Ghost Free & Singularity Free Theory of Gravity

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Phys. Rev. Lett. (2012), JCAP (2012, 2011), JCAP (2006)
CQG (2013)

Einstein's GR is well behaved in IR, but UV is Pathetic; Aim is to address the UV aspects of Gravity

Motivations

- Resolution to Blackhole Singularity
- Resolution for Quantum Mechanics & Gravity
  Blackhole Information Loss Paradox
- Resolution to Cosmological Big Bang Singularity
  Geodesically complete Inflationary Trajectory

While Keeping IR Property of GR Intact

I wish I were a Magician!

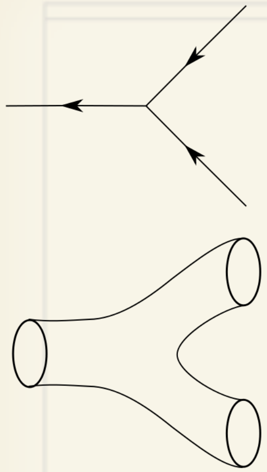
$$S = \int d^4x \, \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box} \right] R - 2R_{\mu\nu} \left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box} \right] R^{\mu\nu} \right]$$

Absence of Cosmological and Blackhole Singularities

Conjecture : The Form of Most General Action

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + \alpha_0(R, R_{\mu\nu}) + \alpha_1(R, R_{\mu\nu}) R \mathcal{F}_1(\Box) R \right.$$
$$\left. + \alpha_2(R, R_{\mu\nu}) R_{\mu\nu} \mathcal{F}_2(\Box) R^{\mu\nu} + \alpha_3(R, R_{\mu\nu}) C_{\mu\nu\lambda\sigma} \mathcal{F}_3 C^{\mu\nu\lambda\sigma} \right]$$

Facts



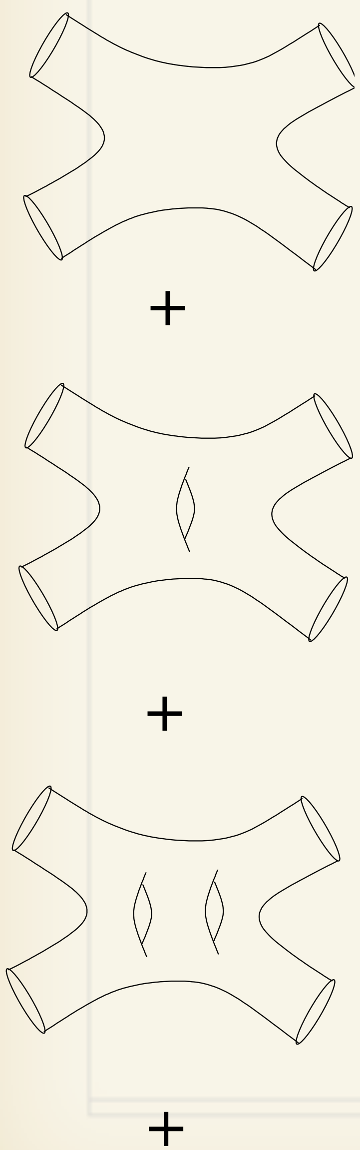
- **String Theory Introduces 2 Parameters**

$$\kappa^2 \approx g_s^2(\alpha')^{12}$$

- **Fundamental Strings are Non-Local**
- **DBI action ameliorates the Point like Singularity of Coulomb Solution**

$$S = -T_p \int d^{p+1}\zeta \sqrt{-\det(\gamma_{ab} + 2\pi\alpha' F_{ab})}$$

- **DBI Action Provides a Description of Open Strings to All Orders in α' at One-Loop**



Challenge for String Theorists:

To Construct a similar Action for Closed Strings with All Orders in α'

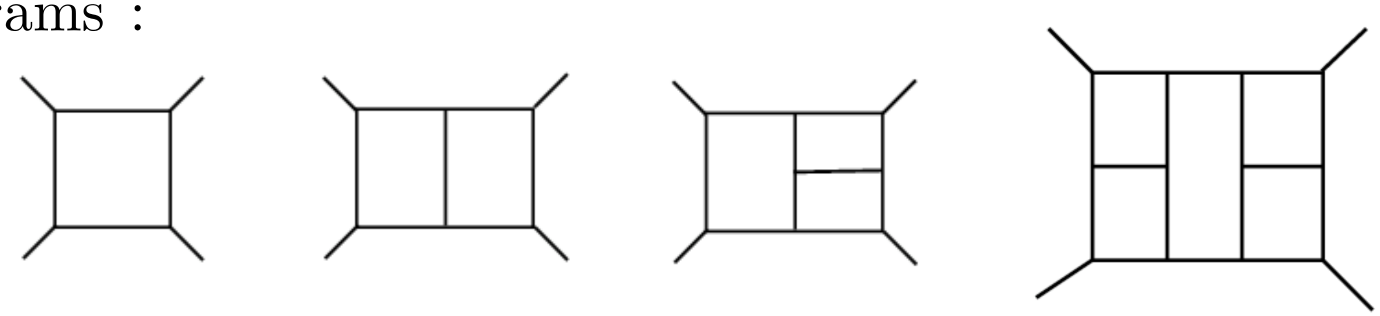
Perturbative Quantum Gravity

Pure Gravity is 1-Loop Renormalizable !

Pure Gravity requires 1 new counter term @ 2 Loops !

Superficial degree of divergence of a Feynman diagram

Loops diagrams :



$D = L\,d + 2V - 2I,$

L : number of loops,

V : number of vertices,

I : number of internal lines in the graph.

Topological relation between V, I and L, $L = 1 + I - V$,

$$D = 2 + (d-2)L. \quad \text{For } d = 4 \;\rightarrow\; D = 2 + 2L$$

GR Propagator in 4 Dimensions:

$$\Pi = \frac{P^2}{k^2} - \frac{P_s^0}{2k^2}$$

4th order Gravity is Renormalizable !

$$I = \int d^4x \sqrt{g} \left[\lambda_0 + k\, R + a\, R_{\mu\nu} R^{\mu\nu} - \tfrac{1}{3}\,(b + a) R^2\right]$$

$$D \propto \frac{1}{k^4 + A k^2} = \frac{1}{A}\left(\frac{1}{k^2} - \frac{1}{k^2 + A}\right)$$

Massive Spin-0 & Massive Spin-2 (Ghost) Stelle (1977)

Utiyama, De Witt (1961), Stelle (1977)

Modification of Einstein's GR

Modification of Graviton Propagator

Extra propagating degree of freedom

Challenge: to get rid of the extra dof

Ghosts

Higher Order Derivative Theory Generically Carry Ghosts (-ve Risidue) with real "m"(No-Tachyon)

$$S = \int d^4x \; \phi\Box(\Box + m^2)\phi \Rightarrow \Box(\Box + m^2)\phi = 0$$

$$\Delta(p^2) = \frac{1}{p^2(p^2+m^2)} \sim \frac{1}{p^2} - \frac{1}{(p^2+m^2)}$$

Propagator with first order poles

Ghosts cannot be cured order by order, finite terms in perturbative expansion will always lead to Ghosts !!

Higher Derivative Action around Minkowski

$$S = S_E + S_q$$

$$S_q = \int d^4x\sqrt{-g}\,[R_{....}\mathcal{O}^{....}_{....}R^{....} + R_{....}\mathcal{O}^{....}_{....}R^{....}\mathcal{O}^{....}_{....}R^{....} + R_{....}\mathcal{O}^{....}_{....}R^{....}\mathcal{O}^{....}_{....}R^{....}\mathcal{O}^{....}_{....}R^{....} + \cdots]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad R \sim \mathcal{O}(h)$$

$$S_q = \int d^4x\sqrt{-g}R_{\mu_1\nu_1\lambda_1\sigma_1}\mathcal{O}^{\mu_1\nu_1\lambda_1\sigma_1}_{\mu_2\nu_2\lambda_2\sigma_2}R^{\mu_2\nu_2\lambda_2\sigma_2}$$

Covariant derivatives

Unknown Infinite Functions of Derivatives

$$\infty \longrightarrow \infty$$

$$
\begin{aligned}
S_q &= \int d^4x\sqrt{-g}[RF_1(\Box)R + RF_2(\Box)\nabla_\mu\nabla_\nu R^{\mu\nu} + R_{\mu\nu}F_3(\Box)R^{\mu\nu} + R_\mu^\nu F_4(\Box)\nabla_\nu\nabla_\lambda R^{\mu\lambda} \\
&+ R^{\lambda\sigma}F_5(\Box)\nabla_\mu\nabla_\sigma\nabla_\nu\nabla_\lambda R^{\mu\nu} + RF_6(\Box)\nabla_\mu\nabla_\nu\nabla_\lambda\nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\lambda}F_7(\Box)\nabla_\nu\nabla_\sigma R^{\mu\nu\lambda\sigma} \\
&+ R_\lambda^\rho F_8(\Box)\nabla_\mu\nabla_\sigma\nabla_\nu\nabla_\rho R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1}F_9(\Box)\nabla_{\mu_1}\nabla_{\nu_1}\nabla_\mu\nabla_\nu\nabla_\lambda\nabla_\sigma R^{\mu\nu\lambda\sigma} \\
&+ R_{\mu\nu\lambda\sigma}F_{10}(\Box)R^{\mu\nu\lambda\sigma} + R^\rho_{\mu\nu\lambda}F_{11}(\Box)\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\rho_1\nu\sigma_1}F_{12}(\Box)\nabla^{\rho_1}\nabla^{\sigma_1}\nabla_\rho\nabla_\sigma R^{\mu\rho\nu\sigma} \\
&+ R_\mu^{\nu_1\rho_1\sigma_1}F_{13}(\Box)\nabla_{\rho_1}\nabla_{\sigma_1}\nabla_{\nu_1}\nabla_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1\rho_1\sigma_1}F_{14}(\Box)\nabla_{\rho_1}\nabla_{\sigma_1}\nabla_{\nu_1}\nabla_{\mu_1}\nabla_\mu\nabla_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma}
\end{aligned}
$$

$$F_i(\Box) = \sum_{n\geq 0} f_{i,n}\Box^n$$

What Have We Gained ?

Fundamental Theory Must have Finite Parameters

Redundancies

$$
\begin{aligned}
S_q \;=\; & \int d^4x\sqrt{-g}[RF_1(\Box)R + RF_2(\Box)\nabla_\mu\nabla_\nu R^{\mu\nu} + R_{\mu\nu}F_3(\Box)R^{\mu\nu} + R^\nu_\mu F_4(\Box)\nabla_\nu\nabla_\lambda R^{\mu\lambda} \\
&+ R^{\lambda\sigma}F_5(\Box)\nabla_\mu\nabla_\sigma\nabla_\nu\nabla_\lambda R^{\mu\nu} + RF_6(\Box)\nabla_\mu\nabla_\nu\nabla_\lambda\nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\lambda}F_7(\Box)\nabla_\nu\nabla_\sigma R^{\mu\nu\lambda\sigma} \\
&+ R^\rho_\lambda F_8(\Box)\nabla_\mu\nabla_\sigma\nabla_\nu\nabla_\rho R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1}F_9(\Box)\nabla_{\mu_1}\nabla_{\nu_1}\nabla_\mu\nabla_\nu\nabla_\lambda\nabla_\sigma R^{\mu\nu\lambda\sigma} \\
&+ R_{\mu\nu\lambda\sigma}F_{10}(\Box)R^{\mu\nu\lambda\sigma} + R^\rho_{\mu\nu\lambda}F_{11}(\Box)\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\rho_1\nu\sigma_1}F_{12}(\Box)\nabla^{\rho_1}\nabla^{\sigma_1}\nabla_\rho\nabla_\sigma R^{\mu\rho\nu\sigma} \\
&+ R^{\nu_1\rho_1\sigma_1}_\mu F_{13}(\Box)\nabla_{\rho_1}\nabla_{\sigma_1}\nabla_{\nu_1}\nabla_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1\rho_1\sigma_1}F_{14}(\Box)\nabla_{\rho_1}\nabla_{\sigma_1}\nabla_{\nu_1}\nabla_{\mu_1}\nabla_\mu\nabla_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma}
\end{aligned}
$$

$$
= \int d^4x\sqrt{-g}\left[R + R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\Box)R^{\mu\nu\alpha\beta}\right]
$$

$$
\Delta\mathcal{L} = \sqrt{-g}\,(\alpha R^2 + \beta R^2_{\mu\nu} + \gamma R^2_{\alpha\beta\mu\nu})
$$

$$
\int \mathrm{d}^4x\sqrt{-g}(R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\alpha\beta})
$$

Gauss-Bonet Gravity

$$= \int d^4x \sqrt{-g}\left[R + R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\Box)R^{\mu\nu\alpha\beta}\right]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$S_q = -\int d^4x \Big[\frac{1}{2}h_{\mu\nu}a(\Box)\Box h^{\mu\nu} + h_\mu^\sigma b(\Box)\partial_\sigma\partial_\nu h^{\mu\nu} \qquad (3)$$

$$+ hc(\Box)\partial_\mu\partial_\nu h^{\mu\nu} + \frac{1}{2}hd(\Box)\Box h + h^{\lambda\sigma}\frac{f(\Box)}{\Box}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\mu\nu}\Big]$$

$$\begin{aligned}
a(\Box) &= 1 - \frac{1}{2}\mathcal{F}_2(\Box)\Box - 2\mathcal{F}_3(\Box)\Box \\
b(\Box) &= -1 + \frac{1}{2}\mathcal{F}_2(\Box)\Box + 2\mathcal{F}_3(\Box)\Box \\
c(\Box) &= 1 + 2\mathcal{F}_1(\Box)\Box + \frac{1}{2}\mathcal{F}_2(\Box)\Box \\
d(\Box) &= -1 - 2\mathcal{F}_1(\Box)\Box - \frac{1}{2}\mathcal{F}_2(\Box)\Box \\
f(\Box) &= -2\mathcal{F}_1(\Box)\Box - \mathcal{F}_2(\Box)\Box - 2\mathcal{F}_3(\Box)\Box.
\end{aligned}$$

$$\begin{aligned}
R_{\mu\nu\lambda\sigma} &= \frac{1}{2}(\partial_{[\lambda}\partial_\nu h_{\mu\sigma]} - \partial_{[\lambda}\partial_\mu h_{\nu\sigma]}) \\
R_{\mu\nu} &= \frac{1}{2}(\partial_\sigma\partial_{(\nu}h^\sigma_{\mu)} - \partial_\nu\partial_\mu h - \Box h_{\mu\nu}) \\
R &= \partial_\nu\partial_\mu h^{\mu\nu} - \Box h
\end{aligned}$$

$$a + b = 0$$

$$c + d = 0$$

$$b + c + f = 0$$

$\mathcal{F}_3(\Box)$ is redundant around Minkowski

Graviton Propagator

$$
\begin{aligned}
a(\Box)\Box h_{\mu\nu} \quad &+ \quad b(\Box)\partial_\sigma\partial_{(\nu}h^\sigma_{\mu)} + c(\Box)(\eta_{\mu\nu}\partial_\rho\partial_\sigma h^{\rho\sigma} + \partial_\mu\partial_\nu h) \\
+\eta_{\mu\nu}d(\Box)\Box h \quad &+ \quad \frac{1}{4}f(\Box)\Box^{-1}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\lambda\sigma} = -\kappa\tau_{\mu\nu}
\end{aligned}
$$

$$
-\kappa\tau\nabla_\mu\tau^\mu_\nu = 0 = (c \overset{=0}{+} d)\Box\partial_\nu h + (a \overset{=0}{+} b)\Box h^\mu_{\nu,\mu} + (b + c \overset{=0}{+} f)h^{\alpha\beta}_{,\alpha\beta\nu}
$$

Bianchi Identity

$$a + b = 0$$

$$c + d = 0$$

$$b + c + f = 0$$

$$\Pi^{-1\lambda\sigma}_{\mu\nu} h_{\lambda\sigma} = \kappa\tau_{\mu\nu} \qquad h \;=\; h^{TT} + h^L + h^T$$

$$\Pi = \frac{P^2}{ak^2} + \frac{P^0_s}{(a-3c)k^2} + \cancel{\frac{P^0_w}{(c-a+f)k^2}}$$

Remarks on f(R) Gravity & 4th Order Gravity

$$= \int d^4x \sqrt{-g}\left[R + R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\Box)R^{\mu\nu\alpha\beta}\right]$$

$$\mathcal{L} \approx R + c_1 R^2 + c_2 R^3 + c_2 R^4 + c_3 R^5 + c_6 R^6 + \cdots$$

Scalar Ghost (Massive Spin 0)

$$\Pi \sim -1/2k^2(k^2 - m^2) + \ldots.$$

$$I = \int d^4x\, \sqrt{g}\left[\lambda_0 + k\, R + a\, R_{\mu\nu} R^{\mu\nu} - \tfrac{1}{3}\,(b+a) R^2\right]$$

Massive Spin-2 Ghost

$$\Pi \sim P_2/k^2(k^2 - m^2) + \ldots.$$

f(R) type model can be made Ghost Free but they do not improve UV behavior

4th Order Gravity can Improve UV behavior but has a Ghost

Covariant Modification of a Graviton Propagator : Only 1 Entire Function

UV

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a-3c)k^2} \qquad = \frac{1}{a}\left[\frac{P^2}{k^2} - \frac{P_s^0}{2k^2}\right]$$

Demand: $a(k^2) = c(k^2)$

Recovers GR $\lim_{k^2 \to 0} \Pi^{\mu\nu}{}_{\lambda\sigma} = (P^2/k^2) - (P_s^0/2k^2)$

$$a(0) = c(0) = -b(0) = -d(0) = 1$$

IR

ONLY 1 Non-Singular, Analytic functions at k=0, is required to Ameliorate the UV property of GR

'a' should be an Entire Function & cannot contain non-local operators, such as $a(\Box) \sim 1/\Box$

Ghost Free Gravity

$$a(\Box) = c(\Box) \Rightarrow 2\mathcal{F}_1(\Box) + \mathcal{F}_2(\Box) + 2\mathcal{F}_3(\Box) = 0$$

Entire Function

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a-3c)k^2} = \frac{1}{a}\left[\frac{P^2}{k^2} - \frac{P_s^0}{2k^2}\right]$$

$$a(\Box) = c(\Box) = e^{-\Box/M^2}$$

Some function of k which falls faster than $1/k^2$

$$a(\Box) = e^{-\frac{\Box}{M^2}} \text{ and } \mathcal{F}_3 = 0 \Rightarrow \mathcal{F}_1(\Box) = \frac{e^{-\frac{\Box}{M^2}} - 1}{\Box} = -\frac{\mathcal{F}_2(\Box)}{2}$$

UV Gravity Simplified

$$= \int d^4x\sqrt{-g}\left[R + R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\Box)R^{\mu\nu\alpha\beta}\right]$$

$$S = \int d^4x\,\sqrt{-g}\left[\frac{R}{2} + R\left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box}\right]R - 2R_{\mu\nu}\left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box}\right]R^{\mu\nu}\right]$$

Applications

Black Hole Singularity, i.e. Schwarzschild Type

$\mathcal{F}_3(\Box) = 0$

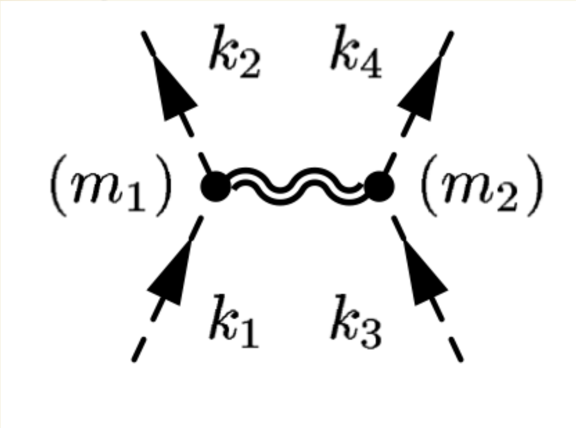
Biswas, Gerwick, Koivisto, AM, PRL (2012)
(gr-qc/1110.5249)

Cosmological Singularity, i.e. Big Bang Type

$\mathcal{F}_3(\Box) \neq 0$

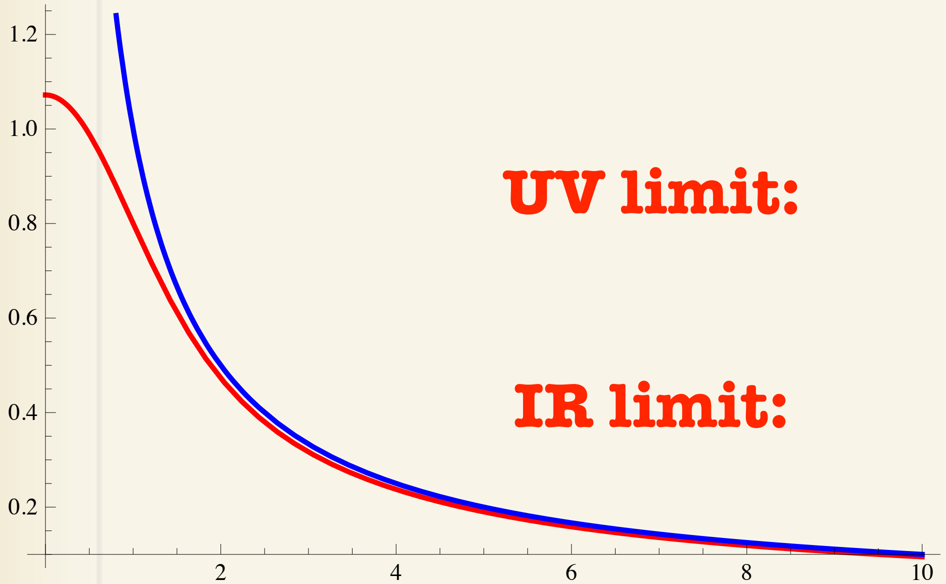
Biswas, AM, Siegel, JCAP (hep-th/0508194),
Brandenberger, Biswas, AM, Siegel, JCAP (hep-th/0610274)
Biswas, AM, Koivisto, JCAP (1005.0590)

Newtonian Potential



$$ds^2 = -(1-2\Phi)dt^2 + (1+2\Psi)dr^2$$

$$\Phi(r) = \Psi(r) = -\frac{m_1 m_2}{4\pi M_p^2 r}\mathrm{erf}\left(\frac{rM}{2}\right) \ll 1$$



UV limit: $r \to 0, \quad \mathrm{erf}(r) \to r \qquad \Phi(r) \to \mathrm{const.}$

IR limit: $r \to \infty, \quad \mathrm{erf}(r) \to 1 \qquad \Phi(r) \to \frac{1}{r}$

No Singularity, No Horizon, No Information Loss for Mini-Bhs

$$ds^2 = (1 - \frac{2Gm}{r}\mathrm{erf}\,(rM/2))dt^2 - \frac{dr^2}{(1-\frac{2Gm}{r}\mathrm{erf}\,(rM/2))}$$

$$mM \ll M_p^2 \implies m \ll M_p$$

Non-Singular Bouncing Solution

$$= \int d^4x\sqrt{-g}\left[R + R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\Box)R^{\mu\nu\alpha\beta}\right]$$

$$\begin{aligned} a(\Box)[\Box h_{\mu\nu} &- \partial_\sigma\partial_{(\nu}h^\sigma_{\mu)}] + c(\Box)[\eta_{\mu\nu}\partial_\rho\partial_\sigma h^{\rho\sigma} + \partial_\mu\partial_\nu h - \eta_{\mu\nu}\Box h] \\ &+ [a(\Box) - c(\Box)]\,\Box^{-1}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\lambda\sigma} = -\kappa\tau_{\mu\nu} \end{aligned}$$

$$h \sim \mathtt{diag}(0, A\sin\lambda t, A\sin\lambda t, A\sin\lambda t) \text{ with } A \ll 1$$

$$c(\Box) \equiv \frac{a(\Box)}{3}\left[1 + 2\left(1 - \frac{\Box}{m^2}\right)\tilde{c}(\Box)\right]$$

Non- Singular Bouncing, Homogeneous & Isotropic Universe

Such a solution is not possible in GR

Biswas, Gerwick, Koivisto, AM, (gr-qc/1110.5249)

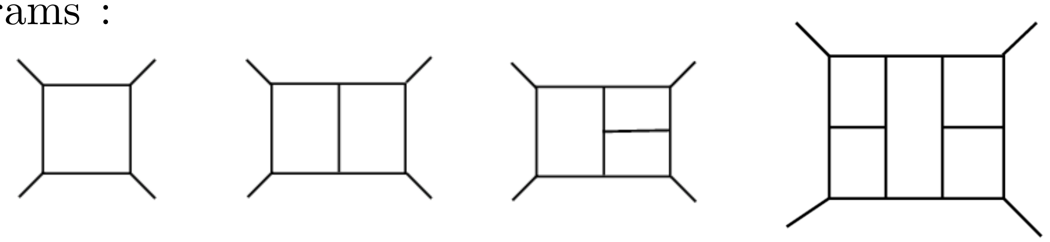
Conclusions

- **We have constructed a Ghost Free & Singularity Free Theory of Gravity**
- **If we can show higher loops are finite then it is a great news --** this is what we are working now
- **But, studying singularity theorems, positive energy theorems, Hawking radiation, Non-Singular Bouncing Cosmology , ..., many interesting problems can be studied in this framework**

Superficial Degree of Divergence: GR vs BGKM Gravity

Superficial degree of divergence of a Feynman diagram

Loops diagrams :



$D = L\,d + 2V - 2I,$

L : number of loops,

V : number of vertices,

I : number of internal lines in the graph.

Topological relation between $V,\ I$ and $L,\ L = 1 + I - V.$

$$D = 2 + (d-2)L. \quad \text{For } d = 4 \ \rightarrow \ D = 2 + 2L$$

$$E = n_h - I_h$$

$n_h = \#$ **of graviton vertices**

$I_h = \#$ **of internal graviton propagator**

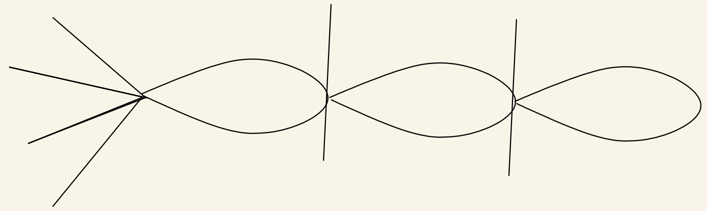
Using Topological Identity : $E = 1 - L$

For $L \geq 2 \ \Rightarrow E < 0$

Higher Loops are well behaved

Non-locality & Quantum Gravity

$$S = \int d^4x \left[\phi \Box e^{\frac{-\Box}{M^2}} \phi - \frac{\phi^6}{M_6^2}\right]$$



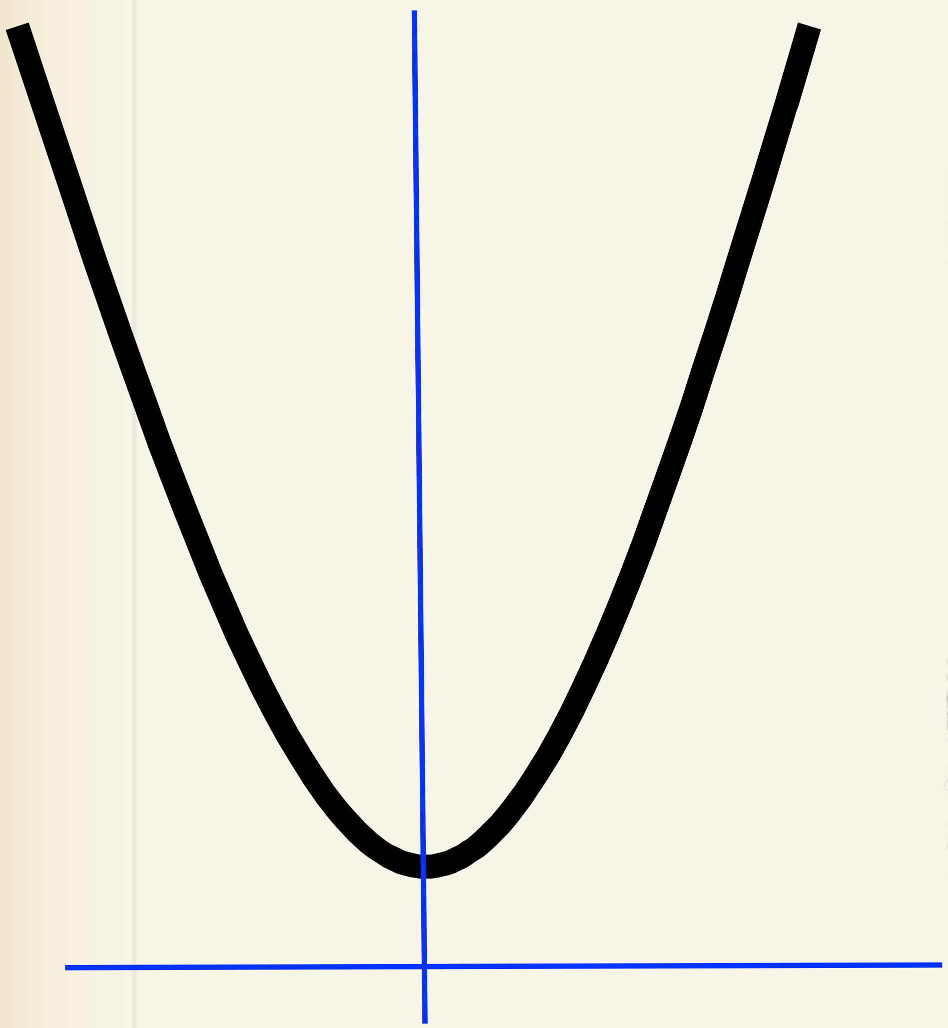
$$\frac{1}{M_6^2}\left[\int d^4p \frac{e^{\frac{-p^2}{M^2}}}{p^2}\right] \sim \frac{M^2}{M_6^2}$$

- Gravity is a Gauge Theory : Free kinetic action is tangled with interactions
- Vertices have the same exponential enhancement as the suppression in the propagator : One has to do the calculation ...
- Effective description : Arising from the integrations of quantum fluctuations of some unknown degrees of freedom -- the question of quantisation has no meaning, and one has to use the classical solutions as master fields (collective variables) for the quantum dynamics of the unknown degrees of freedom.

$$S = \int d^4x \sqrt{-g}\left[\frac{R}{2} + \alpha_0(R, R_{\mu\nu}) + \alpha_1(R, R_{\mu\nu}) R \mathcal{F}_1(\Box) R + \alpha_2(R, R_{\mu\nu}) R_{\mu\nu} \mathcal{F}_2(\Box) R^{\mu\nu} + \alpha_3(R, R_{\mu\nu}) C_{\mu\nu\lambda\sigma} \mathcal{F}_3 C^{\mu\nu\lambda\sigma}\right]$$

Full Non-Singular Solution

$$S = \int d^4x \, \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box} \right] R - 2R_{\mu\nu} \left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box} \right] R^{\mu\nu} \right]$$



$$\Box R = r_1 R + r_2$$

$$\Lambda = -\frac{r_2 M_P^2}{4 r_1}$$

$$a(t) = \cosh\left(\sqrt{\frac{r_1}{2}}\, t\right)$$

$$\ddot{a} > 0 \Longrightarrow \Lambda > 0$$

Does Not Contribute to Dynamics But to Perturbations

Biswas, AM, Siegel, JCAP (hep-th/0508194)

Biswas, Koivisto, AM, JCAP (hep-th/1005.0590)

Gravitational Waves

- In GR we solve:

$$\Box \bar{h}_{\mu\nu} = -\kappa \tau_{\mu\nu}$$

$$\bar{h}_{\mu\nu} \approx -\frac{\kappa}{4\pi r} \int \tau_{\mu\nu}(t, r') d^3 r'$$

- In Our Case:

$$a(\Box)\bar{h}_{\mu\nu} = -\kappa \tau_{\mu\nu}$$

$$\bar{h}_{\mu\nu} = -\frac{\kappa}{4\pi r} \mathrm{erf}\left(\frac{r M_p}{2}\right) \tau_{\mu\nu}(r)$$

$$r \Longrightarrow 0, \ \textbf{No Singularity}$$

No Schwarzschild Singularity

Linearized Solution

$$ds^2 = -(1-2\Phi)dt^2 + (1+2\Psi)dr^2$$

$$\begin{aligned}(a(\Box)-3c(\Box))\Box h + (4c(\Box)-2a(\Box)+f(\Box))\partial_\mu\partial_\nu h^{\mu\nu} &= \kappa\rho \\ a(\Box)\Box h_{00} + c(\Box)\Box h - c(\Box)\partial_\mu\partial_\nu h^{\mu\nu} &= -\kappa\rho\end{aligned}$$

For $f = 0$ and $a(\Box) = c(\Box)$

$$4a(\nabla^2)\nabla^2\Phi = 4a(\nabla^2)\nabla^2\Psi = \kappa\rho = \kappa m\delta^3(\vec{r})$$

$$a(\Box) = e^{-\Box/M^2}$$ **Varying slowly with time** $$\Box \longrightarrow \nabla^2$$

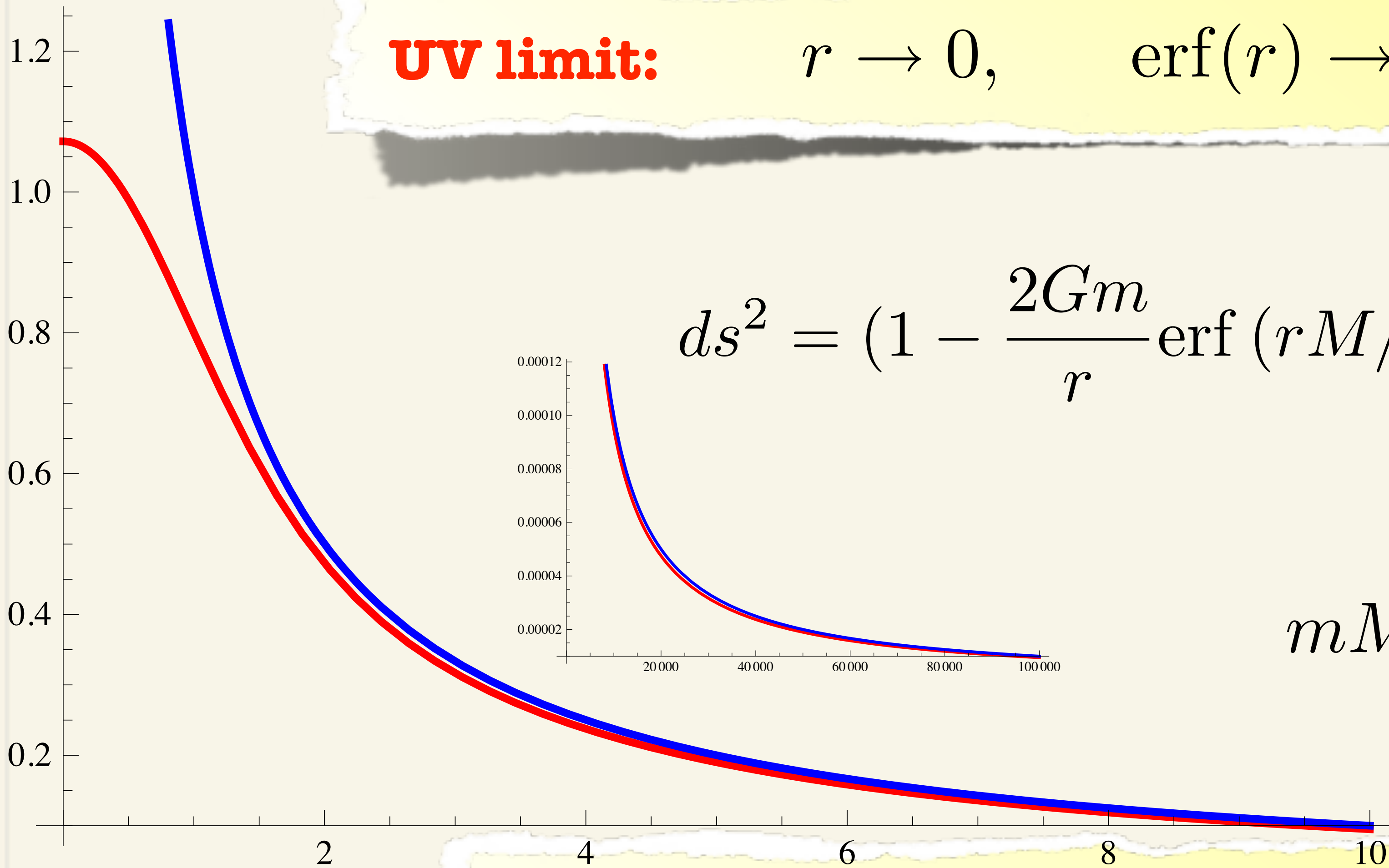
$$\Phi(r) \sim \kappa m \int \frac{dp}{p} e^{-p^2/M^2} \sin(p\,r) = \kappa \frac{m\pi}{4\pi^2 r} \mathrm{erf}\left(\frac{rM}{2}\right) = \frac{Gm}{r}\mathrm{erf}\left(\frac{rM}{2}\right) = \frac{m}{4\pi M_p^2 r}\mathrm{erf}\left(\frac{rM}{2}\right)$$

$$mM \ll M_p^2 \implies m \ll M_p$$

Biswas, Gerwick, Koivisto, AM, PRL, (gr-qc/1110.5249)

No Mini-Black Holes

UV limit: $r \to 0, \quad \mathrm{erf}(r) \to r \qquad \Phi(r) \to \mathrm{const.}$



$$ds^2 = \left(1 - \frac{2Gm}{r}\mathrm{erf}\left(rM/2\right)\right)dt^2 - \frac{dr^2}{\left(1 - \frac{2Gm}{r}\mathrm{erf}\left(rM/2\right)\right)}$$

$$mM \ll M_p^2 \implies m \ll M_p$$

IR limit: $r \to \infty, \quad \mathrm{erf}(r) \to 1 \qquad \Phi(r) \to \frac{1}{r}$

No Singularity → No Horizon

No Information Loss Paradox

Background Independent Action : de & Anti-de Sitter

$$S=\int d^4x\,\sqrt{-g}\left[\frac{R}{2}+\alpha_1(R)R\left[\frac{e^{-\frac{\Box}{M^2}}-1}{\Box/M^2}\right]R-2\alpha_2(R)R_{\mu\nu}\left[\frac{e^{-\frac{\Box}{M^2}}-1}{\Box/M^2}\right]R^{\mu\nu}-\Lambda\right]$$

$$\alpha_1(0)=\alpha_2(0)=1$$

$$g_{\mu\nu}=\bar{g}_{\mu\nu}+h_{\mu\nu}\,,$$

$$\lambda\left[1-\frac{32\lambda^2\alpha_1'(\lambda)}{M_p^2}-\frac{16\lambda^2\alpha_2'(\lambda)}{M_p^2}\right]=\frac{\Lambda}{M_p^2}$$

$$\bar{R}_{\mu\nu}=\lambda\bar{g}_{\mu\nu}\ ;\ \bar{R}=4\lambda \text{ and } \bar{\nabla}_\mu\bar{g}_{\nu\rho}=0$$

Generic Form of Action

$$S=\int d^4x\sqrt{-g}\left[\frac{R}{2}+\alpha_0(R,R_{\mu\nu})+\alpha_1(R,R_{\mu\nu})R\mathcal{F}_1(\Box)R\right.$$
$$\left.+\alpha_2(R,R_{\mu\nu})R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu}+\alpha_3(R,R_{\mu\nu})C_{\mu\nu\lambda\sigma}\mathcal{F}_3C^{\mu\nu\lambda\sigma}\right]$$

Biswas, AM, 2012 (Unpublished)

Looking for Ghosts

$$F(R) = R + \sum_{n=0}^{\infty} c_n R \Box^n R$$

$$S = \int d^4x \sqrt{-g} \left[\Phi R + \psi \sum_1^{\infty} c_i \Box^i \psi - \{\psi(\Phi - 1) - c_0 \psi^2\} \right]$$

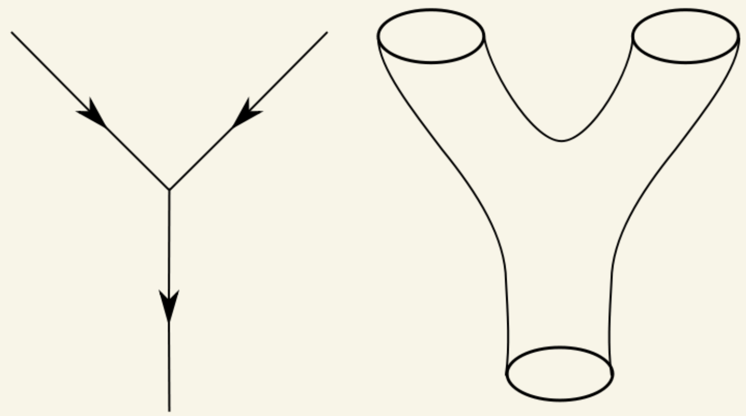
$$\frac{\delta S}{\delta \Phi} = 0 \Rightarrow \psi = R \qquad S \approx \int d^4x \, \sqrt{-g'} \left[R' + \frac{3}{2} \phi \Box' \phi + \psi \sum_1^{\infty} c_i {\Box'}^i \psi - \{\psi \phi - c_0 \psi^2\} \right]$$

$$\psi = 3 \Box \phi \, ,$$

$$\phi = 2 \left[\sum_1^{\infty} c_i \Box^i \psi + c_0 \psi \right]$$

$$\left(1 - 6 \sum_0^{\infty} c_i \Box^{i+1} \right) \phi \equiv \Gamma(\Box) \phi = 0$$

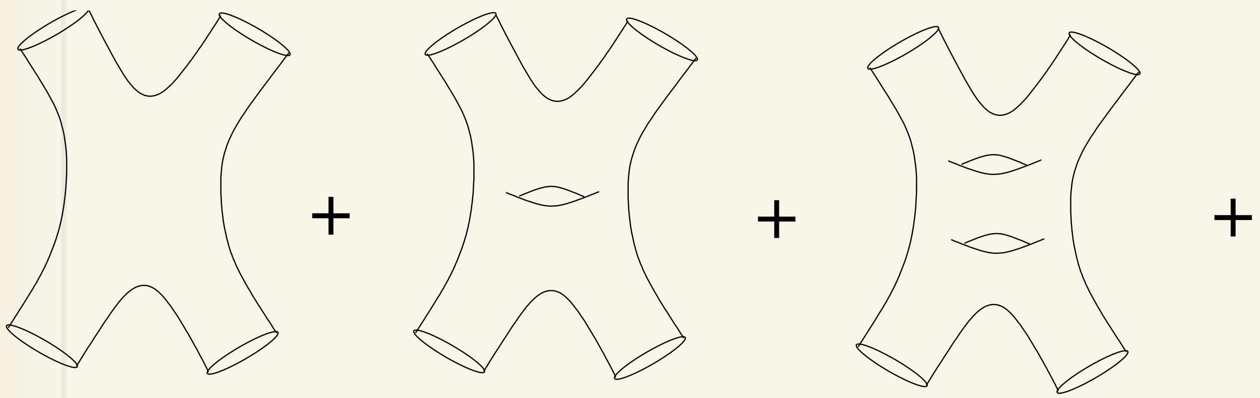
Local $\longrightarrow$ Non-Local



Mostly we Deal with Free Strings

$$\alpha' = l_s^2 \qquad T = \frac{1}{2\pi\alpha'}$$

String Interactions (summing over Topologies)



$$S_{\text{string}} = S_{\text{Poly}} + \lambda\chi$$

$$\chi = 2 - 2h = 2(1-g)$$

$$g_s = e^{\lambda}$$

@ the lowest order

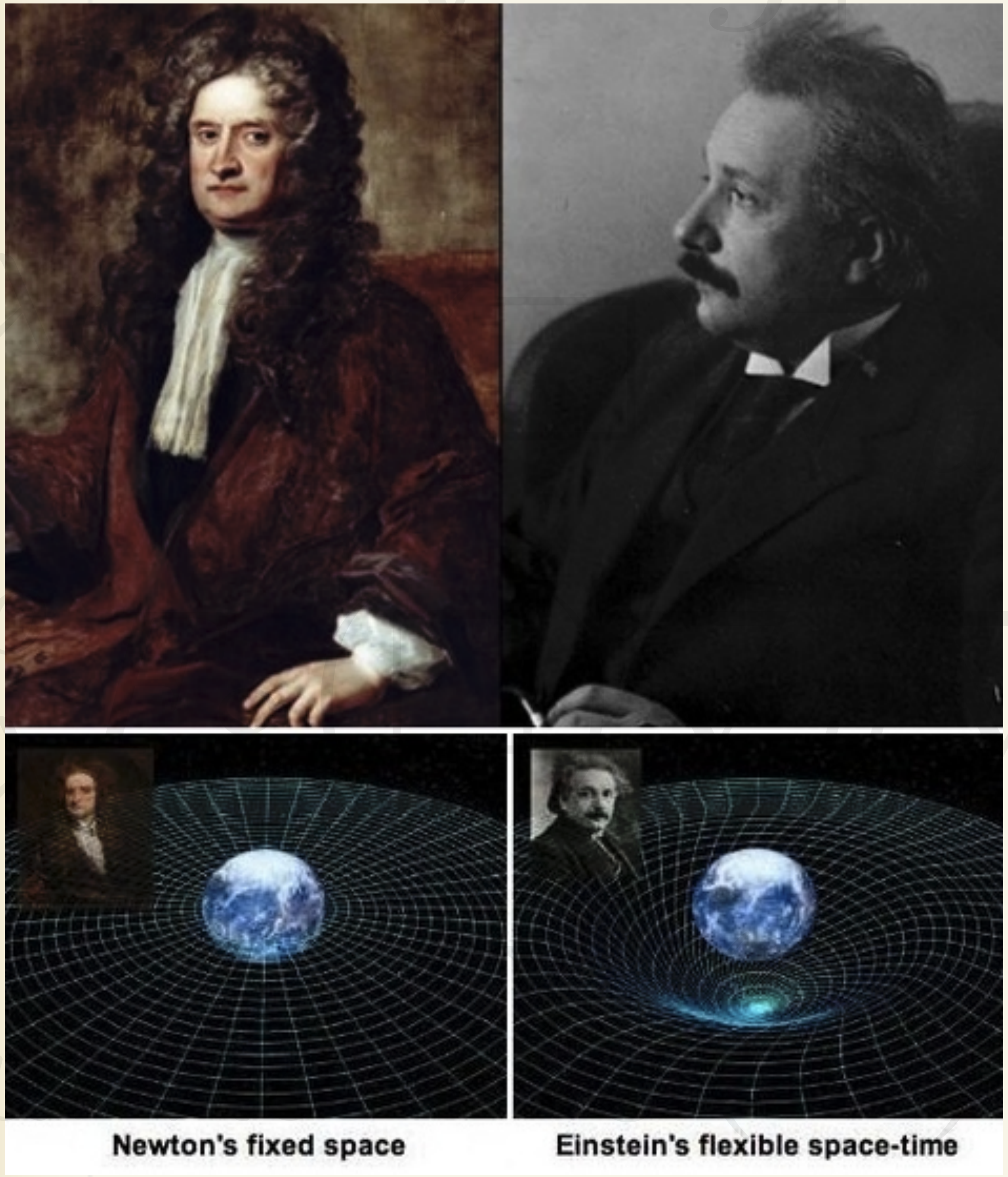
$$S = \frac{1}{2\kappa^2}\int d^{26}X\sqrt{-G}\,\mathcal{R} \qquad \kappa^2 \approx g_s^2(\alpha')^{12}$$

String Theory Inevitably Introduces 2 Parameters !

Plan of the talk

- Motivation
- Higher derivative gravity & ghosts
- Covariant extension of higher derivative ghost-free gravity
- Asymptotically free theory of gravity
- Background independent action of UV gravity

Classical Singularities



UV is Pathological

IR Part is Safe

String theory is non-local

Short distance divergences should be absent

Perturbative Quantum Gravity

$$S = \int \mathcal{L}(x)\,\mathrm{d}^4x \;; \qquad \mathcal{L}(x) = \sqrt{-g}\left(\frac{R}{16\pi G} + \mathcal{L}^{\mathrm{matter}}\right)$$

$$x^\mu \to x^\mu + \varepsilon\eta^\mu(x) \qquad \varepsilon = \sqrt{16\pi G}$$

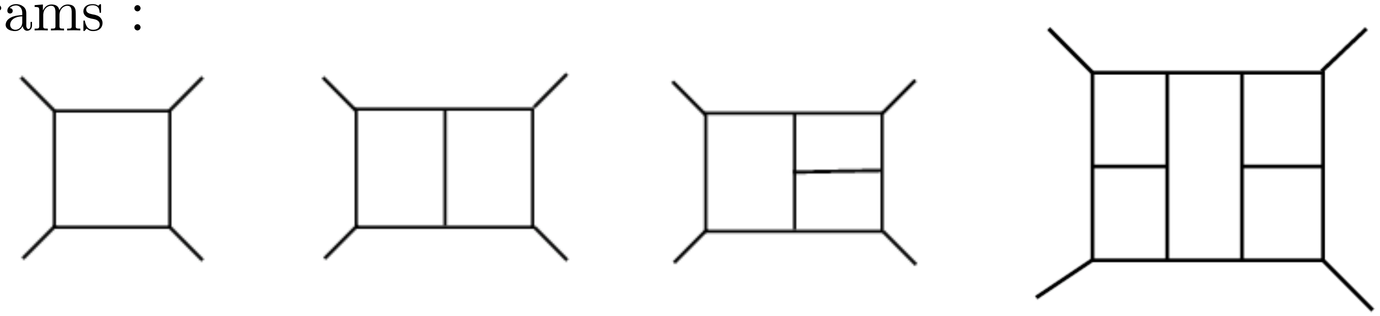
$$\Delta\mathcal{L} = \sqrt{-g}\,(\alpha R^2 + \beta R^2_{\mu\nu} + \gamma R^2_{\alpha\beta\mu\nu}) \equiv \int \mathrm{d}^4x\sqrt{-g}(R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\alpha\beta})$$

Pure Gravity is 1-Loop Renormalizable !

Pure Gravity requires 1 new counter term @ 2 Loops !

Superficial degree of divergence of a Feynman diagram

Loops diagrams :



$D = L\,d + 2V - 2I,$

L : number of loops,

V : number of vertices,

I : number of internal lines in the graph.

Topological relation between V, I and L, $L = 1 + I - V$,

$$D = 2 + (d-2)L. \quad \text{For } d = 4 \;\to\; D = 2 + 2L$$

Graviton Propagator in G-R

$$\partial_\sigma h_{\mu\nu}\,\partial^\sigma h^{\mu\nu} \quad , \quad \partial^\nu h_{\mu\nu}\,\partial_\sigma h^{\mu\sigma} \qquad \mathcal{L}_{sym} = -\tfrac{1}{4}\,\partial_\sigma h_{\mu\nu}\,\partial^\sigma h^{\mu\nu} + \tfrac{1}{2}\,\partial^\nu h_{\mu\nu}\,\partial_\sigma h^{\mu\sigma}$$

$$\partial^\nu h_{\mu\nu}\,\partial^\mu h_\sigma{}^\sigma \quad , \quad \partial^\mu h_\nu{}^\nu\,\partial_\mu h_\sigma{}^\sigma \qquad -\tfrac{1}{2}\,\partial^\nu h_{\mu\nu}\,\partial^\mu h_\sigma{}^\sigma + \tfrac{1}{4}\,\partial^\mu h_\nu{}^\nu\,\partial_\mu h_\sigma{}^\sigma$$

$$\Box\, h_{\mu\nu} - 2\,\partial_\nu\,\partial^\sigma\,\bar{h}_{\mu\sigma} = -\kappa\,\bar{T}_{\mu\nu}$$

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \tfrac{1}{2}\,\eta_{\mu\nu}\,h_\sigma{}^\sigma$$

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \tfrac{1}{2}\,\eta_{\mu\nu}\,h_\sigma{}^\sigma$$

$= 0$ **Harmonic Gauge**

$$h_{\mu\nu} = \kappa\,\frac{1}{k^2}\left(T_{\mu\nu} - \tfrac{1}{2}\,\eta_{\mu\nu}T_\sigma{}^\sigma\right)$$

P Van Nieuwenhuizen (1973)

$$\mathcal{L} = -\tfrac{1}{4}\,\partial_\mu h_{\alpha\beta}\,\partial^\mu h^{\alpha\beta} + \tfrac{1}{8}\,(\partial_\mu h^\alpha{}_\alpha)^2 + \tfrac{1}{2}\,C_\mu^2 + \tfrac{1}{2}\,\kappa\,h_{\mu\nu}\,T^{\mu\nu} + \mathcal{L}_{\rm gf} + \ldots$$

$$\mathcal{L}_{\rm gf} = -\tfrac{1}{2}\,C_\mu^2 + \mathcal{L}_{\rm ghost}$$

$$D_{\mu\nu\alpha\beta}(k) = \frac{\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \frac{2}{d-2}\eta_{\mu\nu}\,\eta_{\alpha\beta}}{k^2}$$

De Donder Gauge