



# Phase reconstruction near the high-symmetry point of the $O(2) \times O(2)$ nonlinear sigma model

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J.M. Fellows, S.T. Carr, CAH, and J. Schmalian, *Phys. Rev. Lett.* **109**, 155703 (2012);  
CAH, S.T. Carr, J.M. Fellows, and J. Schmalian, arXiv:1311.5344 (to appear in JPSJ).

# Outline

- Phase transitions at ‘zero’ temperature.
- ‘Phase reconstruction’ near quantum critical points.
- Special features of 2+1 dimensions: topological order and BKT physics.
- Suppression of BKT transitions near high-symmetry points: an alternative route to phase reconstruction.

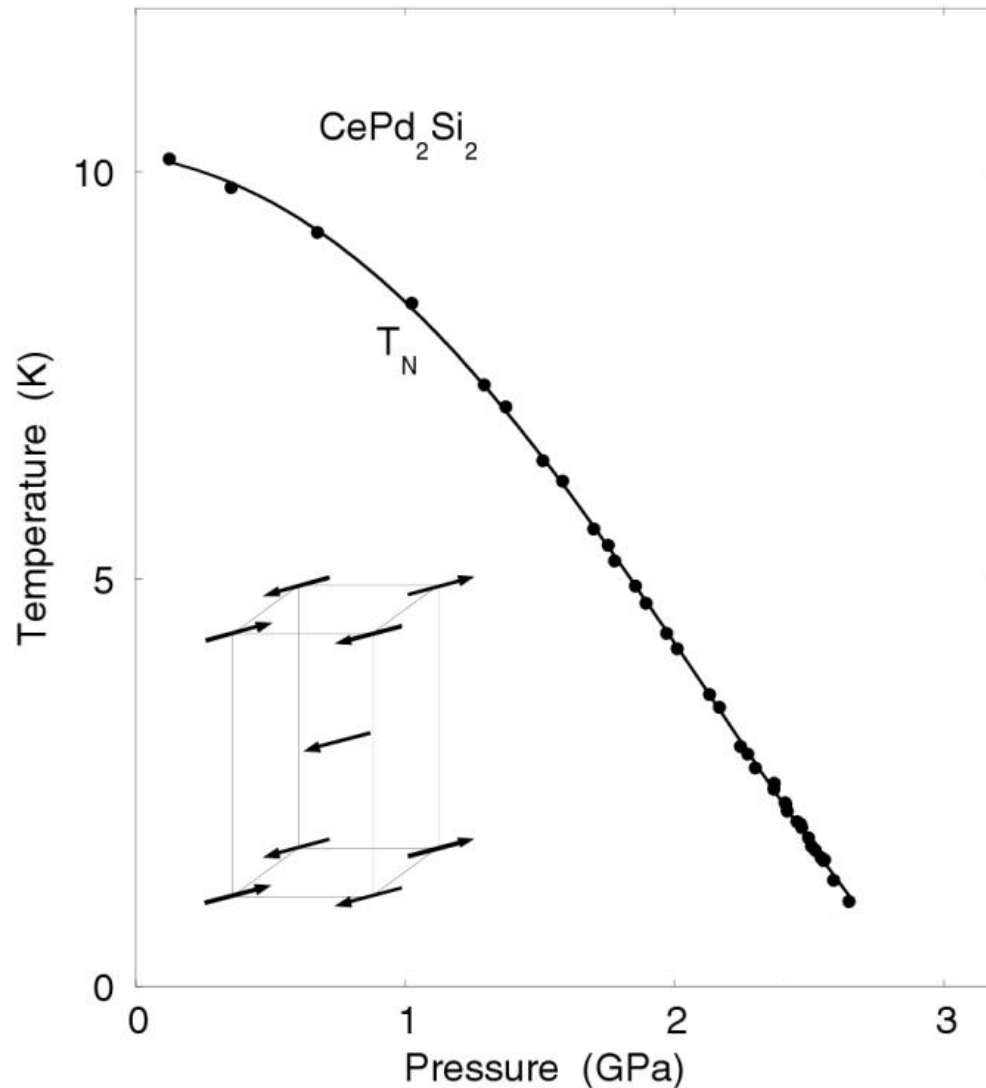
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# From phases to phase transitions

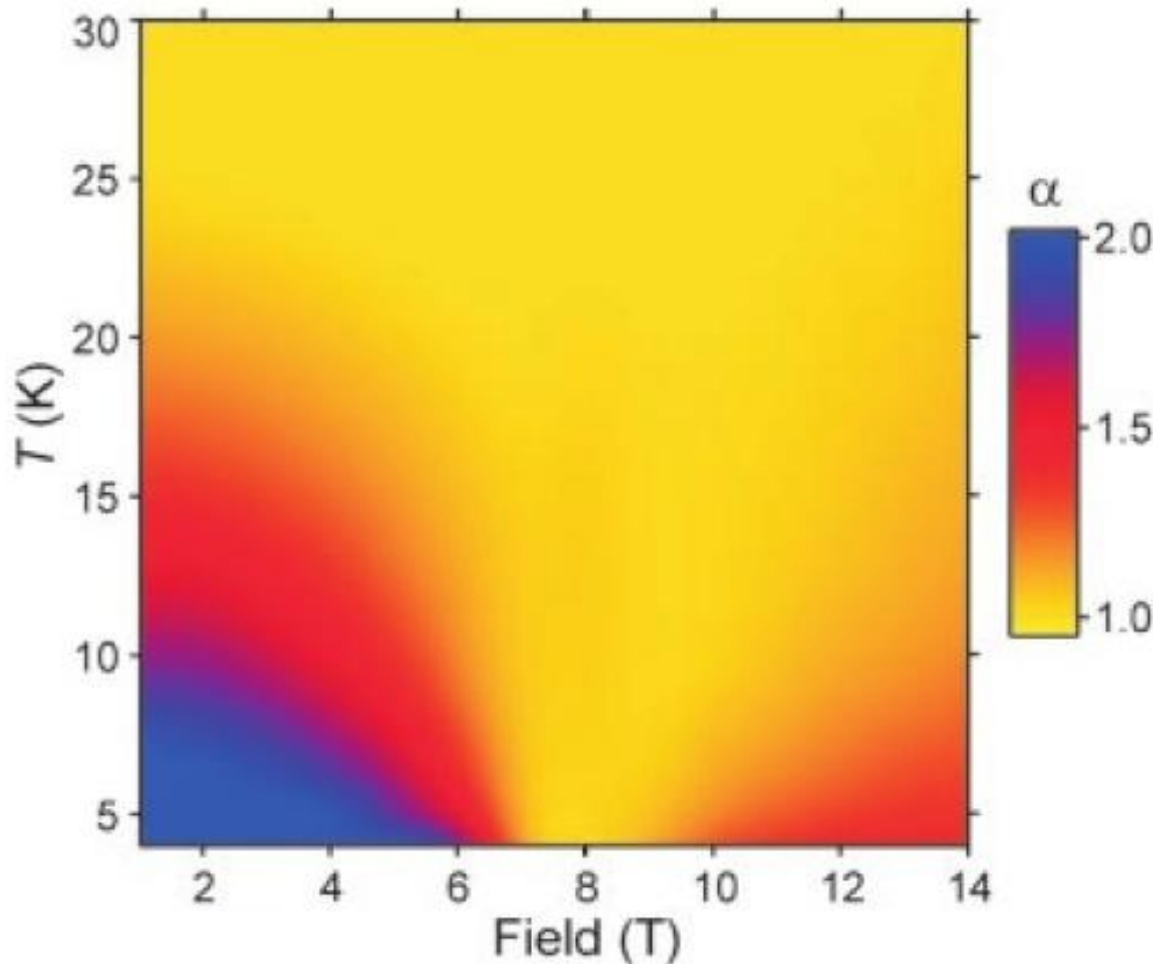
- 1960s and 1970s: shift from classifying phases to classifying phase transitions – particularly continuous phase transitions. (Critical exponents, Widom scaling, etc.)
- Late 1970s, 1980s, and 1990s: shift from thermally ‘driven’ phase transitions to low- (essentially zero-?) temperature phase transitions ‘driven’ by the variation of pressure, magnetic field, or chemical doping.

# Pressure: heavy-fermion materials



adapted from  
F.M. Grosche *et al.*,  
*J. Phys.: Cond. Matt.* **13**,  
2845 (2001).

# Magnetic field: strontium ruthenate



$$\rho(T) = \rho_0 + AT^\alpha$$

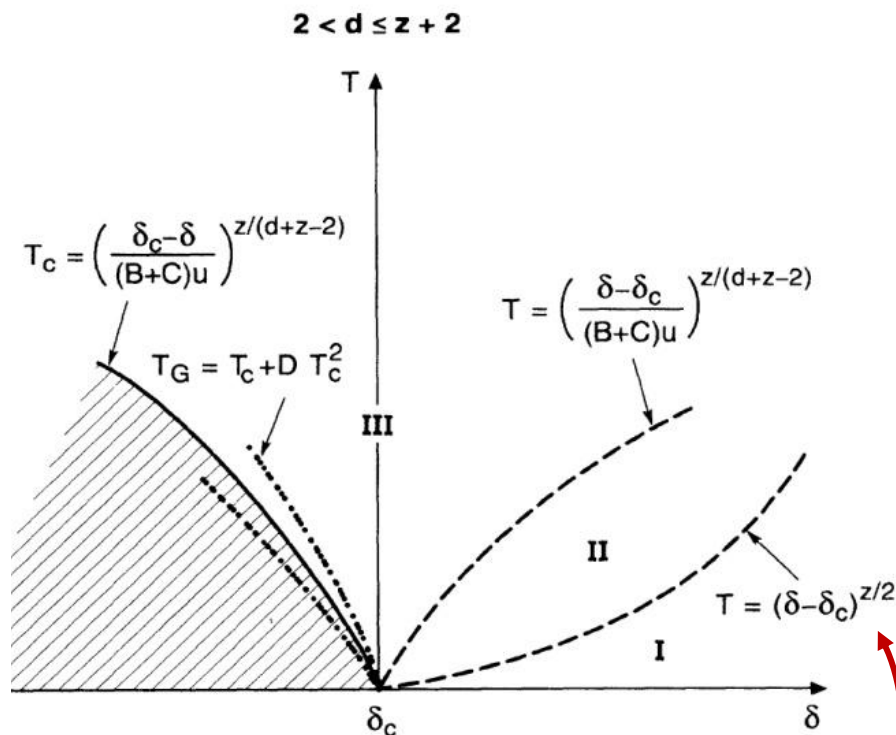
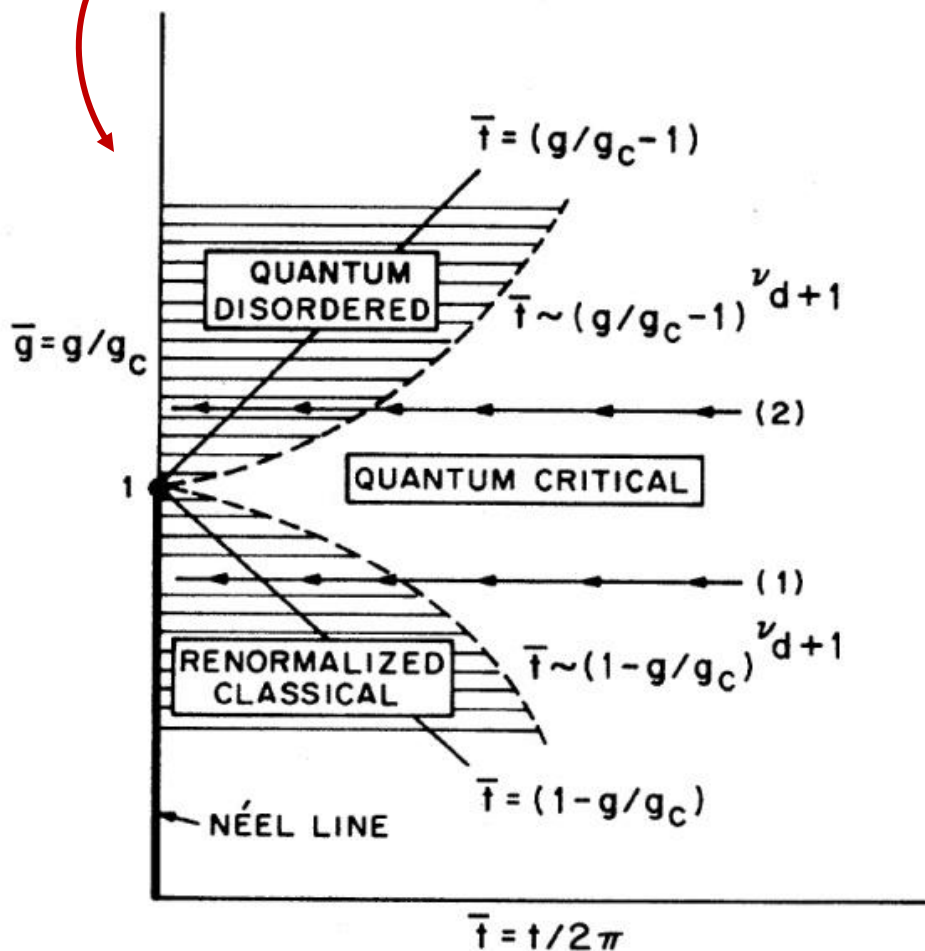
(Well, really it's

$$\alpha = \frac{d \ln (\rho(T) - \rho_0)}{d \ln T} .)$$

*S.A. Grigera et al.,  
Science* **294**, 329 (2001).

# Theories of quantum criticality

Insulators: S. Chakravarty, B.I. Halperin, and D.R. Nelson, *PRB* **39**, 2344 (1989).



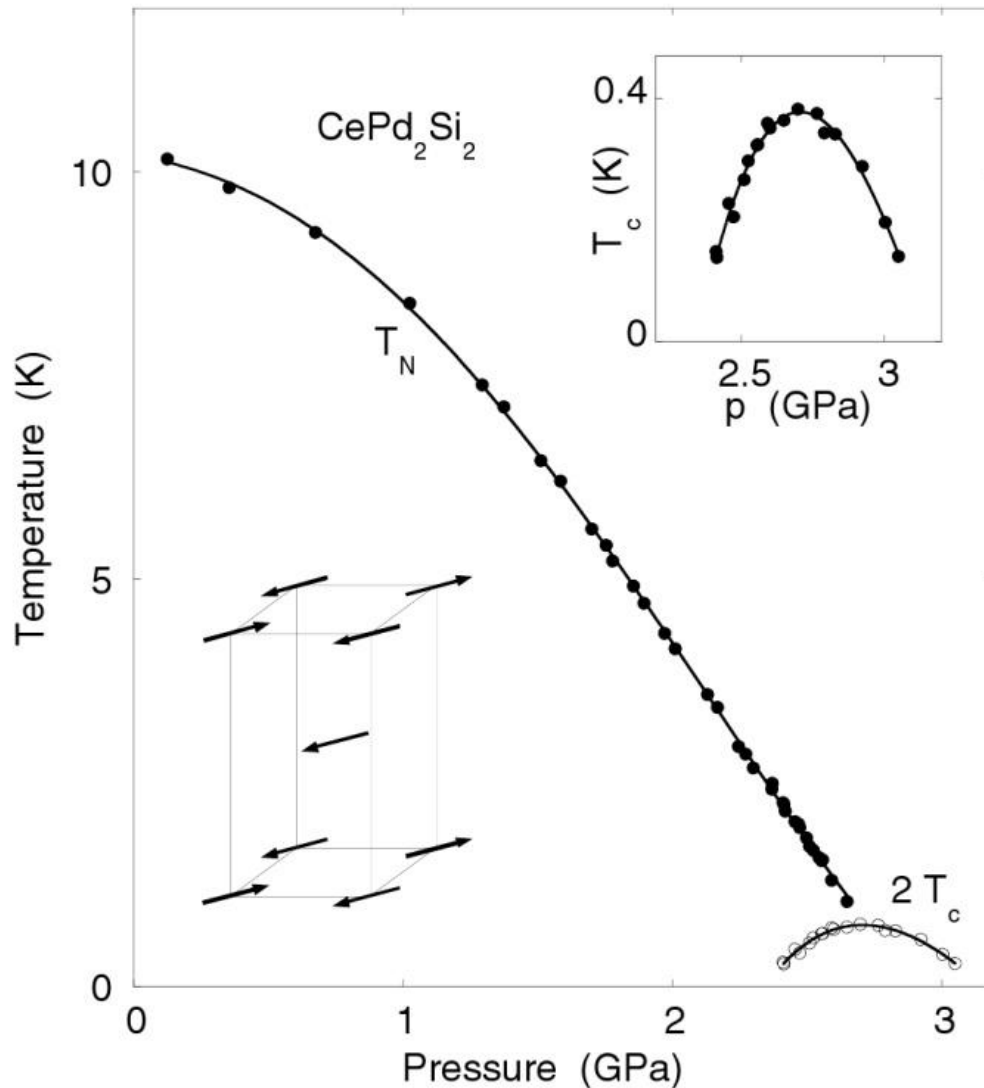
Metals: J.A. Hertz, *PRB* **14**, 1165 (1976); A.J. Millis, *PRB* **48**, 7183 (1993).

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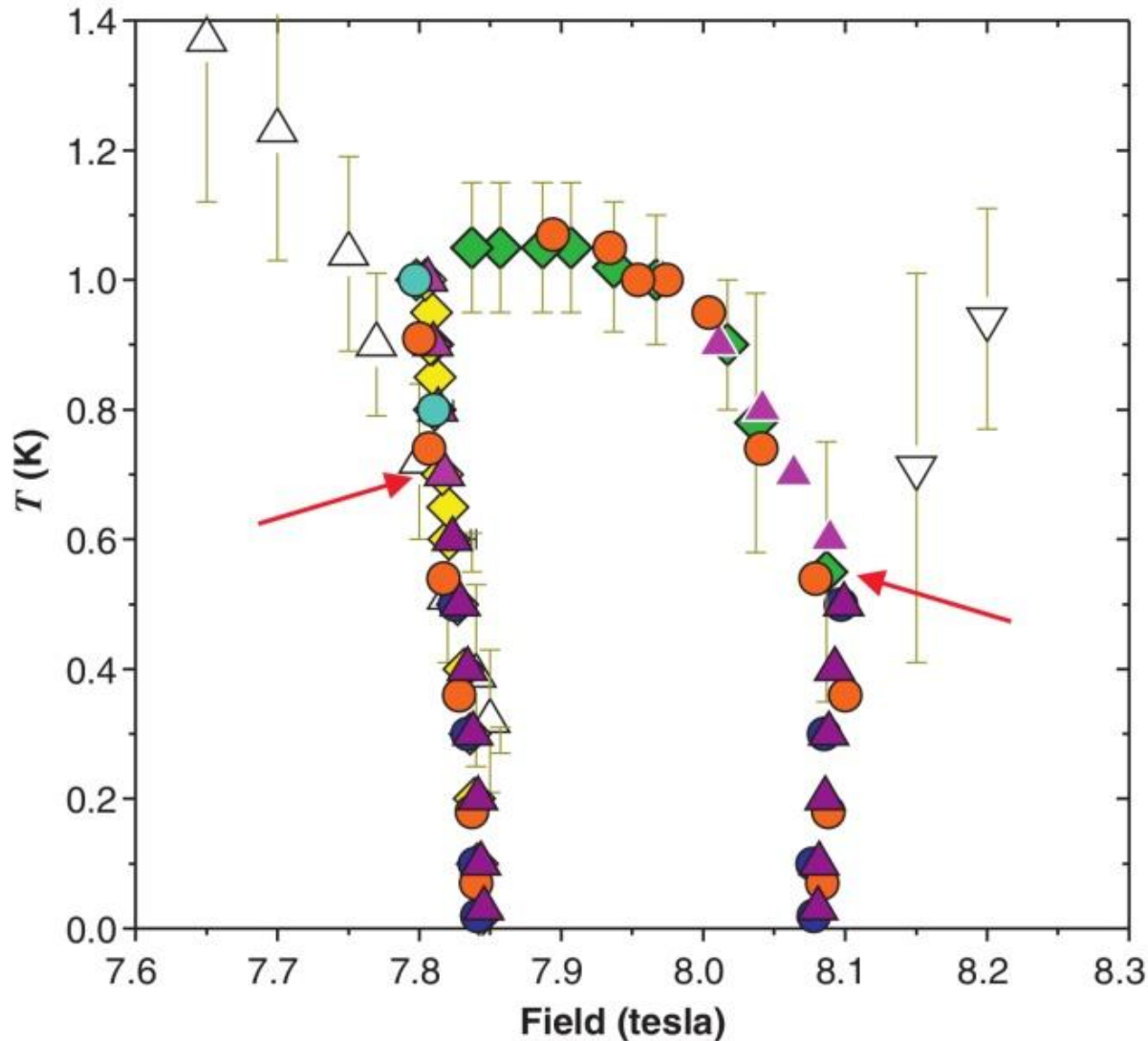


# Heavy-fermion materials (again)



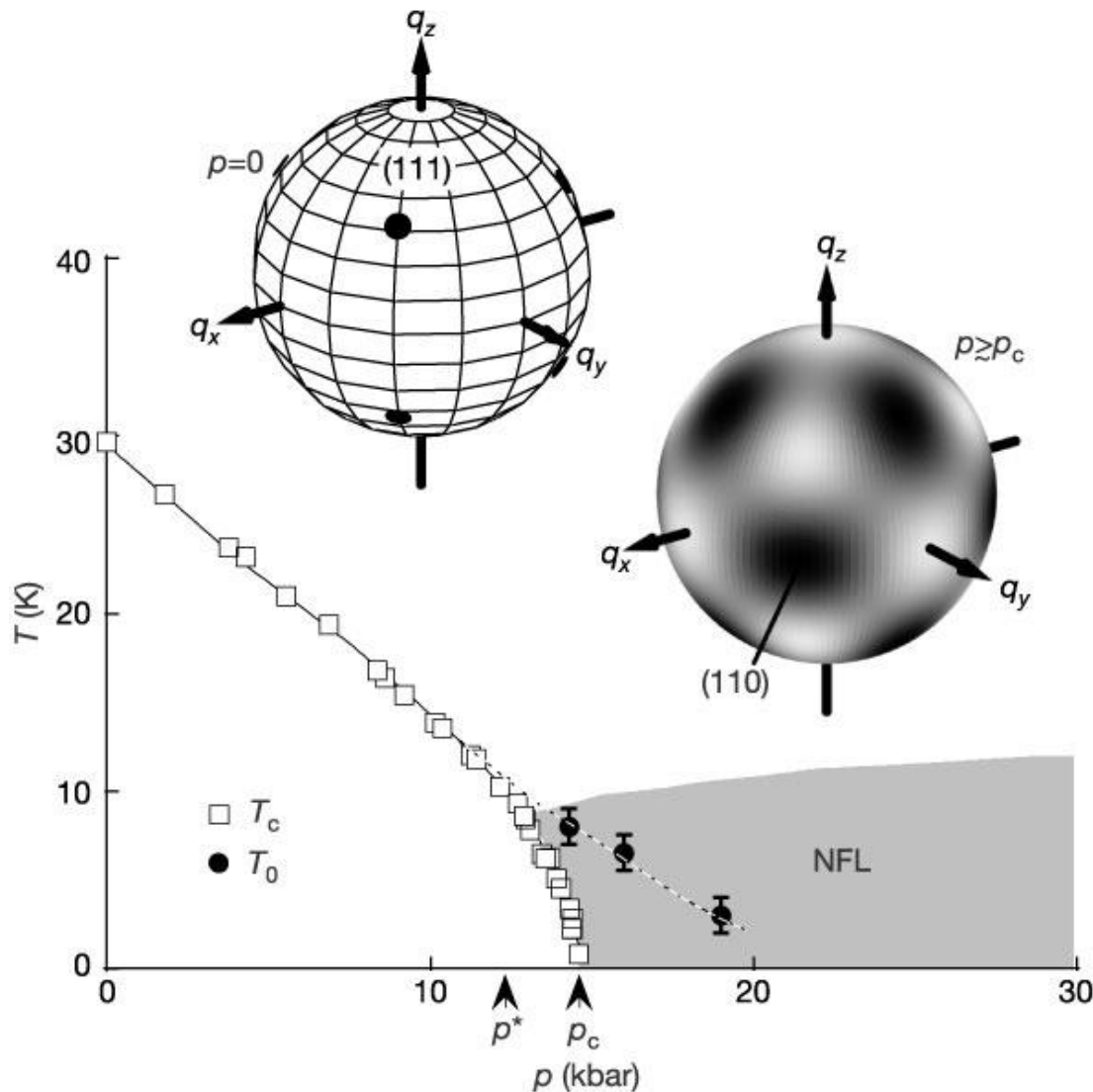
F.M. Grosche *et al.*,  
*J. Phys.: Cond. Matt.* **13**,  
2845 (2001).

# Strontium ruthenate (again)



S.A. Grigera *et al.*,  
*Science* **306**, 1154 (2004).

# Manganese silicide



C. Pfleiderer *et al.*,  
*Nature* **427**, 227 (2004).

# Outline

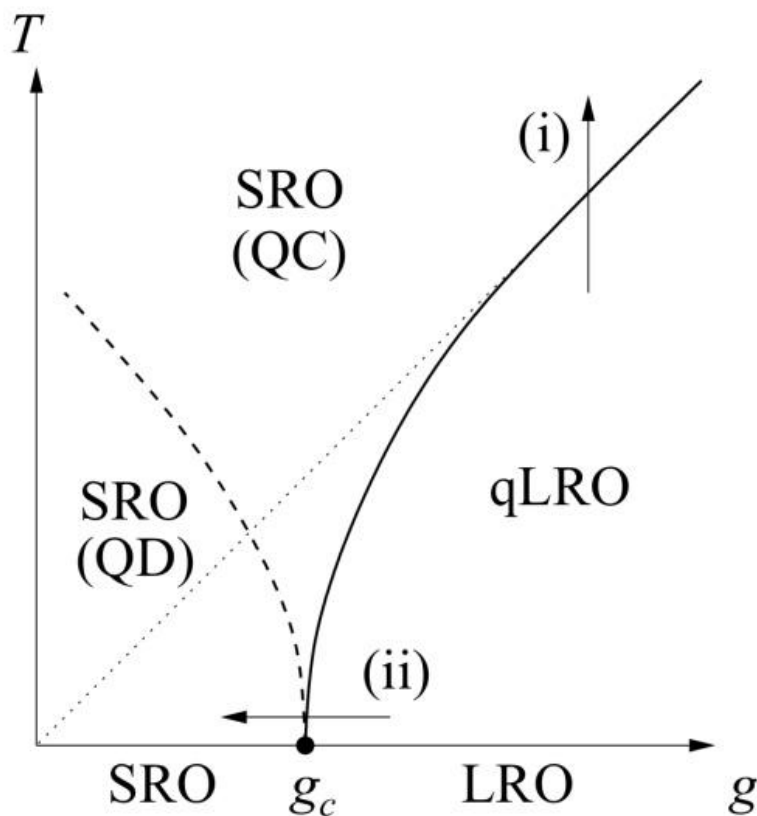
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# Topological order and BKT

- Mermin-Wagner theorem: no spontaneous breaking of a continuous symmetry at  $T > 0$  in  $d \geq 2$ .
- ‘Loophole’ in  $d=2$ : can have a finite- $T$  transition from quasi-long-range (or “topological”) order (algebraically decaying correlations) to short-range order (exponentially decaying correl’ns).
- Mechanism: vortex-antivortex unbinding.

# O(2) nonlinear sigma model

$$S = \frac{g}{2} \int_0^\beta d\tau \int d^2x \left[ (\partial_\tau \mathbf{n})^2 + (\partial_\mu \mathbf{n})^2 \right] \quad \mathbf{n}^2 = 1$$



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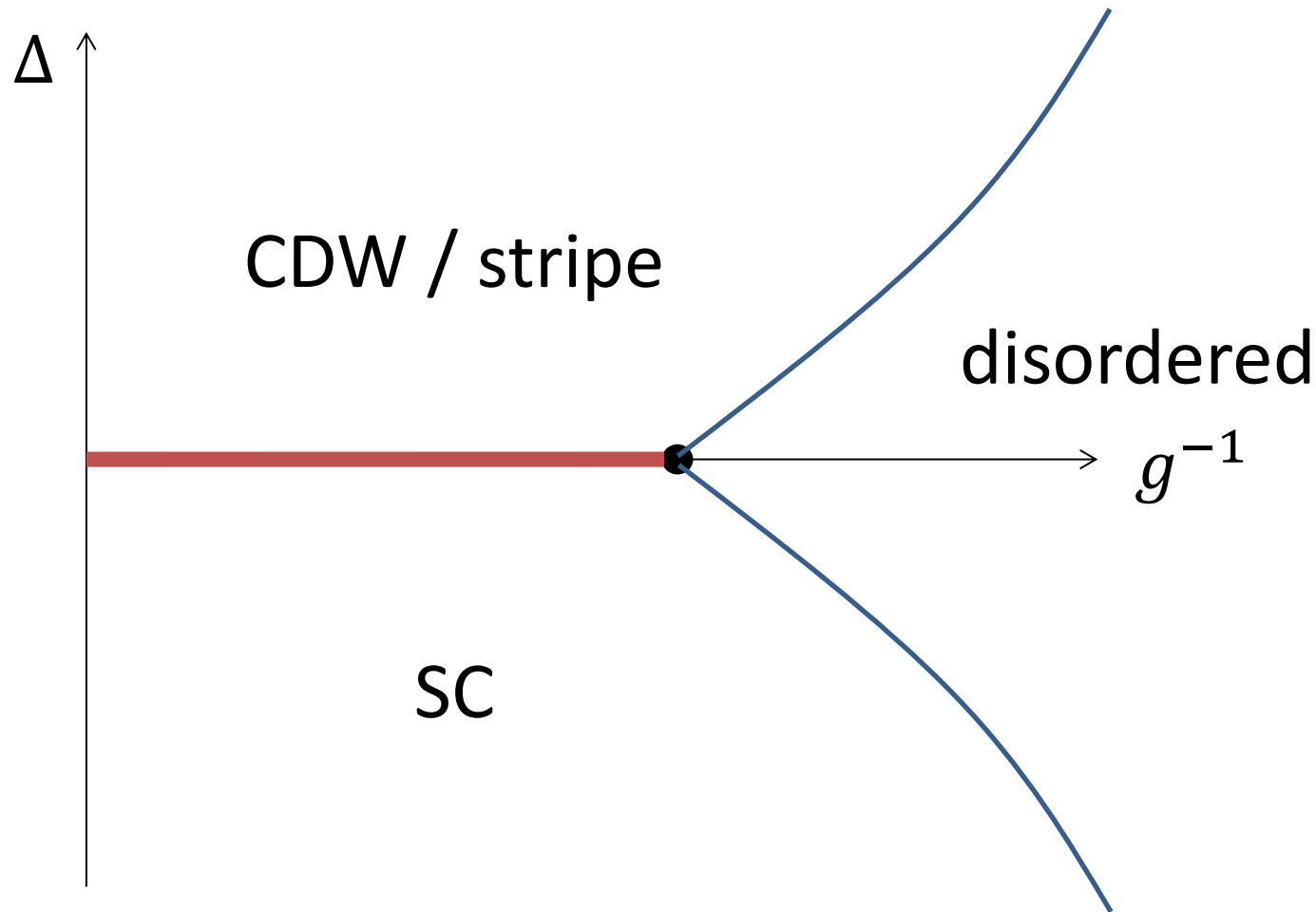
# $O(2) \times O(M)$ quantum NL $\sigma$ M: a simple model of phase competition

$$S = g \int_0^\beta d\tau \int d^d x \left[ (\partial_\tau \mathbf{n})^2 + (\nabla \mathbf{n})^2 + \frac{\Delta}{a^2} \mathbf{n}^T \mathbf{D} \mathbf{n} \right]$$

$$\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \\ \vdots \\ n_{M+2} \end{pmatrix} \quad \mathbf{n}^2 = 1 \quad \mathbf{D} = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}$$

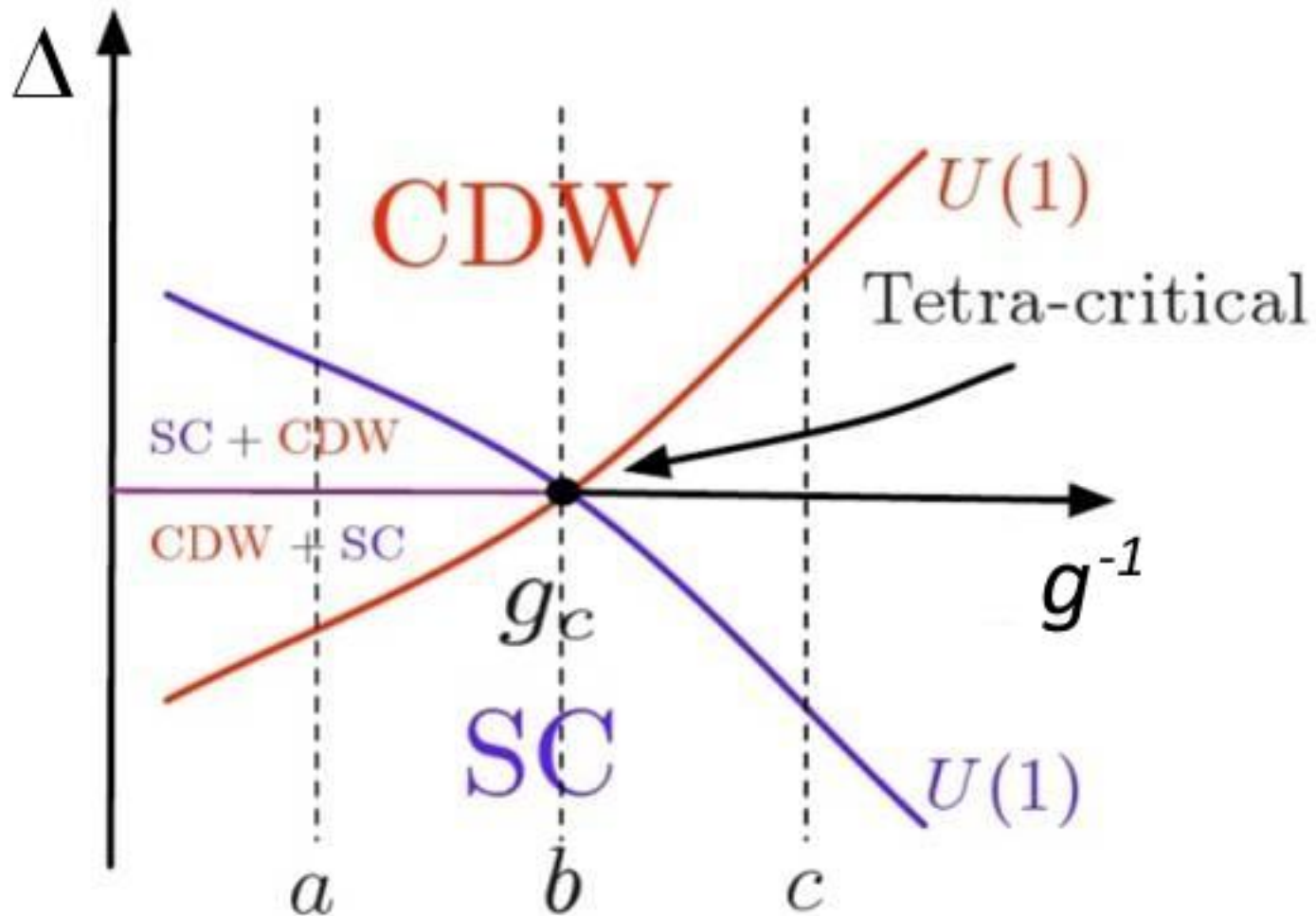


# $d=2$ $O(2) \times O(2)$ NL $\sigma$ M: pre-2010 wisdom



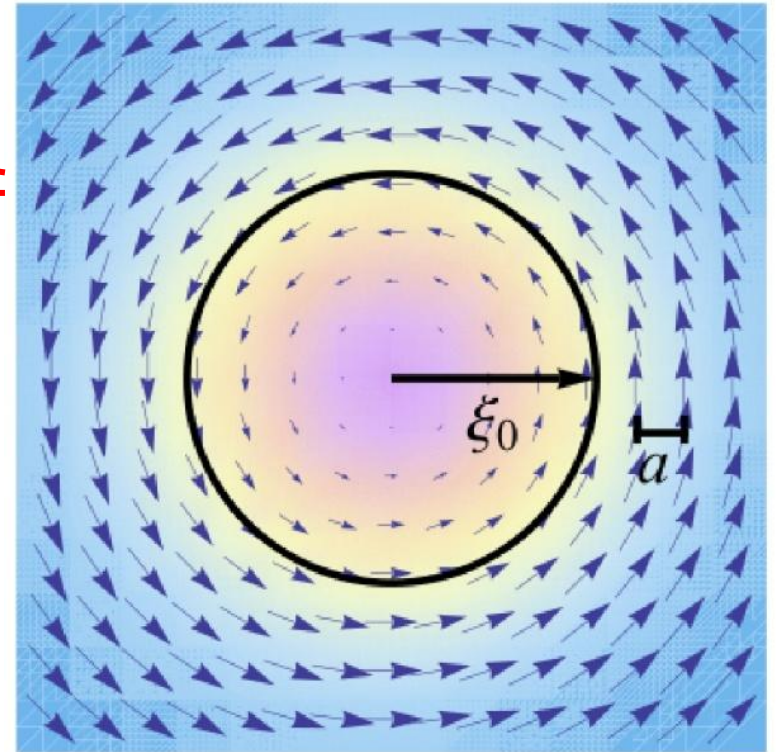
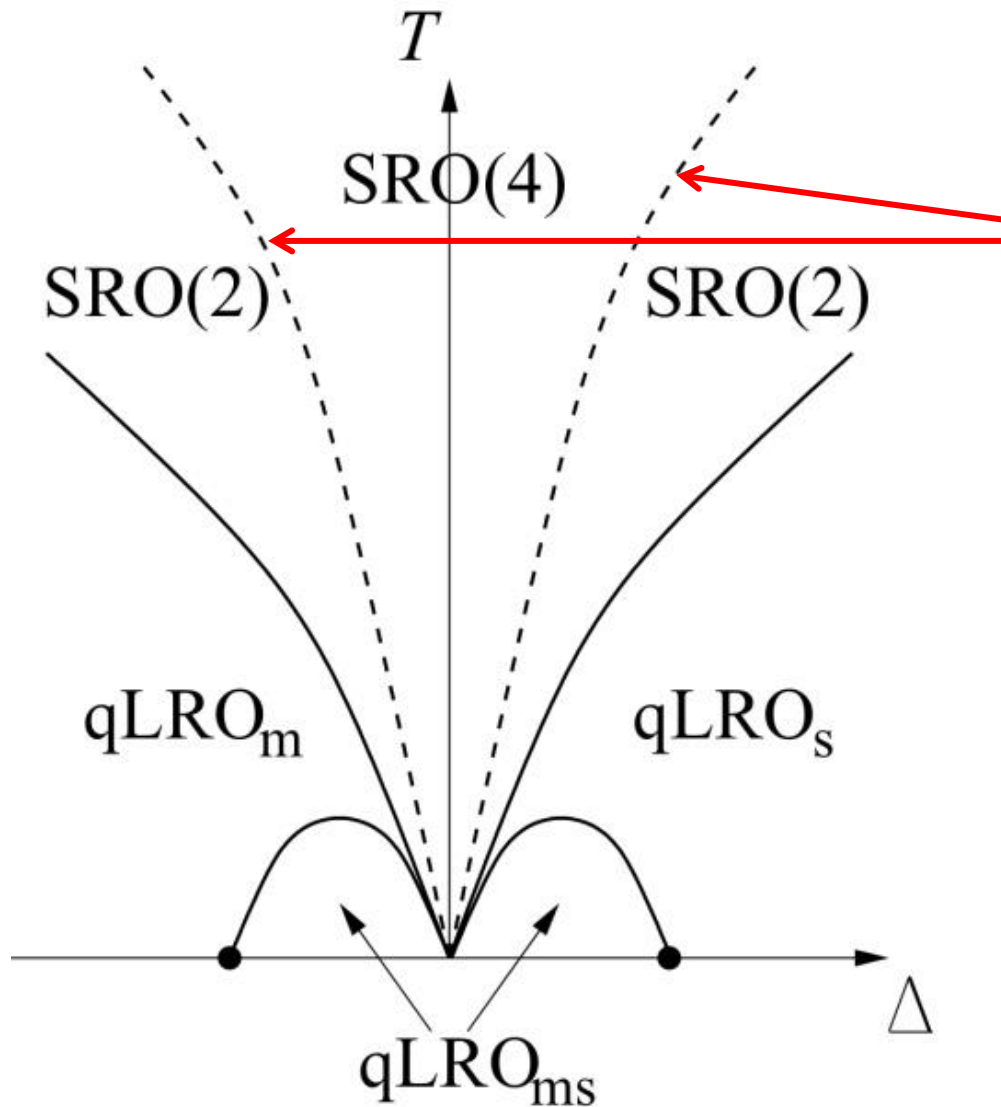
see: D. J. Scalapino, Y. Imry, and P. Pincus, *Phys. Rev. B* **11**, 2042 (1975);  
E. W. Carlson, D. Orgad, S. A. Kivelson, and V. J. Emery, *Phys. Rev. B* **62**, 3422 (2000);  
F. H. L. Essler and A. M. Tsvelik, *Phys. Rev. B* **65**, 115117 (2002).

# $O(2) \times O(2)$ NL $\sigma$ M: Jaefari, Lal, and Fradkin



adapted from A. Jaefari, S. Lal, and E. Fradkin, *Phys. Rev. B* **82**, 144531 (2010)

# $O(2) \times O(2)$ : conjectured phase diagram



# Conclusions

- It is possible, at least in  $(2+1)$ -dim. systems, for finite-temperature phase transitions to be suppressed to  $T=0$  without connecting to a quantum critical point.
- The  $O(2) \times O(2)$  nonlinear sigma model is probably an example of this.
- Conjecture: in  $2+1$  dimensions, even when a finite-temperature line connects to a quantum critical point, it may not necessarily be 'controlled' by it.