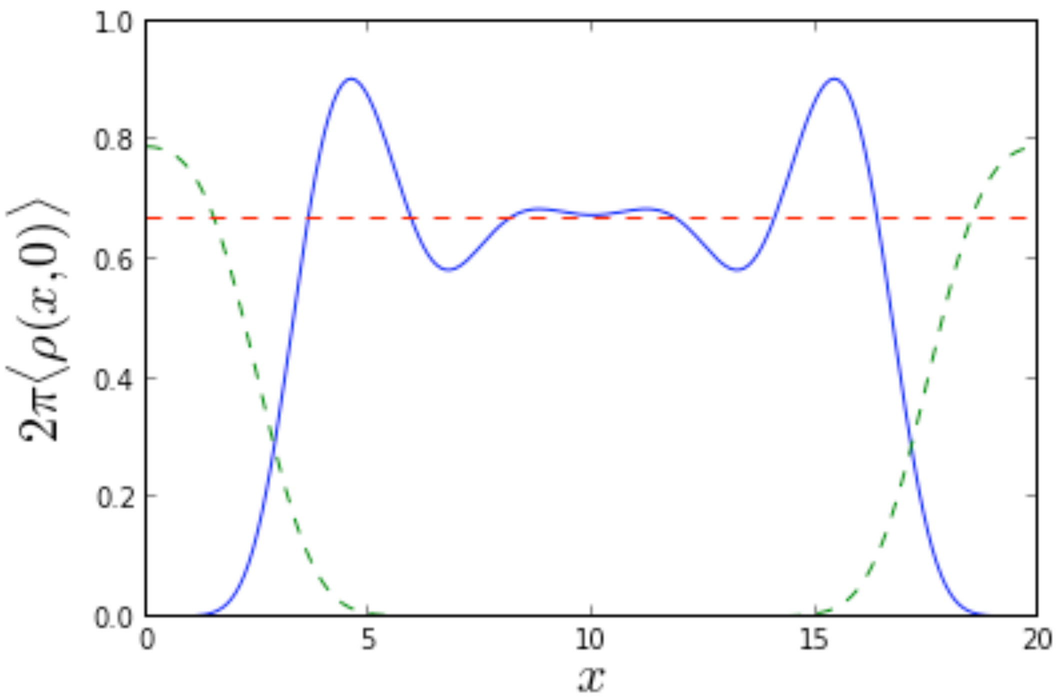


entanglement scaling to count number of edge modes

DMRG for FQHE edges

Can observe more complicated multicondensate behavior and *nonuniversality* of edge exponent: 2/3 spin-polarized state

2/3 density:
1/3 gas of holes
in filled Landau level



State of the art with DMRG-type methods

(Some) ground states in 2D:
Quantum Hall states
Spin liquids
(not yet) doped Hubbard model

(Some) dynamical problems in 1D:
Quenches and sweeps
Many-body localization
Linear-response transport

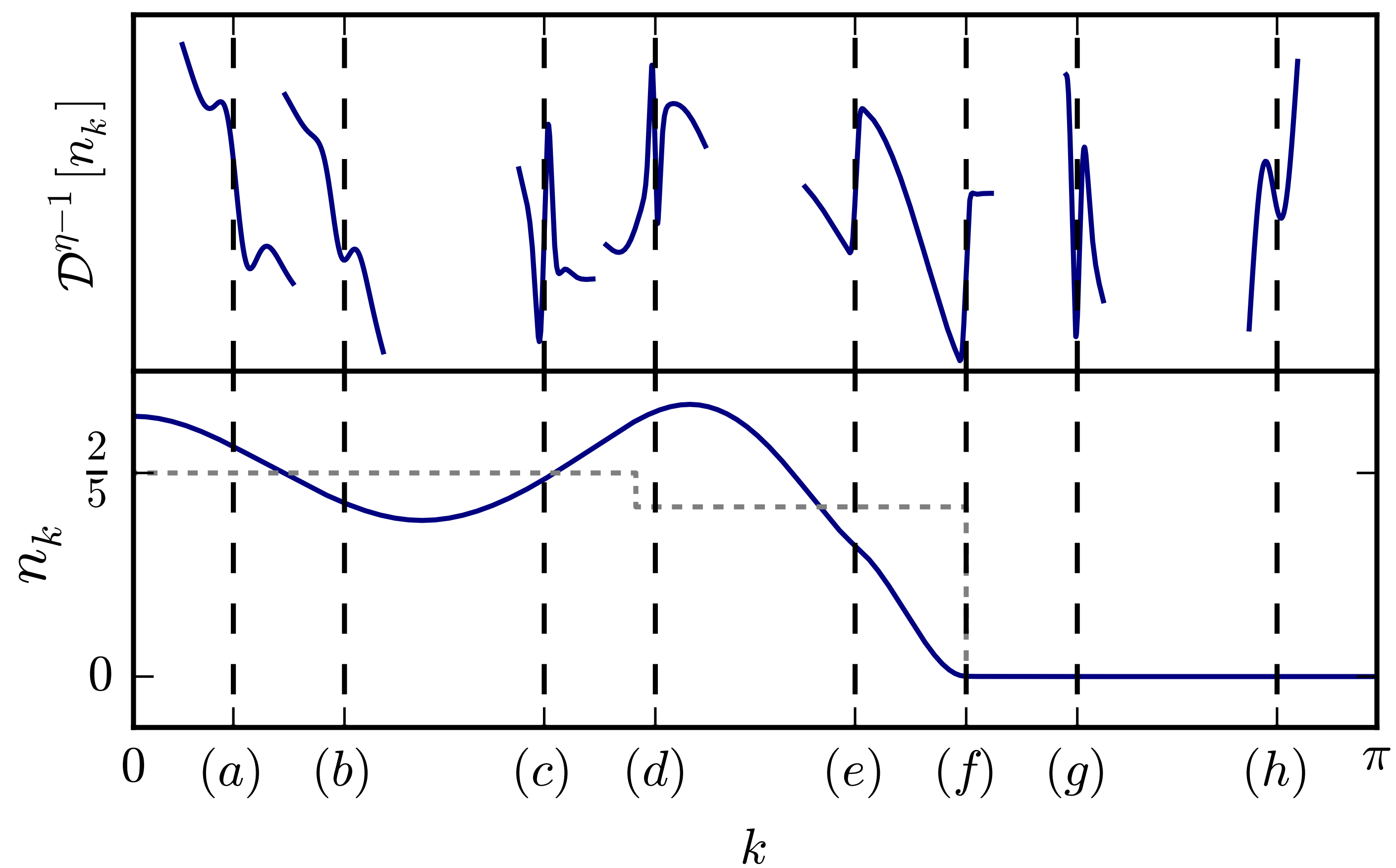
Can we say anything about non-equilibrium transport (steady states)?

Challenge: at least classically, there are few guiding principles for steady states; each is non-equilibrium in its own way.

Some work near quantum critical points (cf. Green, Sondhi et al.)
Let's start as simply as possible...

Luttinger's theorem and reconstruction

Generalized Luttinger's theorem: there are singularities (1D versions of Fermi surfaces) at certain momentum values. There is a sum rule relating their locations to the density.

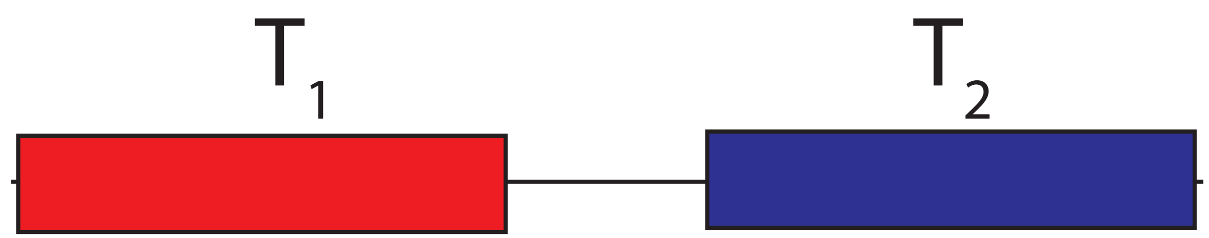


Central charge estimate lets us figure out how many propagating modes there are.

Current work: quantify how edge potential leads to extra modes suggested by experiments.

What about non-equilibrium transport?

1. Create two different temperatures in two disconnected, infinite 1D "leads".
2. Connect them by a finite region (e.g., one bond).
3. Evolve in time for as long as possible.



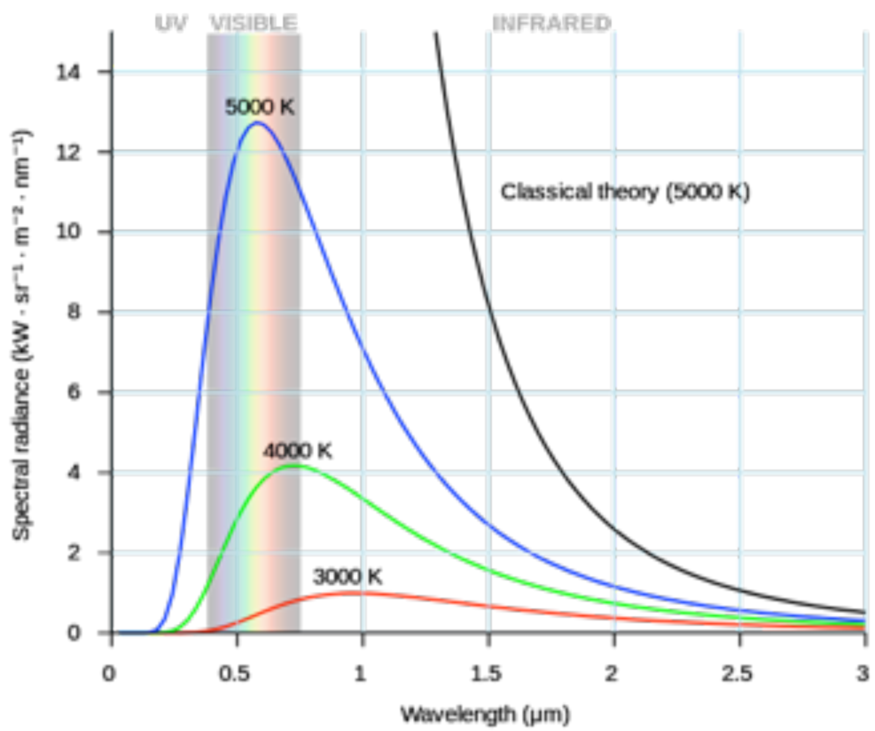
Is a steady-state heat current reached?

When is non-equilibrium (finite bias) thermal transport determined by linear-response thermal conductance?

We observe two different outcomes, depending on integrability of the leads and whether the connected system is homogeneous.

Non-equilibrium in the early days of quantum mechanics

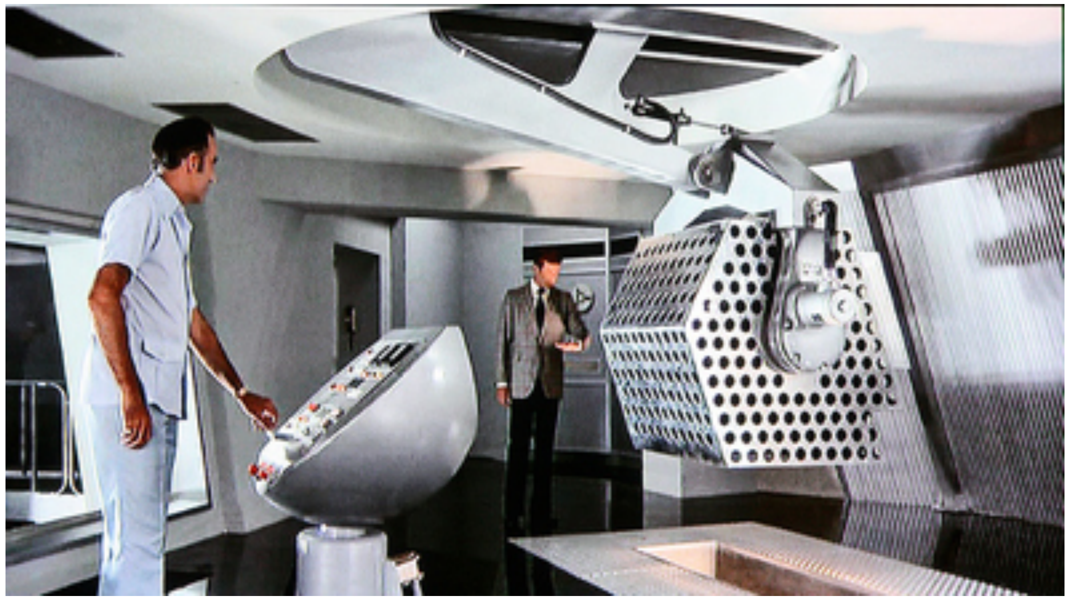
Stefan-Boltzmann law = integrated Planck distribution



$$P = \sigma T^4 \text{ in } d = 3$$

$$P \propto T^2 \text{ in } d = 1$$

Practical efficiency limit on concentrating solar plants: concentration point radiates



Linear and non-linear transport: summary

When the final *H is* a homogeneous XXZ model, (integrable; conserved energy current) there is a "generalized Stefan-Boltzmann law" to high accuracy, to be defined in a moment.

Main goal: analytical explanation and calculation

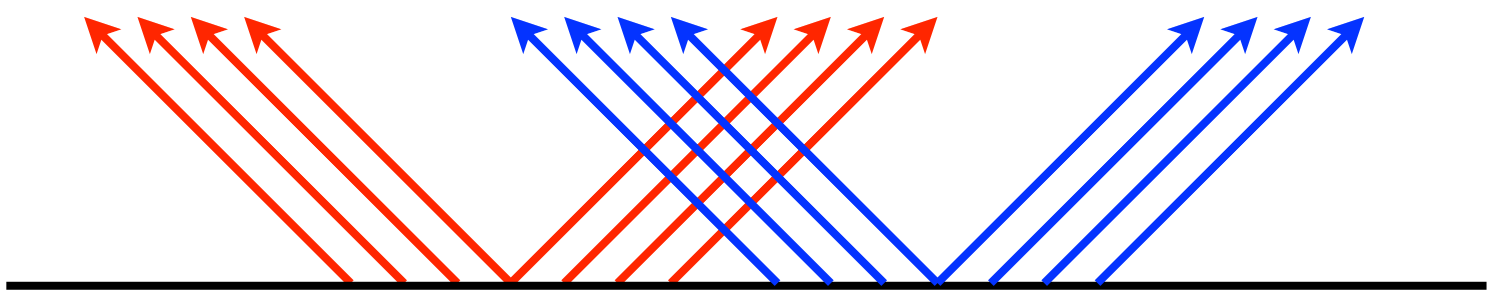
For final *H* homogeneous and non-integrable, we do not observe a steady state. We believe that the temperature gradient is decreasing and Fourier's law is setting in, but cannot access very long times.

For final H inhomogeneous, there can be a steady state if the leads are integrable, but there is not an *f*-function; *J* is a function of both temperatures separately.

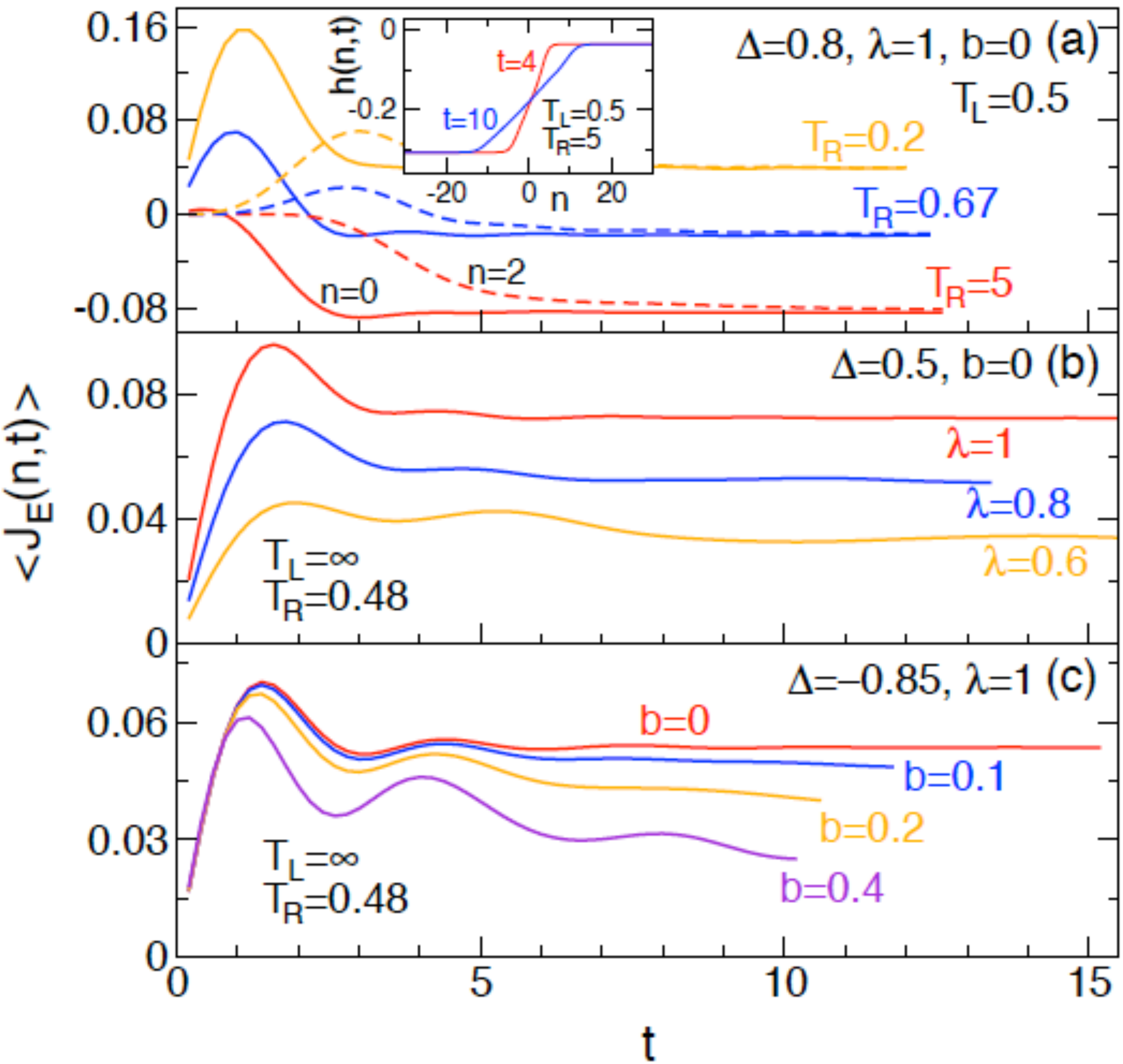
We can see the onset of the nontrivial power-laws in tunneling between Luttinger liquids as temperature is lowered.

Stefan-Boltzmann picture

Idea: the right lead is prepared at one temperature and the left lead at a different temperature.



In a ballistic system like a CFT, there is no local temperature at x=0 at later times; rather the right-movers are at a different temperature than the left-movers. The thermal current is the difference between total radiation from left and right.



Δ=0.8, λ=1, b=0 (a)
T_L=0.5
T_R=0.2
T_R=0.67
T_R=5
n=0
n=2
h(n,t)
t=4
t=10
T_L=0.5
T_R=5
n
Δ=0.5, b=0 (b)
λ=1
λ=0.8
λ=0.6
T_L=∞
T_R=0.48
Δ=−0.85, λ=1 (c)
b=0
b=0.1
b=0.2
b=0.4
T_L=∞
T_R=0.48
<J_E(n,t)>
t

Linear and non-linear response

When the finite system is homogeneous and integrable, with a conserved energy current, we find:

1. there is a steady state;
2. there is a Stefan-Boltzmann function *f* such that

$$\lim_{t\to\infty} \langle J_E(n,t)\rangle = f(T_L) - f(T_R)$$

In other words, linear response $G = \partial_T f$ is sufficient to determine non-linear response.

For a CFT (Sotiriadis & Cardy, Bernard & Doyon), this was known, and *f* goes as T^2 for small T, 1/T for large T. ("1D black-body")

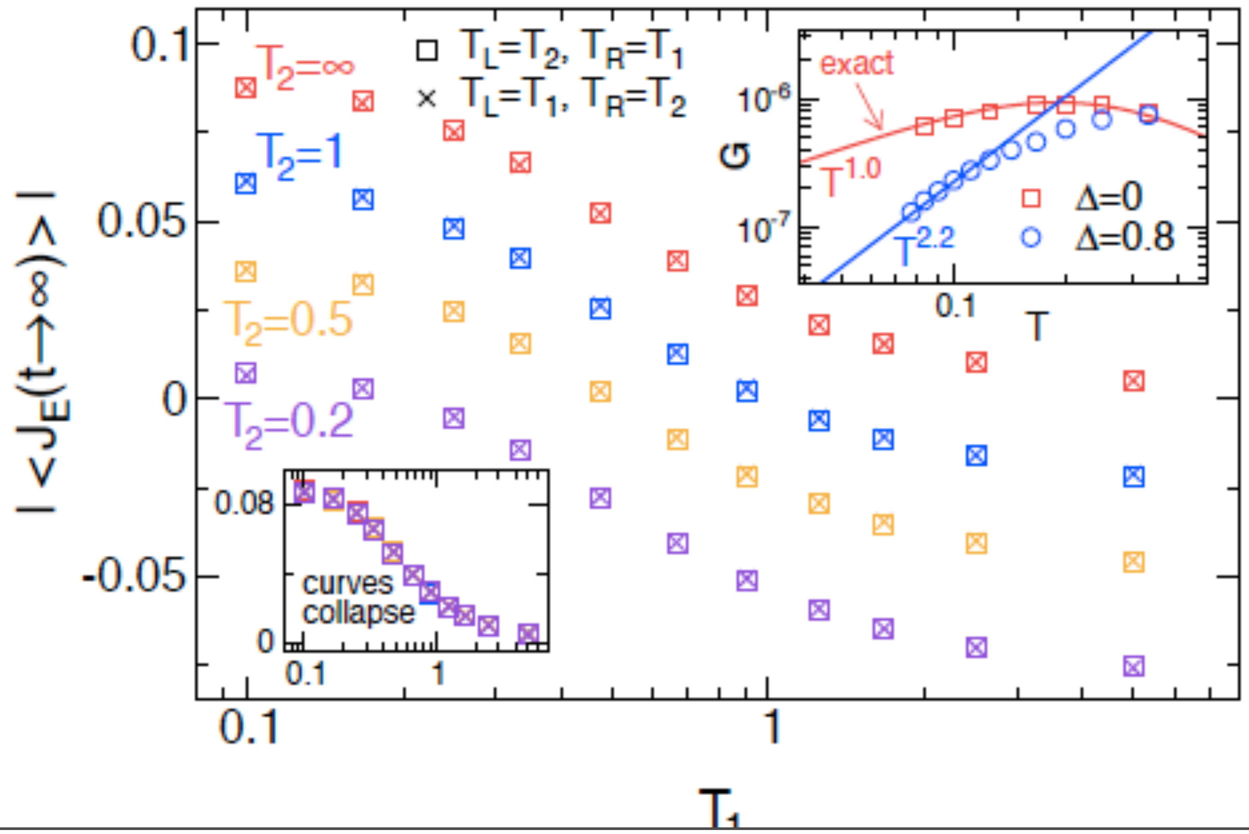
Linear and non-linear response

2. there is a Stefan-Boltzmann function *f* such that

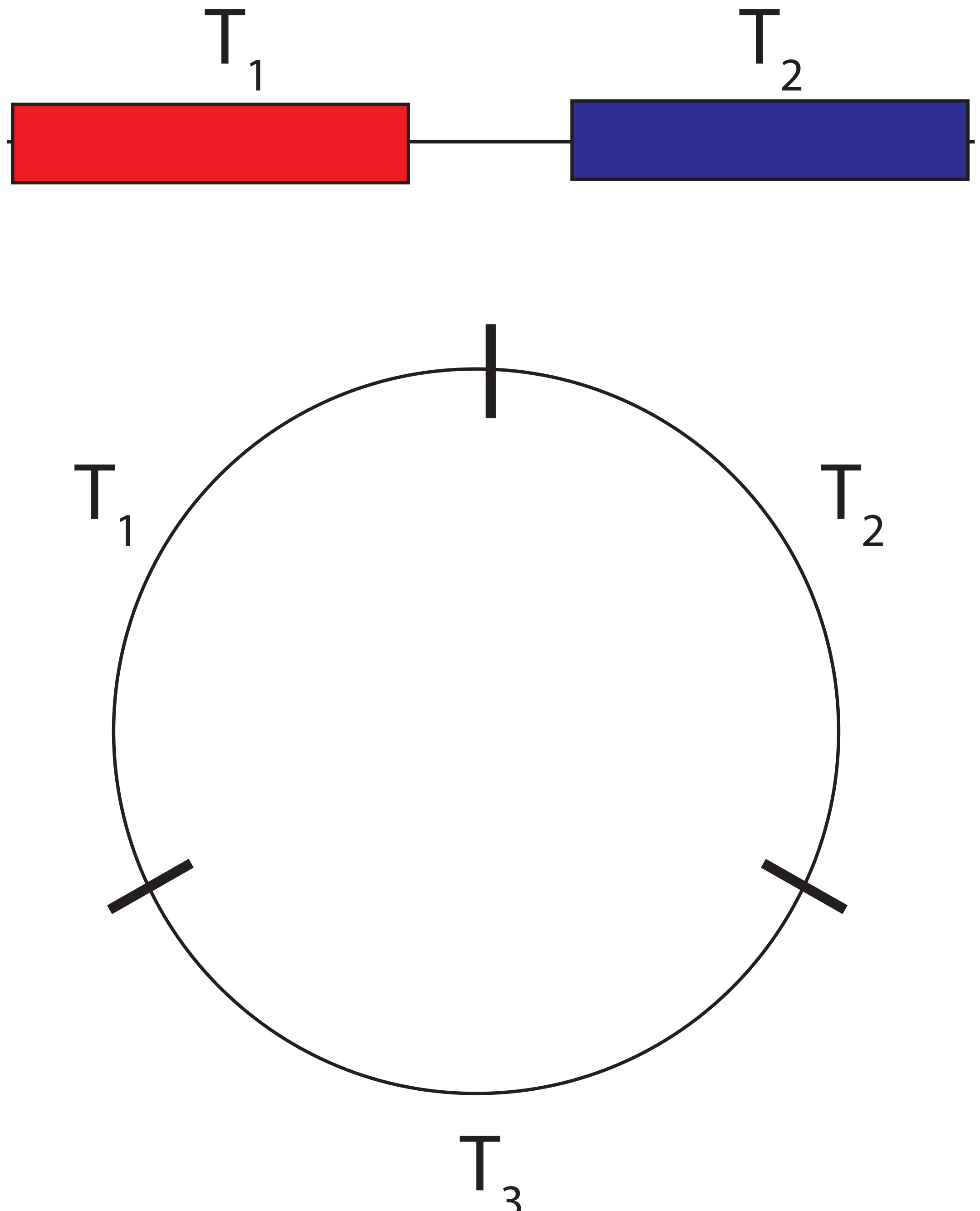
$$\lim_{t\to\infty} \langle J_E(n,t)\rangle = f(T_L) - f(T_R)$$

Makes testable predictions, e.g.,

$$J_E(T_1 \to T_3) = J_E(T_1 \to T_2) + J_E(T_2 \to T_3)$$



One quantity with a "cyclic rule"



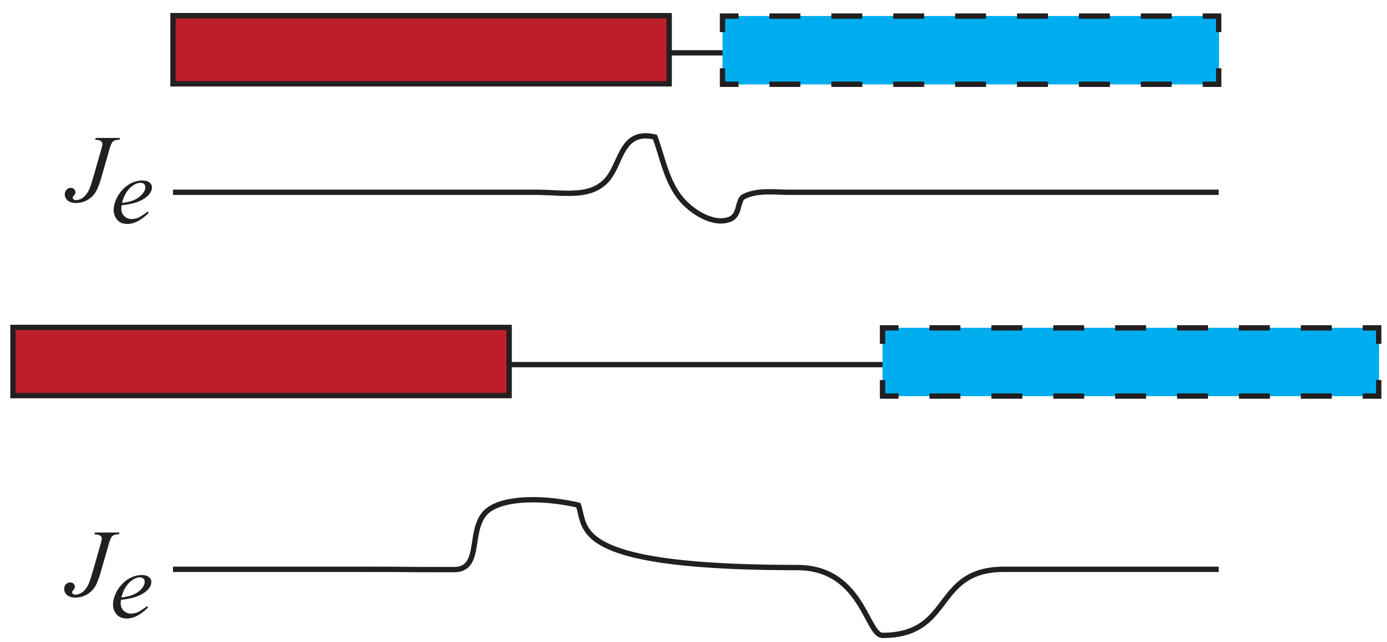
Global energy current conservation connects what happens at 3 interfaces:

any change at just 1 interface cannot affect spatially integrated current at that interface

Cyclic form of integrated currents:
(exact at all times for large reservoirs)

$$j_E^{\Sigma}(T_1 \to T_2, t) + j_E^{\Sigma}(T_2 \to T_3, t) + j_E^{\Sigma}(T_3 \to T_1, t) = 0$$

Adding a "spacer" region makes no difference in the time evolution of integrated energy current:



$$j_E^{\Sigma}(T_1 \to T_2, t) = j_E^{\Sigma}(T_1 \to 0, t) - j_E^{\Sigma}(T_2 \to 0, t)$$

There is an interesting new universal function.
But can we say anything about steady-state current (non-integrated)?

But can we say anything about steady-state current at a point (i.e., non-integrated)?

Getting from spatially integrated current to a point requires a length scale: natural guess is

$$f = \frac{\text{Total right-moving energy current as } t \to \infty}{\text{Length of reservoir}}$$

For free systems, even with a dispersion of velocities, differences of *f* indeed describe the steady-state.

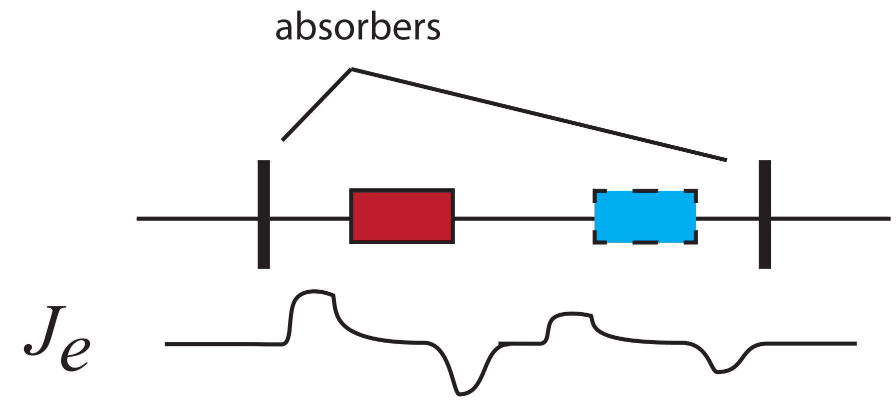
Might think interactions must violate this: effective velocity of one excitation is modified by presence of others, for example.

But energy current conservation is very powerful...

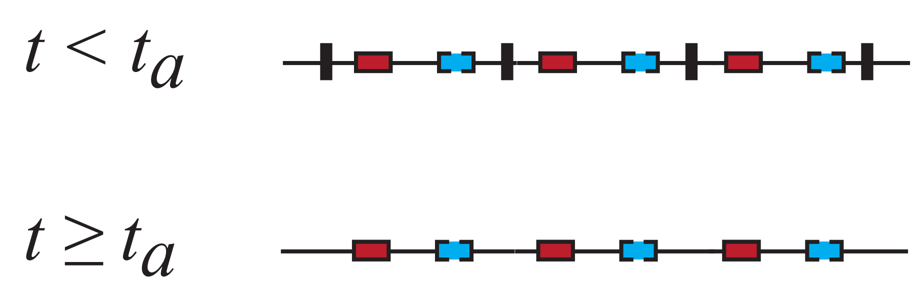
Claim: there is a translation-invariant steady state whose energy current is given precisely by differences of *f, even in the presence of interactions.*

An explicit protocol to make a final state with this current:

1. absorb hot left-movers and cold right-movers



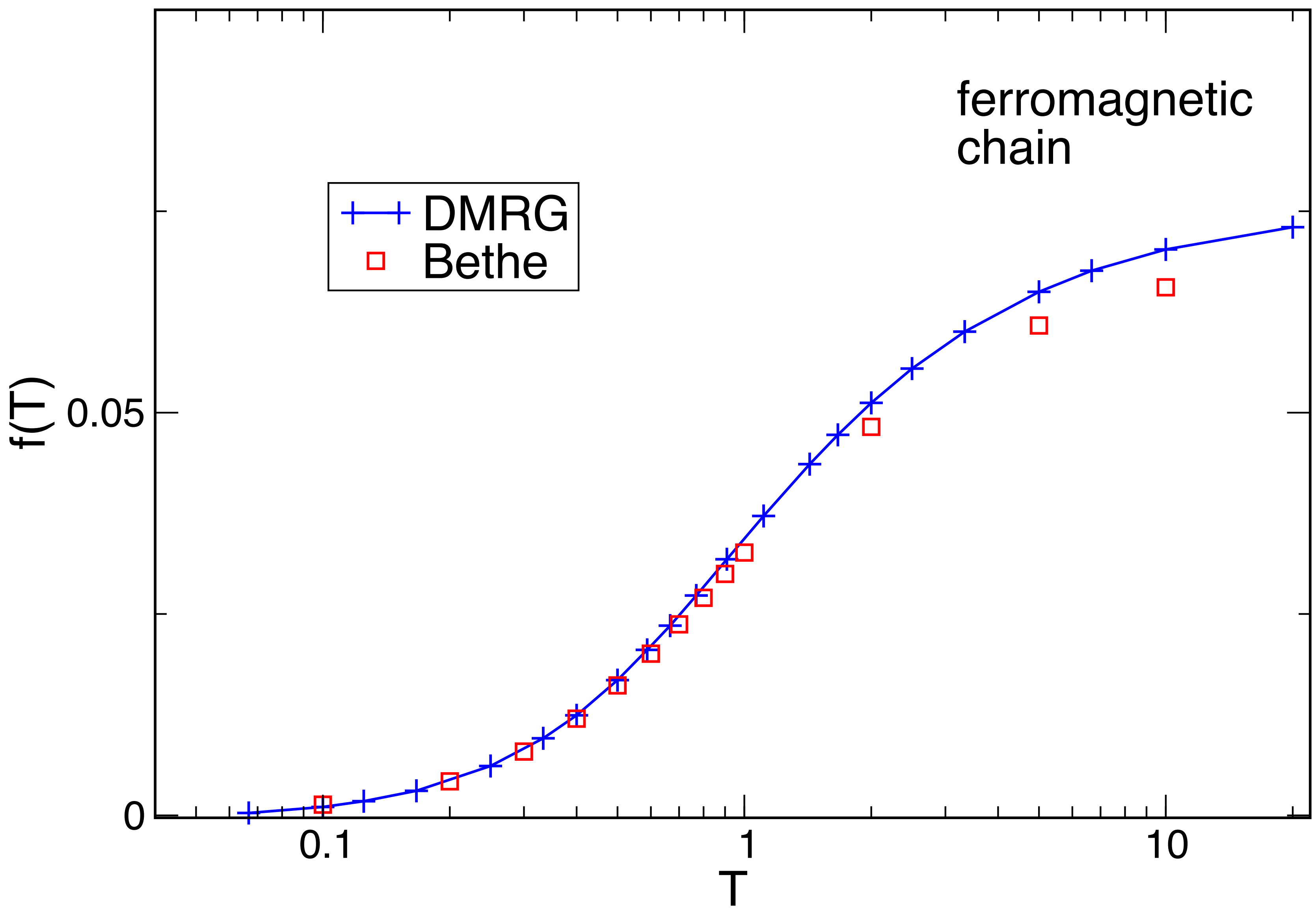
2. remove absorbers and let system evolve; energy current conservation uniquely fixes final translation-invariant steady-state.



Cannot prove that the same steady-state current applies as the long-time limit of original geometry, but the two initial conditions look very similar in momentum space.

Bethe-ansatz estimate of SB *f* for XXX ferromagnet

Compute using “bare” magnon/string velocity



(But fails even at low T for XXX antiferromagnet--not “elementary excitations”)

Bare/dressed equivalence for $\int dx\, \rho_n g_n, \quad \text{not} \int dx\, \rho_n v_n g_n$

Conclusions

1. 3DTI nanowires in magnetic fields offer a useful platform for both Majorana physics and unusual normal-state transport.

2. Transport at the QSHE edge with a magnetic impurity is a realization of the Fendley-Ludwig-Saleur problem, with some interesting differences from the FQHE realization.

3. The chiral Luttinger liquid theory of Abelian edges is almost certainly correct for a broad class of short-range interactions.

4. Non-equilibrium steady-states are generated by a thermal boundary in the XXZ model. Within numerical accuracy, these are consistent with a generalized Stefan-Boltzmann law.
Analytical arguments prove "cyclic law" for a related quantity and suggest possible steady state satisfying SB law. (C. Karrasch, R. Ilan, JEM, arXiv 2013)