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How to make an invisibility cloak

Work done with Courtial, Oxburgh, Antoniou, Orife, Mertens, Mullen

New Directions in Theoretical Physics

Overview

- Transformation optics: metamaterials.
- Transformation optics: windows.
- Generalised Confocal Lenslet Arrays (gCLAs).
- Invisibility cloak designs.
- General transformation optics of gCLAs.

Transformation optics

The curved space Maxwell equations are equivalent to flat space equations in a nontrivial medium.

- Example:

$$\underbrace{\frac{1}{\sqrt{g}}\partial_i\left(\sqrt{g}g^{ij}E_j\right)=0}_{\text{Gauss' Law in curved space}} \qquad \underbrace{\partial_i\left(\epsilon_0\epsilon^{ij}E_j\right)=0}_{\text{Gauss' Law in dielectric (permittivity } \epsilon^{ij})}$$

  These are the same if $g^{ij} \propto \epsilon^{ij}$.

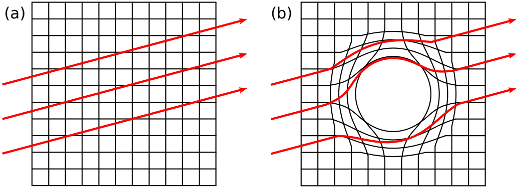
- Consistent with other Maxwell equations provided $\epsilon^{ij}=\mu^{ij}$ ("impedance matched medium" - no reflections).

Transformation optics

- A material with spatially varying optical properties can be described by a metric tensor: *physical space* (Pendry, Schurig, Smith).
- If the Riemann curvature vanishes, this metric is a coordinate transformation of a flat Cartesian empty space: *electromagnetic space*.
- Materials that do this are called *transformation media*.
- Useful for two reasons:
  1. Can analyse non-trivial media by transforming vacuum solutions to Maxwell's equations.
  2. Can design novel materials based on desired optical properties.

Cloaking

- Flagship application: invisibility cloaks.



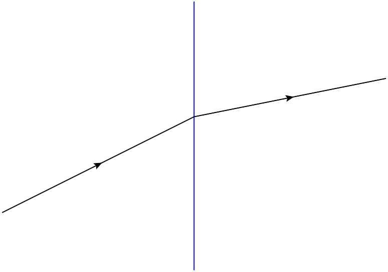
- A point in electromagnetic space is blown up to a finite region.
- Space is Cartesian-like at large distances: contents of central region becomes invisible!

Metamaterials

- Transformation optical (TO) devices can be realised with *metamaterials*.
- Periodic structures, whose cells are much smaller than the wavelength of the light.
- Whilst exciting and interesting, there are a number of problems:
  (i) Can be limited to a single wavelength, typically in the microwave range.
  (ii) Can be limited to a single polarisation.
  (iii) Difficult and expensive to make large amounts of metamaterial.
- Suggests an alternative approach could be useful.

TO using windows

- Consider a two-dimensional window with an exotic refraction law:



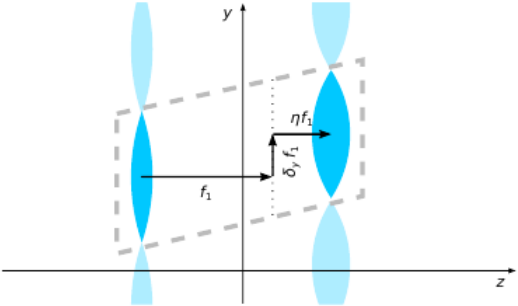
- Subject to the right properties, one can make TO devices by stacking windows together.
- Need a flexible enough refraction law.
- Also, the windows need to be *perfectly imaging*.

TO and Imaging

- An optical component (lens, window, ...) is *perfectly imaging* if an intersecting bundle of rays in object space also intersects in image space.
- In TO language, this means that the rays must intersect in both the electromagnetic and physical spaces.
- This is necessary for TO, because the physical space must be a local coordinate transformation of the electromagnetic space.
- Local coordinate transformations are perfectly imaging as defined here.

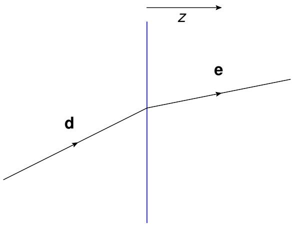
Lenslet arrays

- The most general homogeneous, planar imaging sheet has been described by Oxburgh, Courtial.
- An approximate realisation of such a sheet is provided by *generalised confocal lenslet arrays* (gCLAs).
- Two arrays of small (w.r.t. wavelength) lenses, which form “telescopelets”:



gCLAs

- The second array of lenses may be offset w.r.t. the first.
- Also, the lenses may be rotated w.r.t. the gCLA plane.
- Novel refraction law, with seven degrees of freedom (Hamilton, Courtial; Oxburgh, White, Antoniou, Courtial).



- Take lenses to be in (u, v) plane, with a direction $\perp$ to lens.
- Can then write refraction law as:

$$e_u = \frac{d_u/d_a - \delta_u}{\eta_u}, \quad e_v = \frac{d_v/d_a - \delta_v}{\eta_v}, \quad e_a = d_a.$$

- There are a further 3 rotation angles back to (x, y, z) system.

Refraction with gCLAs

- The refraction law includes rotations, scalings (η_u, η_v) and offsets (δ_u, δ_v).
- This is possible due to the pixellated nature of the gCLAs (c.f. metamaterials).
- The refraction is exotic in that it can create light configurations that appear to be wave-optically forbidden.
- Also perfectly imaging: object positions **P** and image positions **P**′ related by e.g.

$$\mathbf{P}' = \mathbf{P} - z \begin{pmatrix} \delta_x \\ \delta_y \\ 1 - \eta \end{pmatrix}$$

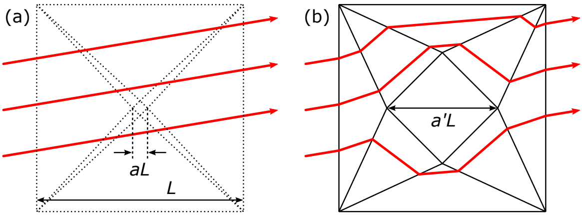
(here have taken $(u, v) = (x, y)$).

Why use gCLAs?

- Using gCLAs has a number of benefits:
  1. They will work for a range of wavelengths (incl. visible range).
  2. Independent of polarisation.
  3. Can make large devices ($\mathcal{O}(10^0)$ m).
  4. Potentially lightweight, durable and cheap (similar arrays are used in 3D postcards).
  5. Windows can do things 3D metamaterials cannot.
- Disadvantages include:
  1. Limited field of view.
  2. Limitations to quality of view.
- However, precision optical engineering has yet to be carried out.

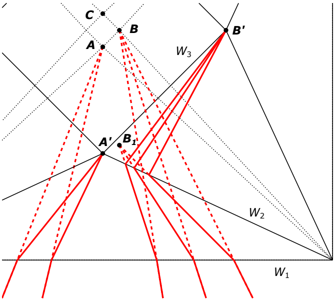
Building a cloak

- It is possible to build cloaks with gCLAs (Oxburgh, White, Antoniou, Orife, Courtial).
- Similar to birefringent crystal designs (Chen, Zheng).



- Small square in e.m. space $\rightarrow$ larger square in physical space.
- Can calculate parameters of each interface in terms of a, a' (taking $L = 1$).

Cloaking with gCLAs



- Interface W_1 images A to A'.
- B imaged to B_1 by W_1, then $B_1 \to B'$ by W_2.
- Then get

$$\eta_1 = \frac{a'}{a}, \quad \delta_{x,1} = \delta_{y,1} = 0.$$

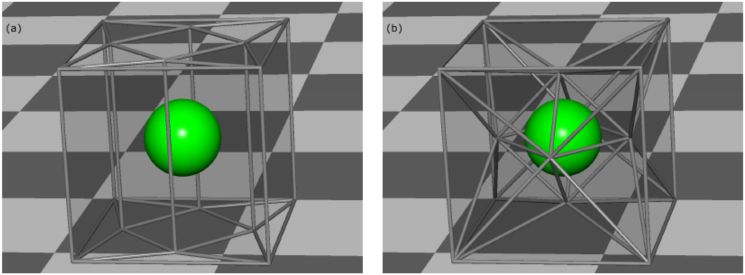
- For interface W_2:

$$\eta_2 = \frac{a(4a'^2 - 1)}{a'(4a^2 - 1)}, \quad \delta_{x,2} = \frac{2(a^2 - 2aa' + a'^2)}{a'(4a^2 - 1)}, \quad \delta_{y,2} = 0.$$

- Practically achievable for useful values of a, a'.

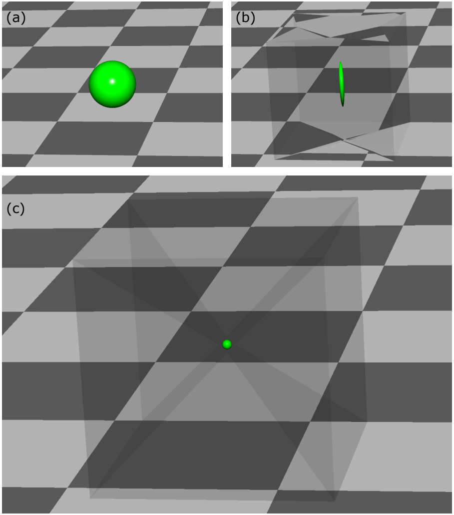
Ray tracing simulations

- Further support for the feasibility of gCLA cloaking comes from computer simulation.
- We implemented the relevant gCLA interfaces in the raytracer `Dr TIM` (Oxburgh, Tyc, Courtial).
- Considered two different 3D cloak designs:



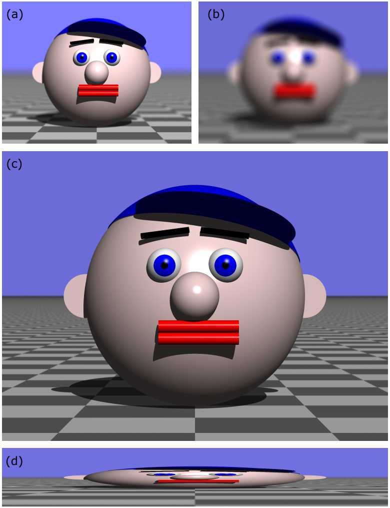
- How does the sphere look from outside the cloak?

Results



- Have chosen $a'/a = 10$.
- The central region of the cloak appears shrunken, according to the topology of the cloak.
- Proof of principle of the design.
- Ideal cloak corresponds to $a'/a \to \infty$.

Results



- Can also sit inside the cloak, and look at `TIM` (outside).
- The octahedral cloak scales all space dimensions.
- `TIM` becomes $a'/a = 10$ times bigger, but 10 times further away!
- Similar results for square prismatic cloak (lower panel).

General transformation optics of gCLAs

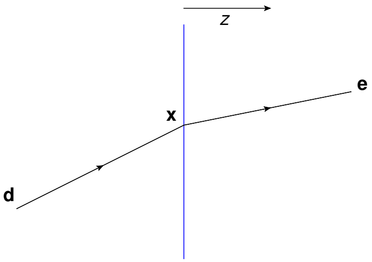
- Cloaks are only one application of transformation windows.
- We foresee many other applications.
- To look for them, the transformation optics language is particularly useful.
- A gCLA transforms the space behind it so that it looks like an homogeneous medium.
- Can then ask: what metric tensor describes the passage of light rays in this medium?
- Can solve for this using Fermat's principle, given that we know the law of refraction.

Fermat's Principle

- The optical path length

$$s = \sqrt{(\mathbf{d}-\mathbf{x})^T\mathbf{g}(\mathbf{d}-\mathbf{x})} + \sqrt{(\mathbf{e}+\mathbf{x})^T\mathbf{h}(\mathbf{e}+\mathbf{x})}$$

must be extremised.



- That is,

$$\left.\frac{\partial s}{\partial x}\right|_{x=y=0} = \left.\frac{\partial s}{\partial y}\right|_{x=y=0} = 0.$$

- Here **g** and **h** are metric tensors on either side of the window, which include the optical properties of the space.

A metric for gCLAs

- Recall the refraction law for gCLA (e.g. for unrotated lenses, and $\eta_x = \eta_y$):

$$e_x = \frac{d_x/d_z - \delta_x}{\eta}, \quad e_y = \frac{d_y/d_z - \delta_y}{\eta}, \quad e_z = d_z.$$

- We are free to choose $e_z = d_z = 1$, so that we can write the law as

$$\mathbf{e} = \mathbf{N}\mathbf{d},$$

with

$$\mathbf{N} = \begin{pmatrix} \frac{1}{\eta} & 0 & -\frac{\delta_x}{\eta} \\ 0 & \frac{1}{\eta} & -\frac{\delta_y}{\eta} \\ 0 & 0 & 1 \end{pmatrix}.$$

- Can substitute this into Fermat's principle, and solve for $\mathbf{h}$ in terms of $\mathbf{g}$.

A metric for gCLAs

- The solution is

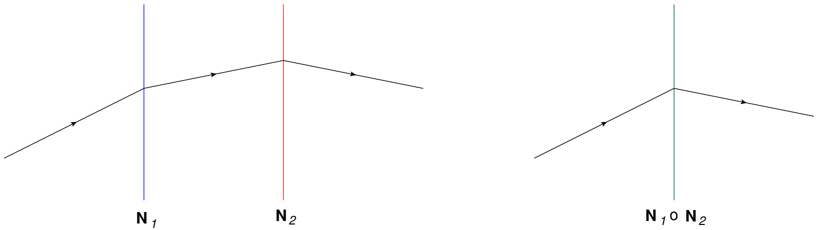
$$\mathbf{h} = \frac{(\mathbf{N}^{-1})^T \, \mathbf{g} \, \mathbf{N}^{-1}}{\det(\mathbf{N}^{-1})}.$$

- Makes sense: the gCLA performs a similarity transformation of the metric.
- Normalisation factor due to the fact that volume element has scaled (due to η).
- Example of gCLA applied to air:

$$\mathbf{g} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{h} = \begin{pmatrix} 1 & 0 & \frac{\delta_x}{\eta} \\ 0 & 1 & \frac{\delta_y}{\eta} \\ \frac{\delta_x}{\eta} & \frac{\delta_y}{\eta} & \frac{1+\delta_x^2+\delta_y^2}{\eta^2} \end{pmatrix}.$$

A metric for gCLAs

- Similar solutions apply if one includes rotations of the window and / or lenses.
- The solutions have a group theoretic structure, as follows from the TO language:



- Elegant way of analysing possible applications.

Engineering issues

- The limitations of gCLAs have been explored by Maceina, Juzeliūnas, Courtial.
- There is plenty of scope, however, for precision optical engineering.
- Typically, lens arrays are produced by indenting stainless steel with sapphire to make a mould.
- Limitations to surface quality, and achievable lens shape.
- Instead could use high-speed (and high-precision) diamond micro-milling techniques (Girkin, Love, Robertson).
- Further investigation being carried out.

Conclusions

- Transformation optics is a highly active field, usually involving metamaterials.
- By using *windows*, one can significantly extend possible applications.
- Generalised confocal lenslet arrays (gCLAs) realise the most general, homogeneous, fully imaging refraction law.
- Can be used to construct an invisibility cloak.
- Precision optical engineering underway.
- Many possible applications.