Vacuum decay and the end of the universe

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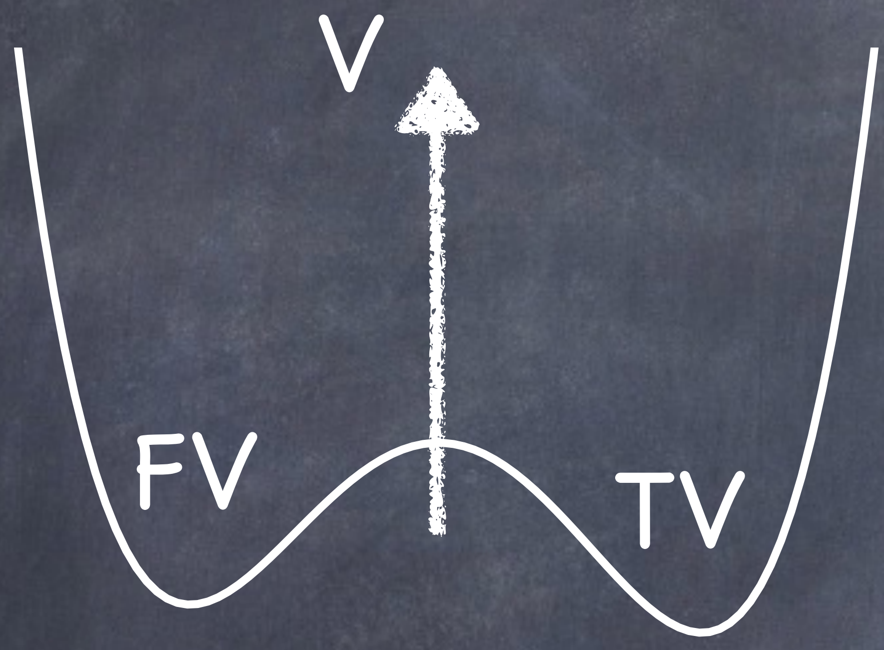
Outline

- metastable vacuum decay
- in curved space
- seeded nucleation*
- late universe

*Ruth Gregory, Ian Moss, Ben Withers, arXiv:1401.0017

Metastable-vacuum decay



Coleman: vacuum decay can be described by an instanton $\phi(r^2+\tau^2)$ representing the nucleation of a bubble of the true vacuum.

Callan,Coleman, Phys Rev D15 2929 1977

Nucleation rate

The bubble nucleation rate is determined by the Euclidean action, $\Gamma = Ae^{-B}$ $B = I_b - I_v$

In the thin wall approximation

$$B = 2\pi^2 R^3 \sigma - \frac{1}{2}\pi^2 R^4 \epsilon$$

Energy density ε Tension $\sigma = \int (2V)^{1/2} d\phi$

Bubble radius $R_c = 3\sigma/\epsilon$

Thermal bubbles

$$B = 4\pi\beta R^2\sigma - \frac{4}{3}\pi\beta R^3\epsilon$$

B changes by a factor R_C/T - but ϵ changes

In curved space

The decay time can be larger than the Hubble time and then we get inflation.

In an expanding universe, both first and second order phase transitions can supercool.

old inflation

(Guth)

new inflation

(Linde,Albrecht,Steinhart)

CDL instanton

Compexify the time in the scalar field and the metric: Coleman De Luccia instanton.

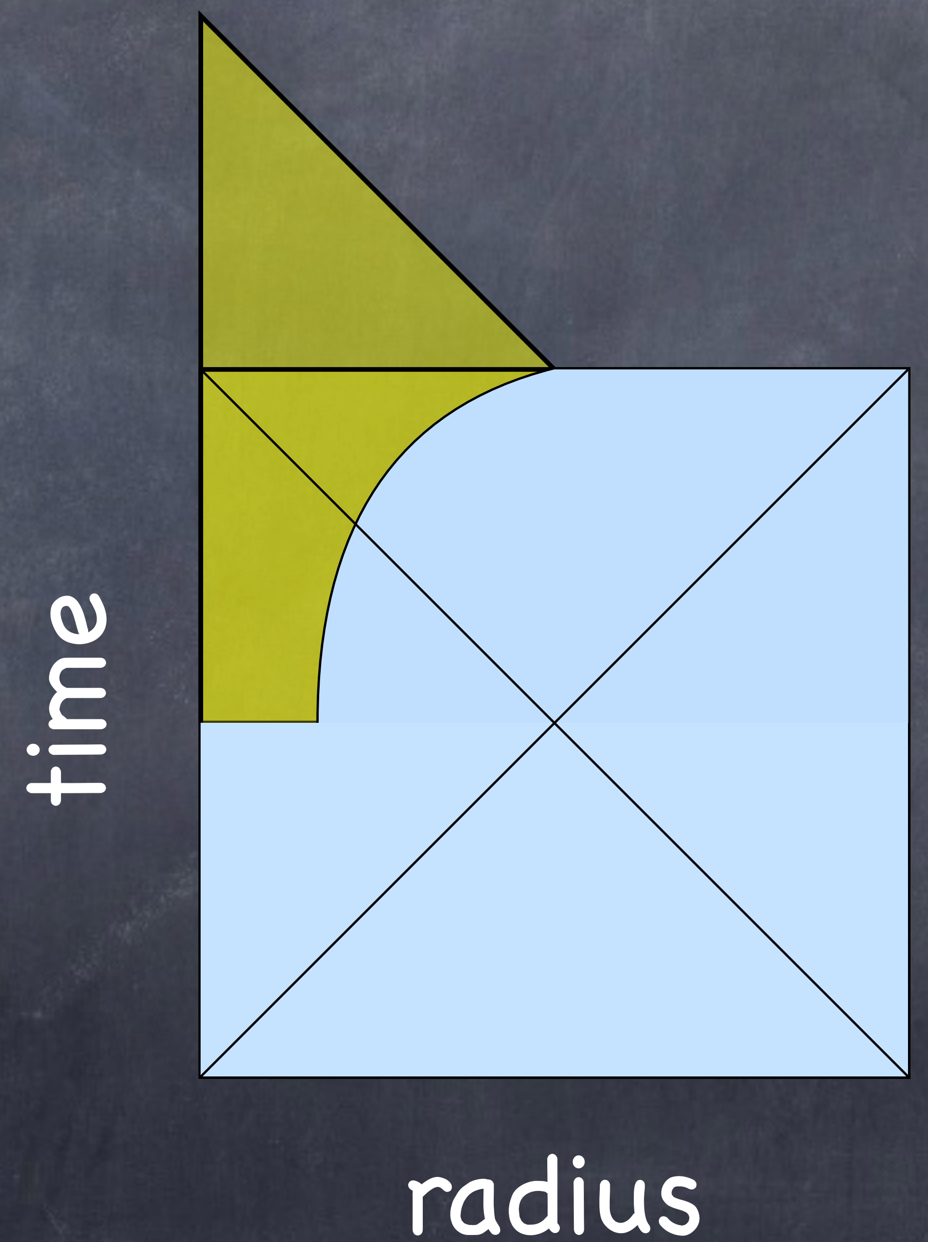
False vacuum 4-sphere

CDL instanton

$$B_{CDL} = \frac{\pi\sigma^4}{2\epsilon^3}(1 + 4\bar{\sigma}^2 l^2)^{-2} \quad \bar{\sigma} l = (8\pi G\sigma^2/16\epsilon)^{1/2}$$

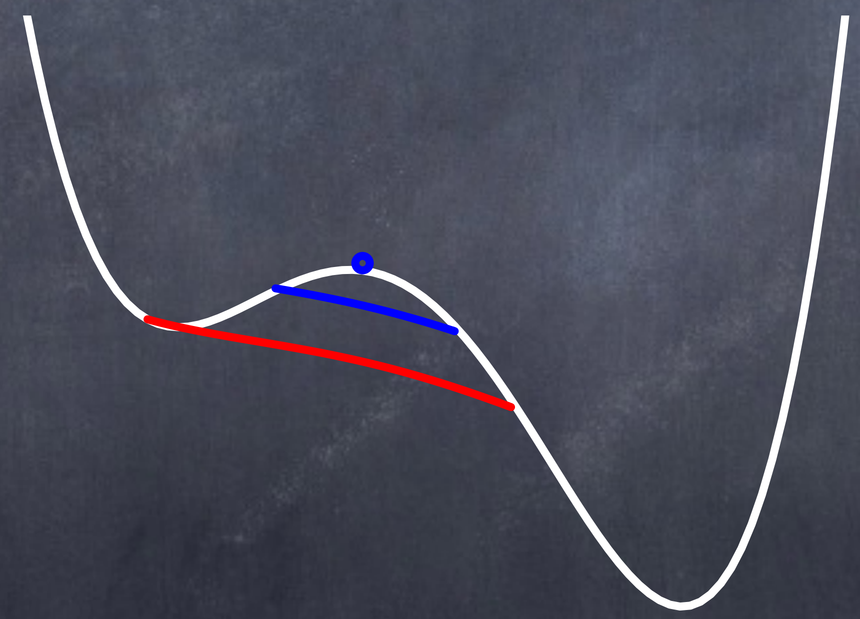
Coleman, De Luccia Phys Rev D21 3305 1980

Lorentzian picture



Thick wall limit

Can have a 'weakly' first order phase transition by reducing the barrier height

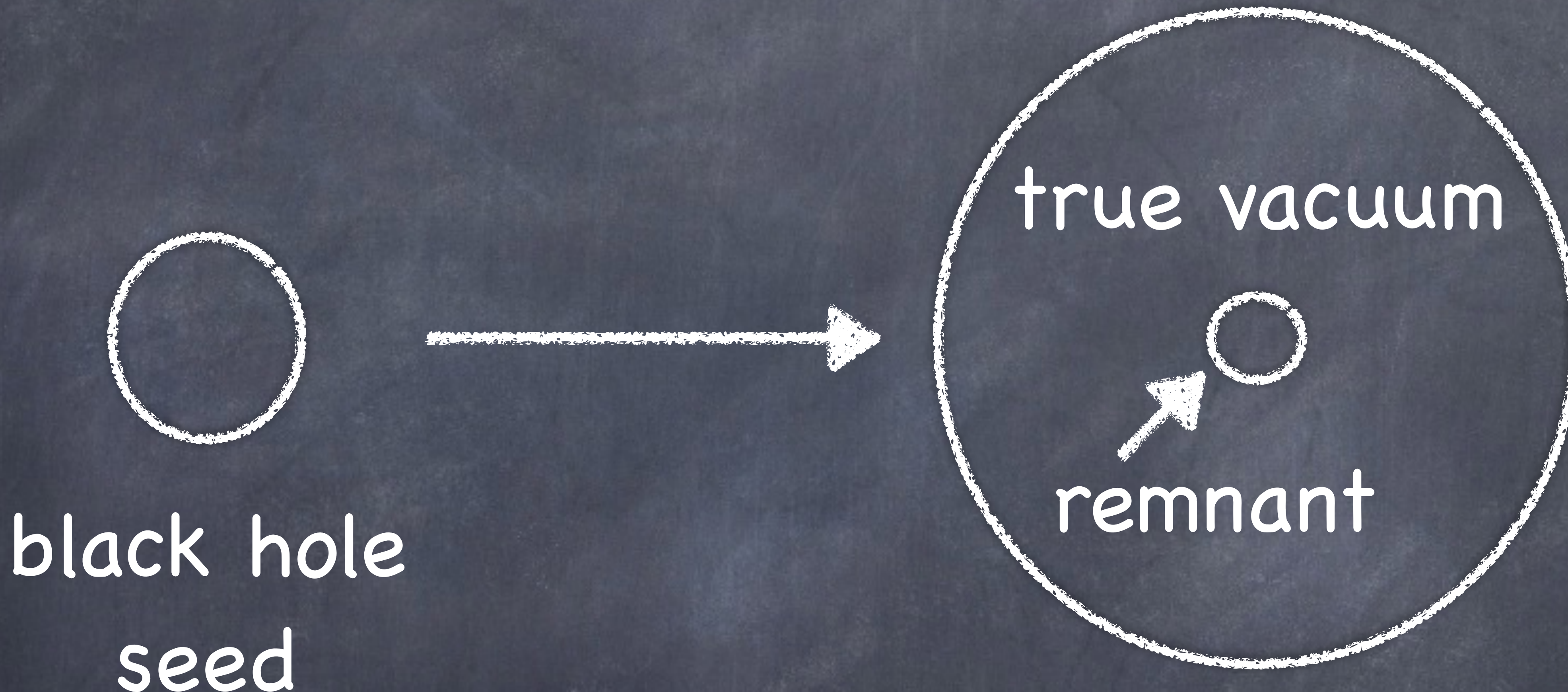


Seeded nucleation

Most supercooled phase transitions are nucleated by impurities and imperfections.

Hiscock, Phys Rev D35 1161 1987; Gregory, Moss, Withers 1401.0017

Black hole seeds



Hiscock, Phys Rev D35 1161 1987; Gregory, Moss, Withers 1401.0017

Suppose bubbles nucleate in the false vacuum on de Sitter space-time with a black hole of mass M and de Sitter radius $l^2 = 3/(8\pi G\epsilon)$

In static coordinates

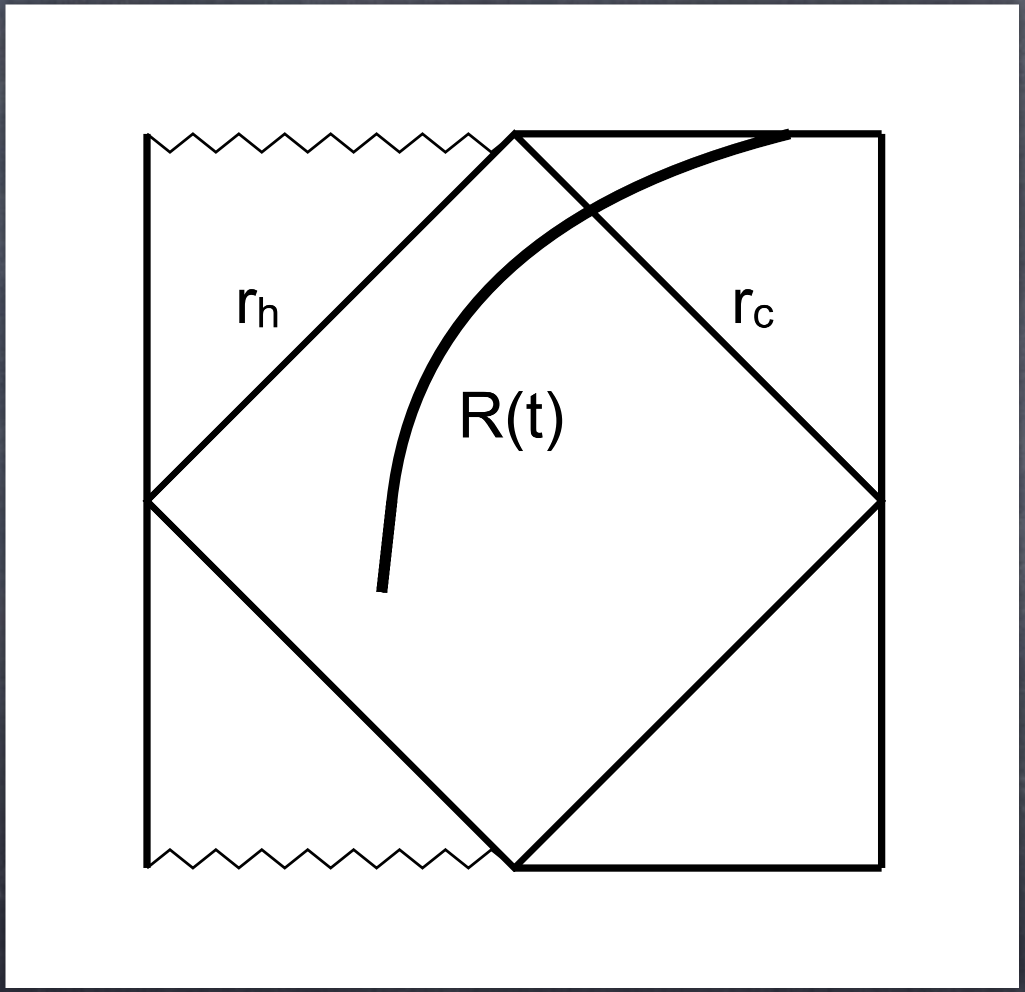
$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

where

$$f(r) = 1 - \frac{2GM}{r} - \frac{r^2}{l^2}$$

In the thin wall limit the wall is at R(t)

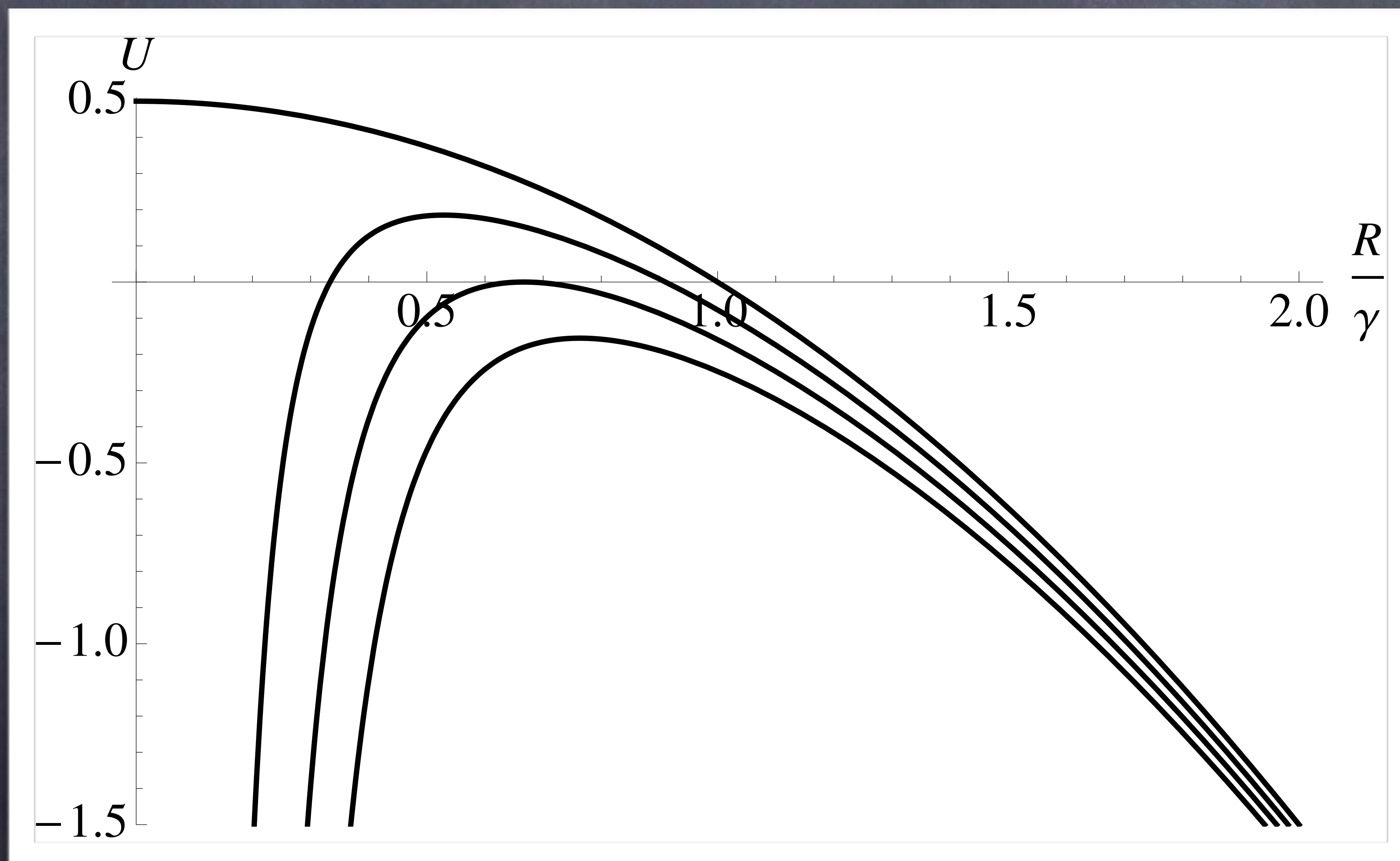
Lorenzian spacetime



Static patch or
causal diamond

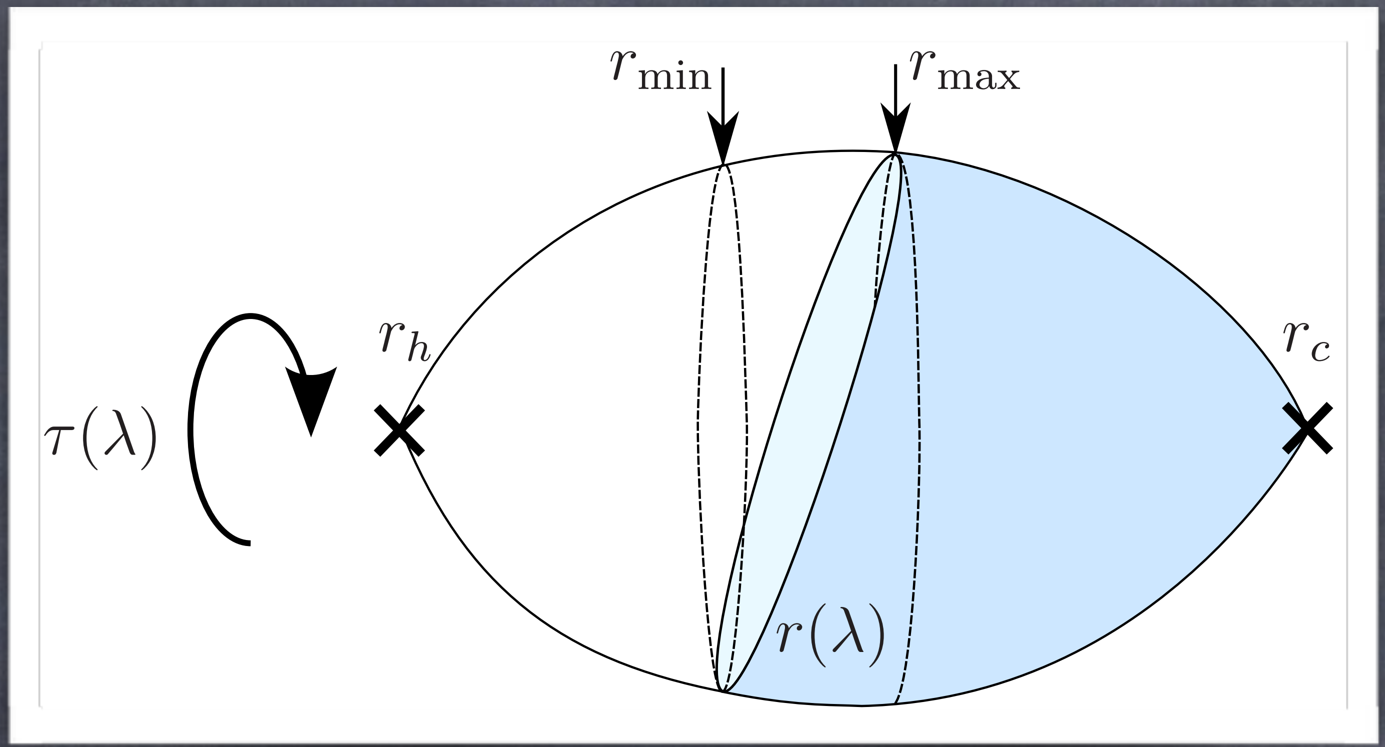
Integrate the Einstein equations to get R(t)

$$\dot{R}(t)^2 + U(R) = 0$$



Parameters: M_S, M_R, ε, σ Static case M_S^*

The instanton is obtained by sending t to -iτ



As usual, assume $\Gamma = Ae^{-B}$ $B = I_b - I_v$

- This gives the correct result for the CDL case.
- In the static case, action is independent of the conical deficit.
- A WKB argument backs up the result.

General case

There is a critical seed mass

$$M_C = \frac{16\pi}{27}\frac{\sigma^3}{\epsilon^2}(1+4\bar{\sigma}^2 l^2)^{-2}$$

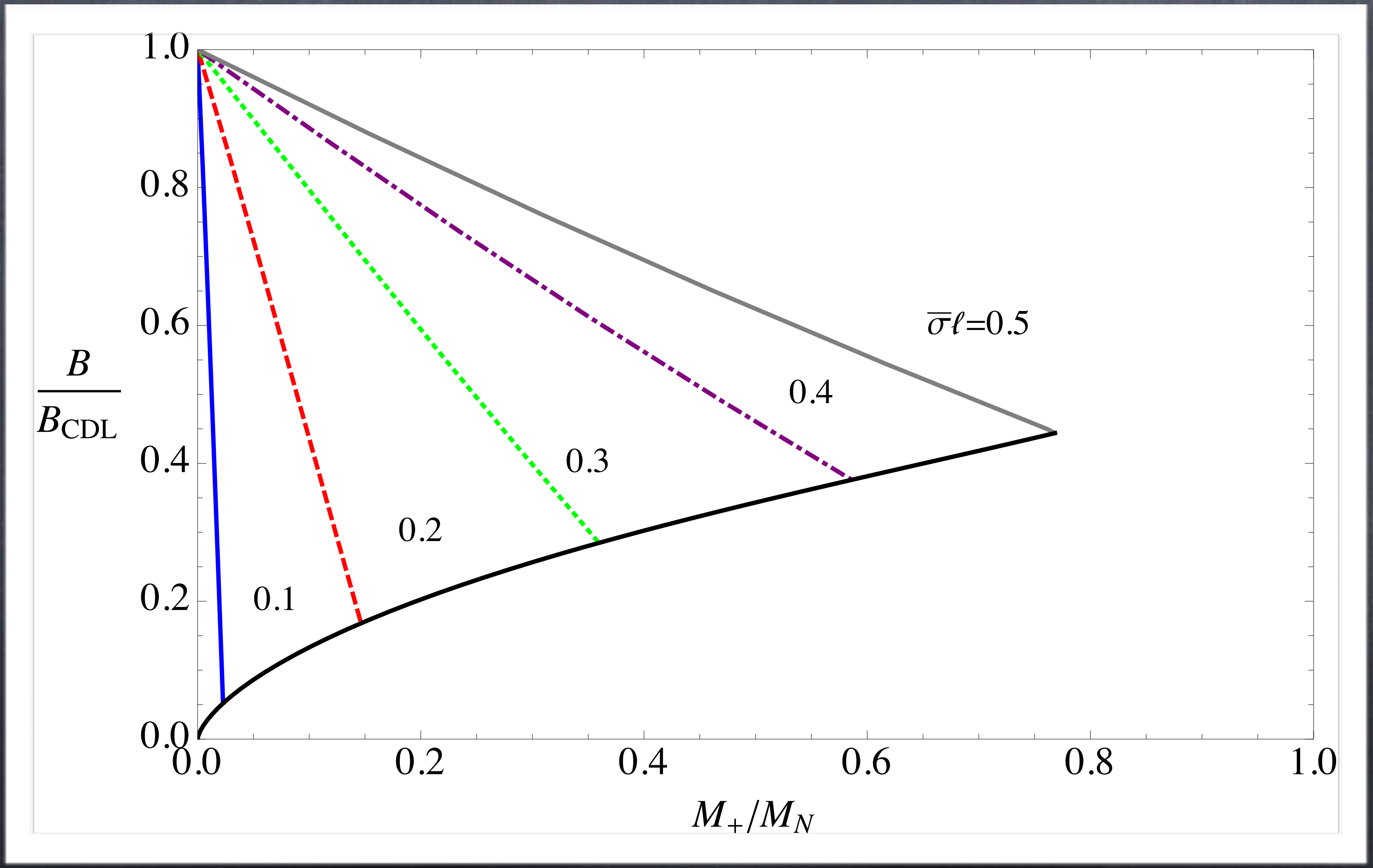
$M_S < M_C$: no remnant

$M_S > M_C$: remnant with mass $M_R^{\ast}$

$\bar{\sigma} l = (8\pi G\sigma^2/16\epsilon)^{1/2}$

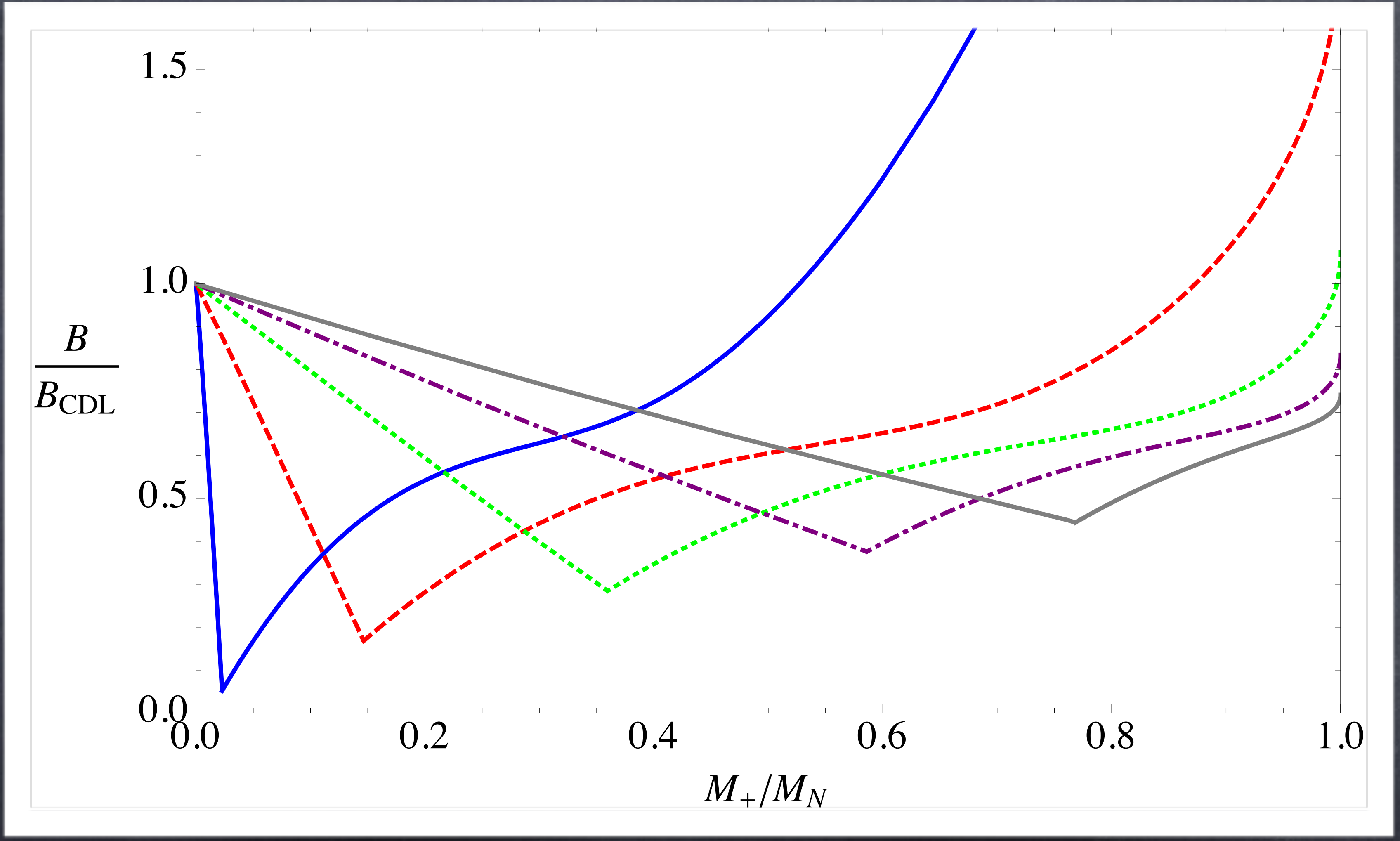
The nucleation rate in enhanced compared to the CDL case

$M_S < M_C$



$$GM_N = l/\sqrt{27}$$

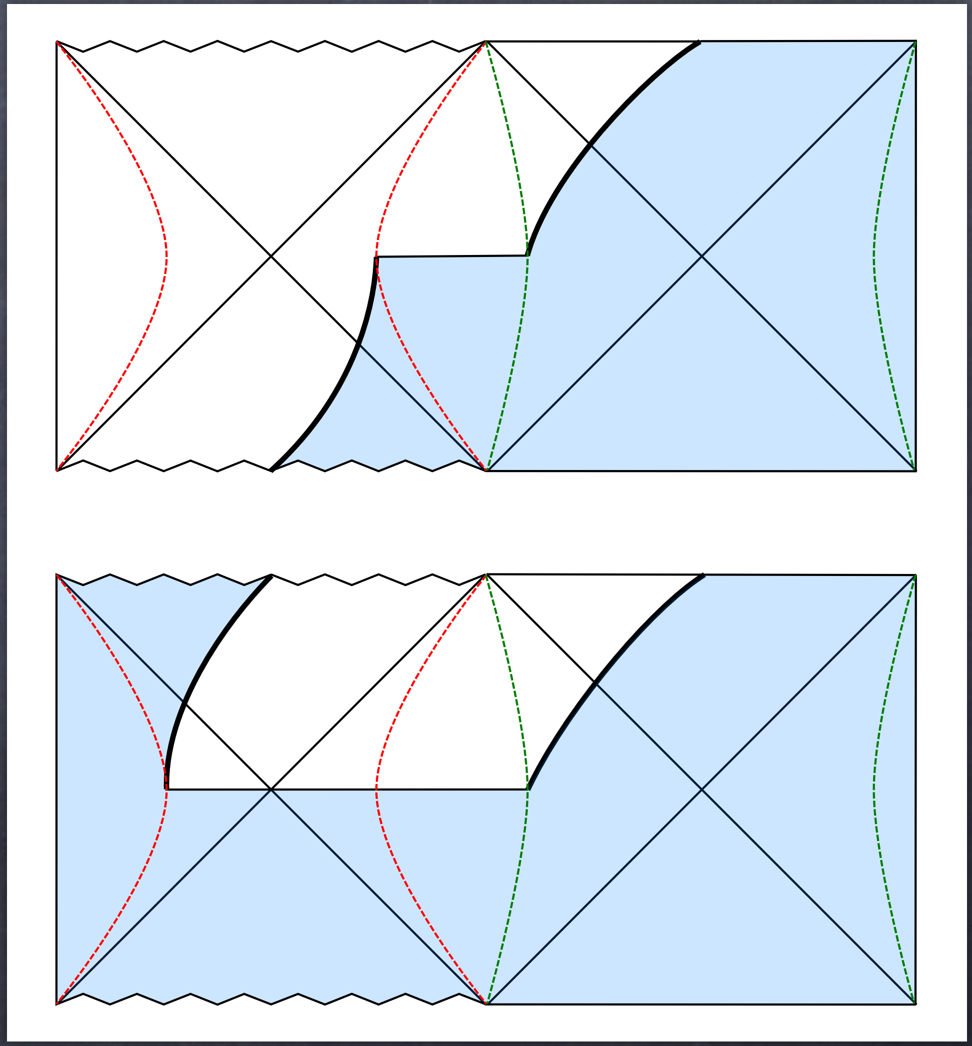
$M_S > M_C$



WKB approach

$$\dot{R}(t)^2 + U(R) = 0 \longrightarrow \{P^2 + U(R)\}\Psi = 0$$

Tunnelling probability is given by the same instanton.



Conjecture that there is a crossing symmetry.

Farhi,Guth,Guven, Nucl Phys B339 417 1990,
Fischler,Morgan,Polchinski Phys Rev D42 4042 1990

Evaporation vs nucleation

For the static bubble with no remnant, the nucleation exponent is $B^*=4\pi G\, M^{*2}$.

The nucleation pre-factor A contains $\sqrt{(B/2\pi)}$ for each zero mode and determinant $1/(GM^*)$:

$$\Gamma_* = \sqrt{2/G}\, e^{-4\pi G M^{*\,2}}$$

Hawking evaporation rates are roughly

$$\Gamma_H = 3.6 \times 10^{-4} (G^2 M^{*\,3})^{-1}$$

$\Gamma_H \approx \Gamma^*$ for masses around the Planck mass.

Correcting for Hawking evaporation

The potential is modified*:

$$V(\phi)_M = V(\phi)_0 + y\phi\langle tt\rangle_M$$

Calculate on a black hole background.

*IGM Phys Rev D32 1333 1985

The late universe

We may live in a metastable vacuum.

Phase transitions can be triggered by thermal processes–but nature provides limits

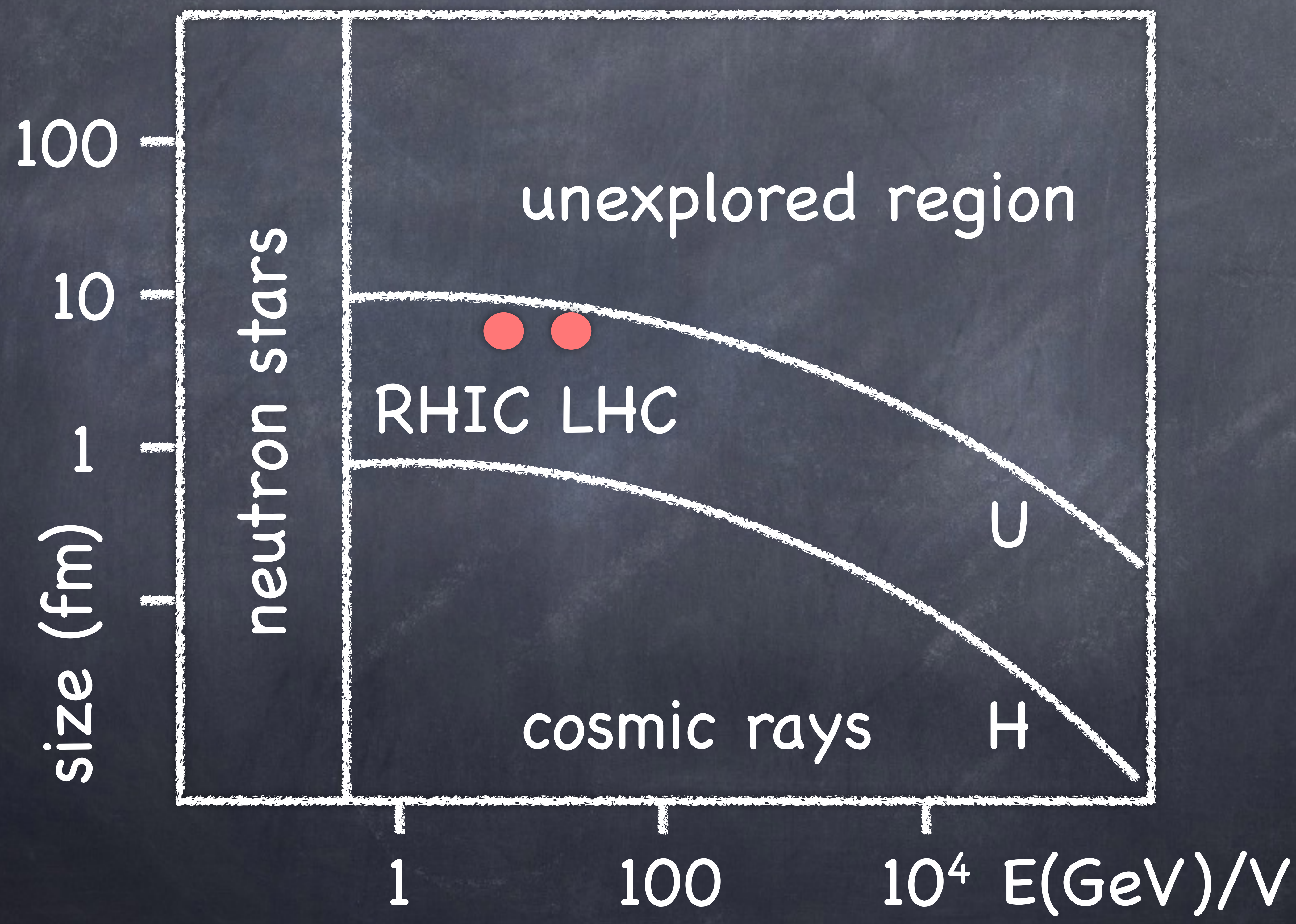
Size matters: high temperatures can stabilise the vacuum.

Shape matters: spherical shapes are optimal for transitions.

Cosmic rays: 4000 FeFe TeV collisions per cubic parsec per second*.

*Hut, Rees, Nature 302 508 1983, Buzra et al Rev Mod Phys 72 1125 2000

Safe regions



Outlook

- What is the theory of vacuum decay in curved space-time (beyond just an analogy)?
- Do singular instantons play a role?
- How stable is the string landscape?
- Can black holes seed nucleation in low Planck mass scenarios?