$\boxed{\text{Axions}}$

Lectures at
Higgs Centre School
of Theoretical Physics

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DESY

Plan:

- Strong CP problem
- Dynamical solution of Strong CP problem
- Laboratory and stellar constraints
- Axion dark matter
- Axion models

Lecture II

Dynamical Solution of Strong CP Problem

- Dynamical solution of Strong CP problems based on the fact:
- Vacuum energy density of QCD, $\mathcal{E}_0(\bar{\theta})$, which can be inferred from vacuum to vacuum amplitude $Z(\bar{\theta})$ by

$$Z(\bar{\theta}) = \exp\Big(-\underbrace{V}_{\text{4 volume of the spacetime}}\mathcal{E}_0(\bar{\theta})\Big)$$

has absolute minimum at $\bar{\theta} = 0$

∘ Reason:

$$Z(\bar{\theta}) = \sum_{\nu=-\infty}^{+\infty} Z_\nu \exp(i\bar{\theta}\nu)$$

positive

Fourier series with positive coefficients

$\Longrightarrow$

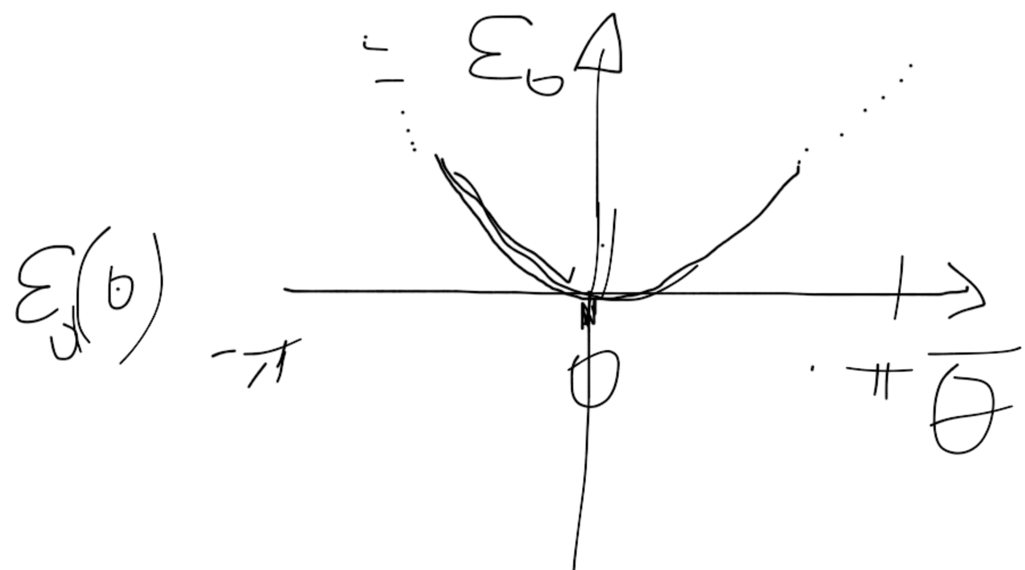
$\mathcal{E}_0(\bar{\theta})$ has Taylor expansion

$$\mathcal{E}_0(\bar{\theta}) = \mathcal{E}_0(0) + \frac{1}{2}\chi\bar{\theta}^2 + O(\bar{\theta}^4)$$

$$\boxed{\chi > 0}$$

χ is the topological susceptibility, i.e. the variance of the topological charge (winding number) distribution.

$$\chi = \frac{d^2\epsilon_0}{d\theta^2}\Big|_{\theta=0} = -\lim_{V\to\infty}\frac{1}{V}\frac{1}{Z}\frac{d^2 Z}{d\theta^2}\Big|_{\theta=0} = \lim_{V\to\infty}\frac{1}{V}\frac{1}{Z}\sum_{\nu=-\infty}^{+\infty}\nu^2 Z_\nu = \lim_{V\to\infty}\frac{\langle\nu^2\rangle|_{\theta=0}}{V}$$



If $\bar{\theta}$ where a dynamical angular field, then its vev would be zero $\langle \theta_A \rangle = 0$ and therefore $d_n \sim \langle \theta_A \rangle \rightsquigarrow 0$.

Moreover, the mass of the associated particle excitation around $\theta_A = 0$ would be related to the topological susceptibility as

$$m_A^2 f_A^2 = \chi$$

where the canonically norm. scalar field A and the angular field θ_A are related by $\theta_A = \frac{A}{f_A}$.

A axion field

com. particle: axion

f_A decay constant

- more progress can be made in chiral limit $M \to 0$, exploiting the fact that
- Z depends on mass M and θ only via

$$\boxed{M \exp\left(i \frac{\theta}{n_f}\right)}$$

follows from anomalous Ward identity of singlet axial current

- For large V and small masses, properties of Z can be analyzed by means of LEFT
- Z dominated by contributions from lightest particles of theory:

Strongly dependent on n_f! and magnitude of m_q!

<u>For $n_f = 1$</u> expect that mass gap persists in chiral limit and lightest particle, $\bar{q}q$, has mass $\gtrsim \Lambda$

$$Z = \exp\left(-V \mathcal{E}_0\left(\underset{\substack{\uparrow \\ \text{quark mass}}}{m}\, e^{i\theta}\right)\right)$$

Since for $n_f = 1$ spectrum of the theory does not contain massless particles in chiral limit:

Perturbation expansion of $\mathcal{E}_0$ in powers of m does not give rise to infrared divergences, such that ordinary Taylor series is obtained:

$$\mathcal{E}_0(me^{i\theta})$$

$$= \mathcal{E}_0(0) - \Sigma \, \mathrm{Re}(me^{i\theta}) + \mathcal{O}(m^2)$$

with Σ related to quark condensate in chiral limit:

$$-\frac{1}{2}\Sigma e^{-i\theta} \equiv -\frac{1}{V}\frac{\partial}{\partial m^*}\ln Z$$

$$= \langle \bar{q}_L q_R \rangle \Big|_{m=0}$$

NB :

For a torus, $V = L_1 L_2 L_3 L_4$, the partition function can be written in terms of a Fourier series[1],

$$Z(\theta) = \sum_{\nu=-\infty}^{+\infty} \exp[i\theta\nu] Z_\nu, \tag{1.4}$$

of Euclidean path integrals over gauge fields with fixed topological charge $\nu = \int d^4x \omega(x)$,

$$Z_\nu = \int_\nu [dG][dq][d\bar{q}] \exp\left[- \int d^4x \left\{ \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + i\bar{q}\gamma_\mu D_\mu q - \bar{q}_R \mathcal{M} q_L - \bar{q}_L \mathcal{M}^\dagger q_R \right\} \right]. \tag{1.5}$$

The latter can be formally further evaluated by performing the integration over the quarks, taking into account the fact that the Euclidean Dirac operator $\gamma_\mu D_\mu$ has $|\nu|$ left-handed (right-handed) zero modes, if the winding number is positive (negative) and that the non-zero eigenvalues occur in pairs $(\lambda_n, -\lambda_n)$, resulting in

$$Z_\nu = \int_\nu [dG] \exp\left[- \int d^4x \left\{ \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} \right\} \right] (\det_f \mathcal{M})^\nu {\prod_n}' \det_f \left(\lambda_n^2 + \mathcal{M}\mathcal{M}^\dagger \right), \tag{1.6}$$

for $\nu > 0$. Here, $\det_f \mathcal{M}$ stands for the determinant of the $n_f \times n_f$ mass matrix $\mathcal{M}$, which with a suitable choice of the quark-field basis, can be brought to a diagonal form with real positive entries $m_u, m_d, \ldots$, while the product occuring on the right hand side only extends over the positive eigenvalues. For $\nu < 0$, the factor $(\det_f \mathcal{M})^\nu$ is to be replaced by $(\det_f \mathcal{M}^\dagger)^{-\nu}$.

For small m:

$$\Sigma^2 = \langle \bar{q} q \rangle^2 + \langle \bar{q} i \gamma_5 q \rangle^2 \Big|_{m=0} + O(m)$$

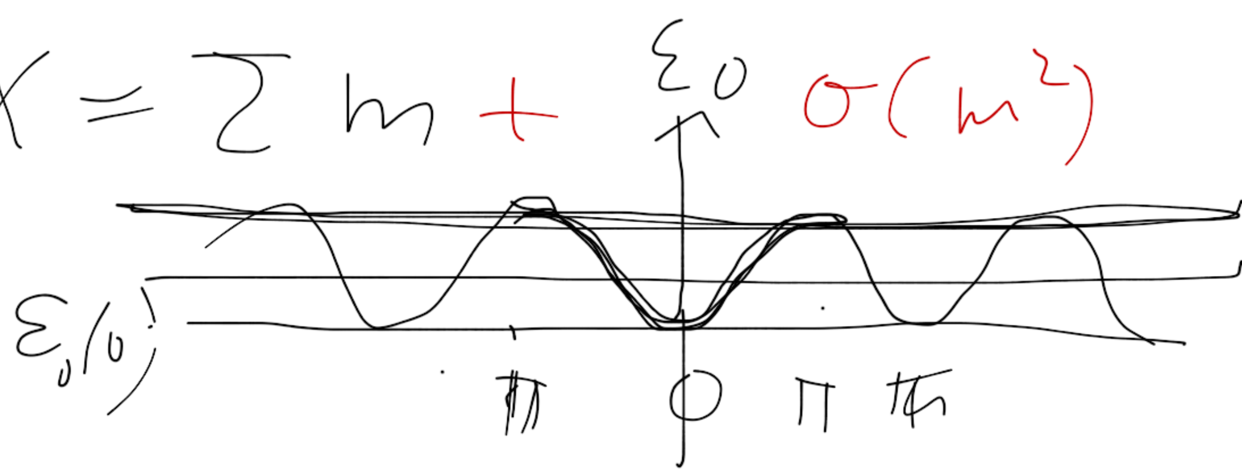
Thus:

$$\mathcal{E}_0(\bar{\theta}) = \mathcal{E}_0(0) - \Sigma m \cos\bar{\theta} + O(m^2)$$

in basis where m real and positive

Topological susceptibility:

$$\chi = \Sigma m + O(m^2)$$



- Generalization to realistic case $N_f \geq 2$ possible (→ see Leutwyler + Smilga) in Tutorial

$$Z = A \int_{SU(N_f)} d\mu(U_0) \times \exp\left(V \Sigma \, \mathrm{Re}\{\mathrm{tr}(\mathcal{M} U_0^\dagger) \exp\left(i \frac{\theta}{N_f}\right)\}\right)$$

appropriate repr. for large volume! Since lighter particles are Goldstone bosons with masses $\ll 1$!

Results :

$$\mathcal{E}_0(\bar{\theta}) = \mathcal{E}_0(0) - \Sigma \left(m_u^2 + m_d^2 + 2 m_u m_d \cos\bar{\theta} \right)^{1/2}$$

$$\Longrightarrow$$

$$\chi = \Sigma \frac{m_u m_d}{m_u + m_d} + \mathcal{O}(m^2)$$

↑
reduced
quark mass

$$= m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2} + \mathcal{O}(m^2)$$

Dynamic solution of Strong CP problem:

Assume existence of scalar field $\theta_A(x) = \frac{A(x)}{f_A}$,

$\theta_A \in (-\pi, \pi)$, which enjoys a continuous shift Symmetry $\theta_A \to \theta_A + \text{const.}$

(that is acts Goldstone field of the spontaneous breaking of a $U(1)$ symmetry) and couples to the topological charge density of QCD like the θ - parameter:

$$S \supset \frac{1}{16\pi^2} \int d^4x \underset{A}{\theta(x)} \, \mathrm{tr}\, G_{\mu\nu} {}^*G^{\mu\nu}$$

notation to distinguish from em gauge fields

Then we can eliminate the $\bar{\theta}$ parameter by shift

$$\theta_A \rightarrow \theta_A - \bar{\theta}$$

The shifted field will have a vev 0 - this strong CP problem solved - and a mass

$$m_A^2 = \frac{m_\pi^2 f_\pi^2}{f_A} \frac{m_u m_d}{(m_u + m_d)^2}$$

Numerically:

$$m_A \simeq 6 \text{ meV} \left(\frac{10^9 \text{ GeV}}{f_A} \right)$$

NNLO chiral perturbation theory + $\mathcal{O}(\alpha_{em})$ corrections: [Gorghetto, Villadoro]

$$\chi^{1/4} = 75.44(34) \text{ MeV}$$

$$m_A = 5.691(51) \text{ meV} \left(\frac{10^9 \text{ GeV}}{f_A} \right)$$

Best lattice results (include isospin break., no α_{em}!)

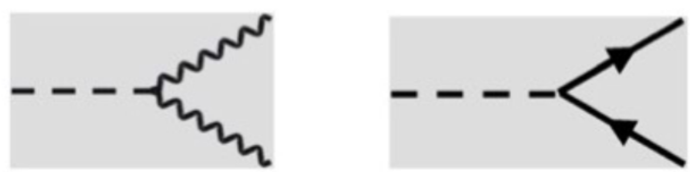
$$\chi^{1/4} = 75.6(1.8)(0.9) \text{ MeV}$$

[Borsanyi et al. 16] stat. system.

Strong CP problem solved for any value of f_A !

Axion couplings to SM at energies below QCD scale:

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu A \partial^\mu A - \frac{1}{2} m_A^2 A^2 - \frac{\alpha}{8\pi}\frac{C_{A\gamma}}{f_A} A\, F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{1}{2}\frac{C_{Af}}{f_A}\partial_\mu A\, \overline{\psi}_f \gamma^\mu \gamma_5 \psi_f$$



- Couplings of axion to SM suppressed by inverse powers of f_A
- LEFT valid for energies $\ll f_A$!

- Couplings of axion to SM suppressed by powers of

$$f_A = v_{\mathrm{PQ}}/N \gg v = 246\,\mathrm{GeV}$$

rendering the axion „invisible“

[Kim 79;Shifman,Vainshtein,Zakharov 80;Zhitnit
80;Dine,Fischler,Srednicki 81;...]

- Photon coupling: $C_{A\gamma} = \frac{E}{N} - 1.92(4)$

[Kaplan 85;Srednicki `8

- Nucleon couplings:

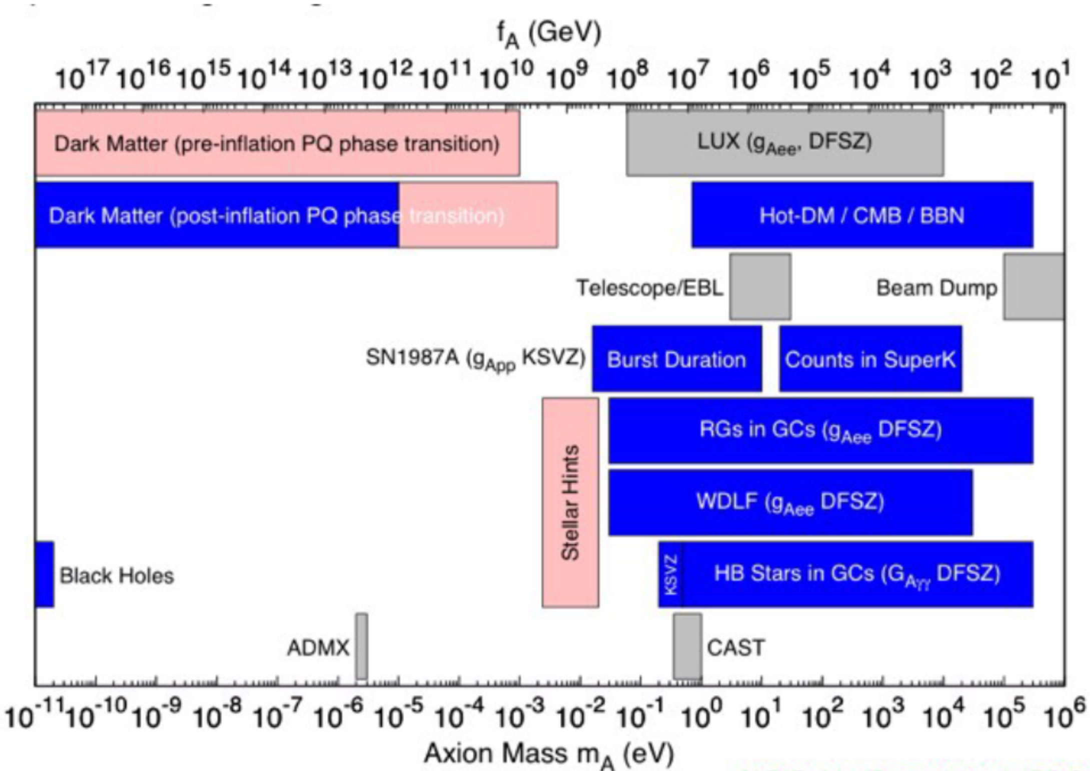
[Grilli di Cortona et al.

$$\begin{aligned} C_{Ap} &= -0.47(3) + 0.88(3)C_{Au} - 0.39(2)C_{Ad} - 0.038(5)C_{As} \\ &\qquad - 0.012(5)C_{Ac} - 0.009(2)C_{Ab} - 0.0035(4)C_{At}\,, \\ C_{An} &= -0.02(3) + 0.88(3)C_{Ad} - 0.39(2)C_{Au} - 0.038(5)C_{As} \\ &\qquad - 0.012(5)C_{Ac} - 0.009(2)C_{Ab} - 0.0035(4)C_{At} \end{aligned}$$

- Electron coupling very model-dependent

- Photon coupling and nucleon coupling are model independently from axion pion mixing!!
- Electron coupling model dependent!

Current constraints:



[AR,Rybka,Rosenberg in PDG RPP 2018]

- Exercises Lecture II

Study

PHYSICAL REVIEW D VOLUME 46, NUMBER 12 15 DECEMBER 1992

Spectrum of Dirac operator and role of winding number in QCD

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(Received 4 August 1992)

We show that very general considerations based on the properties of the partition function of QCD allow one to extract information about the eigenvalues of the Dirac operator in vacuum gauge fields. In particular, we demonstrate that the familiar suppression of field configurations with a nontrivial topology occurring for small quark masses is a finite size effect which disappears if the four-dimensional volume V is large enough. The formation of a quark condensate is connected with the occurrence of small eigenvalues of order $\lambda_n \propto 1/V$.

PACS number(s): 11.15.Tk, 11.30.Qc, 11.30.Rd, 12.38.Aw

- Verify (4.1) – (4.4)

- Verify (9.6)