Quantum mechanics without quasiparticles

New Directions in Theoretical Physics Higgs Centre, University of Edinburgh January 9, 2014

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An even number of electrons per unit cell









Famous examples:

The <u>fractional quantum Hall</u> effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge.

Famous examples:

Electrons in one dimensional wires form the <u>Luttinger liquid</u>. The quanta of density oscillations ("phonons") are a *quasiparticle* basis of the lowenergy Hilbert space. Similar comments apply to magnetic insulators in one dimension.

Outline

I. The simplest models without quasiparticles A. Superfluid-insulator transition of ultracold bosons in an optical lattice B. Conformal field theories in 2+1 dimensions and the AdS/CFT correspondence 2. Metals without quasiparticles "Nematic" order in the high temperature superconductors

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2+1 dimensions and

the AdS/CFT correspondence

2. Metals without quasiparticles

"Nematic" order in the high

temperature superconductors

Superfluid-insulator transition



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature 415, 39 (2002).







$$S = \int d^{2}r dt \left[|\partial_{t}\Psi|^{2} - c^{2}|\nabla_{r}\Psi|^{2} - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_{c})|\Psi|^{2} + u \left(|\Psi|^{2}\right)^{2}$$

$$\langle \Psi \rangle \neq 0$$

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$$\int c \left(\frac{|\Psi|^{2}}{|\Psi|^{2}} + \frac{|\Psi|^{2}$$

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Particles and holes correspond
to the 2 normal modes in the
oscillation of Ψ about $\Psi = 0$.

$$\langle \Psi \rangle \neq 0$$
Superfluid
$$\langle \Psi \rangle = 0$$
Insulator
$$\langle \Psi \rangle = 0$$

$$\int_{\Delta c}$$

Insulator (the vacuum) at large repulsion between bosons

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Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

Friday, January 10, 14

















Electrical transport in a free quasiparticle CFT3 for T > 0



Quasiparticle view of quantum criticality: Electrical transport for a (weakly) interacting CFT3


Quasiparticle view of quantum criticality: Electrical transport for a (weakly) interacting CFT3



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Quasiparticle view of quantum criticality: Electrical transport for a (weakly) interacting CFT3



The dynamics of quantum criticality via Quantum Monte Carlo and holography

William Witczak-Krempa, Erik Sorensen, Subir Sachdev

(Submitted on 11 Sep 2013 (v1), last revised 29 Nov 2013 (this version, v2))

Understanding the real time dynamics of quantum systems without quasiparticles constitutes an important yet challenging problem. We study the superfluid-insulator quantum-critical point of bosons on a two-dimensional lattice, a system whose excitations cannot be described in a quasiparticle basis. We present detailed quantum Monte Carlo results for two separate lattice realizations: their low-frequency conductivities are found to have the same universal dependence on imaginary frequency and temperature. We then use the structure of the real time dynamics of conformal field theories described by the holographic gauge/gravity duality to make progress on the difficult problem of analytically continuing the Monte Carlo data to real time. Our method yields quantitative and experimentally testable results on the frequency-dependent conductivity near the quantum critical point, and on the spectrum of quasinormal modes in the vicinity of the superfluid-insulator quantum phase transition. Extensions to other observables and universality classes are discussed.

arXiv.org > cond-mat > arXiv:1309.5635

Condensed Matter > Strongly Correlated Electrons

Universal Conductivity in a Two-dimensional Superfluid-to-Insulator Quantum Critical System

Kun Chen, Longxiang Liu, Youjin Deng, Lode Pollet, Nikolay Prokof'ev

(Submitted on 22 Sep 2013)

We compute the universal conductivity of the (2+1)-dimensional XY universality class, which is realized for a superfluid-to-Mott insulator quantum phase transition at constant density. Based on large-scale Monte Carlo simulations of the classical (2+1)-dimensional *J*-current model and the two-dimensional Bose-Hubbard model, we can precisely determine the conductivity on the quantum critical plateau, $\sigma(\infty) = 0.359(4)\sigma_Q$ with σ_Q the conductivity quantum. The universal conductivity is the schoolbook example of where the AdS/CFT correspondence from string theory can be tested and made to use. The shape of our $\sigma(i\omega_n) - \sigma(\infty)$ function in the Matsubara representation is accurate enough for a conclusive comparison and establishes the particle-like nature of charge transport. We find that the holographic gauge/gravity duality theory for transport properties can be made compatible with the data if temperature of the horizon of the black brane is different from the temperature of the conformal field theory. The requirements for measuring the universal conductivity in a cold gas experiment are also determined by our calculation.

Search or

Quantum Monte Carlo for lattice bosons



FIG. 2. Quantum Monte Carlo data (a) Finite-temperature conductivity for a range of βU in the $L \to \infty$ limit for the quantum rotor model at $(t/U)_c$. The solid blue squares indicate the final $T \to 0$ extrapolated data. (b) Finite-temperature conductivity in the $L \to \infty$ limit for a range of L_{τ} for the Villain model at the QCP. The solid red circles indicate the final $T \to 0$ extrapolated data. The inset illustrates the extrapolation to T = 0 for $\omega_n/(2\pi T) = 7$. The error bars are statistical for both a) and b).

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941 See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

Quantum Monte Carlo for lattice bosons



W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941 See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

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AdS/CFT correspondence





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Compute scattering matrix elements of quasiparticles (or of collective modes)

These parameters are input into a quantum Boltzmann equation

Deduce dissipative and dynamic properties at nonzero temperatures

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Solve Einsten-Maxwell equations. Dynamics of quasinormal modes of black branes.

AdS₄ theory of quantum criticality

Most general effective holographic theory for linear charge transport with 4 spatial derivatives:

$$\begin{aligned} \mathcal{S}_{\text{bulk}} &= \frac{1}{g_M^2} \int d^4 x \sqrt{g} \left[\frac{1}{4} F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd} \right] \\ &+ \int d^4 x \sqrt{g} \left[-\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right], \end{aligned}$$

This action is characterized by 3 dimensionless parameters, which can be linked to data of the CFT (OPE coefficients): 2-point correlators of the conserved current J_{μ} and the stress energy tensor $T_{\mu\nu}$, and a 3-point T, J, J correlator. Constraints from both the CFT and the gravitational theory bound $|\gamma| \leq 1/12 = 0.0833$.

R. C. Myers, S. Sachdev, and A. Singh, *Phys. Rev. D* 83, 066017 (2011)
D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, *Phys. Rev. B* 87, 085138 (2013)

AdS₄ theory of quantum criticality



Good agreement between high precision Monte Carlo for imaginary frequencies, and holographic theory after rescaling effective T and taking $\sigma_Q = 1/g_M^2$.

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941 See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

AdS₄ theory of quantum criticality



Predictions of holographic theory, after analytic continuation to real frequencies

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941 See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

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Iron pnictides:

a new class of high temperature superconductors







Physical Review B 81, 184519 (2010)



Physical Review B 81, 184519 (2010)







S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010) S. Kasahara, H.J. Shi, K. Hashimoto, S. Tonegawa, Y. Mizukami, T. Shibauchi, K. Sugimoto, T. Fukuda, T. Terashima, A.H. Nevidomskyy, and Y. Matsuda, *Nature* **486**, 382 (2012).



Quantum criticality of Ising-nematic ordering in a metal



A metal with a <u>Fermi surface</u> with full square lattice symmetry

Quantum criticality of Ising-nematic ordering in a metal



Spontaneous elongation along y direction:


Spontaneous elongation along x direction:

Ising-nematic order parameter

$$\phi \sim \int d^2 k \left(\cos k_x - \cos k_y\right) c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma}$$

Measures spontaneous breaking of square lattice point-group symmetry of underlying Hamiltonian



Spontaneous elongation along x direction: Ising order parameter $\phi > 0$.



Spontaneous elongation along y direction: Ising order parameter $\phi < 0$.



Pomeranchuk instability as a function of coupling r

Quantum criticality of Ising-nematic ordering















Theory of transport without quasiparticles (inspired by holography):

• Formulate a continuum theory with a conserved momentum.

S. Hartnoll, R. Mahajan, M. Punk, and S. Sachdev, to appear

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- We can then relate the momentum relaxation rate to the resistivity via hydrodynamic/memory matrix methods.
- All steps above can also be implemented in holographic models, and consistent results are obtained *i.e.* solution of gravitational equations provides results consistent with hydrodynamics, and with the breakdown of hydrodynamics due to perturbations that violate momentum conservation.

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Exciting recent progress on the description of transport in metallic states without quasiparticles, via field theory and holography