

# Heavy Fermions



Piers Coleman,

Center for Materials Theory, Rutgers University, NJ, USA

Royal Holloway, University of London, UK.

**TOPNES Meeting, Higgs Centre  
Edinburgh 3rd Sept, 2013**



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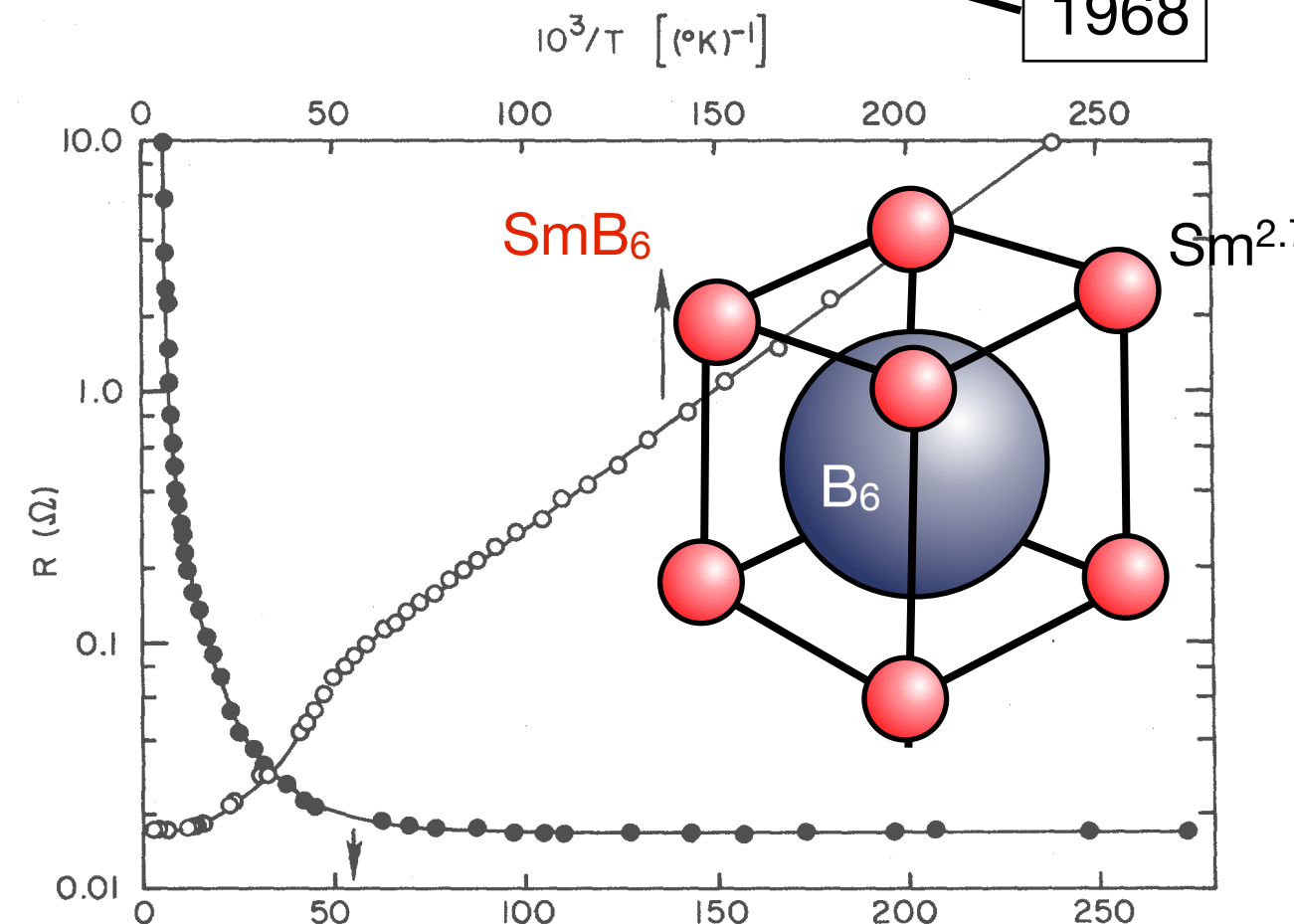
MAGNETIC AND SEMICONDUCTING PROPERTIES OF  $\text{SmB}_6$ <sup>†</sup>

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Bell Telephone Laboratories, Murray Hill, New Jersey

and

T. H. Geballe  
Department of Applied Physics, Stanford University, Stanford, California,  
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(Received 21 November 1968)

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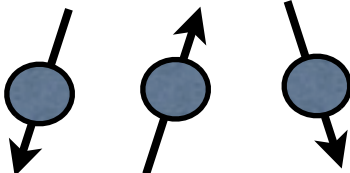
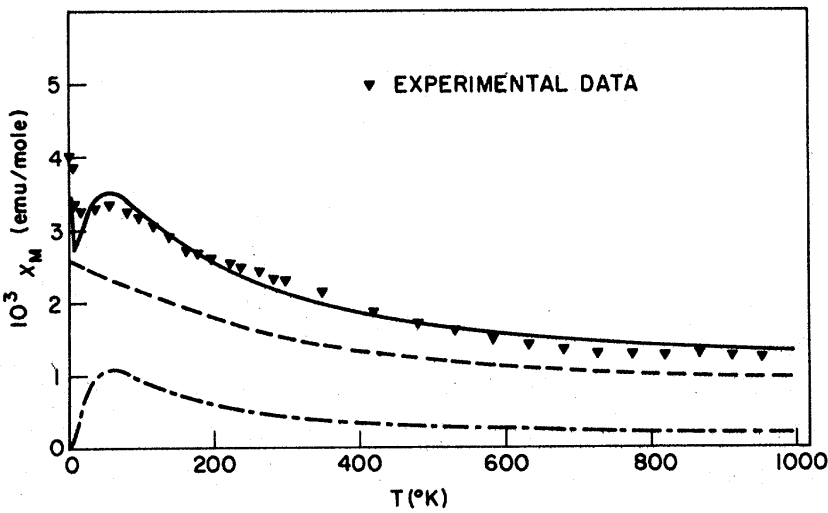




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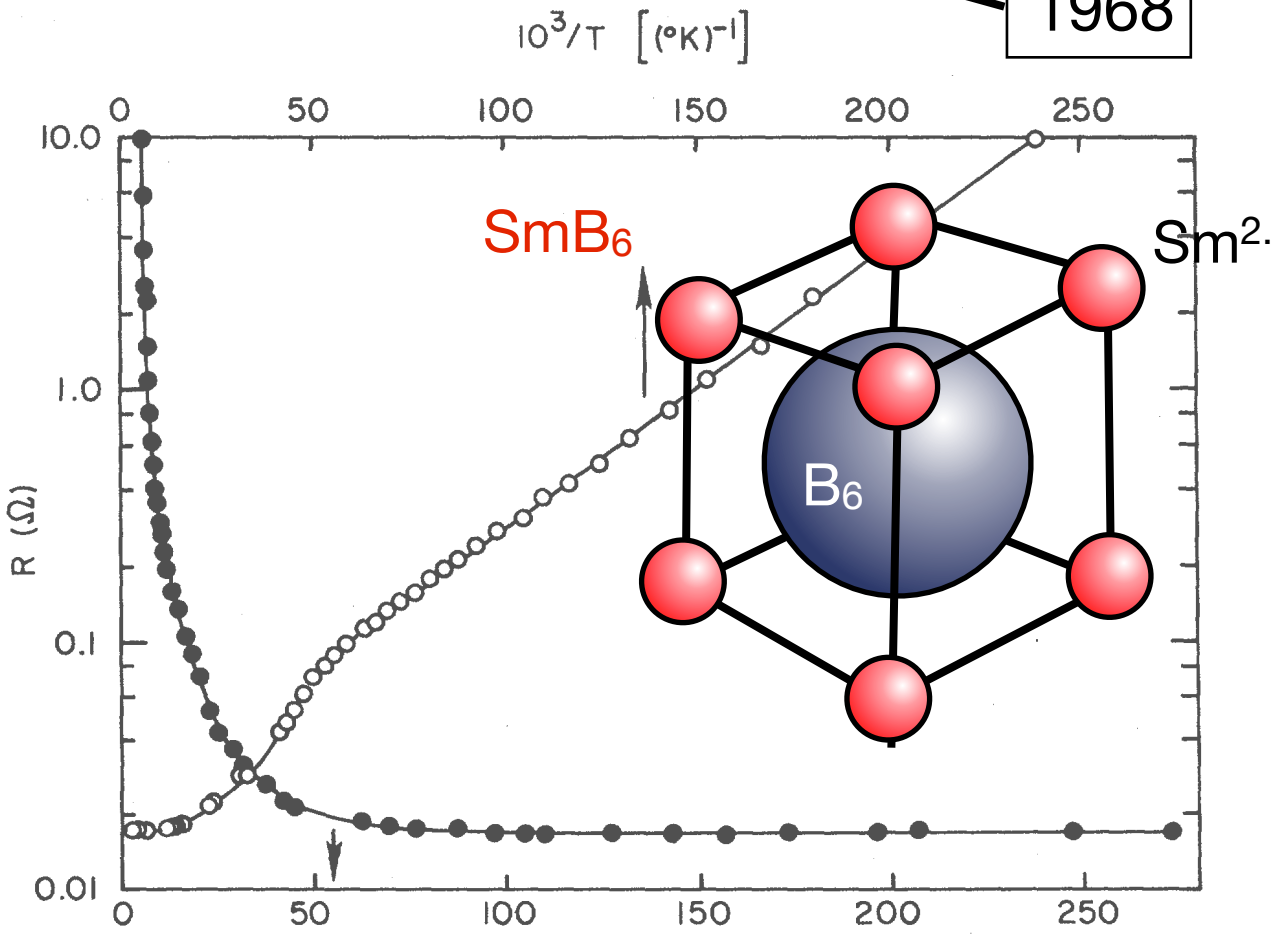
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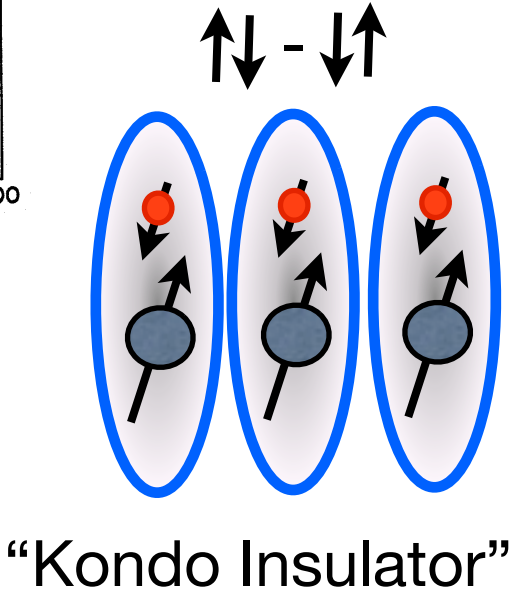
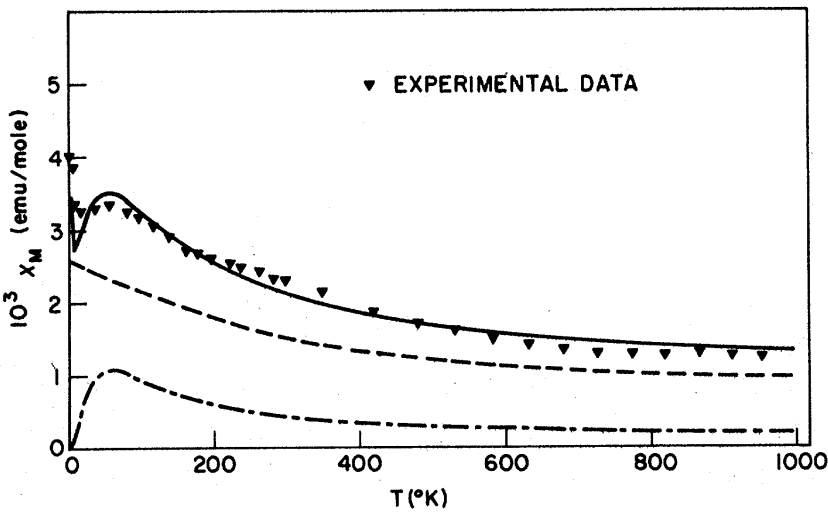
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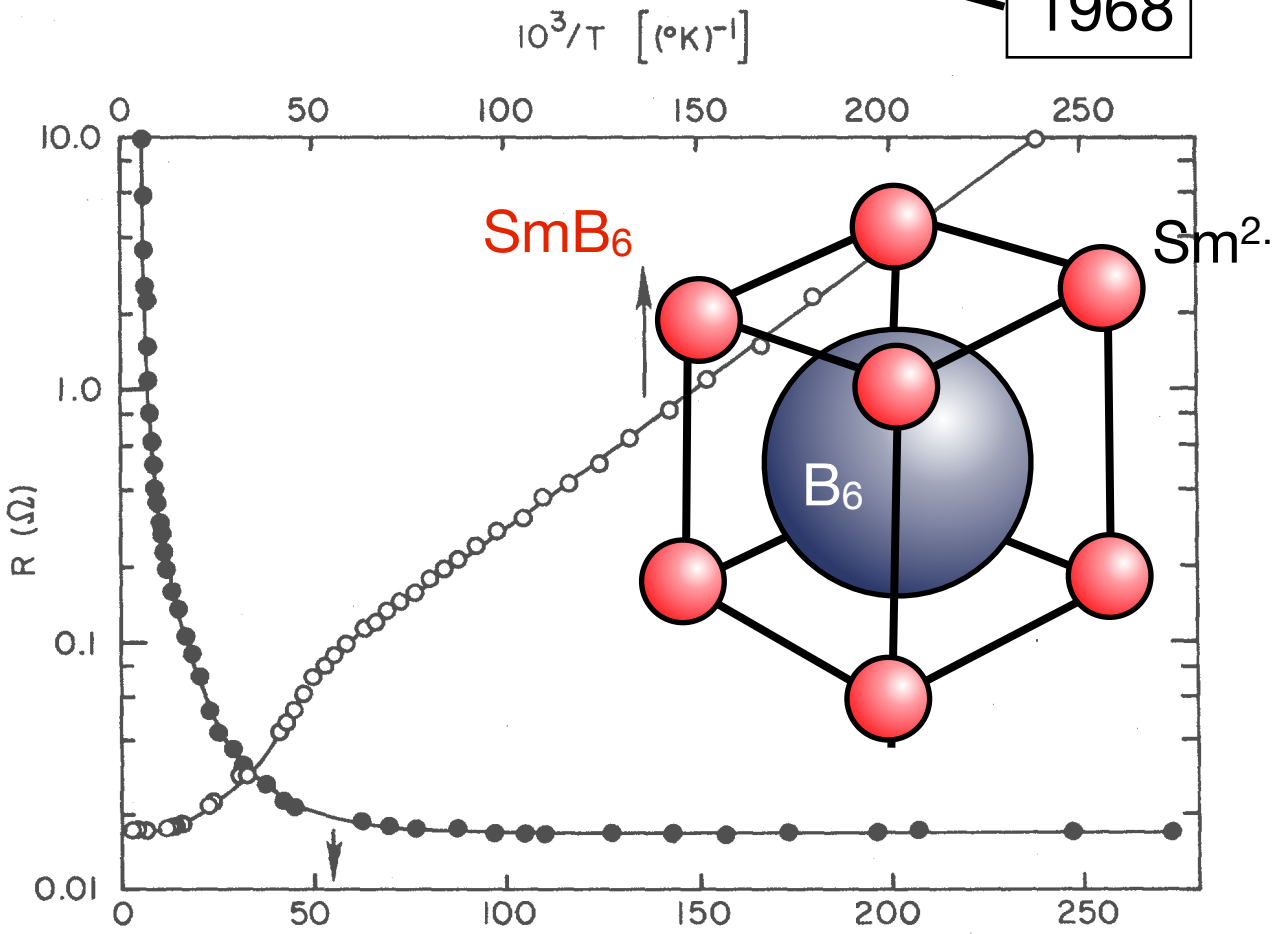
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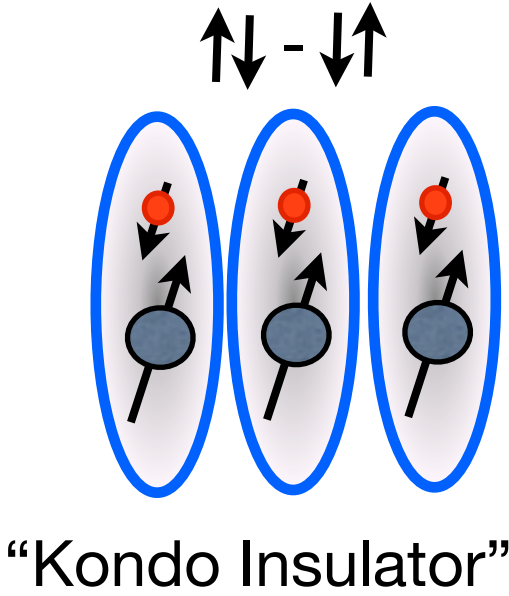
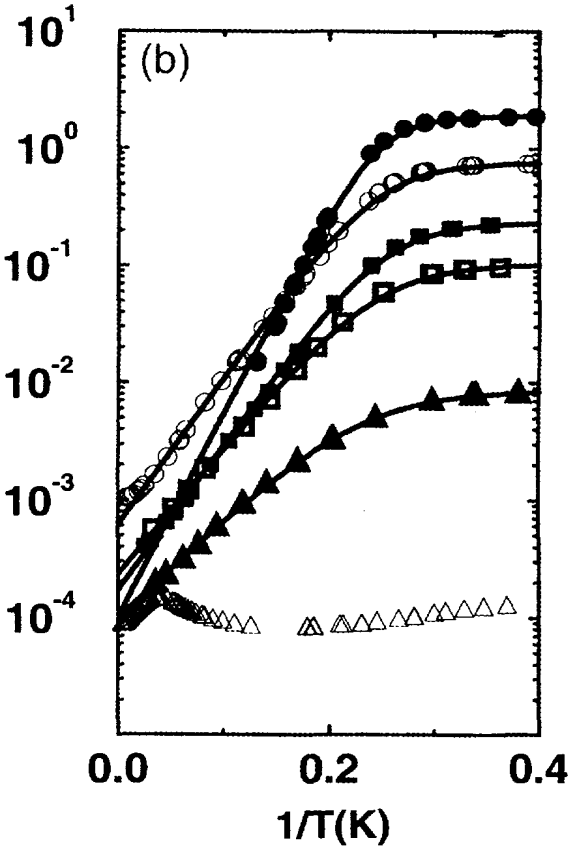


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Persistent conductivity  
Cooley et al (1995)



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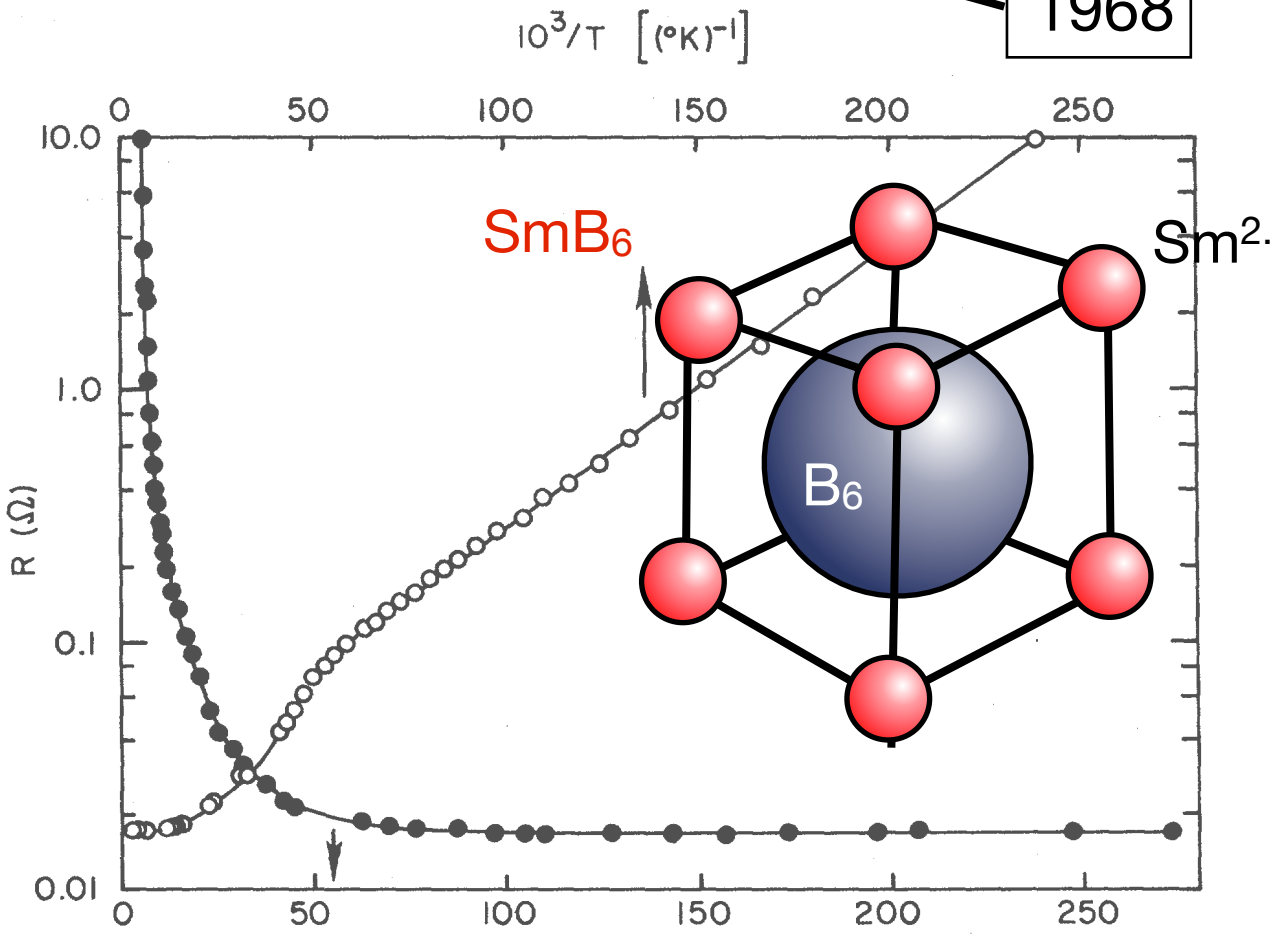
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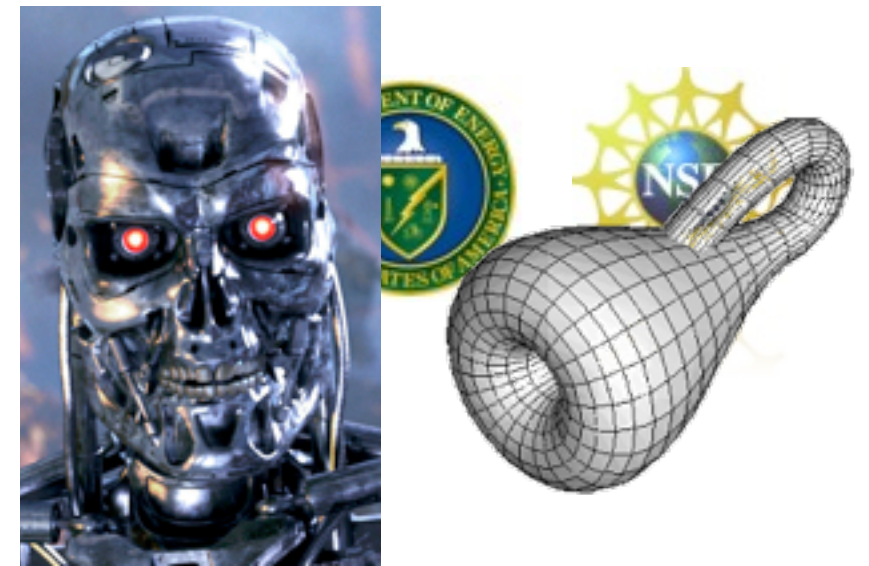
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# Heavy Fermions The Rise of Topology.

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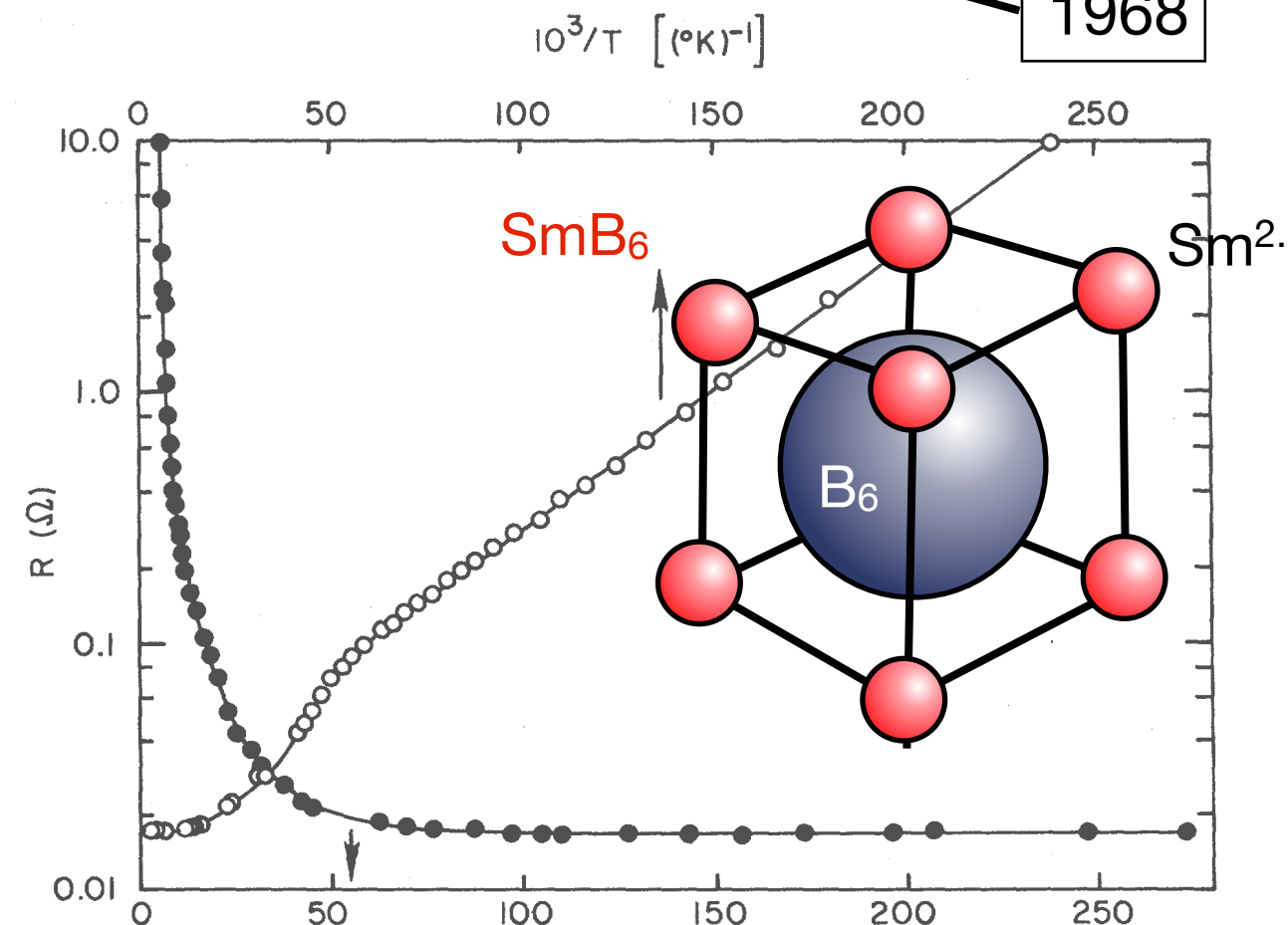
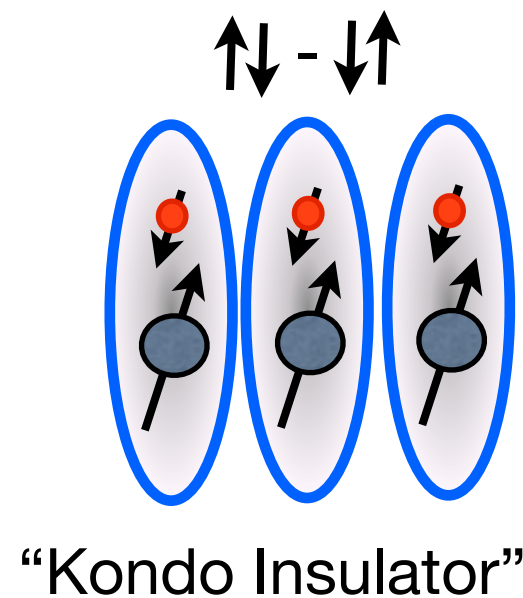
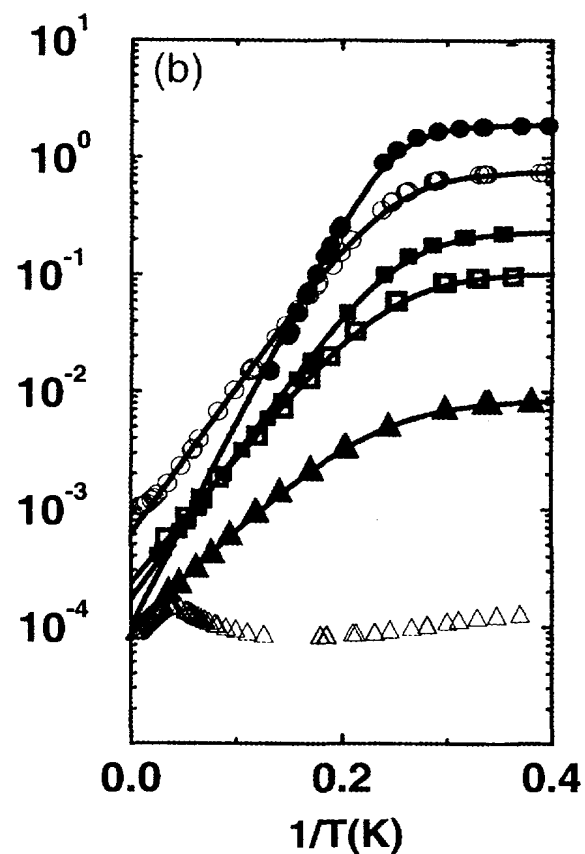
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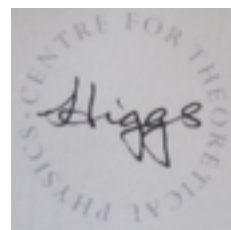
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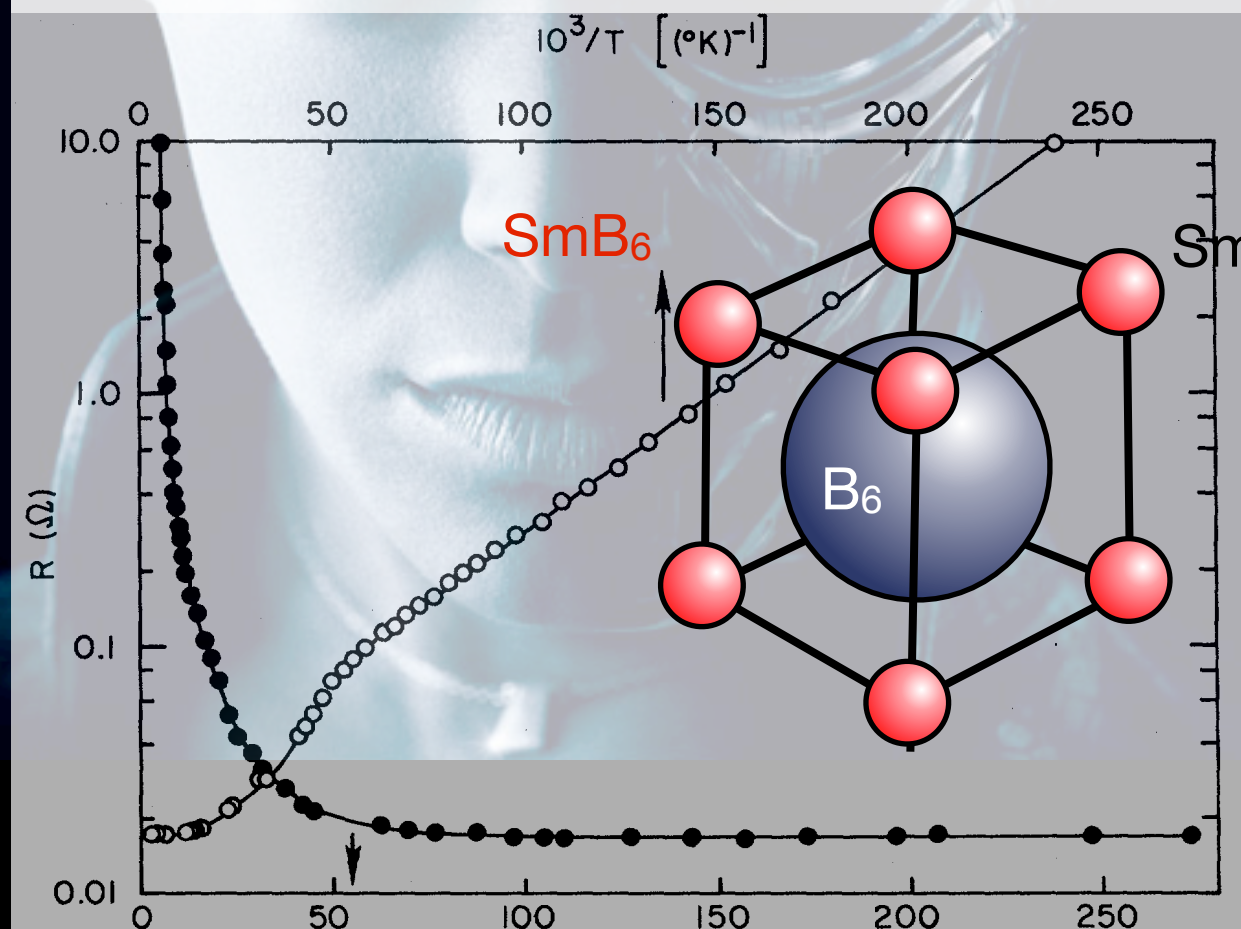


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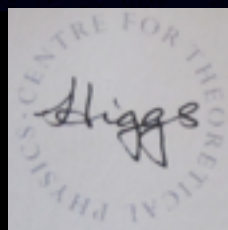
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•Kondo insulators.

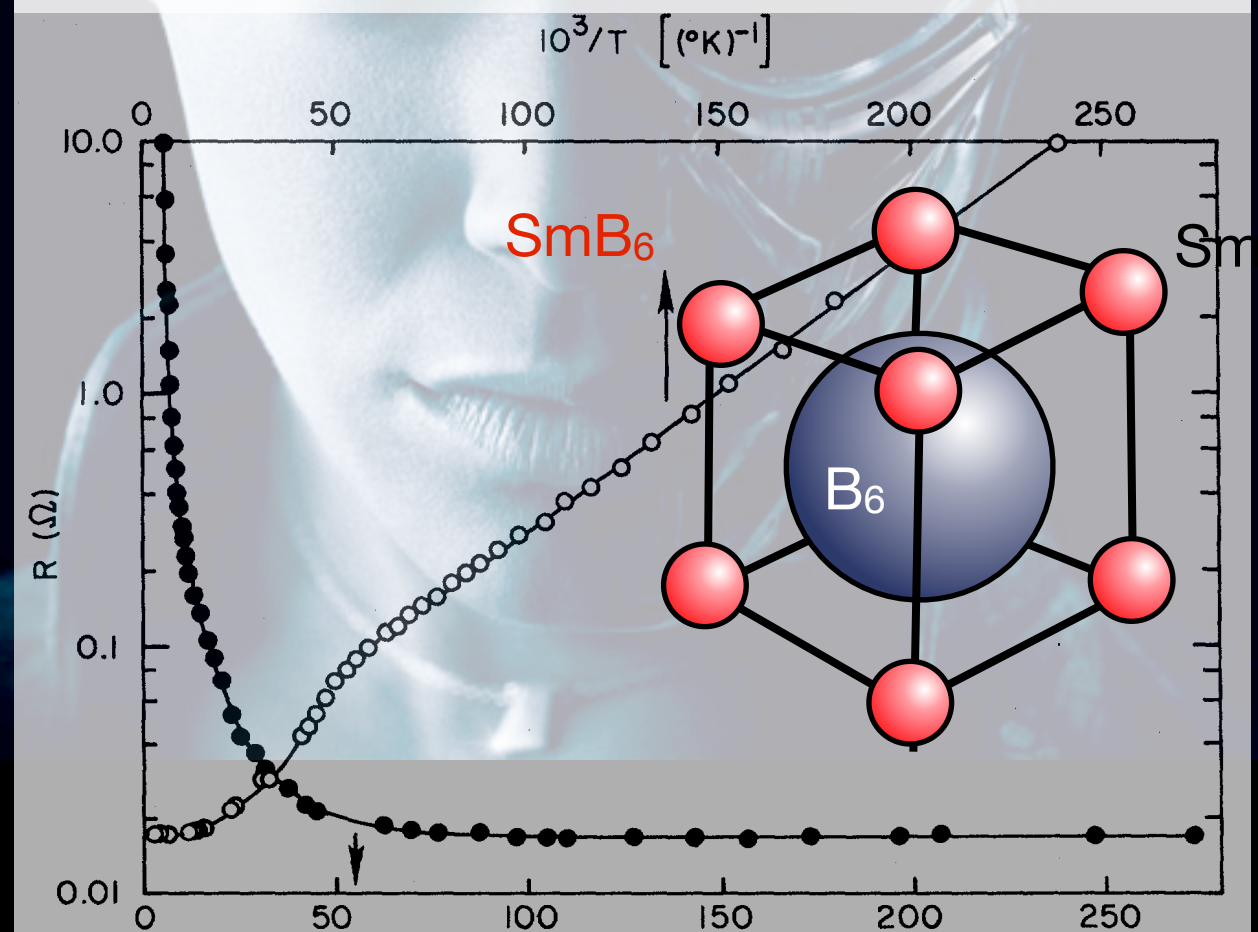


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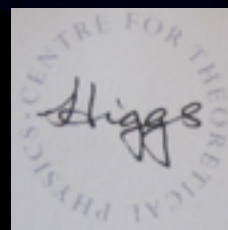
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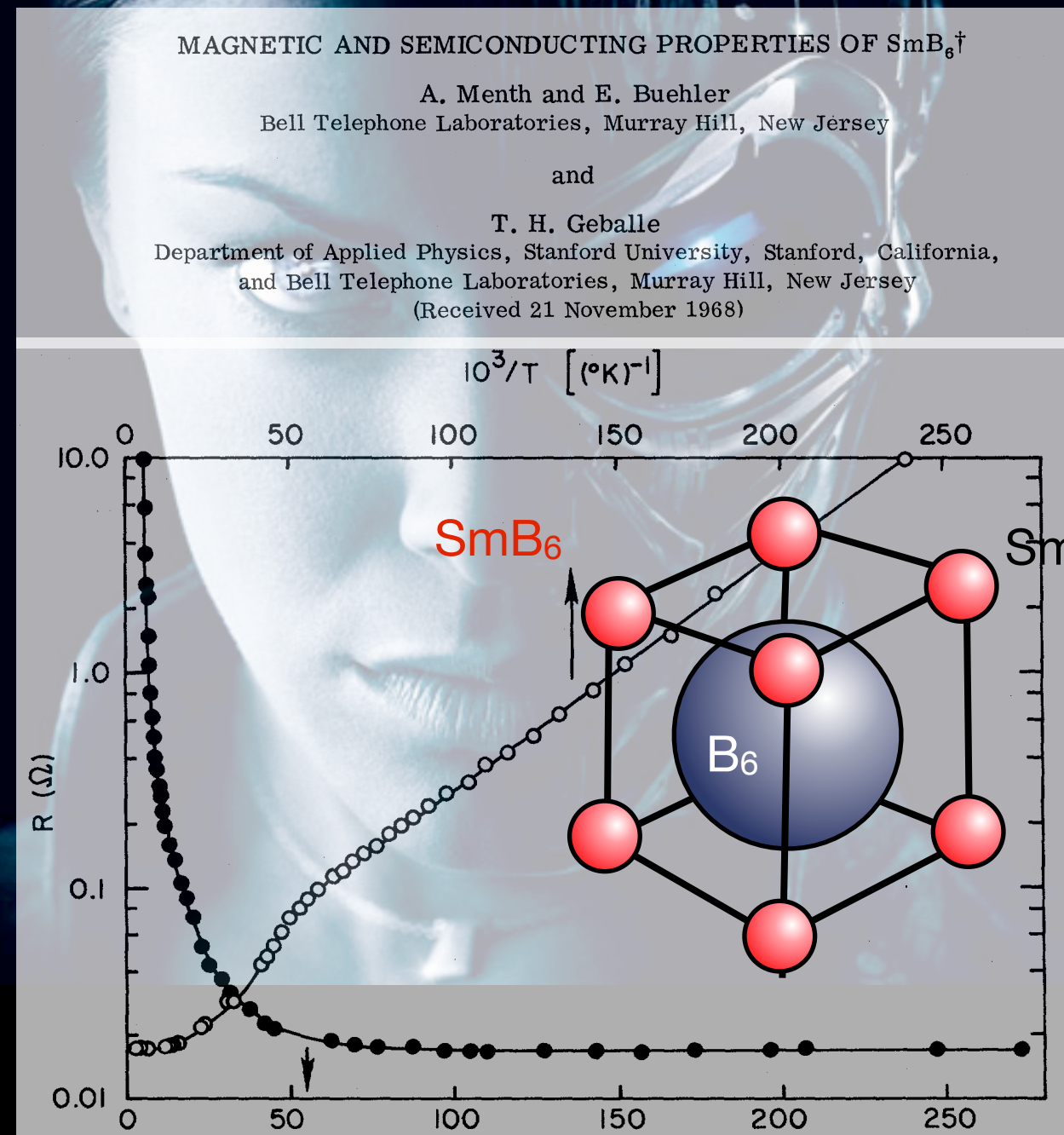
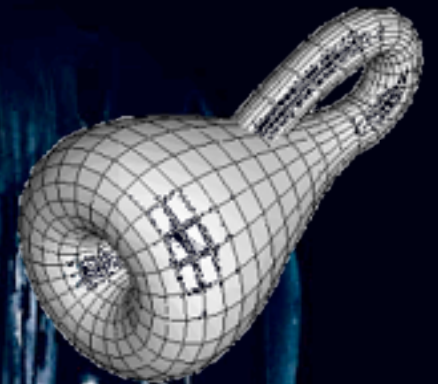
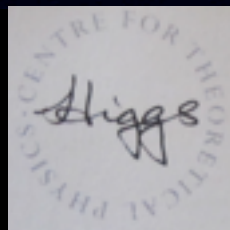


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- Kondo insulators.
- Topological Kondo Insulators

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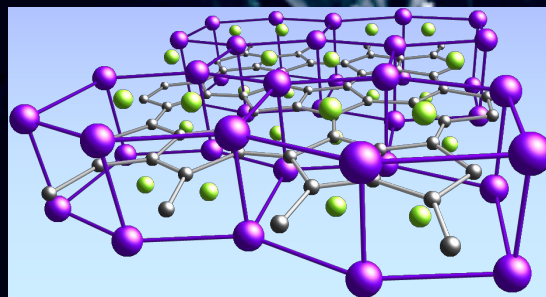




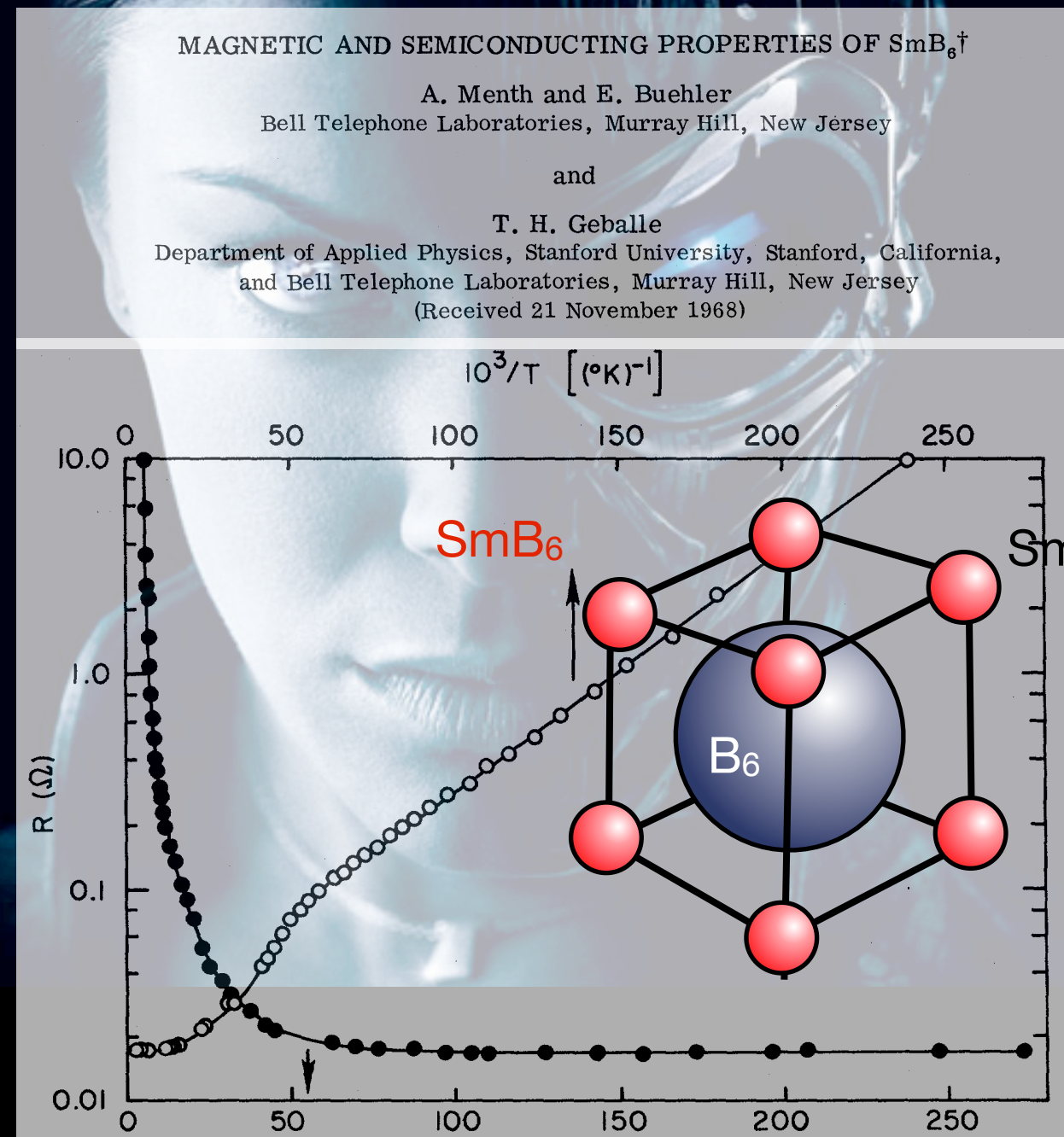
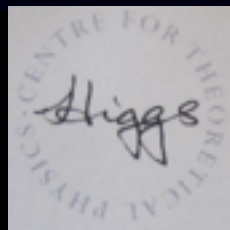
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- Kondo insulators.
- Topological Kondo Insulators
- $\beta$ -YbAlB<sub>4</sub> : intrinsic QC metal.
- YbAuAl: Kondo Quasicrystal.



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# Collaborators

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Kai Sun

Victor Galitski

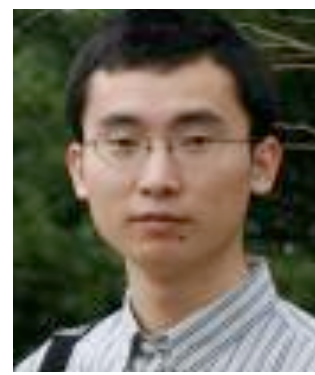
Vic Alexandrov,

Kent State

Michigan

Maryland

Rutgers



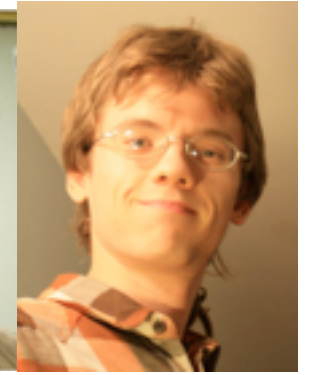
Kai



Maxim



Victor



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Topological Kondo Insulators, Dzero, Sun, Galitski, PC Phys. Rev. Lett. **104**, 106408 (2010)

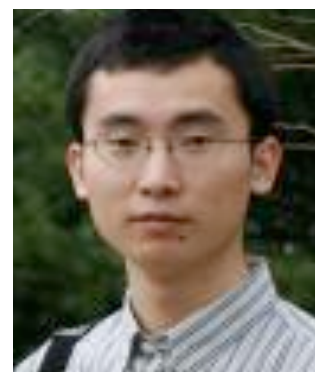
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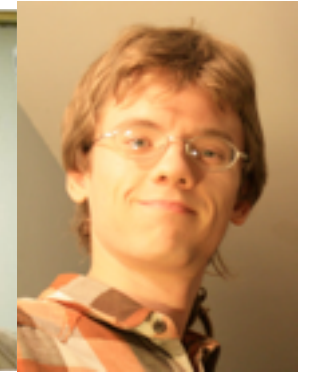
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arXiv.org > cond-mat > arXiv:1211.5104

Condensed Matter > Strongly Correlated Electrons

## Discovery of the First Topological Kondo Insulator: Samarium Hexaboride

Steven Wolgast, Cagliyan Kurdak, Kai Sun, J. W. Allen, Dae-Jeong Kim, Zachary Fisk

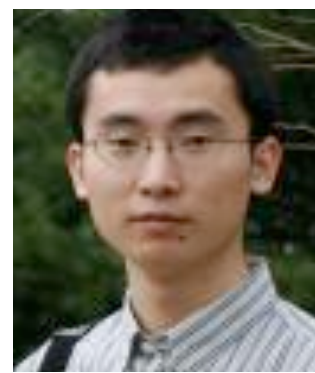
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Aline Ramires      Rutgers  
Andriy Nevidomskyy      Rice  
Alexei Tsvelik      Brookhaven NL  
Satoru Nakatsuji      ISSP Tokyo  
Yosuke Matsumoto      ISSP Tokyo  
Eoin O'Farrell      ISSP Tokyo



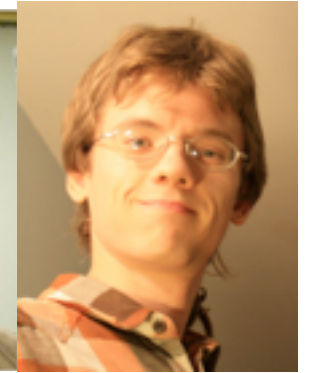
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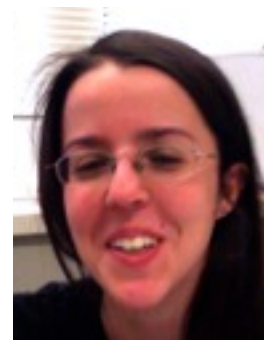
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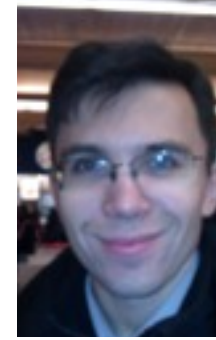
Victor



Vic



Aline



Andriy



Satoru



Yosuke



Alexei

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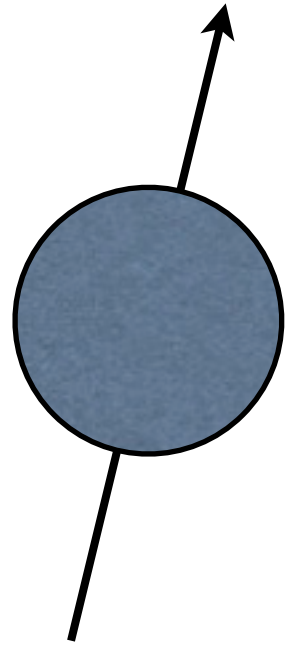
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## $\beta$ -YbAlB<sub>4</sub>: A Critical Nodal Metal

Aline Ramires, PC, Andriy H. Nevidomskyy and A. M. Tsvelik, Phys. Rev. Lett. 109, 176404 (2012).

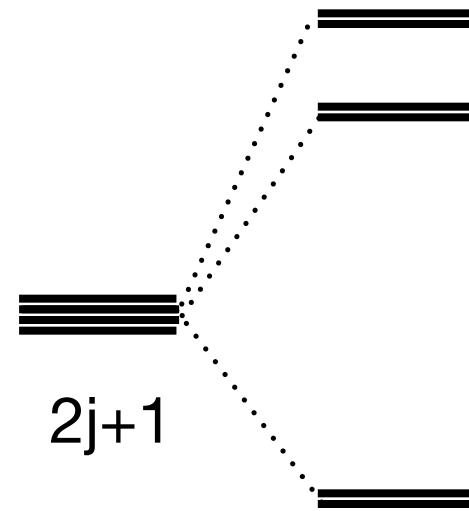
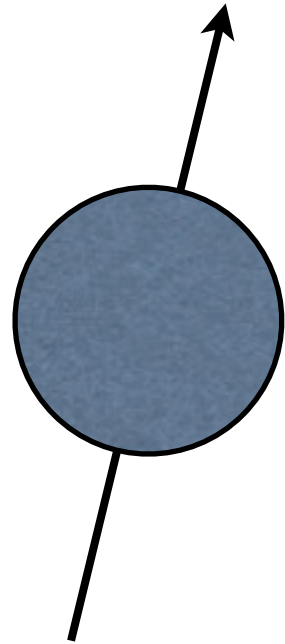


# Heavy Fermion Primer



Spin (4f,5f): basic  
fabric of heavy  
electron physics.

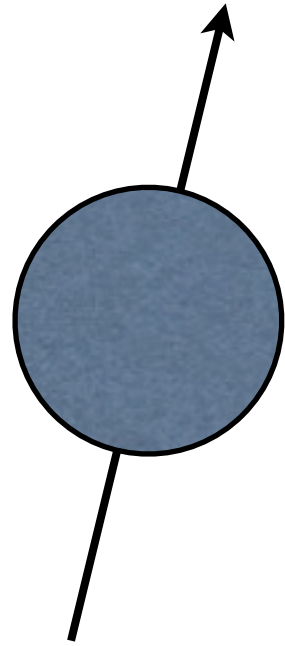
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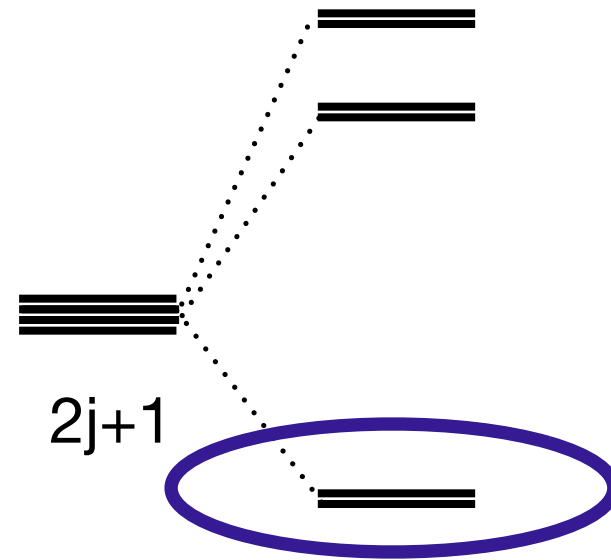
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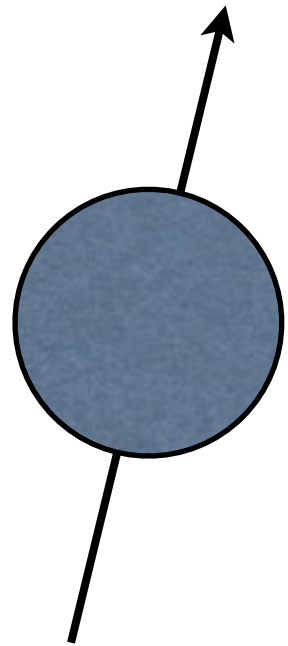
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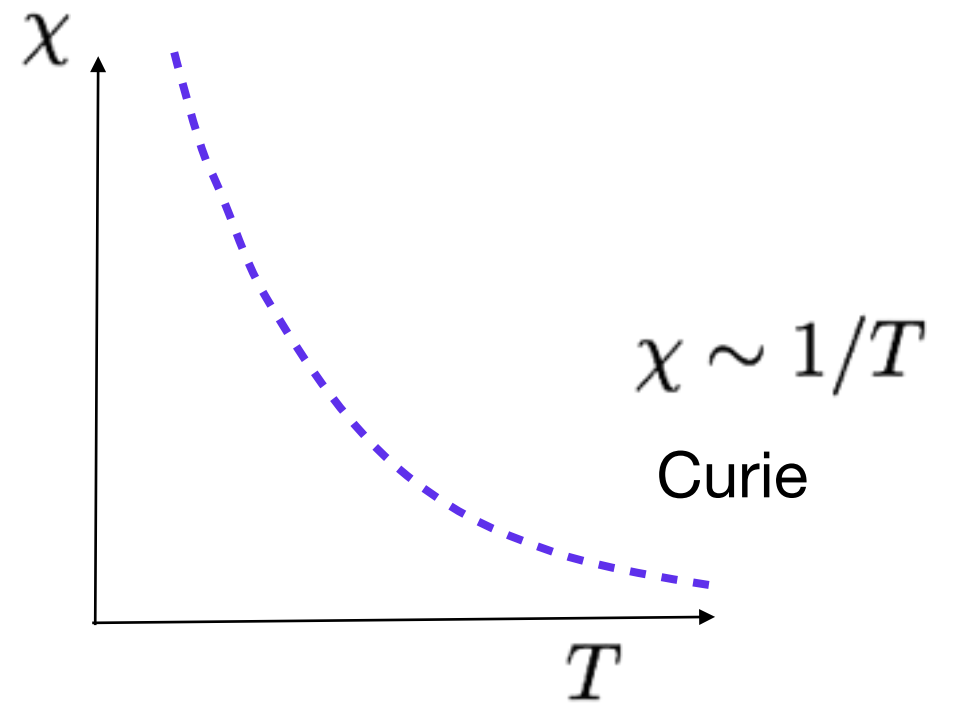
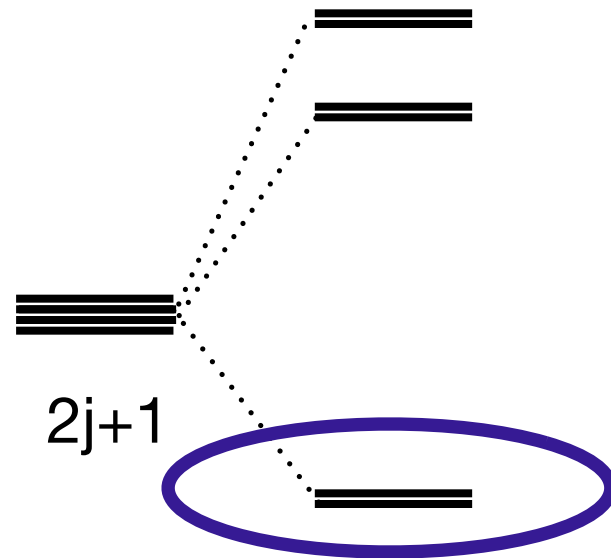
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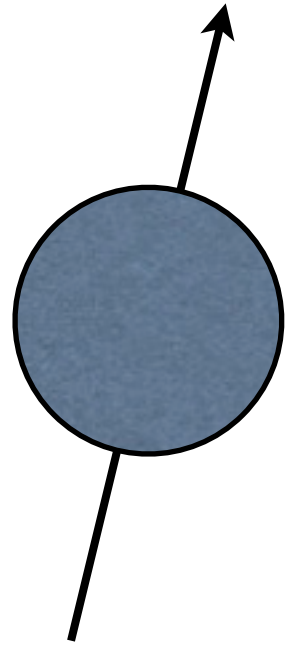
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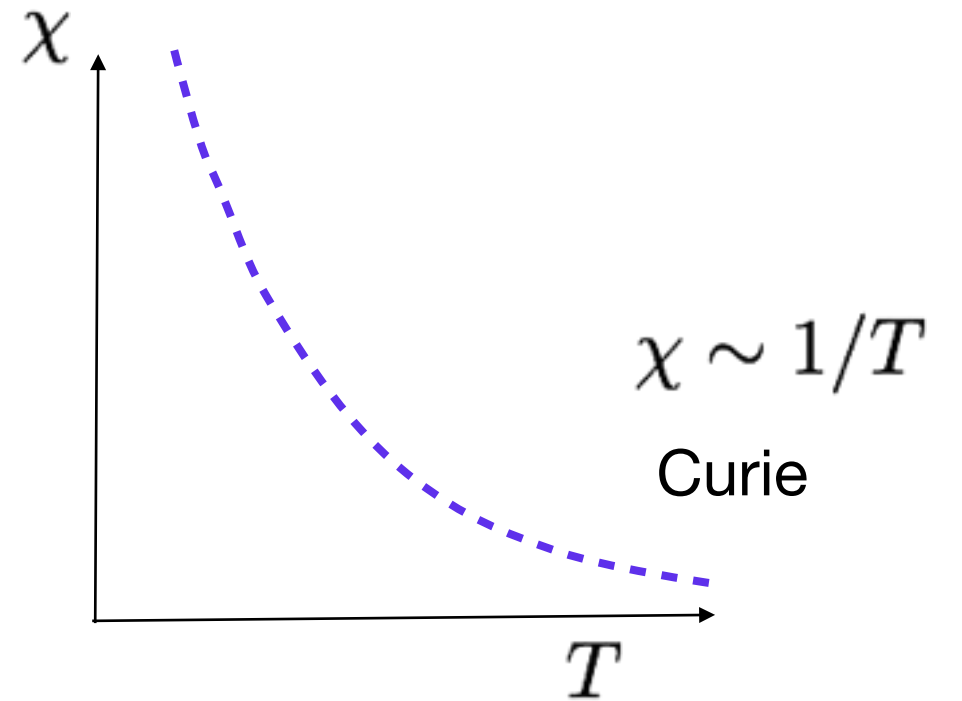
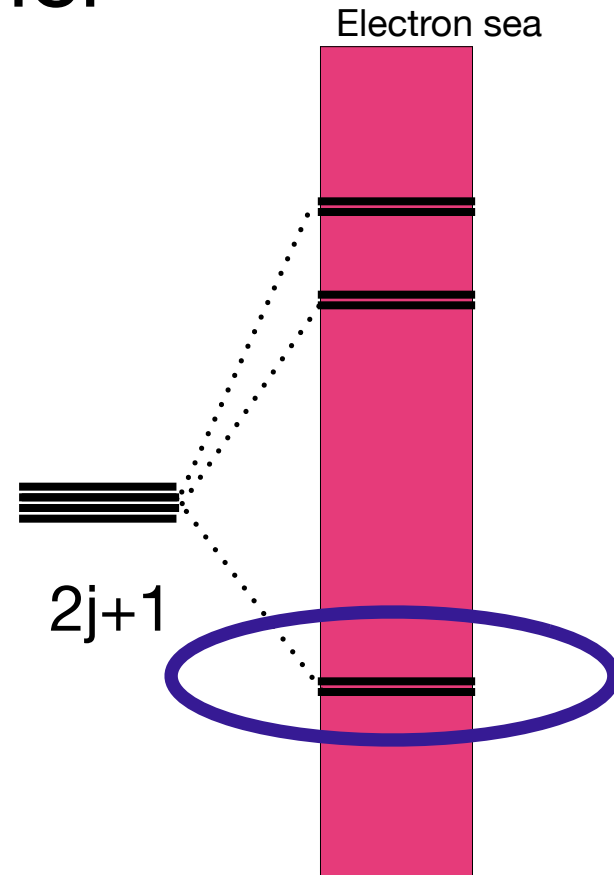
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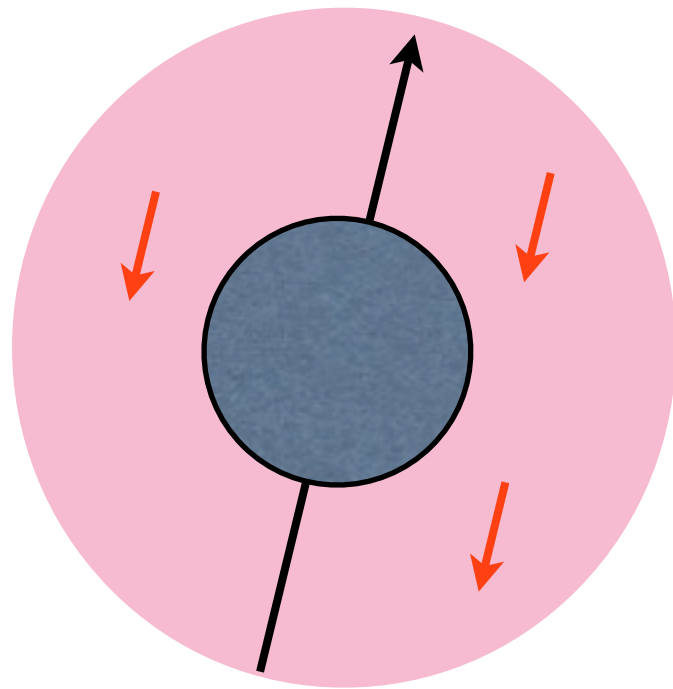
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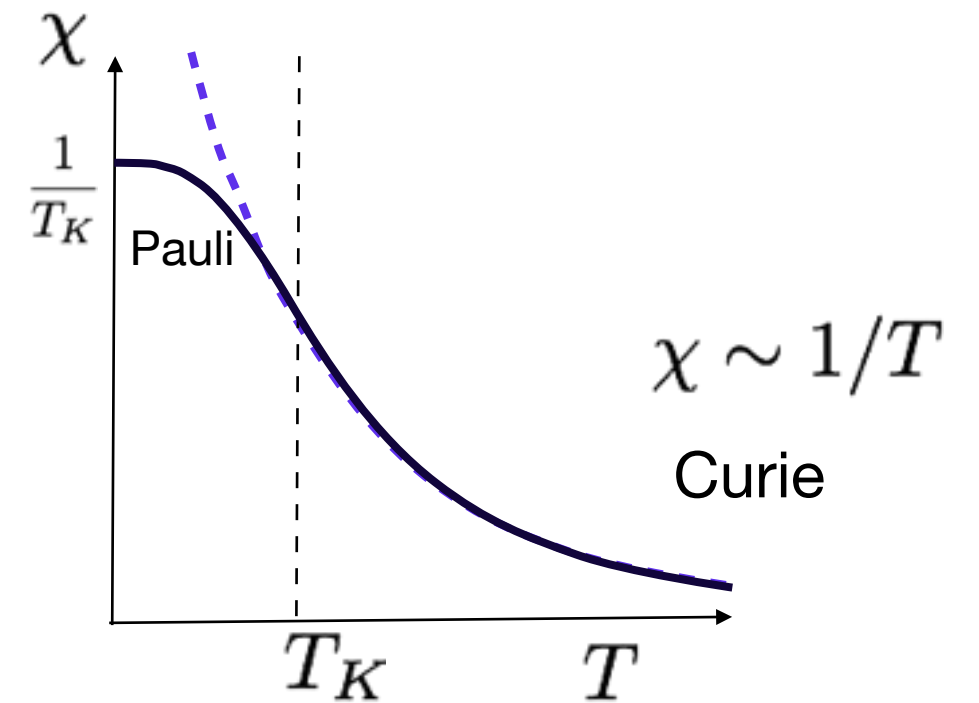
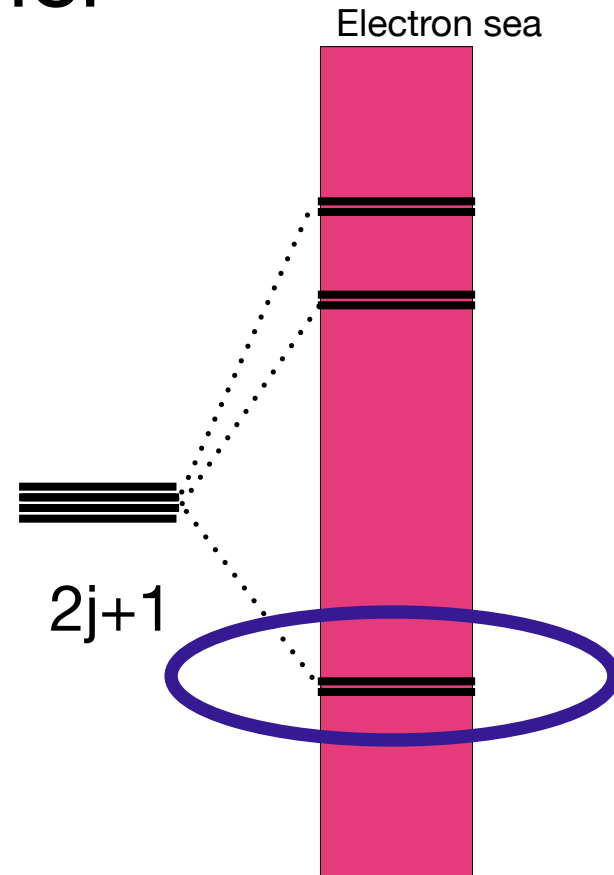
$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + J \vec{S} \cdot \vec{\sigma}(0)$$



# Heavy Fermion Primer



Spin screened by  
conduction  
electrons



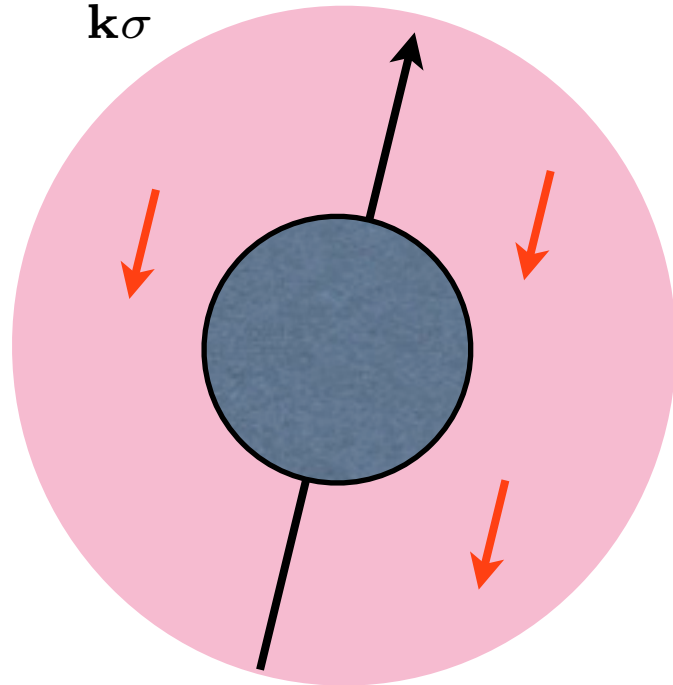
“Kondo temperature”

$$T_K = W \sqrt{J\rho} e^{-\frac{1}{2J\rho}}$$

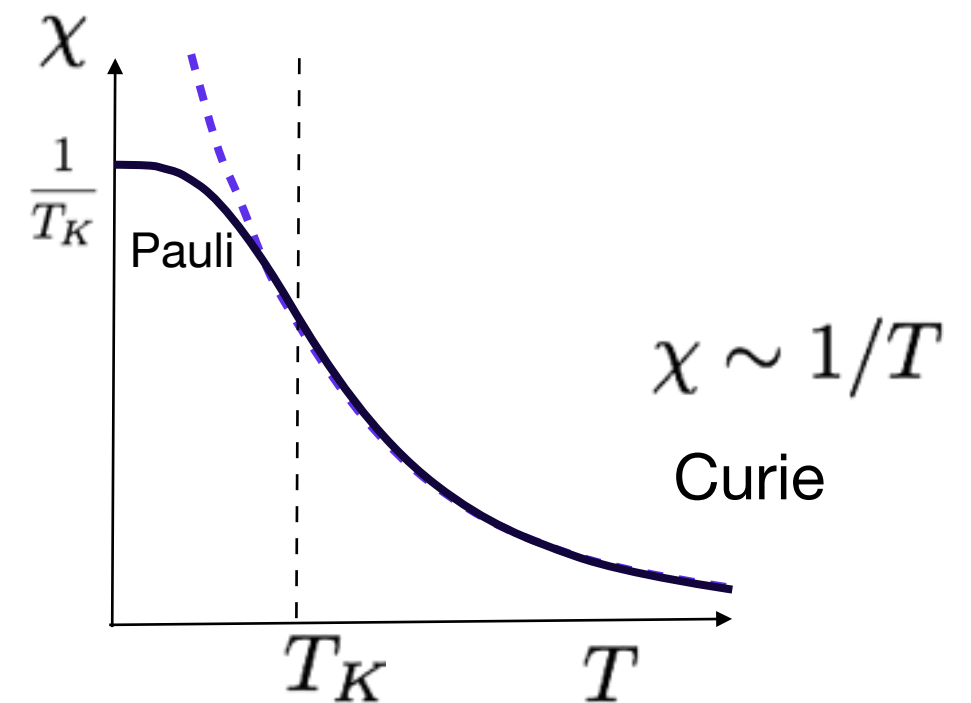
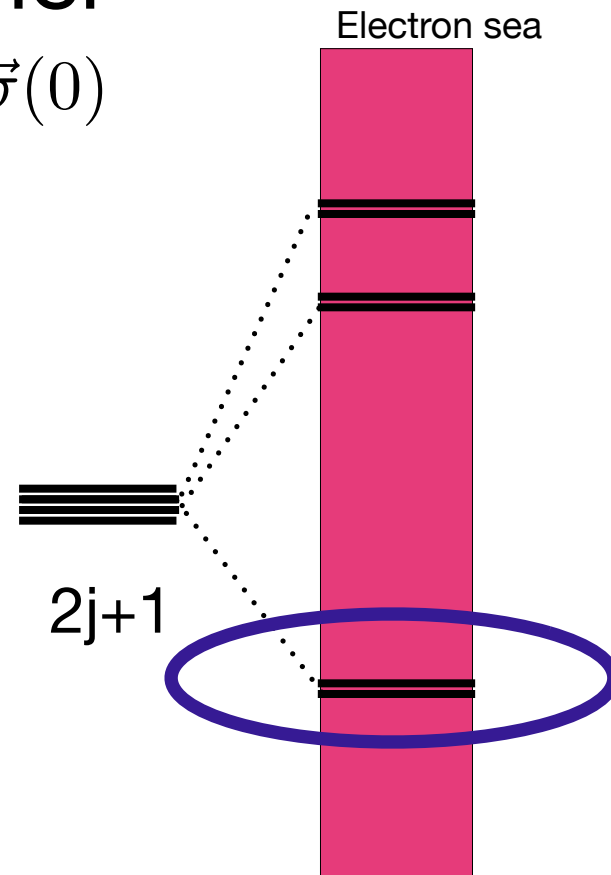
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Spin screened by  
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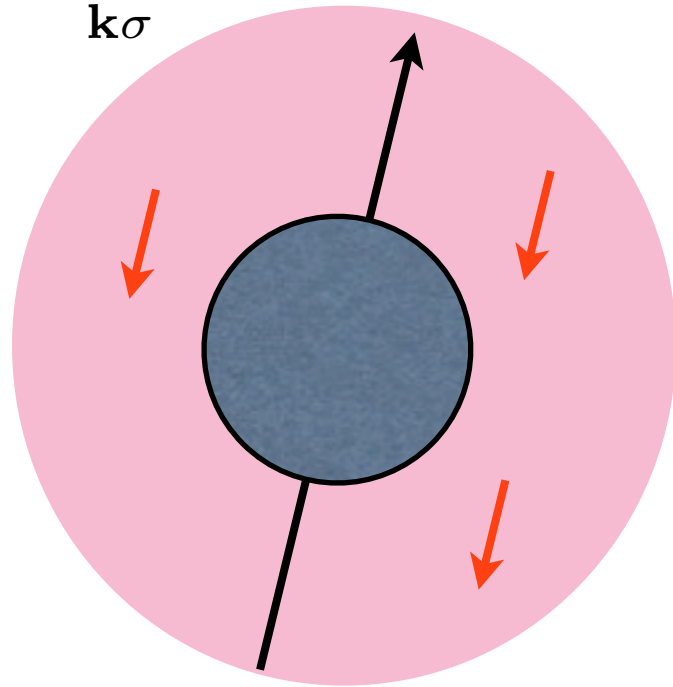


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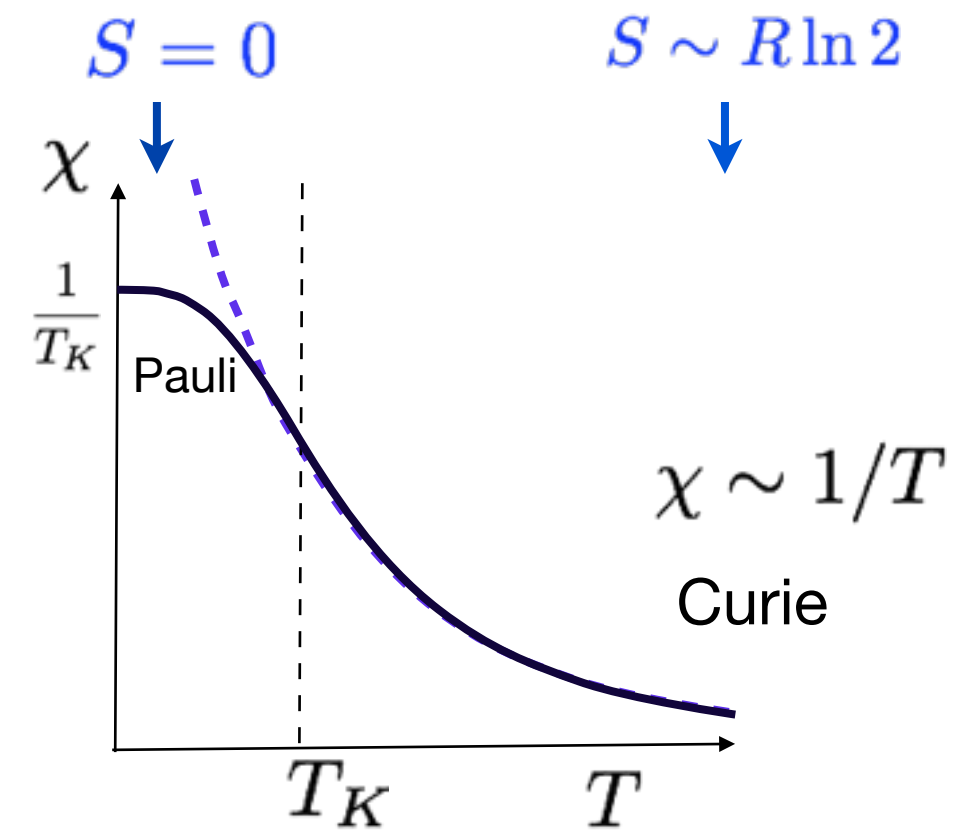
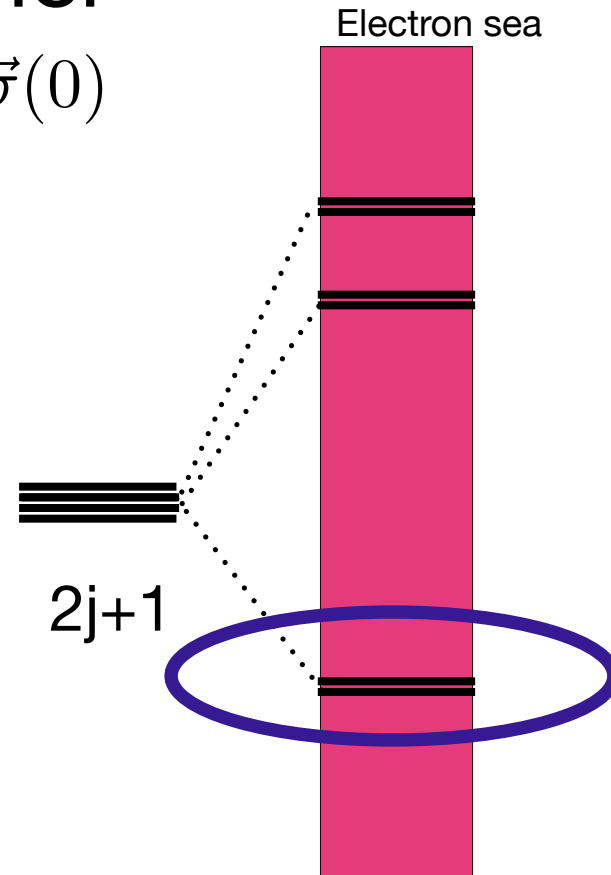
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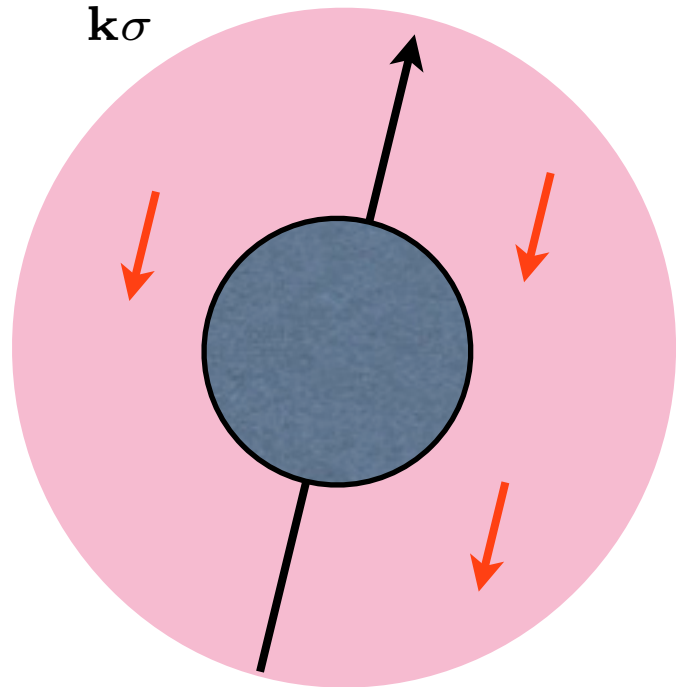


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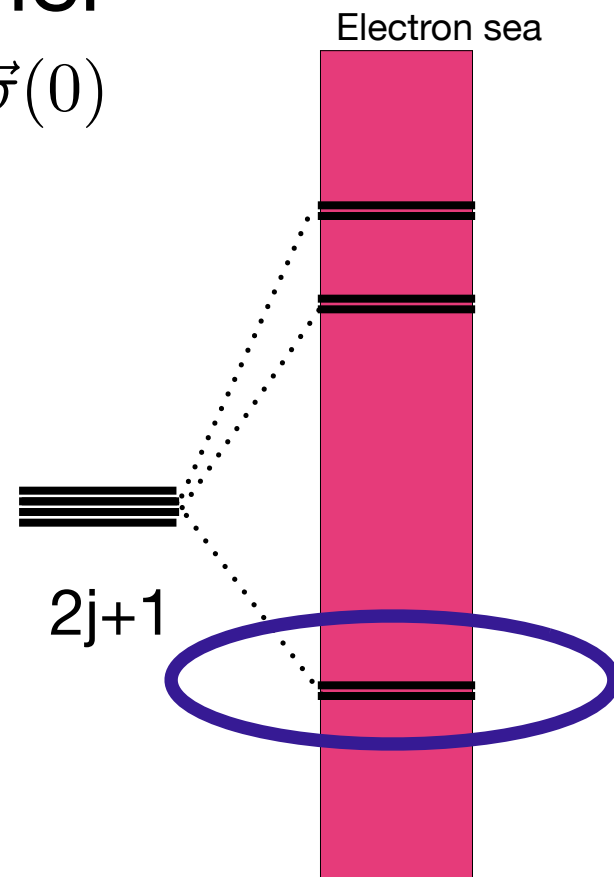
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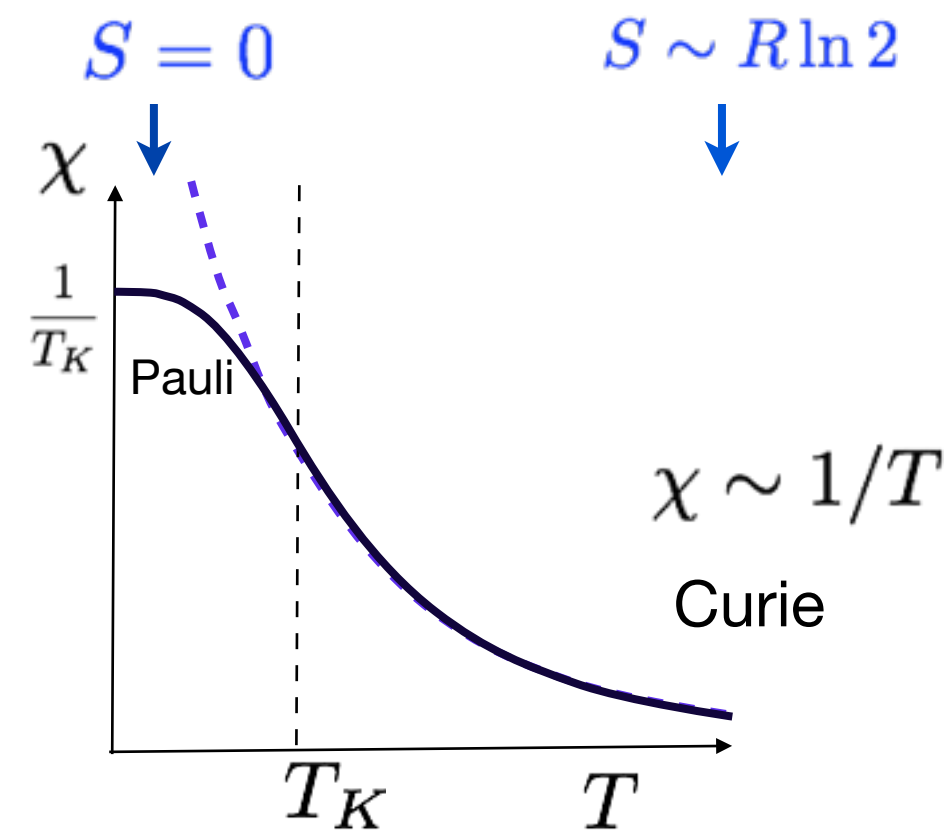


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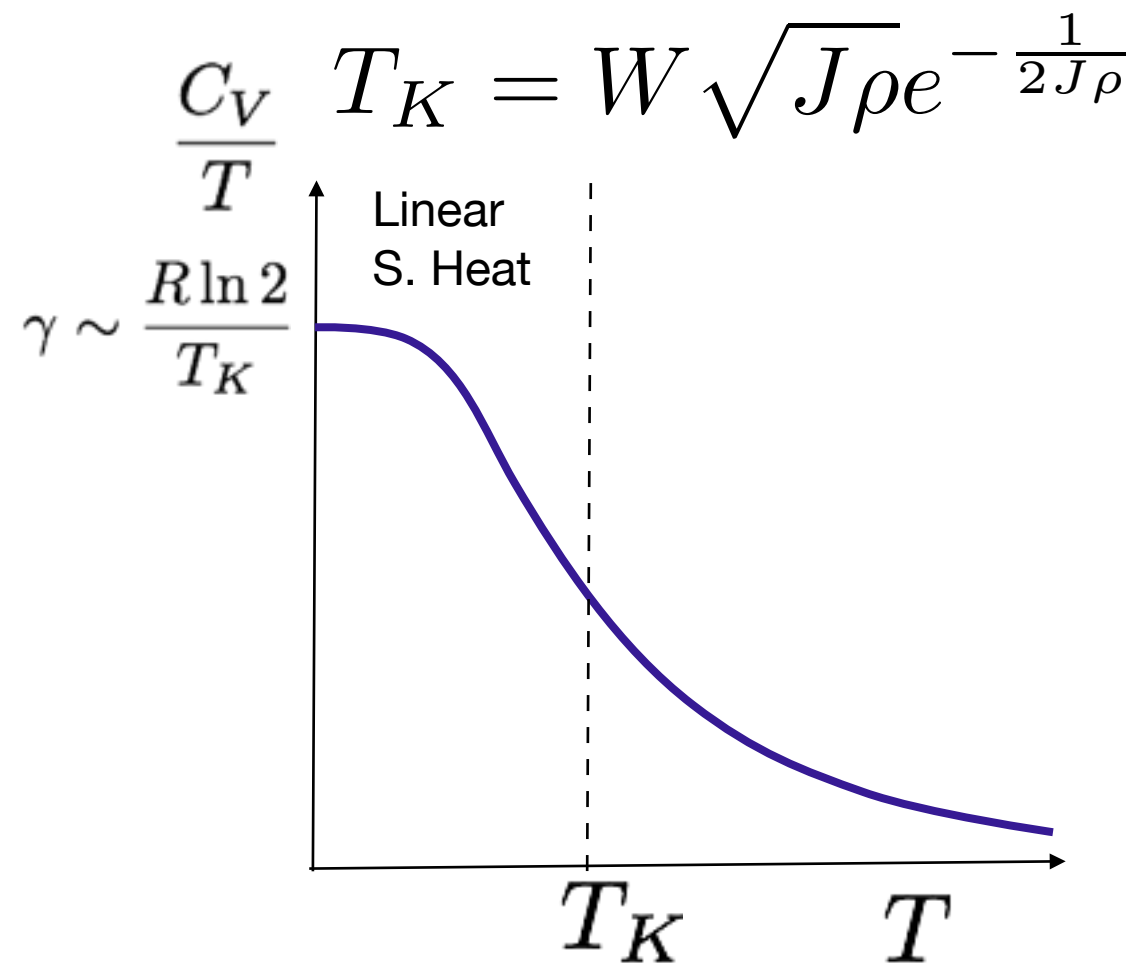


$$S(T) = \int_0^T \frac{C_V}{T'} dT'$$

Spin entropy

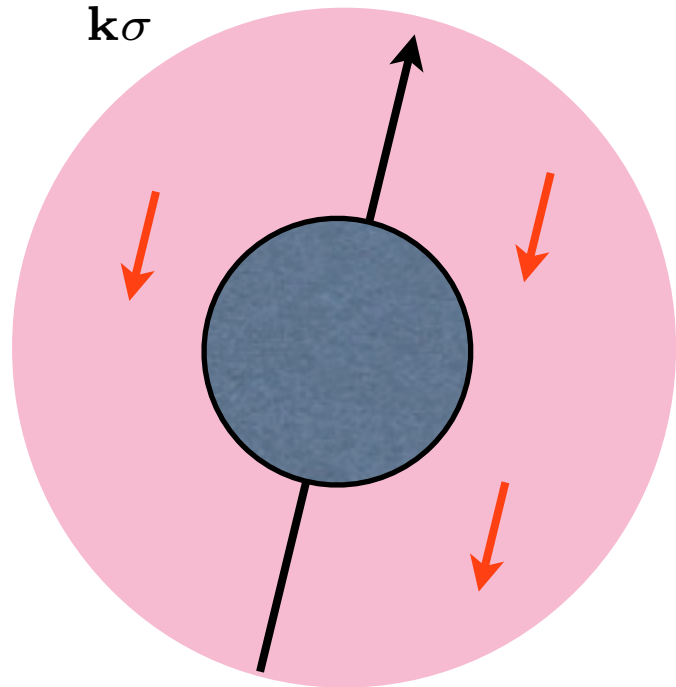


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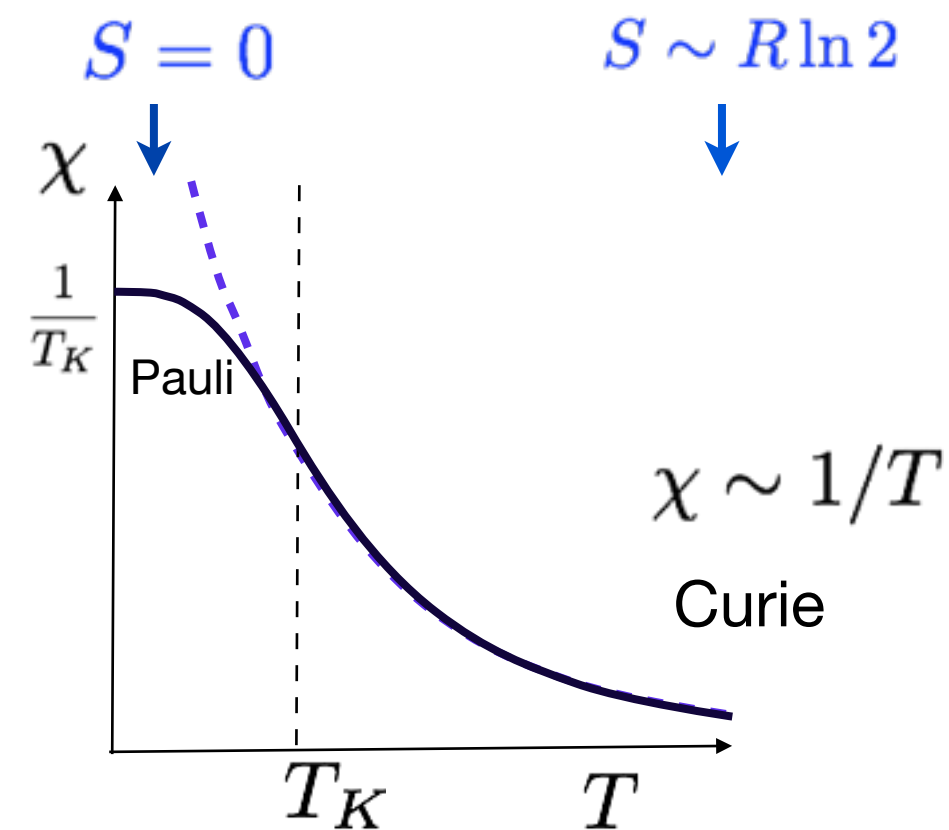
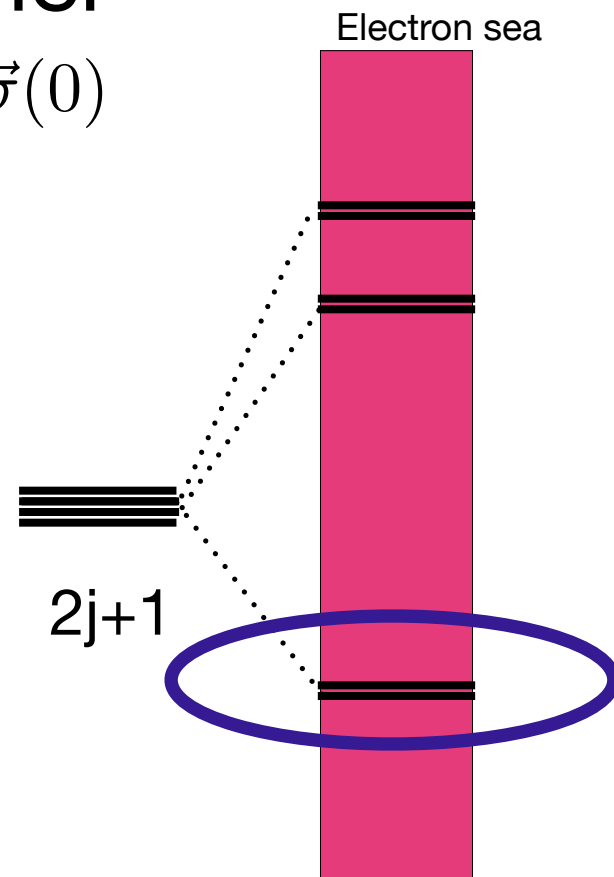


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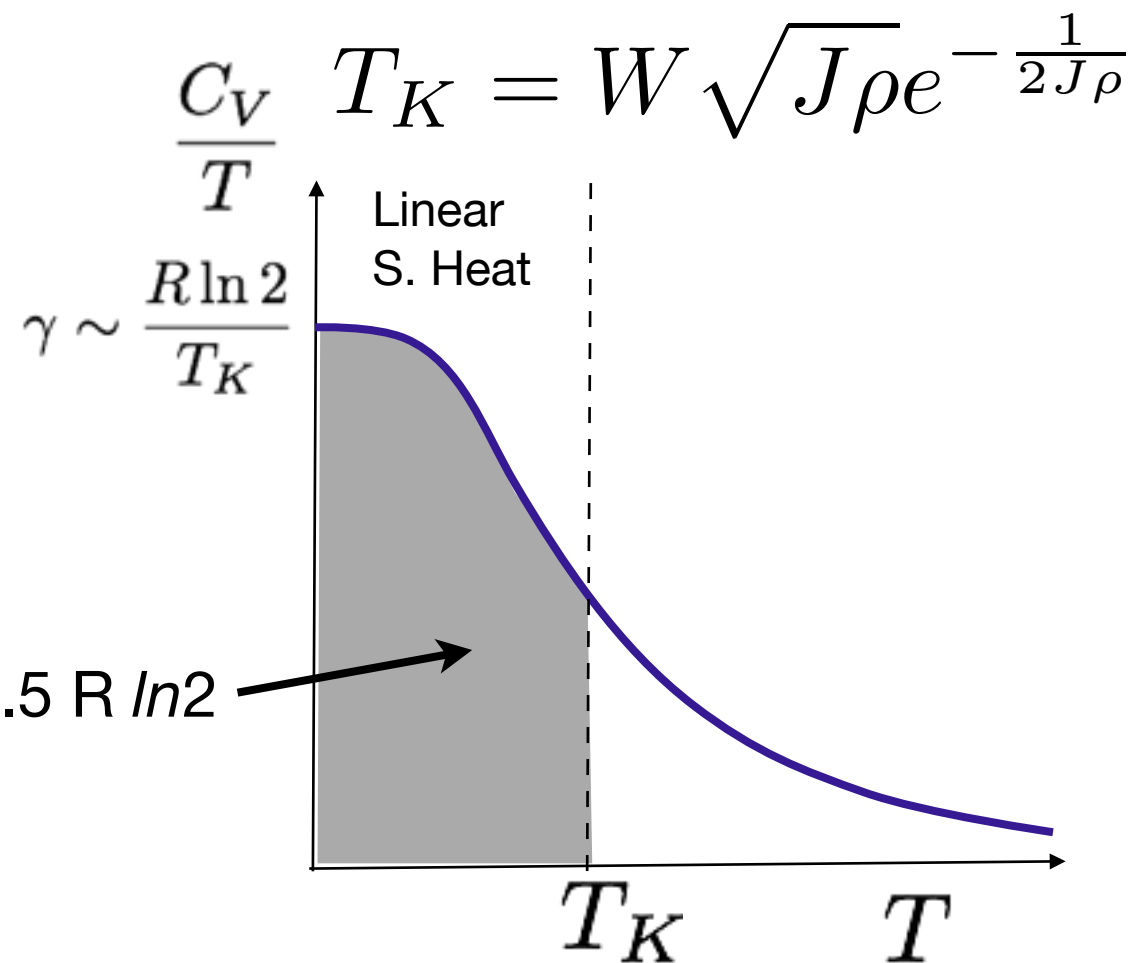
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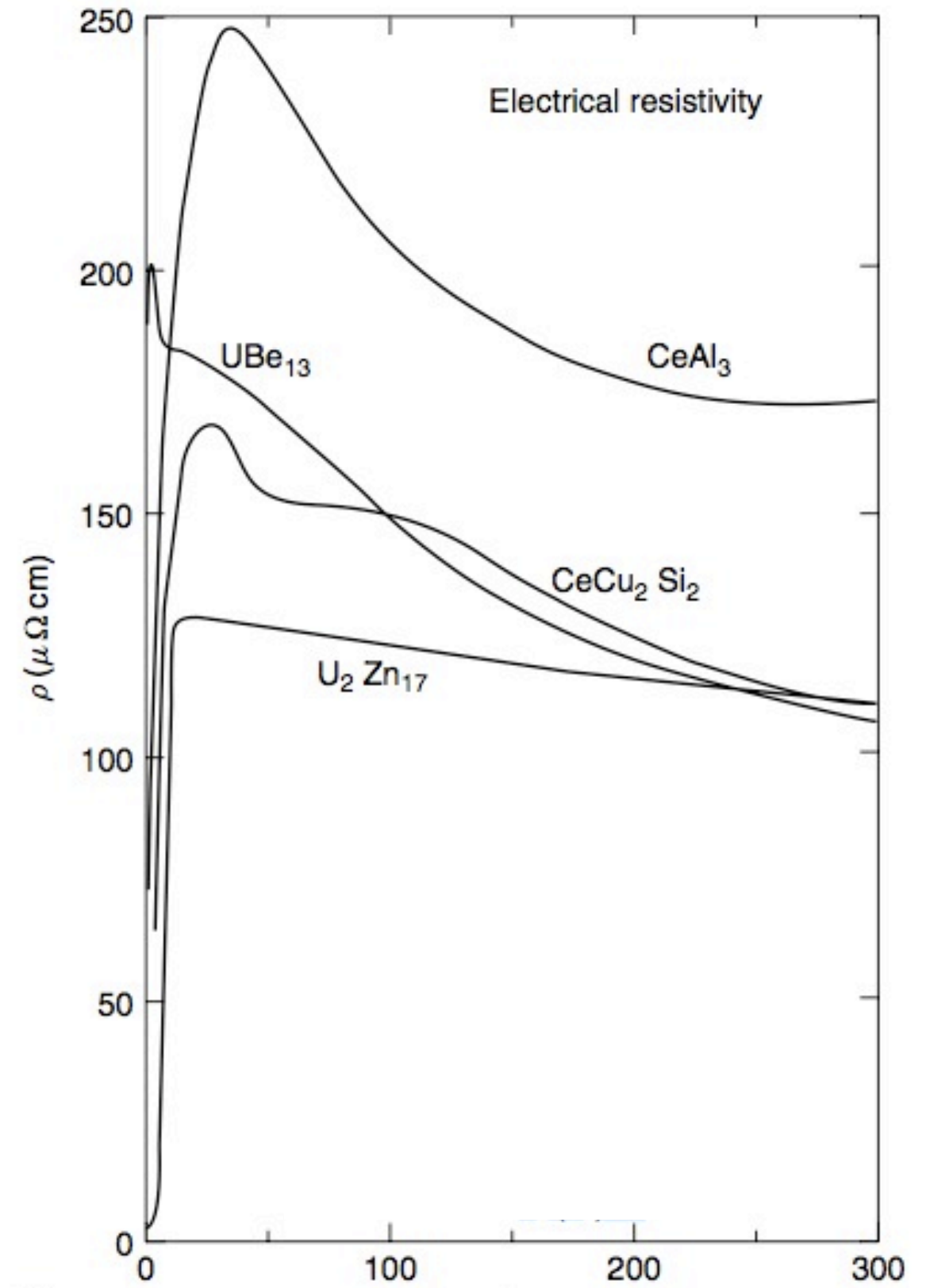
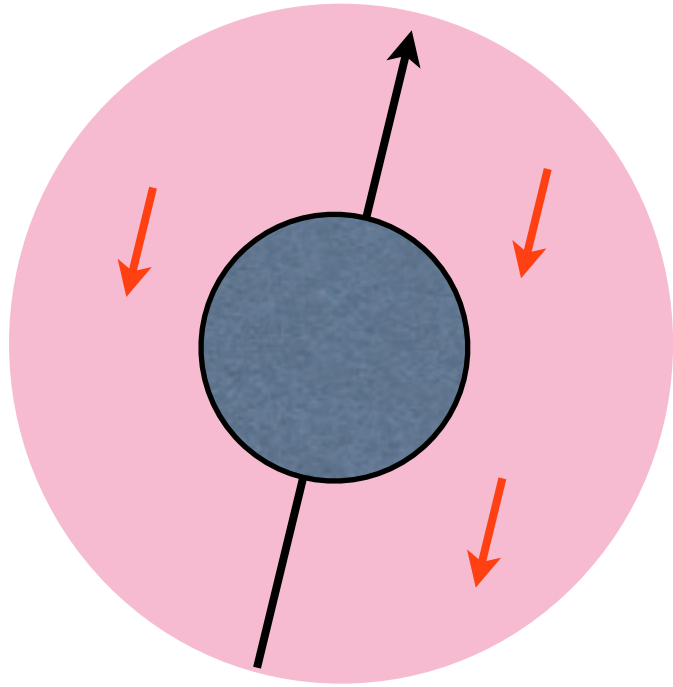
“Kondo temperature”



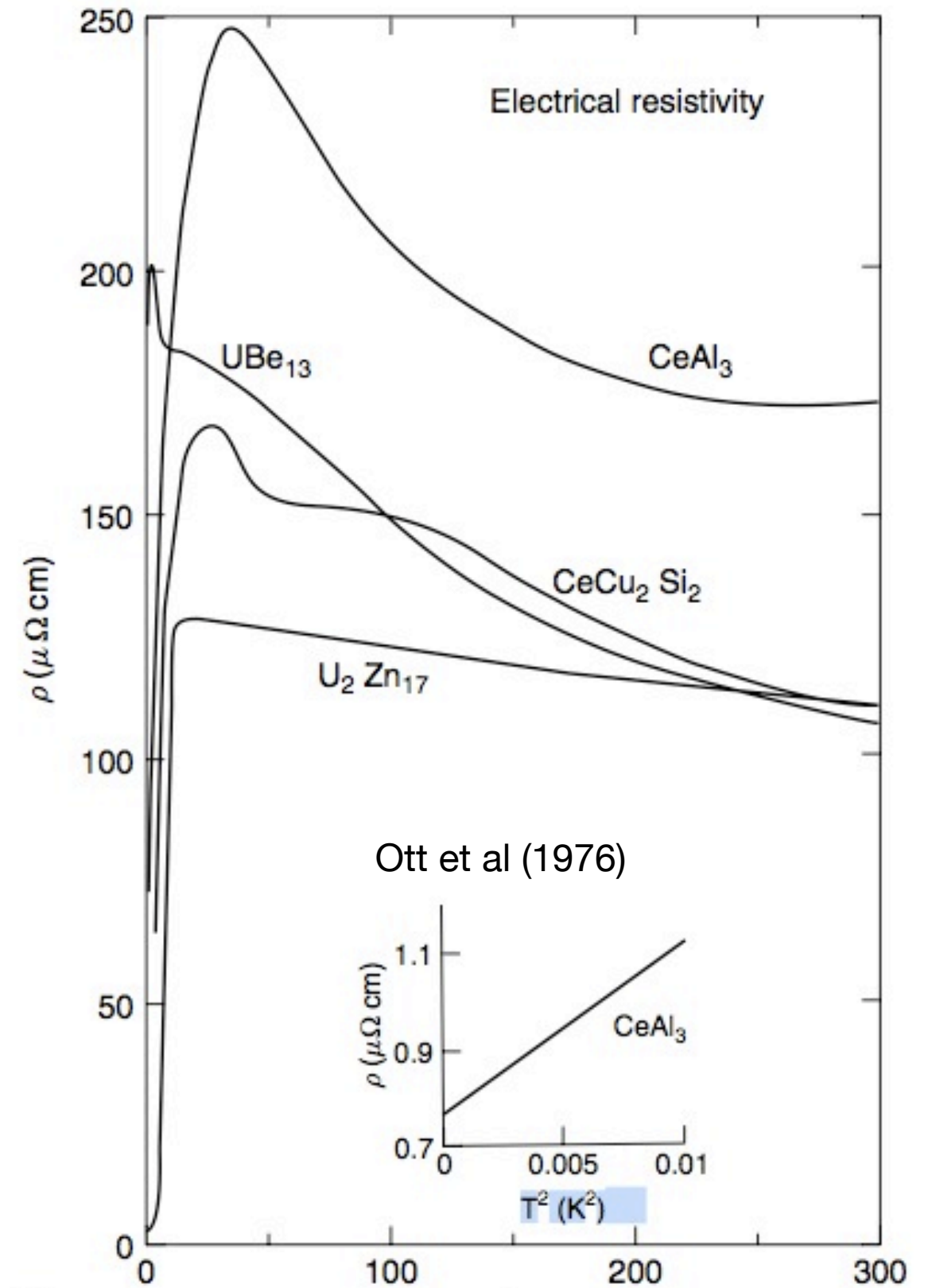
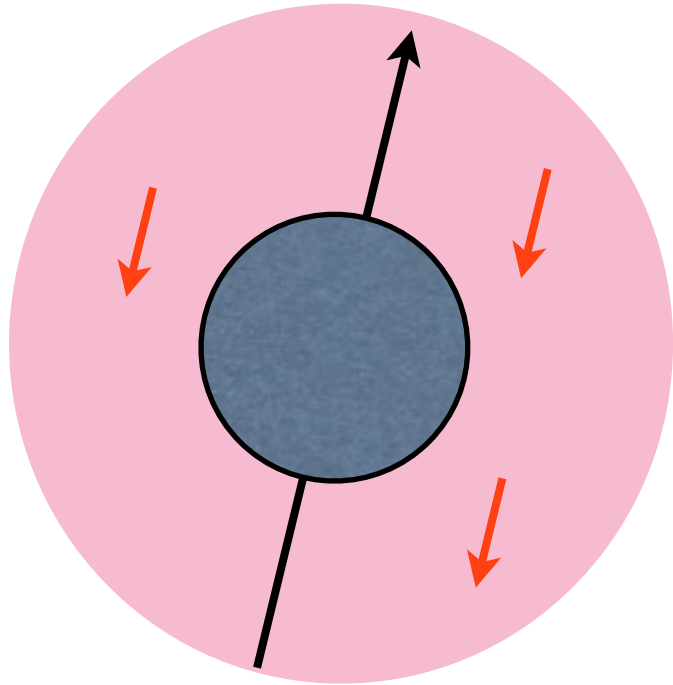
$$S(T) = \int_0^T \frac{C_V}{T'} dT'$$

Spin entropy

# Heavy Fermion Primer



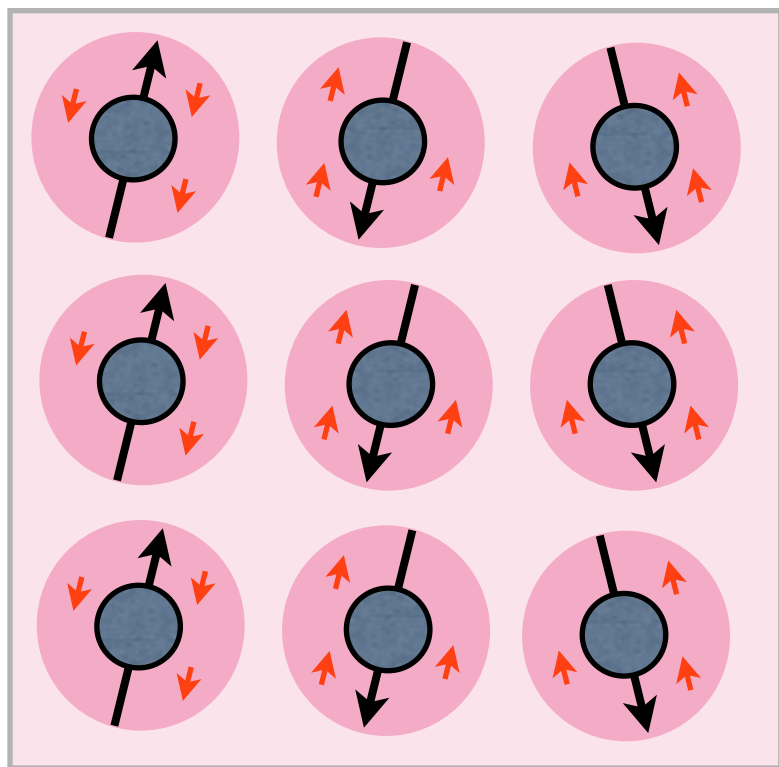
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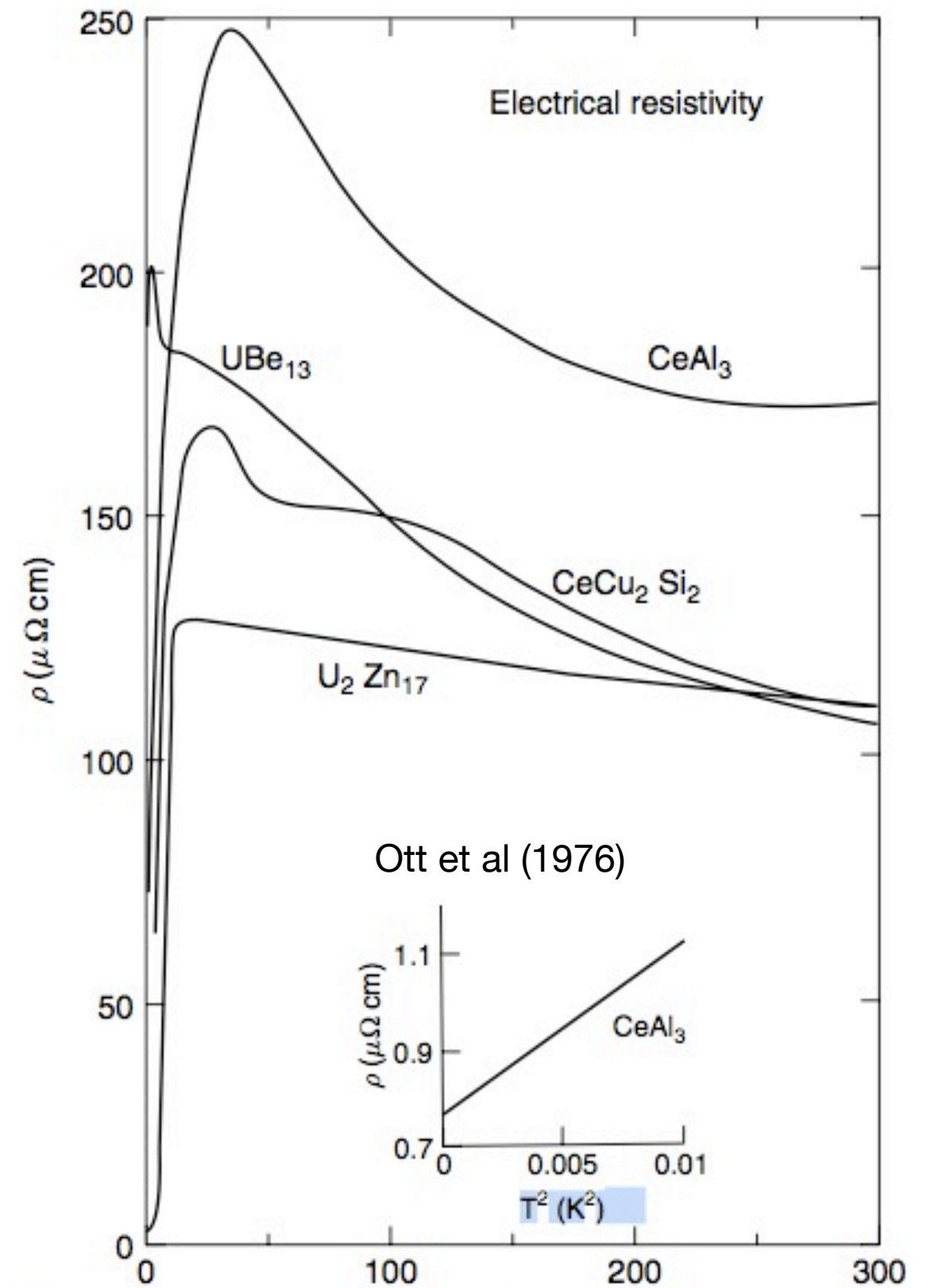
$$\rho(T) = \rho_0 + AT^2$$

Coherent Heavy Fermions

# Heavy Fermion Primer



“Kondo Lattice”

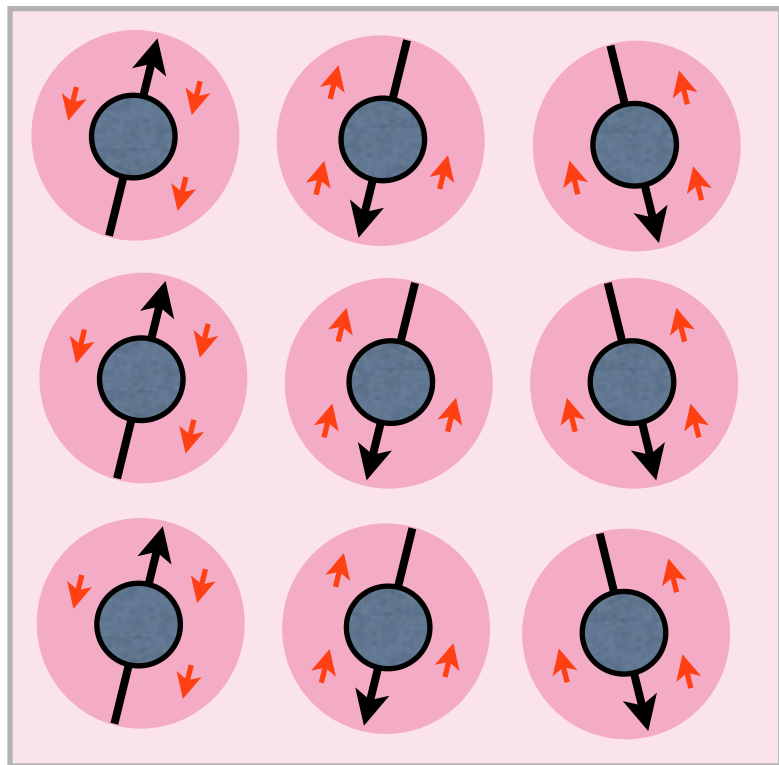


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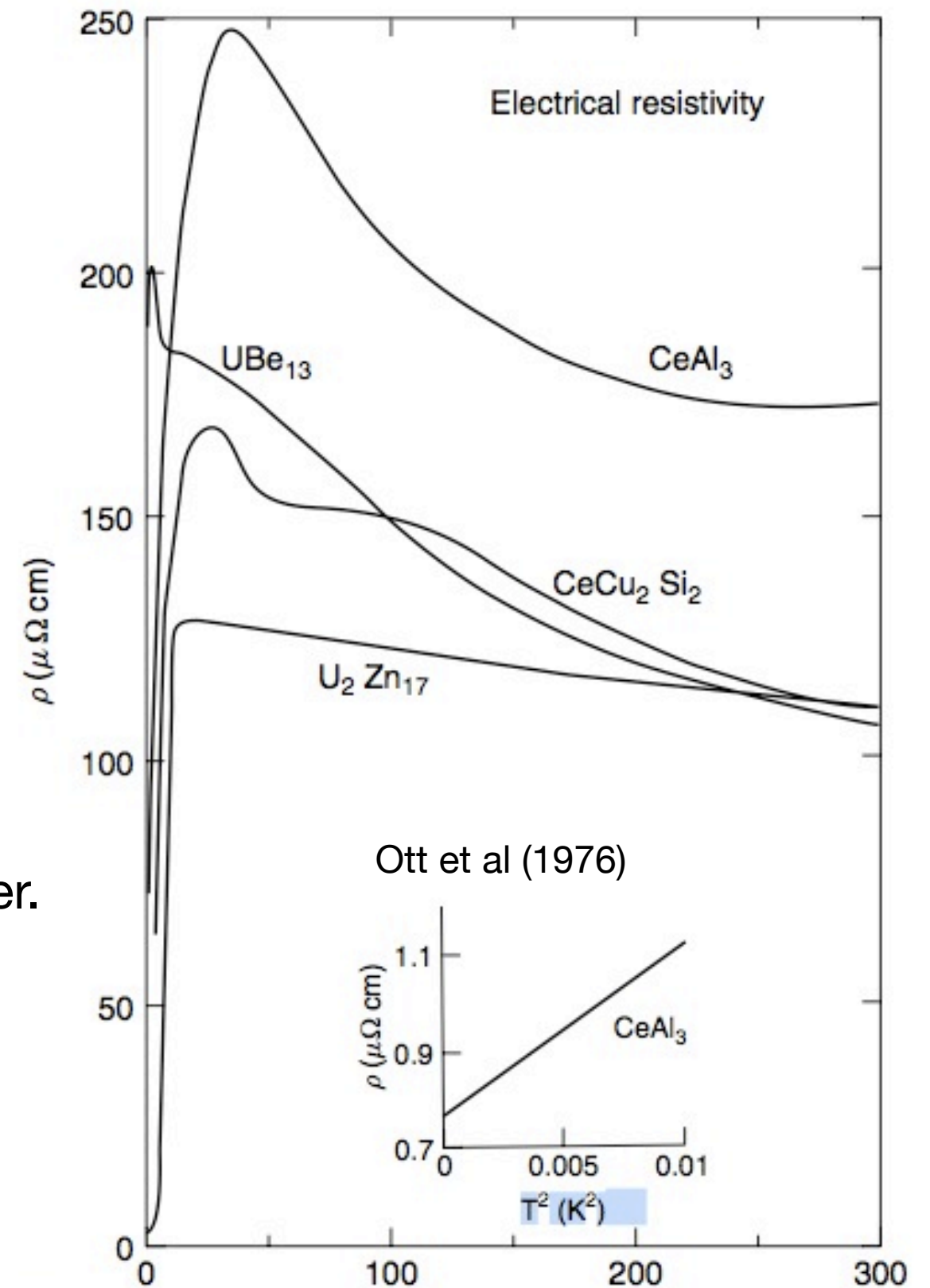


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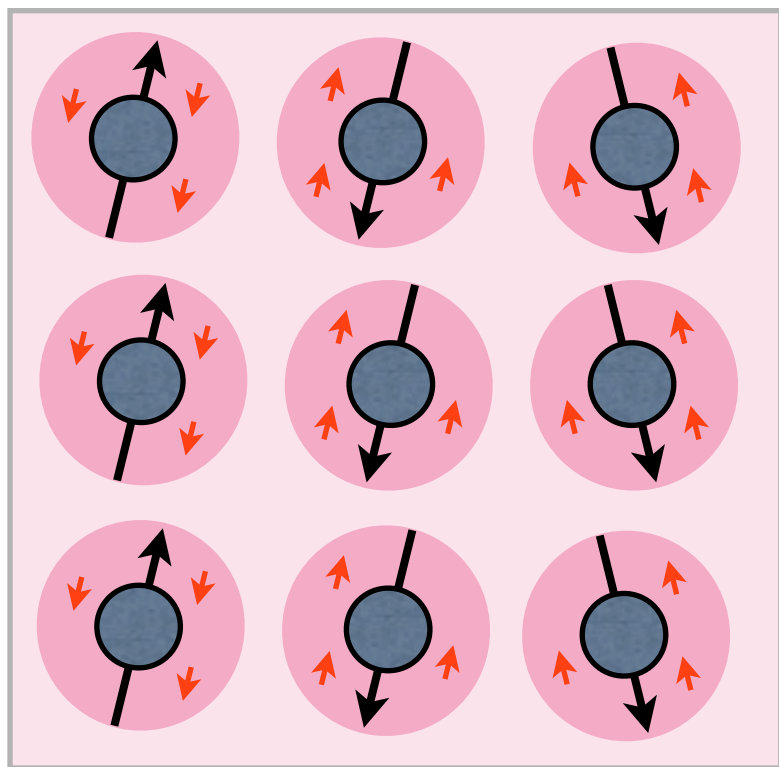
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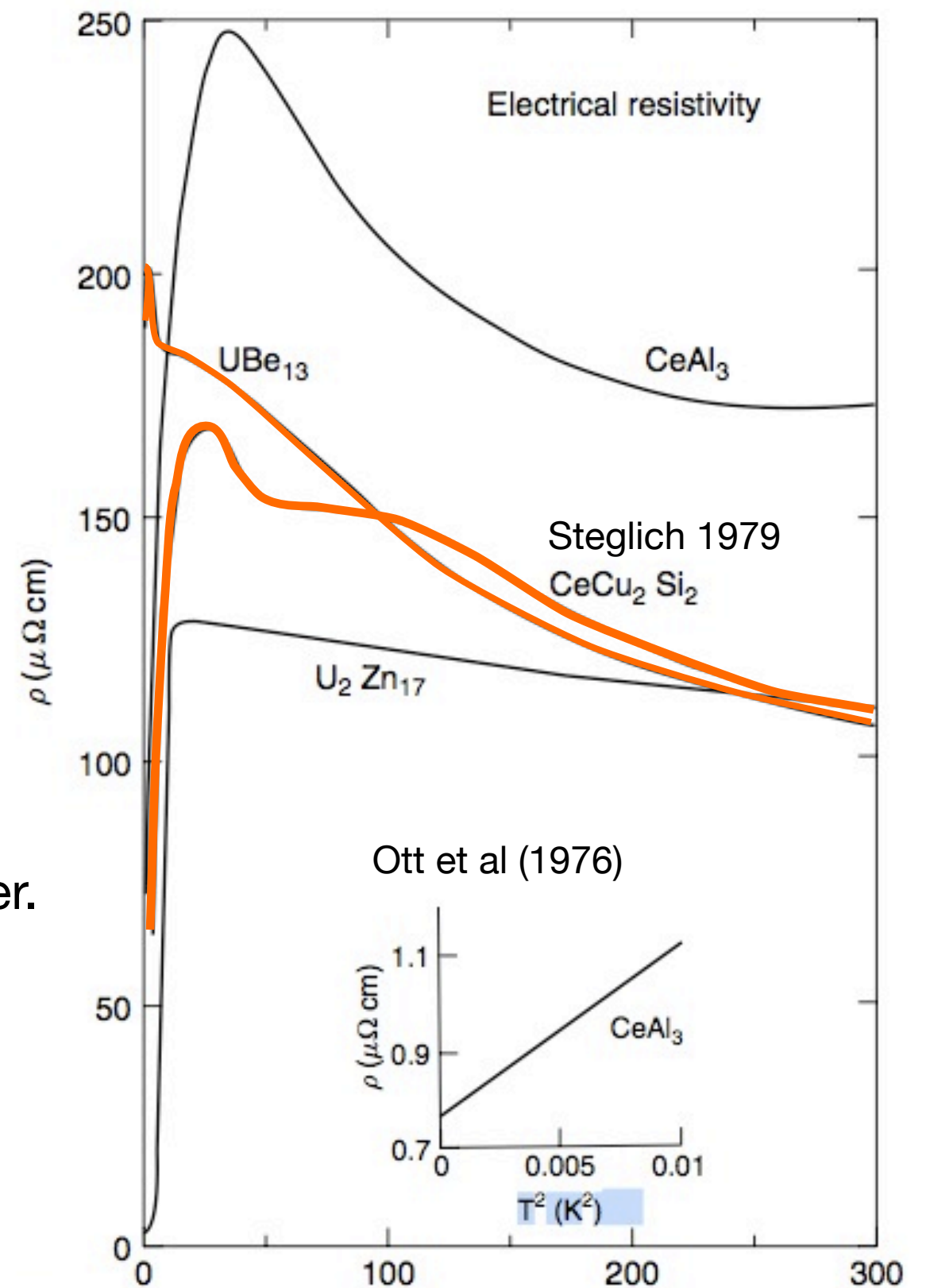
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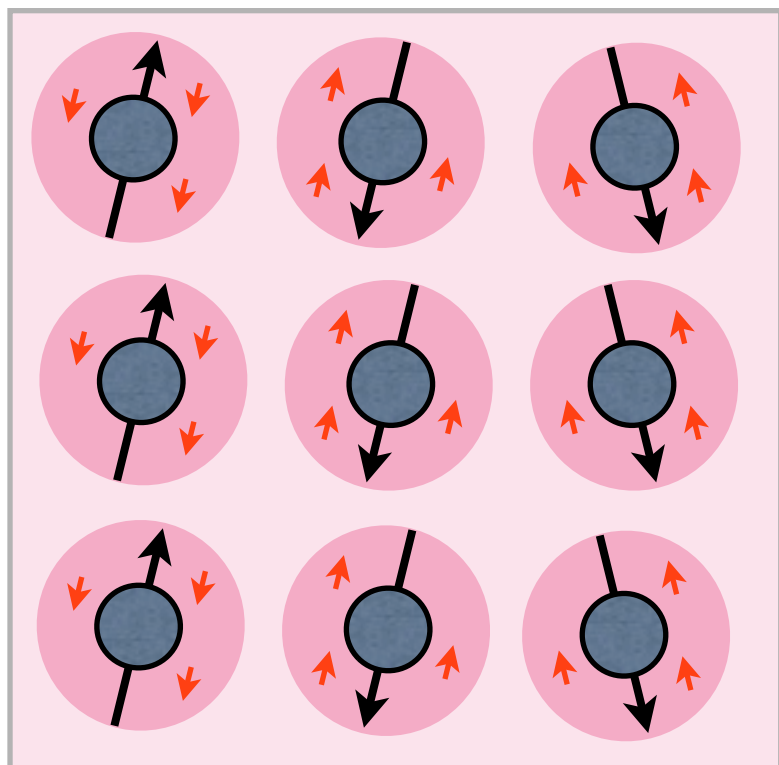
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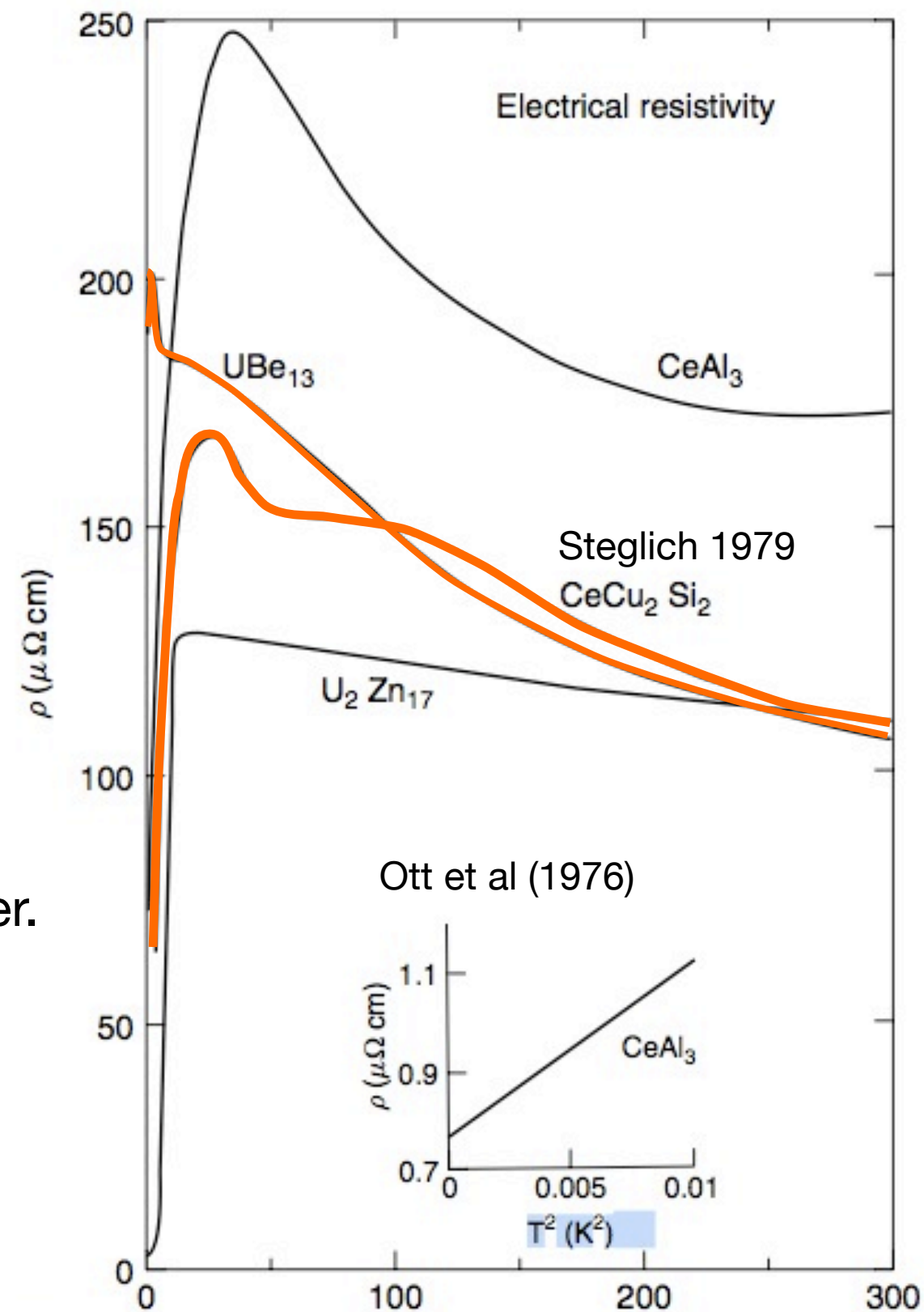
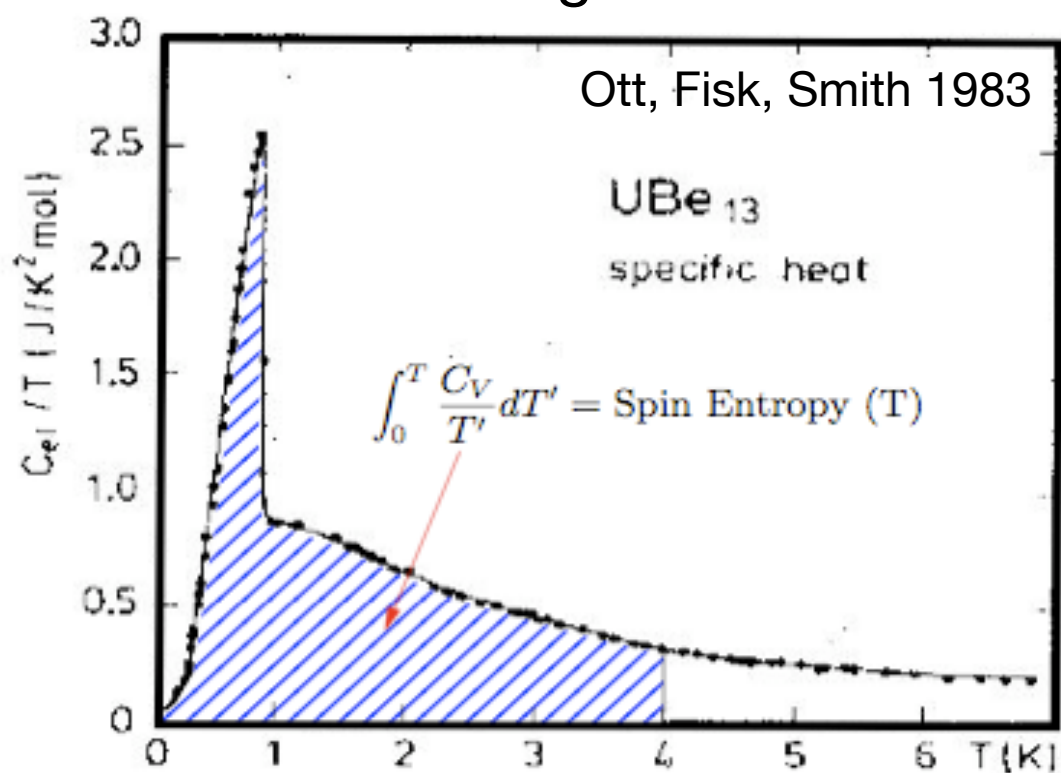
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# Kondo insulators

Menth, Bueller and Geballe (PRL 22,295, 1969)

Aeppli and Fisk (Comments CMP 16, 155, 1992)

Simplest Kondo Lattice

# MAGNETIC AND SEMICONDUCTING PROPERTIES OF $\text{SmB}_6$

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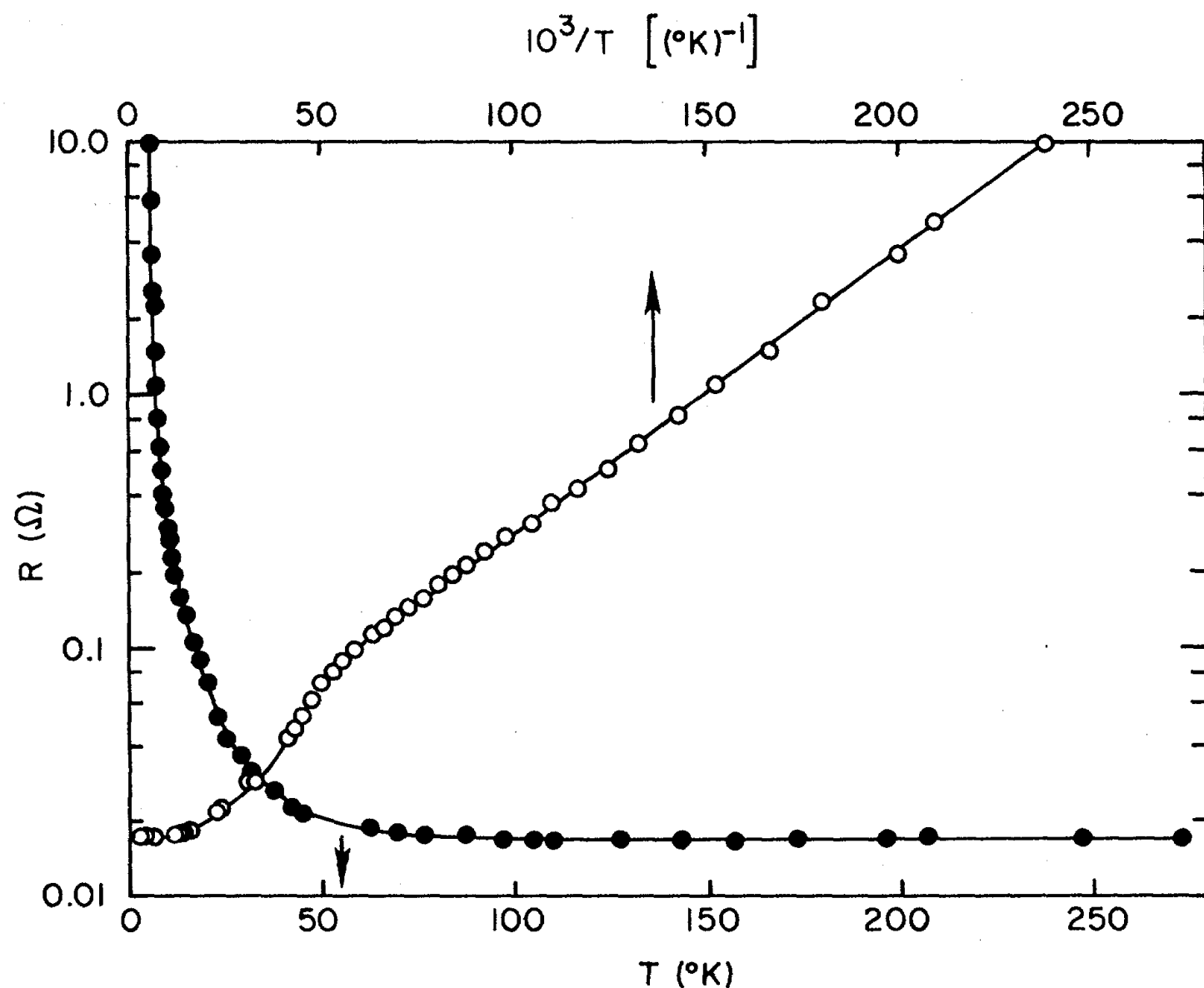


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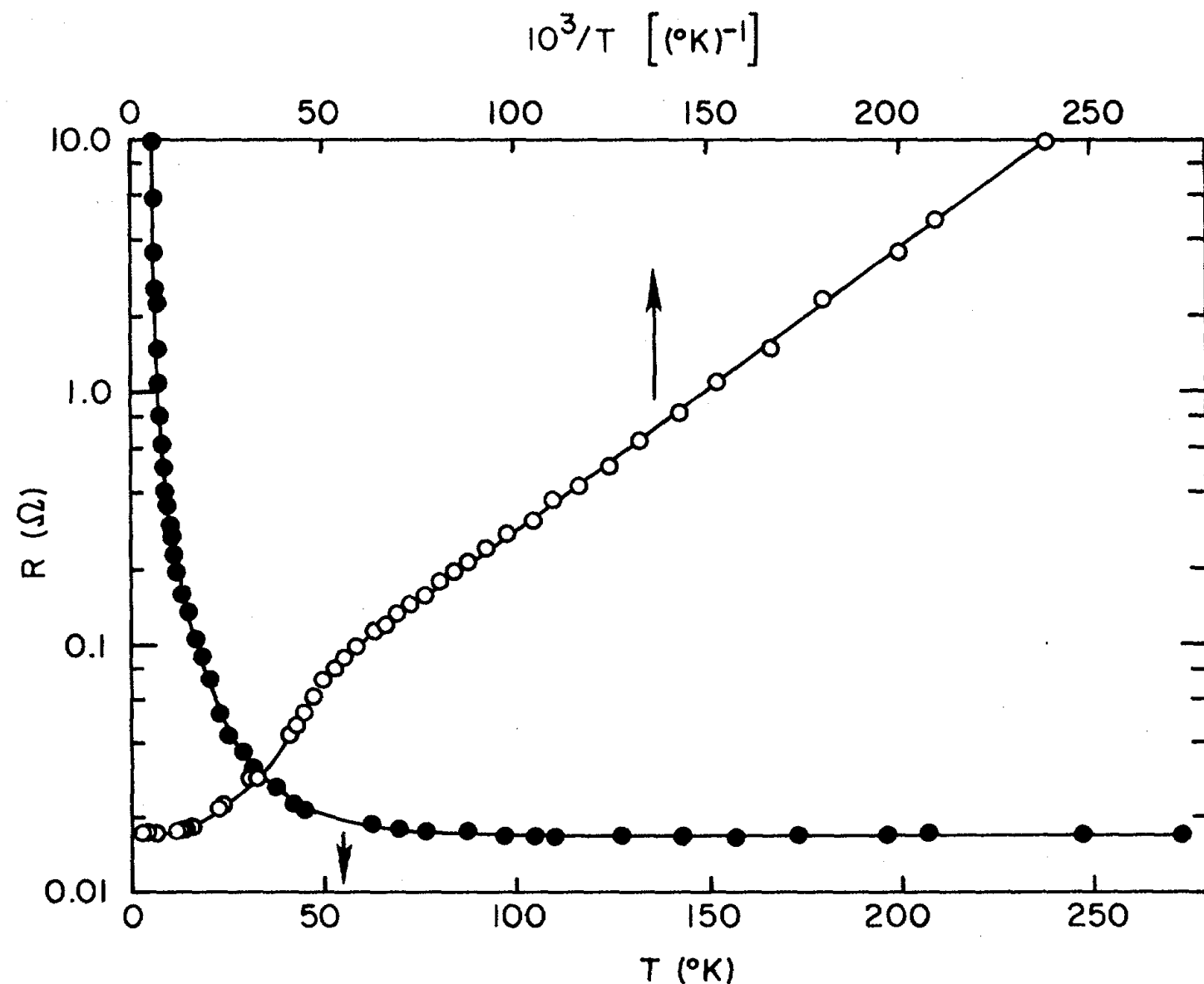
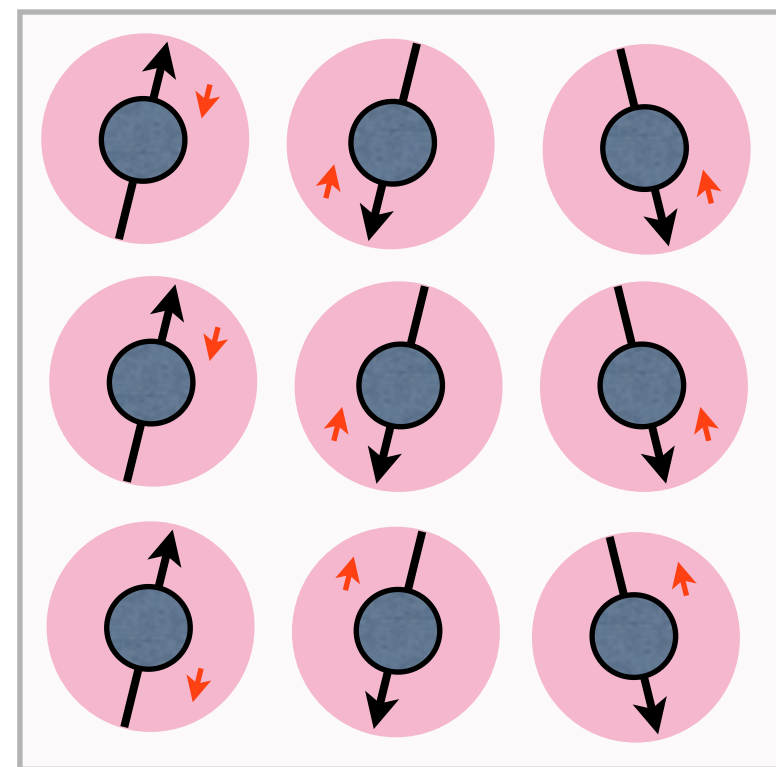


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### Simplest Kondo Lattice



Mott's Hybridization picture. Mott Phil Mag, 30,403,1974



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**Formation of Heavy f-bands:** quasiparticle hybridization of electrons  $|\mathbf{k}\sigma\rangle$  and **localized f** doublets, possibly due to Kondo effect.



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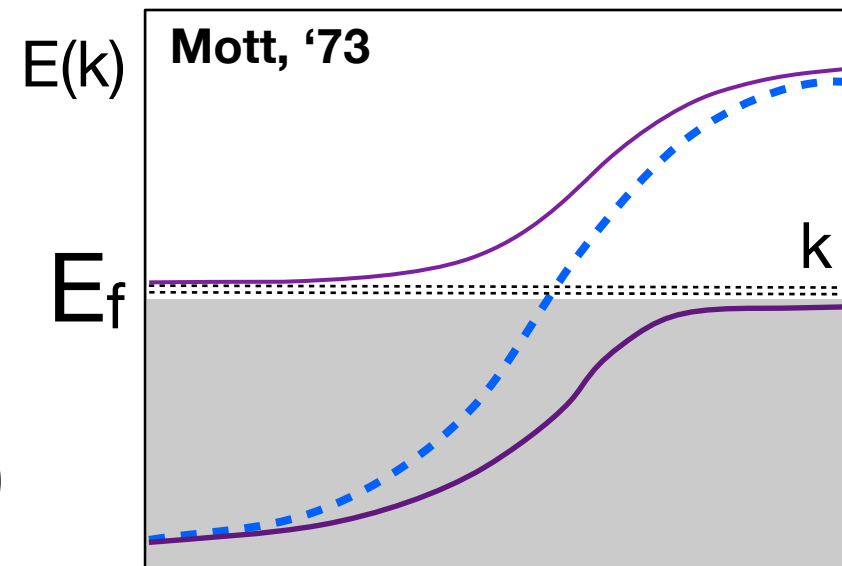
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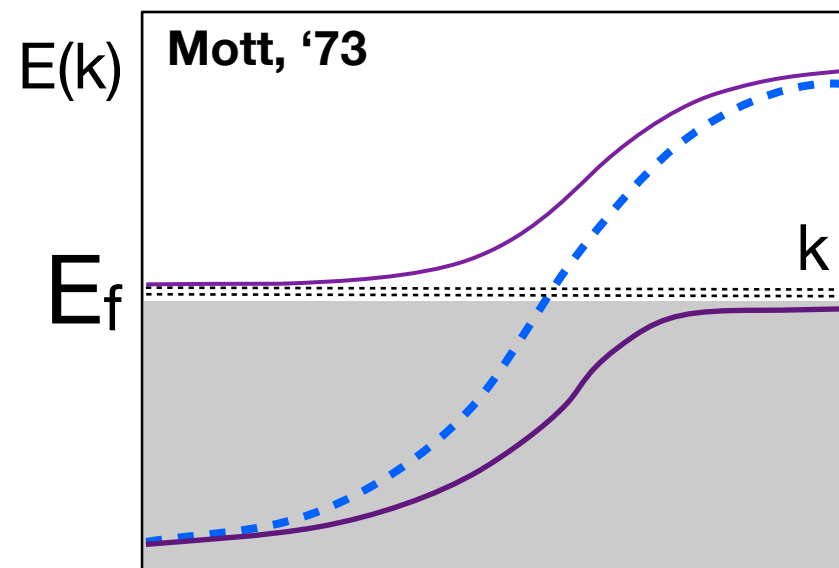
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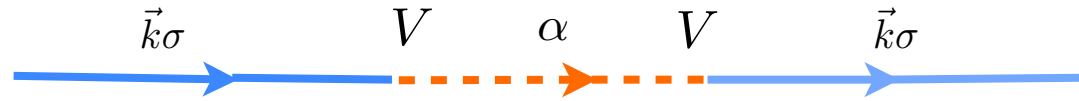
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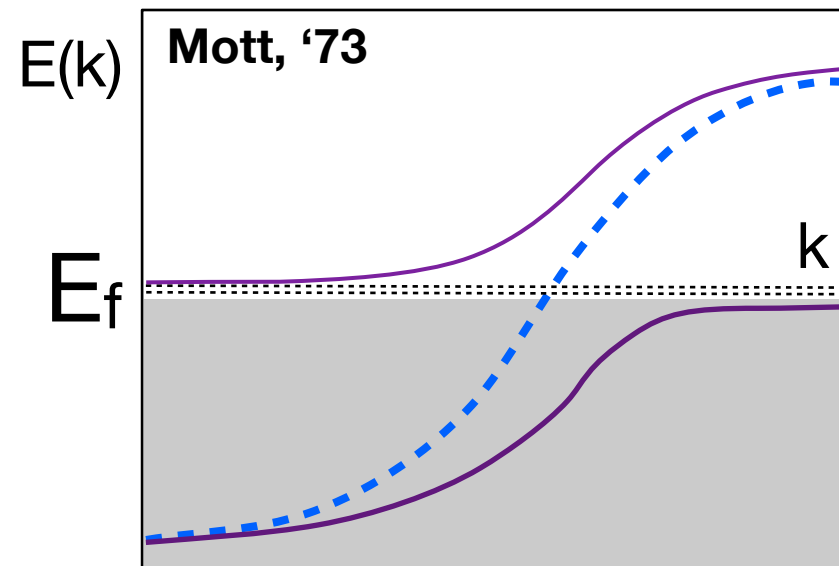


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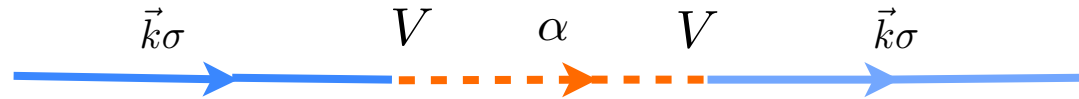


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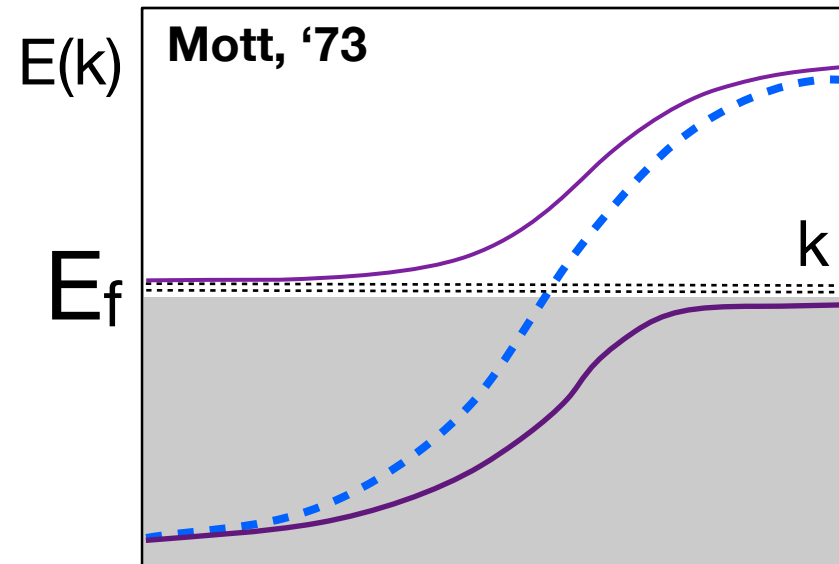


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## Rare-earth compounds with mixed valencies

By N. F. Mott

Cavendish Laboratory, University of Cambridge, England

[Received 23 May 1974]

### ABSTRACT

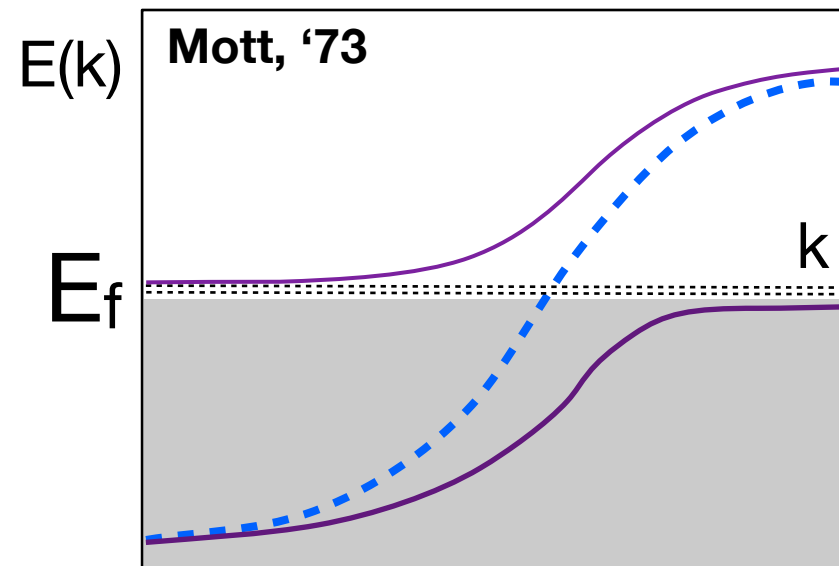
This paper reviews the properties of certain rare-earth compounds in which the 4f band has mixed valency, notably  $\text{SmB}_6$  and the high-pressure forms of  $\text{SmS}$ ,  $\text{SmSe}$  and  $\text{SmTe}$ . The metal-insulator transitions of the last three materials under pressure are discussed. It is suggested that the low-pressure form of  $\text{SmS}$  is an excitonic insulator. In  $\text{SmB}_6$  and high-pressure  $\text{SmS}$  a very small gap separates occupied from unoccupied states, this in our view being due to hybridization of 4f and 5d bands. The electrical properties are discussed; if  $kT$  is greater than the gap energy, then the gap does not affect the metallic behaviour. Finally metallic compounds such as  $\text{CeAl}_3$  are described, in which there is no magnetic ordering at low temperatures, and it is suggested that this must always occur if the Kondo temperature is higher than the  $RKKY$  interaction. In this case, as in compounds with mixed valency, the Fermi energy will pass through the 4f band, and there is a very large enhancement of the effective mass. The relationship to the side-band model is discussed.

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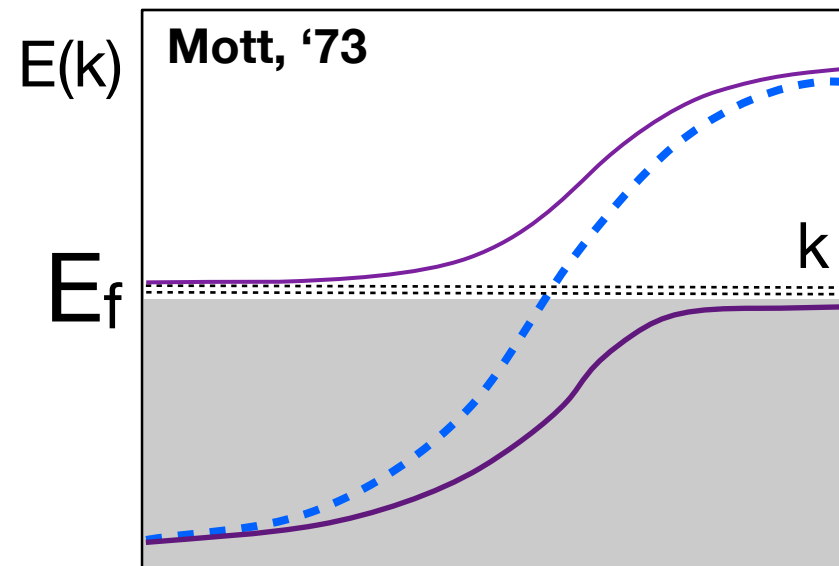


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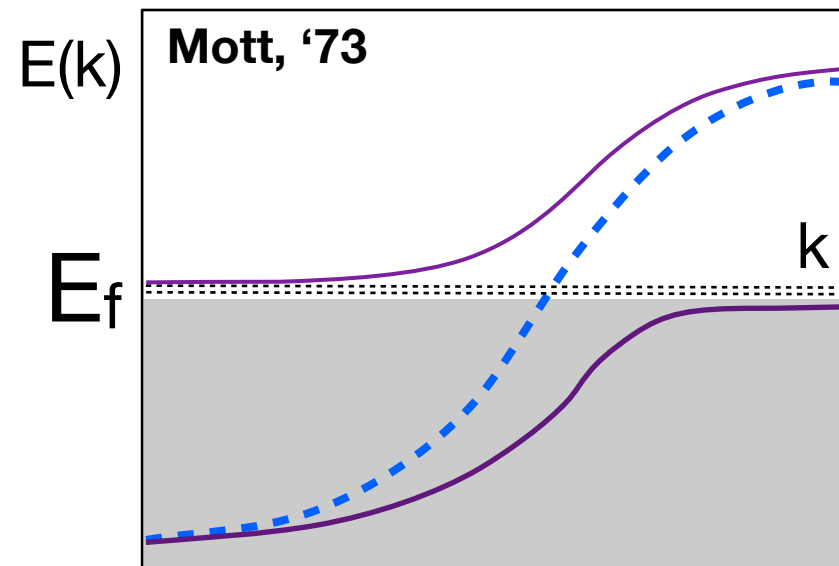
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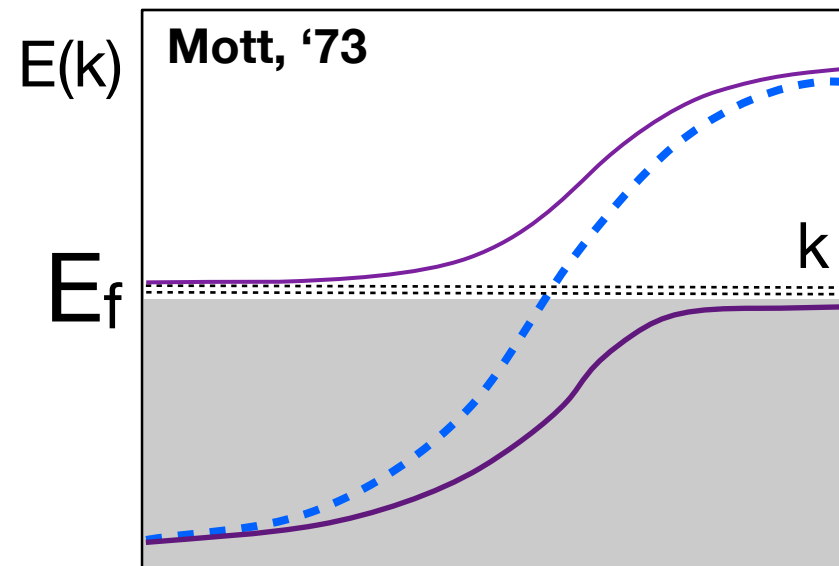


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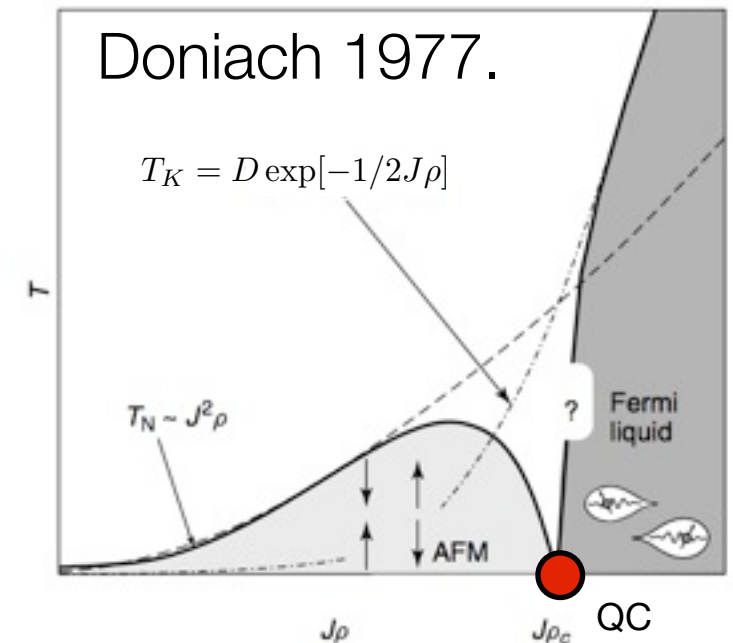
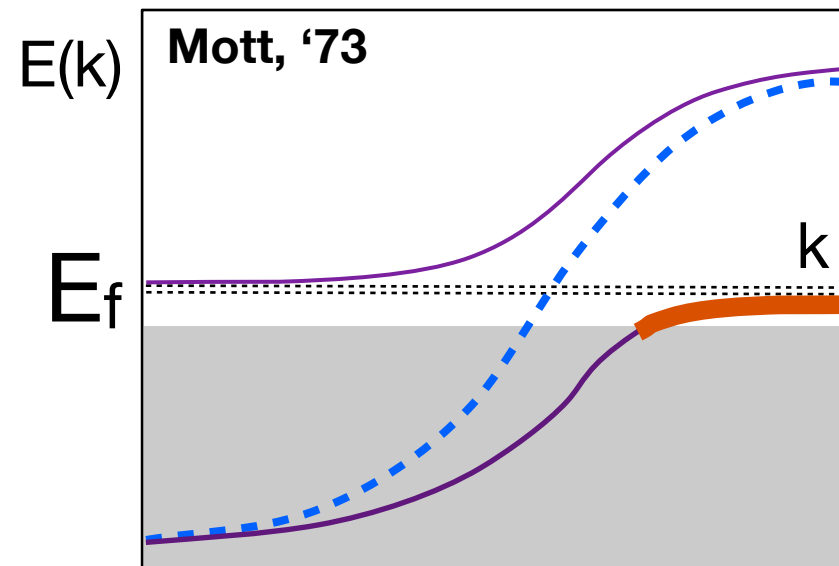
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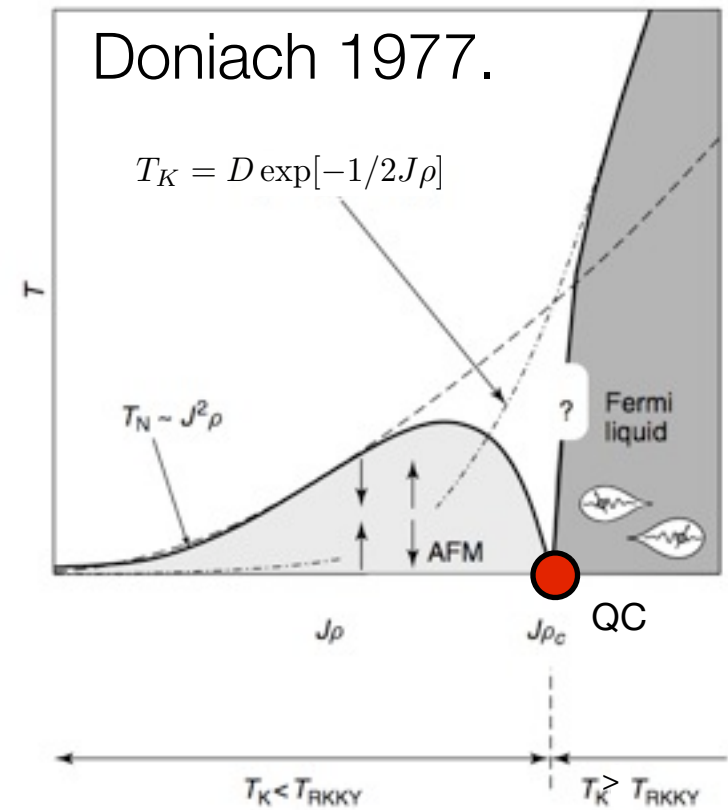
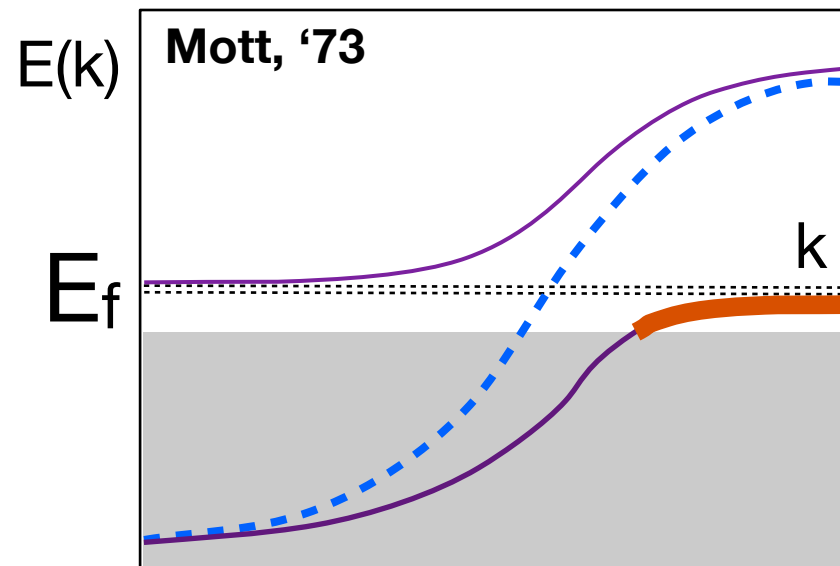
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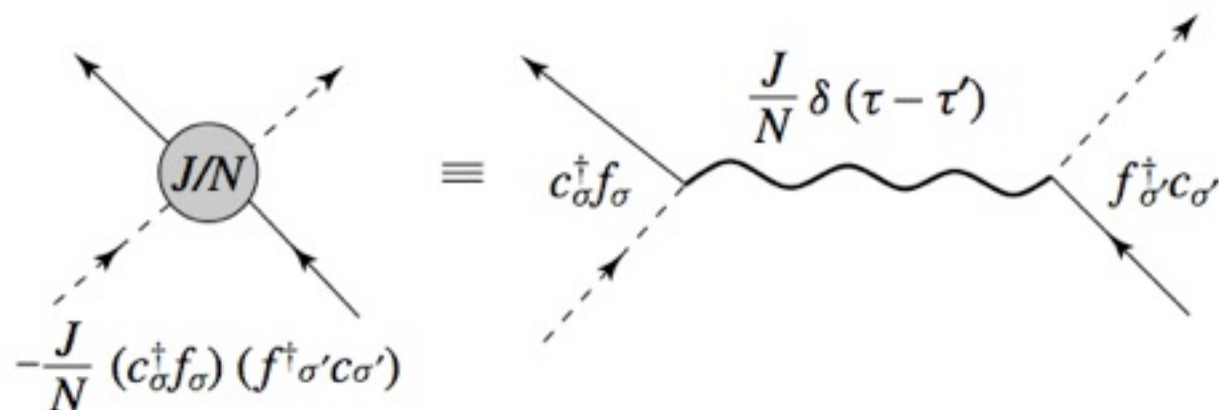
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- The large N approach to the Kondo lattice.  
Spin x conduction = composite fermion

$$H_I(j) = \frac{J}{N} (c_{j\beta}^\dagger c_{j\alpha}) S_{\alpha\beta} \rightarrow \left( \bar{V}_j c_{j\beta}^\dagger f_{j\beta} + \text{H.c} \right)$$



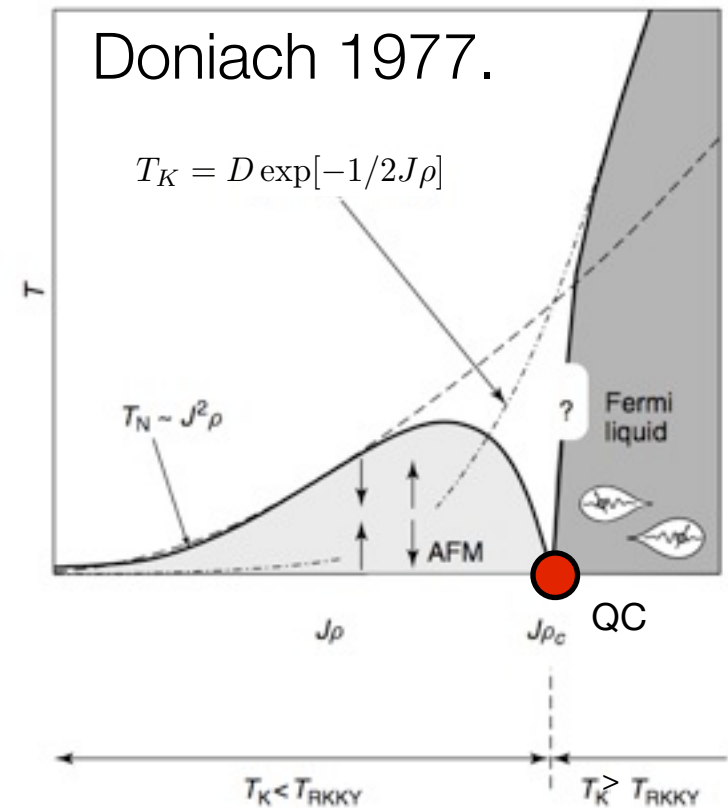
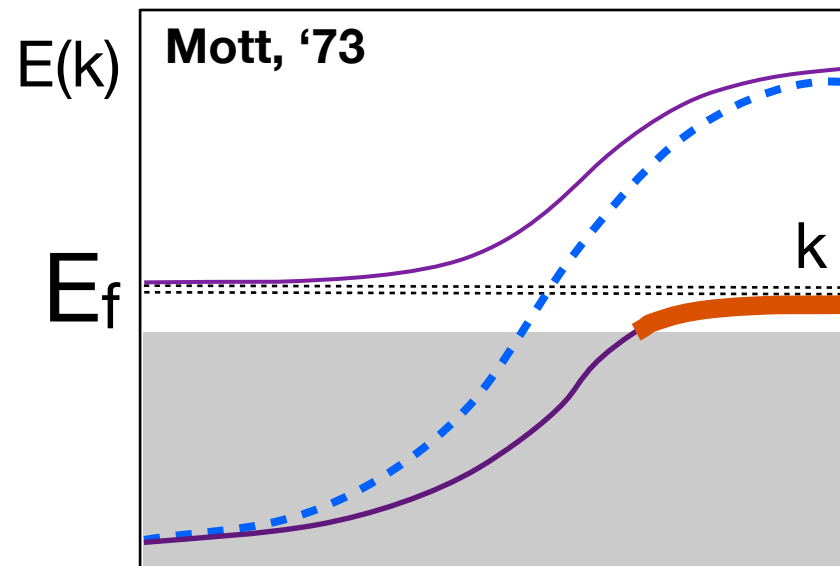


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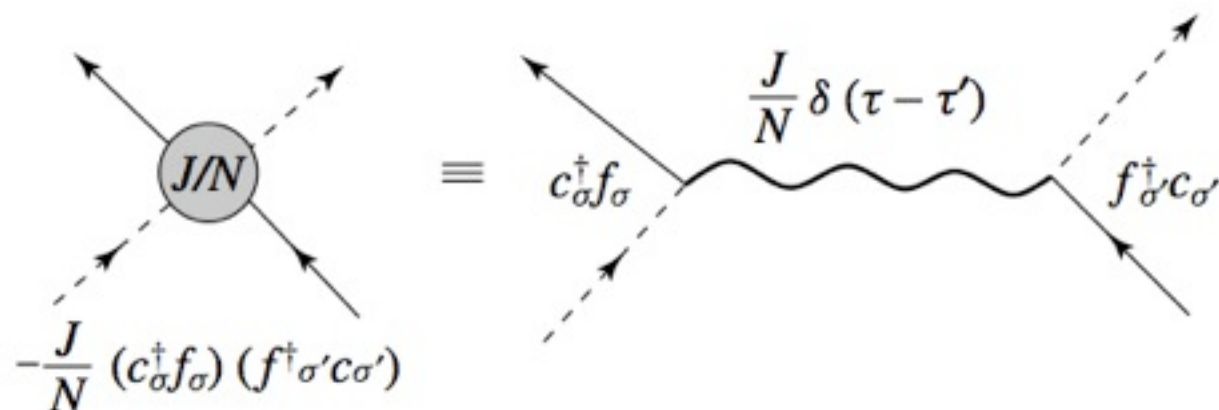
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Composite Fermion

Read & Newns 1983, PC 1983.

# MAGNETIC AND SEMICONDUCTING PROPERTIES OF $\text{SmB}_6$ <sup>†</sup>

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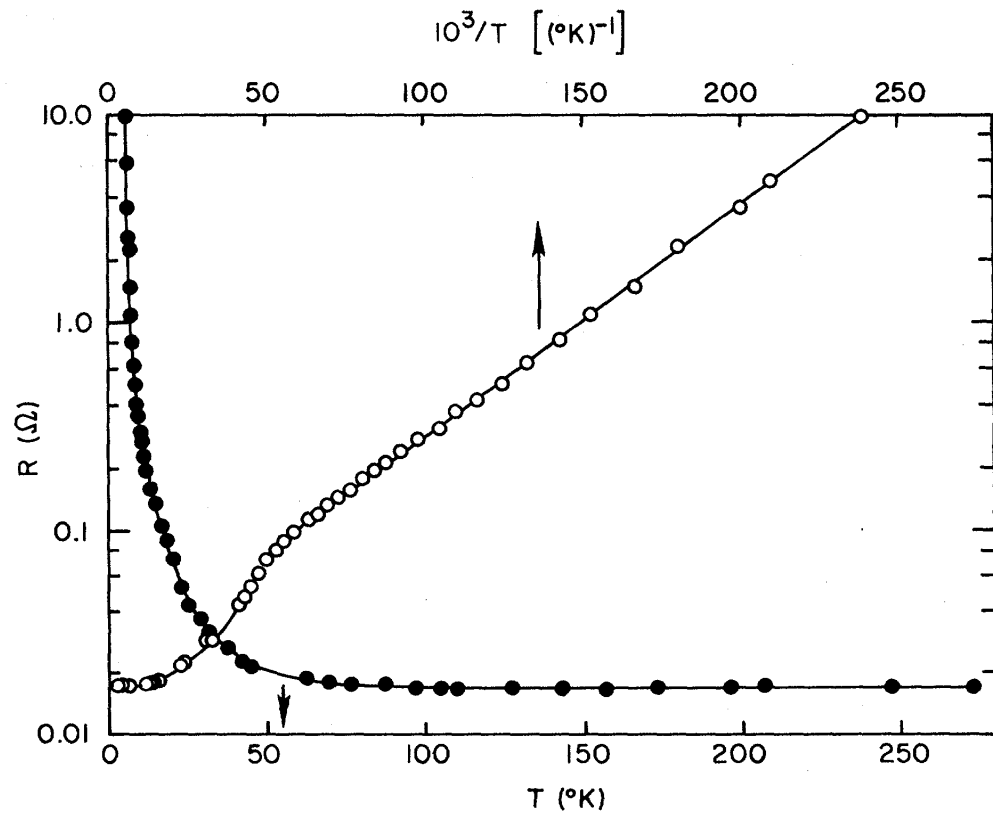


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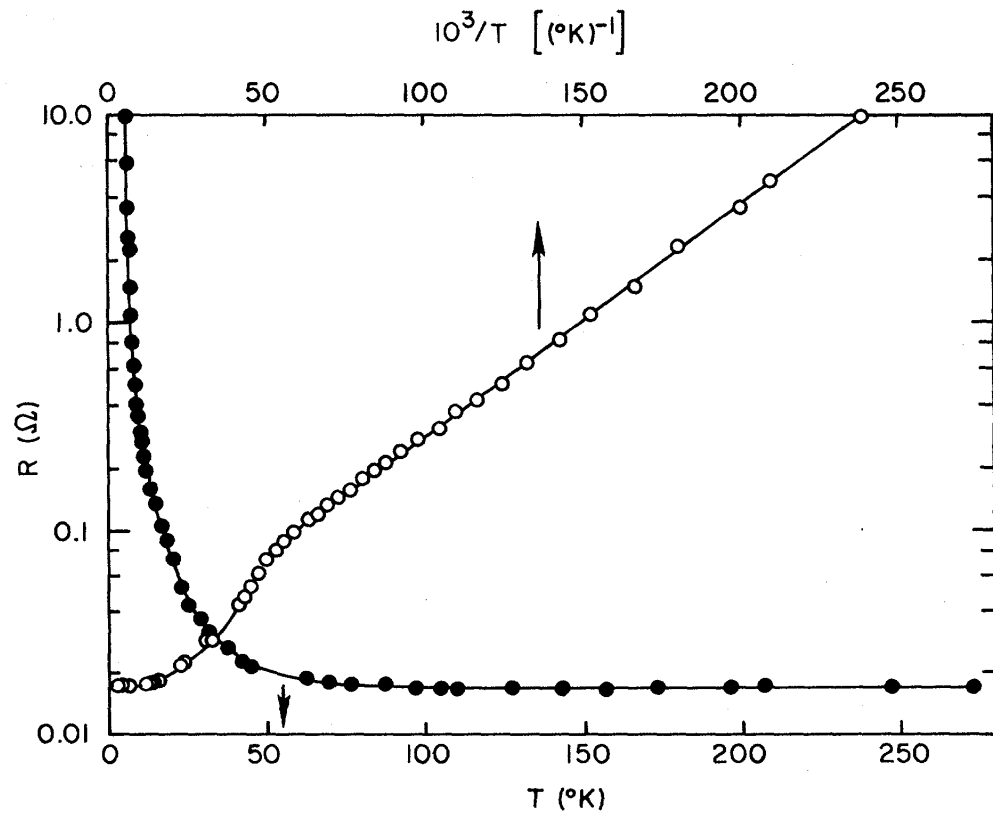
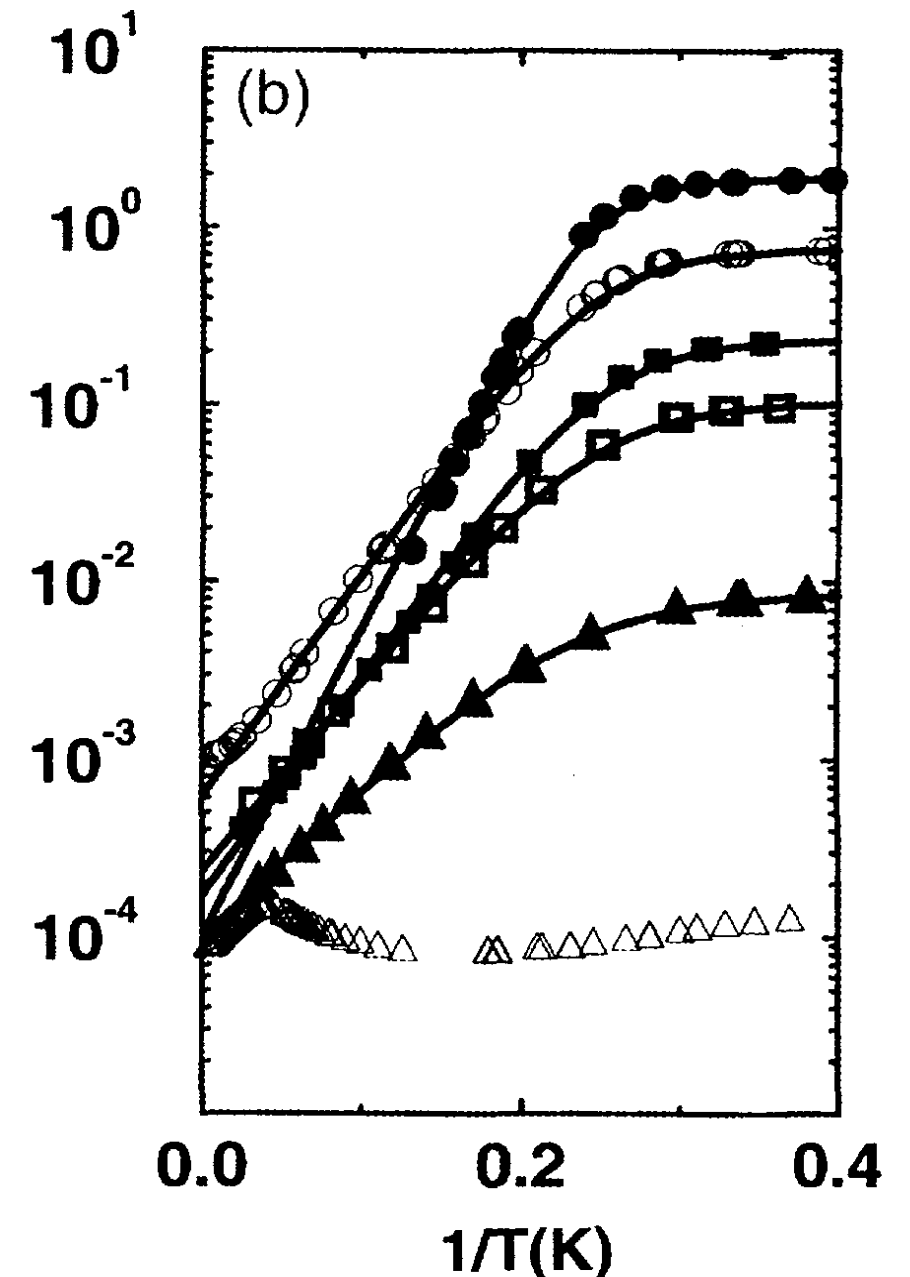


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**Persistent conductivity  
Plateau** Cooley et al (1995)

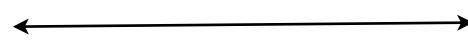


# Topological insulators.

Hasan and Kane (RMP 2009)

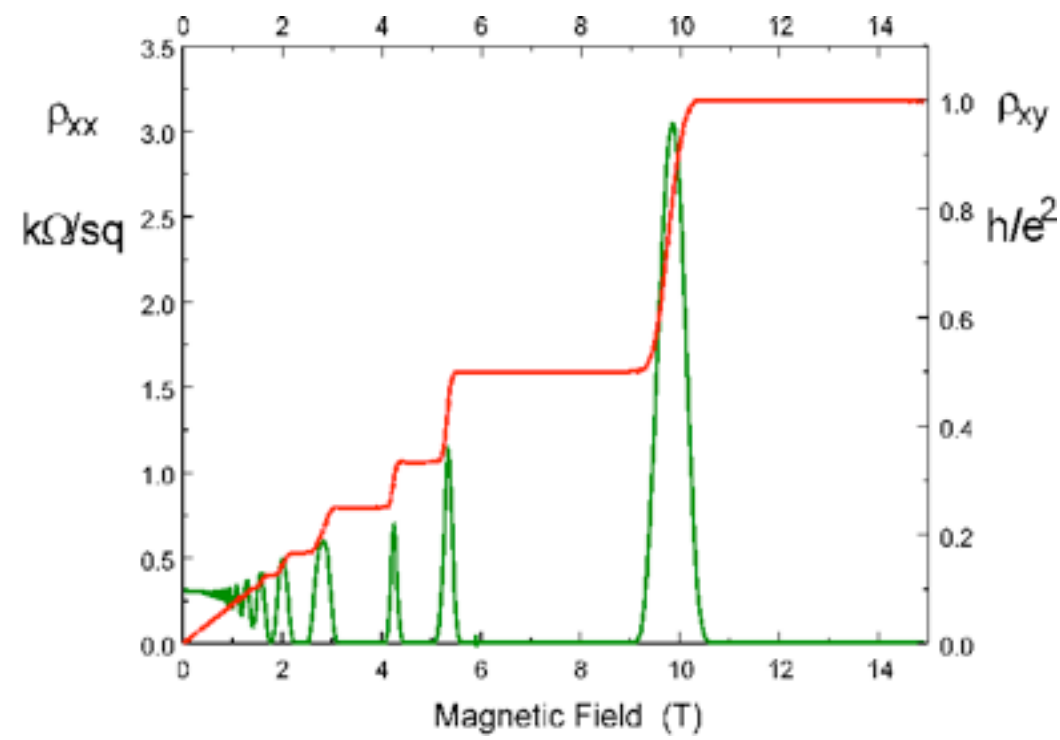
Qi and Zhang (RMP 2010)

Differential Geometry  
of the wavefunction



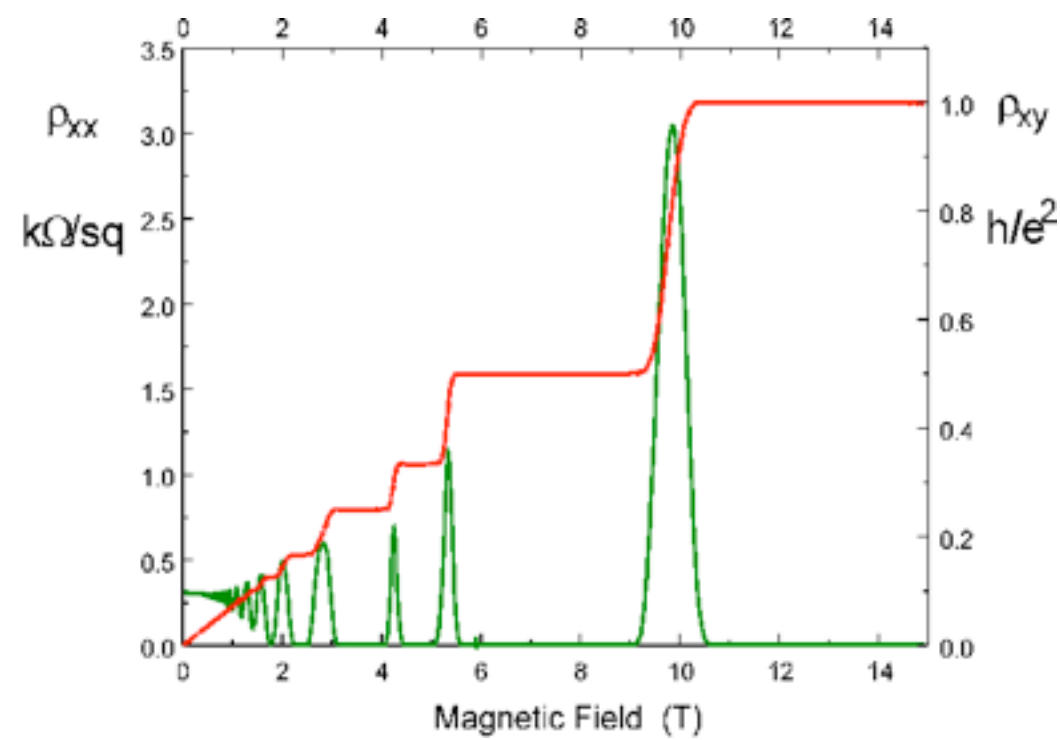
Topological  
states of matter





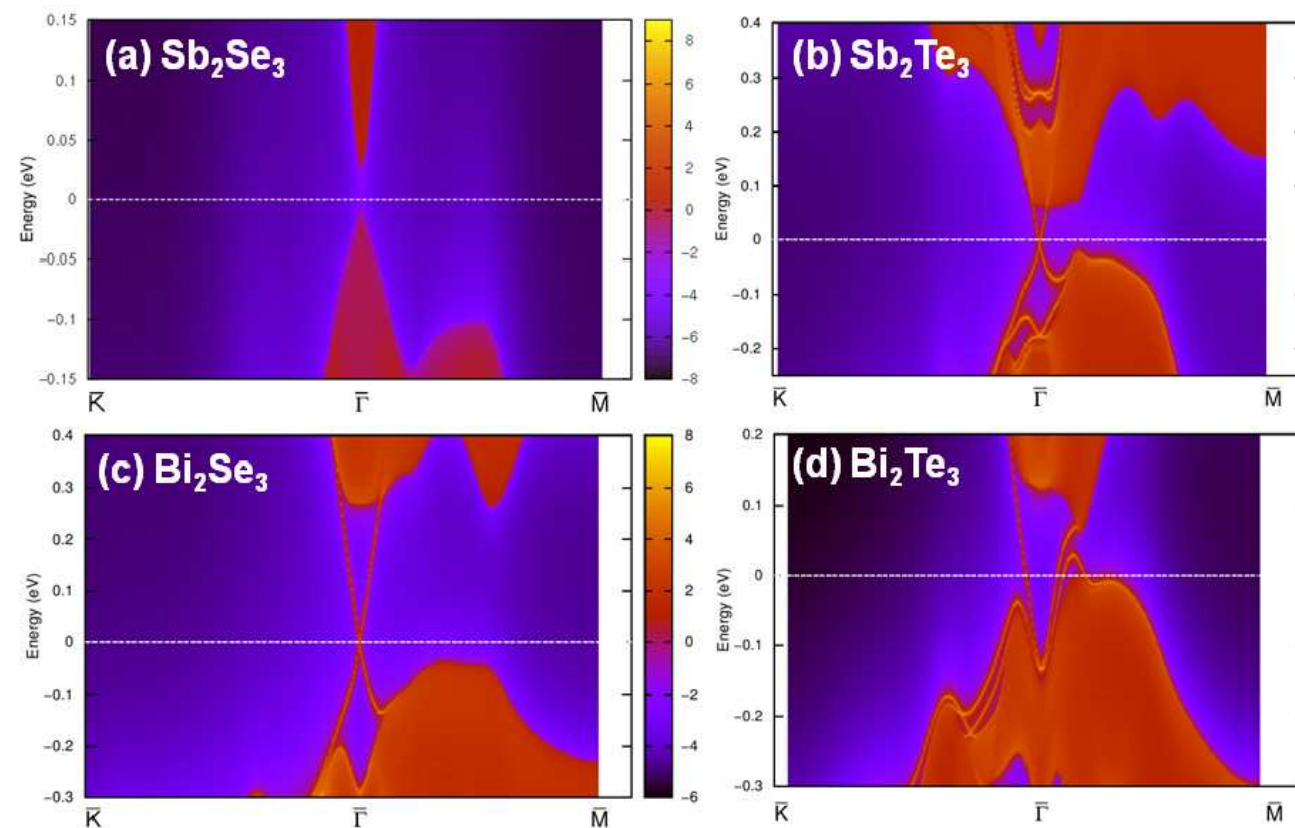
von Klitzing, Dorda & Pepper (1980)

Differential Geometry of the wavefunction  $\longleftrightarrow$  Topological states of matter

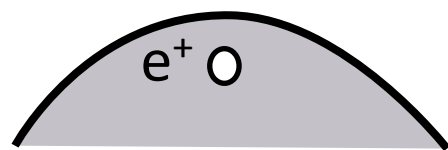
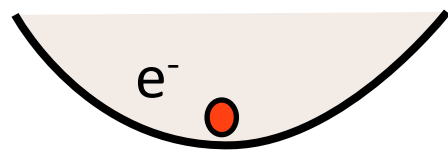


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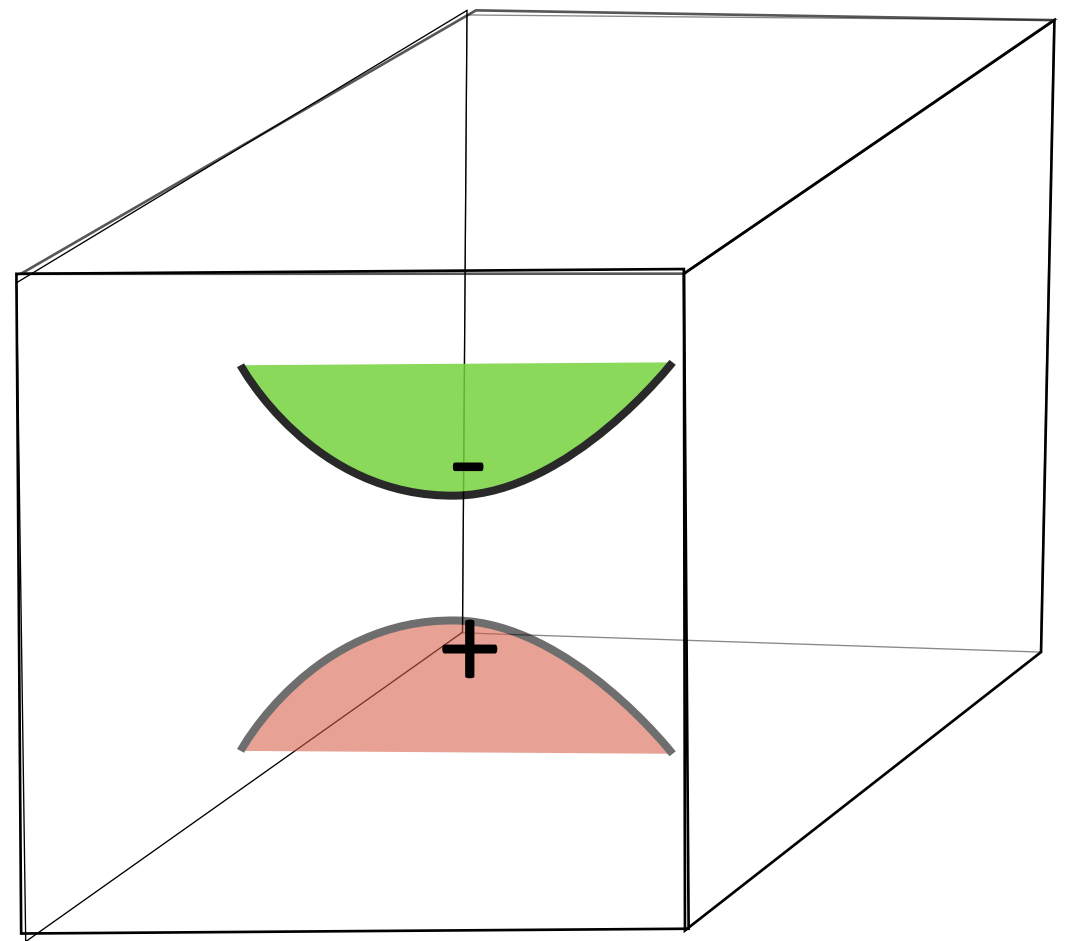
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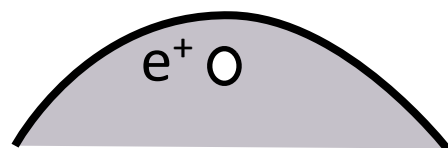


Qi and Zhang, Rev. Mod Phys (2010).



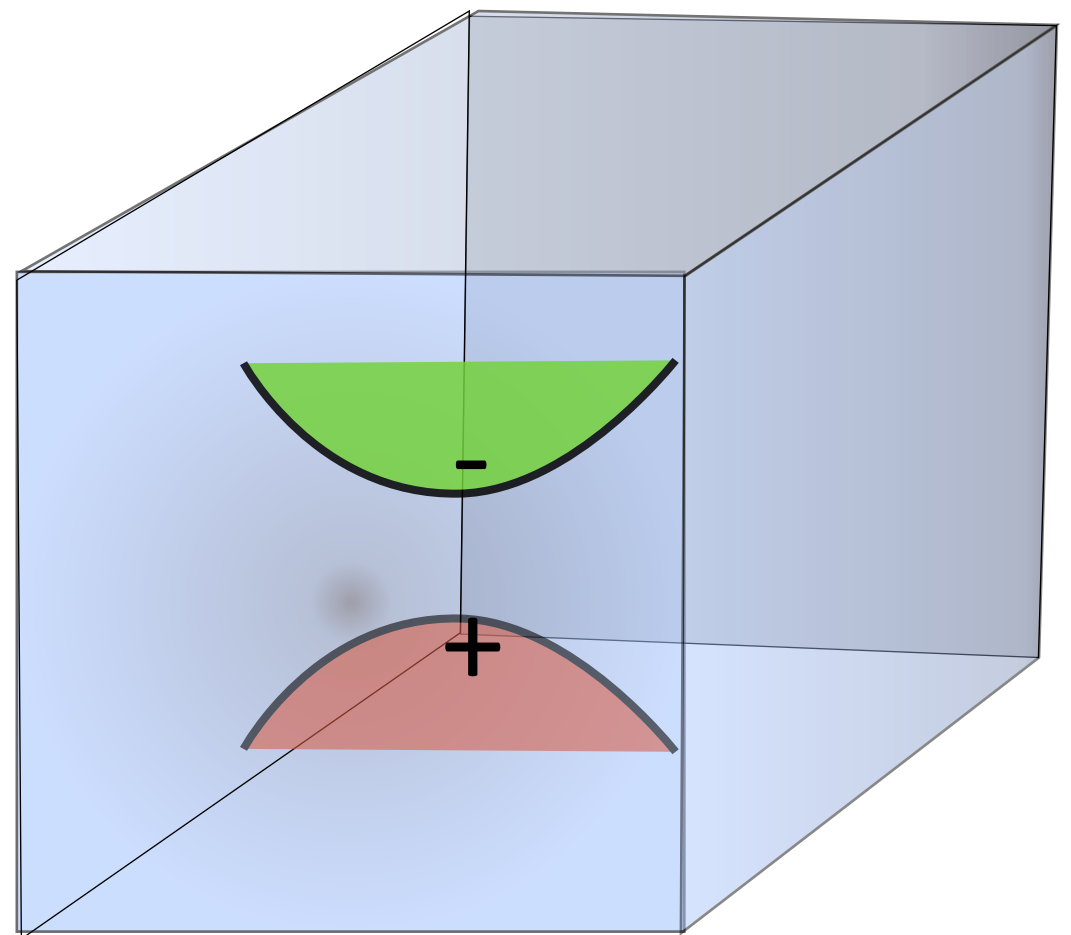
Vacuum



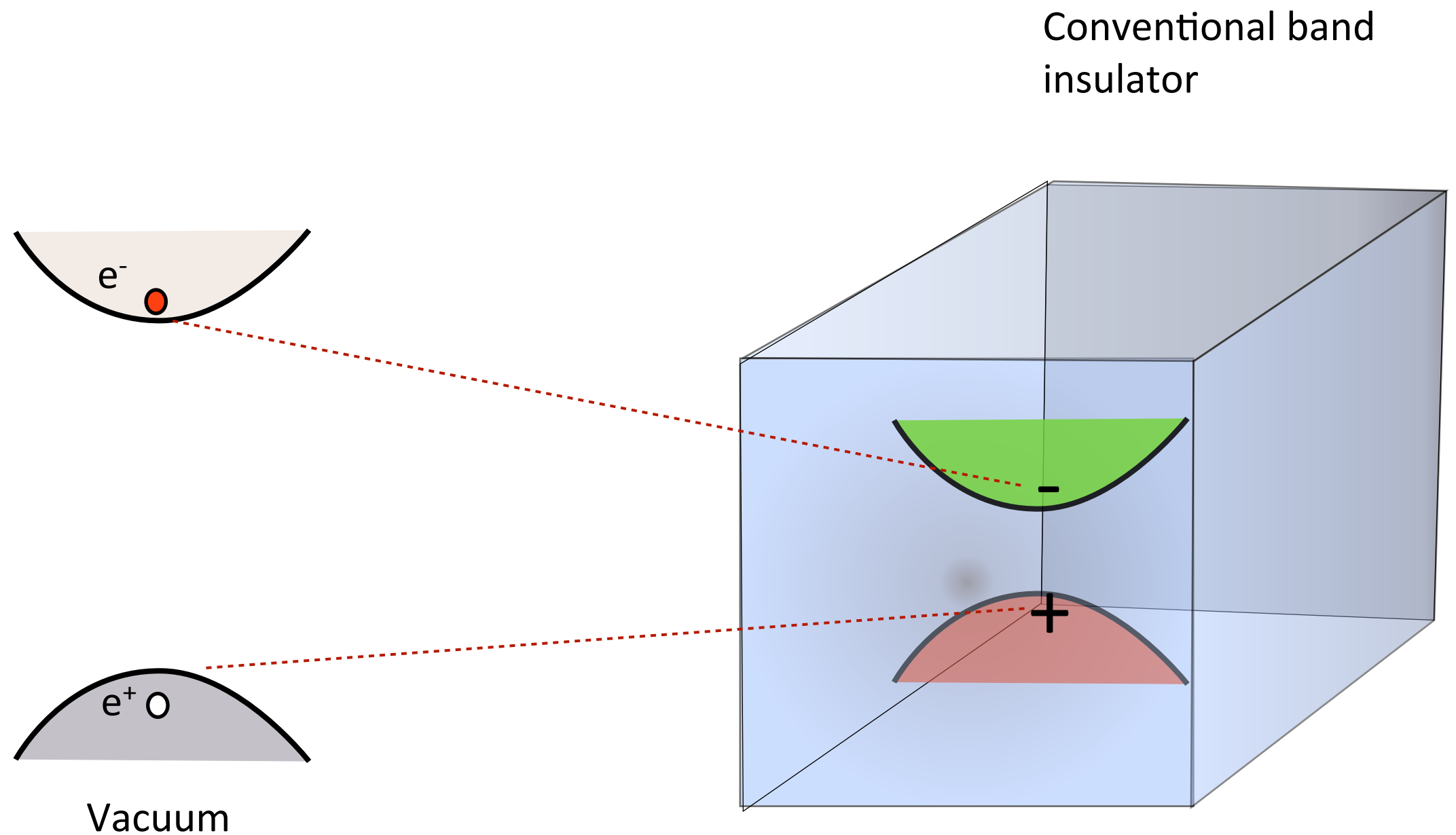


Vacuum

Conventional band  
insulator

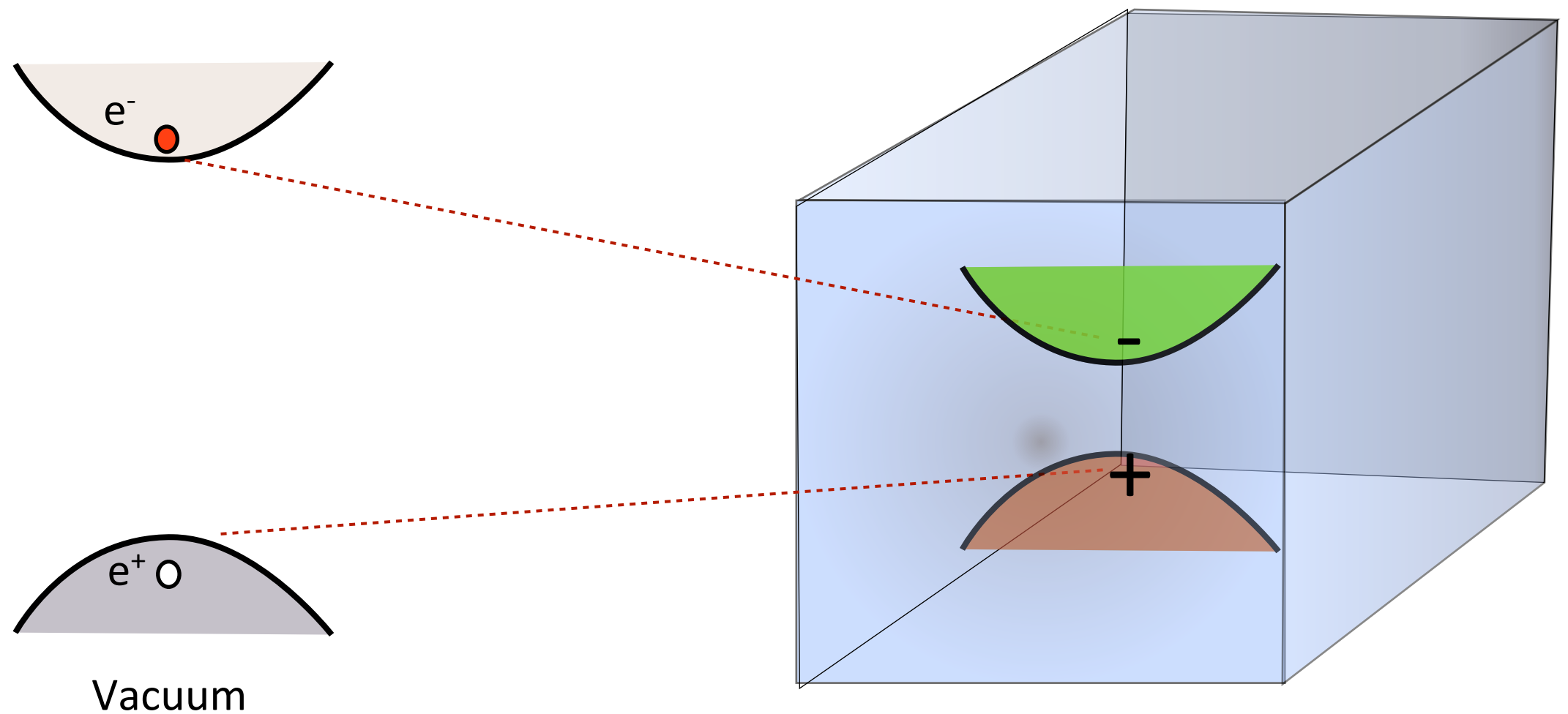


# Conventional band insulator: adiabatic continuation of the vacuum.



Topological insulator: adiabatically disconnected to vacuum.

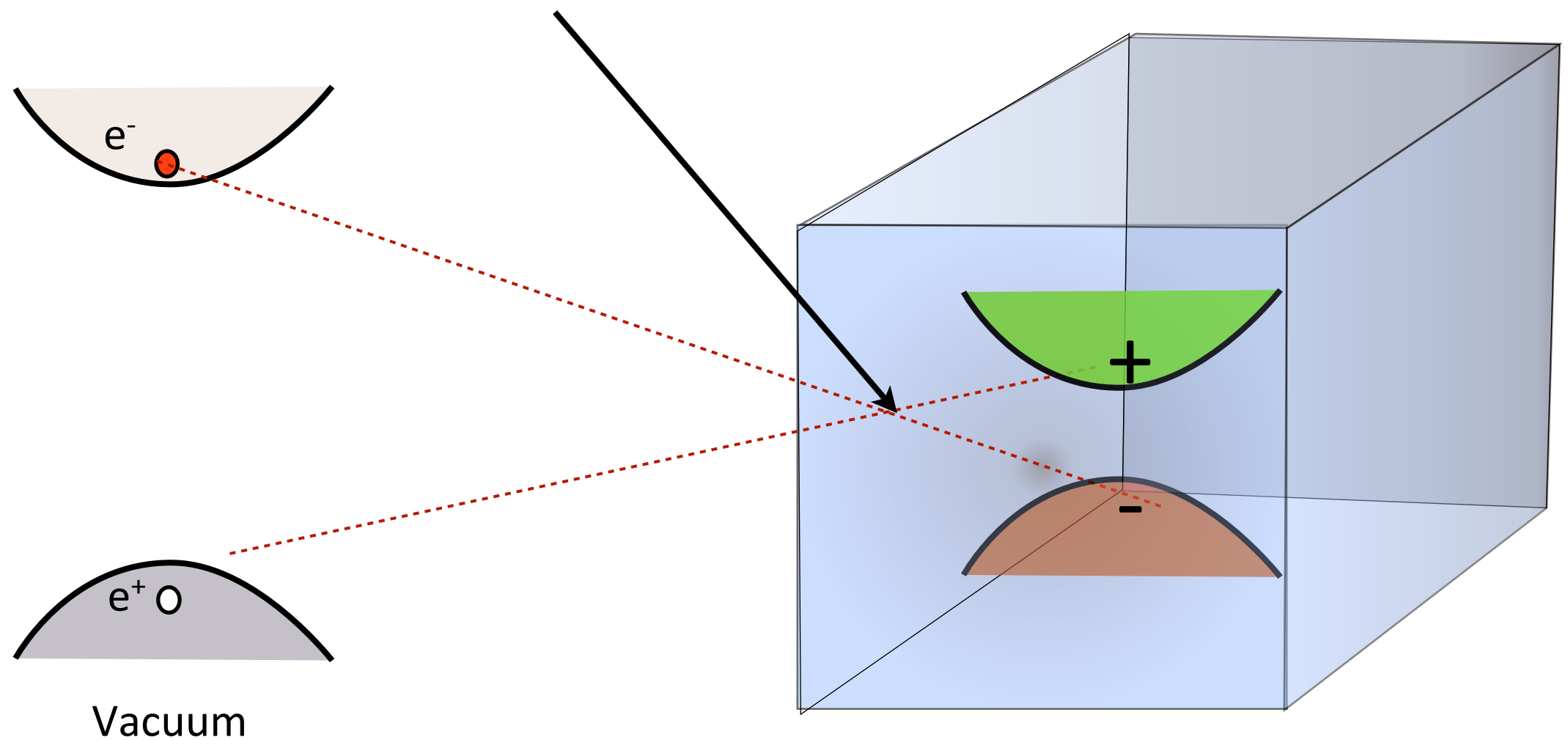
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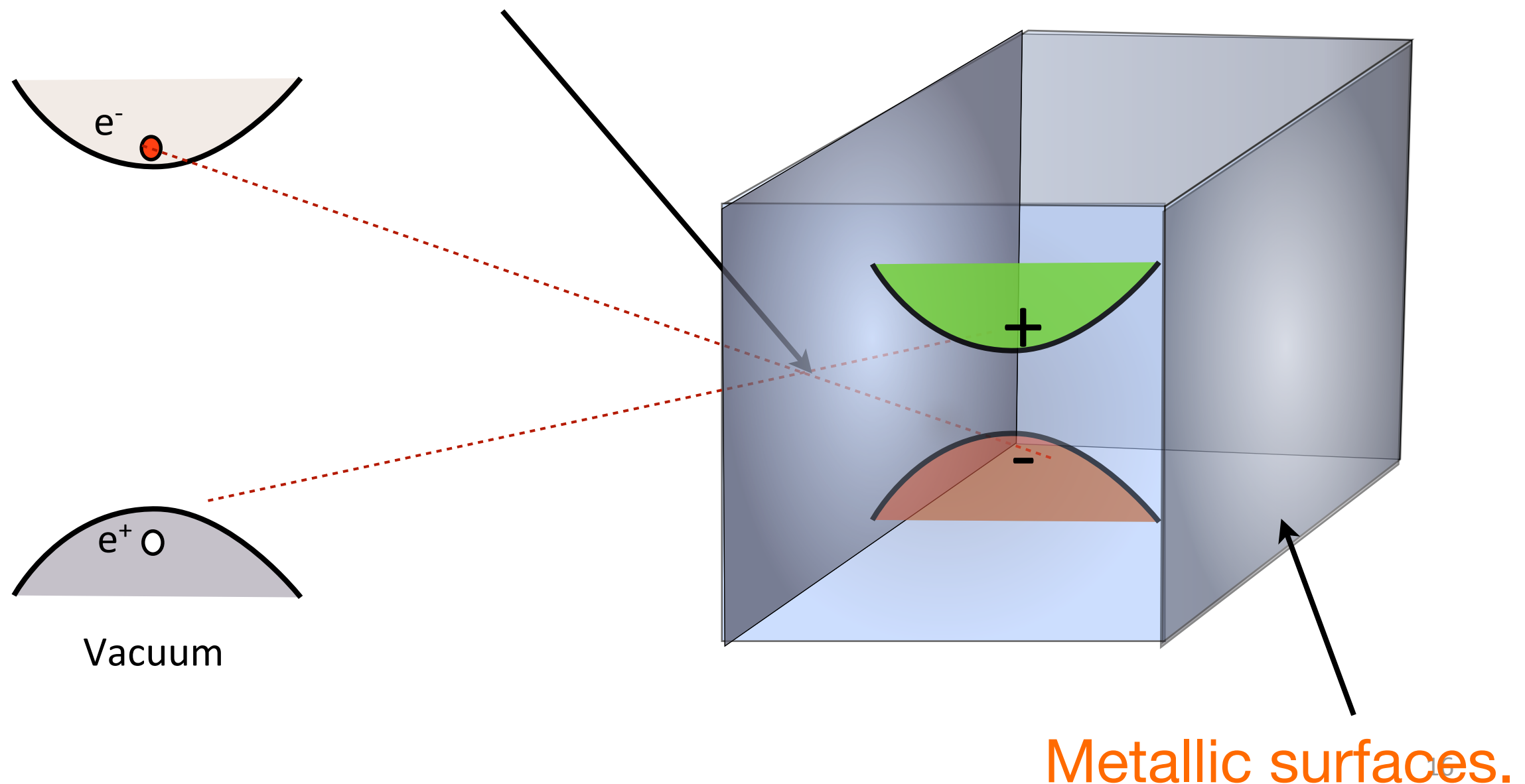
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# So are Kondo insulators topological?

Topological Kondo Insulators, Dzero, Sun, Galitski, PC Phys. Rev. Lett. **104**, 106408 (2010)

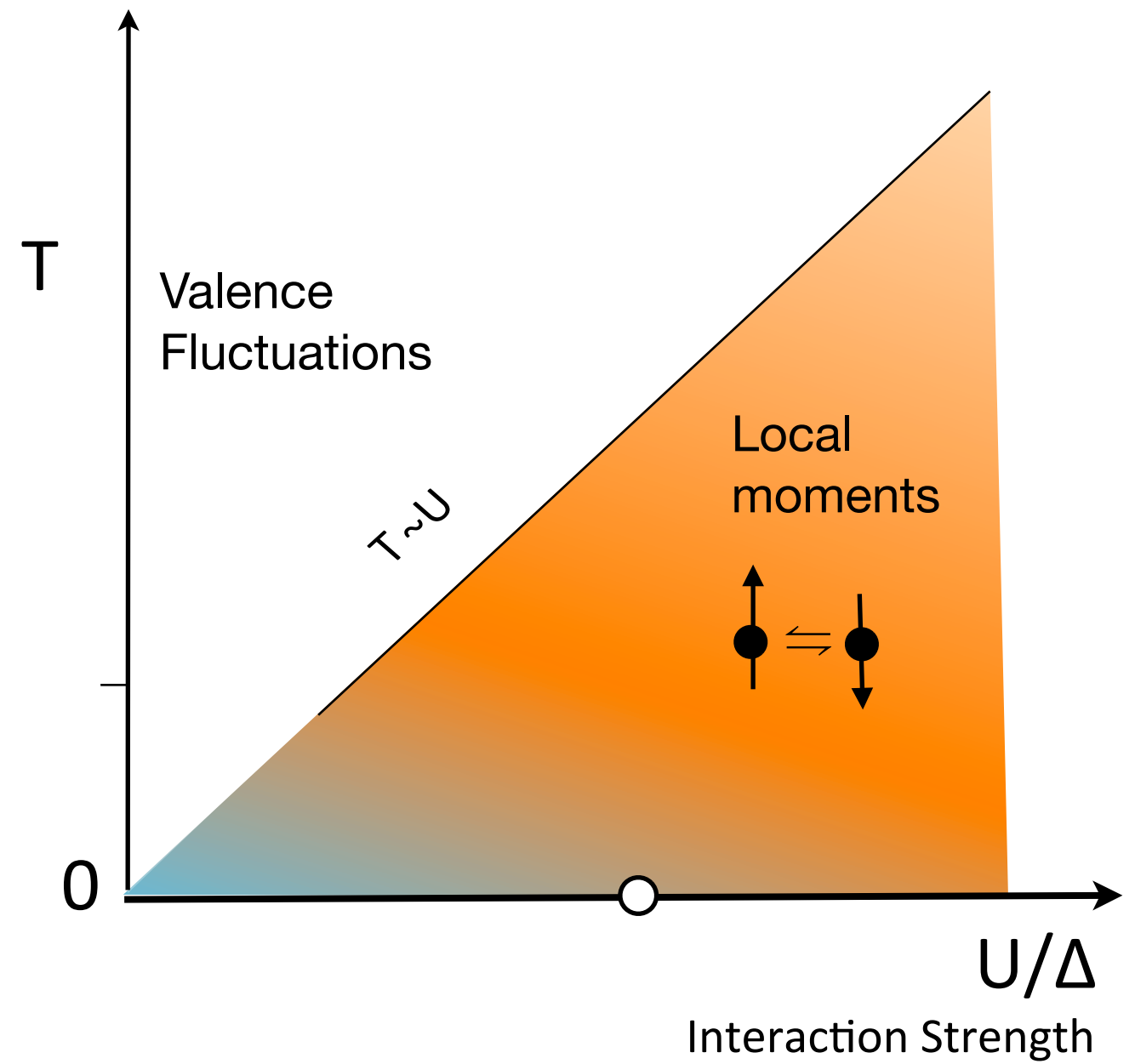
Maxim Dzero, Kai Sun, Piers Coleman and Victor Galitski, Phys. Rev. B 85 , 045130-045140 (2012).

Victor Alexandrov, Maxim Dzero and Piers Coleman preprint (2013).

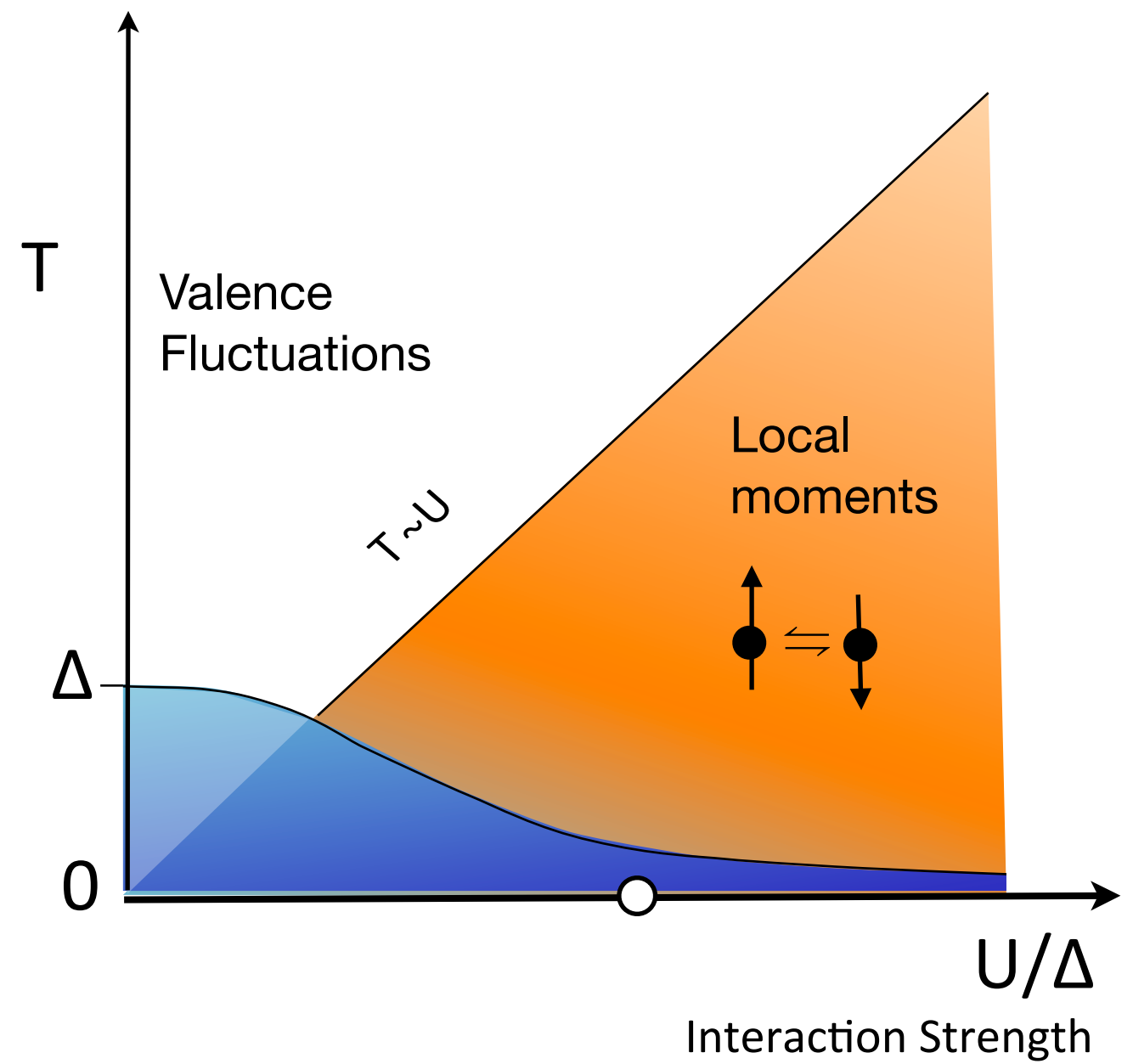
Band Theory: T. Takimoto, J. Phys. Soc. Jpn. 80, 123710 (2011).

Gutzwiller + Band Theory F. Lu, J. Zhao, H. Weng, Z. Fang and X. Dai, Phys. Rev. Lett. 110, 096401 (2013).

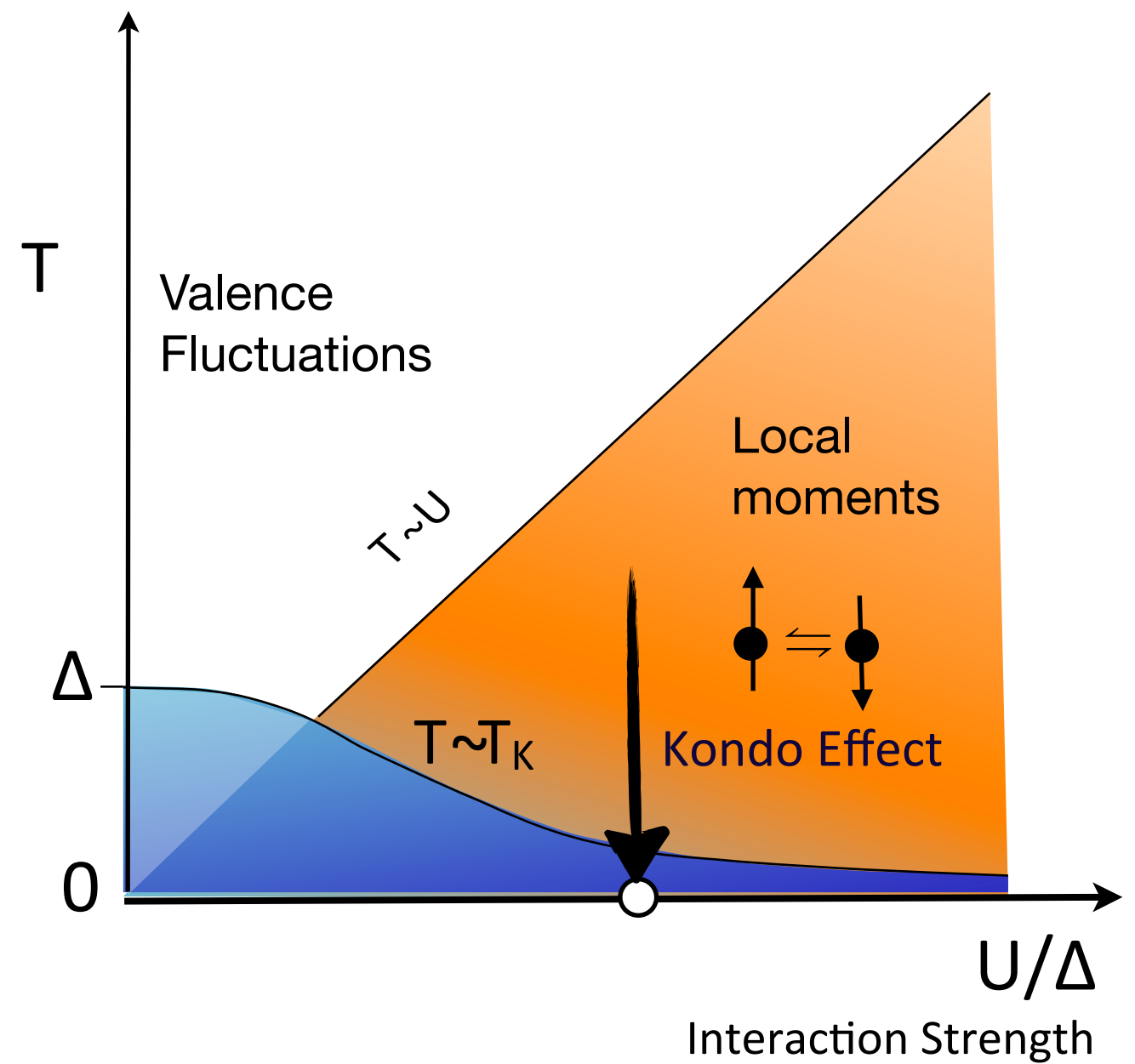
# Adiabatic Considerations about KI's



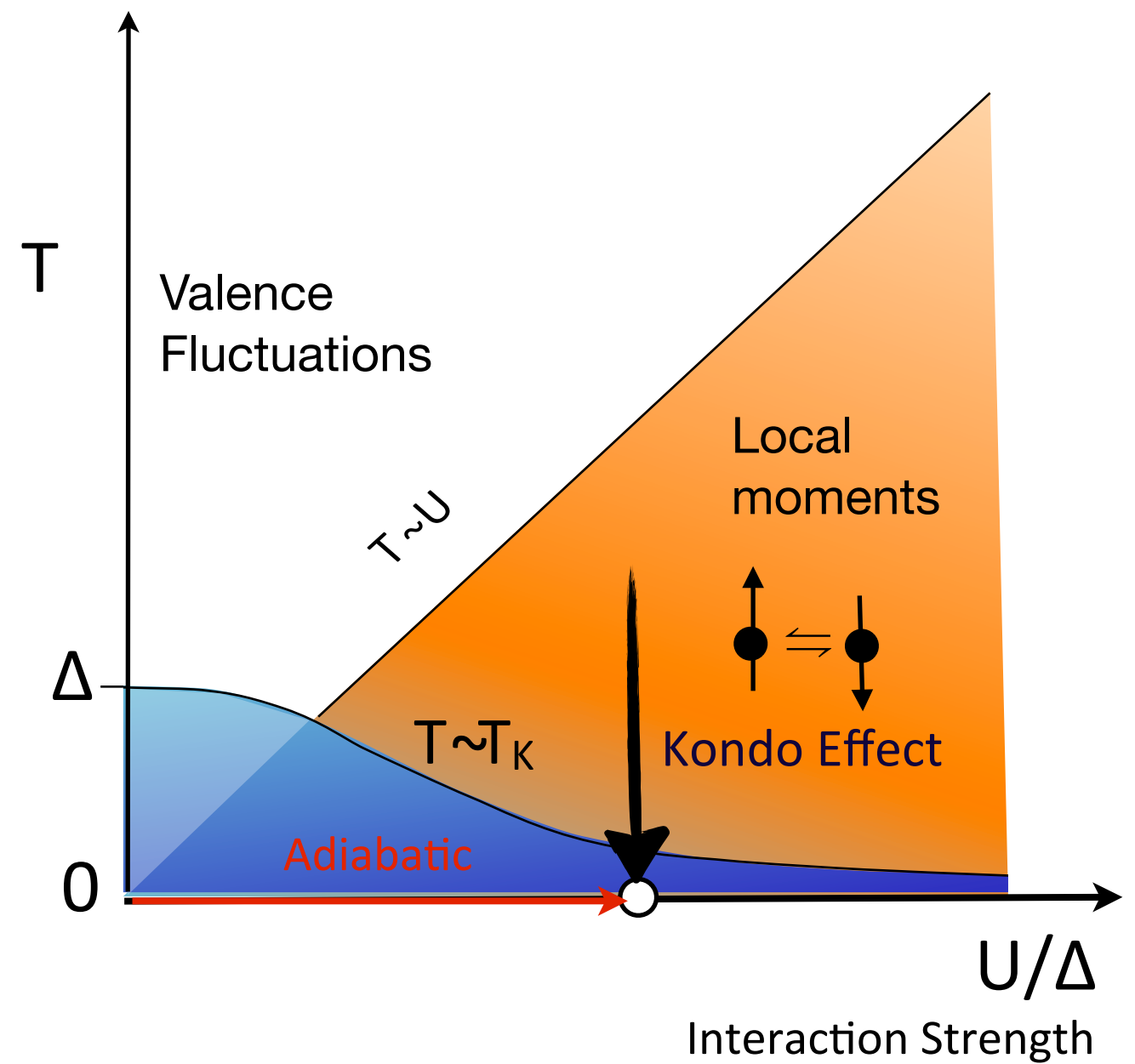
# Adiabatic Considerations about KI's



# Adiabatic Considerations about KI's

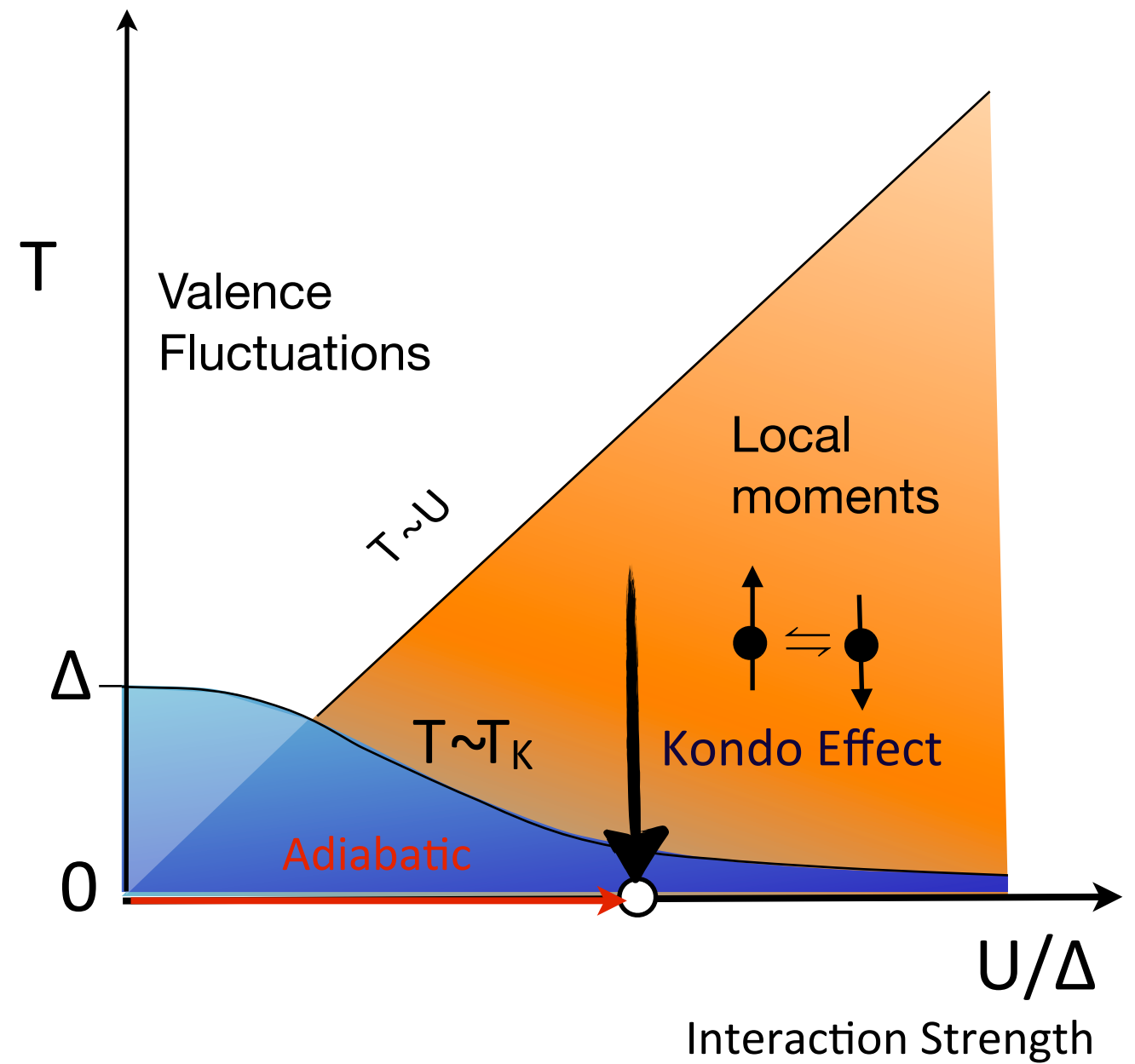
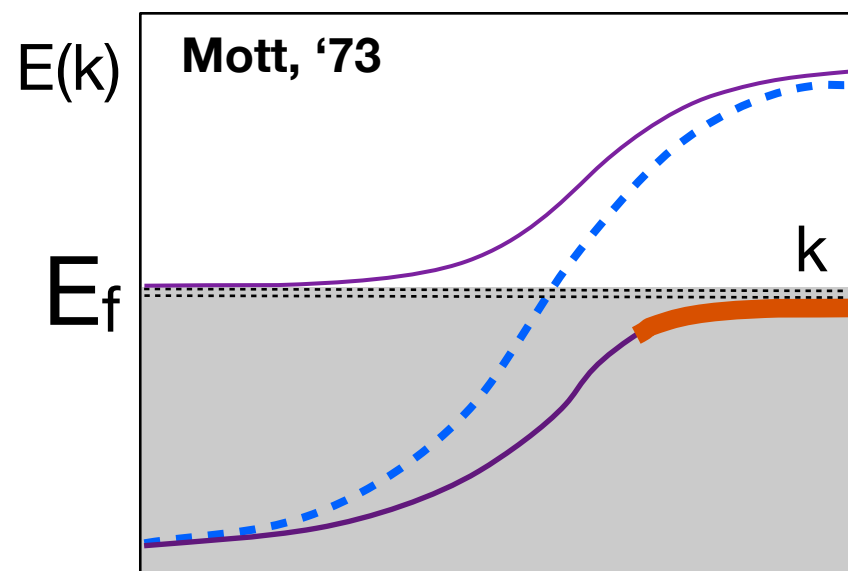


# Adiabatic Considerations about KI's

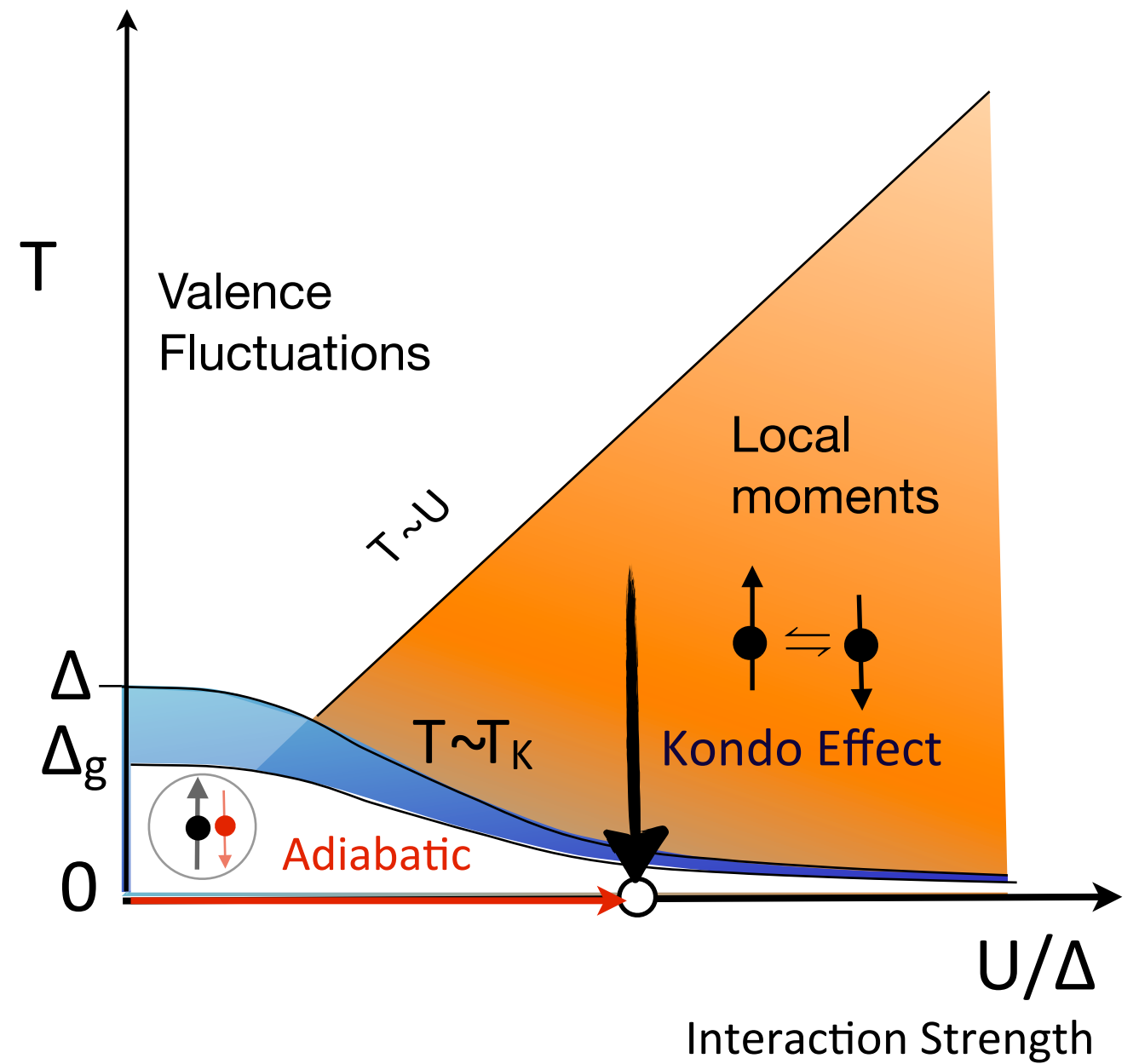
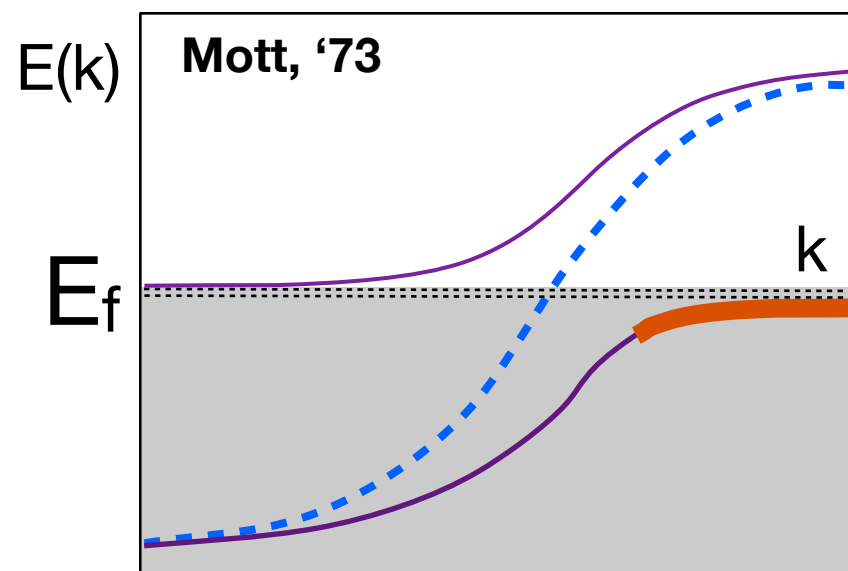




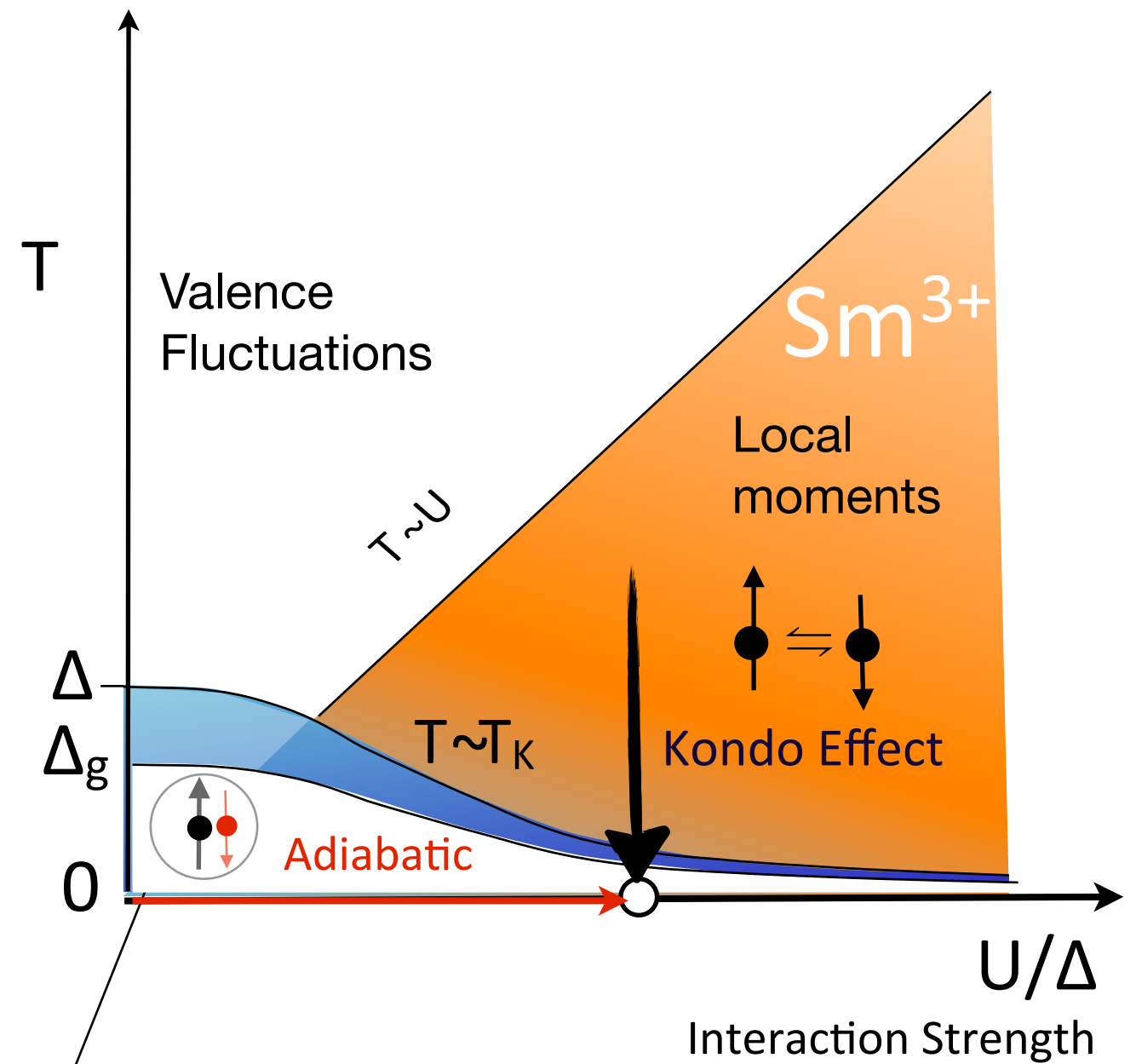
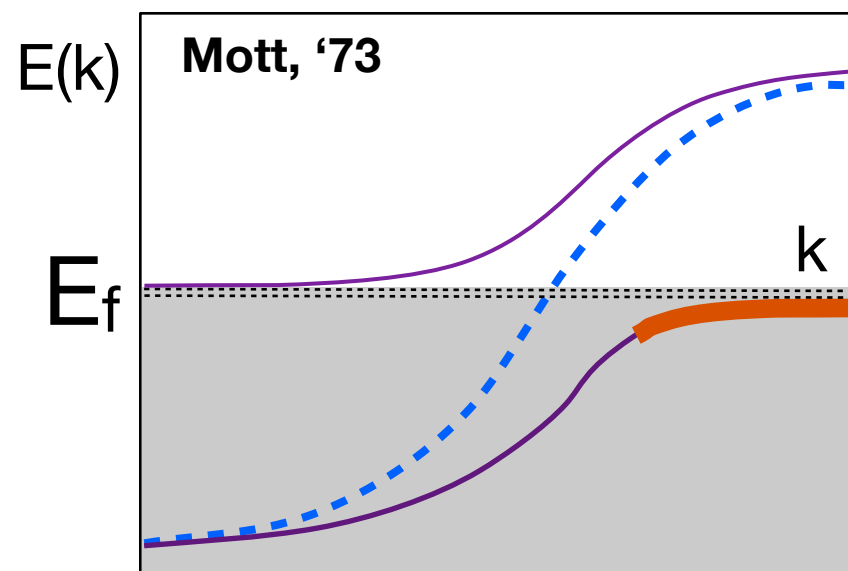
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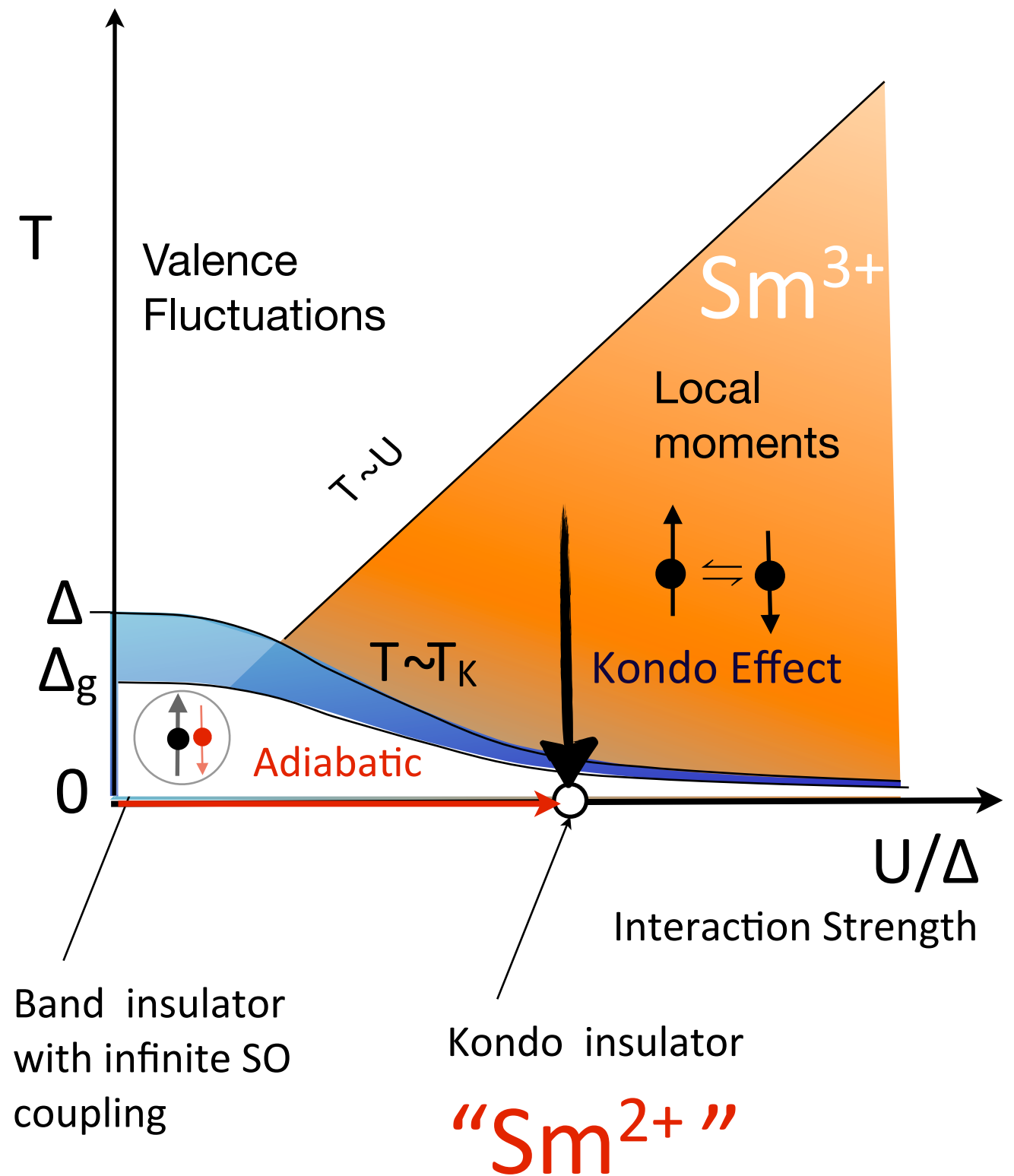
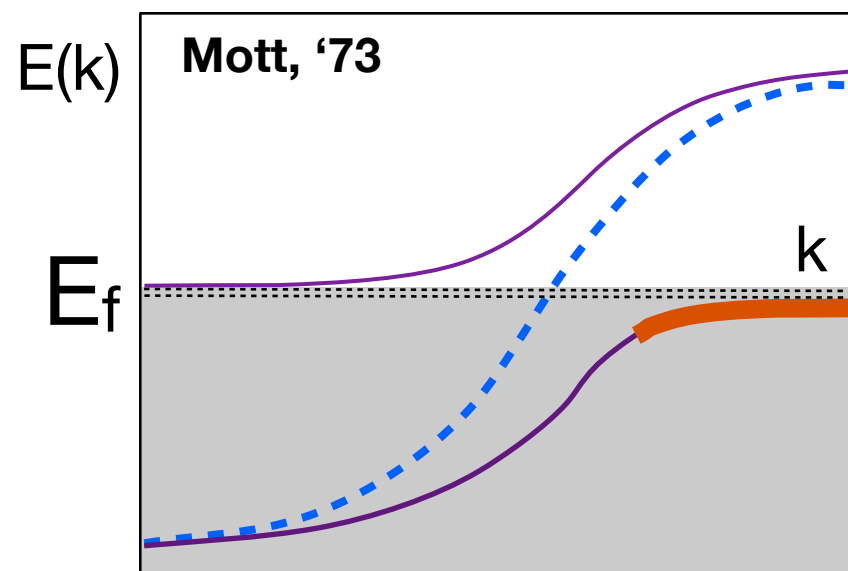


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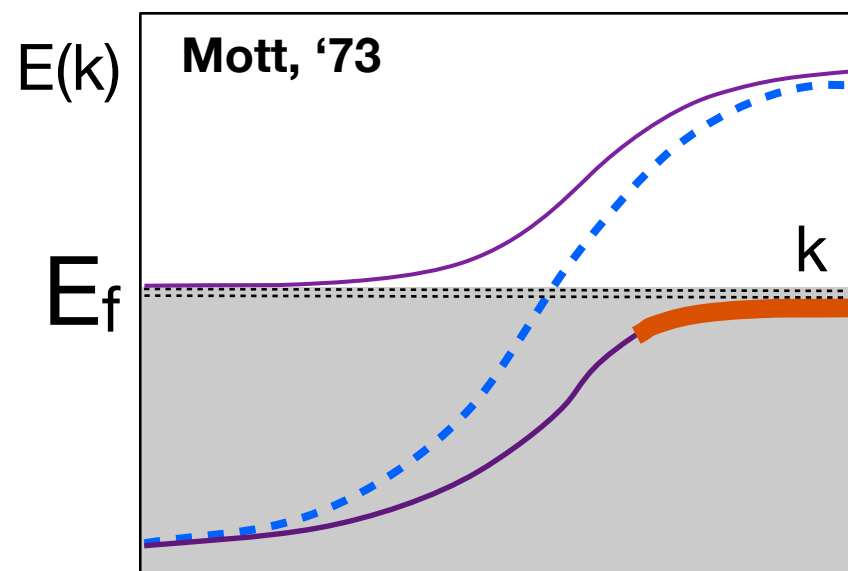


Band insulator  
with infinite SO  
coupling

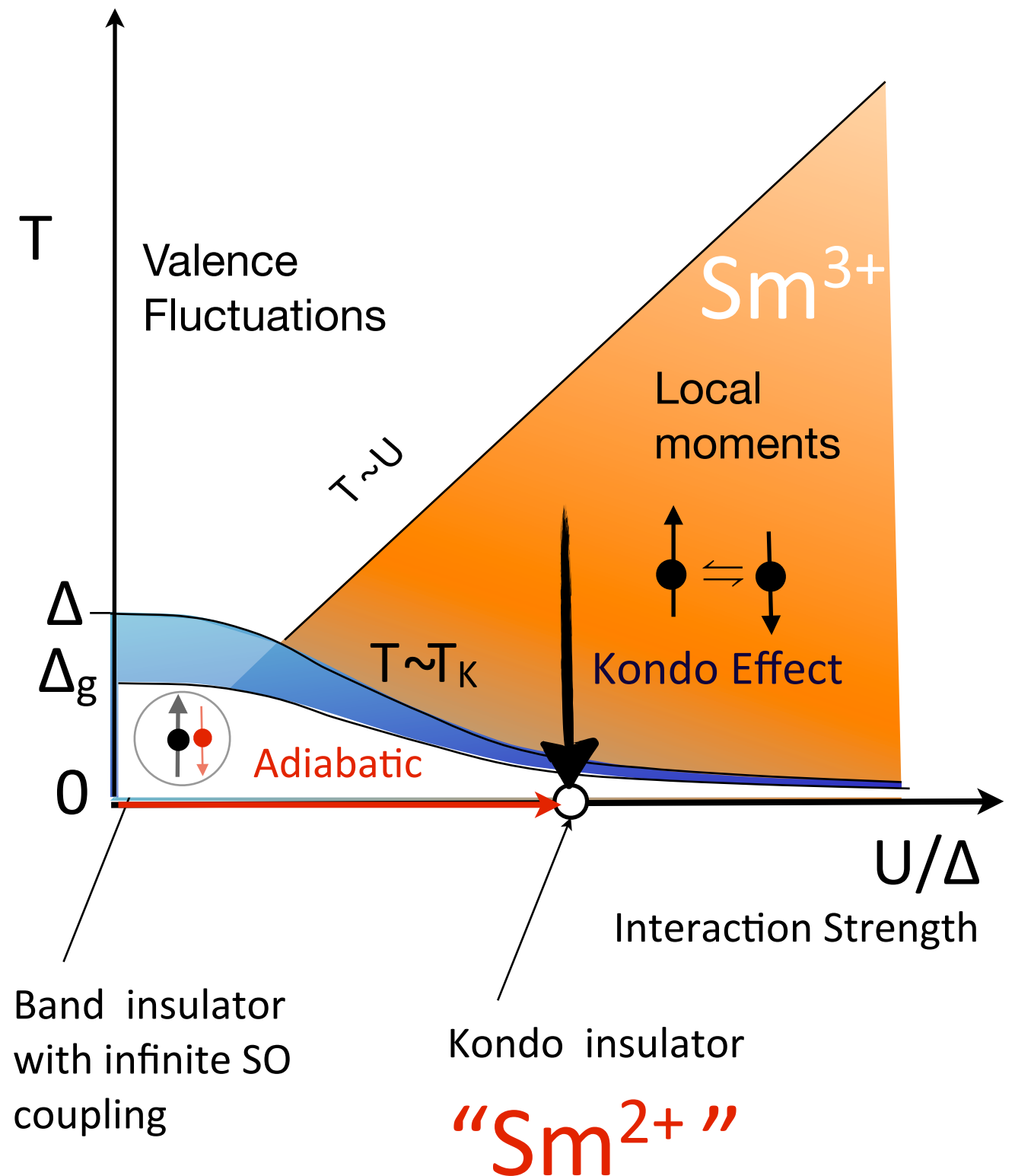
# Adiabatic Considerations about KI's



# Adiabatic Considerations about KI's



If f-states “sink” beneath the Fermi sea, does the Insulator become topological?





# Kondo insulators: theory

- Anderson model:

$$\hat{H} = \sum_{\mathbf{k}, \alpha} \xi_{\mathbf{k}} c_{\mathbf{k}\alpha}^{\dagger} c_{\mathbf{k}\alpha} + \sum_{j\alpha} \left[ V \psi_{j\alpha}^{\dagger} f_{\alpha} + \text{h.c.} \right] + \sum_{j\alpha} \left[ E_f^{(0)} n_{f\alpha} + \frac{U_f}{2} n_{f,j\alpha} n_{f,j\bar{\alpha}} \right]$$

$$\psi_{j\alpha} = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \underbrace{\Phi_{\alpha\sigma}(\hat{\mathbf{k}})}_{\substack{\text{f-electron} \\ \text{form factor}}} e^{-i\mathbf{k} \cdot \mathbf{x}_j} c_{\mathbf{k}\sigma}$$

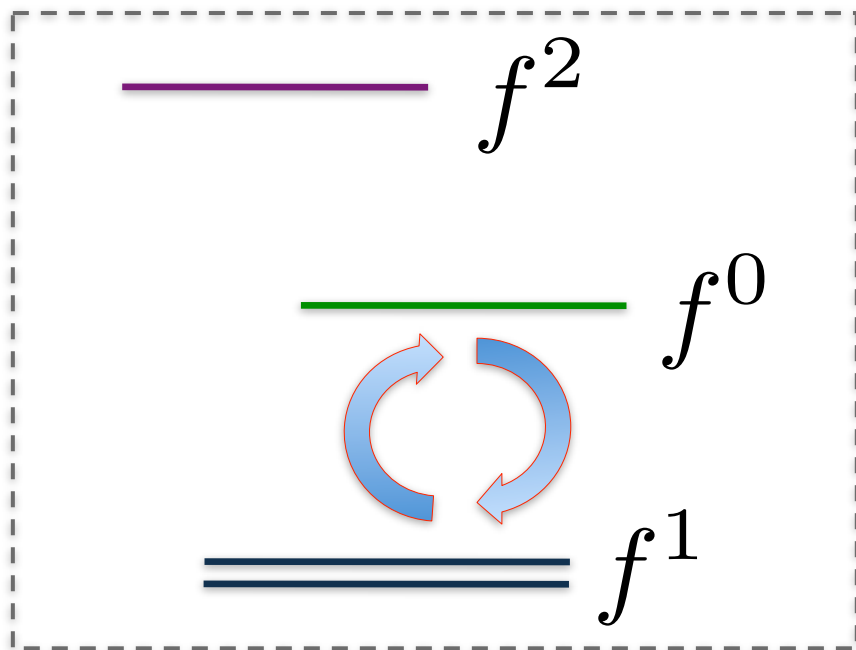
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- Hybridization



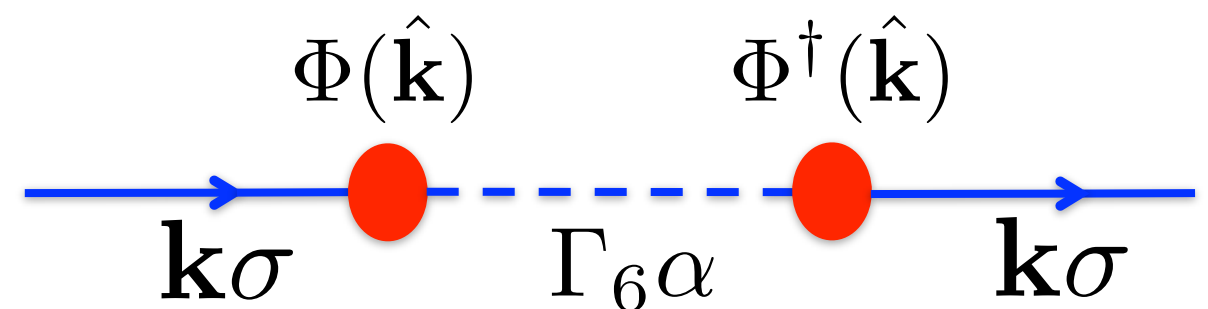
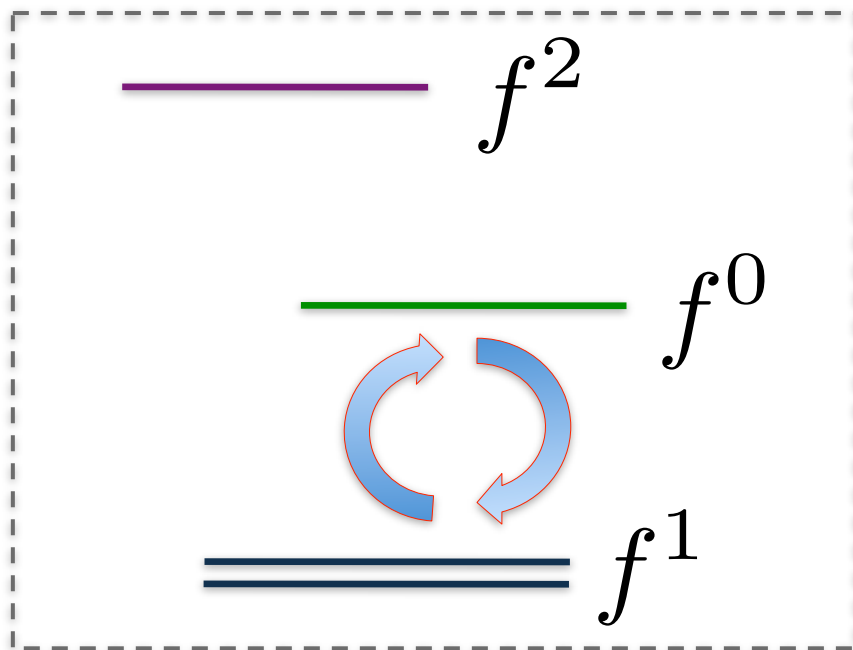
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- Hybridization



Strong spin-orbit coupling is encoded in the hybridization.

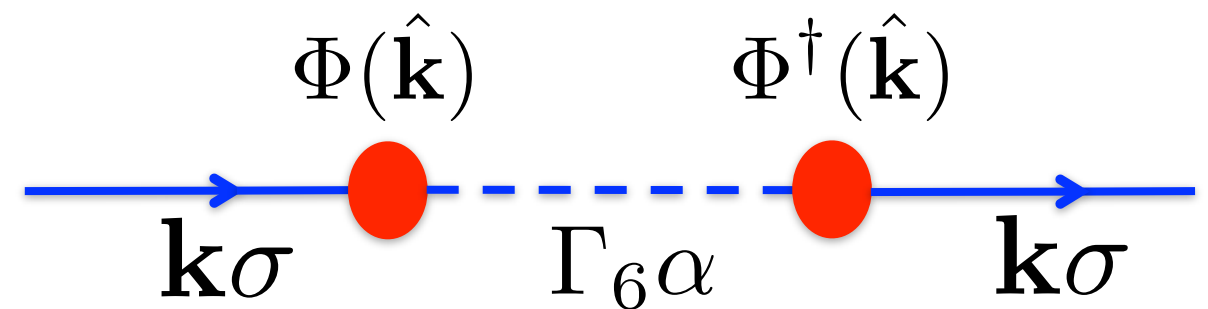
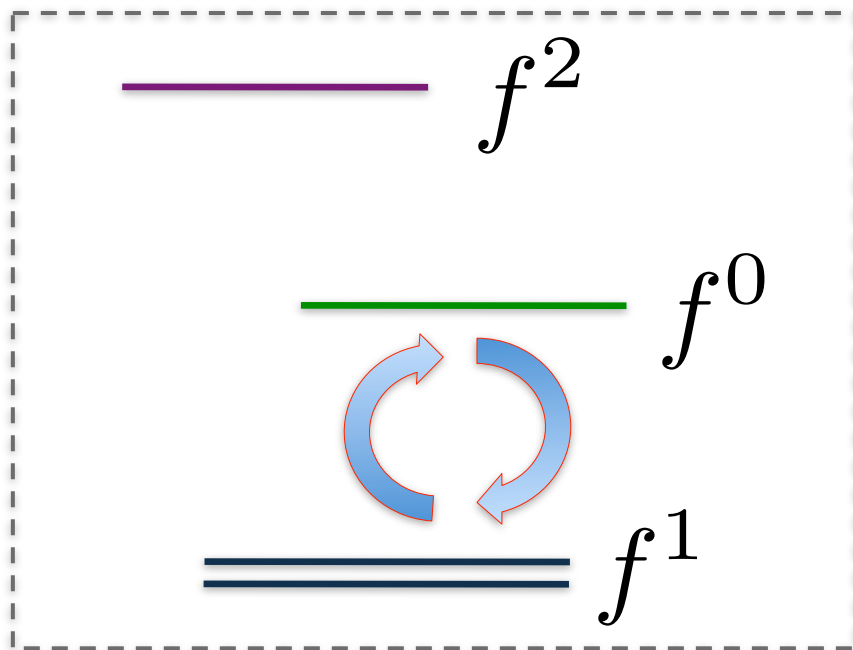
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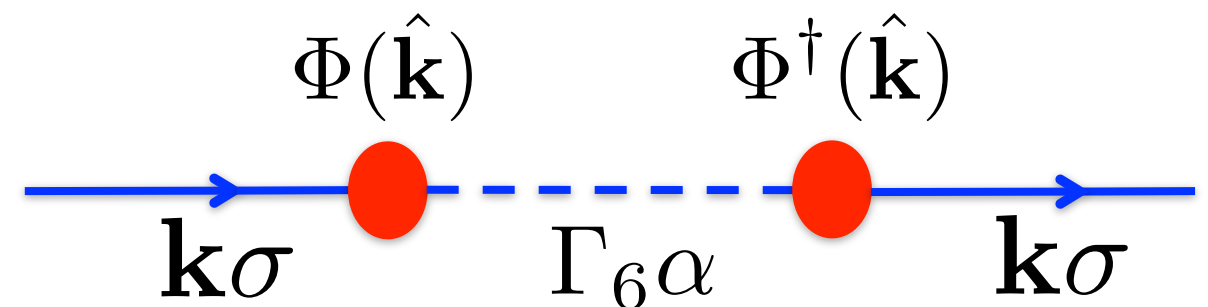
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form factors:

$$[\Phi_{\Gamma\mathbf{k}}]_{\alpha\sigma} = \sum_m \langle \Gamma\alpha | jm \rangle \underbrace{\langle jm | \mathbf{k}\sigma \rangle}_{\text{Matrix element between Bloch and Wannier states}}$$



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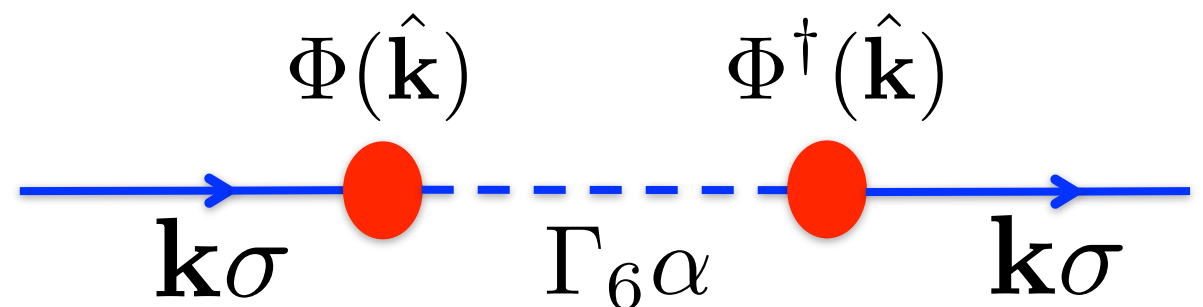
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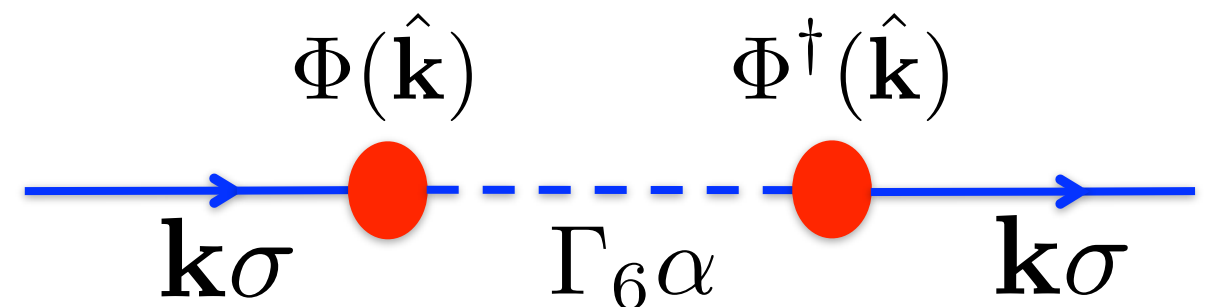
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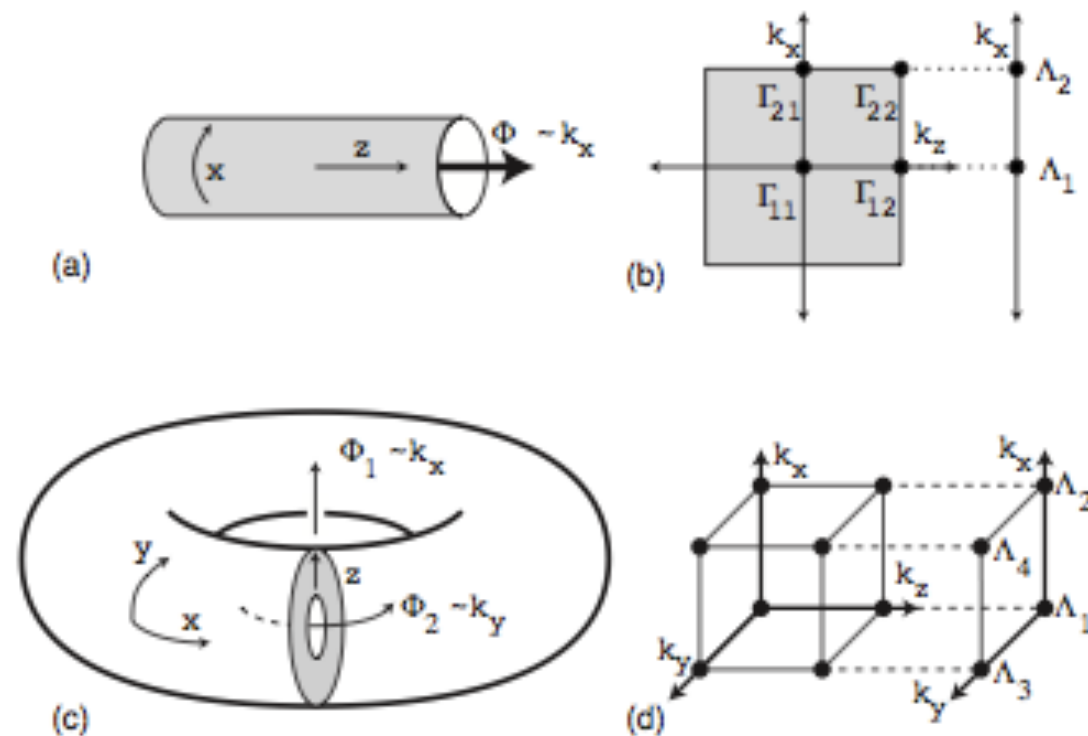
Strong spin-orbit coupling is encoded in the hybridization.

ODD PARITY!

# Topological insulators

L. Fu & C. Kane, "Topological Insulators with Inversion Symmetry", PRB 76, 45302 (2007).

## Response to a fictitious applied flux



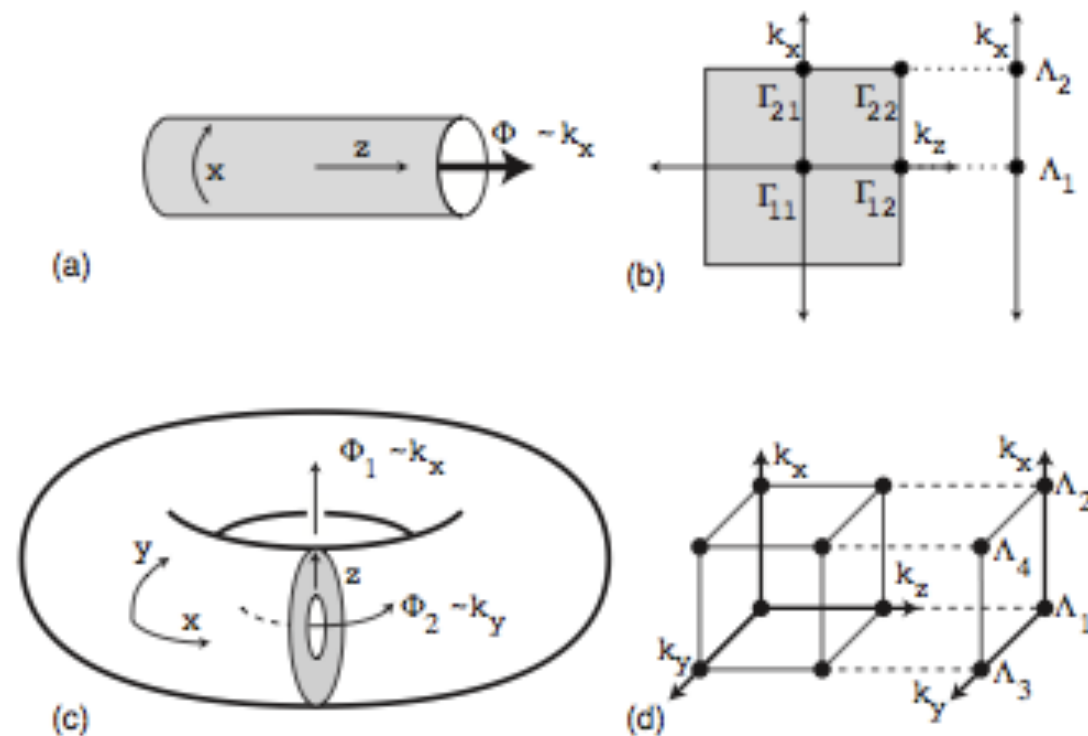
➤ 2D: Flux plays the role of the edge crystal momentum  $k_x$

➤ 3D: two fluxes corresponding to two components of the surface crystal momentum

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## Response to a fictitious applied flux



➤ 2D: Flux plays the role of the edge crystal momentum  $k_x$

➤ 3D: two fluxes corresponding to two components of the surface crystal momentum

$Z_2$  invariants computed from the **parity** of the occupied bands at high SPs.  
Change in time reversal polarization due to changes in bulk Hamiltonian

$$Z_2 = \prod_i \delta(\Gamma_i)$$

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$$V_{\alpha\sigma}(\mathbf{k}_m) = V_{\alpha\sigma}(\mathbf{k}_m + \mathbf{G}) = -V_{\alpha\sigma}(-\mathbf{k}_m) = -V_{\alpha\sigma}(\mathbf{k}_m) = 0$$

Vanishes at high symmetry points

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$$H_{mf}(\mathbf{k}_m) = \frac{1}{2}(\xi_{\mathbf{k}_m} + \varepsilon_f)\underline{1} + \frac{1}{2}(\xi_{\mathbf{k}_m} - \varepsilon_f)P$$

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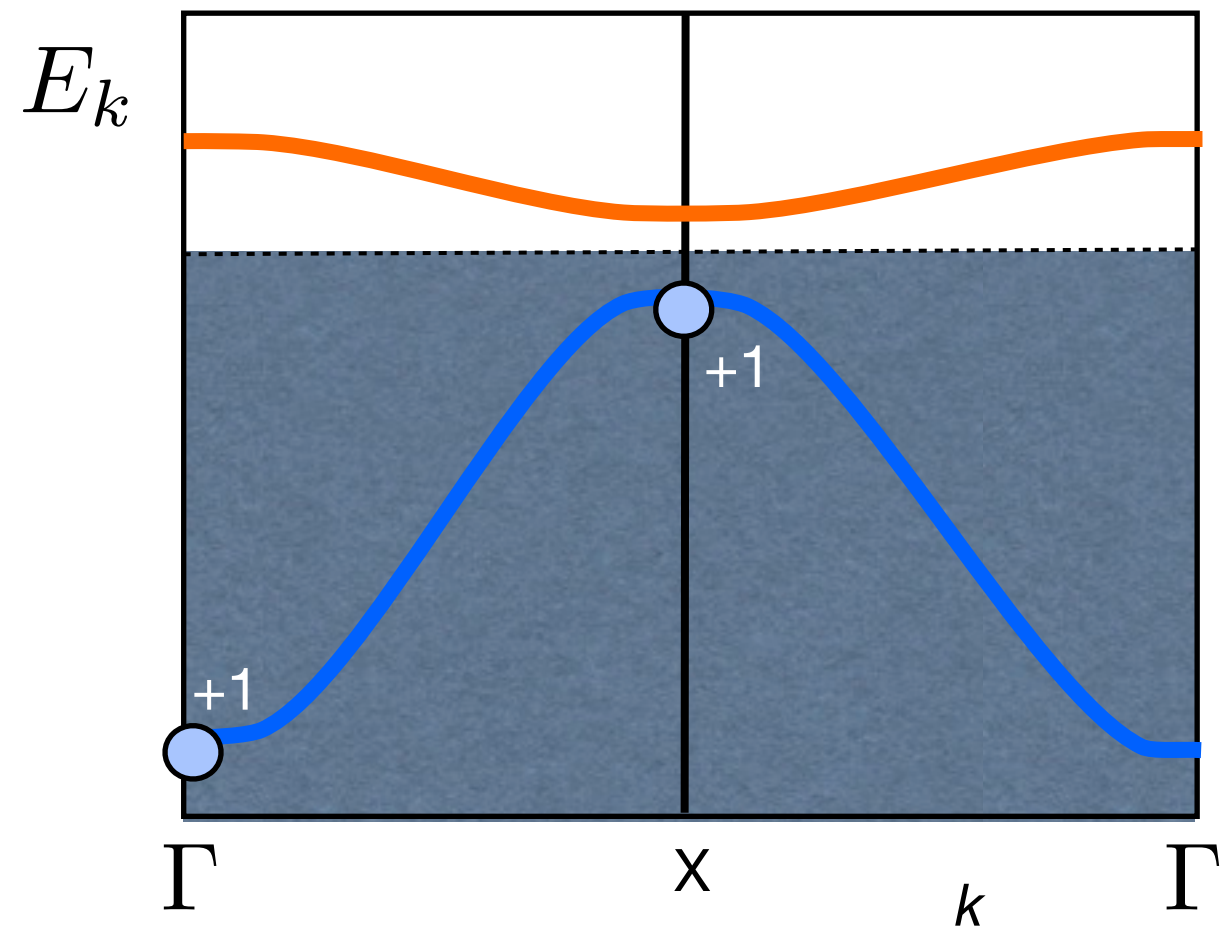
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➤  $Z_2$  invariants are characterized by the parity eigenvalues:

$$\delta(\Gamma_m) = \text{sgn}(E_f^* - \xi_{\mathbf{k}_m})$$

# 1D Kondo Insulator

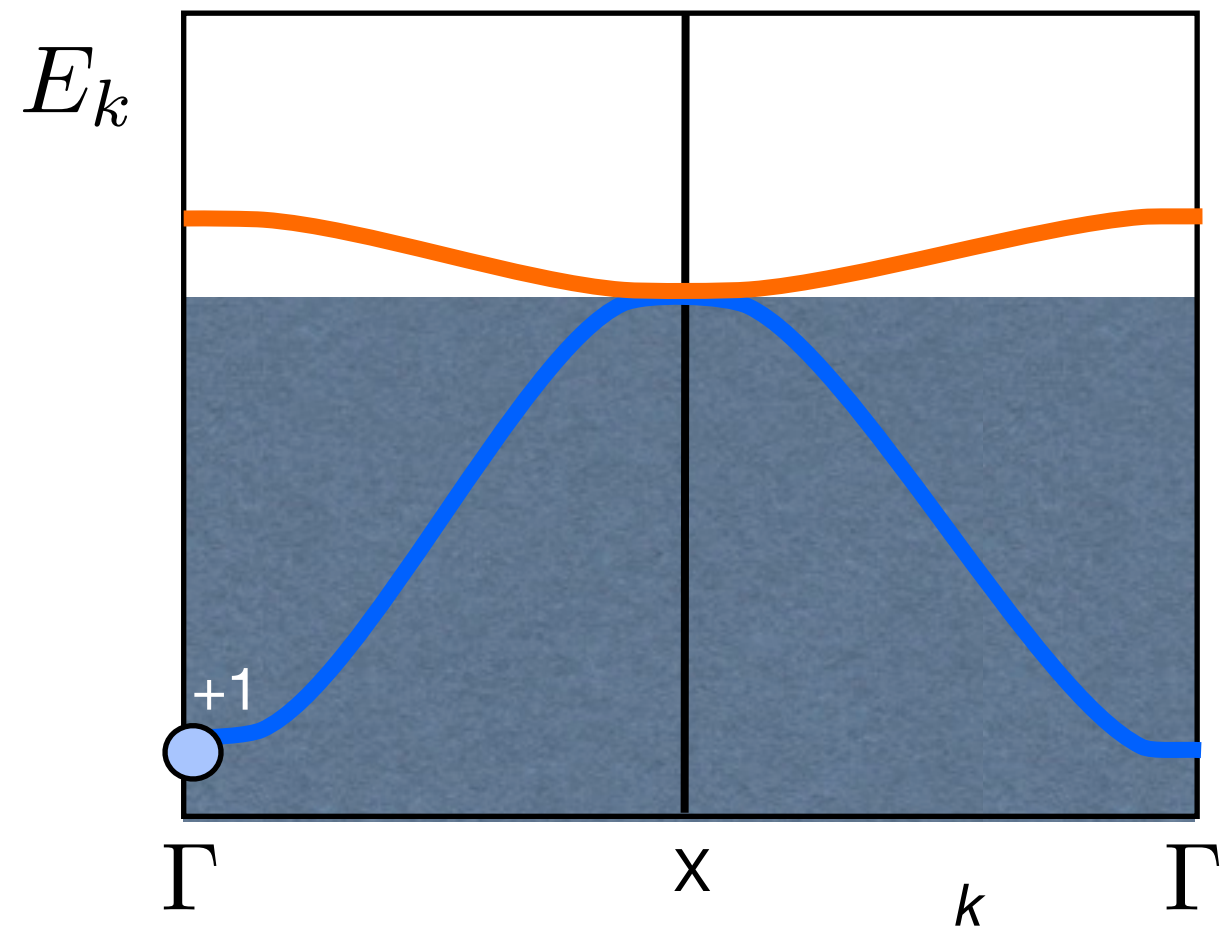
$$d^2$$
$$\nu = +1$$



Alexandrov, Dzero and PC (2013)

# 1D Kondo Insulator

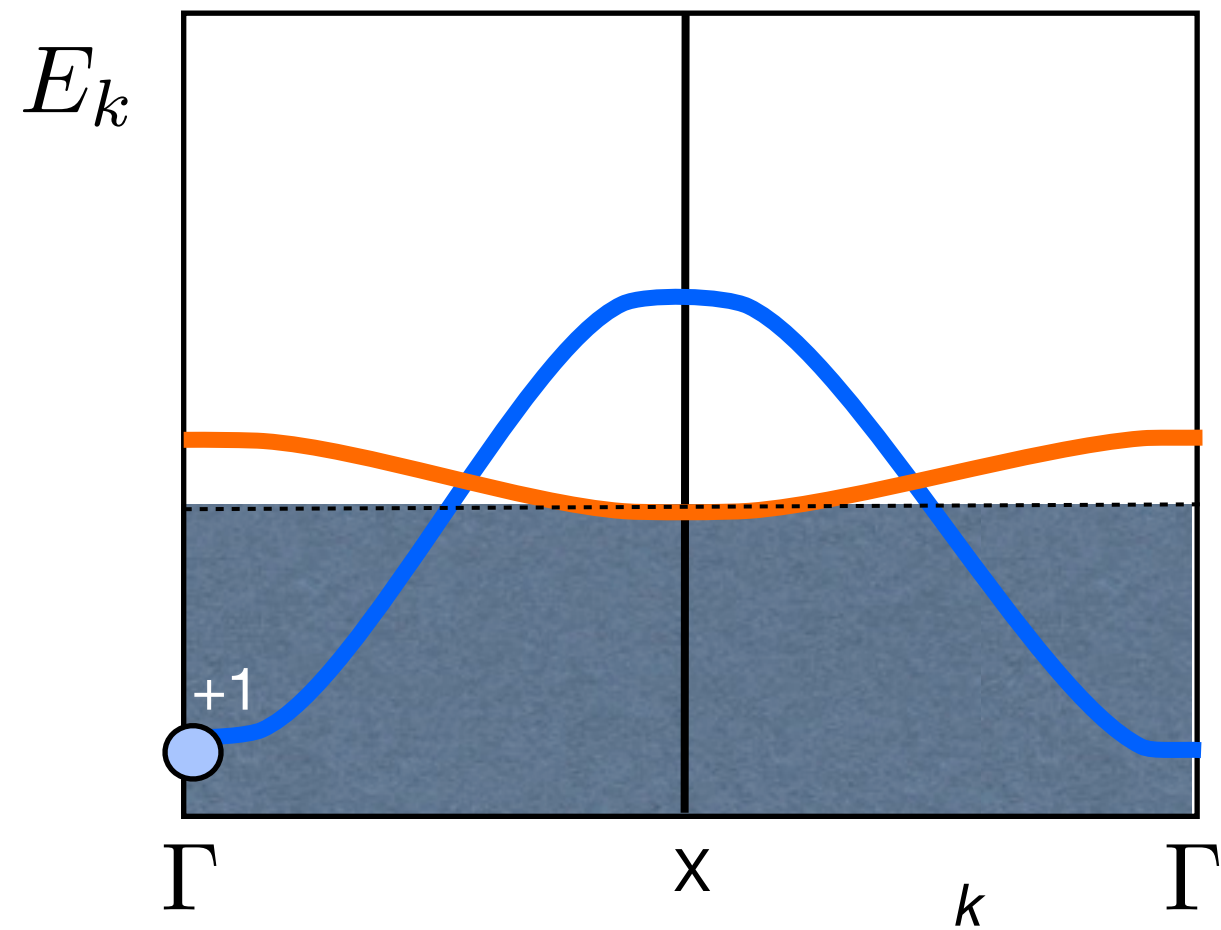
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$$d^2$$
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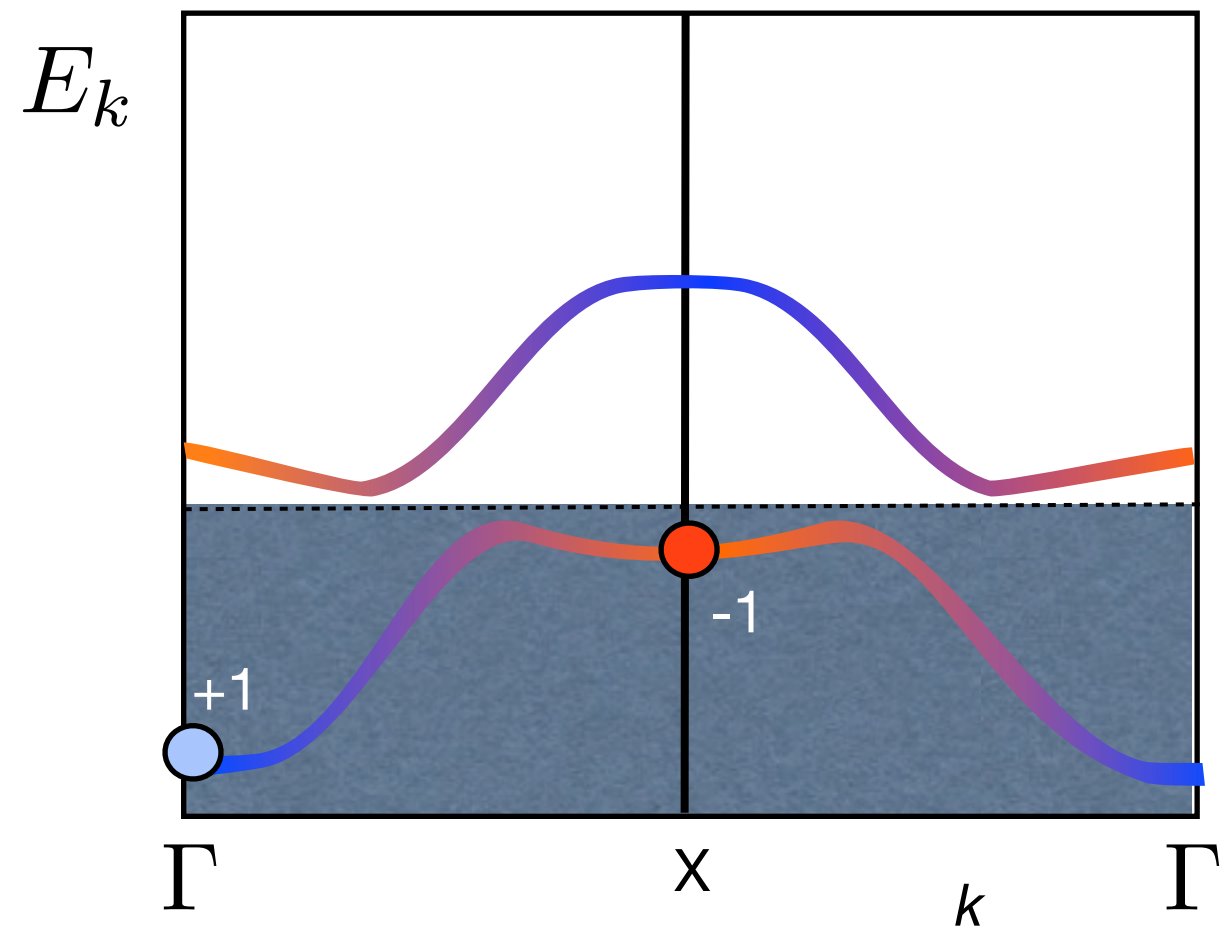


Alexandrov, Dzero and PC (2013)



# 1D Kondo Insulator

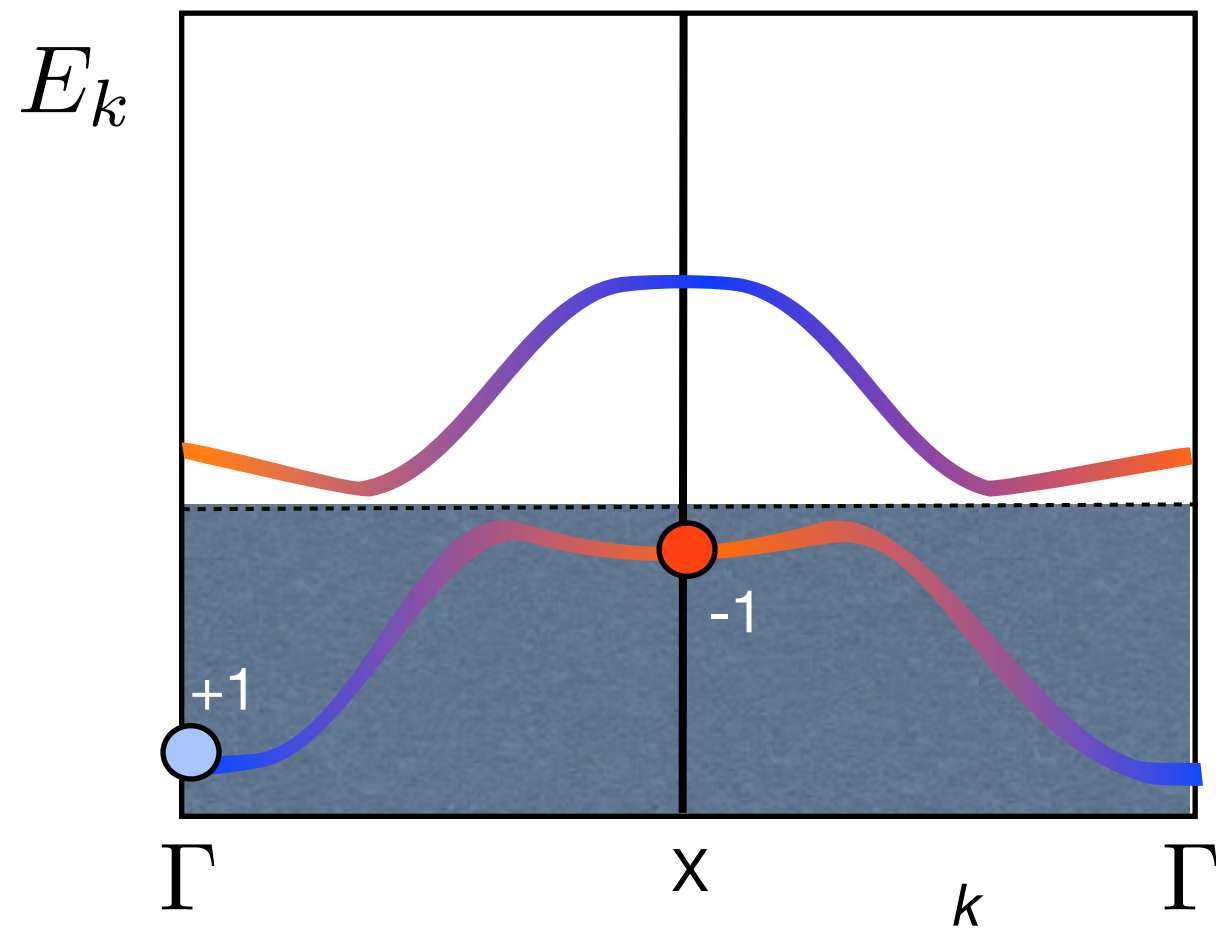
$$\begin{array}{ccc} d^2 & \rightarrow & f^1 d^1 \\ \nu = +1 & & \nu = -1 \end{array}$$



Alexandrov, Dzero and PC (2013)

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Alexandrov, Dzero and PC (2013)

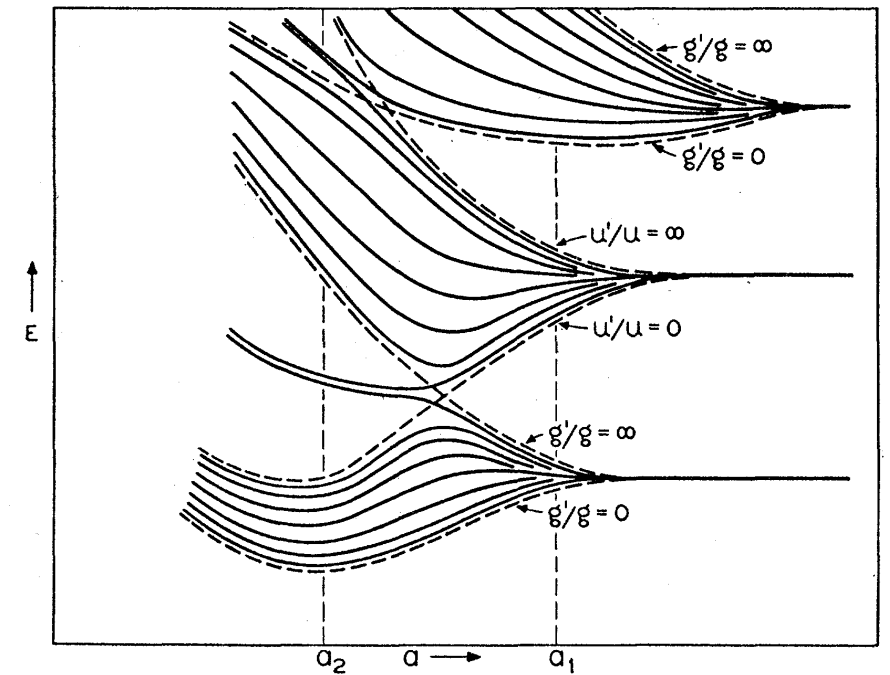


FIG. 2. Energy spectrum for a one-dimensional lattice with eight atoms.

Schockley, Phys Rev, 56, 317 (1939).

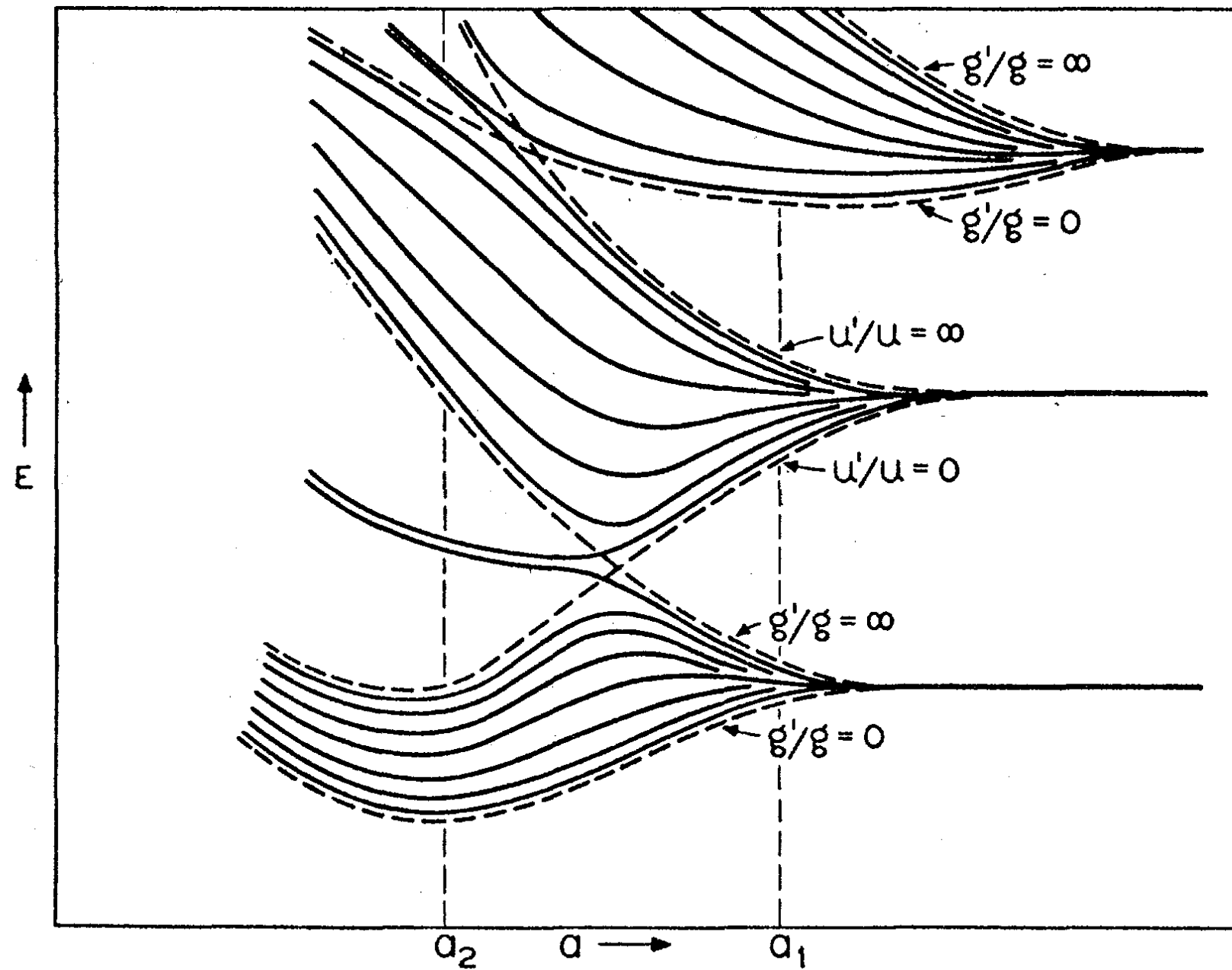
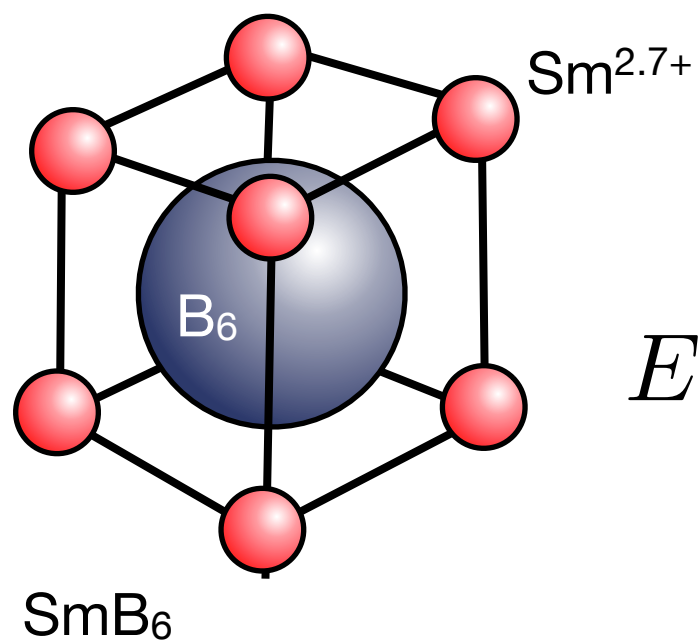


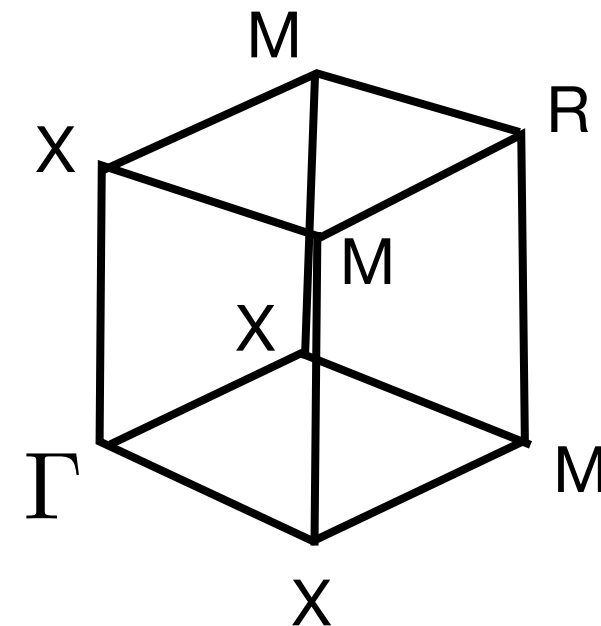
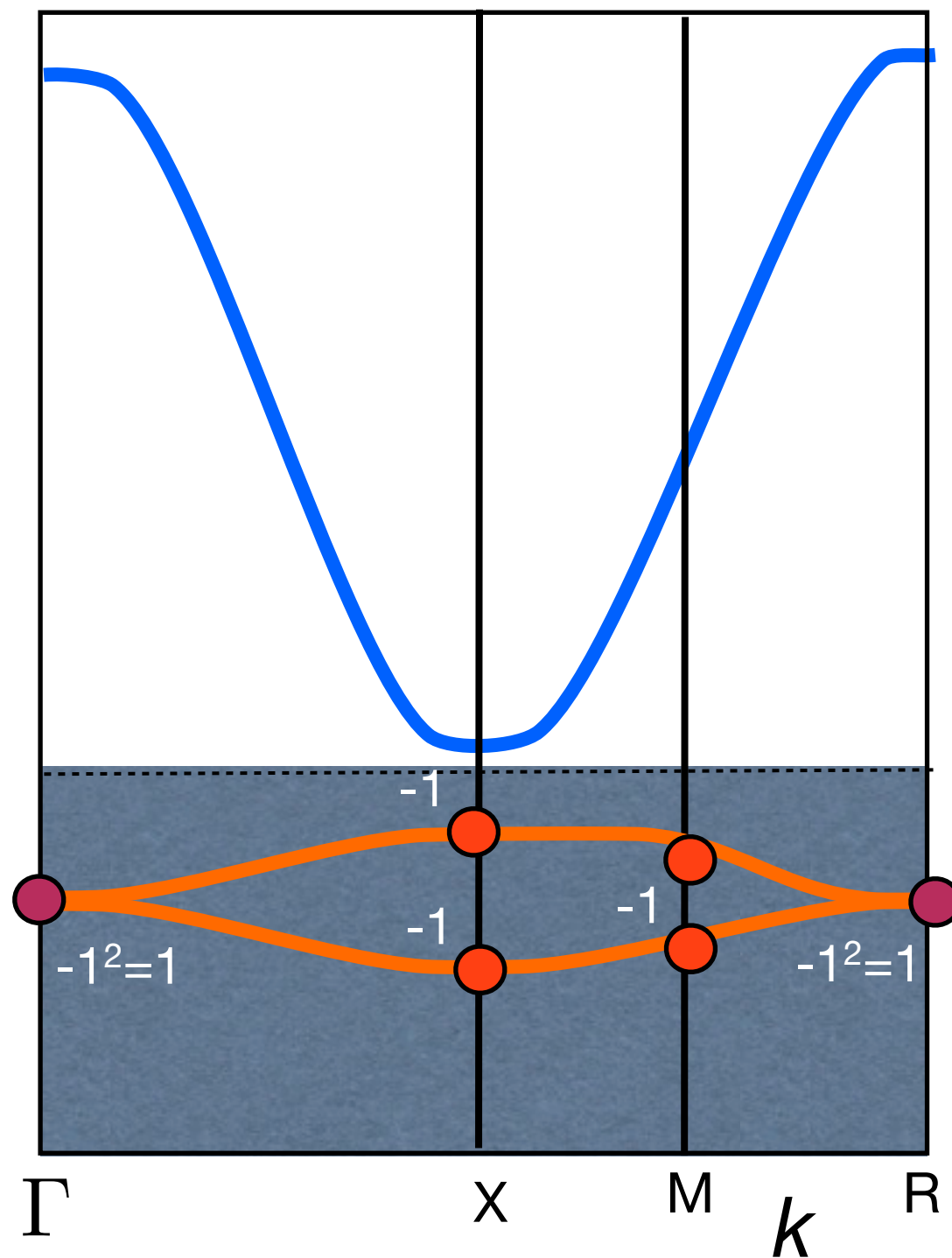
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3D Cubic Kondo  
Insulator



$E_k$



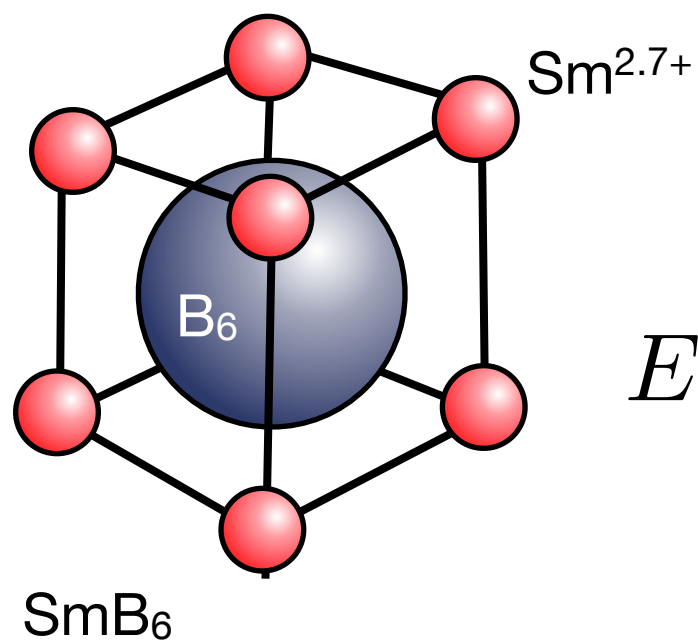
$d^0 f^6$

$\nu = +1$

THREE DIRAC CONES  
ON SURFACE.

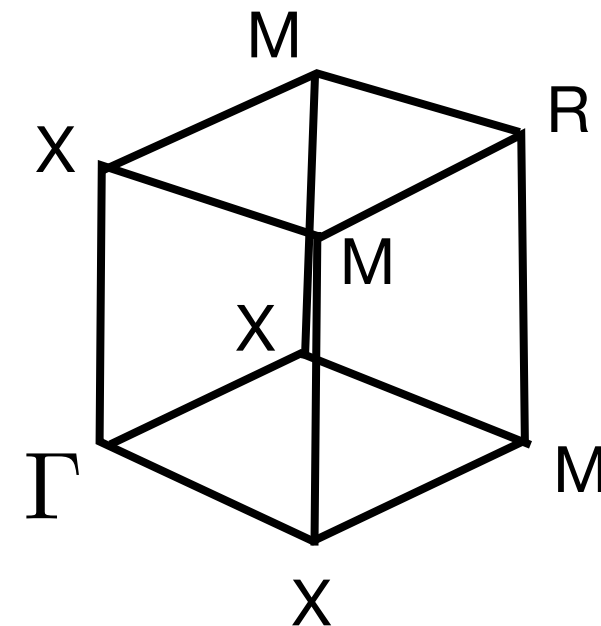
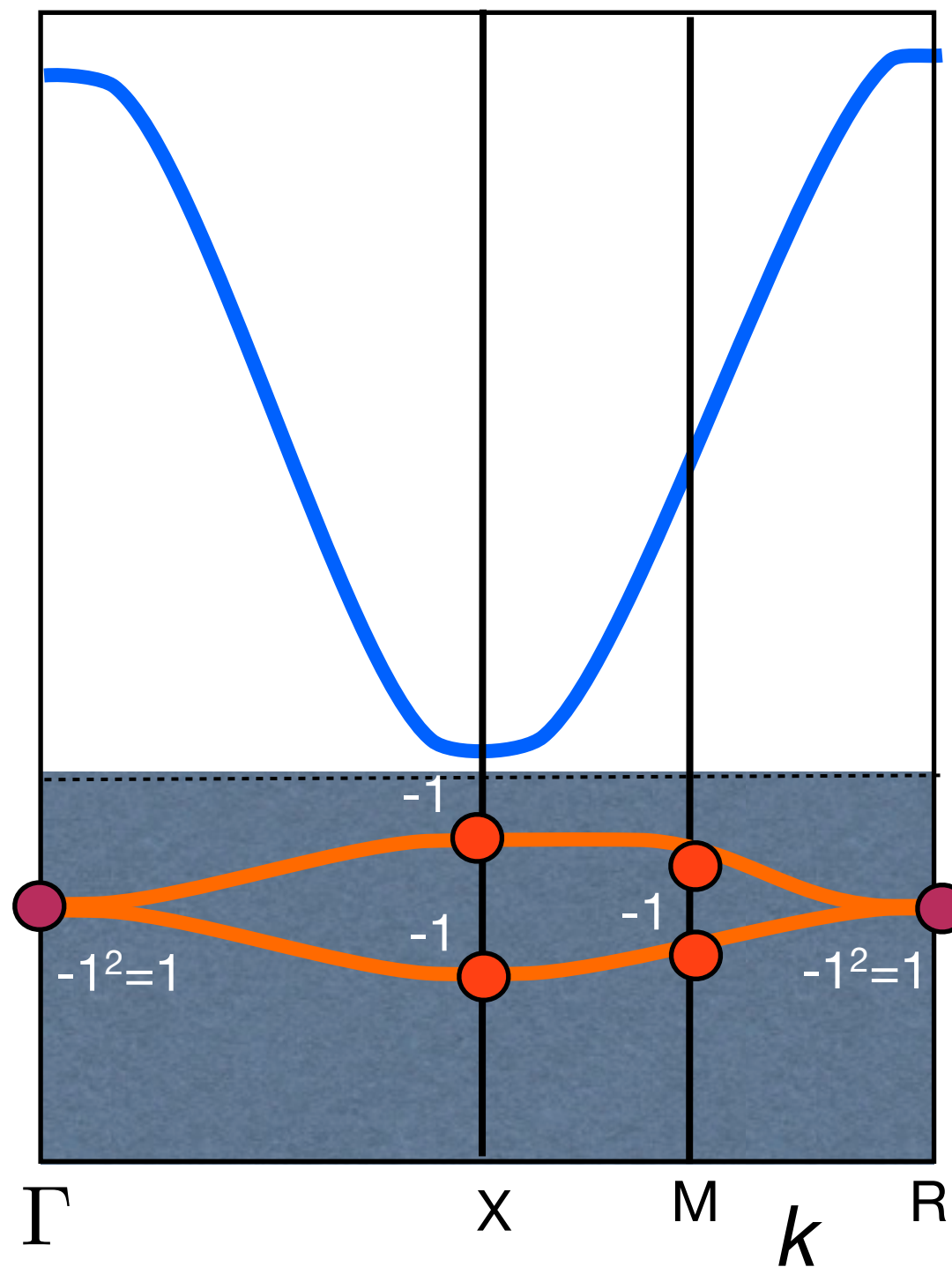
Alexandrov, Dzero and PC (2013)

3D Cubic Kondo  
Insulator



$E_k$

QUARTET:



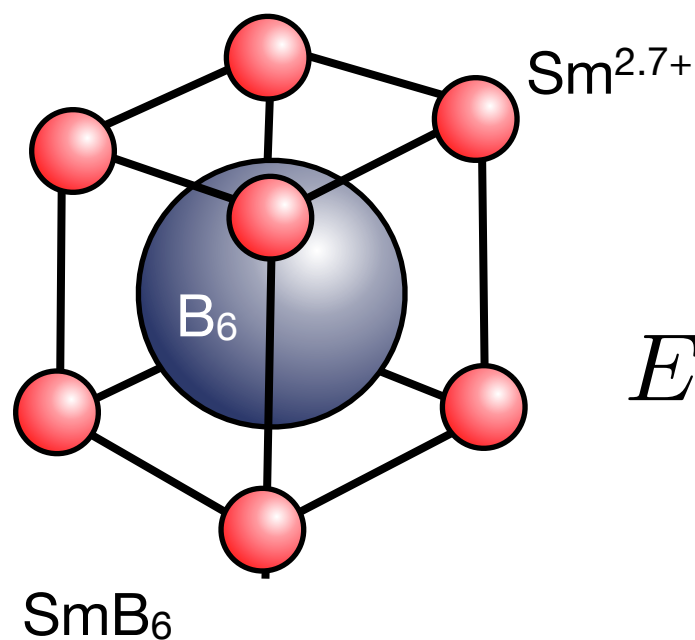
$d^0 f^6$

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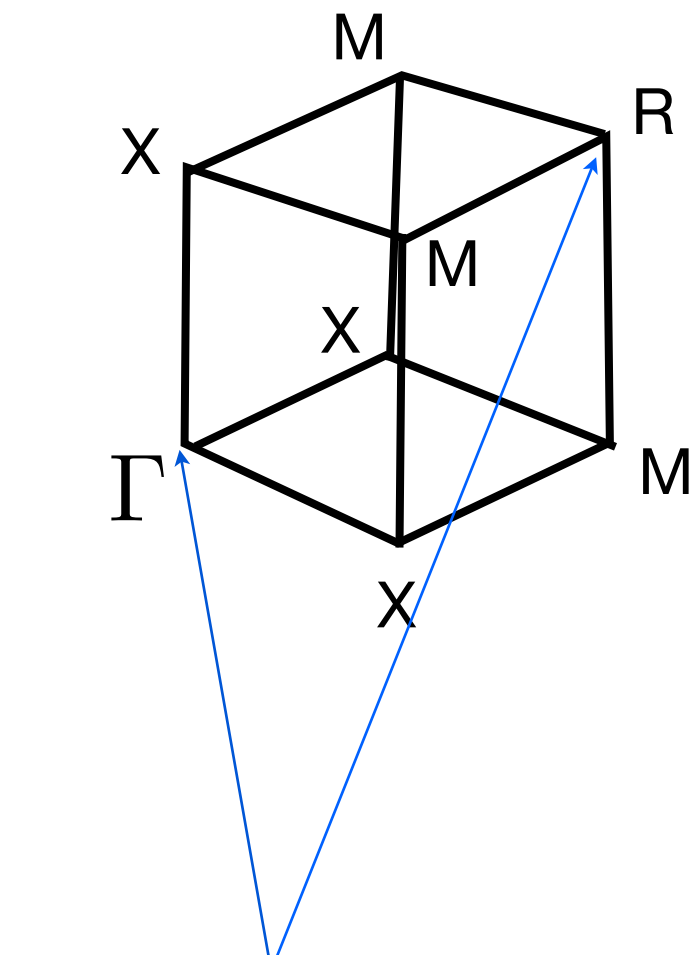
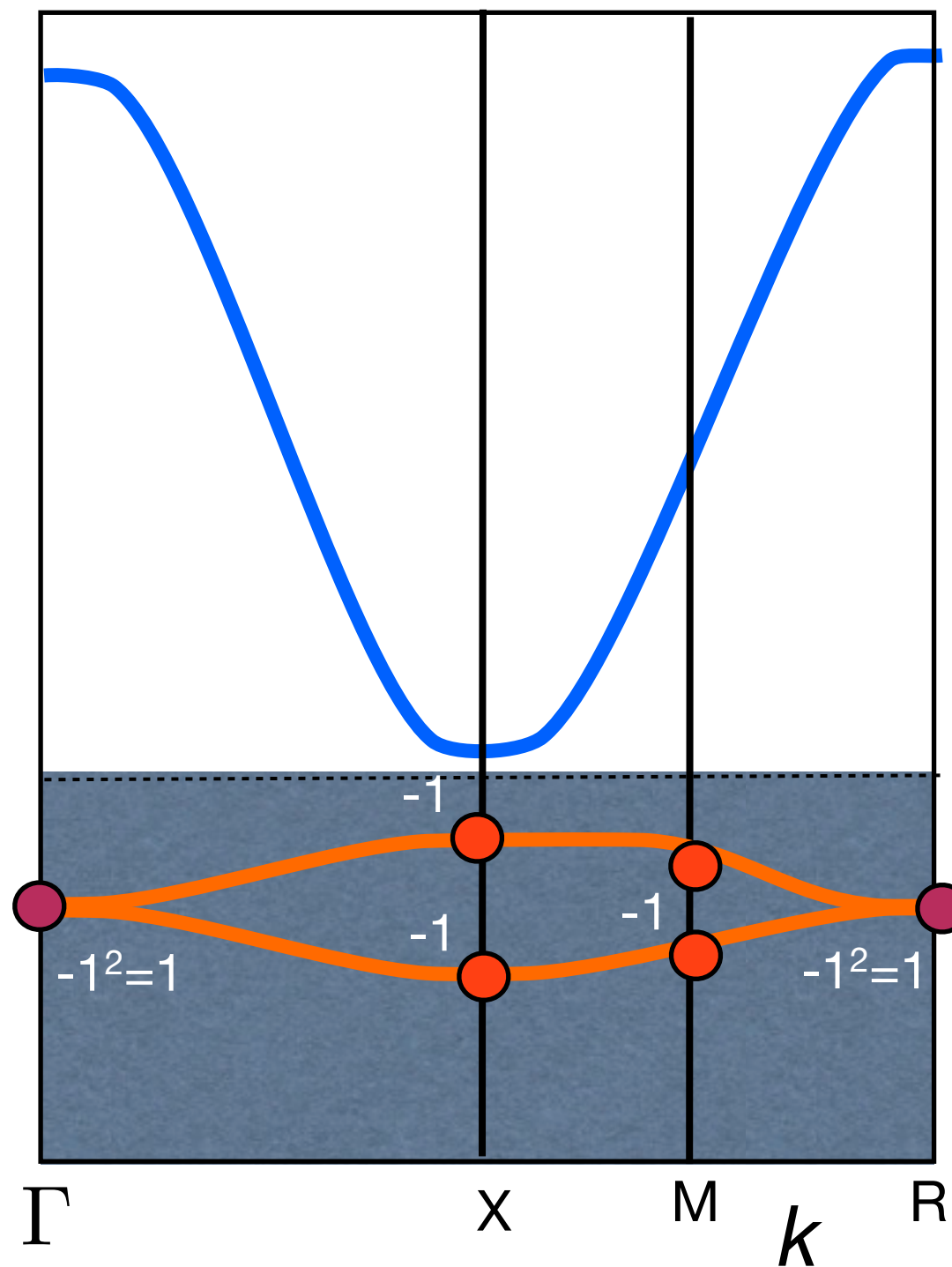
Alexandrov, Dzero and PC (2013)

# 3D Cubic Kondo Insulator



$E_k$

QUARTET:



QUARTET:  
topologically inert

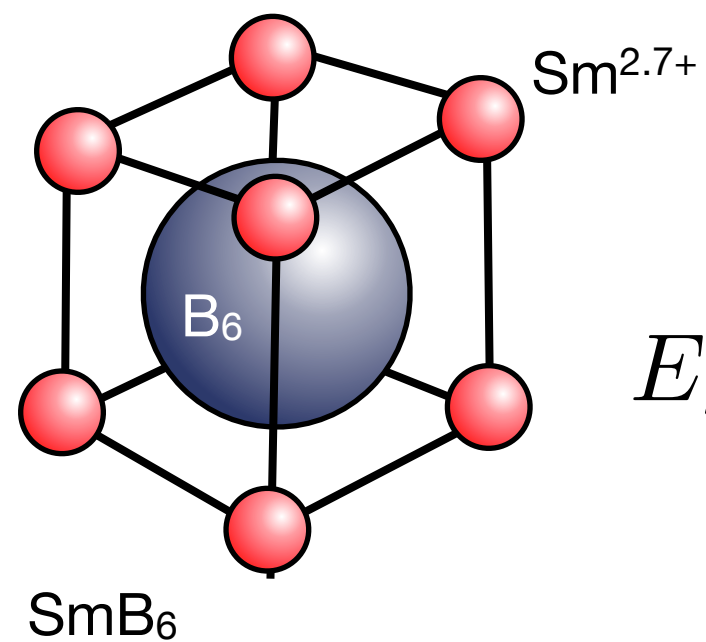
$d^0 f^6$

$\nu = +1$

THREE DIRAC CONES  
ON SURFACE.

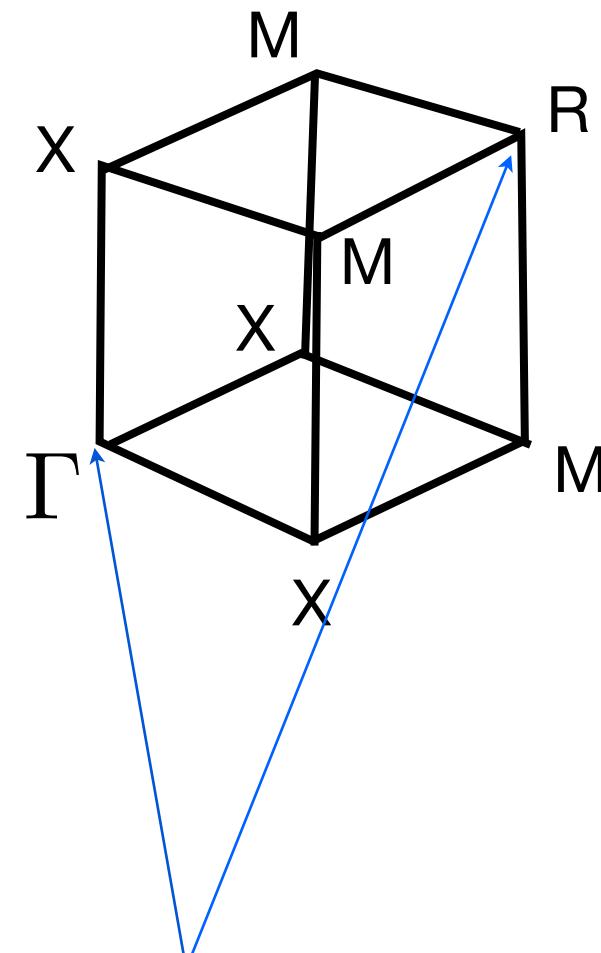
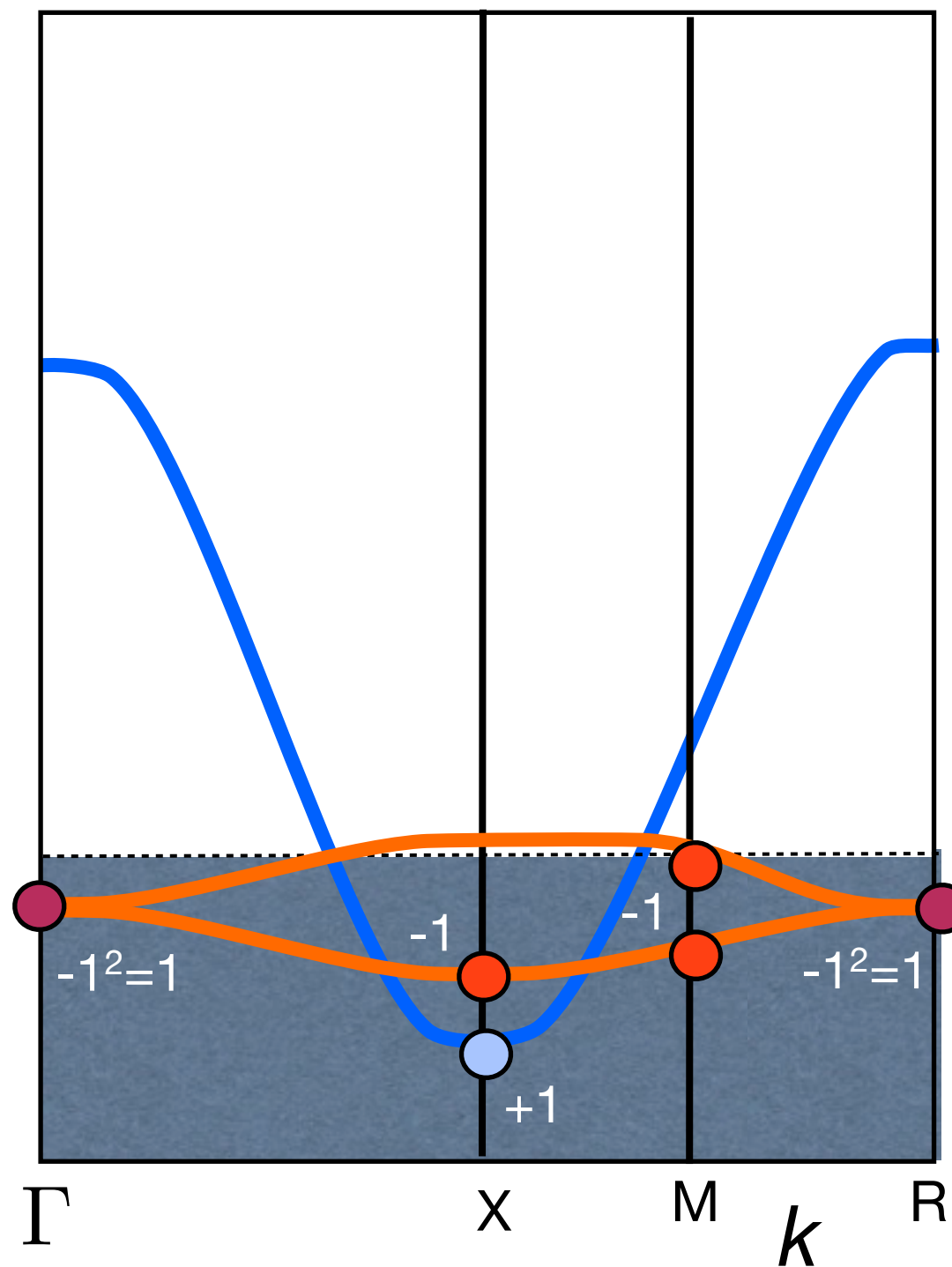


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QUARTET:

$E_k$



QUARTET:  
topologically inert

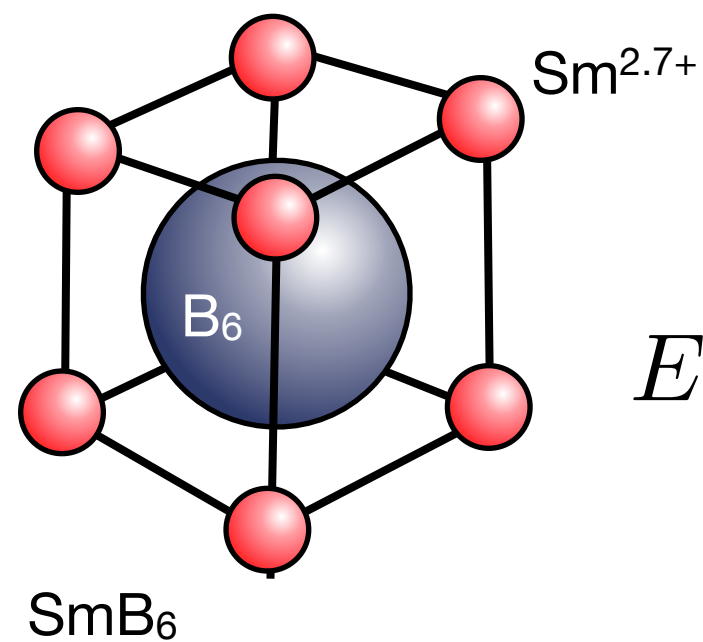
$d^0 f^6$

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THREE DIRAC CONES  
ON SURFACE.

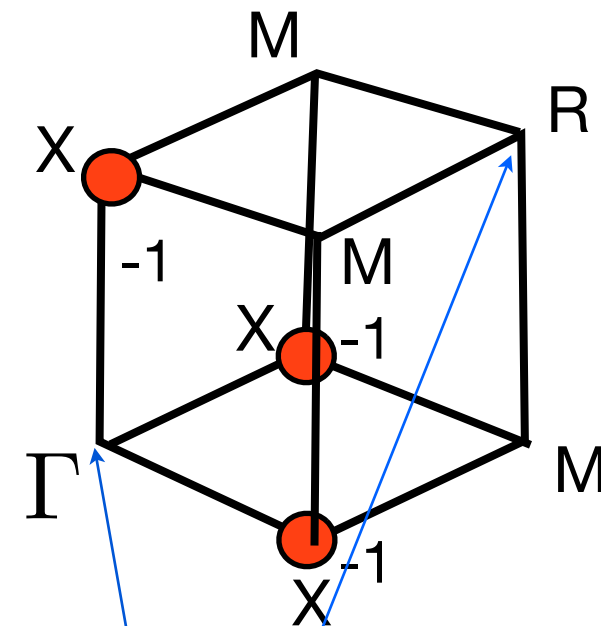
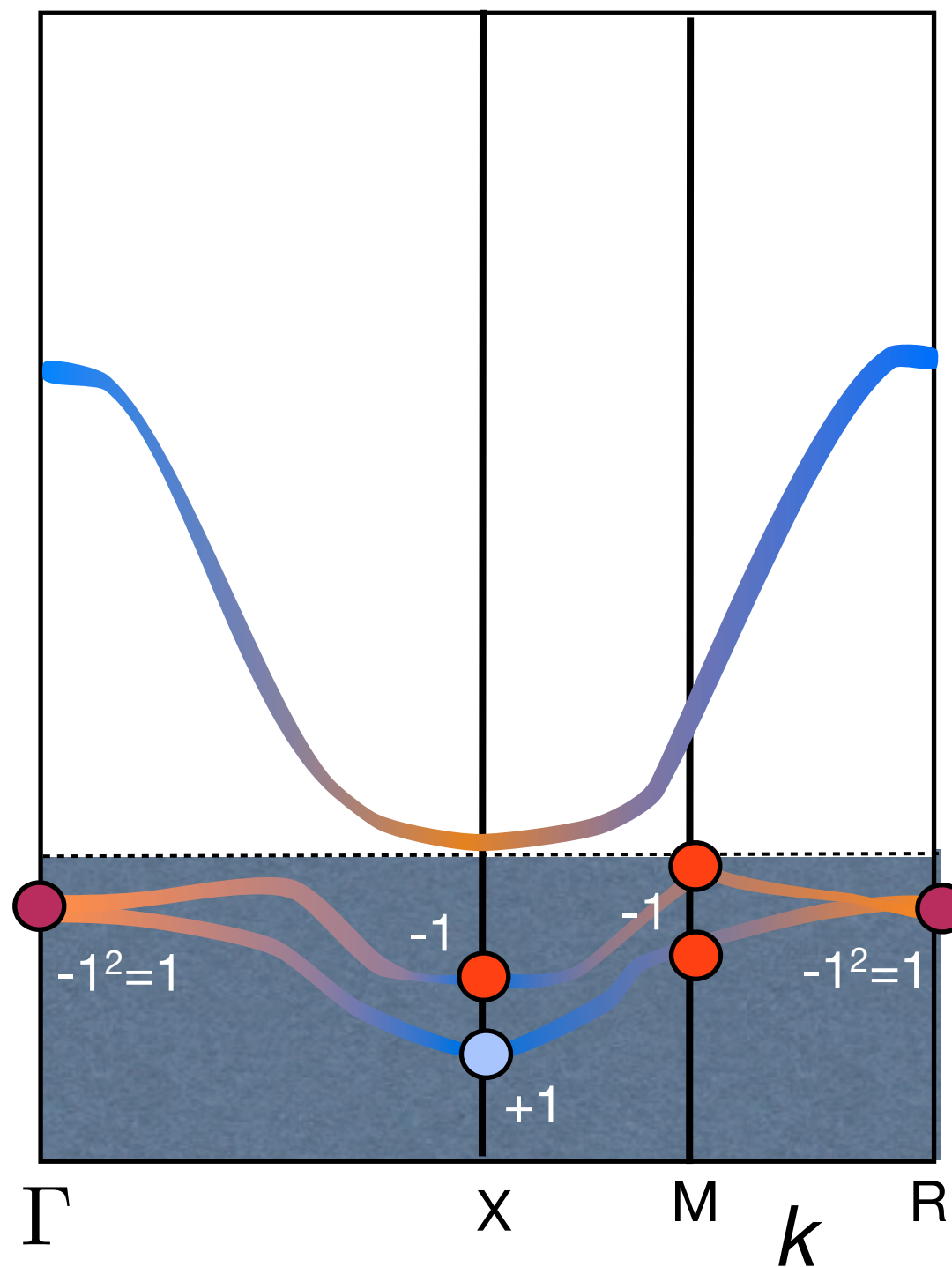
Alexandrov, Dzero and PC (2013)

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$E_k$

QUARTET:



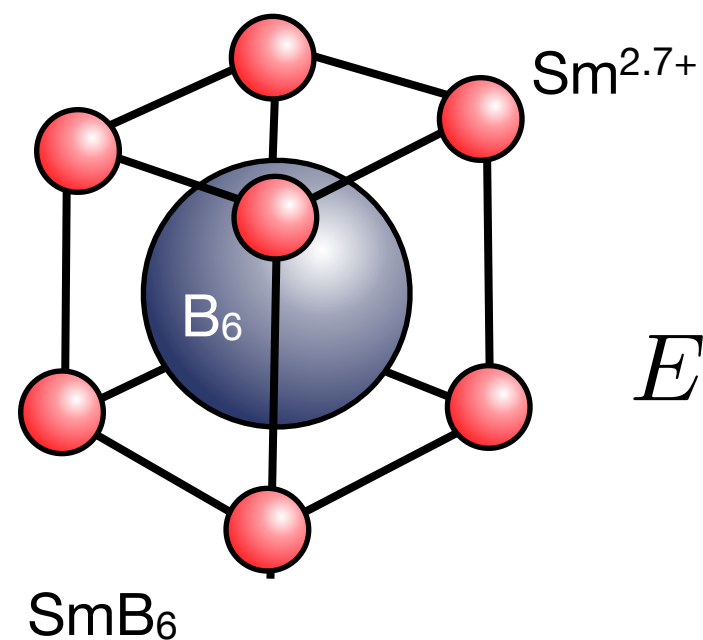
QUARTET:  
topologically inert

THREE DIRAC CONES  
ON SURFACE.

$$d^0 f^6$$

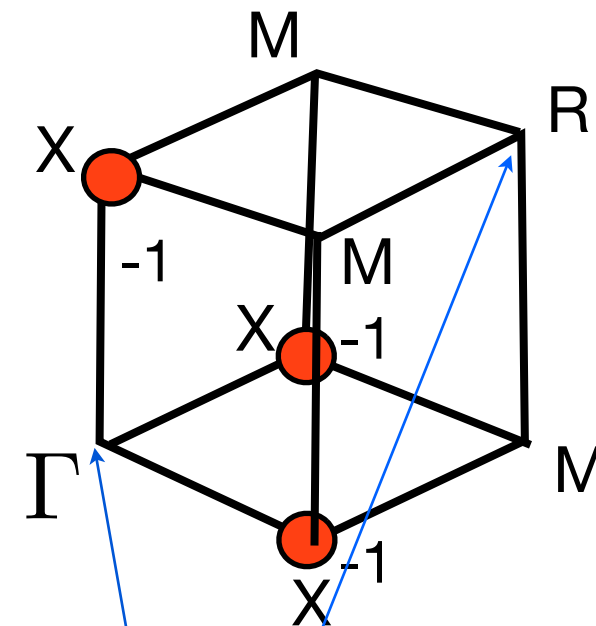
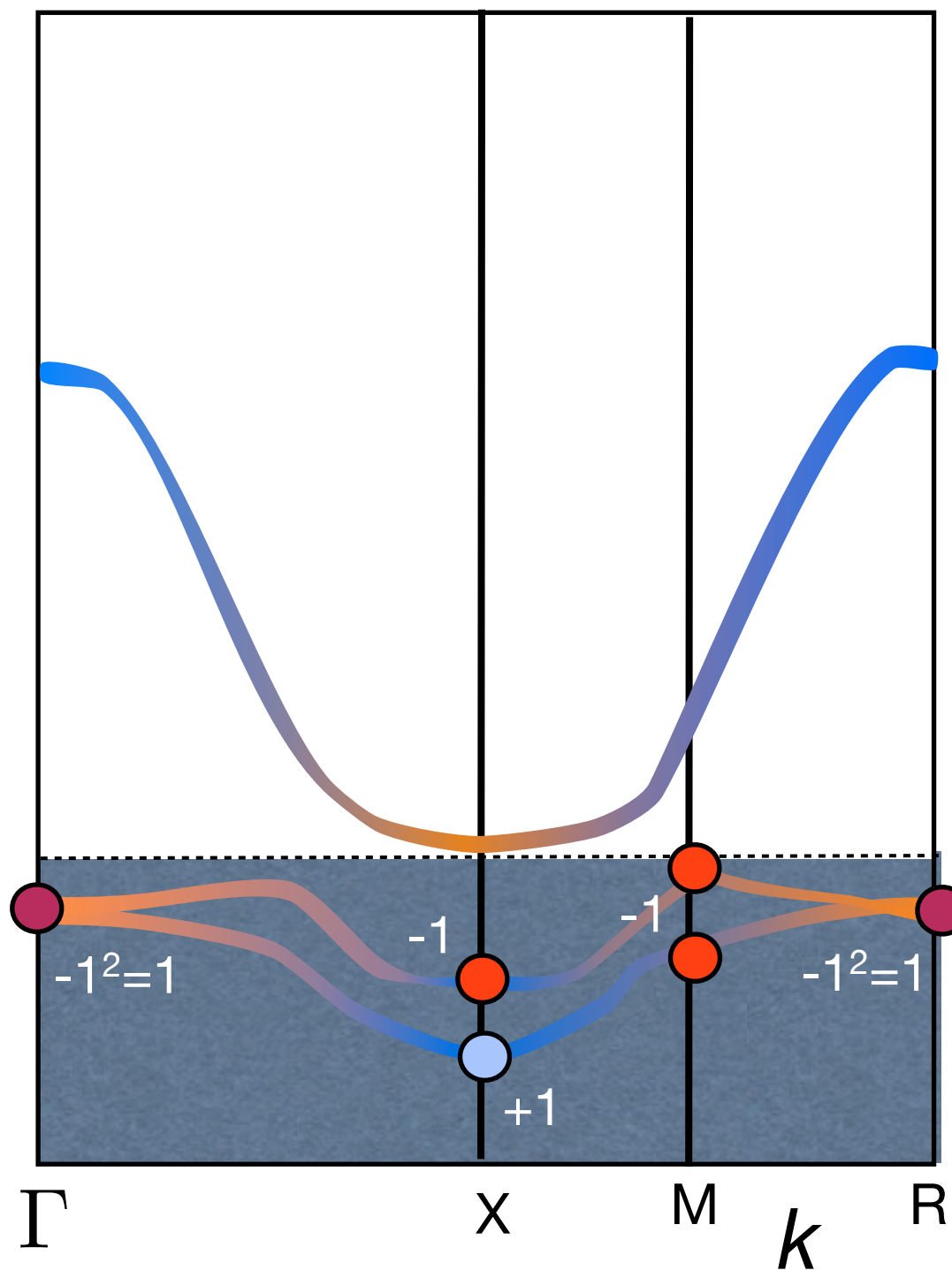
$$\nu = +1$$

# 3D Cubic Kondo Insulator



QUARTET:

$E_k$



QUARTET:  
topologically inert

$$d^0 f^6 \longrightarrow d^1 f^5$$

$$\nu = +1 \quad \nu = -1$$

THREE DIRAC CONES  
ON SURFACE.

# Experiments:

- Non-local conductivity
- Arpes
- dHvA
- Weak Antilocalization

# Discovery of the First Topological Kondo Insulator: Samarium Hexaboride

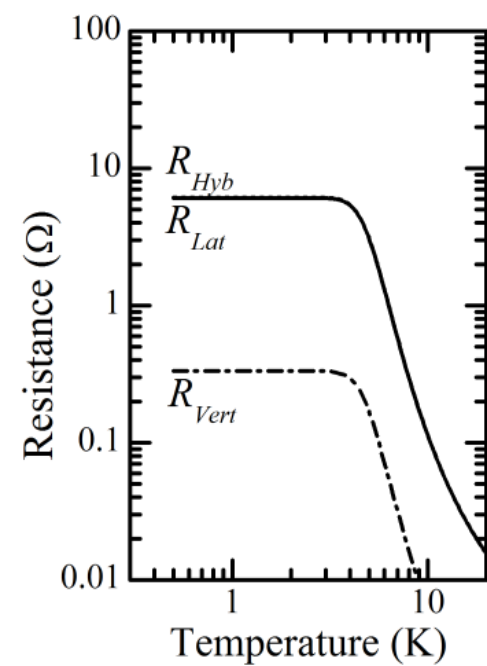
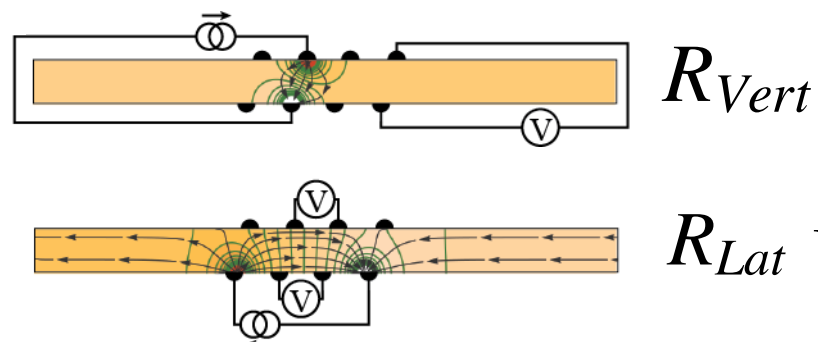
Steven Wolgast, Cagliyan Kurdak, Kai Sun, J. W. Allen, Dae-Jeong Kim, Zachary Fisk

*(Submitted on 21 Nov 2012 (v1), last revised 27 Nov 2012 (this version, v2))*

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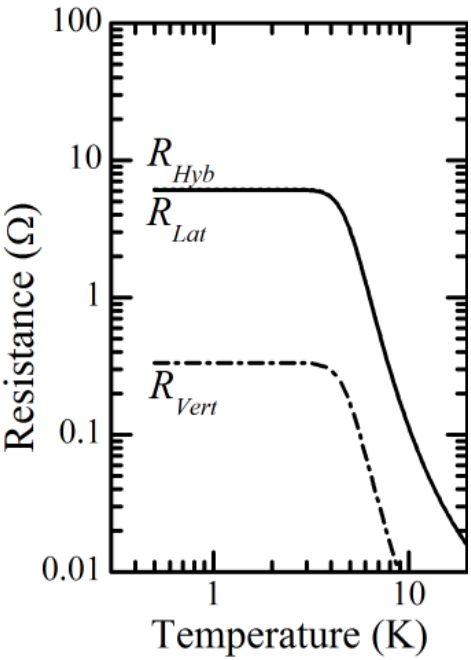
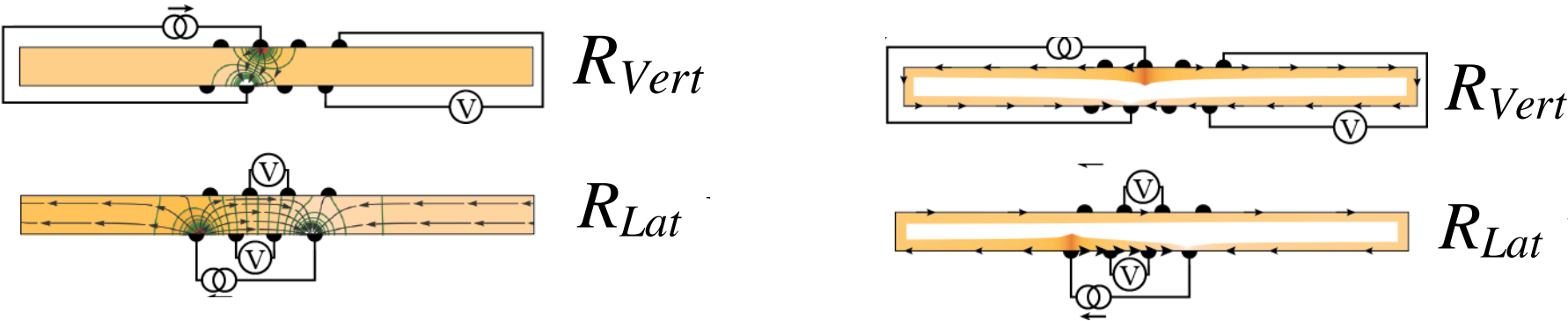
Simulation-bulk scenario



# Discovery of the First Topological Kondo Insulator: Samarium Hexaboride

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(Submitted on 21 Nov 2012 (v1), last revised 27 Nov 2012 (this version, v2))

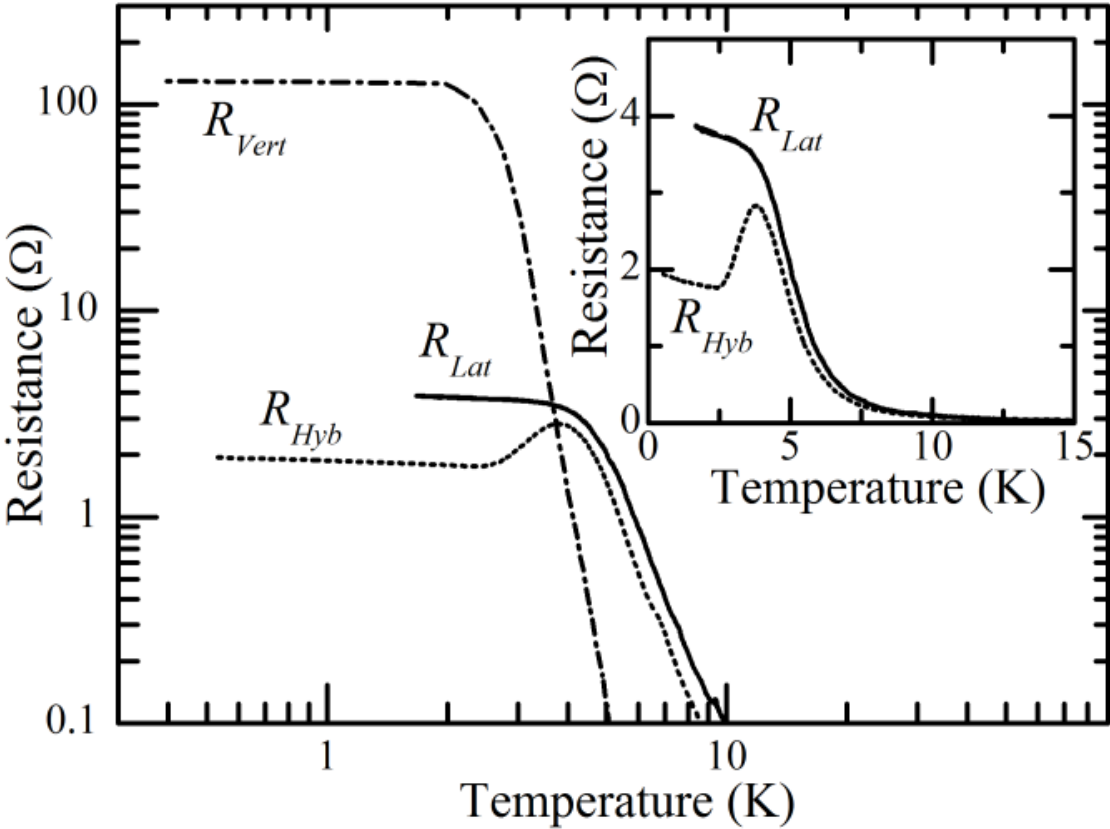
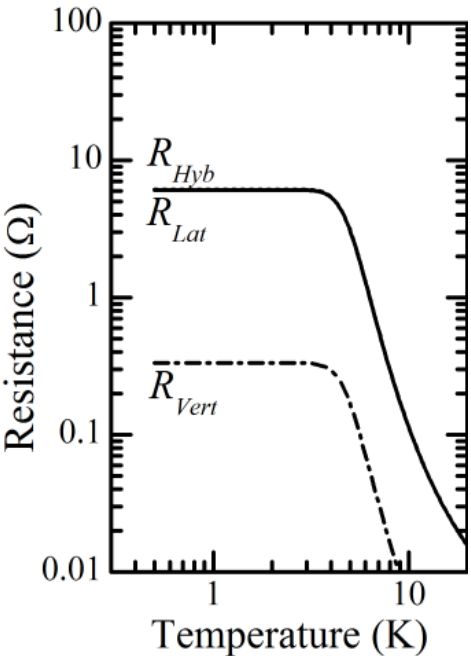
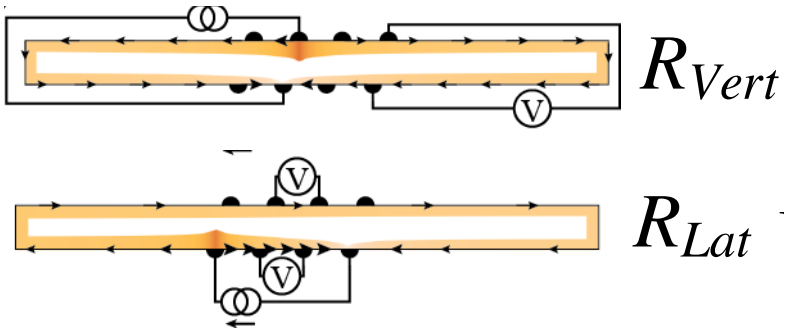
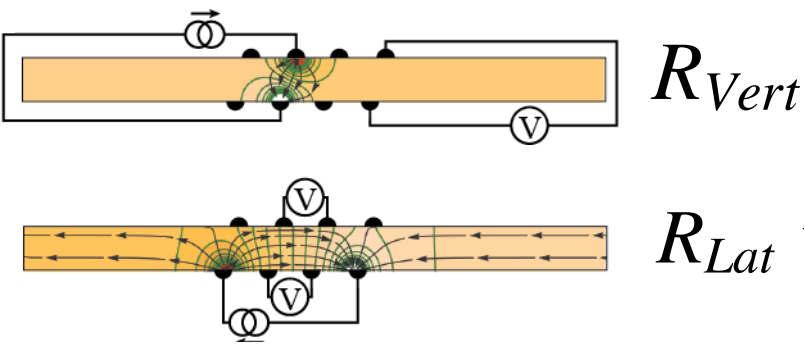


Simulation-bulk scenario

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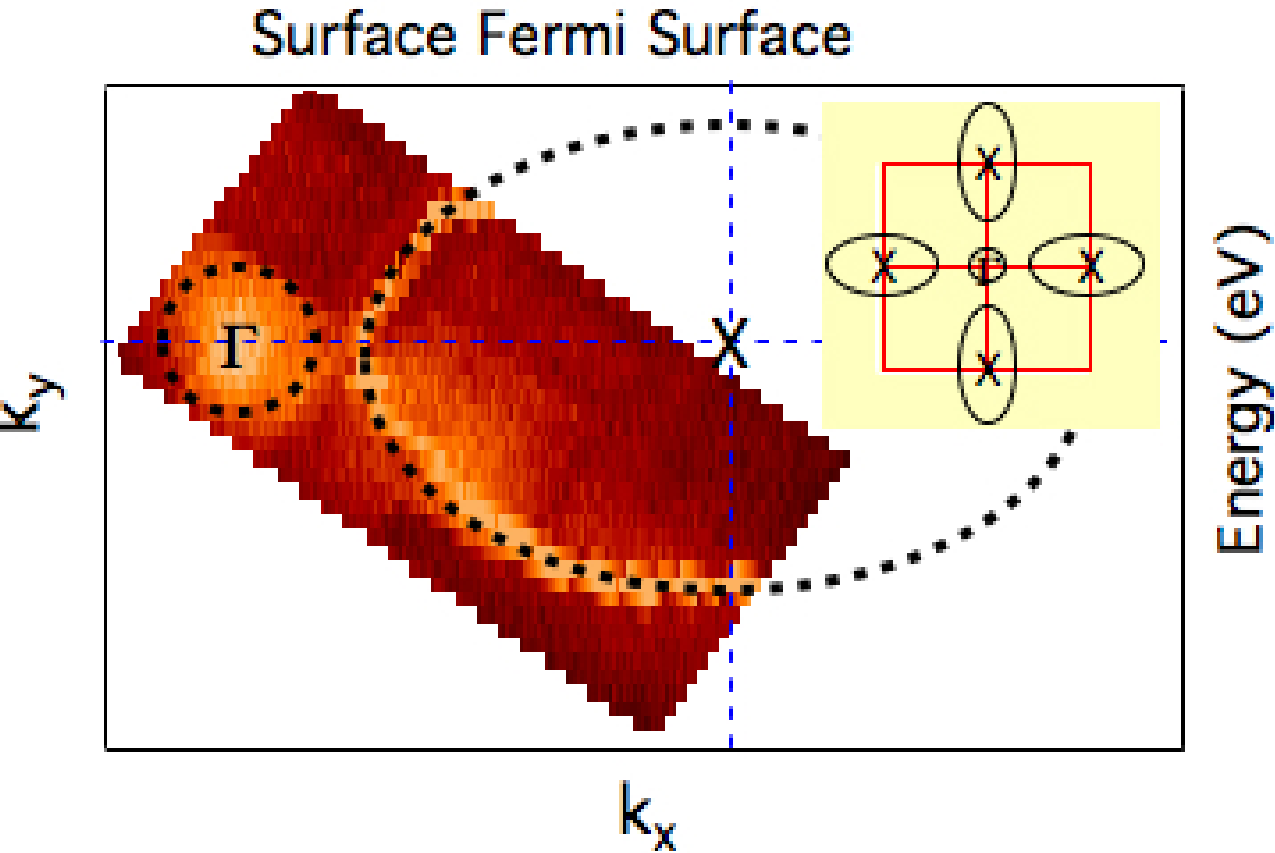


Simulation-bulk scenario

# Surface electronic structure of topological Kondo insulator candidate

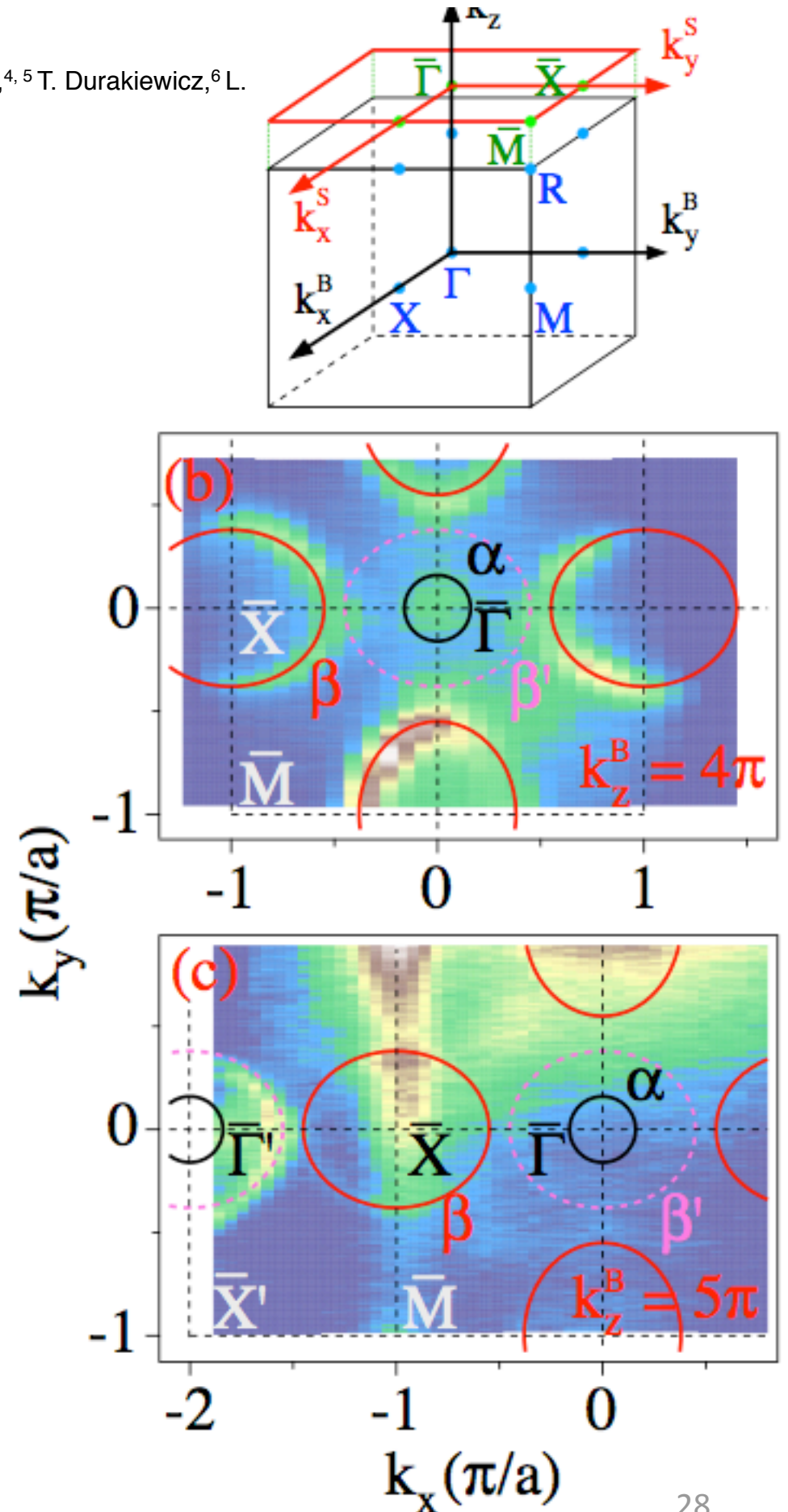
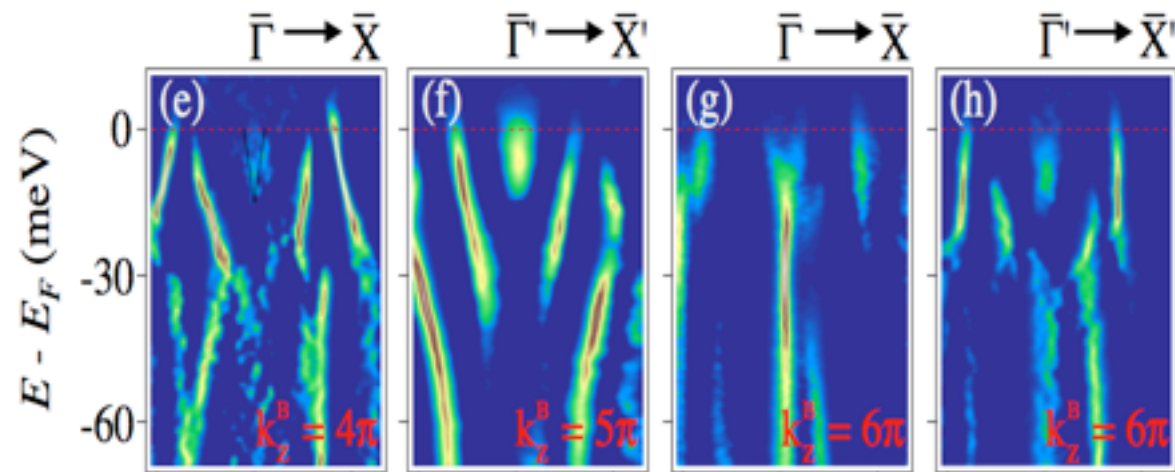
## SmB<sub>6</sub>: A view from high-resolution Laser-ARPES

M. Neupane,<sup>1</sup> N. Alidoust,<sup>1</sup> S.-Y. Xu,<sup>1</sup> T. Kondo,<sup>2</sup> Dae-Jeong Kim,<sup>3</sup> Chang Liu,<sup>1</sup> I. Belopolski,<sup>1</sup> T.-R. Chang,<sup>4</sup> H.-T. Jeng,<sup>4, 5</sup> T. Durakiewicz,<sup>6</sup> L. Balicas,<sup>7</sup> H. Lin,<sup>8</sup> A. Bansil,<sup>8</sup> S. Shin,<sup>2</sup> Z. Fisk,<sup>3</sup> and M. Z. Hasan<sup>1</sup>



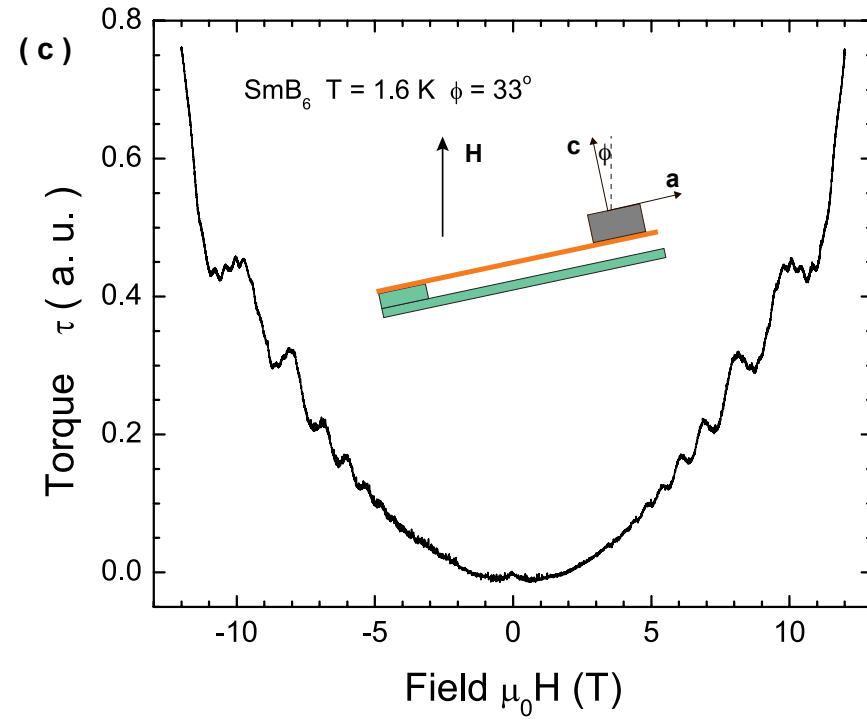
## Surface and Bulk Electronic Structure of the Strongly Correlated System SmB<sub>6</sub> and Implications for a Topological Kondo Insulator

N. Xu,<sup>1, \*</sup> X. Shi,<sup>1, 2</sup> P. K. Biswas,<sup>3</sup> C. E. Matt,<sup>1, 4</sup> R. S. Dhaka,<sup>1</sup> Y. Huang,<sup>1</sup> N. C. Plumb,<sup>1</sup> M. Radovic,<sup>1, 5</sup> J. H. Dil,<sup>6, 1</sup> E. Pomjakushina,<sup>7</sup> A. Amato,<sup>3</sup> Z. Salman,<sup>3</sup> D. McK. Paul,<sup>8</sup> J. Mesot,<sup>1, 9</sup> H. Ding,<sup>2</sup> and M. Shi<sup>1, †</sup>



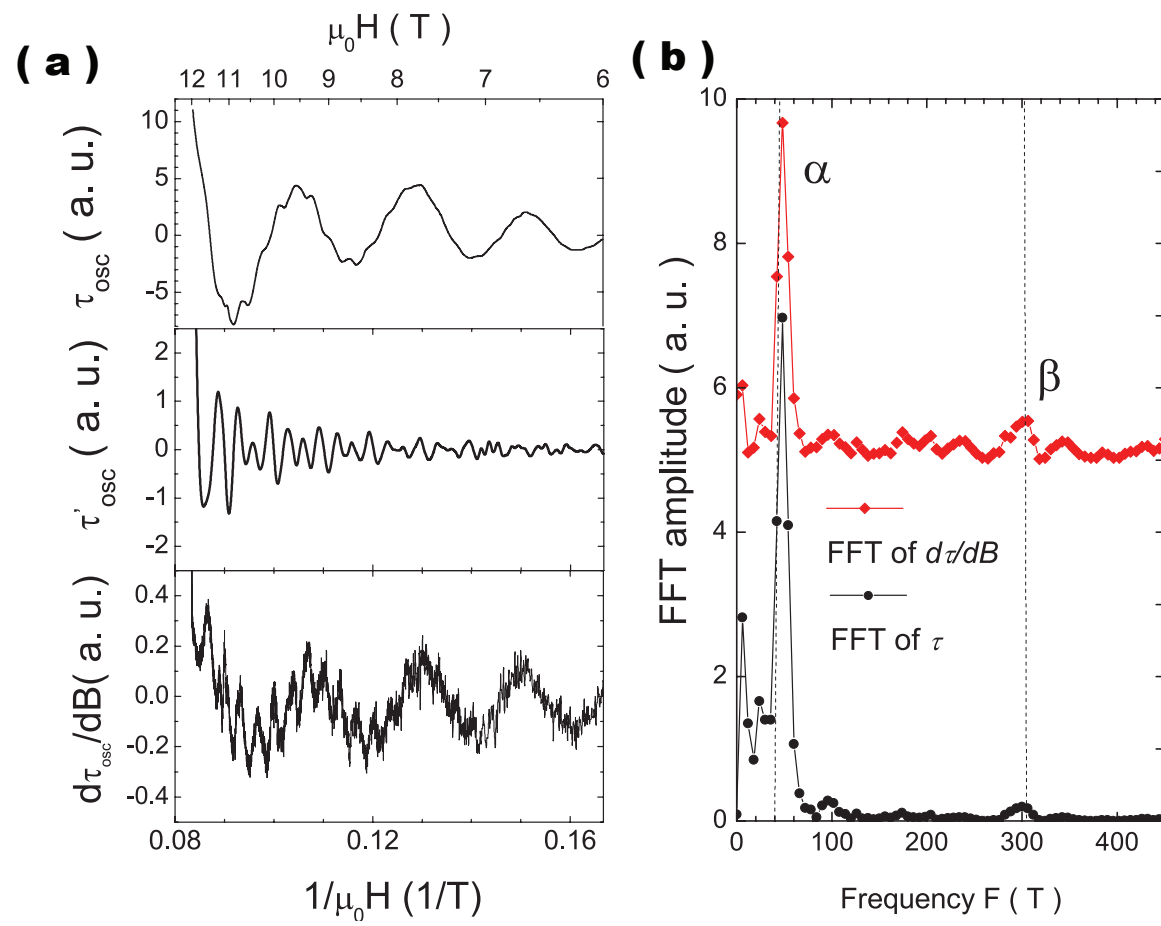
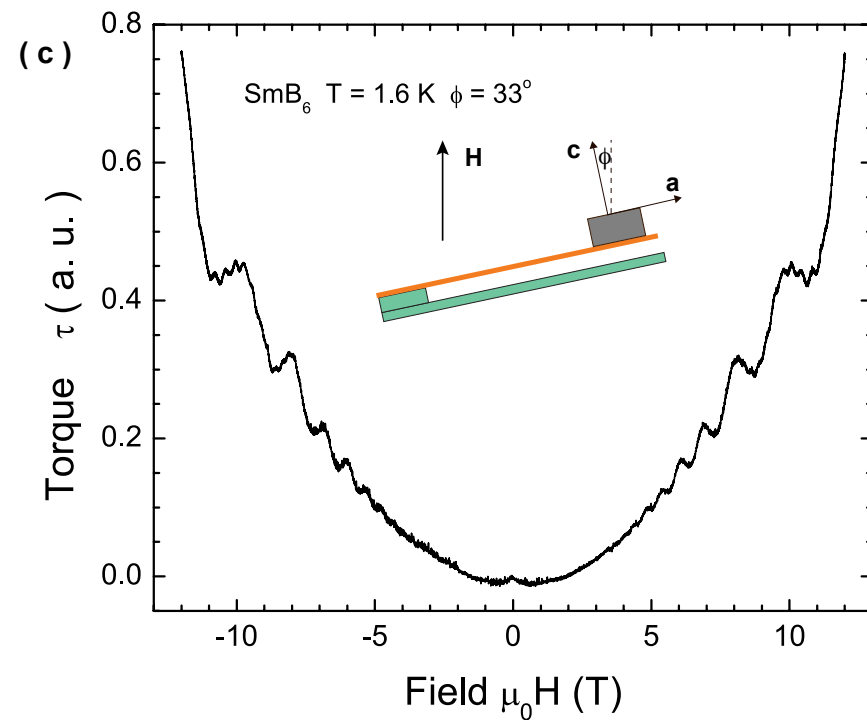
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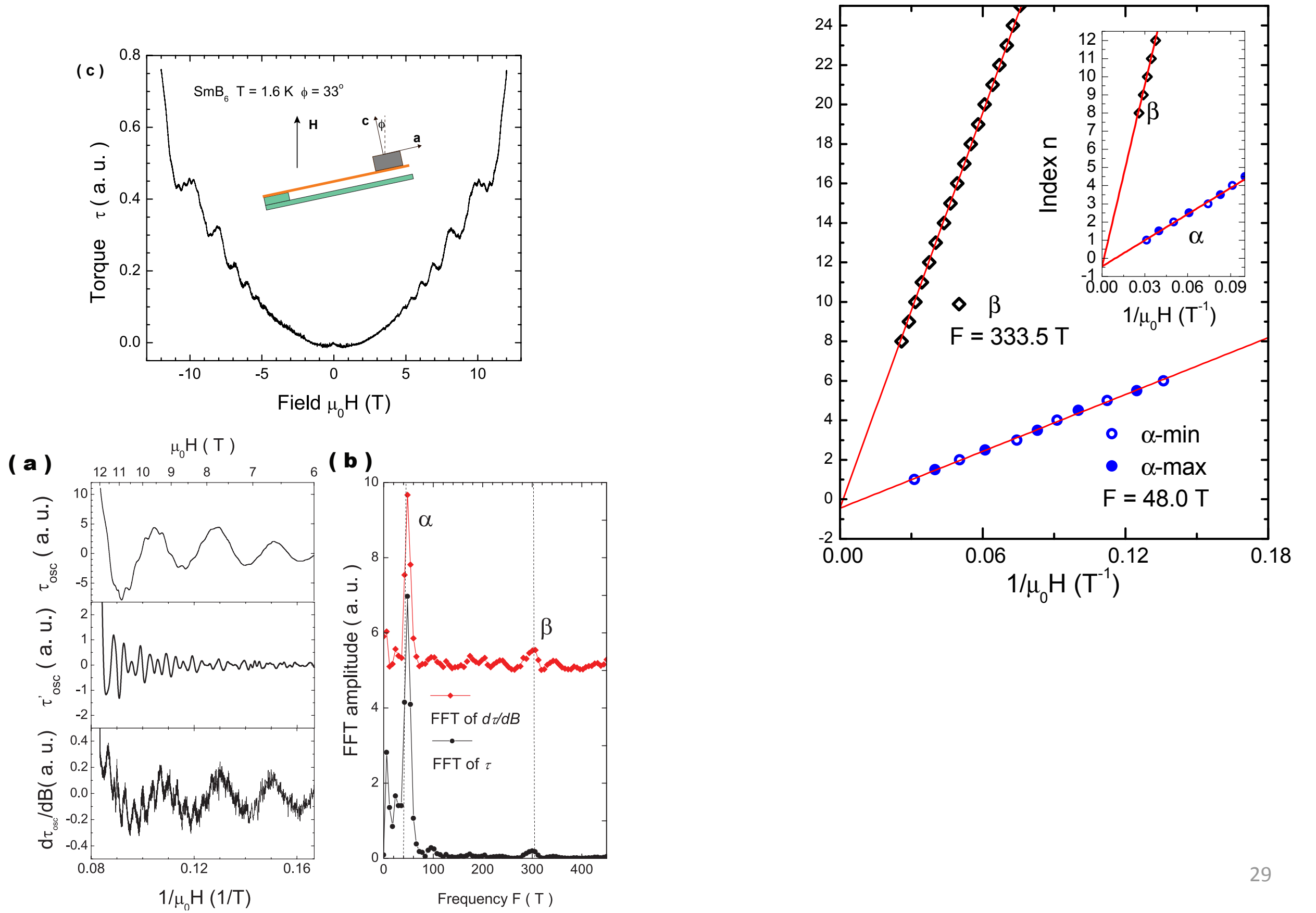
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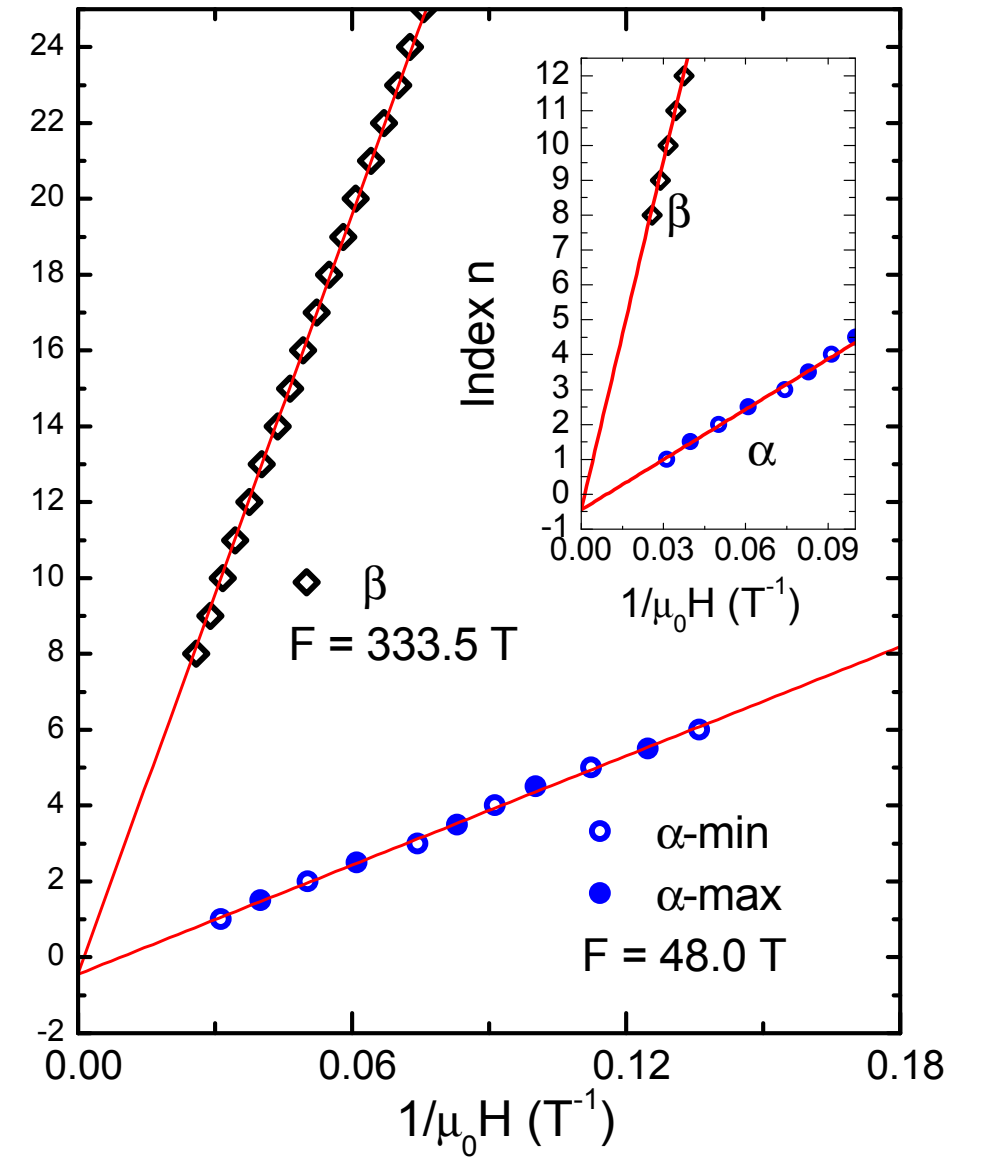
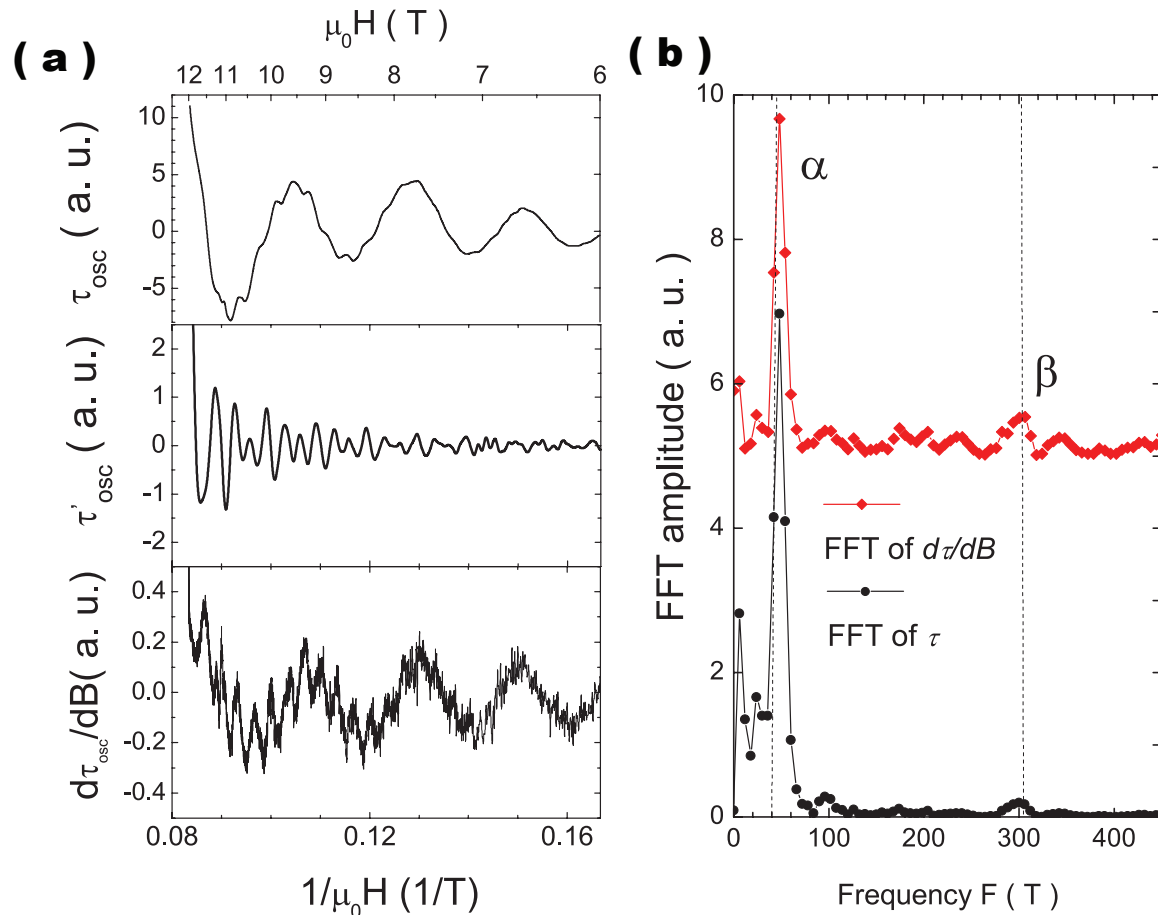




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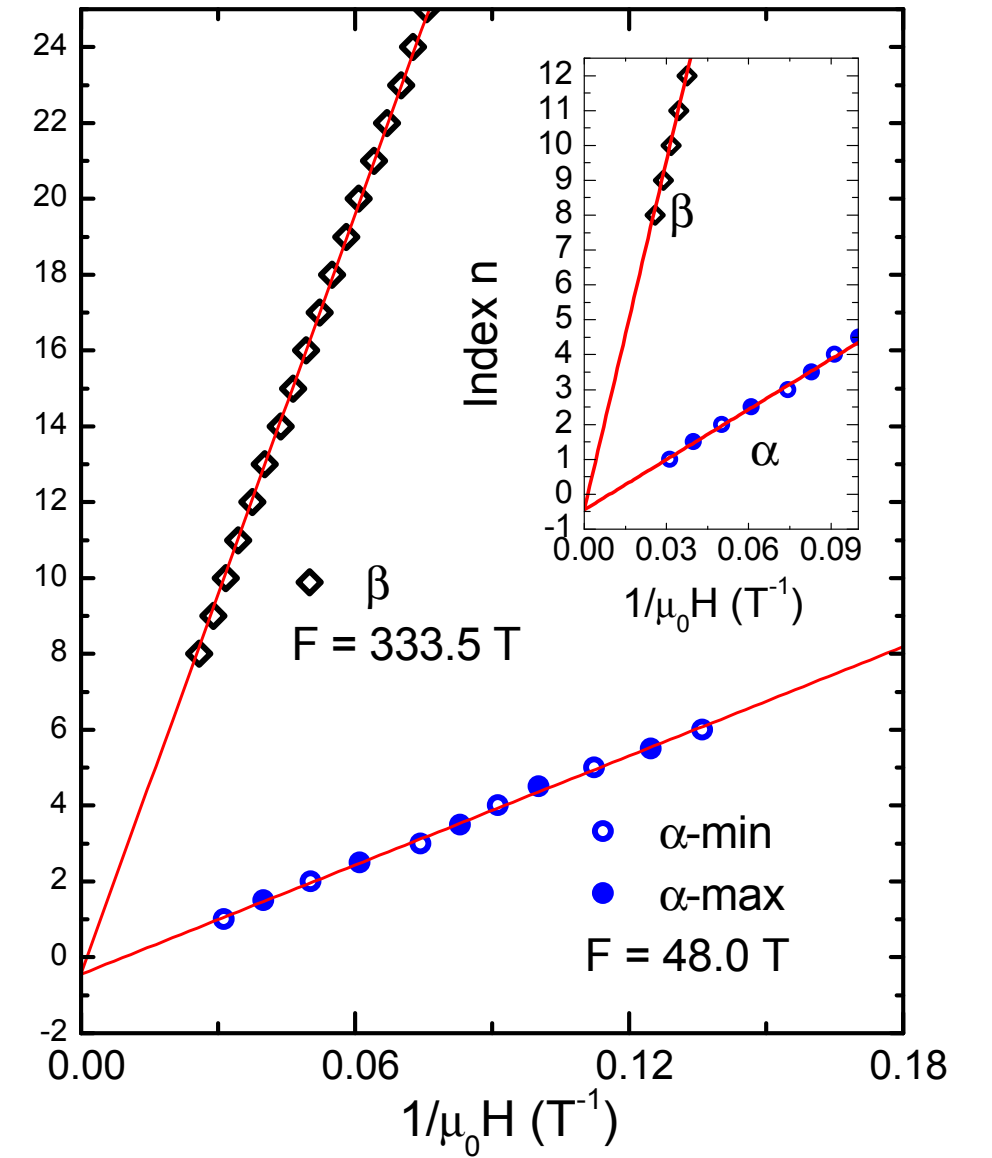
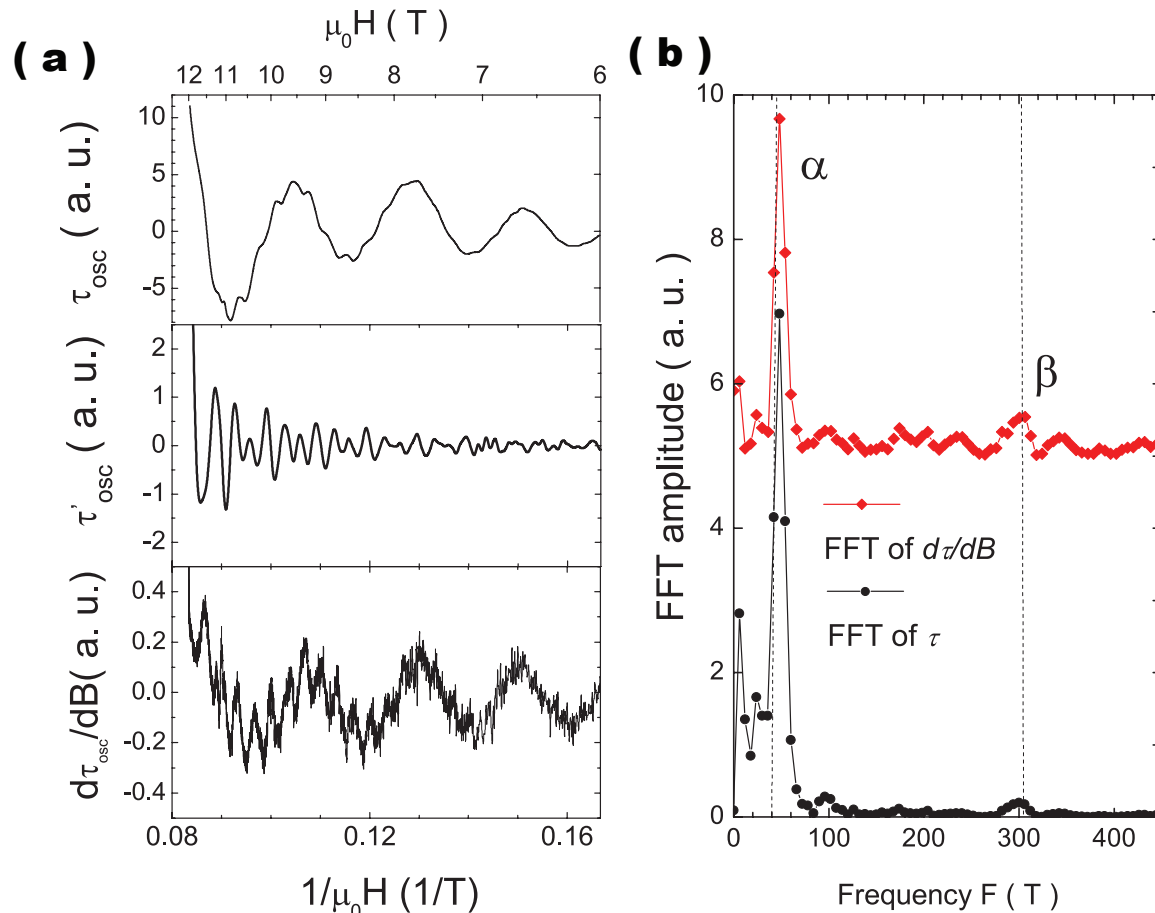
	$\alpha$	$\beta$
$F$ ( T )	$48.3 \pm 1.8$	$300.5 \pm 1.3$
$k_F$ ( nm <sup>-1</sup> )	$0.383 \pm 0.007$	$0.955 \pm 0.002$
$\frac{m}{m_e}$	$0.074 \pm 0.004$	$0.101 \pm 0.012$
$v_F$ (10 <sup>5</sup> m s <sup>-1</sup> )	$6.0 \pm 0.4$	$10.9 \pm 1.3$
$l$ (nm)	$33 \pm 7$	$55 \pm 16$
$\mu$ (×10 <sup>3</sup> cm <sup>2</sup> /V s)	$1.3 \pm 0.3$	$0.86 \pm 0.26$
$k_F l$	$13 \pm 3$	$53 \pm 15$
$\gamma$	$-0.45 \pm 0.07$	$-0.44 \pm 0.06$



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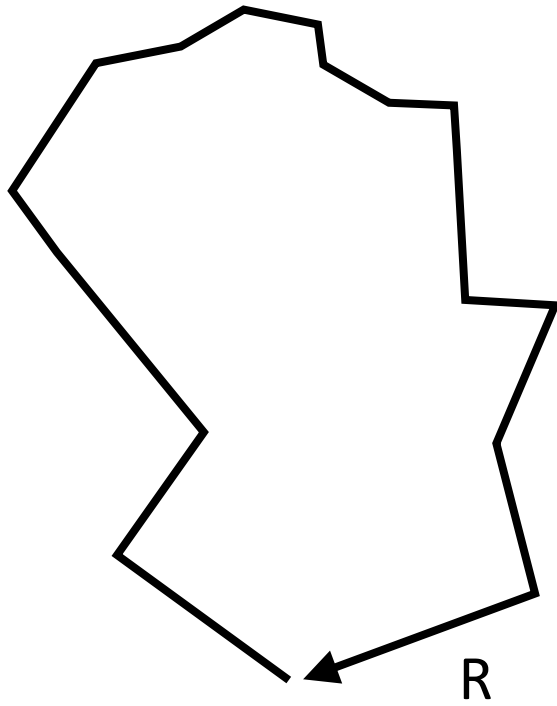
- Two Dirac Cones
- Light quasiparticles

# **Weak Antilocalization and Linear Magnetoresistance in The Surface State of SmB<sub>6</sub>**

S. Thomas<sup>1\*</sup>, D.J. Kim<sup>1\*</sup>, S. B. Chung<sup>2</sup>, T. Grant<sup>1</sup>, Z. Fisk<sup>1</sup> and Jing Xia<sup>1</sup>

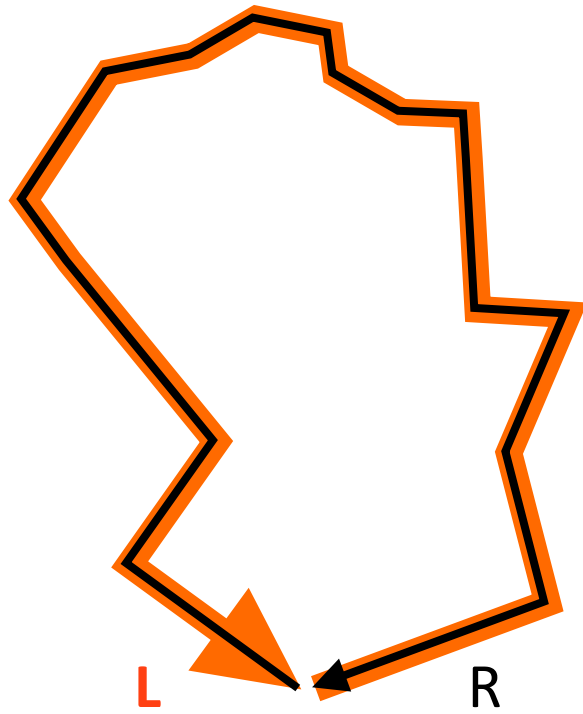
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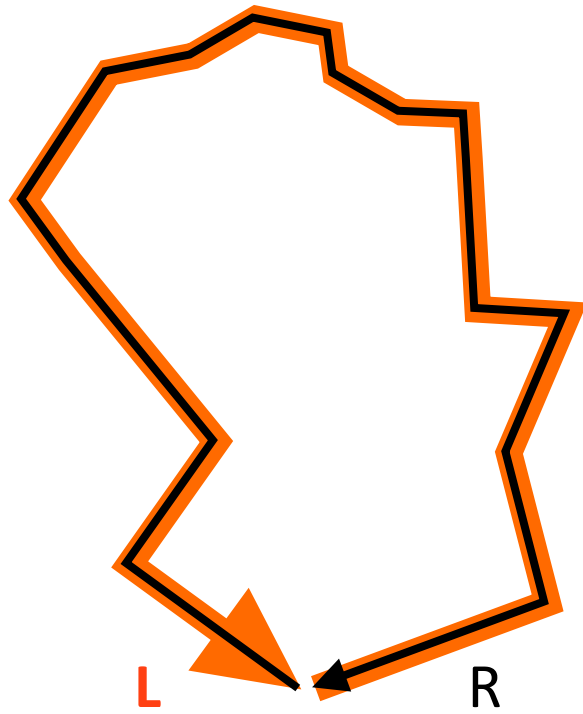
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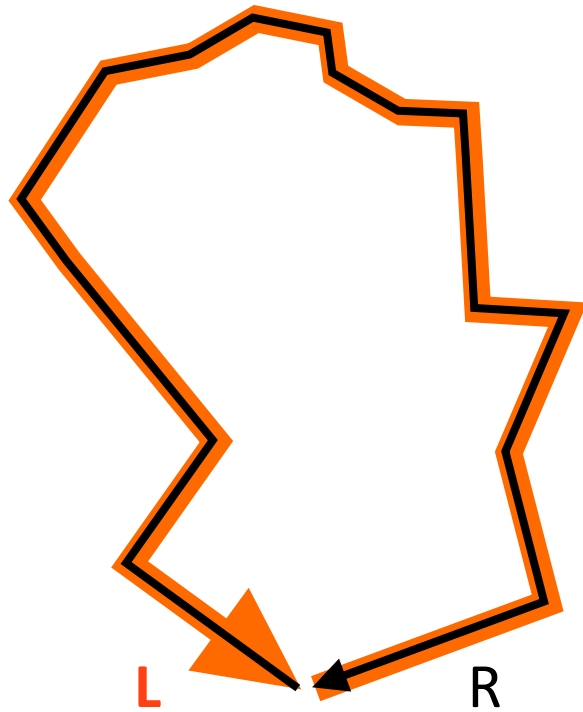
Localization

$$\psi_L = \psi_R$$

$$p = |\psi_L + \psi_R|^2 = |\psi_L|^2 + |\psi_R|^2 + 2|\psi_L||\psi_R|$$

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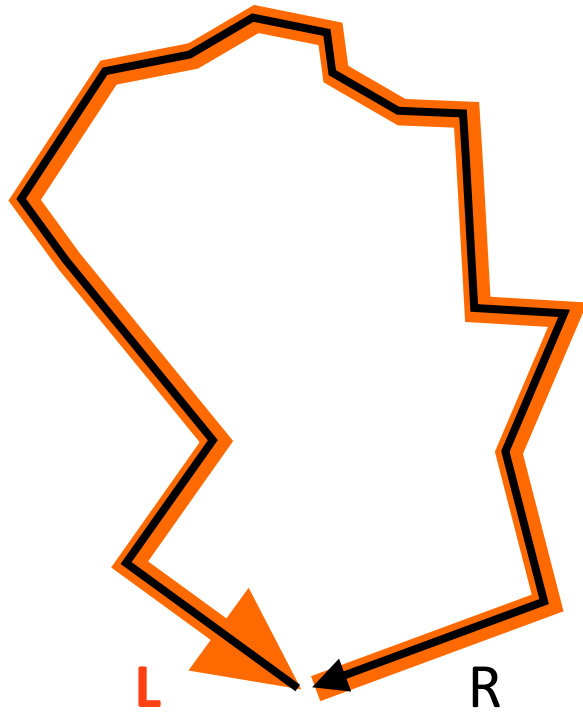
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Anti Localization:  
spin rotates with p.



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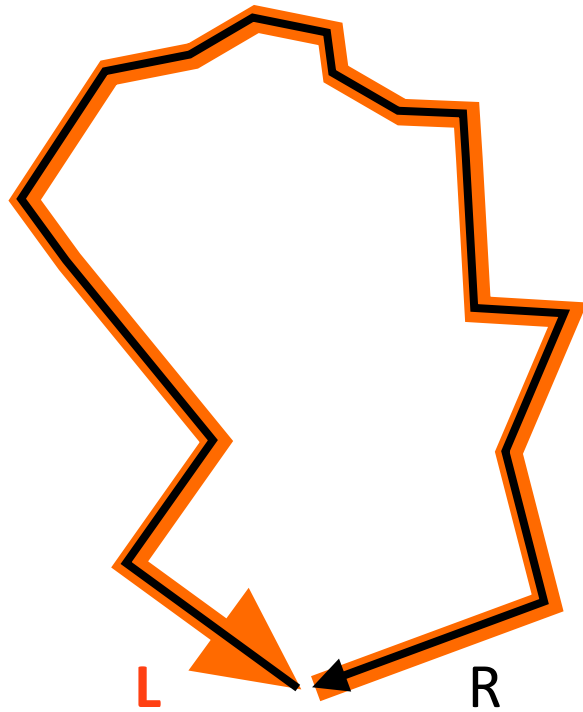
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Hikami, Larkin & Nagaosa, Prof. Th. Physics, 63, 707 (1980)

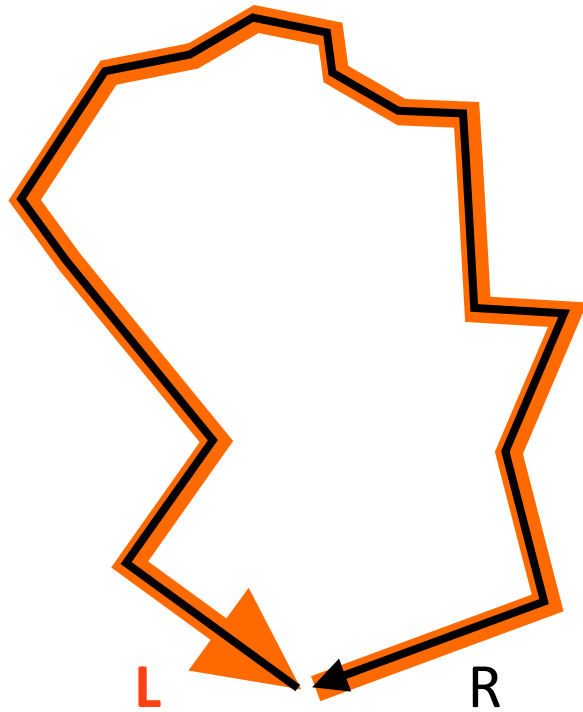
$$\Delta\sigma = \sigma(H) - \sigma(0)$$

$$= -\frac{\alpha e^2}{2\pi^2 \hbar} \left[ \ln \frac{1}{\tau_\epsilon a} - \psi\left(\frac{1}{2} + \frac{1}{\tau_\epsilon a}\right) \right]$$

where  $a = 4DeH/\hbar c$ ,

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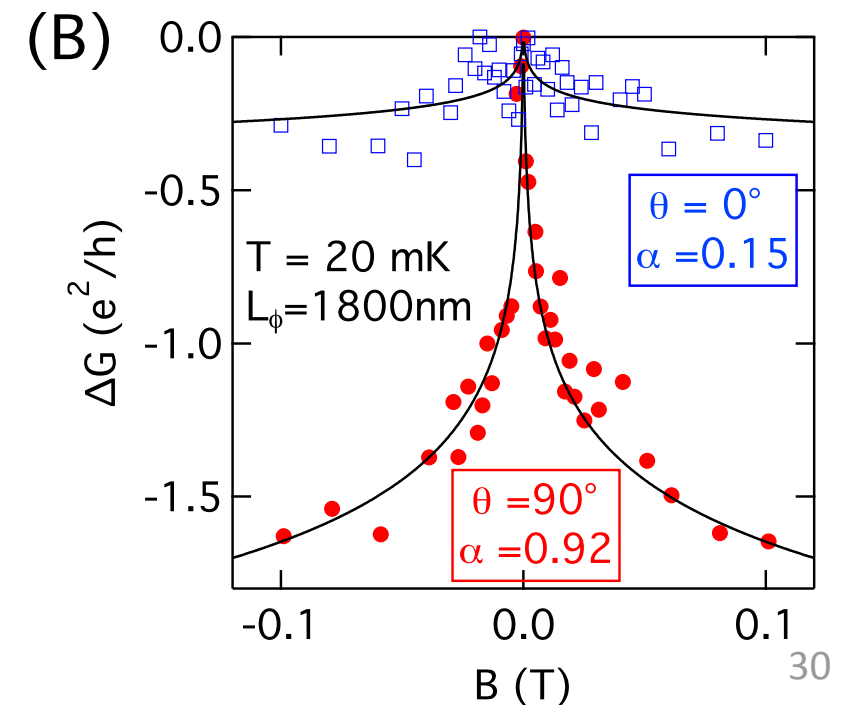
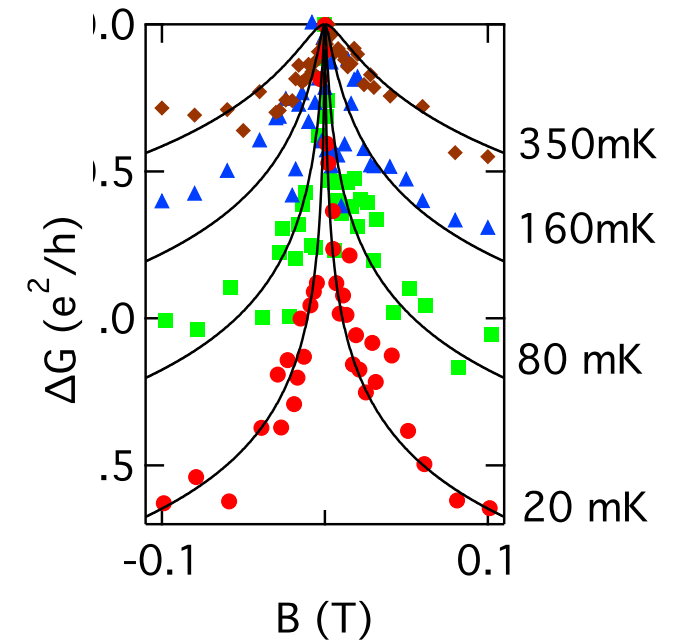
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# SmB<sub>6</sub> : Summary & Questions

- Weak localization, dHvA, surface conductance and Arpes, taken together, indicate that this is a topological insulator - moreover, a TKI.
- The multiplicity of Dirac cones supports the idea that this system derives from a quartet state of SmB<sub>6</sub>.
- The high bulk resistivity may make this the best practical candidate for 3D TI's discovered to date.

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- The high bulk resistivity may make this the best practical candidate for 3D TI's discovered to date.
- But! Why are the surface states light in dHvA and Arpes?
- Can the surface states undergo phase transitions? eg Paired surface states.
- Is topology important for other strongly correlated systems - metals, superconductors?

# Topological Kondo insulators ?

Hasan and Kane (RMP 2009)

Qi and Zhang (RMP 2010)

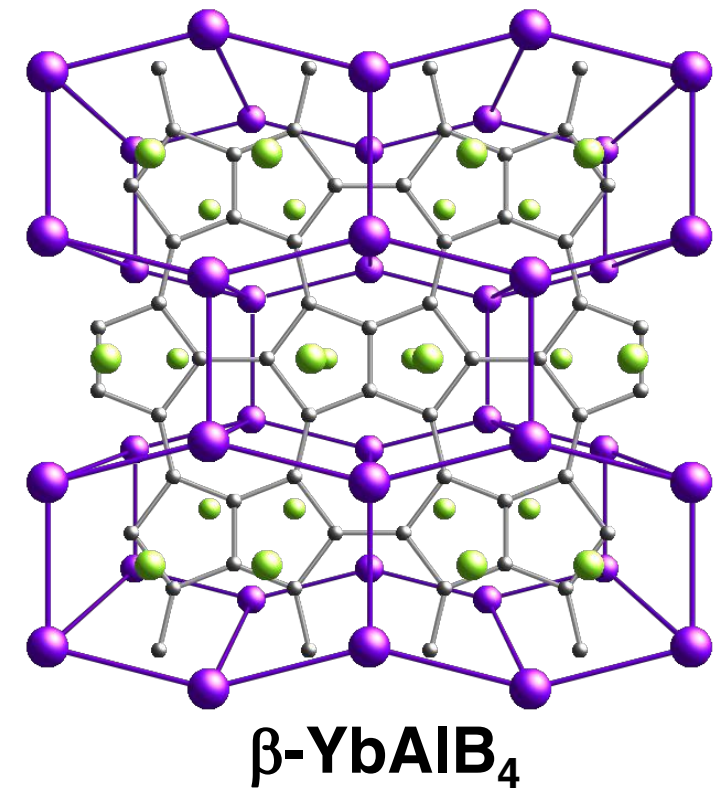
# $\beta$ -YbAlB<sub>4</sub>: Intrinsically Quantum Critical Heavy Fermion Superconductor



# $\beta$ -YbAlB<sub>4</sub>:

Intrinsically Quantum Critical Heavy Fermion SC.

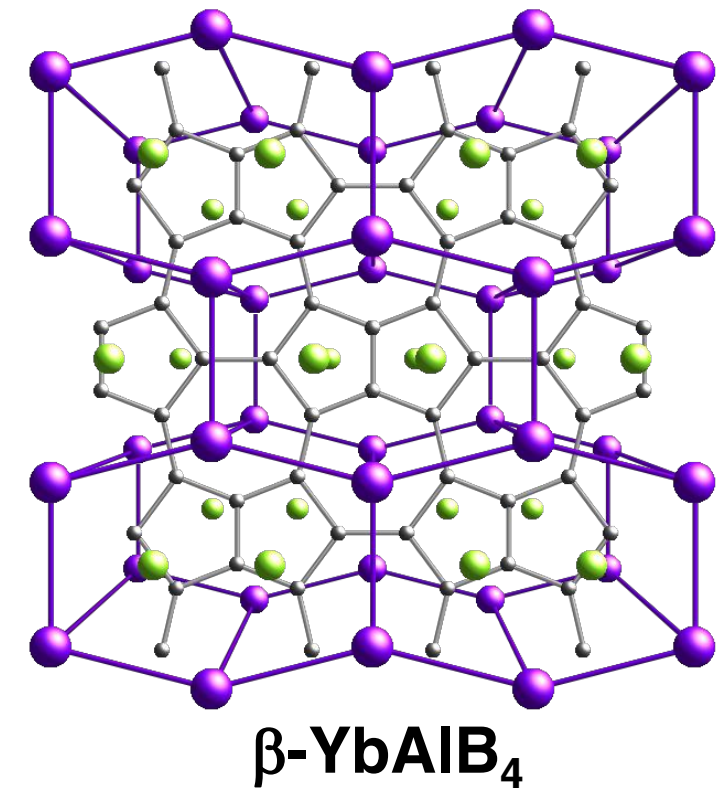
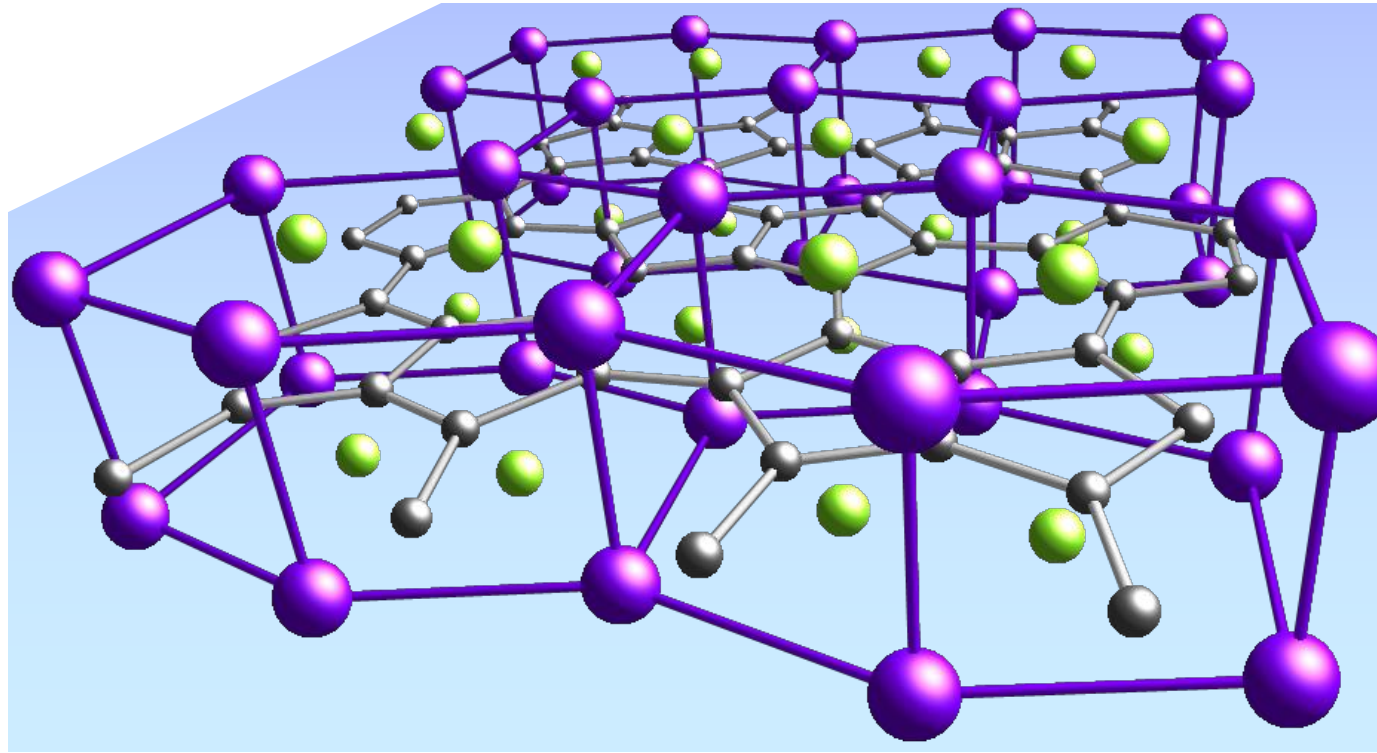
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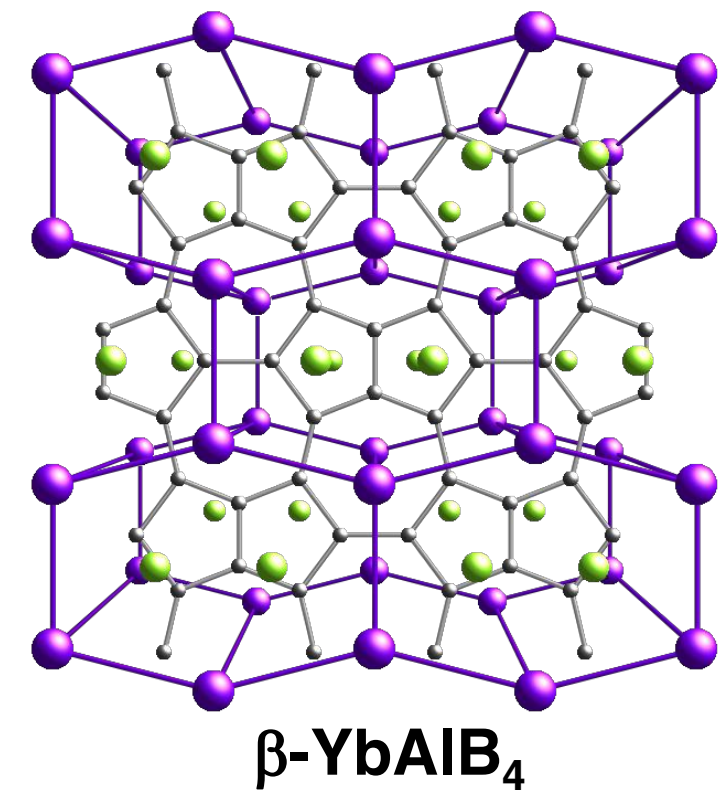
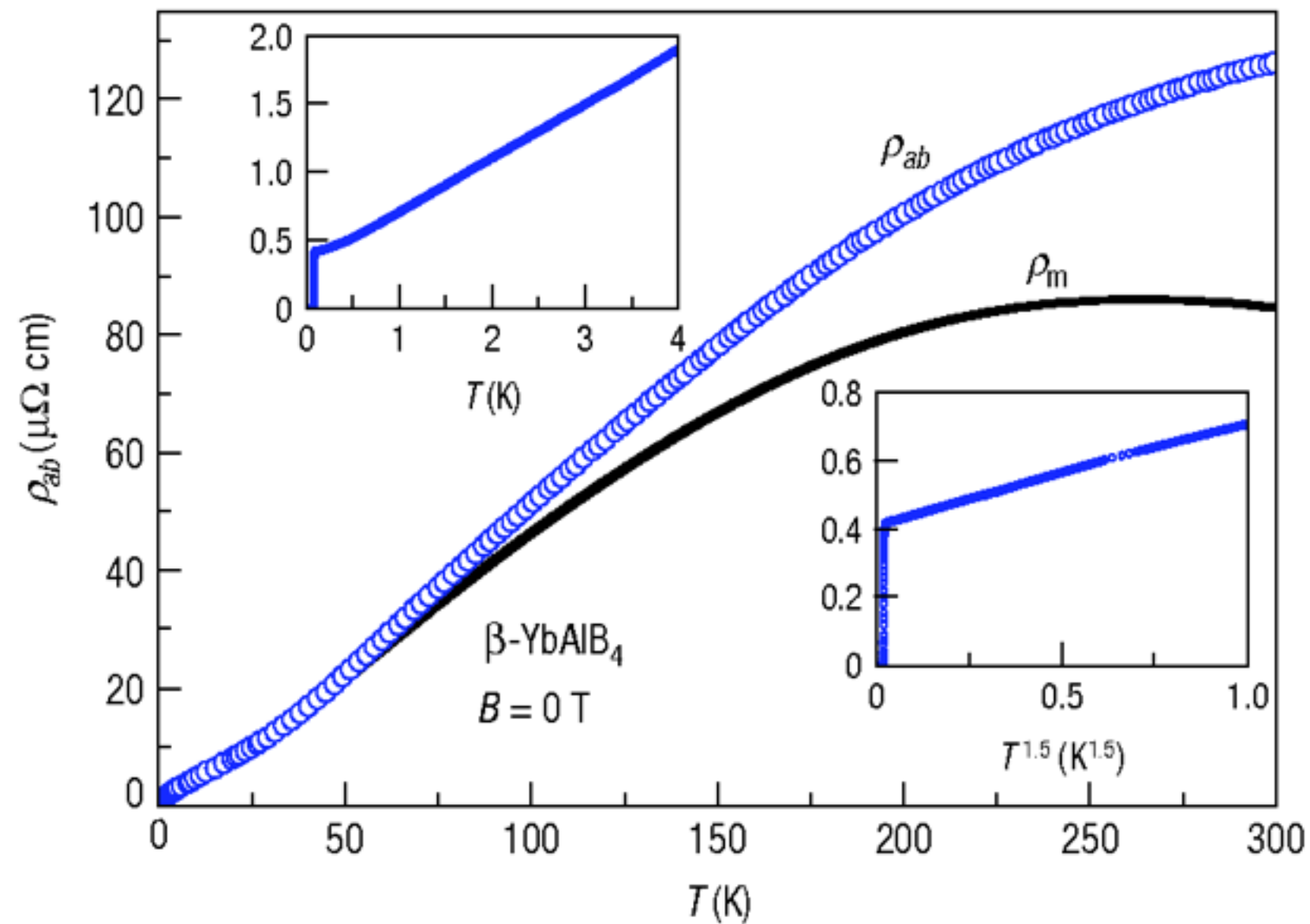
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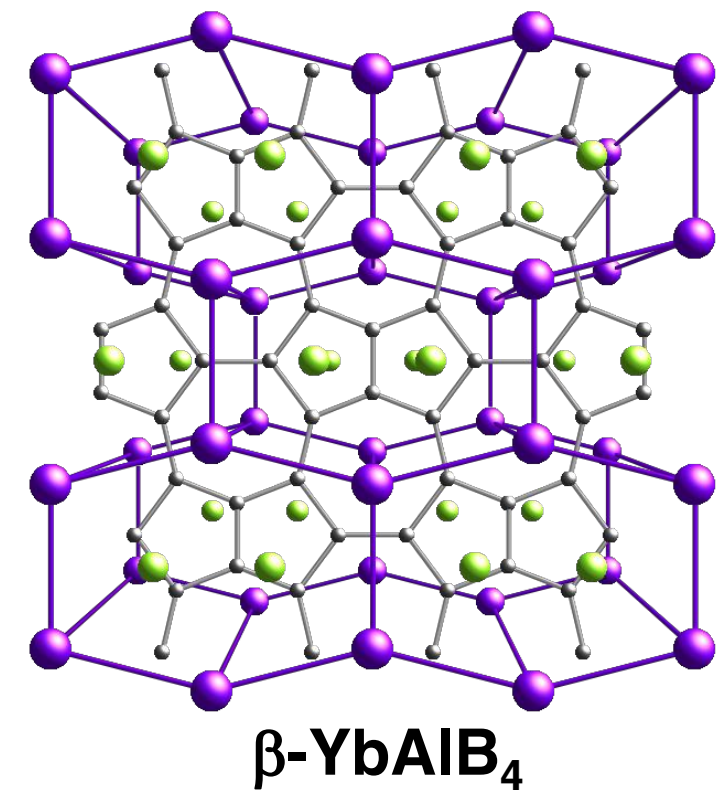
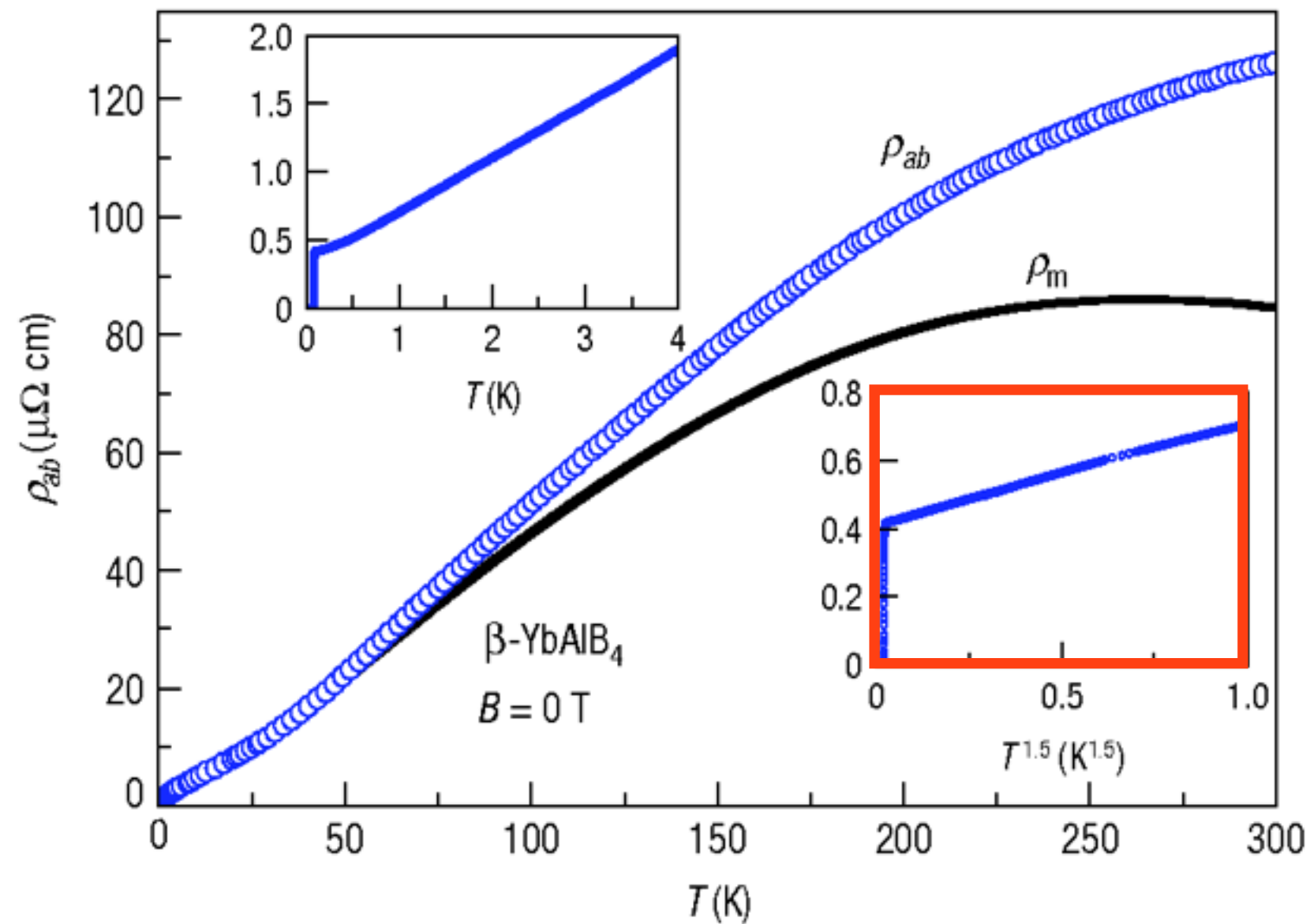
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S. Nakatsuji et al, Nature Physics, 603, (2008)



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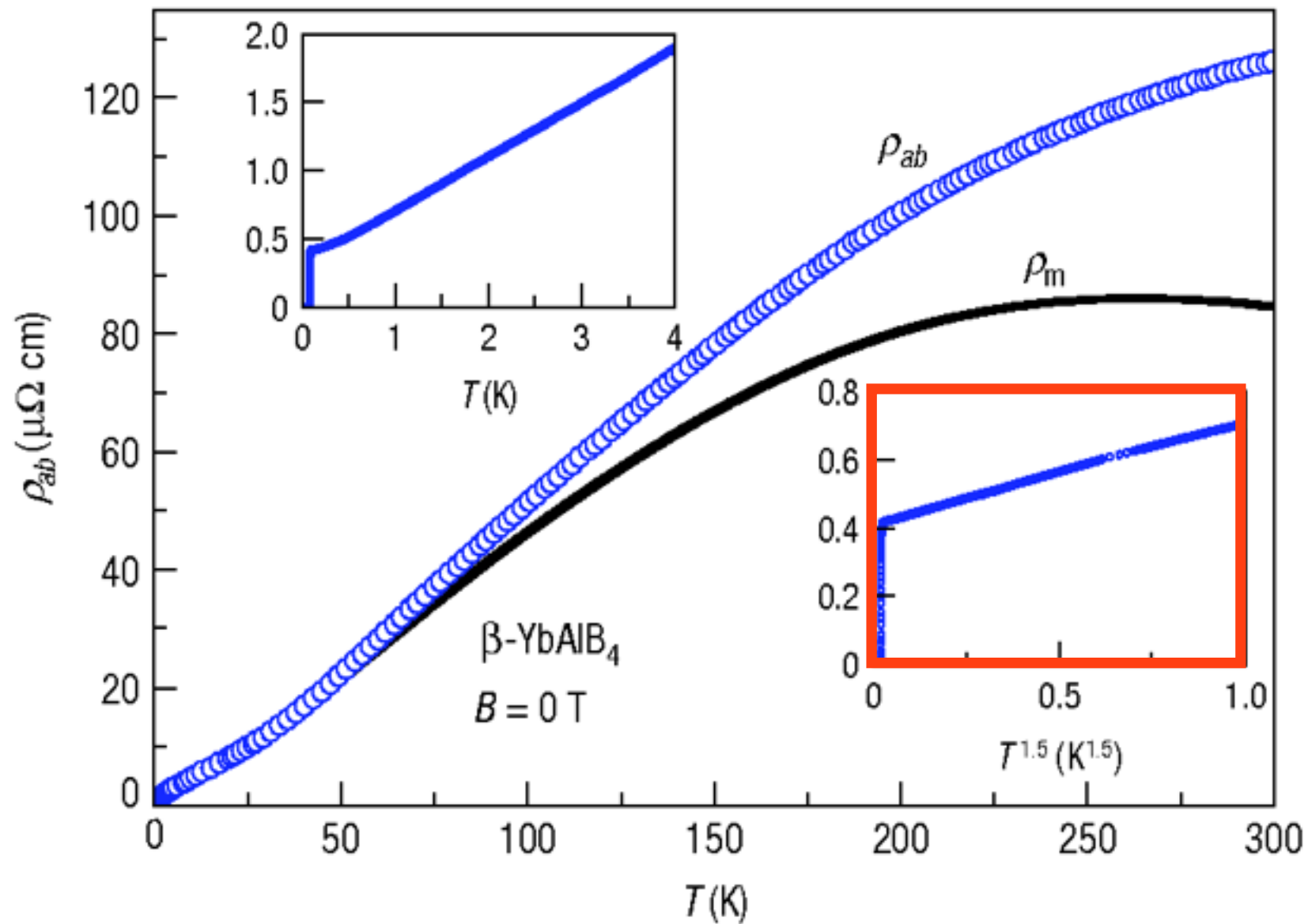
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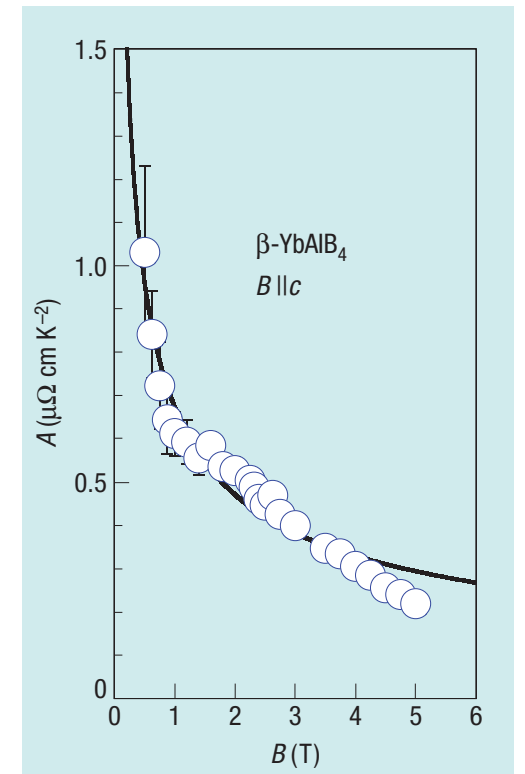
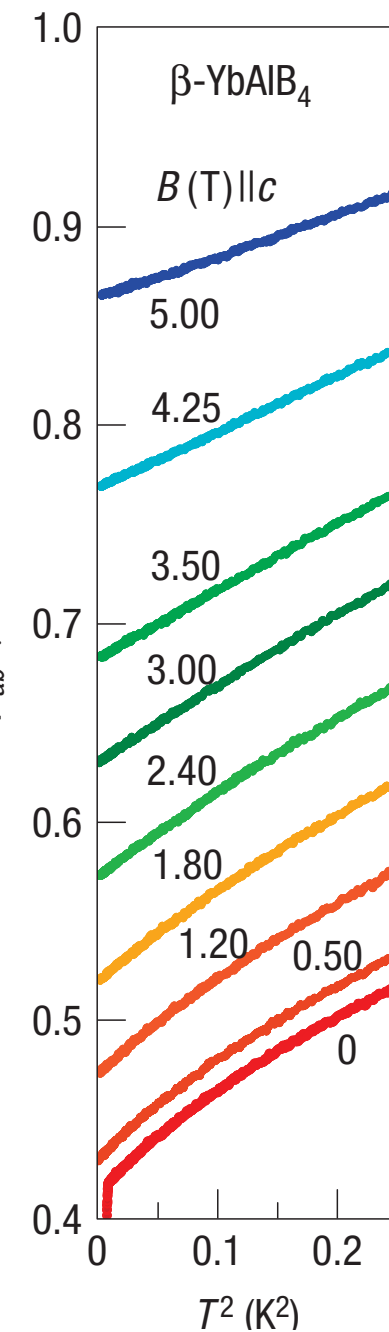
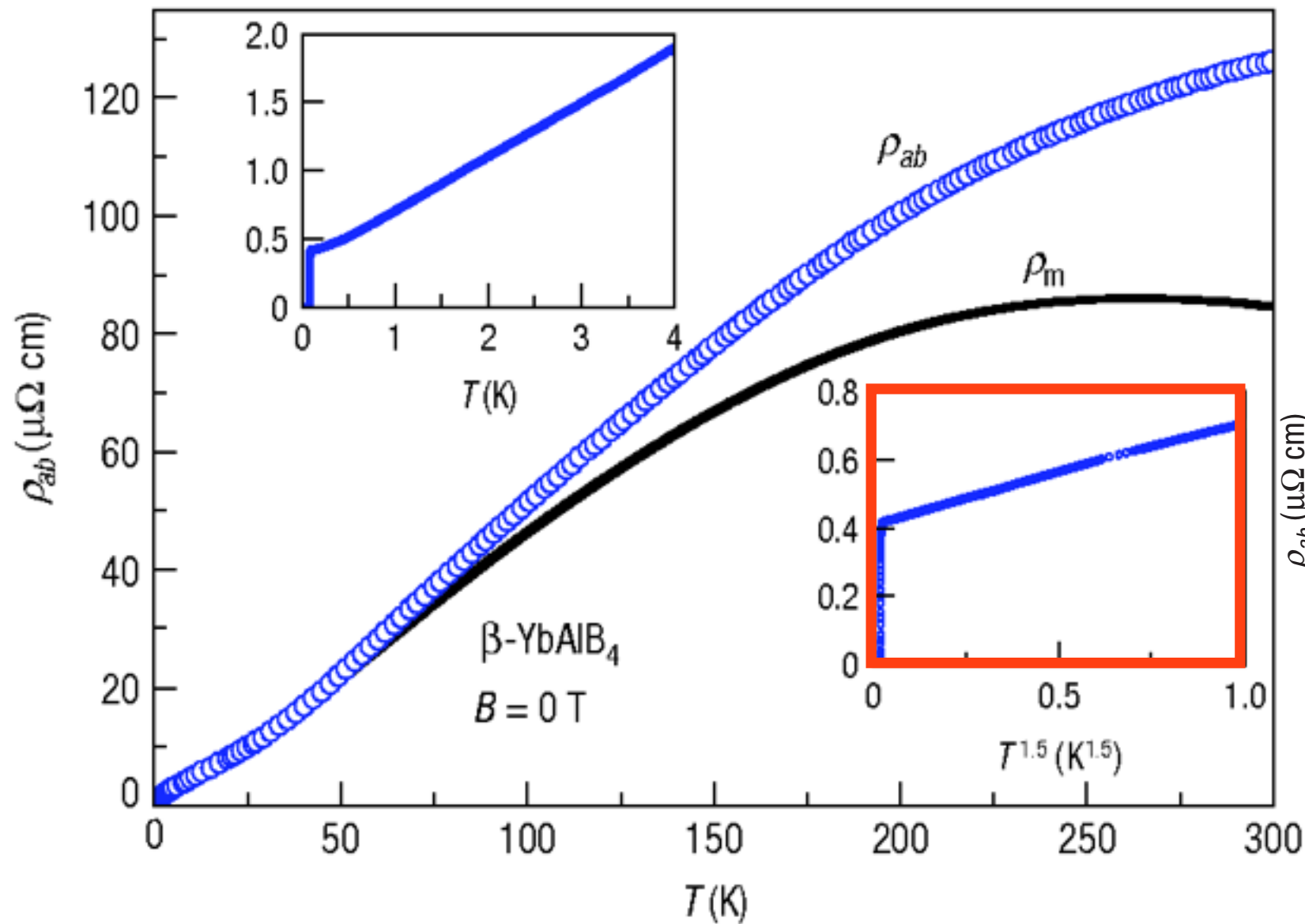
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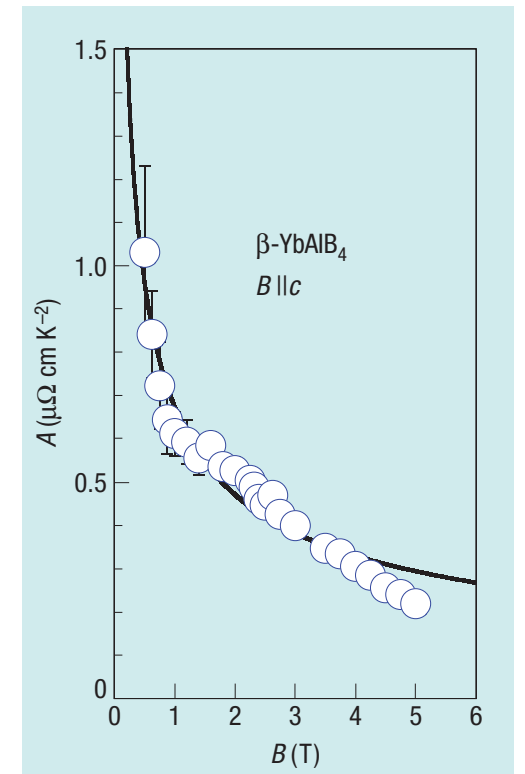
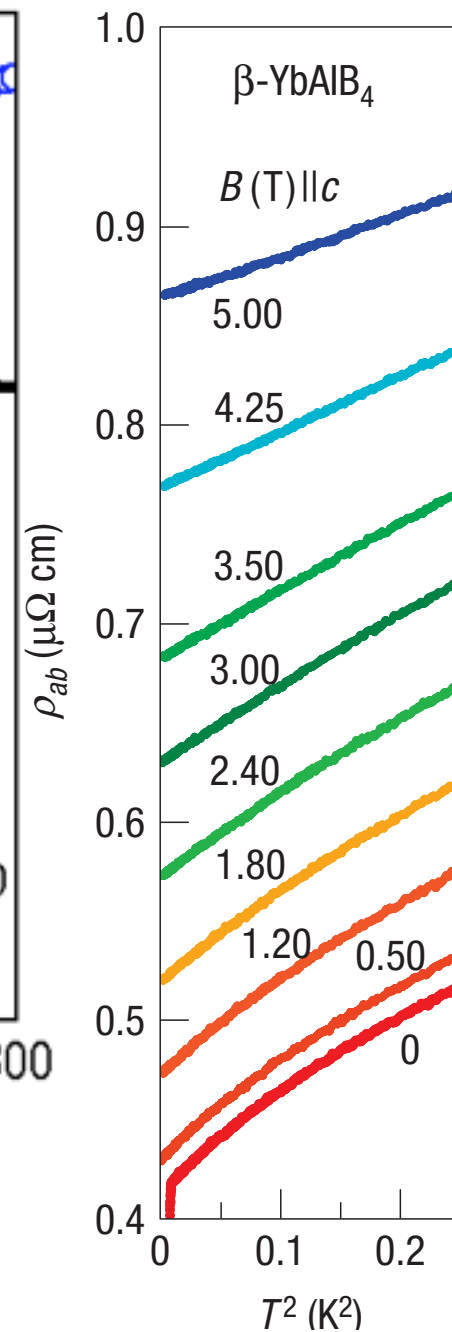
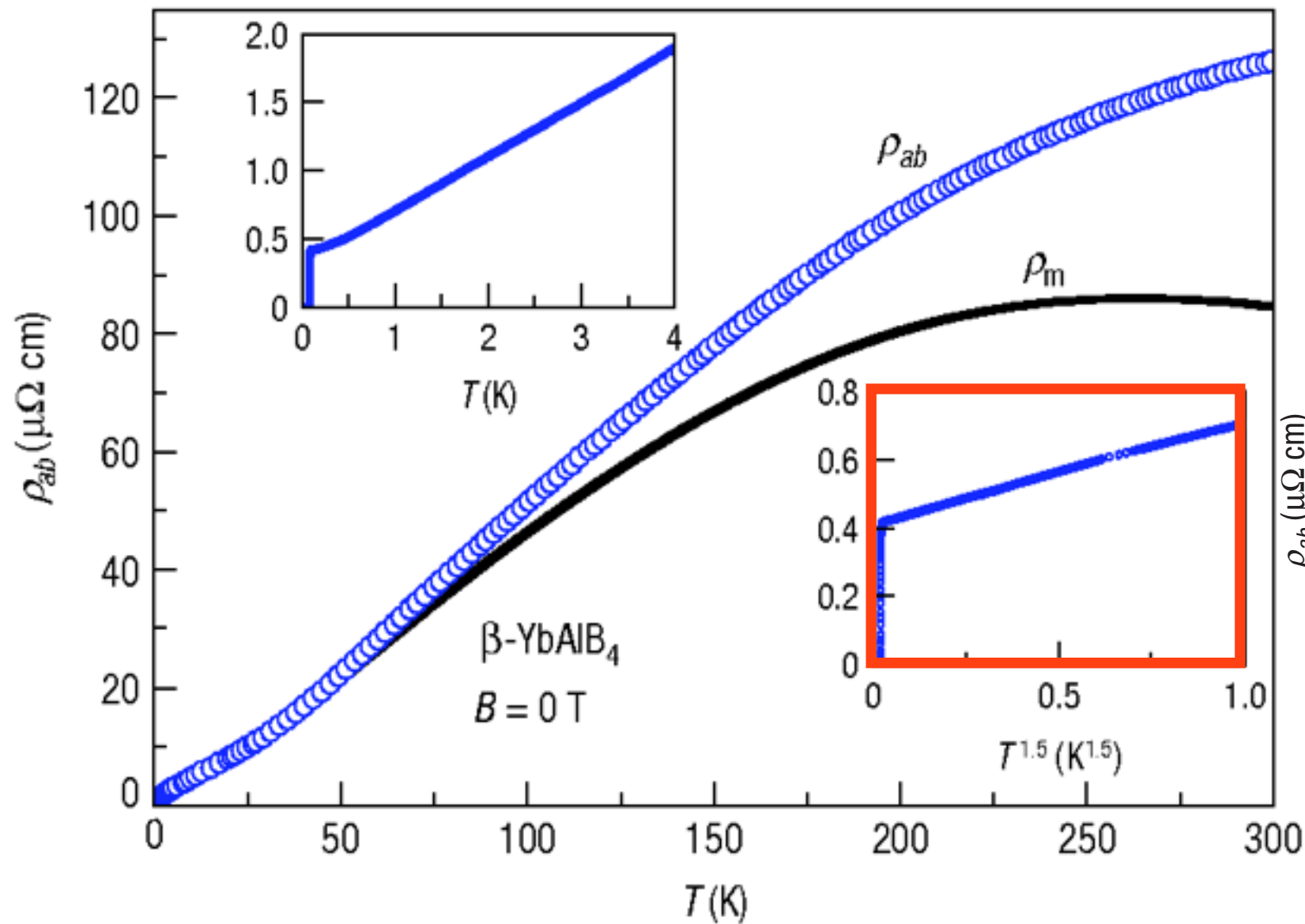
$$B \rightarrow 0 : A(B) \rightarrow \infty$$



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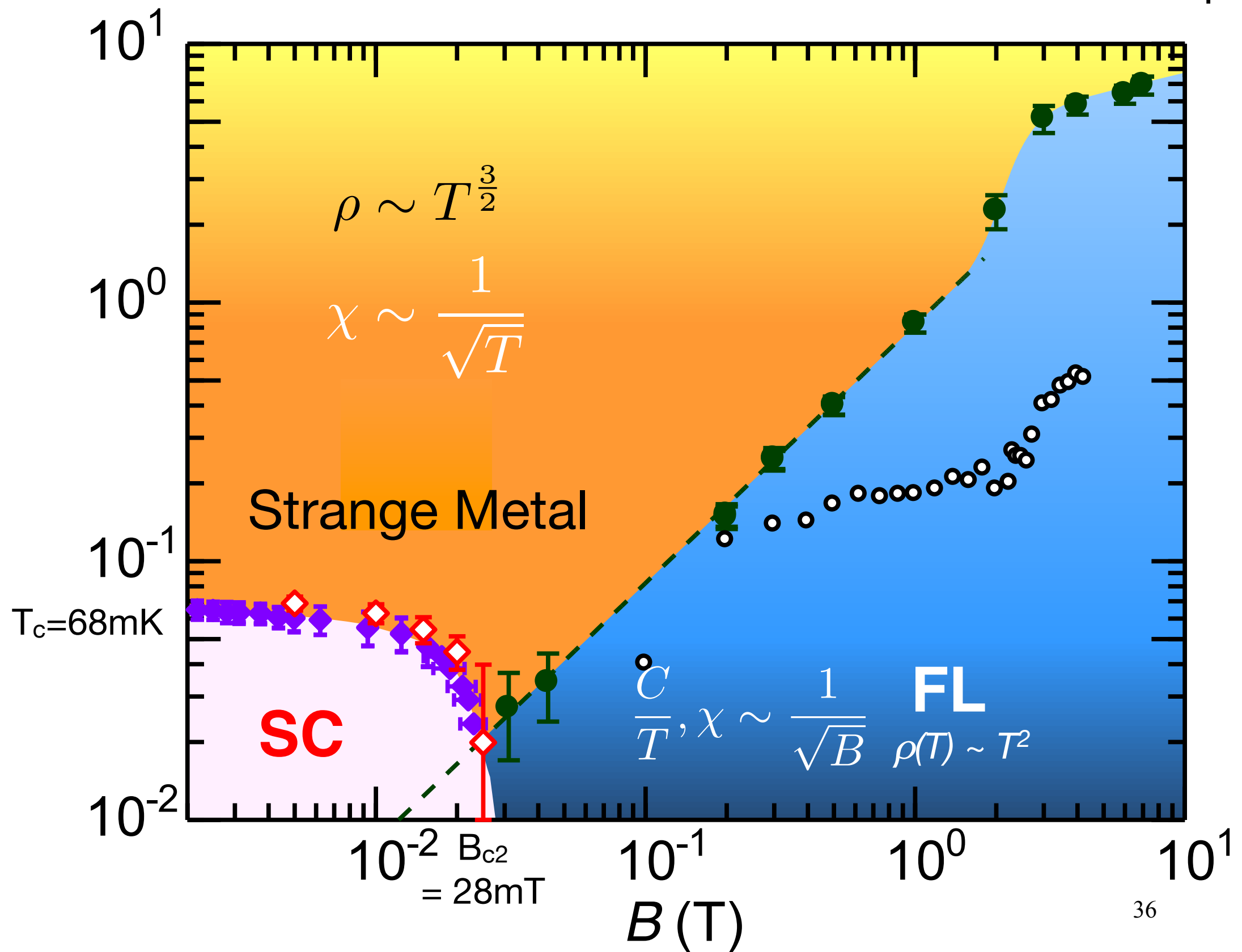
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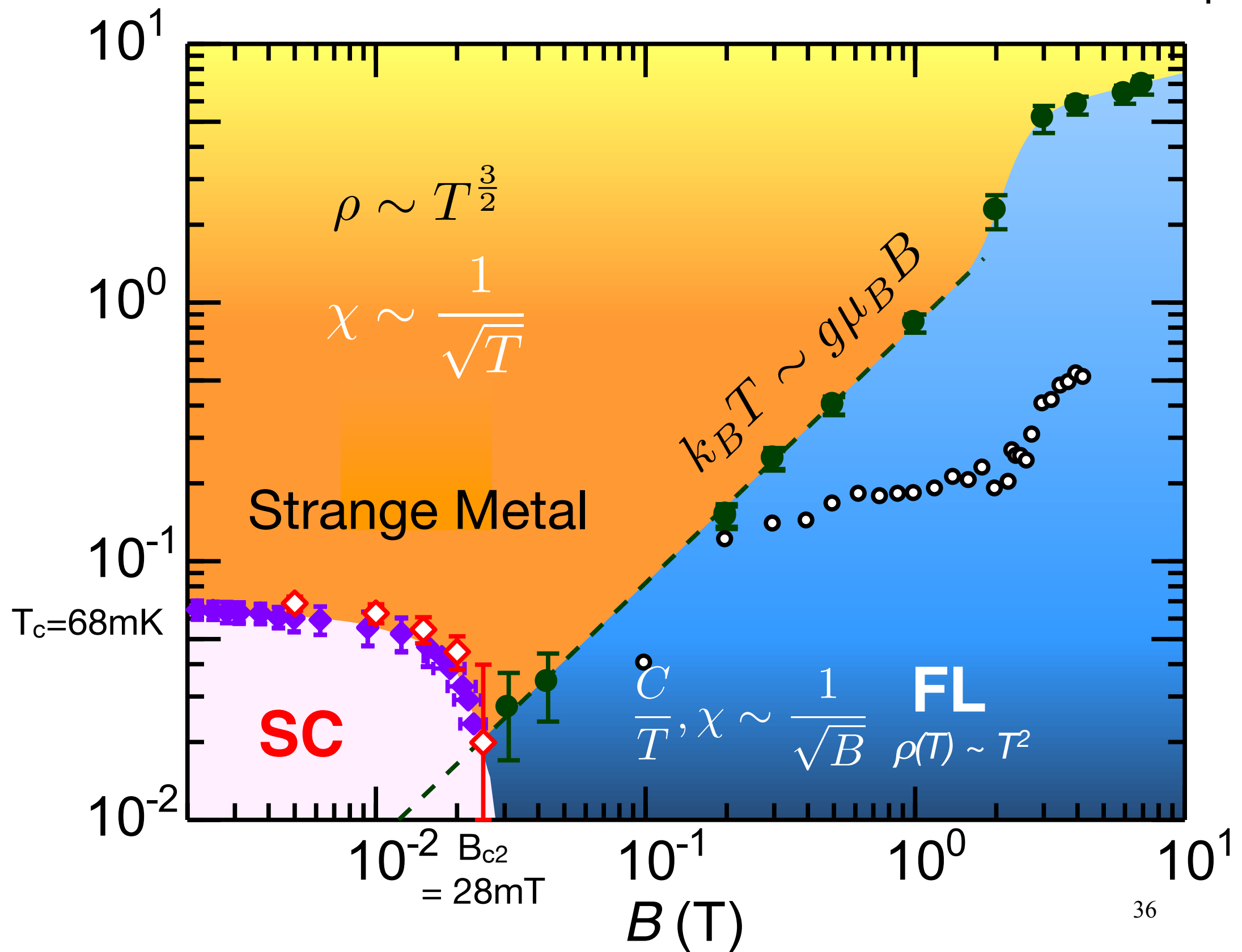
Releasing the field reveals the strange metal.



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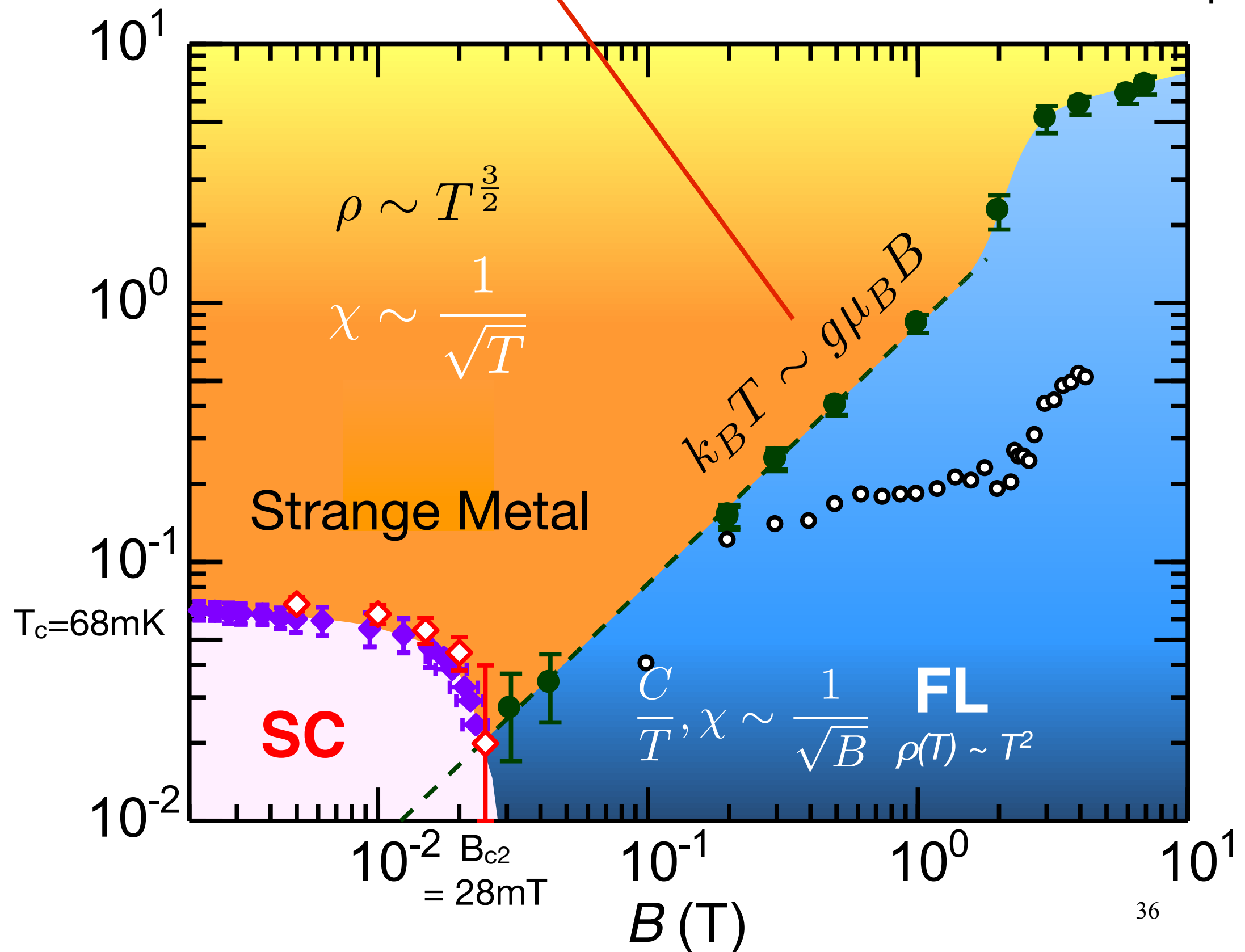


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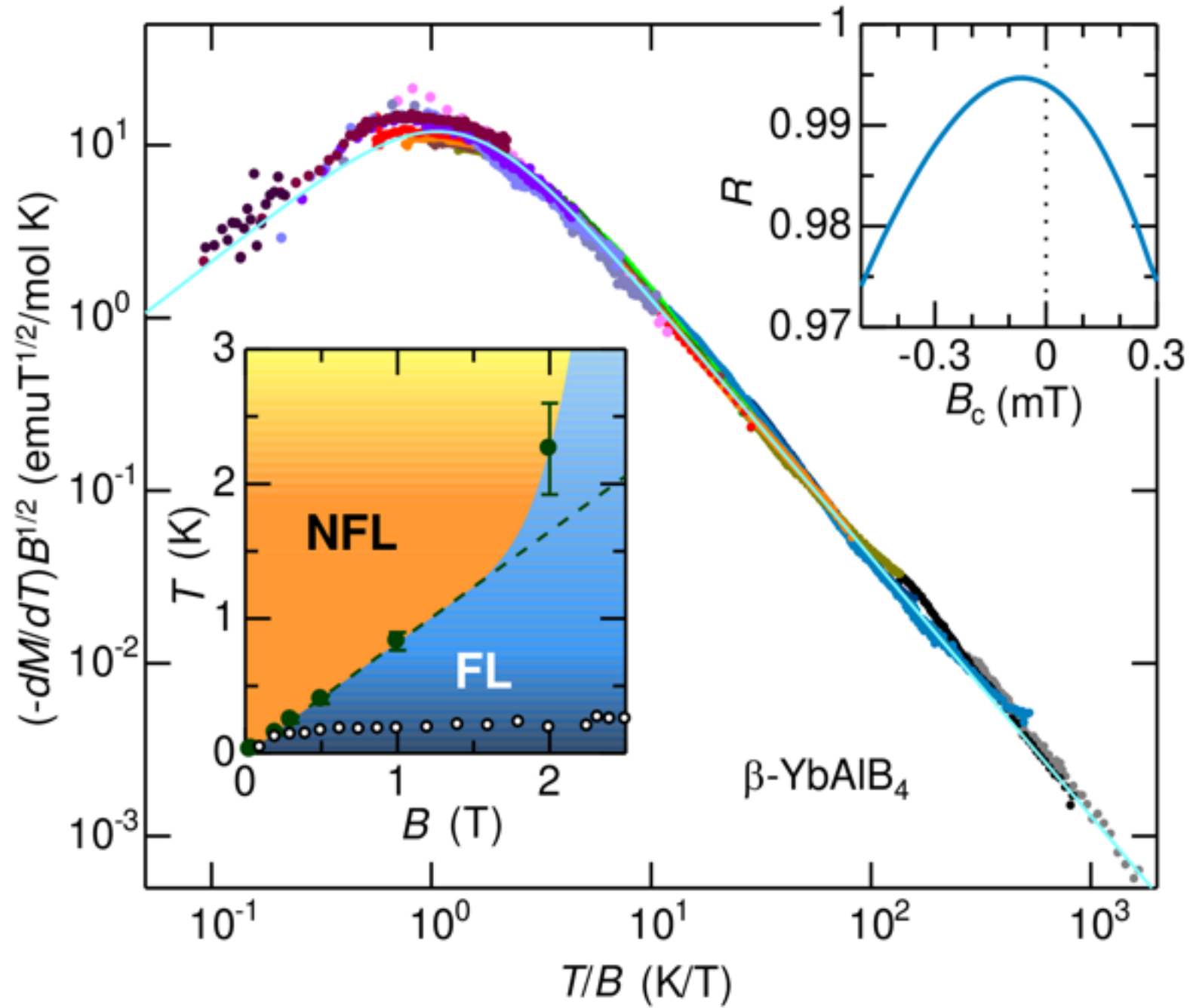


**Zeeman energy is the Fermi energy!**

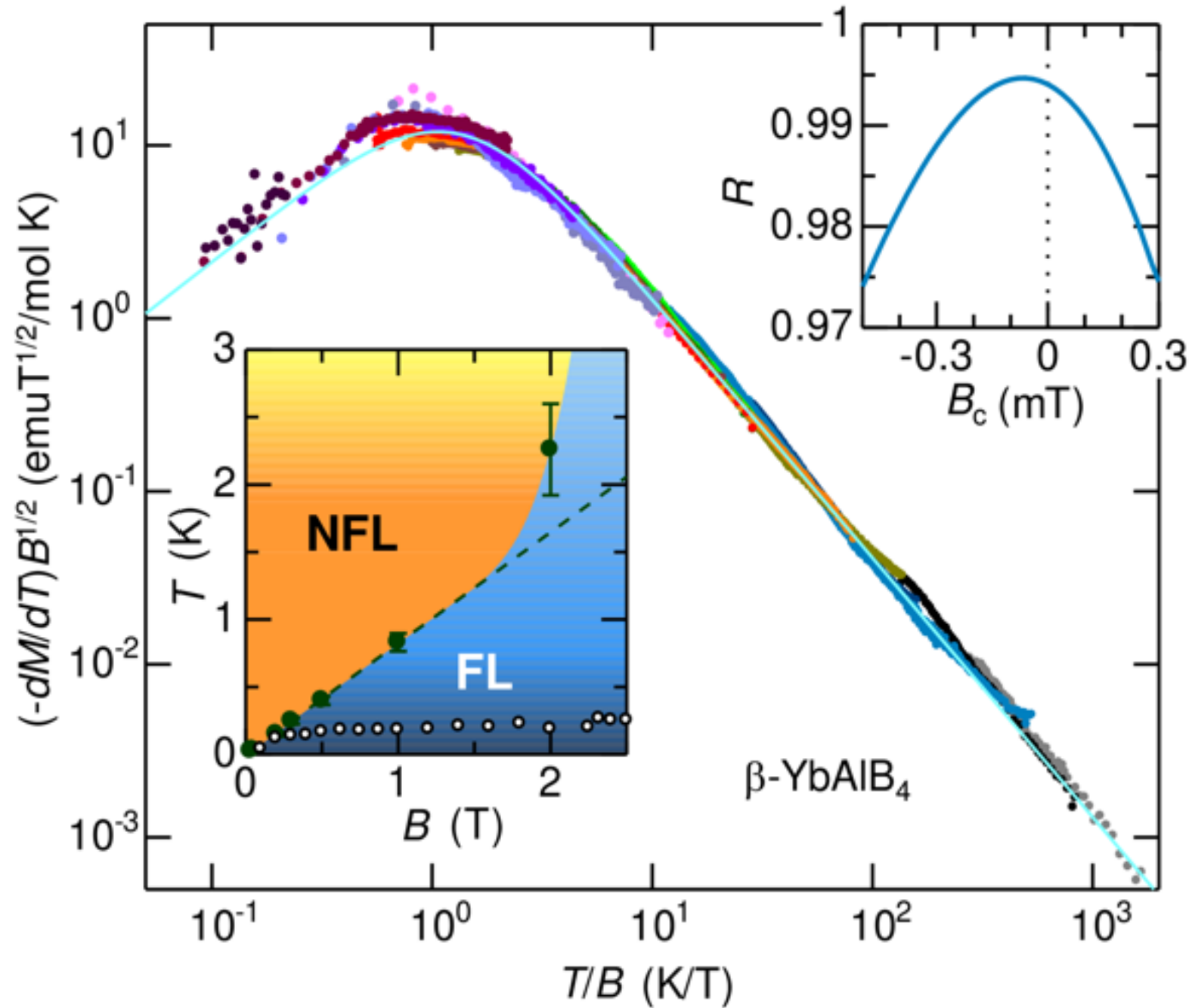
YbAlB<sub>4</sub>



# Scaling in $\beta$ -YbAlB<sub>4</sub>

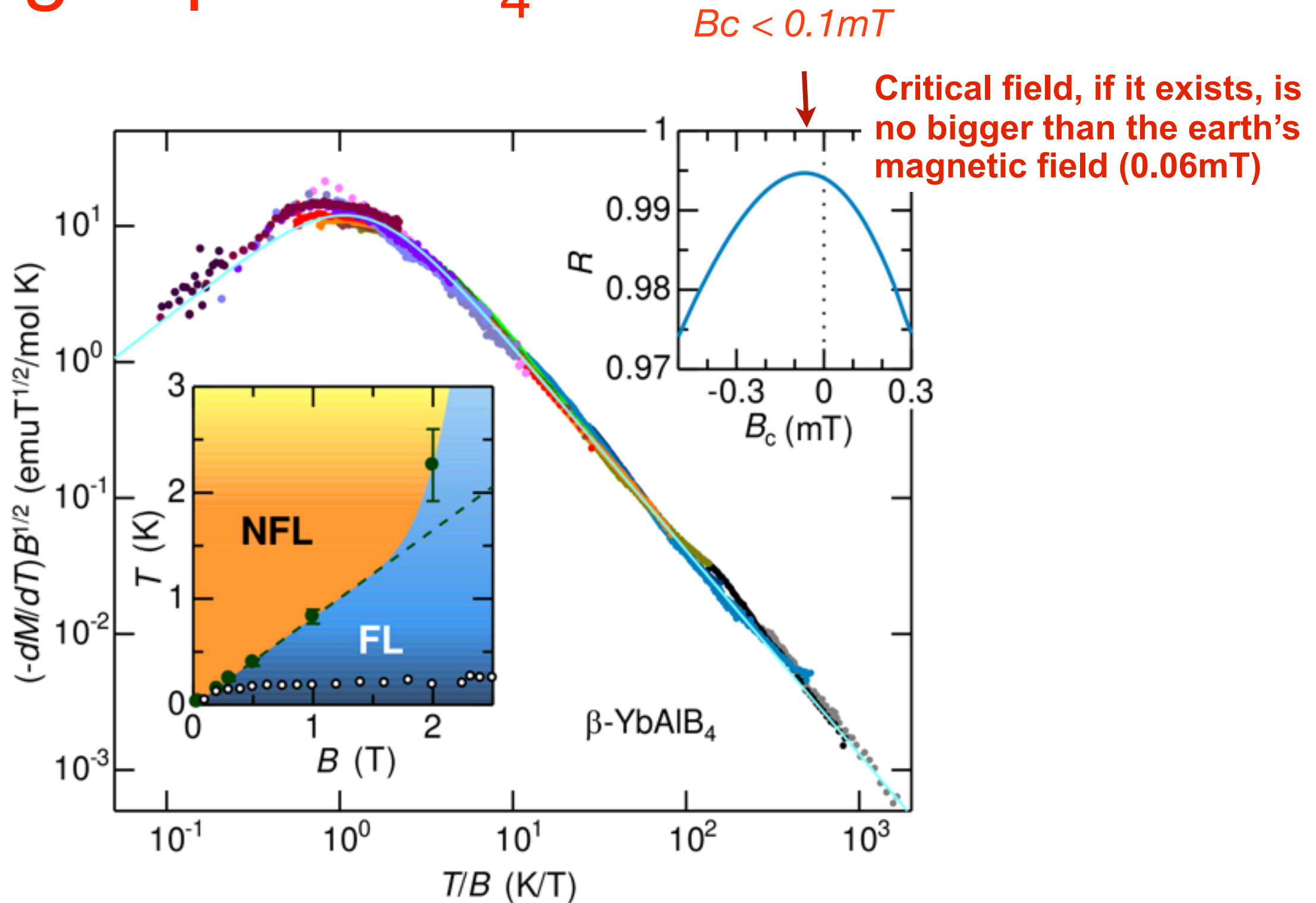


# Scaling in $\beta$ -YbAlB<sub>4</sub>



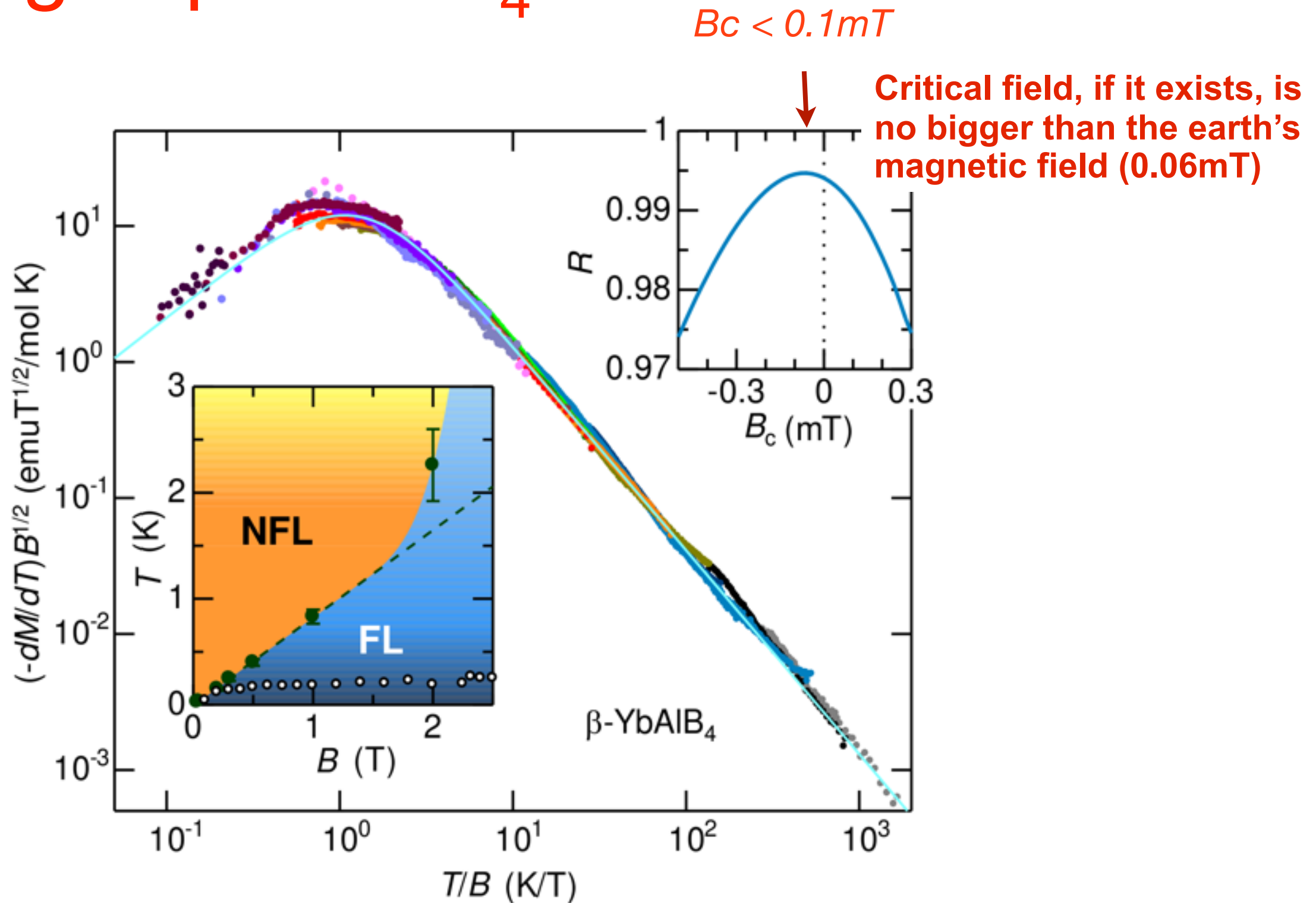
$$-\frac{dM}{dT} = B^{-1/2} f\left(\frac{T}{B}\right), \quad T \gg B$$

# Scaling in $\beta\text{-YbAlB}_4$



$$-\frac{dM}{dT} = B^{-1/2} f\left(\frac{T}{B}\right), \quad T \gg B$$

# Scaling in $\beta\text{-YbAlB}_4$

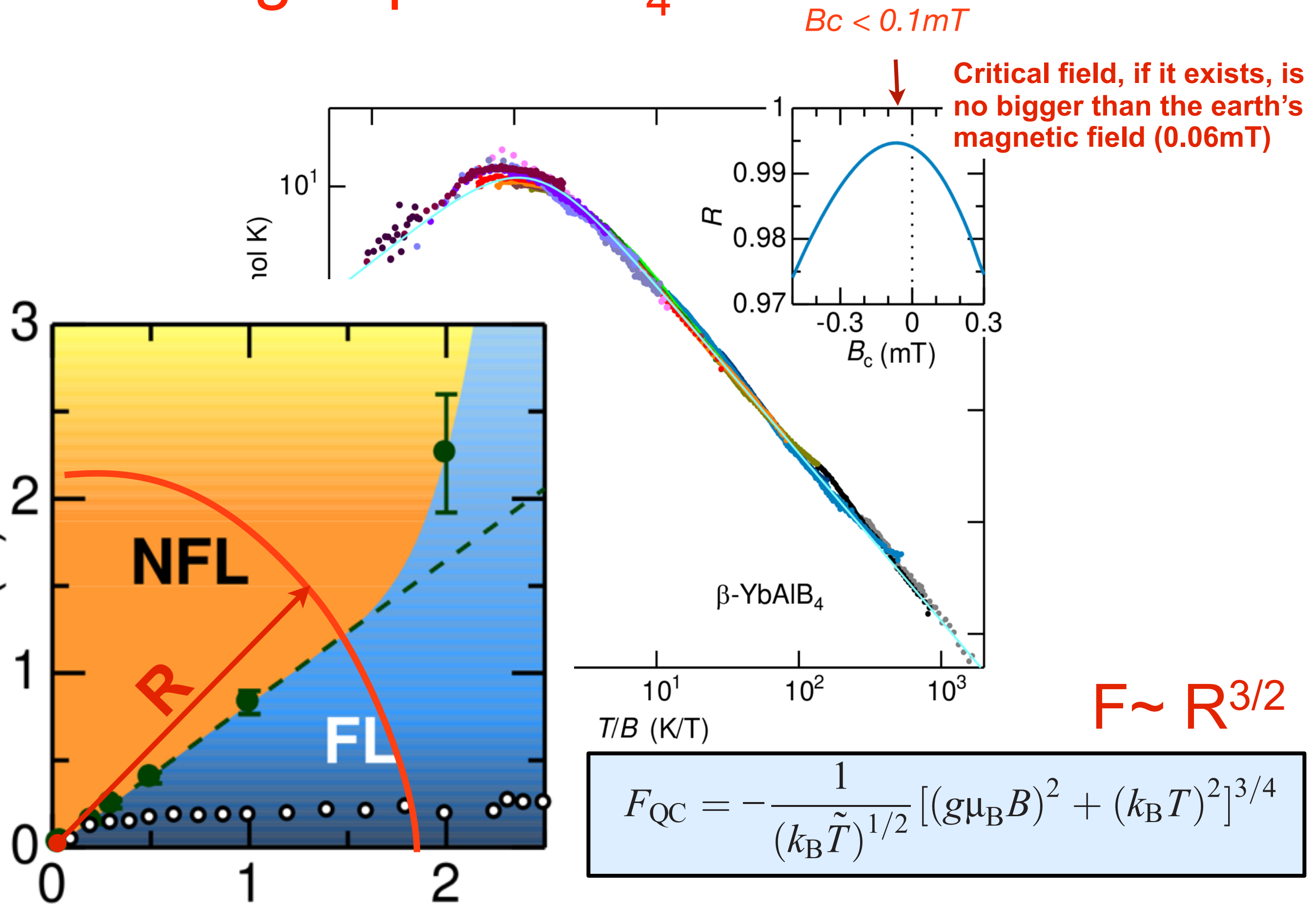


$$-\frac{dM}{dT} = B^{-1/2} f\left(\frac{T}{B}\right), \quad T \gg B$$

$$F_{\text{QC}} = -\frac{1}{(k_B \tilde{T})^{1/2}} [(g\mu_B B)^2 + (k_B T)^2]^{3/4}$$



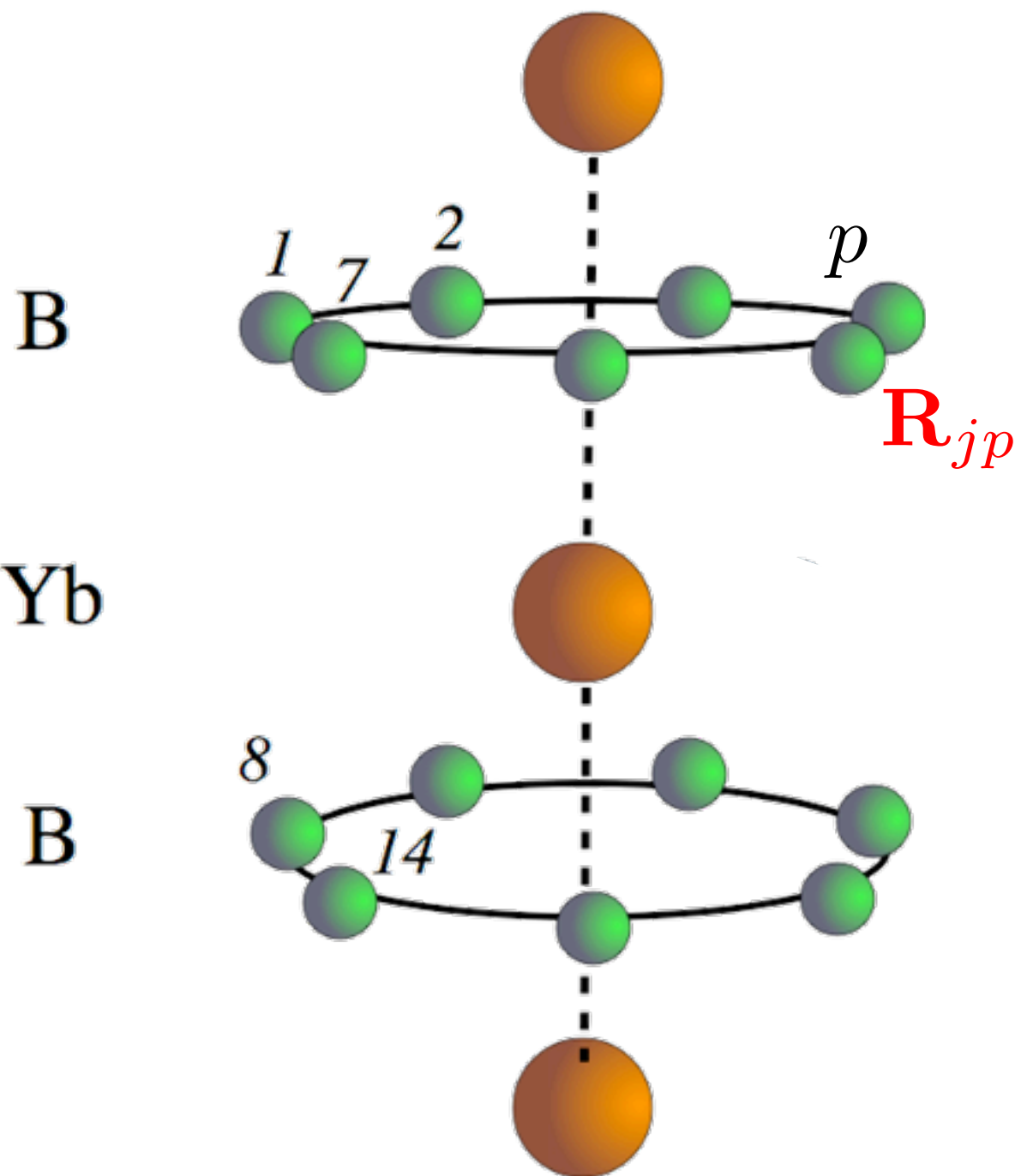
# Scaling in $\beta\text{-YbAlB}_4$

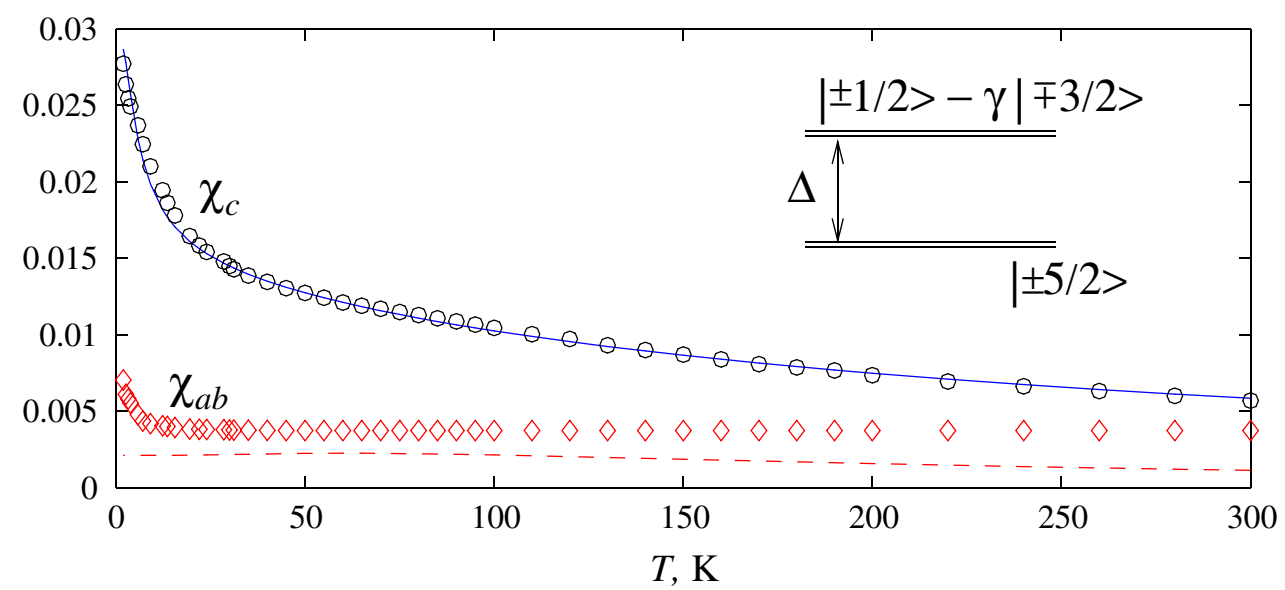
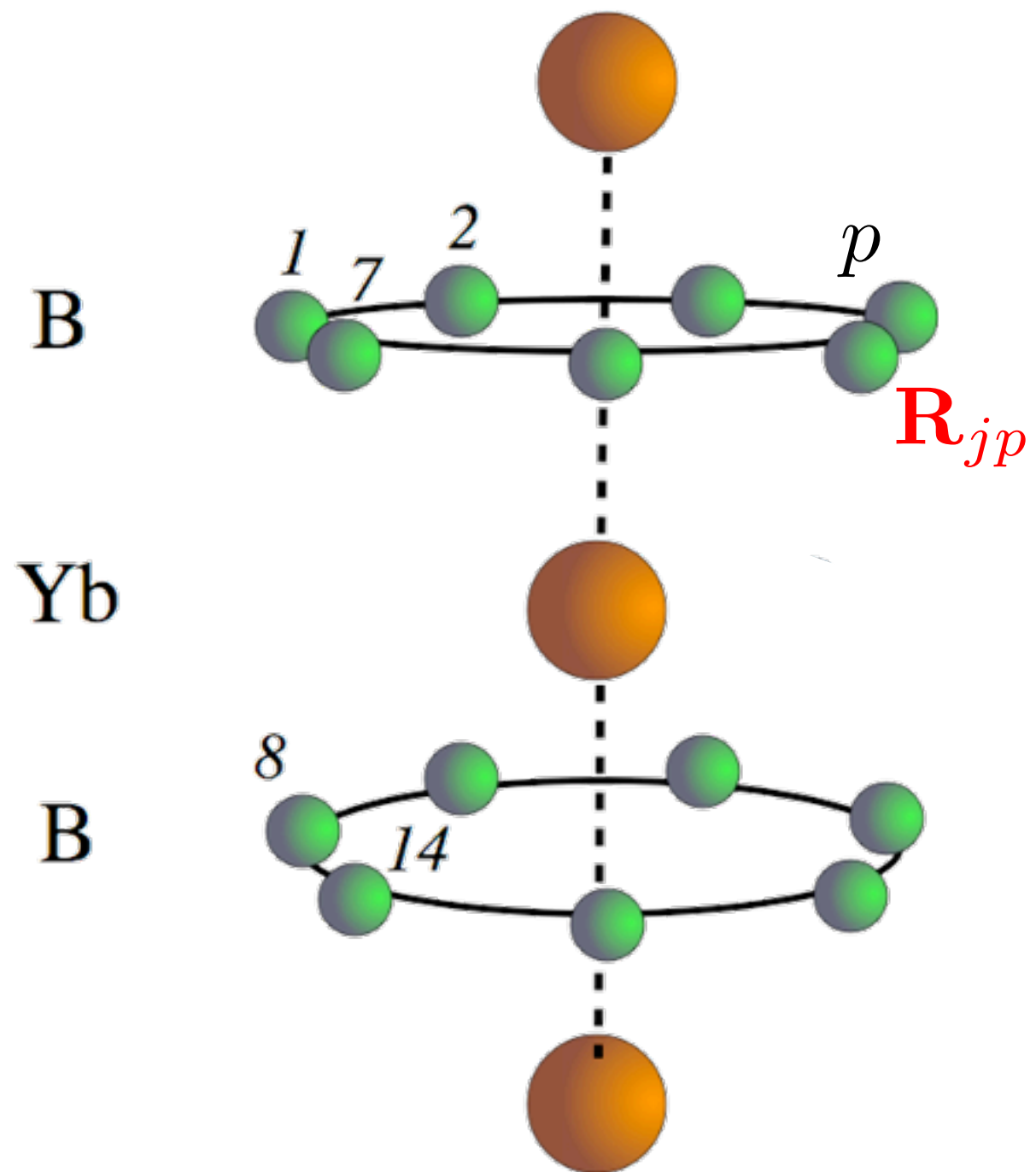


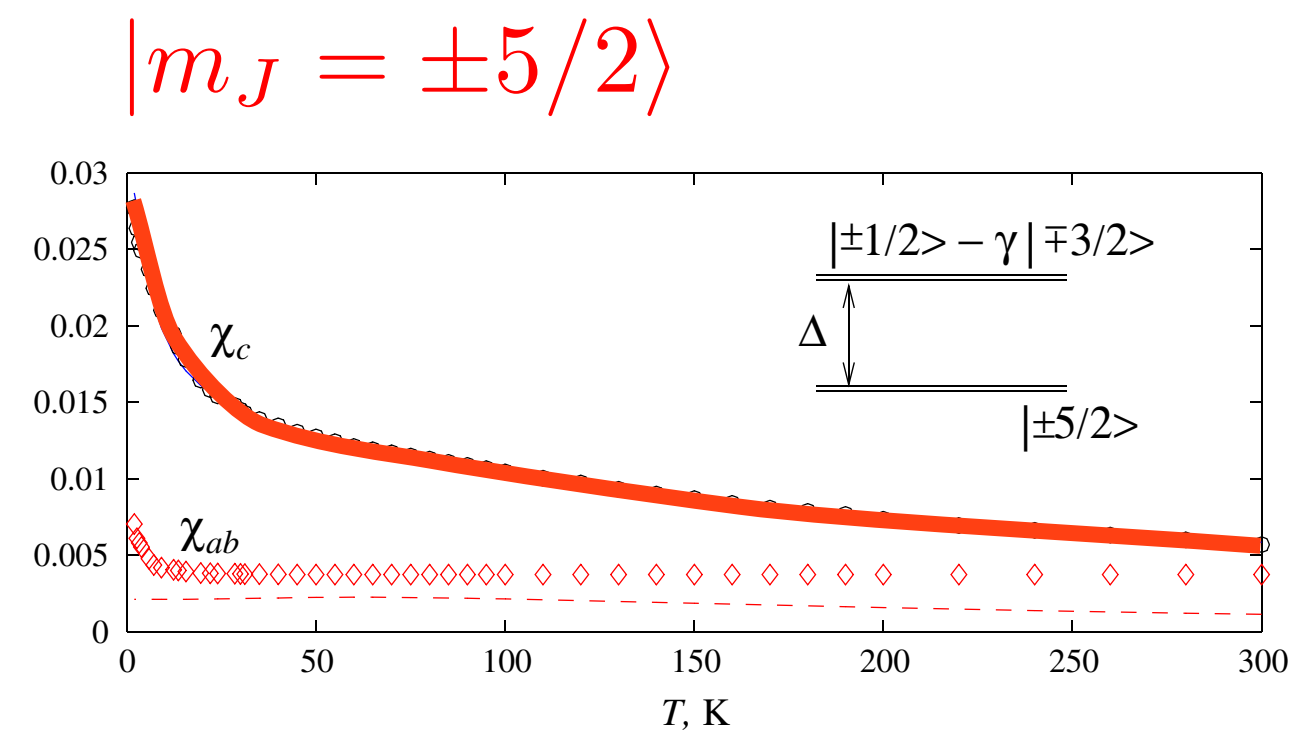
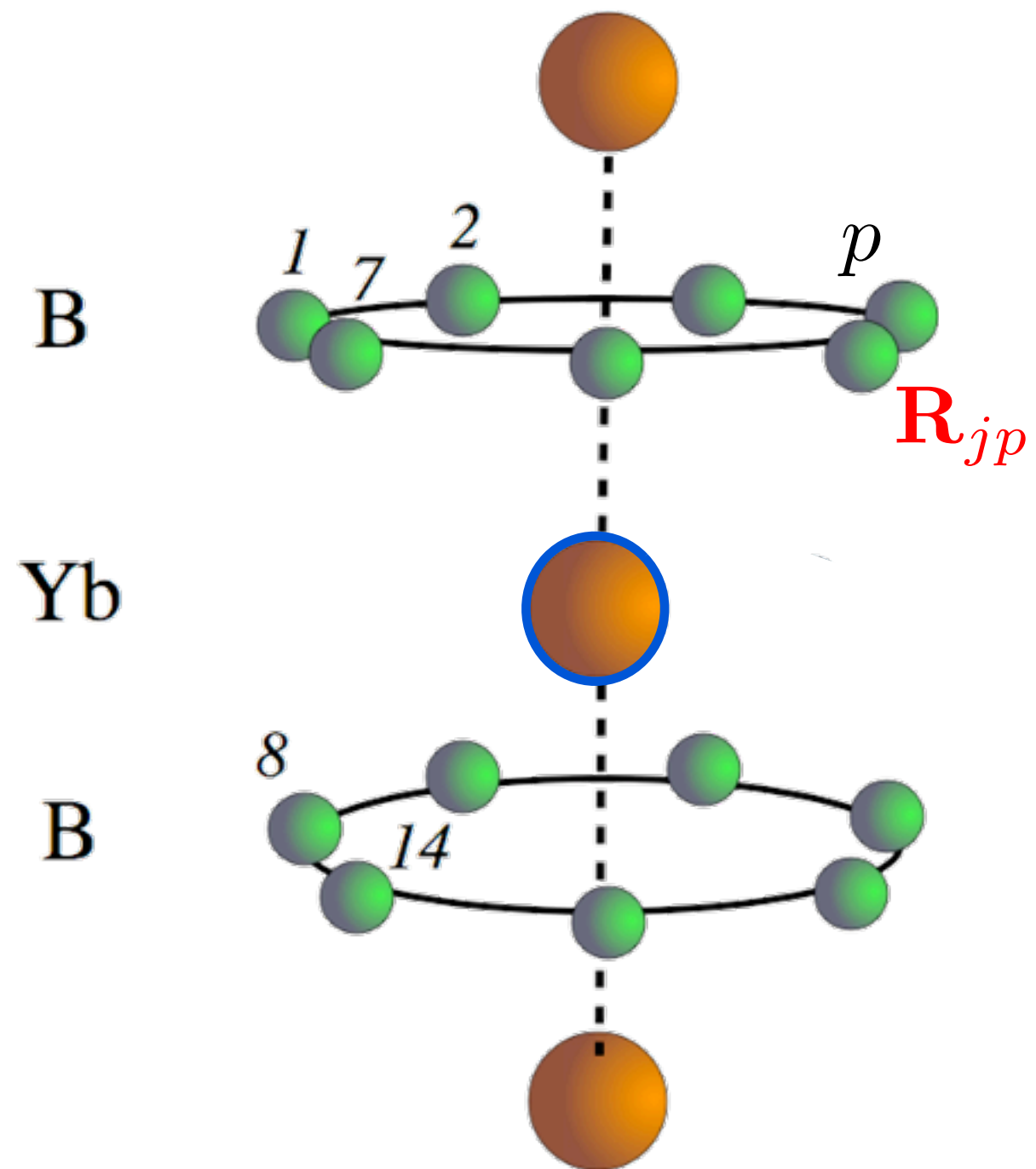
# $\beta$ -YbAlB<sub>4</sub>: Vortex Metal.

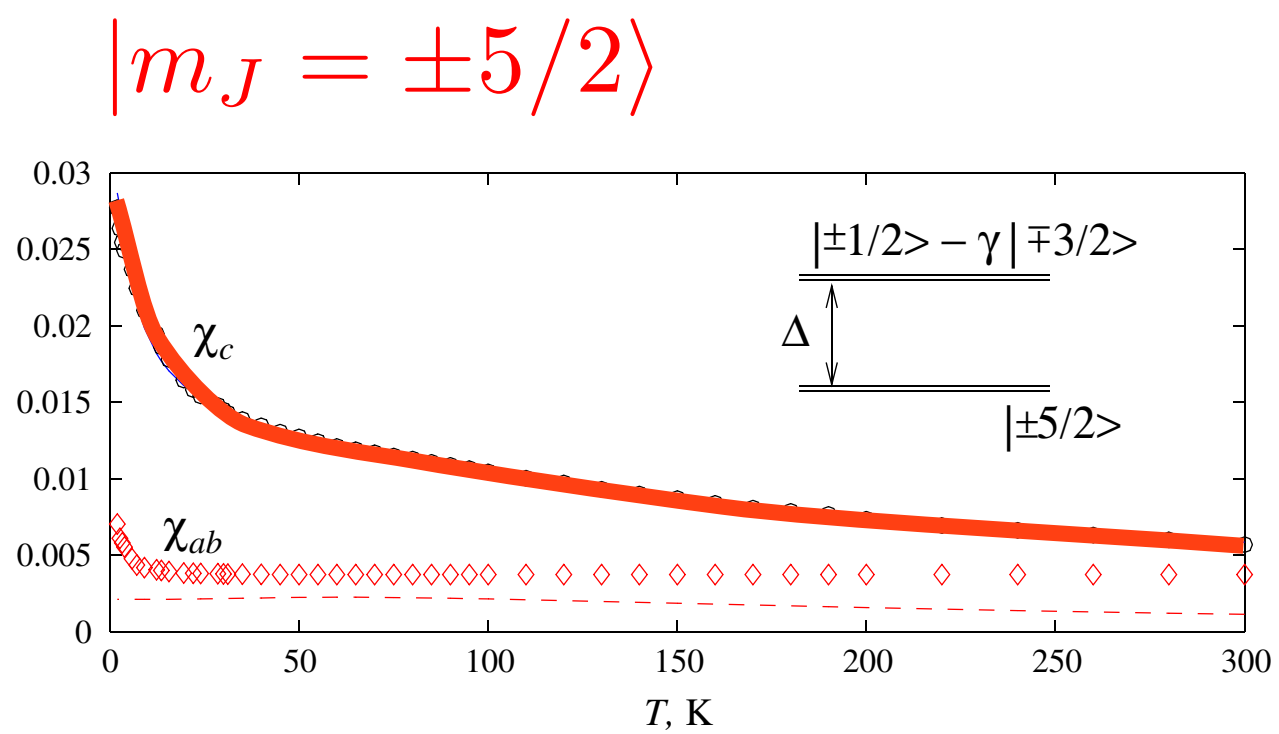
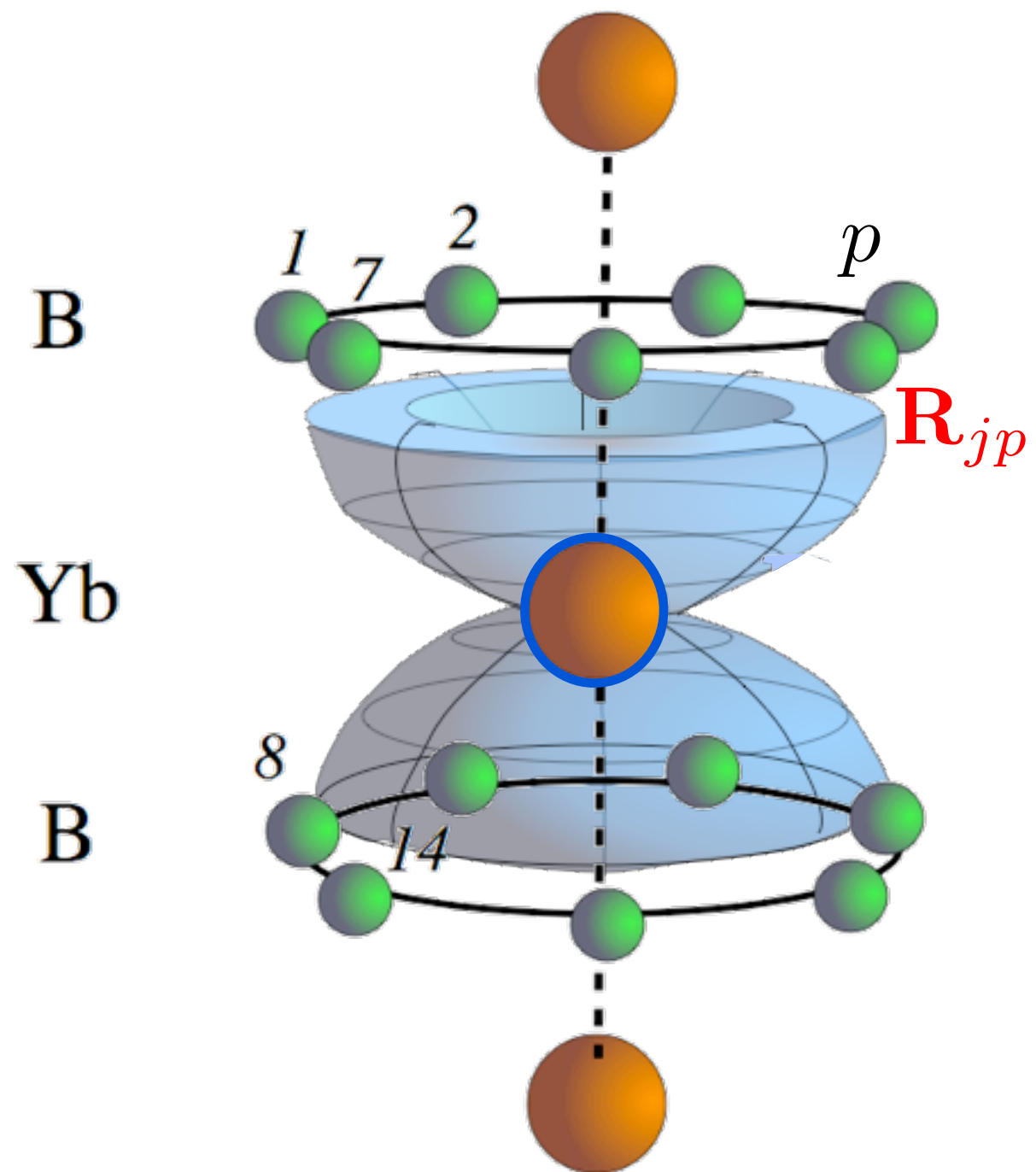
## **$\beta$ -YbAlB<sub>4</sub>: A Critical Nodal Metal**

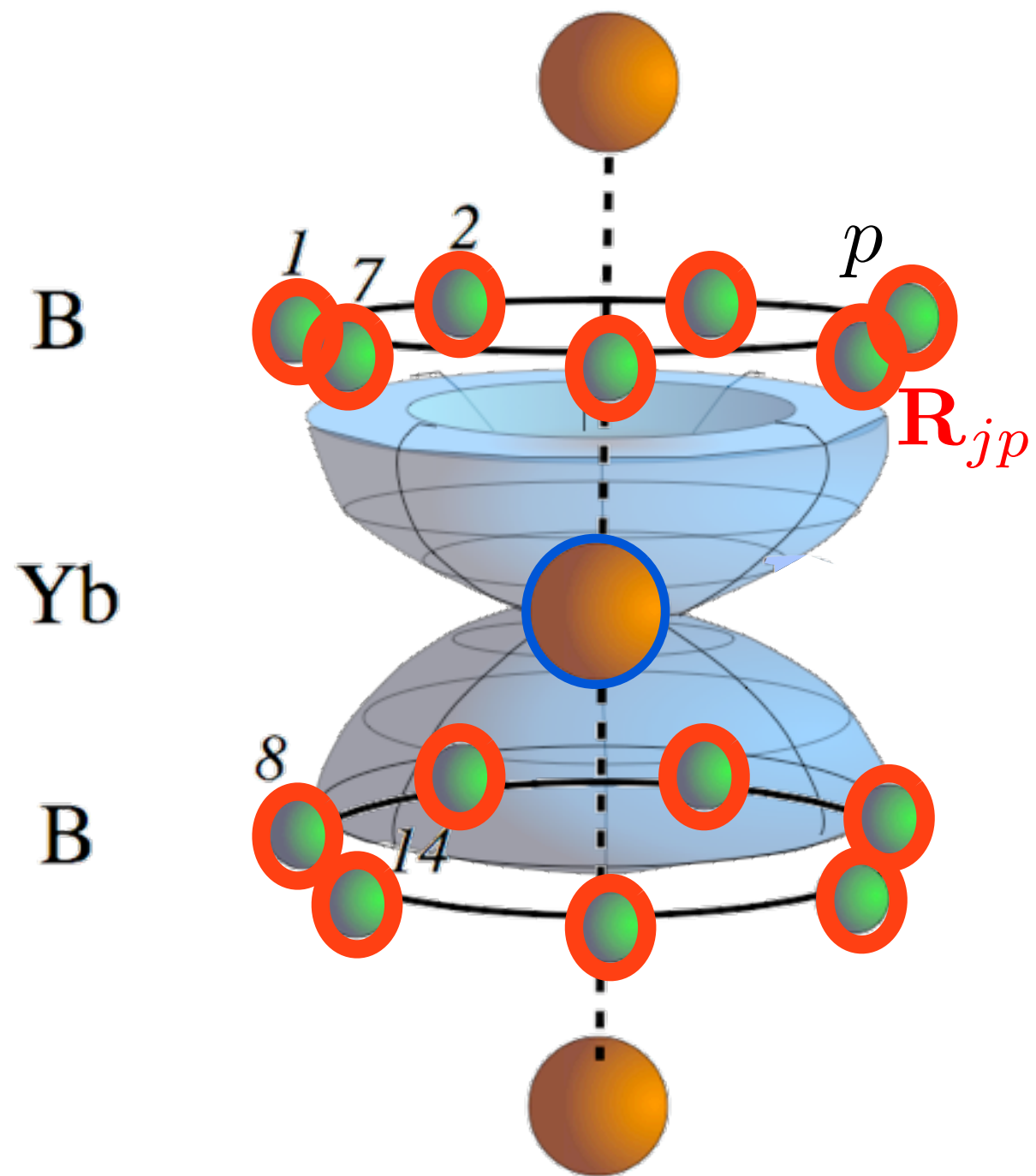
Aline Ramires, PC, Andriy H. Nevidomskyy and A. M. Tsvelik, Phys. Rev. Lett. 109, 176404 (2012).



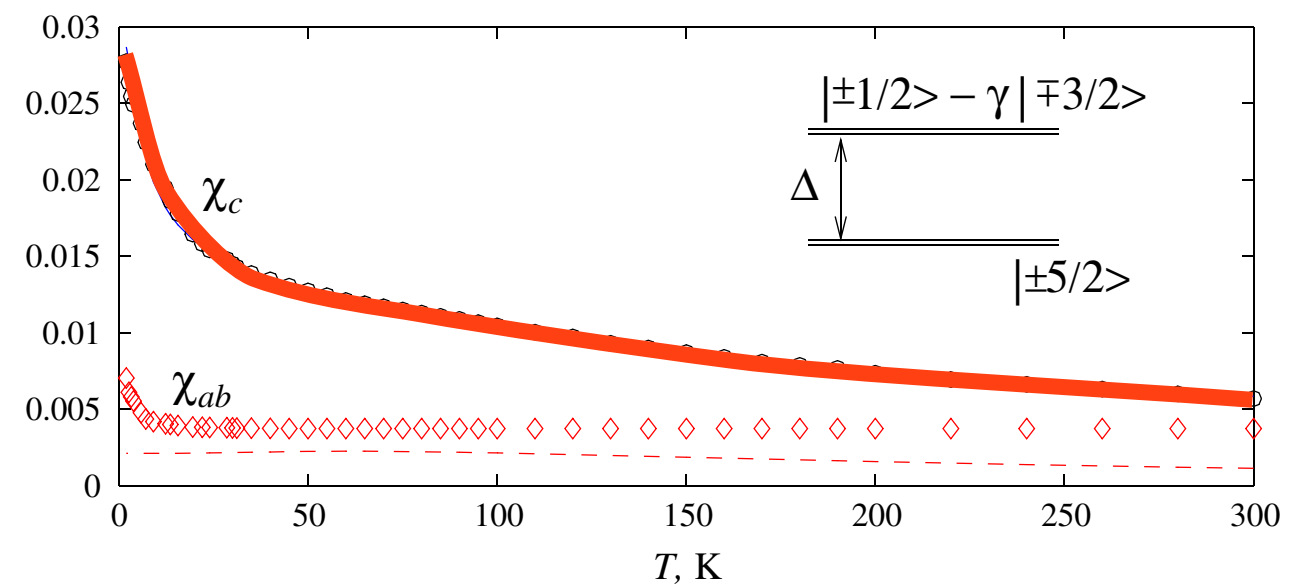








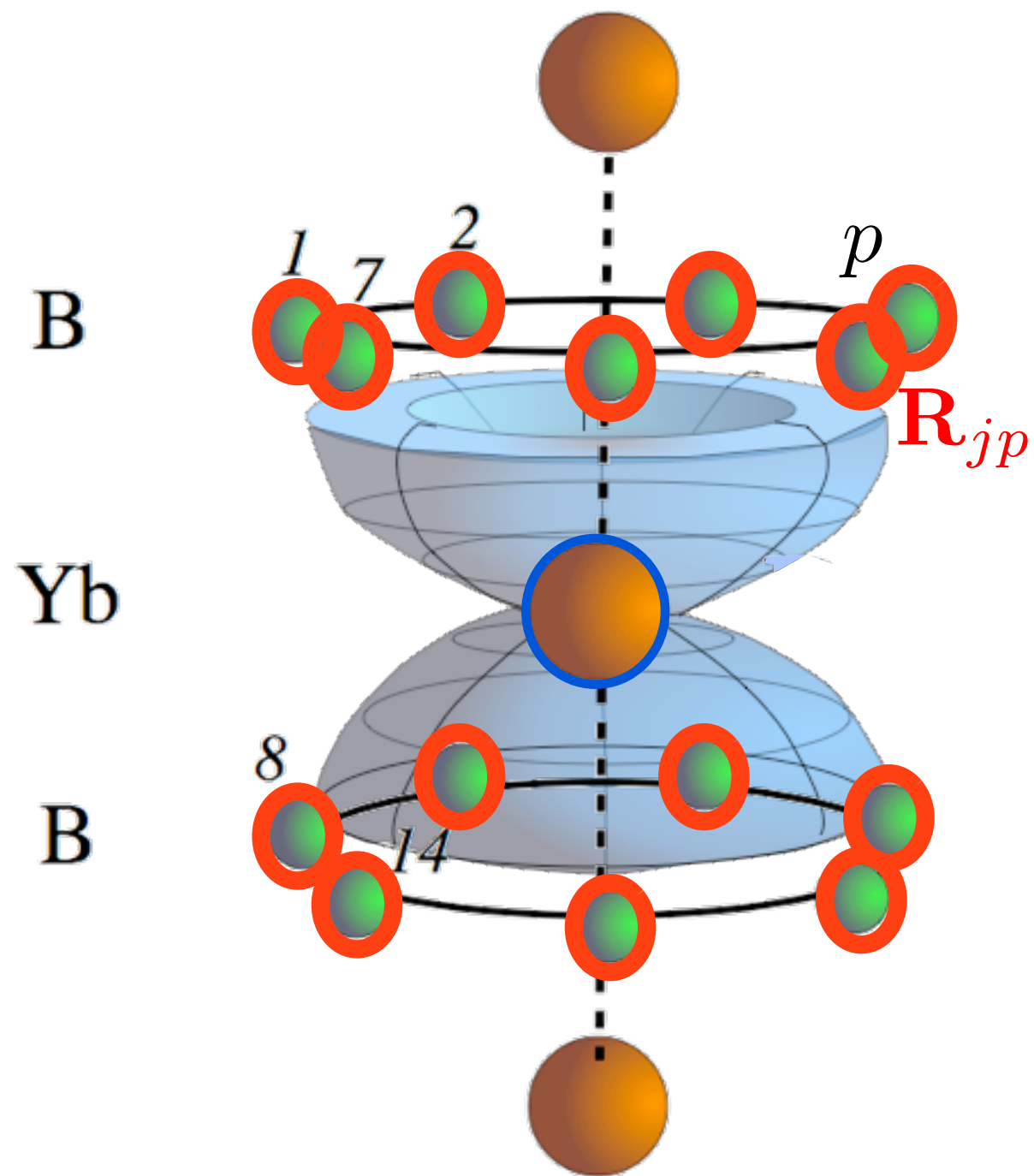
$$|m_J = \pm 5/2\rangle$$



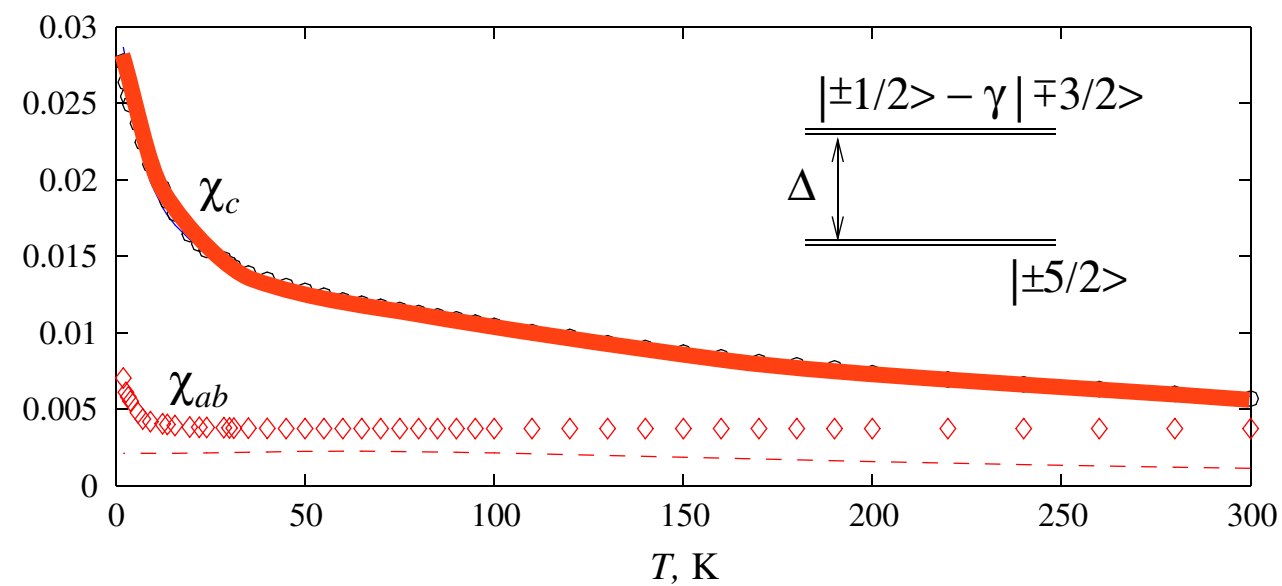
$$c_{j\alpha}^\dagger = \sum_{p \in (1,14), \sigma} c_\sigma^\dagger(\mathbf{R}_{jp}) \mathcal{Y}_{\sigma\alpha}(\mathbf{r}_p)$$

Conduction Wannier state with  $j=7/2$ ,  $m_J=\pm 5/2$





$$|m_J = \pm 5/2\rangle$$

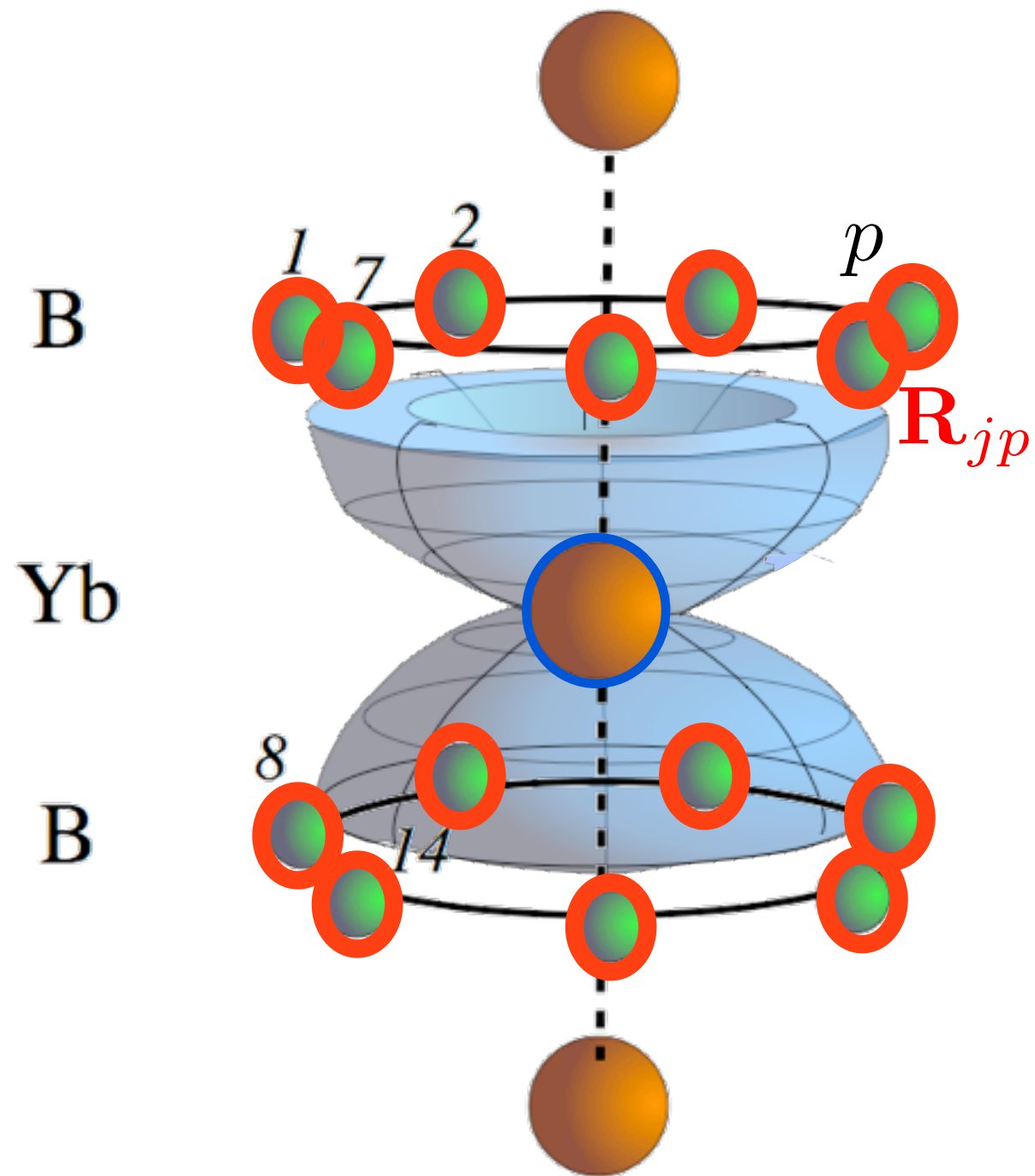


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Conduction Wannier state with  $j=7/2$ ,  $m_J=\pm 5/2$

$$\mathcal{Y}_{\sigma\alpha}(\mathbf{r}) = C_{\sigma\alpha}^{\frac{7}{2}} Y_{\alpha-\sigma}^3(\mathbf{r}) = \frac{1}{\sqrt{7}} \begin{pmatrix} \sqrt{6} Y_2^3 & Y_3^3 \\ Y_{-3}^3 & \sqrt{6} Y_{-2}^3 \end{pmatrix} (\hat{\mathbf{r}}),$$

$$H = \sum_{n,k,\sigma} \epsilon_{\mathbf{k}n} c_{\mathbf{k}n\sigma}^\dagger c_{\mathbf{k}n\sigma} + \sum_j H_m(j)$$



$$c_{j\alpha}^\dagger = \sum_{p \in (1,14), \sigma} c_\sigma^\dagger(\mathbf{R}_{jp}) \mathcal{Y}_{\sigma\alpha}(\mathbf{r}_p)$$

Conduction Wannier state with  $j=7/2$ ,  $m_j=\pm 5/2$

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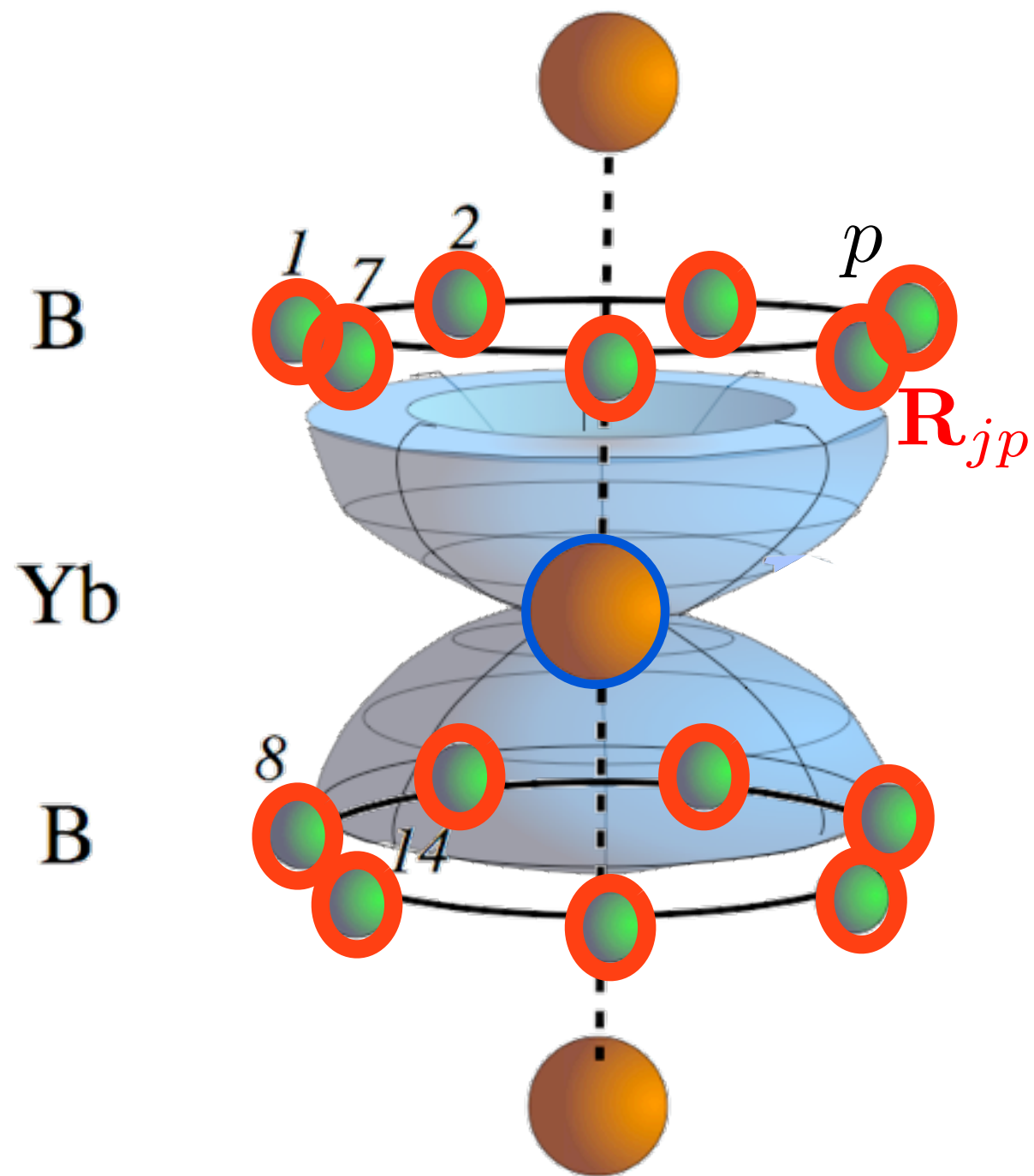


$$H_m(j) = V_0(c_{j\alpha}^\dagger X_{0\alpha}(j) + \text{h.c.}) + E_f X_{\alpha\alpha}(j),$$

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## Conduction Wannier state with $j=7/2$ , $m_J=\pm 5/2$

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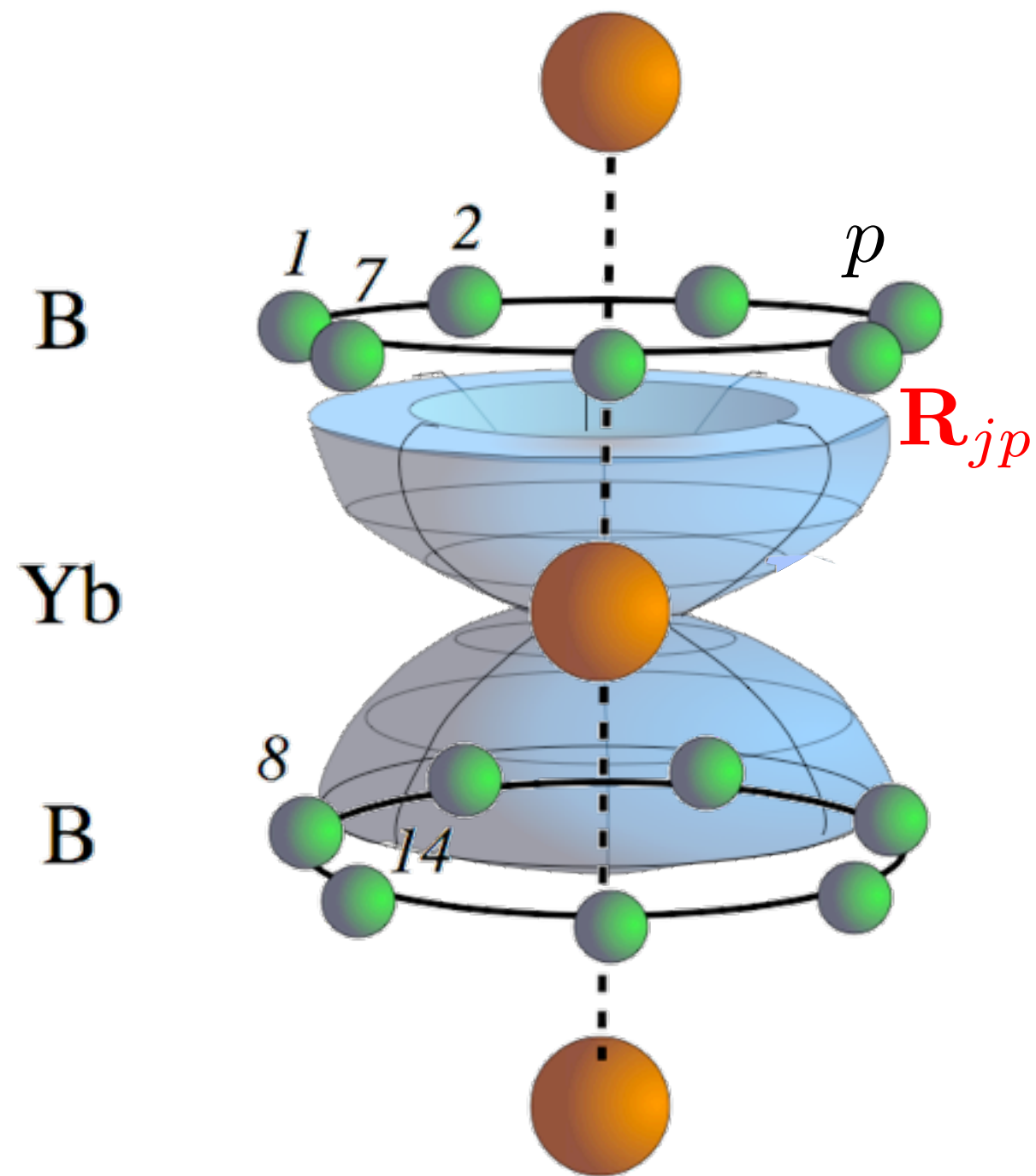
$$H_m(j) = V_0 (c_{j\alpha}^\dagger X_{0\alpha}(j) + \text{h.c.}) + E_f X_{\alpha\alpha}(j),$$

$$H_{eff} = \sum_{\mathbf{k}} (c_{\mathbf{k}}^\dagger, f_{\mathbf{k}}^\dagger) \begin{pmatrix} \epsilon_{\mathbf{k}\mathbb{I}} & \underline{V}(\mathbf{k}) \\ \underline{V}^\dagger(\mathbf{k}) & \tilde{E}_f \mathbb{I} \end{pmatrix} \begin{pmatrix} c_{\mathbf{k}} \\ f_{\mathbf{k}} \end{pmatrix}$$

$$c_{j\alpha}^\dagger = \sum_{p \in (1,14), \sigma} c_\sigma^\dagger(\mathbf{R}_{jp}) \mathcal{Y}_{\sigma\alpha}(\mathbf{r}_p)$$

Conduction Wannier state with  $j=7/2$ ,  $m_J=\pm 5/2$

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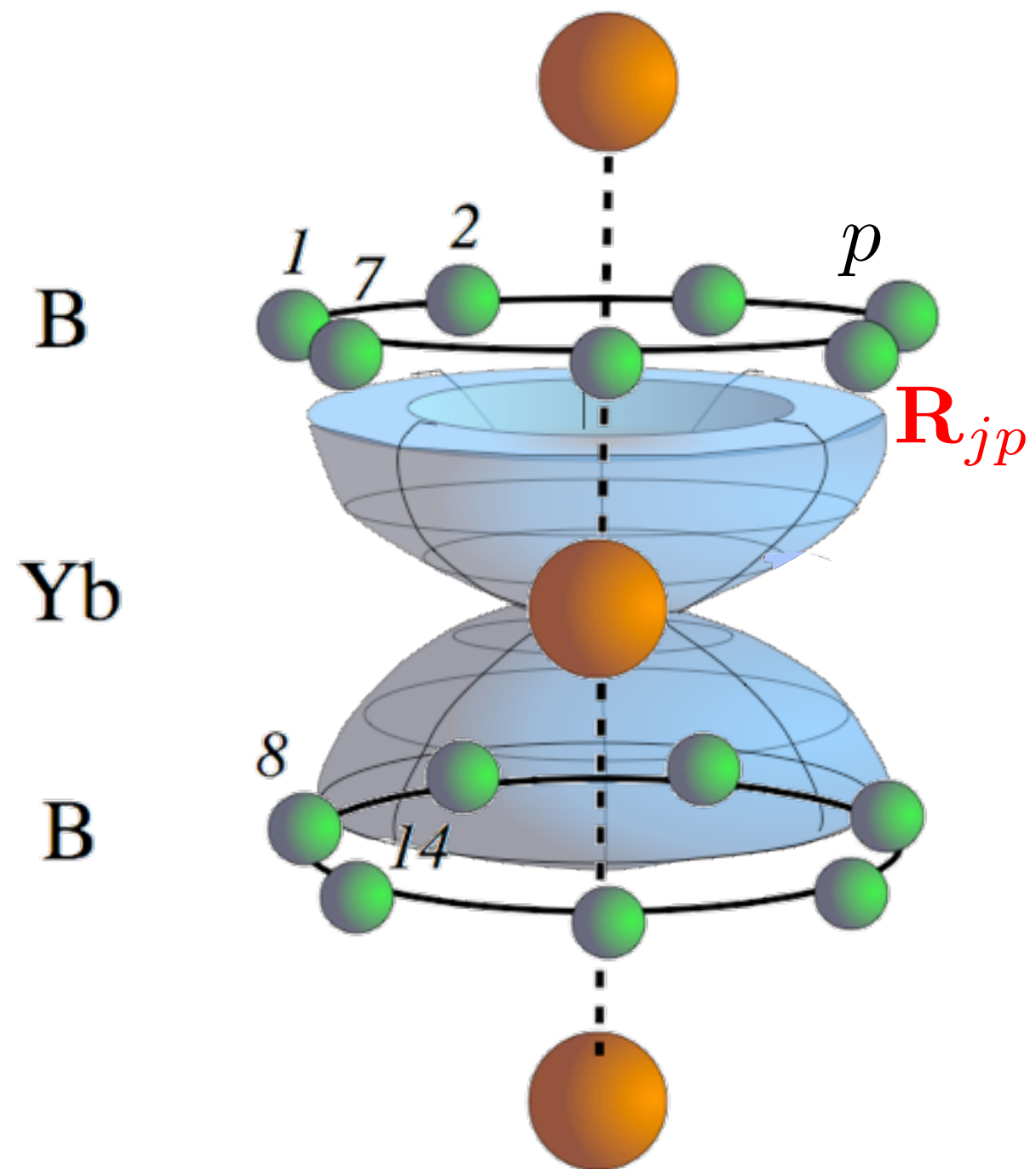
$$H = \sum_{n,k,\sigma} \epsilon_{\mathbf{k}n} c_{\mathbf{k}n\sigma}^\dagger c_{\mathbf{k}n\sigma} + \sum_j H_m(j)$$

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$$H_{eff} = \sum_{\mathbf{k}} (c_{\mathbf{k}}^\dagger, f_{\mathbf{k}}^\dagger) \begin{pmatrix} \epsilon_{\mathbf{k}} \mathbb{I} & \underline{V}(\mathbf{k}) \\ \underline{V}^\dagger(\mathbf{k}) & \tilde{E}_f \mathbb{I} \end{pmatrix} \begin{pmatrix} c_{\mathbf{k}} \\ f_{\mathbf{k}} \end{pmatrix}$$

$$\underline{V}(\mathbf{k}) = V_0^* \underline{\gamma}(\mathbf{k})$$

$$[\underline{\gamma}(\mathbf{k})]_{\sigma\alpha} = \sum_{p=1,14} \mathcal{Y}_{\sigma\alpha}(\mathbf{r}_p) e^{-i\mathbf{k}\cdot\mathbf{r}_p}.$$

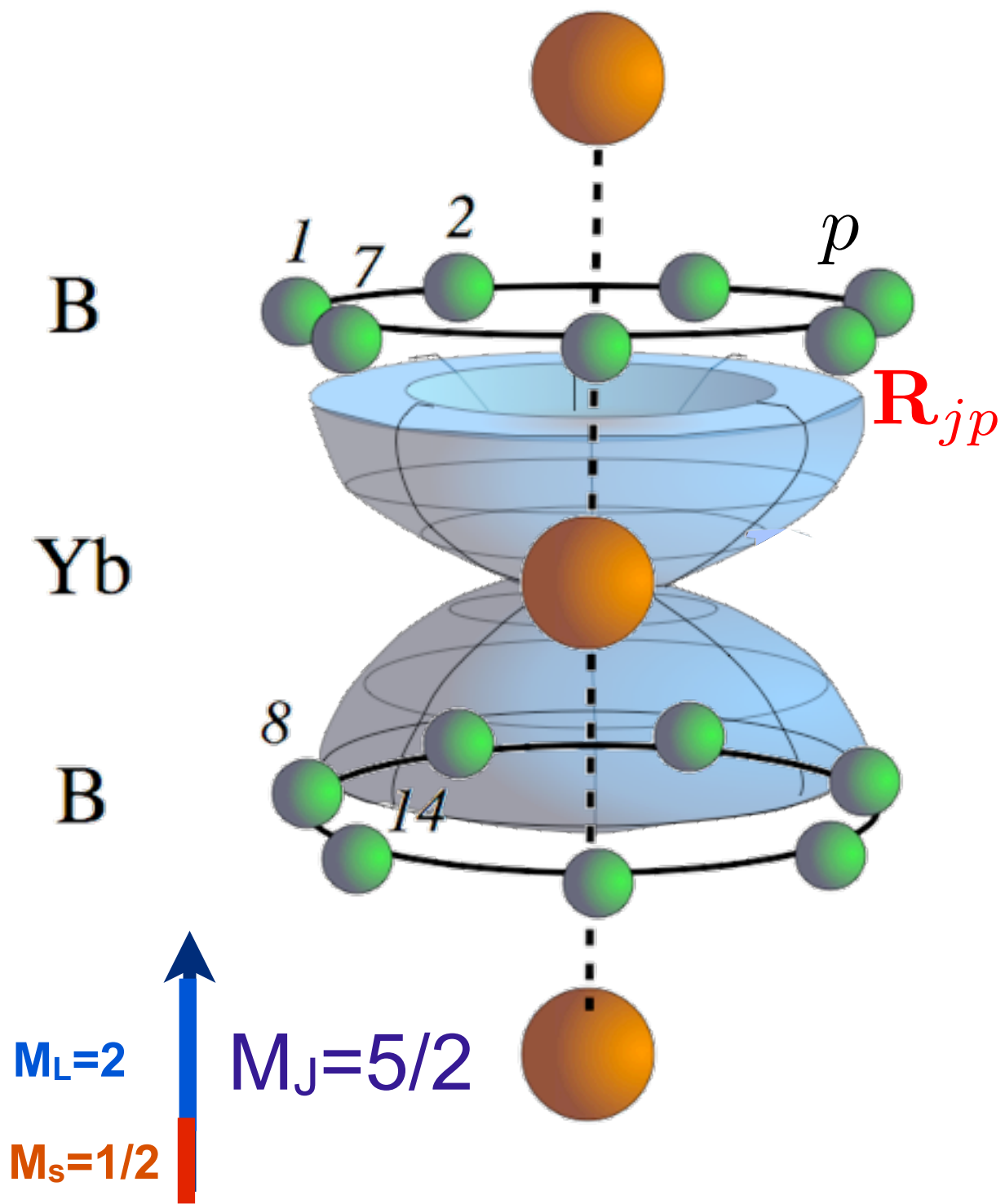


$$H = \sum_{n,k,\sigma} \epsilon_{\mathbf{k}n} c_{\mathbf{k}n\sigma}^\dagger c_{\mathbf{k}n\sigma} + \sum_j H_m(j)$$

$$H_m(j) = V_0 (c_{j\alpha}^\dagger X_{0\alpha}(j) + \text{h.c.}) + E_f X_{\alpha\alpha}(j),$$

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$$V(\mathbf{k}) \sim k_z \begin{pmatrix} (k_x + ik_y)^2 & \\ & (k_x - ik_y)^2 \end{pmatrix}$$



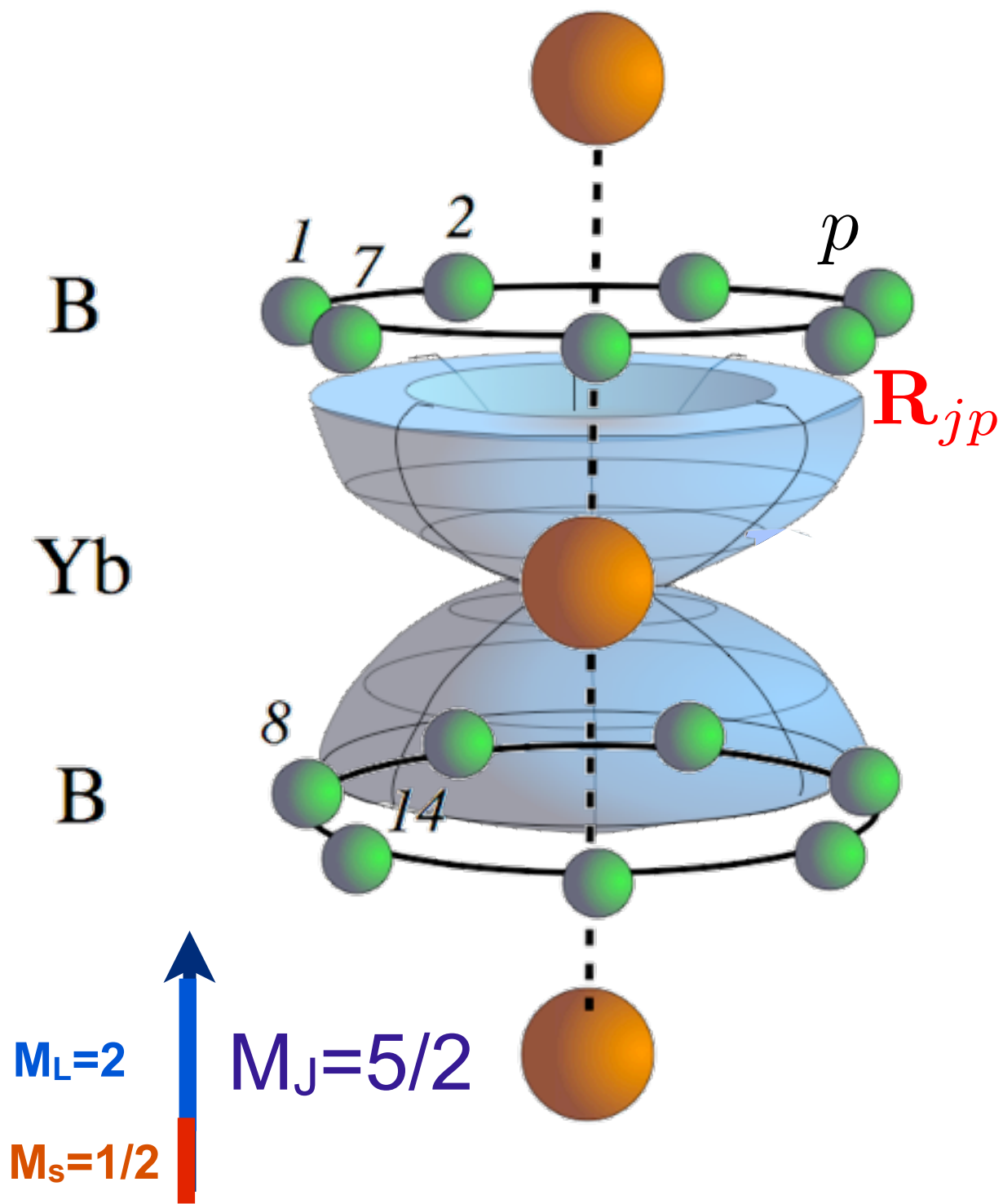
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$$V(\mathbf{k}) \sim k_z \begin{pmatrix} (k_x + ik_y)^2 & \\ & (k_x - ik_y)^2 \end{pmatrix}$$





$$H = \sum_{n,k,\sigma} \epsilon_{\mathbf{k}n} c_{\mathbf{k}n\sigma}^\dagger c_{\mathbf{k}n\sigma} + \sum_j H_m(j)$$

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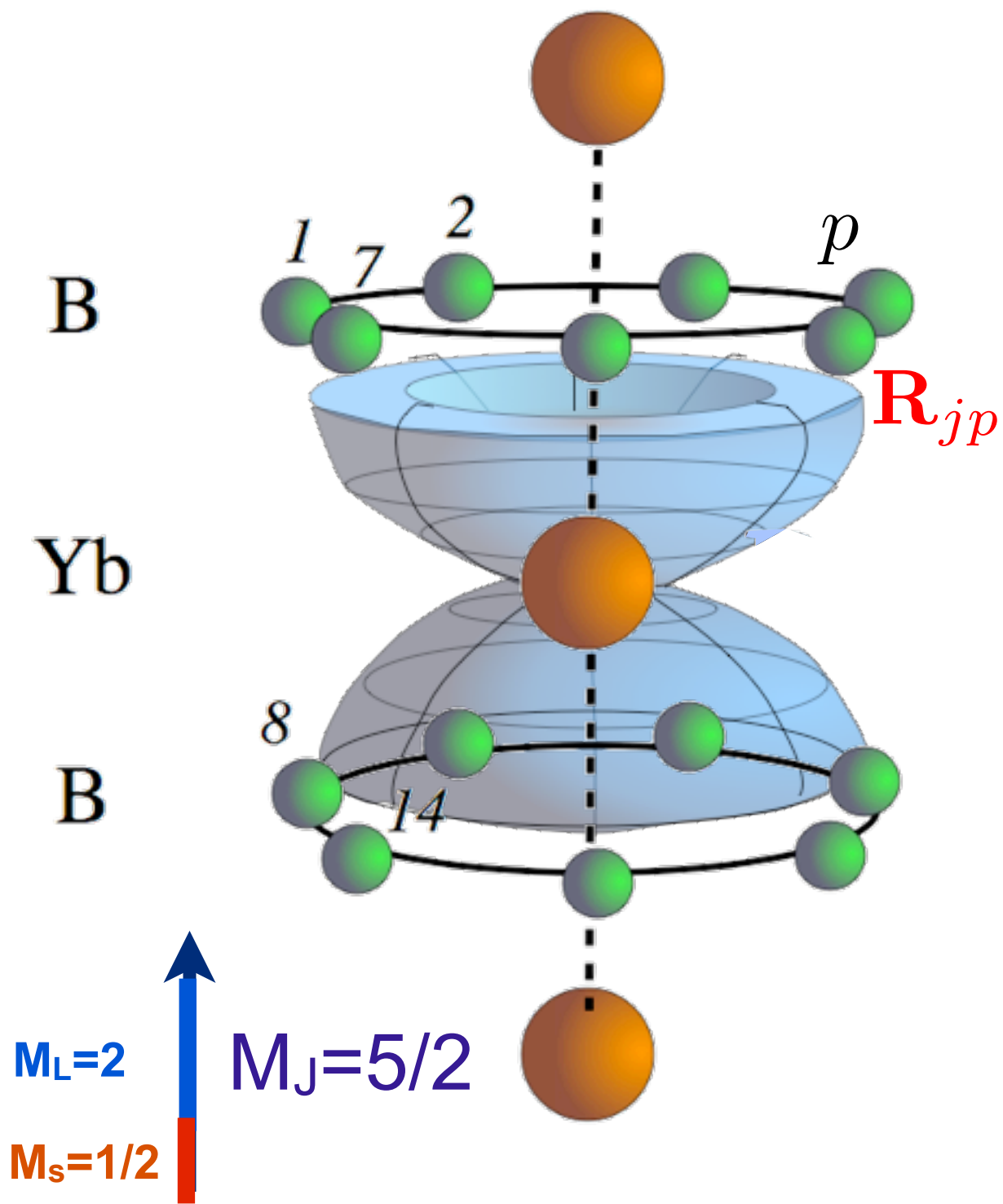
$$H_{eff} = \sum_{\mathbf{k}} (c_{\mathbf{k}}^\dagger, f_{\mathbf{k}}^\dagger) \begin{pmatrix} \epsilon_{\mathbf{k}\mathbb{I}} & \underline{V}(\mathbf{k}) \\ \underline{V}^\dagger(\mathbf{k}) & \tilde{E}_f \mathbb{I} \end{pmatrix} \begin{pmatrix} c_{\mathbf{k}} \\ f_{\mathbf{k}} \end{pmatrix}$$

$$\Delta M=2$$

Diagram illustrating the interaction potential  $V(\mathbf{k})$  and the resulting energy levels. The potential is given by:

$$V(\mathbf{k}) \sim k_z \begin{pmatrix} (k_x + ik_y)^2 & \\ & (k_x - ik_y)^2 \end{pmatrix}$$

The diagram shows four horizontal energy levels (black lines) and a coordinate system with a red arrow labeled  $M_S=1/2$  and a blue arrow labeled  $L$ .



$$H = \sum_{n,k,\sigma} \epsilon_{\mathbf{k}n} c_{\mathbf{k}n\sigma}^\dagger c_{\mathbf{k}n\sigma} + \sum_j H_m(j)$$

$$H_m(j) = V_0(c_{j\alpha}^\dagger X_{0\alpha}(j) + \text{h.c.}) + E_f X_{\alpha\alpha}(j),$$

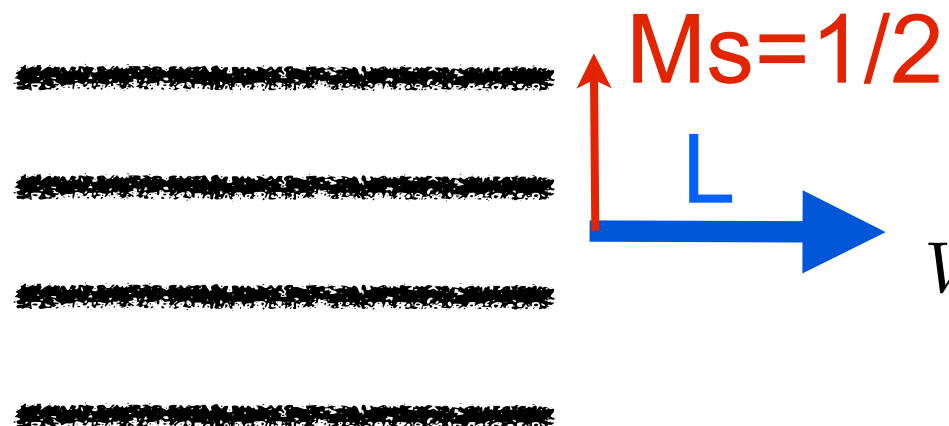
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$$\Delta M=2$$

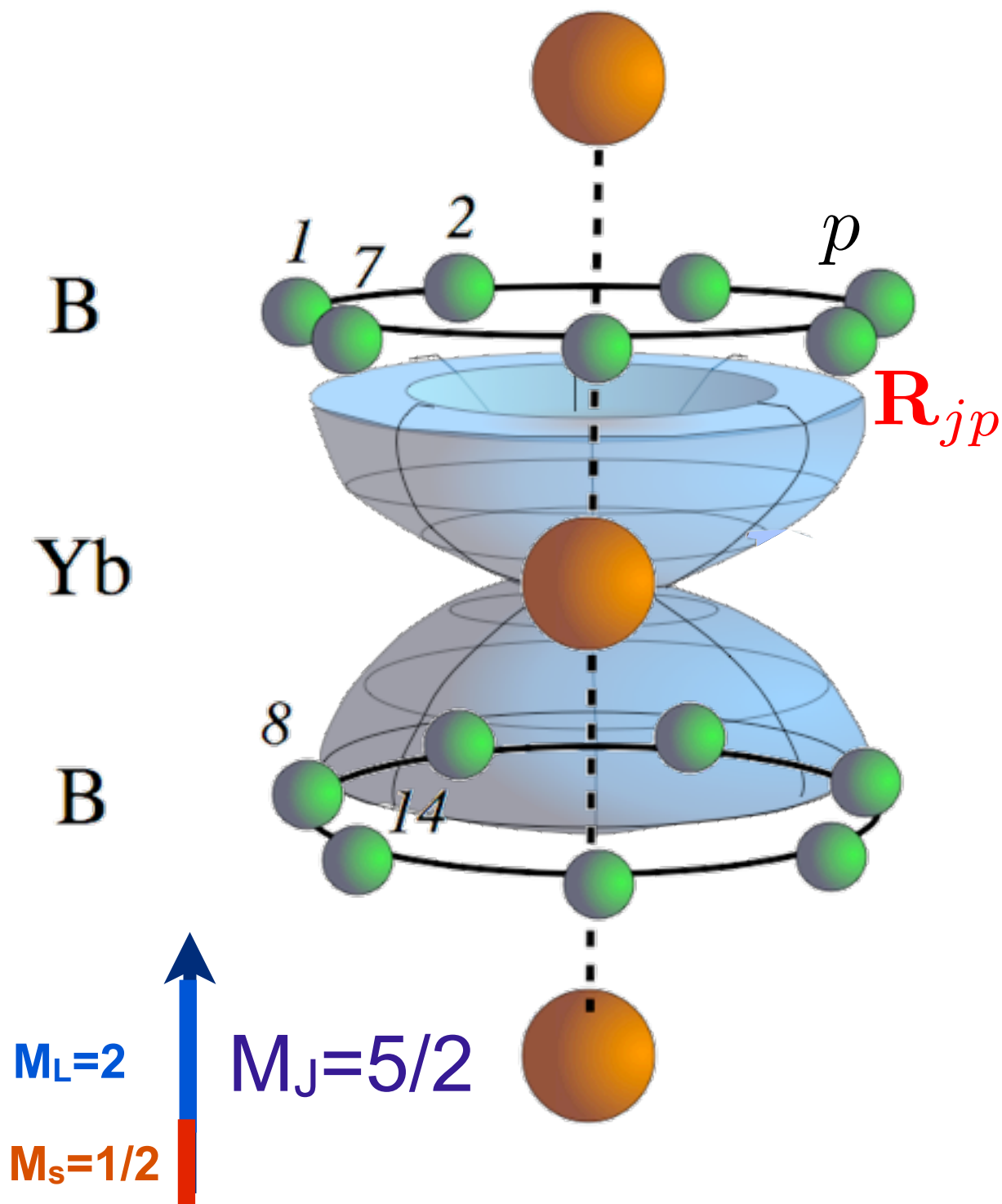
$$V(\mathbf{k})=0$$

**“Spin Blockade”**

(Ikeda and Miyake, JPSJ, 65,1769, **1997**,  
Maignan, Caignaert, Raveau, Khomskii &  
Sawatzky, PRL 93, 026401, **2004**)



$$V(\mathbf{k}) \sim k_z \begin{pmatrix} (k_x + ik_y)^2 & \\ & (k_x - ik_y)^2 \end{pmatrix}$$



$$H = \sum_{n,k,\sigma} \epsilon_{\mathbf{k}n} c_{\mathbf{k}n\sigma}^\dagger c_{\mathbf{k}n\sigma} + \sum_j H_m(j)$$

$$H_m(j) = V_0(c_{j\alpha}^\dagger X_{0\alpha}(j) + \text{h.c.}) + E_f X_{\alpha\alpha}(j),$$

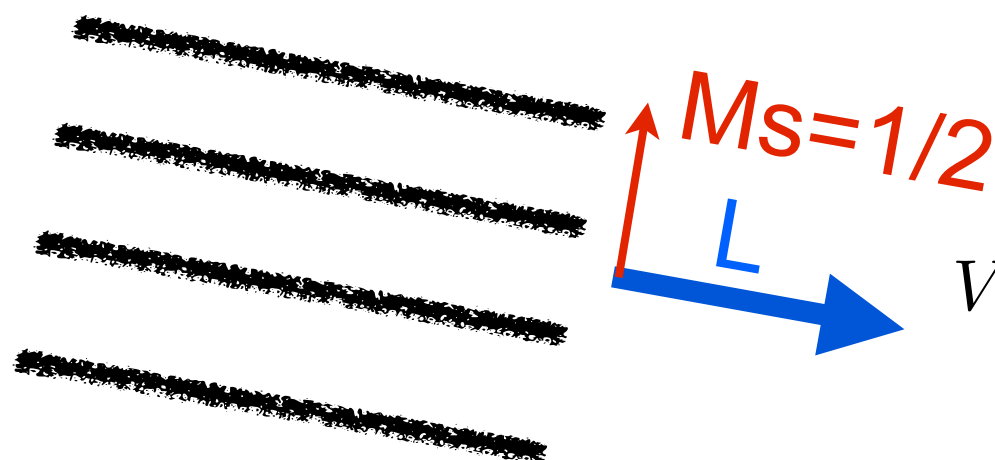
$$H_{eff} = \sum_{\mathbf{k}} (c_{\mathbf{k}}^\dagger, f_{\mathbf{k}}^\dagger) \begin{pmatrix} \epsilon_{\mathbf{k}\mathbb{I}} & \underline{V}(\mathbf{k}) \\ \underline{V}^\dagger(\mathbf{k}) & \tilde{E}_f \mathbb{I} \end{pmatrix} \begin{pmatrix} c_{\mathbf{k}} \\ f_{\mathbf{k}} \end{pmatrix}$$

$$\Delta M=2$$

$$V(\mathbf{k}) \sim (k_x \pm i k_y)^2$$

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$$V(\mathbf{k}) \sim k_z \begin{pmatrix} (k_x + i k_y)^2 & \\ & (k_x - i k_y)^2 \end{pmatrix}$$

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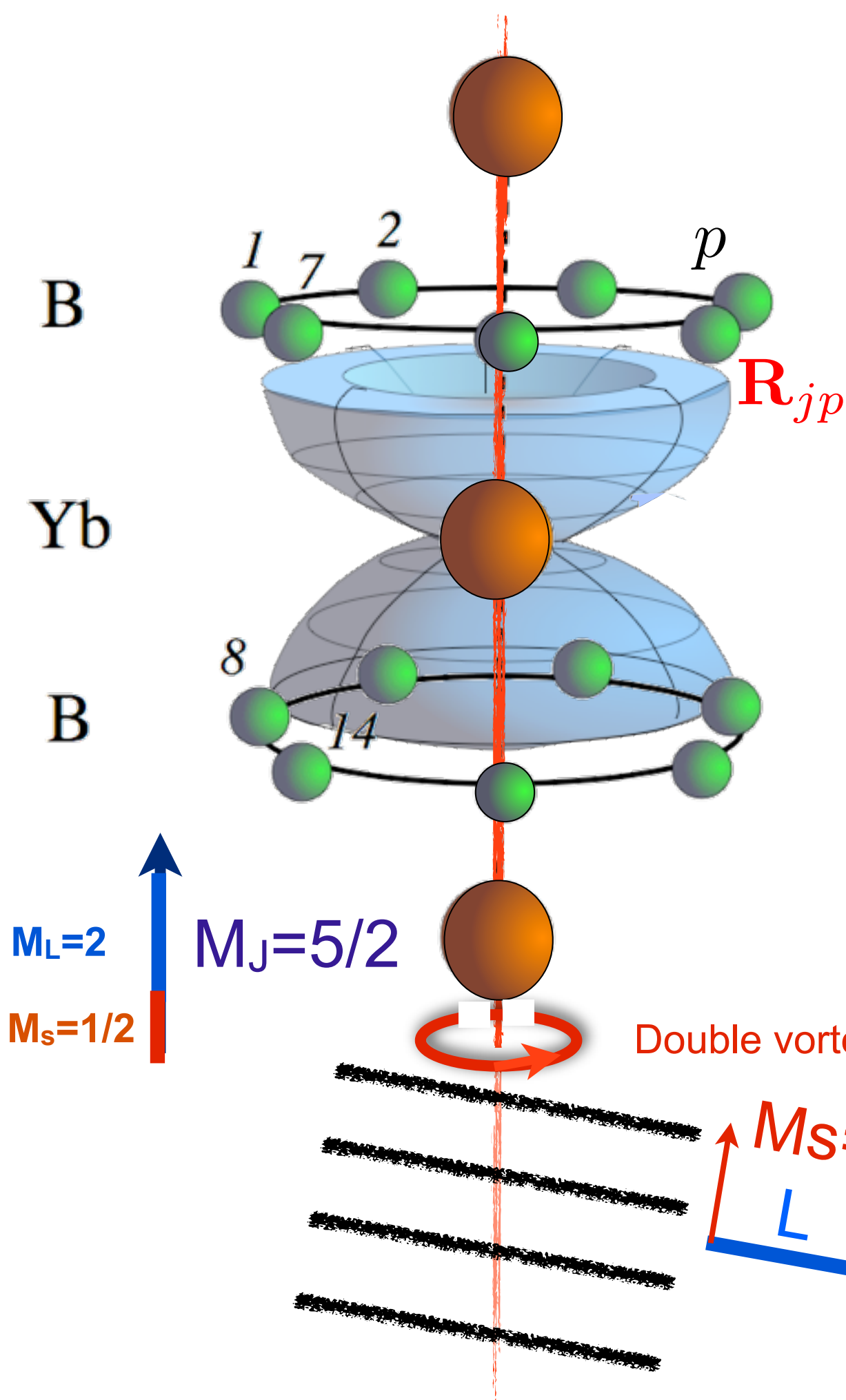
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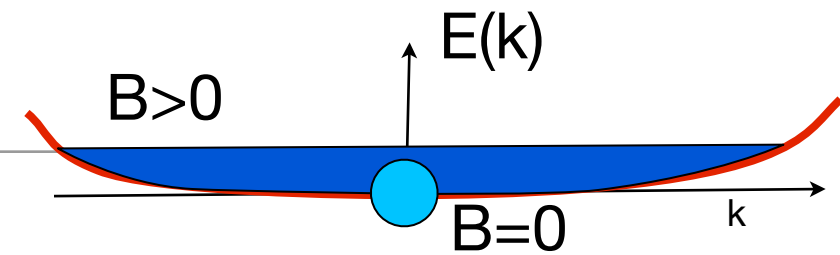
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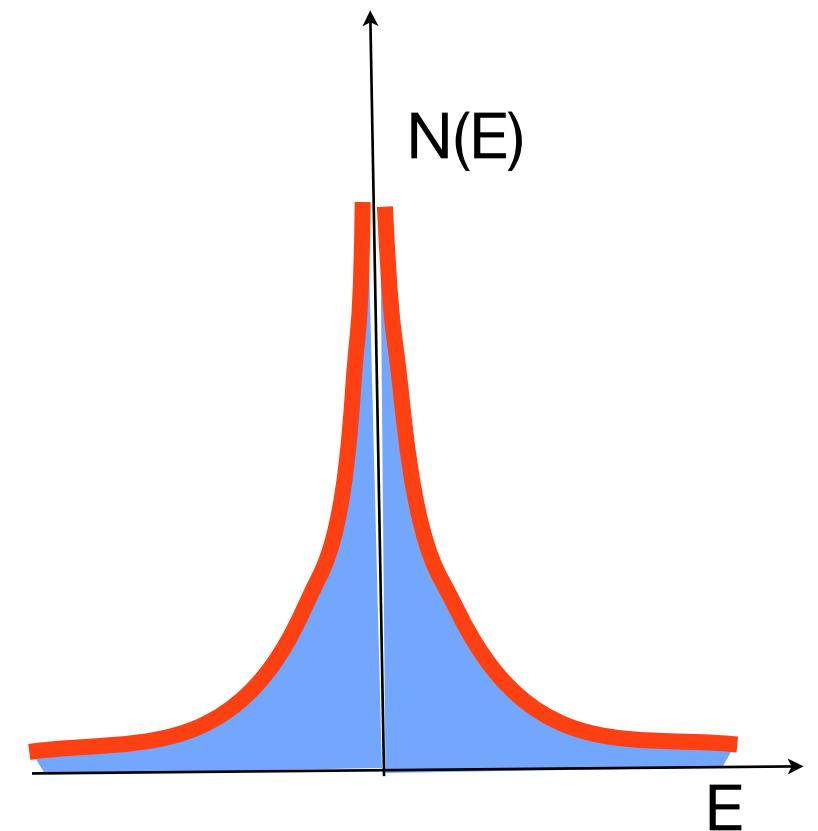


# T/B scaling



## “Vortex” Transition.

Zeeman energy is the Fermi Energy



$$N_{\pm}^*(E) = 2 \int k_{\perp} \frac{dk_{\perp}}{dE_{\pm}} \frac{dk_z}{(2\pi)^2} = \frac{1}{\sqrt{|E|T_0^{\pm}}}$$

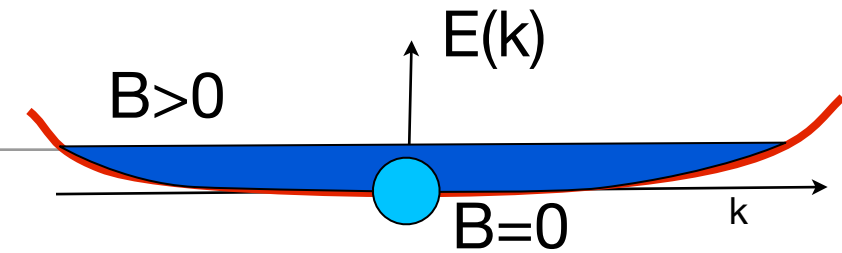
$$\frac{1}{\sqrt{T_0^{\pm}}} = \frac{1}{8\pi^2} \int \frac{dk_z}{\sqrt{|\eta(k_z)|}} \theta[\mp \epsilon(k_z)].$$

# T/B scaling

$$F[B, T] = -T \sum_{\alpha=\pm 5/2} \int_{-\infty}^{\infty} dE N(E) \ln[1 + e^{-\beta(E - g\mu_B B \alpha)}]$$

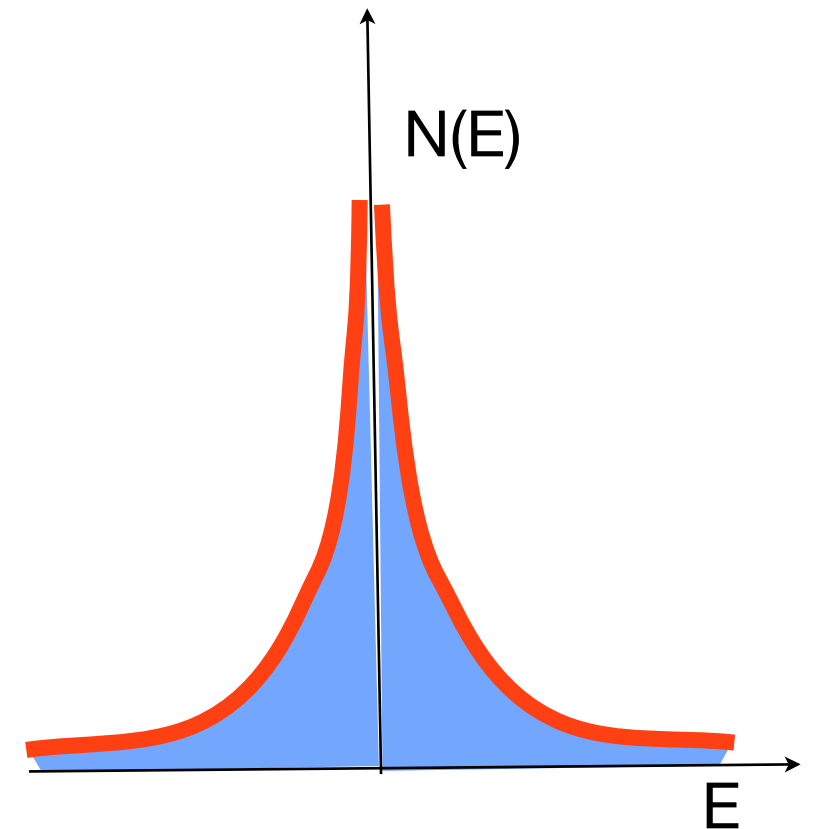
$$= T^{3/2} \Phi \left( \frac{g\mu_B B}{T} \right)$$

$$\Phi(y) = -\frac{1}{\sqrt{T_0}} \int_0^{\infty} \frac{dx}{\sqrt{|x|}} \sum_{\alpha=\pm 5/2} \ln[1 + e^{-x - y\alpha}]$$



**“Vortex” Transition.**

Zeeman energy is the Fermi Energy



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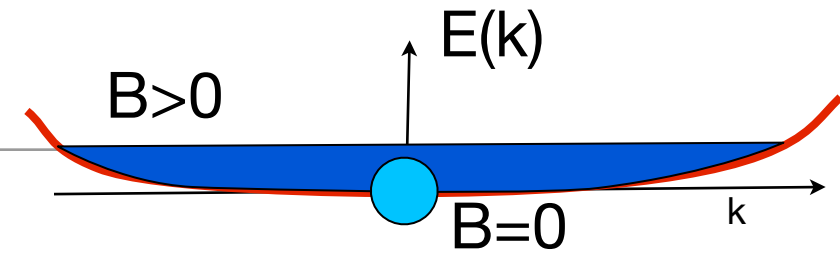
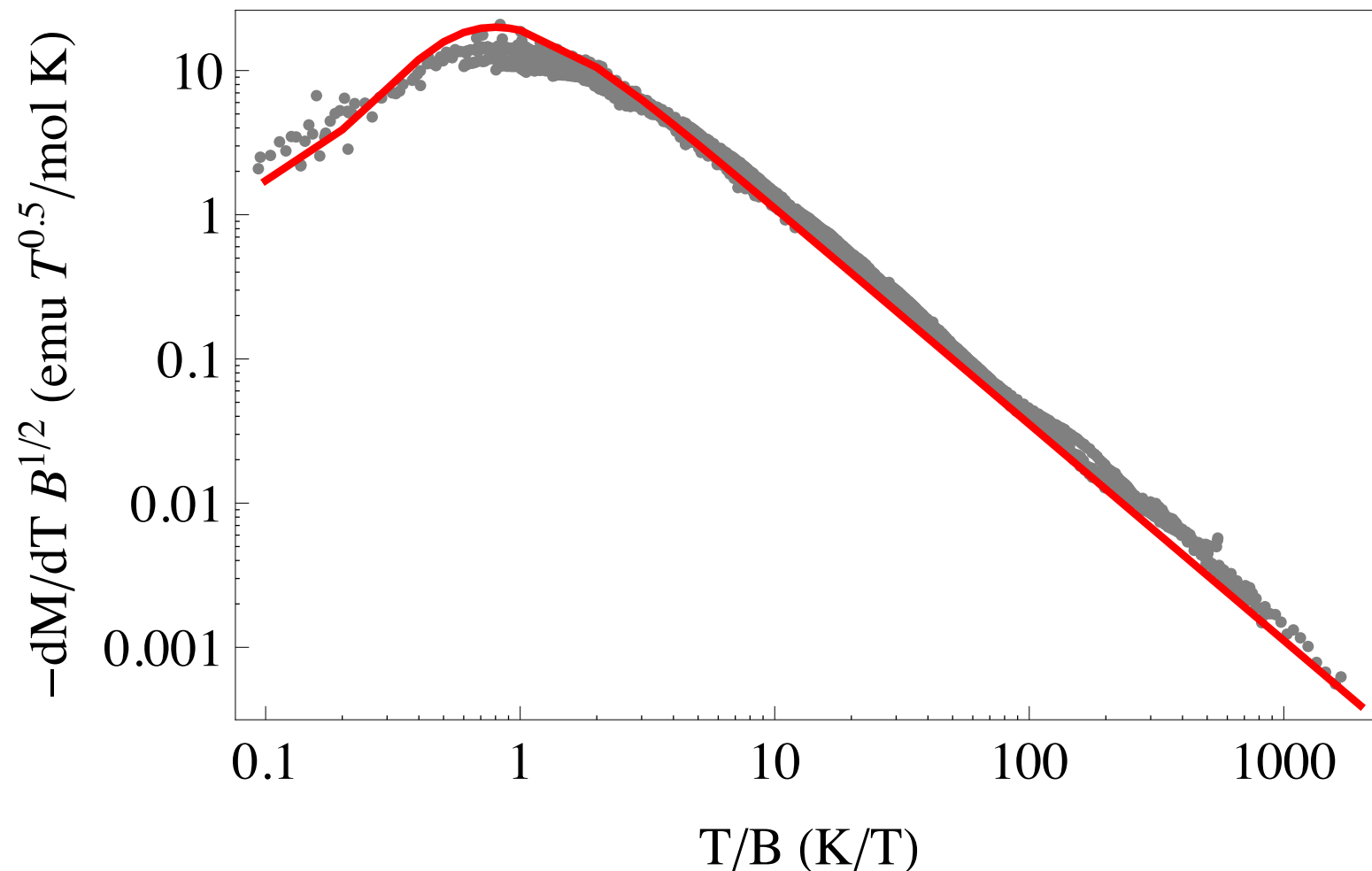
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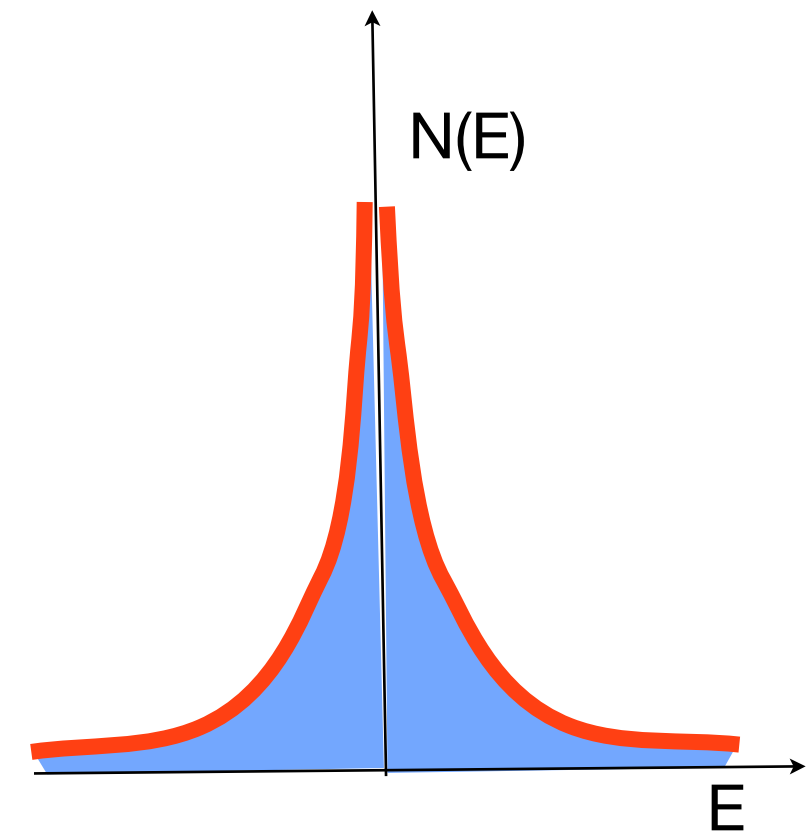
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“Vortex” Transition.

Zeeman energy is the Fermi Energy

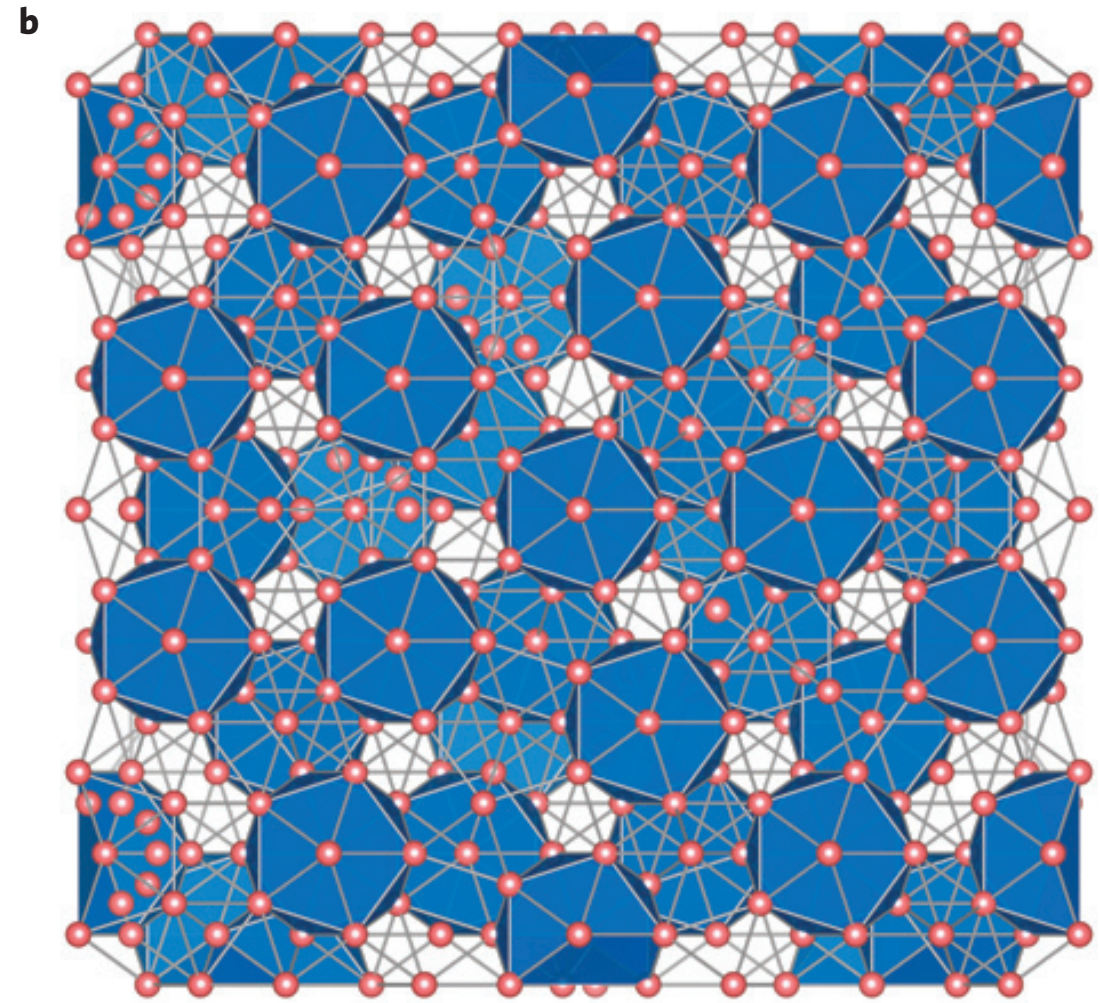


$$N_{\pm}^*(E) = 2 \int k_{\perp} \frac{dk_{\perp}}{dE_{\pm}} \frac{dk_z}{(2\pi)^2} = \frac{1}{\sqrt{|E|T_0^{\pm}}}$$

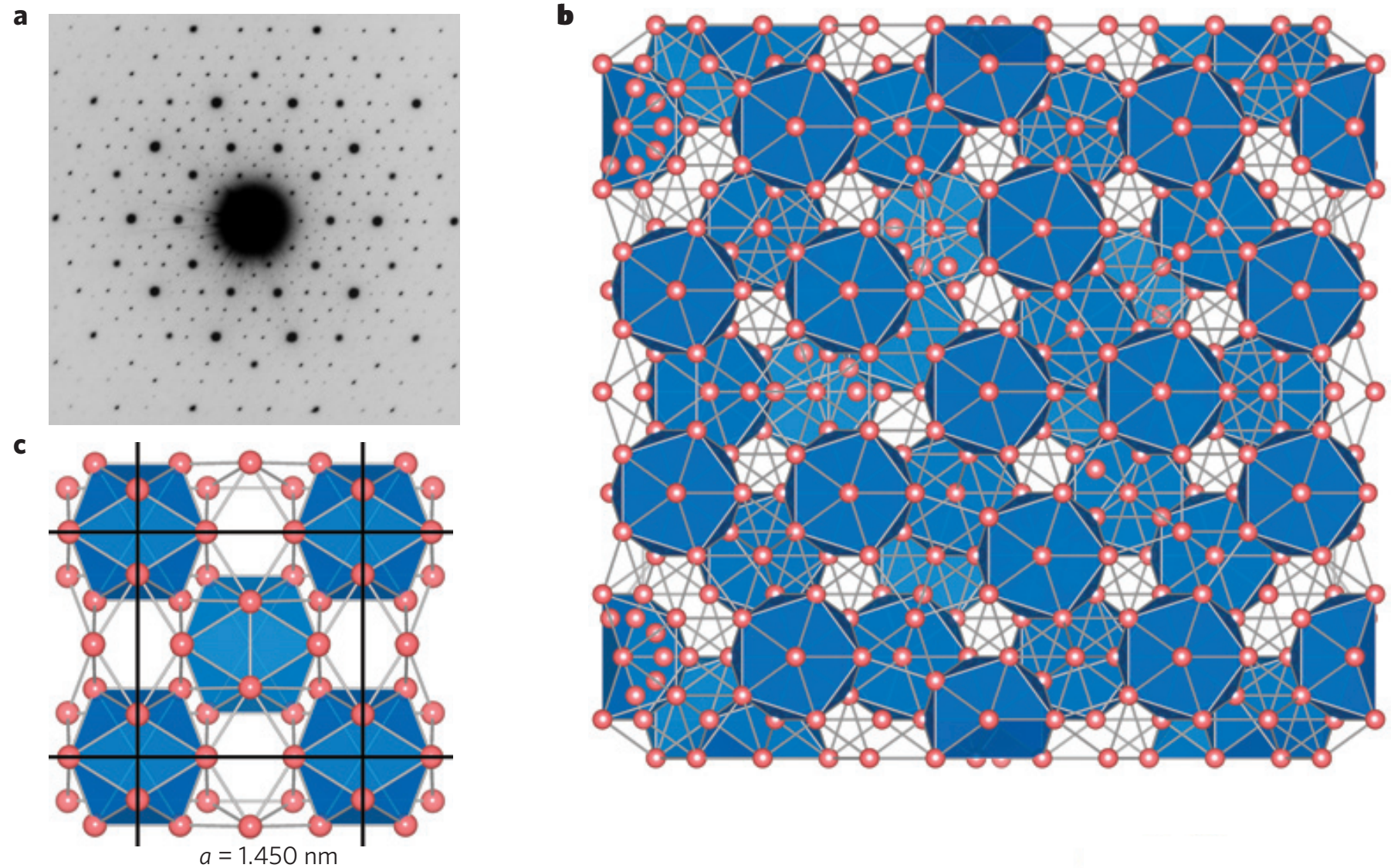
$$\frac{1}{\sqrt{T_0^{\pm}}} = \frac{1}{8\pi^2} \int \frac{dk_z}{\sqrt{|\eta(k_z)|}} \theta[\mp \epsilon(k_z)]$$



# Kondo Quasicrystal.



# Kondo Quasicrystal.



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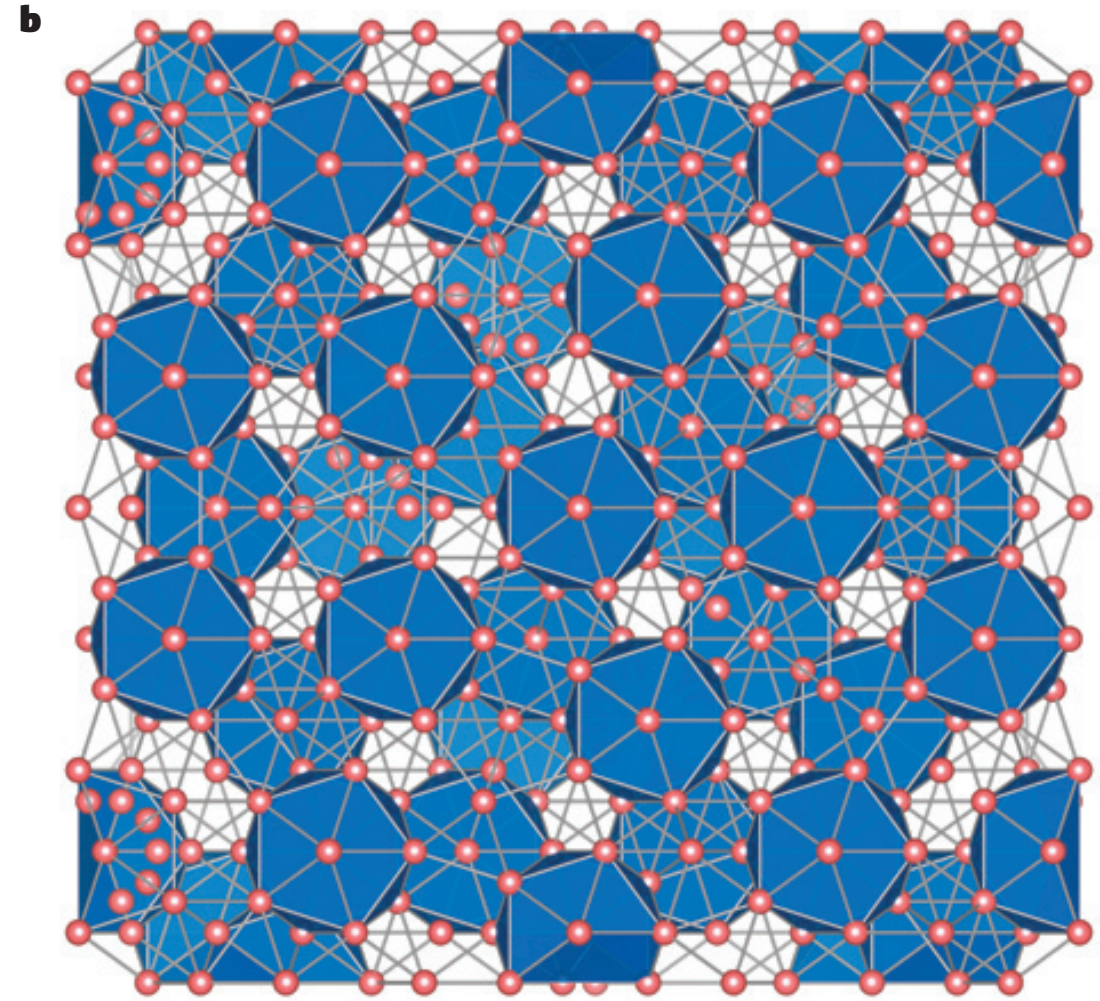
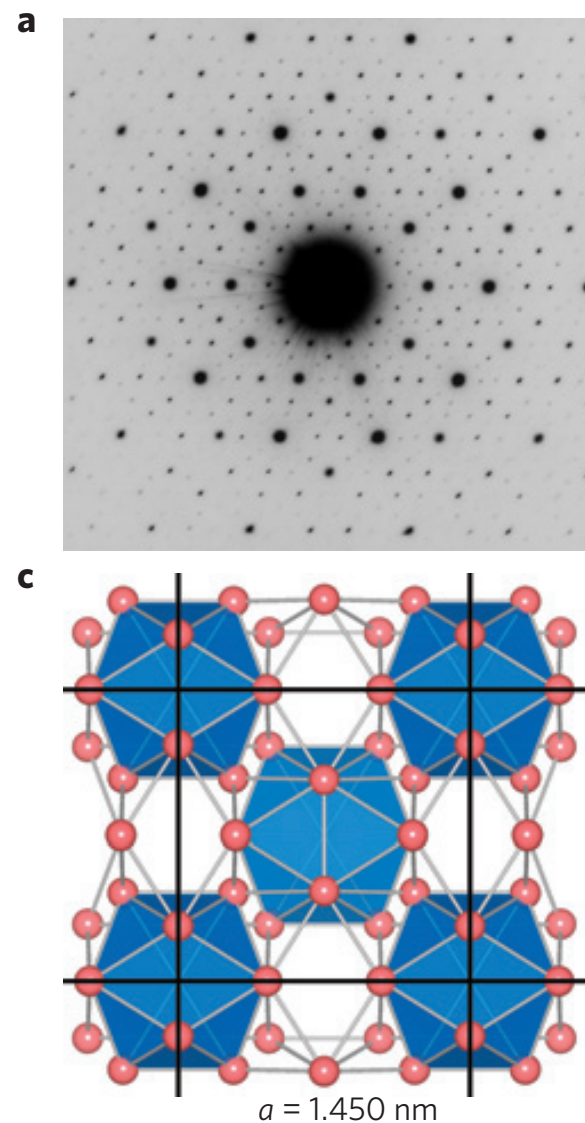
PUBLISHED ONLINE: 7 OCTOBER 2012 | DOI: 10.1038/NMAT3432

## Quantum critical state in a magnetic quasicrystal

Kazuhiko Deguchi<sup>1\*</sup>, Shuya Matsukawa<sup>1</sup>, Noriaki K. Sato<sup>1</sup>, Taisuke Hattori<sup>2</sup>, Kenji Ishida<sup>2</sup>,  
Hiroyuki Takakura<sup>3</sup> and Tsutomu Ishimasa<sup>3</sup>



# Kondo Quasicrystal.



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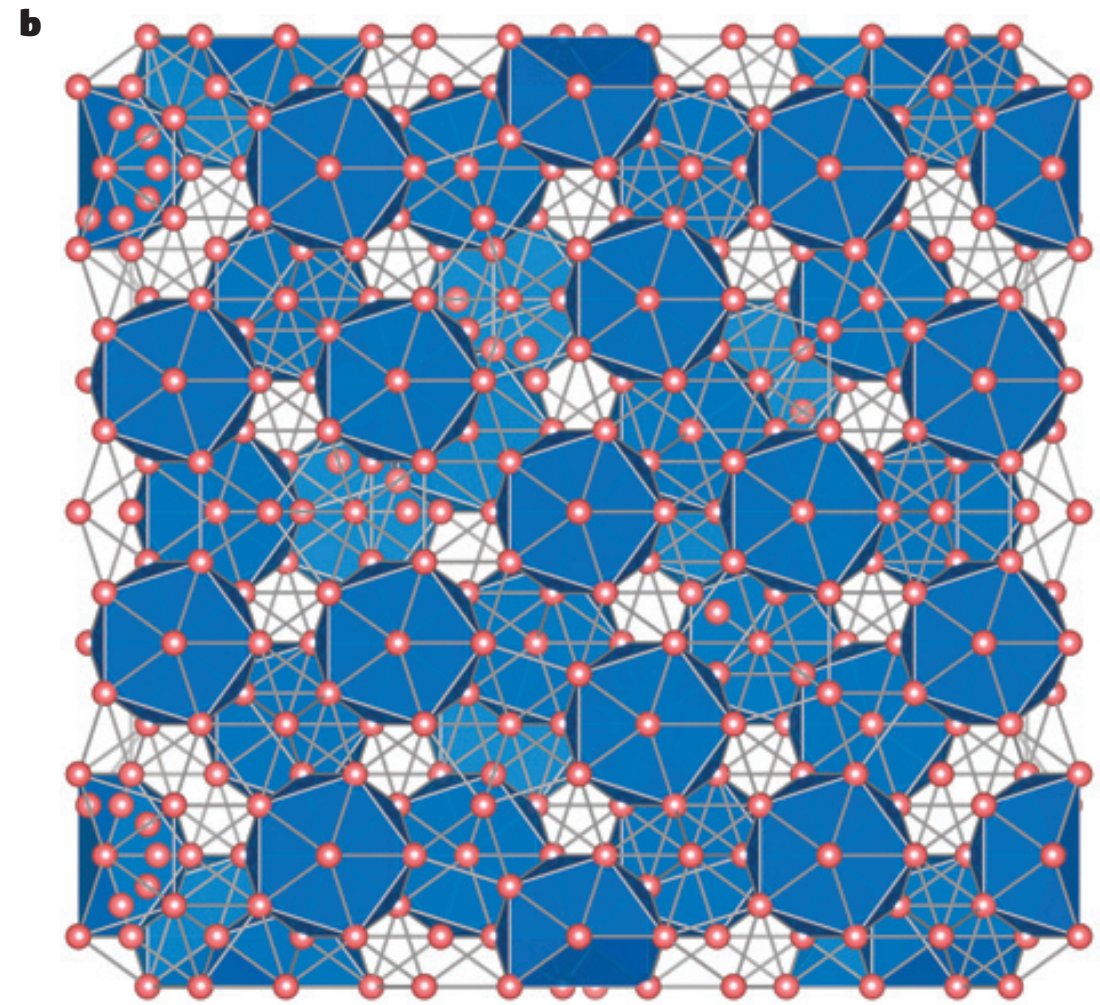
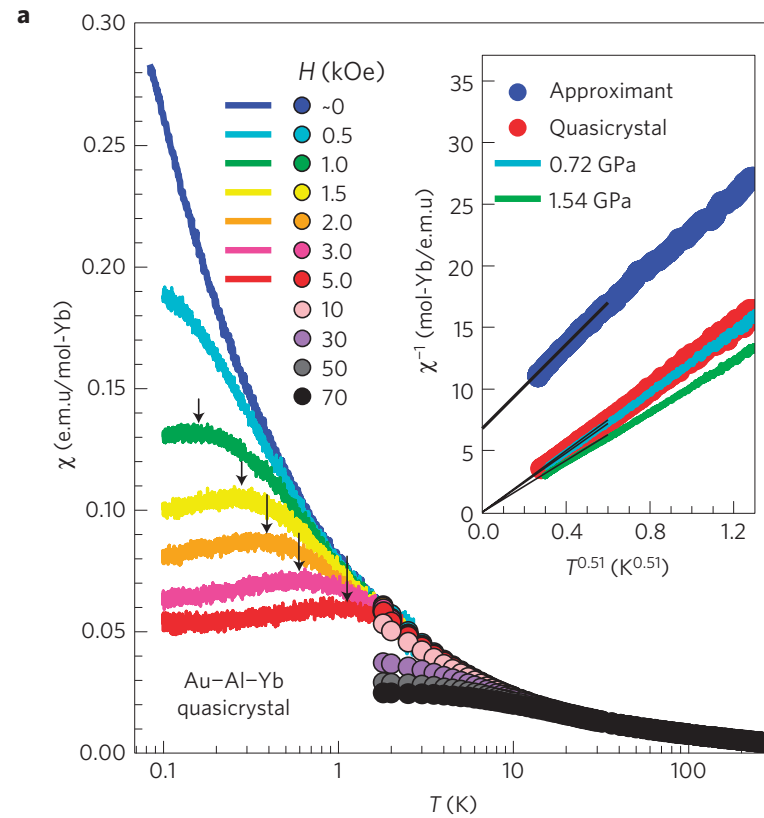
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Approximant:



# Kondo Quasicrystal.



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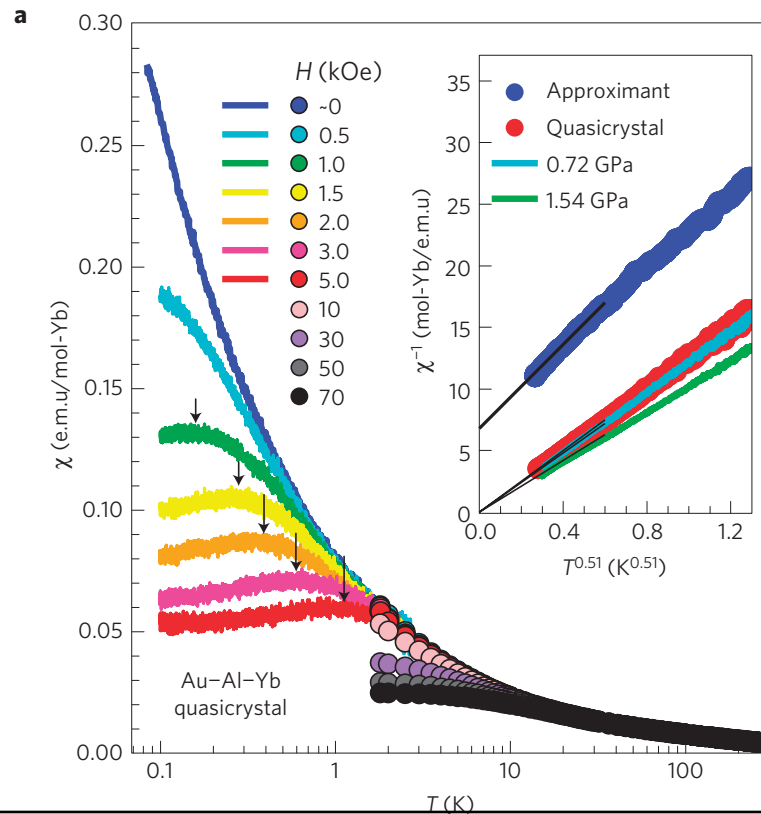
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Approximant:

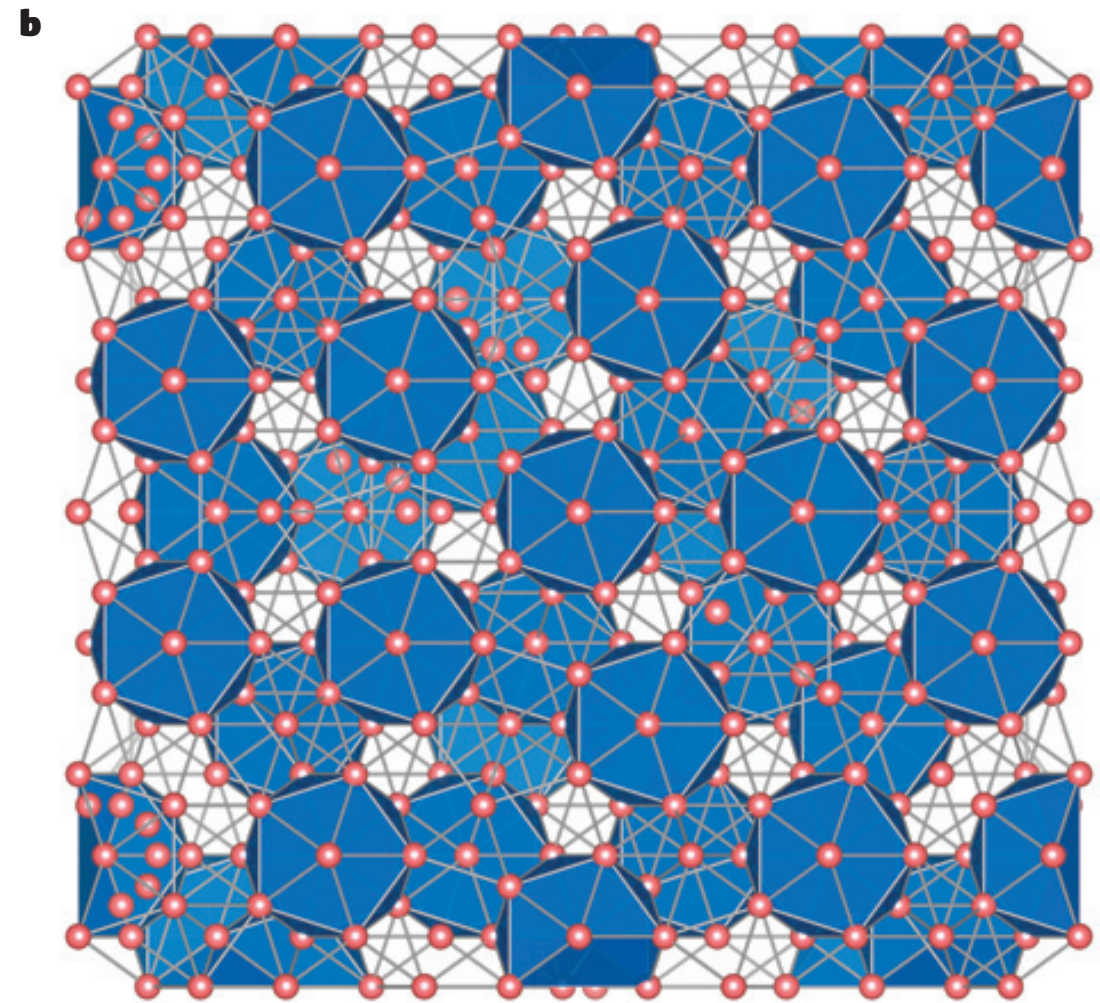




# Kondo Quasicrystal.



Strange metal like YbAlB<sub>4</sub>!



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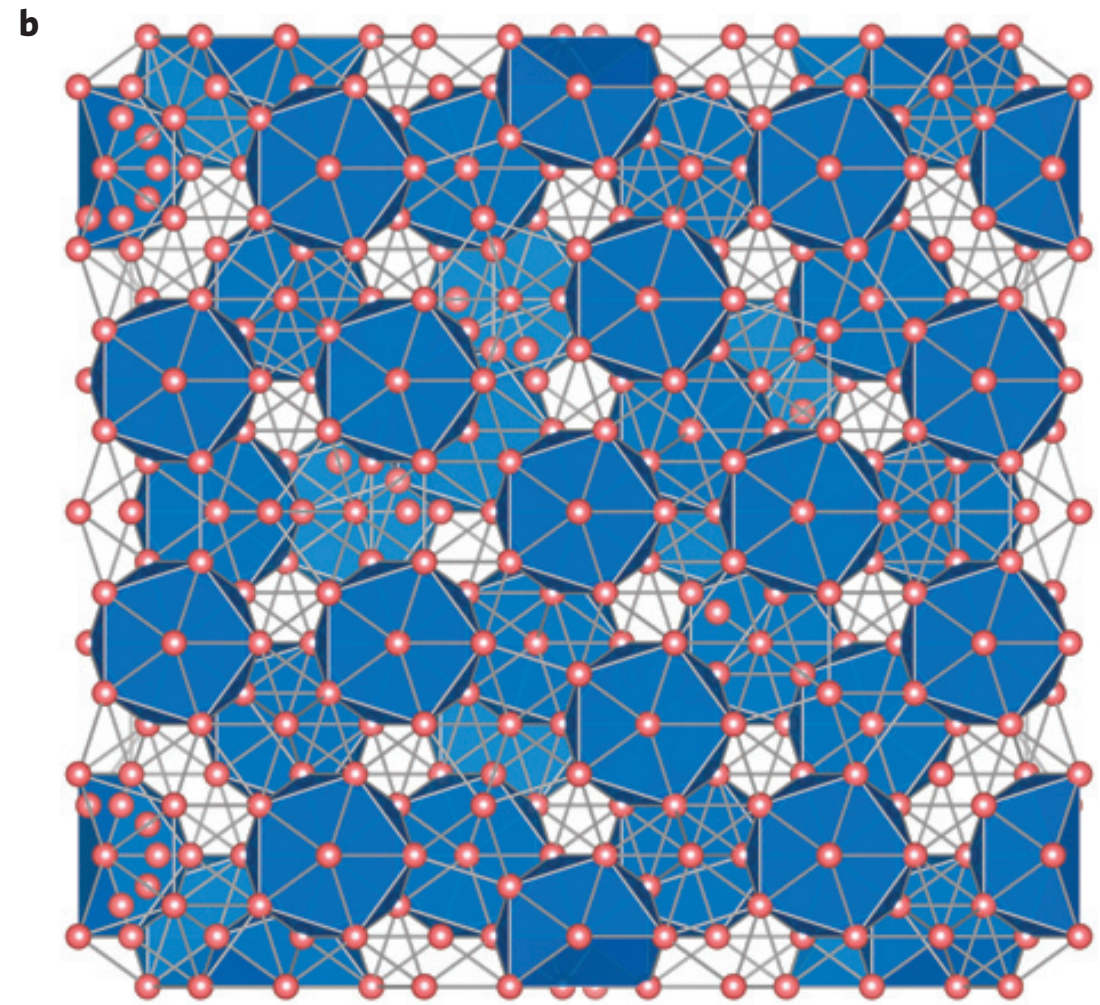
## Quantum critical state in a magnetic quasicrystal

Kazuhiko Deguchi<sup>1\*</sup>, Shuya Matsukawa<sup>1</sup>, Noriaki K. Sato<sup>1</sup>, Taisuke Hattori<sup>2</sup>, Kenji Ishida<sup>2</sup>, Hiroyuki Takakura<sup>3</sup> and Tsutomu Ishimasa<sup>3</sup>

Approximant:

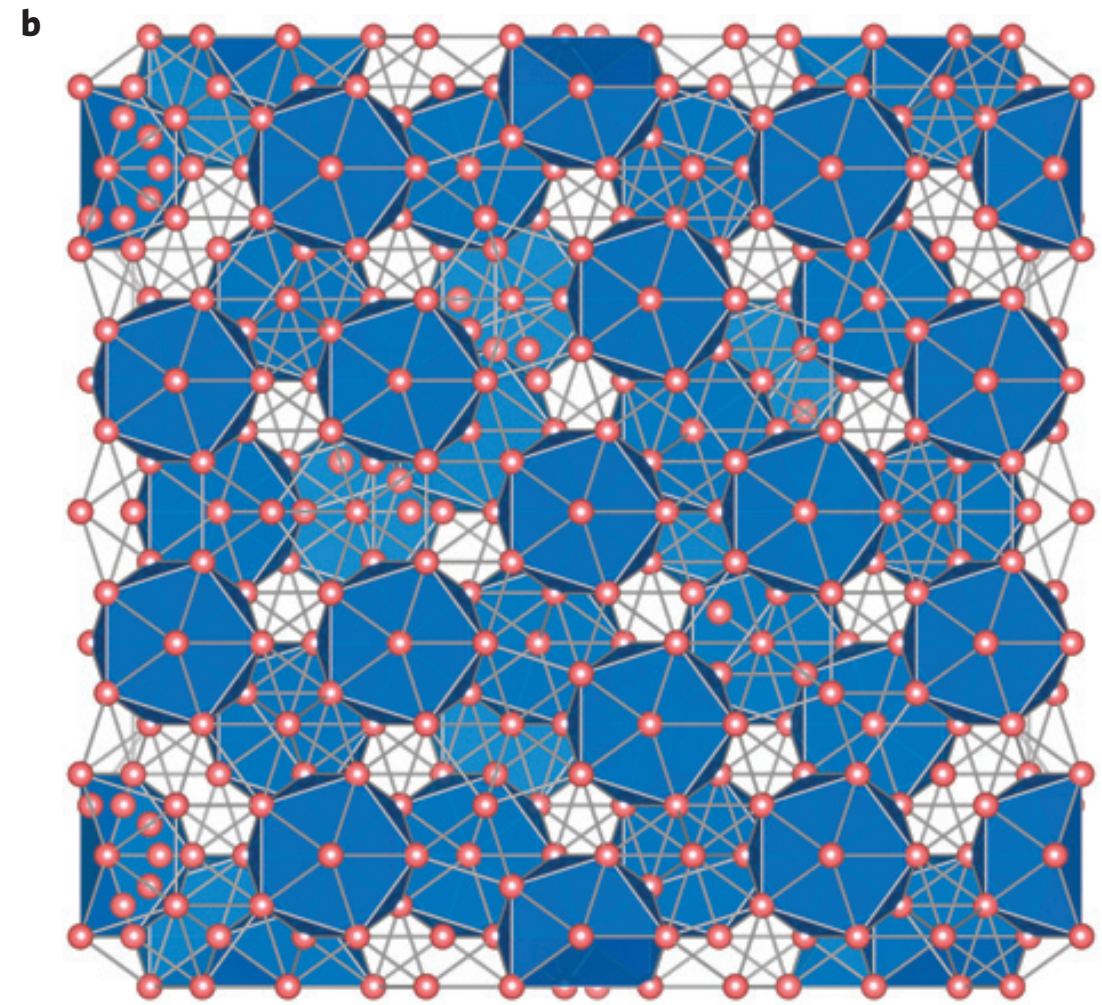
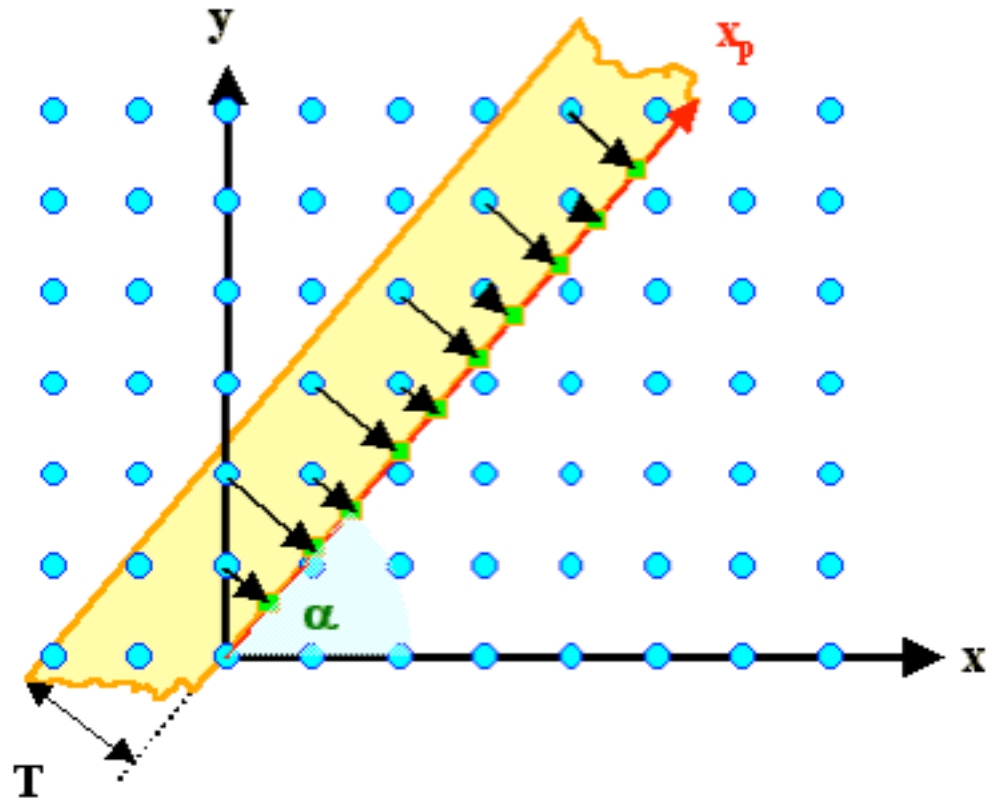


Quasicrystals as “surfaces” (with irrational Miller indices ) of hypercrystals.  
(H. Bohr, 1925, de Bruijn 1981)



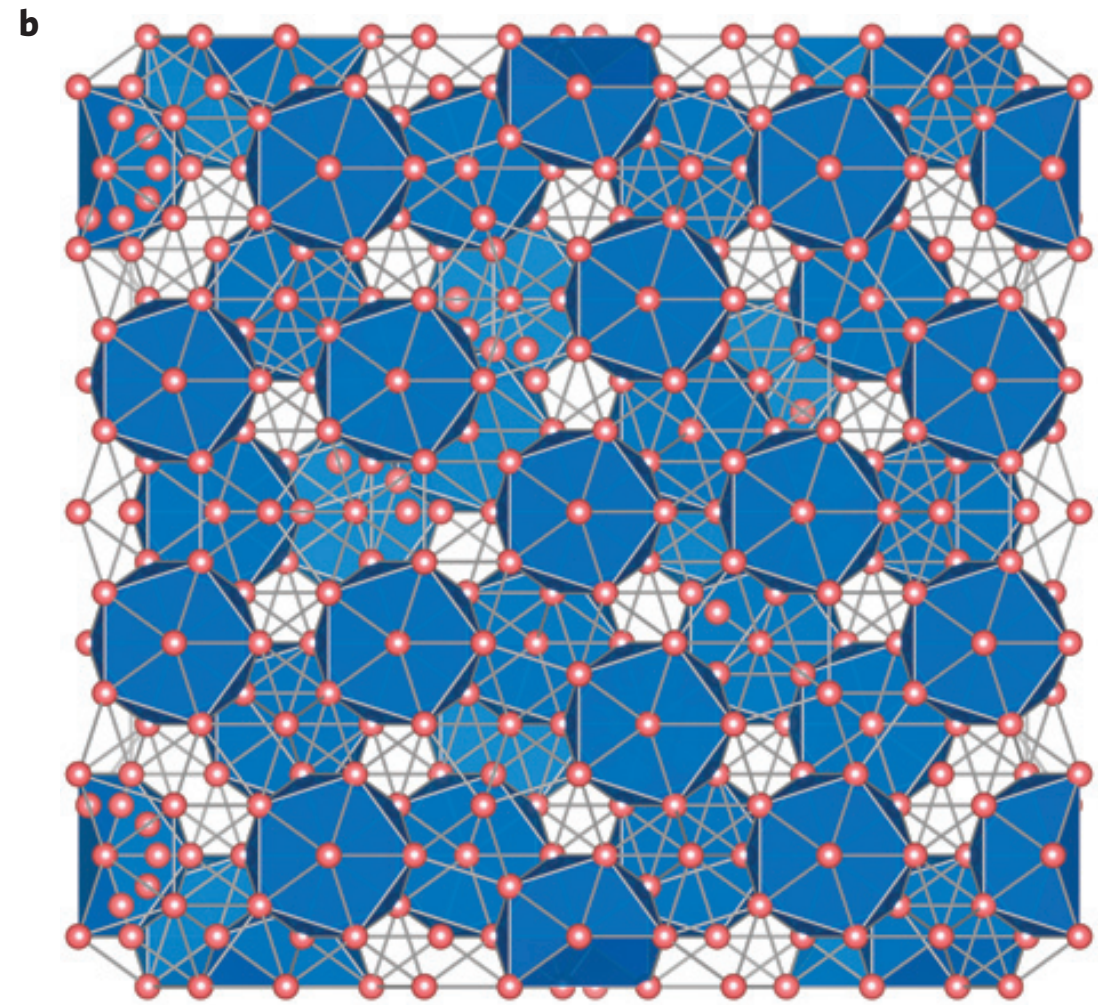
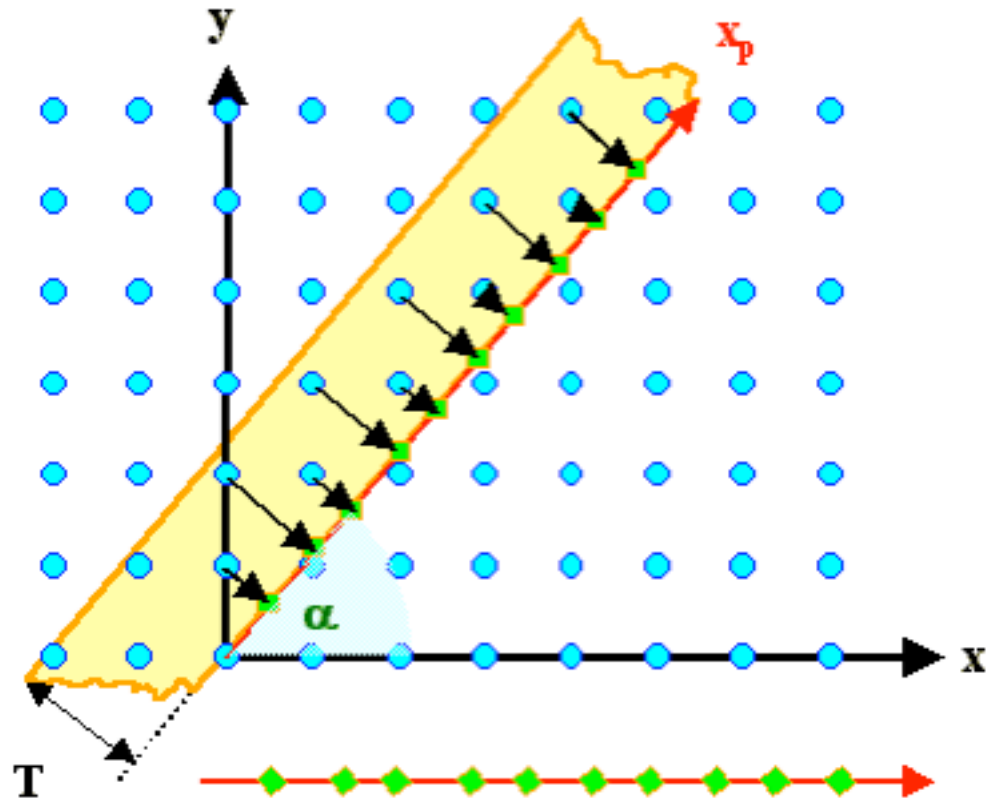


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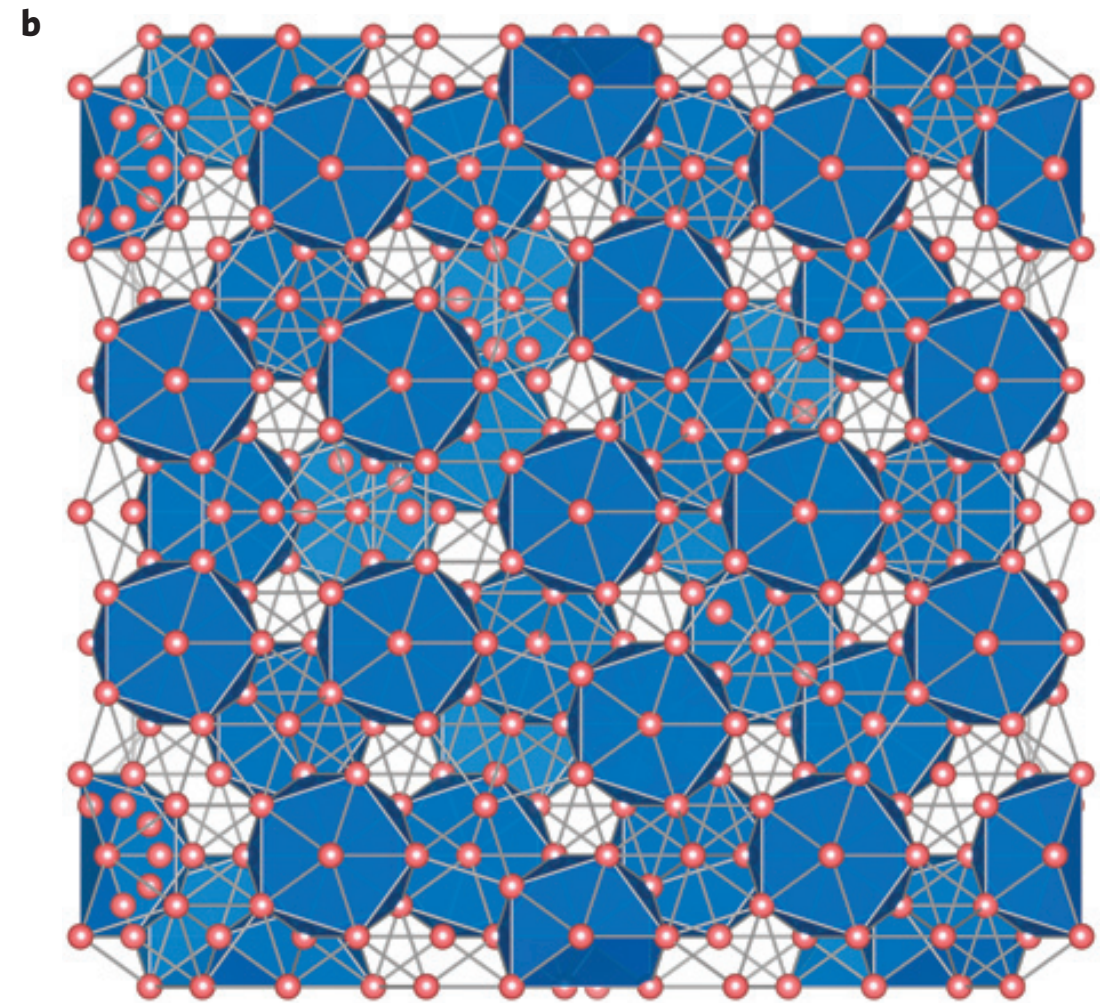
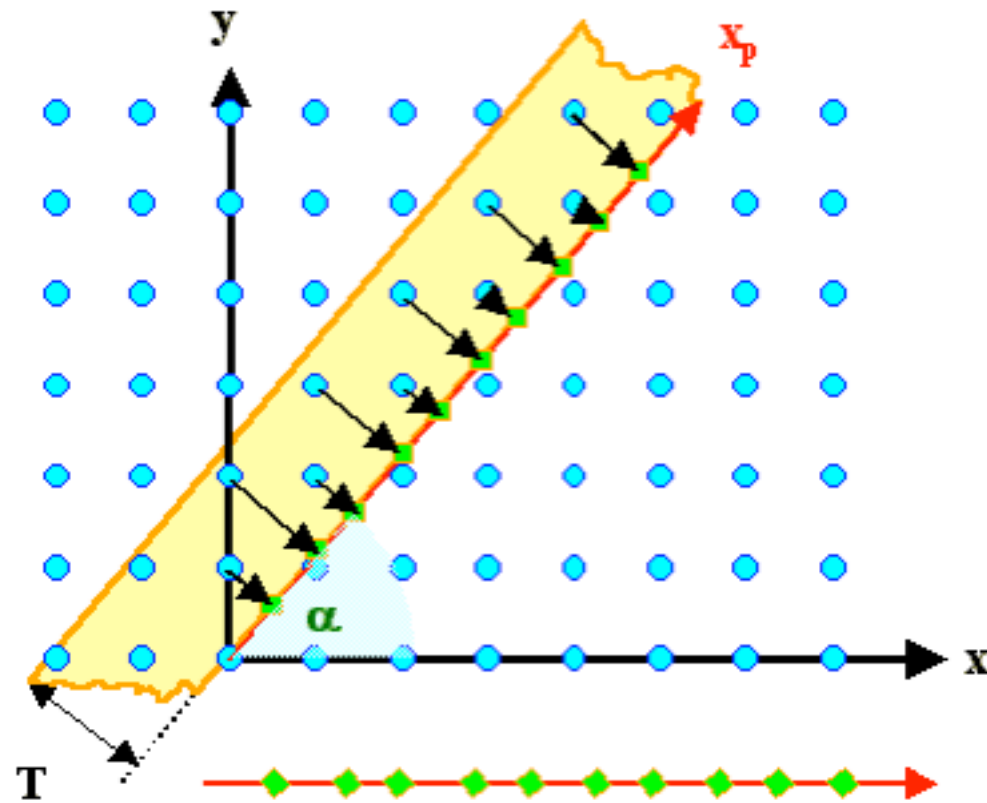




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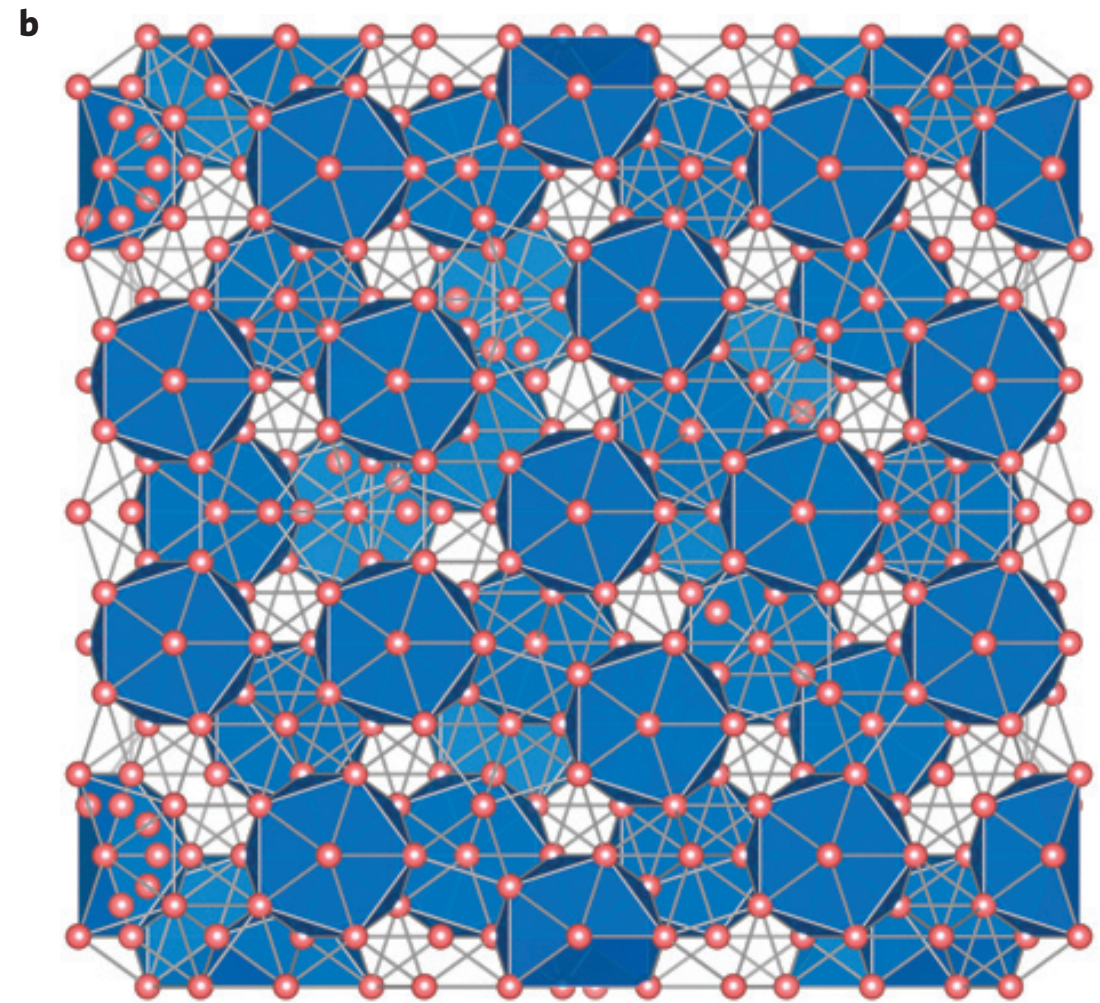
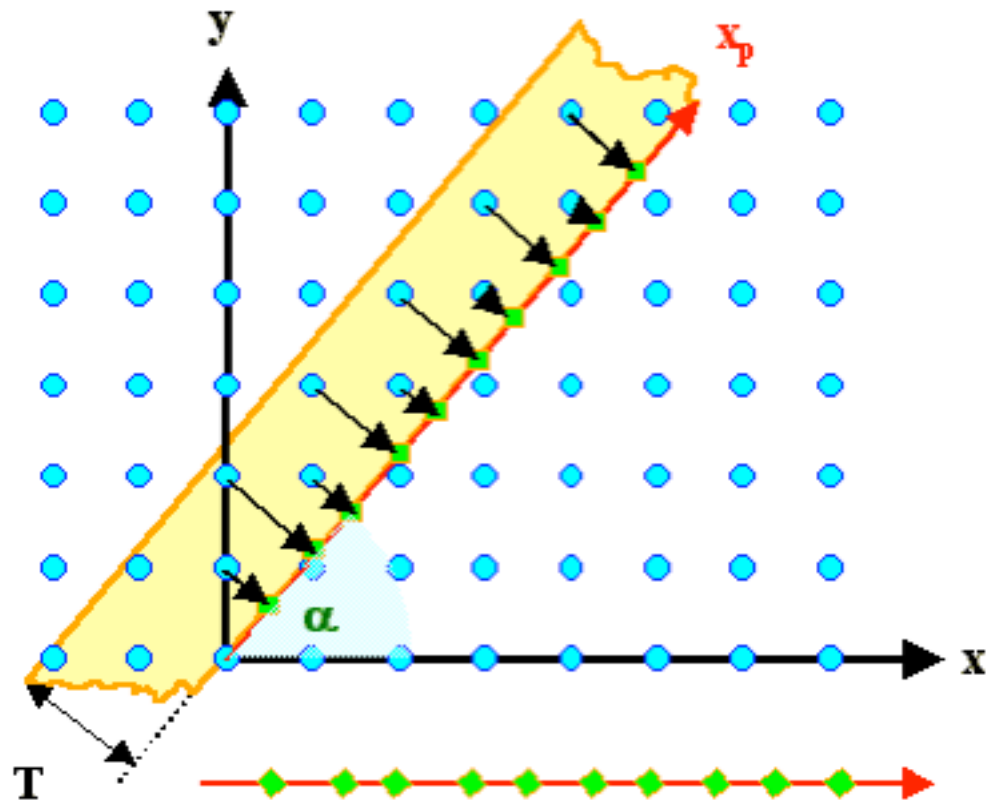
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3- d Qxtal =  
 Surface of 6 d hypercrystal.



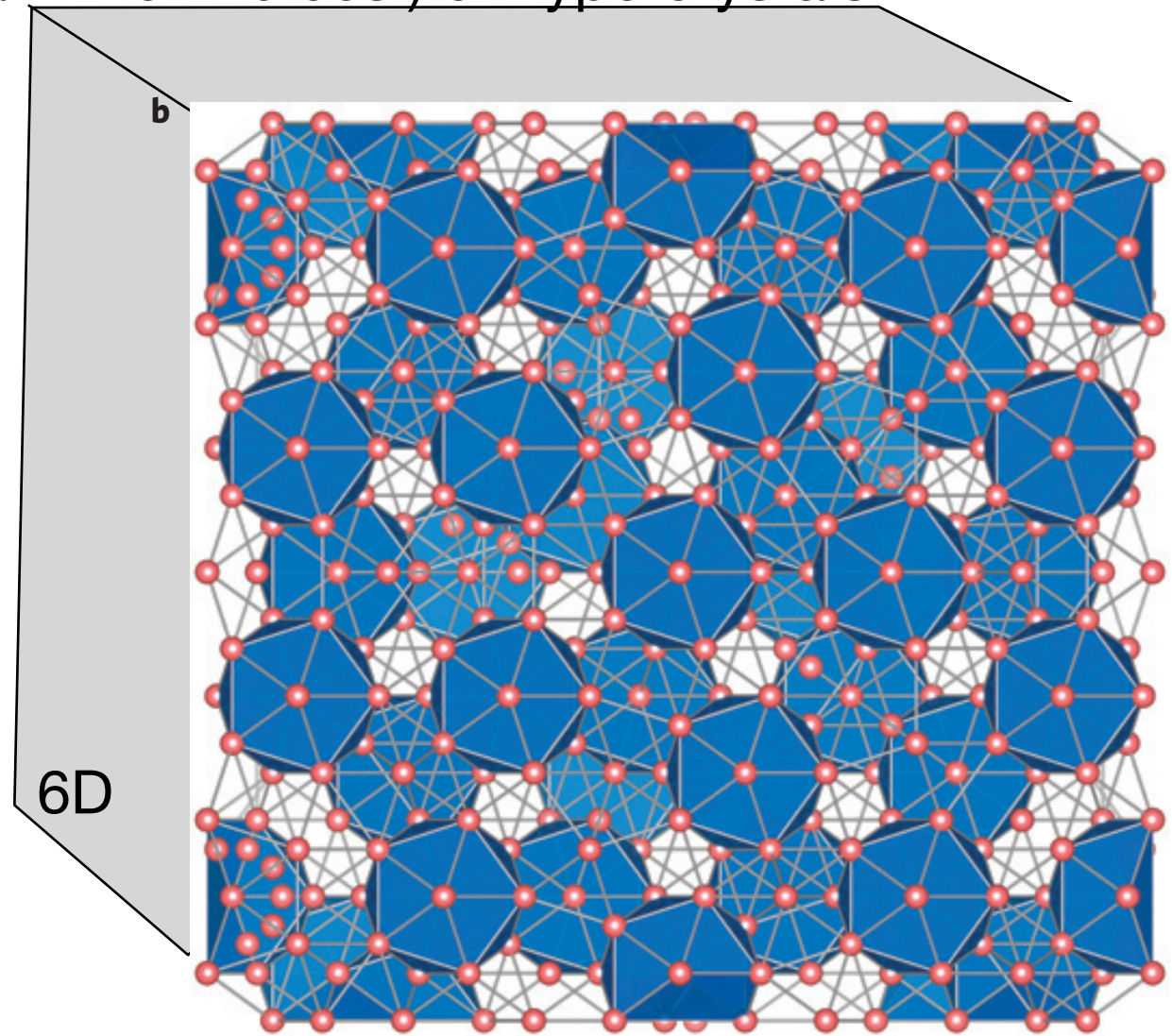
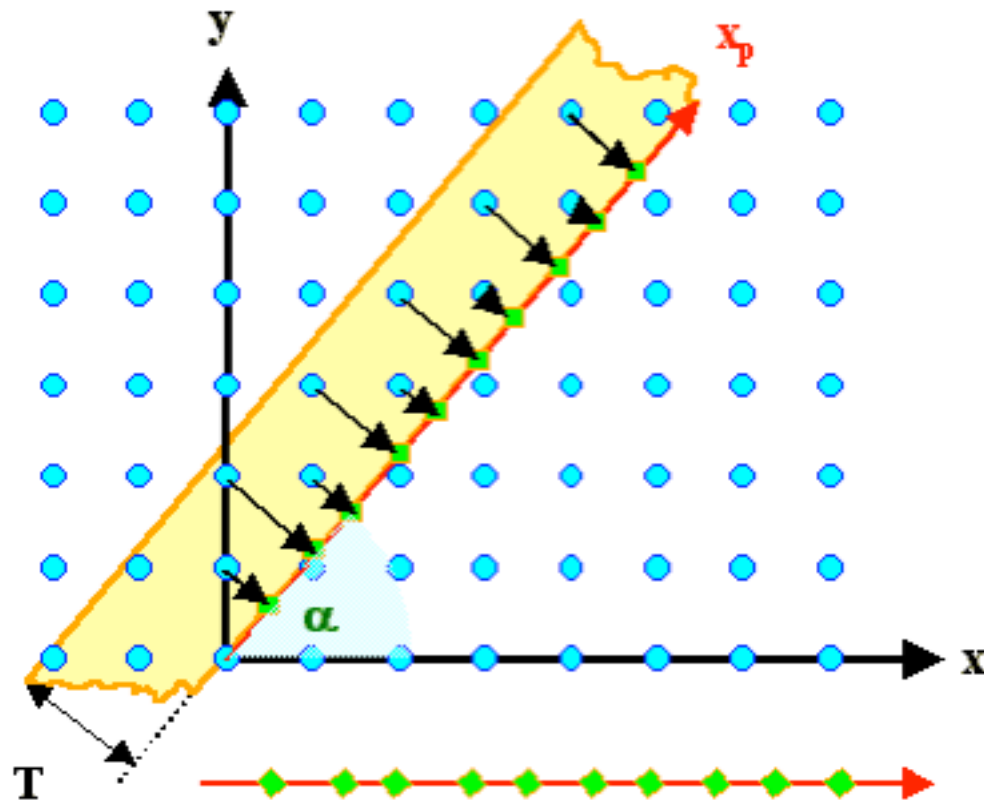
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# Summary & Questions

## SmB<sub>6</sub> :

- Weak localization, dHvA, surface conductance and Arpes, taken together, indicate that this is a topological insulator - moreover, a TKI.
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Thank you!