Stratifying On-Shell Cluster Varieties

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Niels Bohr Institute

based on work in collaboration with

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N. Arkani-Hamed, F. Cachazo, A. Goncharov, A. Postnikov, and J. Trnka

[arXiv:1911.09106] [arXiv:1909.09131] [arXiv:1608.00006] [arXiv:1412.8475] [arXiv:1212.5605]

Enormous Advances in Understanding Scattering Amplitudes

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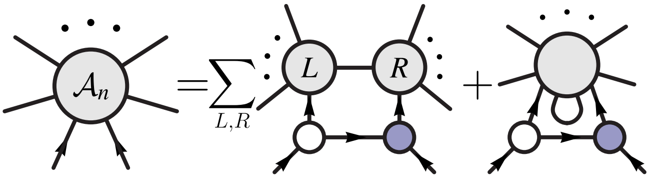

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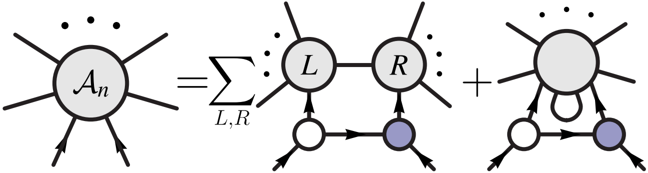

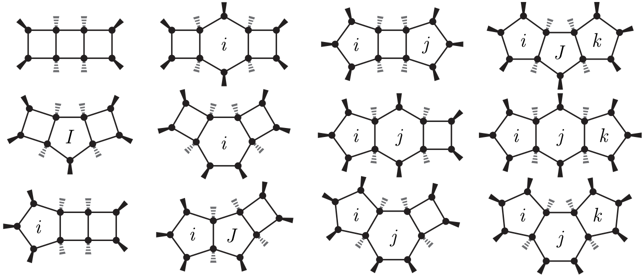

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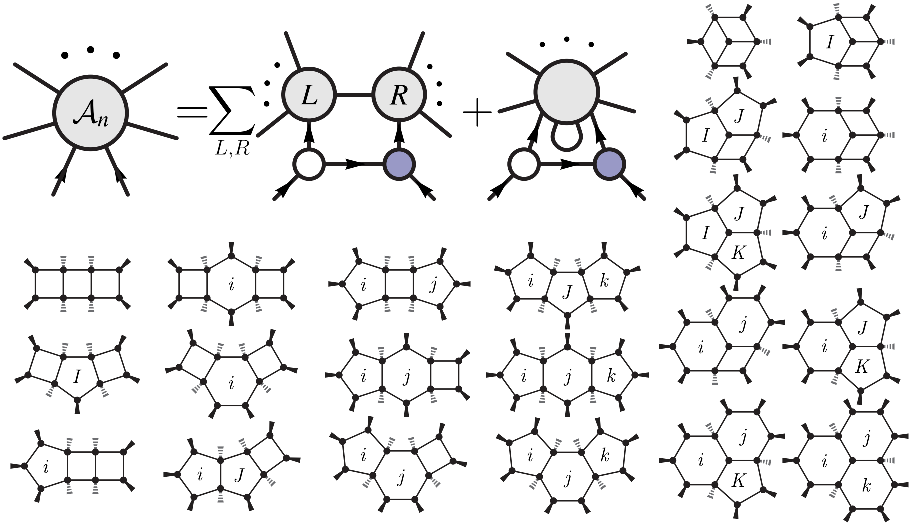

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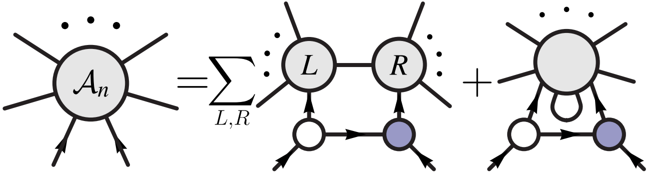

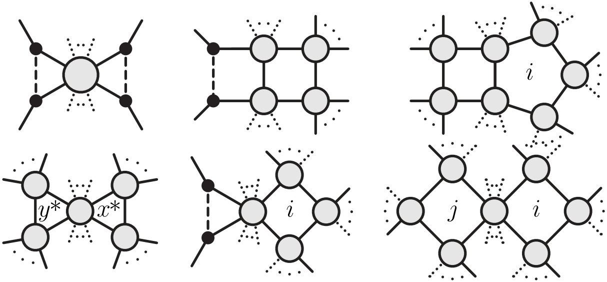

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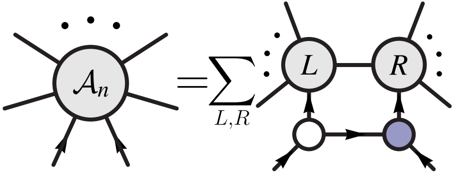

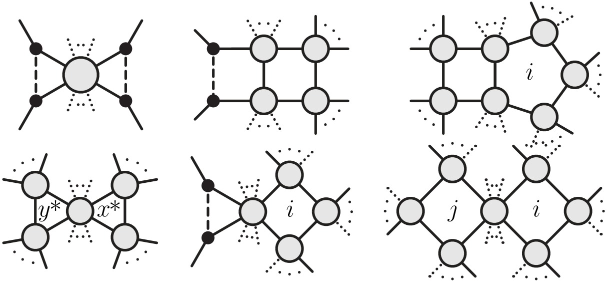

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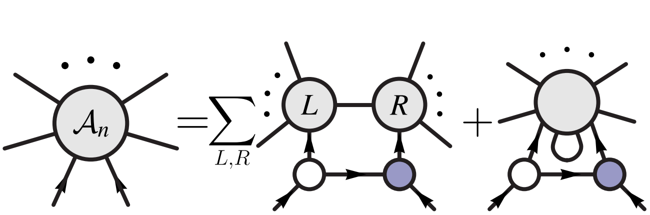

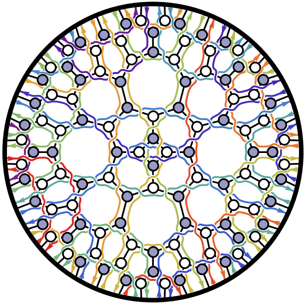

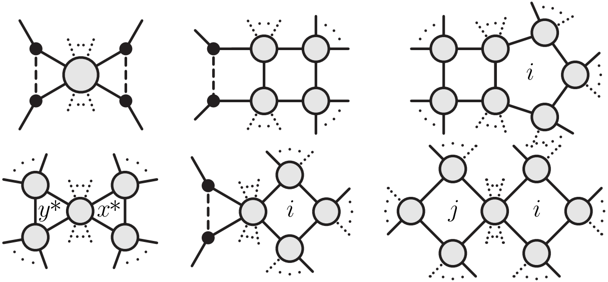

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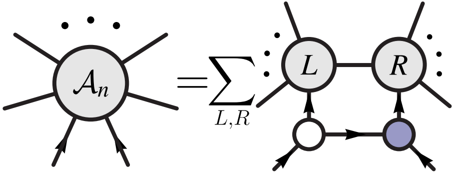

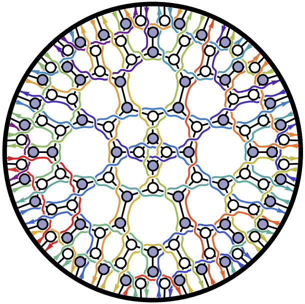

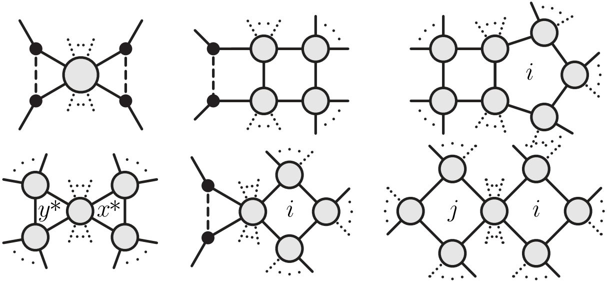

On-Shell Physics/Grassmannian Geometry Correspondence

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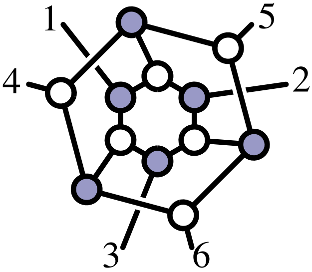

On-Shell Physics/Grassmannian Geometry Correspondence

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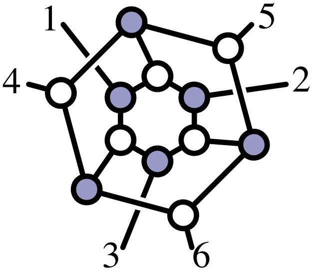

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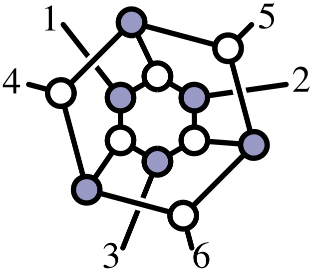

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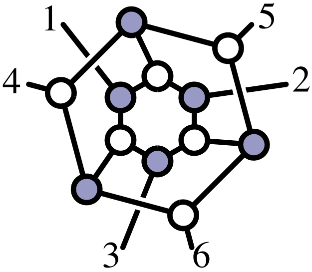

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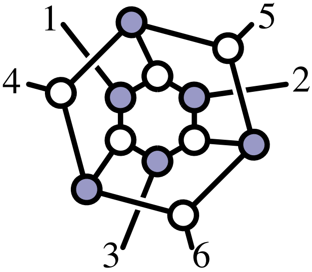

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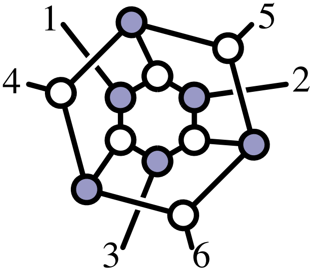

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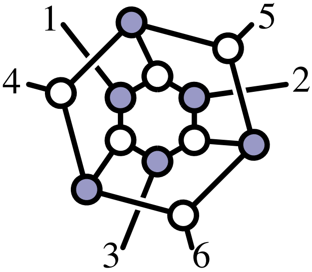

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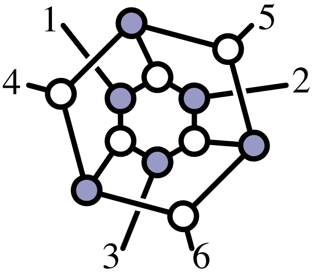

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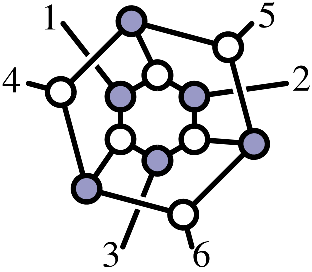

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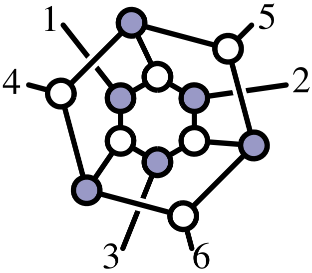

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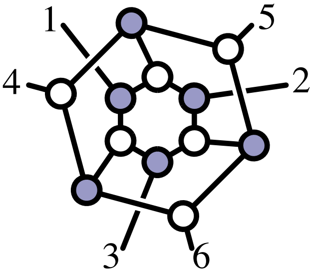

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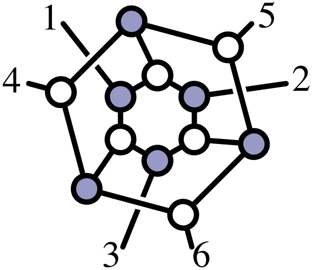

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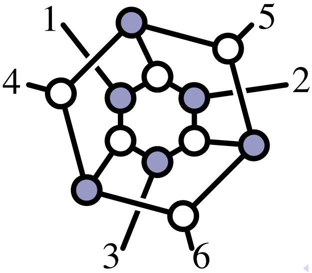

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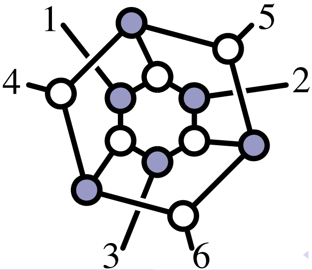

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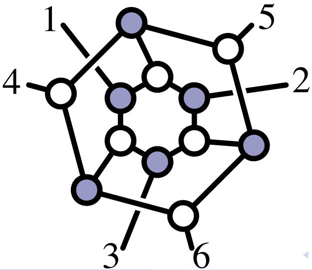

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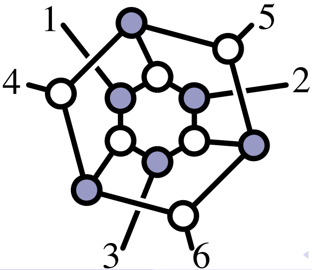

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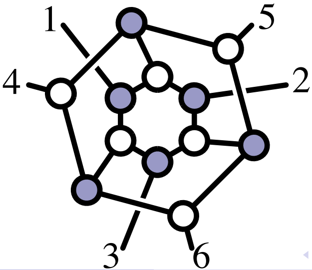

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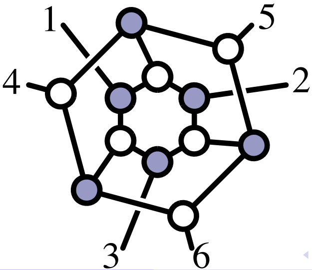

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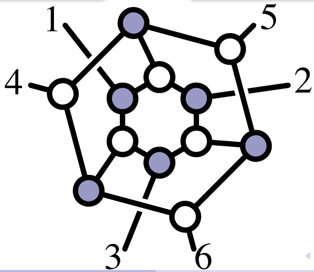
On-Shell Physics

$\Leftrightarrow$

Grassmannian Geometry



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On-Shell Physics

- on-shell diagrams

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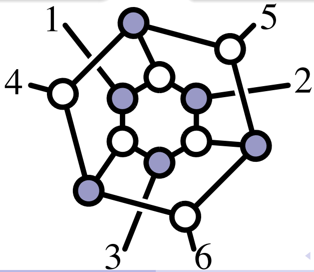

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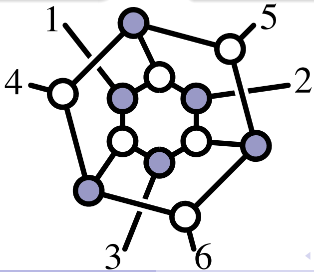
On-Shell Physics

- on-shell diagrams

$\Leftrightarrow$

Grassmannian Geometry

- $\{$strata $C \in G(k,n)$, volume-form $\Omega_C\}$



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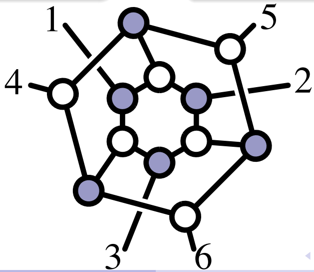

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On-Shell Physics

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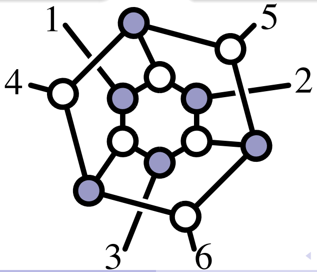
Grassmannian Geometry

- $\{$strata $C \in G(k,n)$, volume-form $\Omega_C\}$
- volume-preserving diffeomorphisms



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On-Shell Physics

- on-shell diagrams
- physical symmetries

$\Leftrightarrow$

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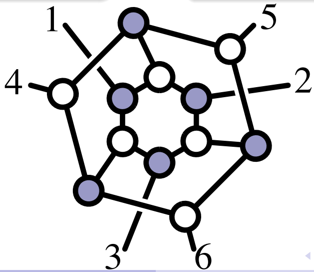

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On-Shell Physics

- on-shell diagrams
- physical symmetries
  - trivial symmetries (identities)

$\Leftrightarrow$

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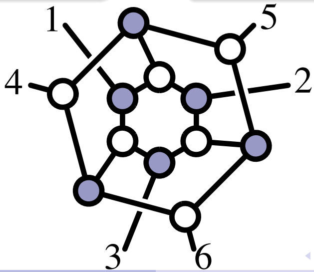

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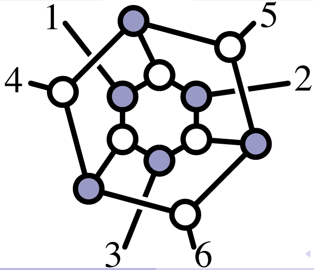
$\Leftrightarrow$

Grassmannian Geometry

- $\{$strata $C \in G(k,n)$, volume-form $\Omega_C\}$

- volume-preserving diffeomorphisms
  - cluster coordinate mutations, …



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On-Shell Physics: planar $\mathcal{N}=4$

- on-shell diagrams
- physical symmetries
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$\Leftrightarrow$

Grassmannian Geometry

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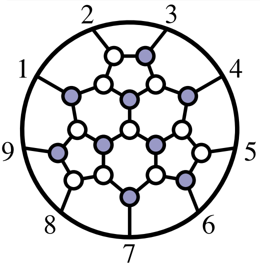

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On-Shell Physics: planar $\mathcal{N}=4$

- on-shell diagrams
  - bi-colored
- physical symmetries
  - trivial symmetries (identities)

$\Leftrightarrow$

Grassmannian Geometry

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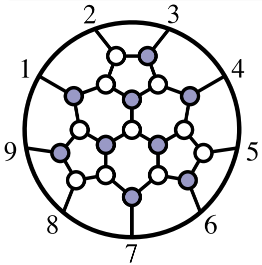

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On-Shell Physics: planar $\mathcal{N}=4$

- on-shell diagrams
  - bi-colored, **un**directed
- physical symmetries
  - trivial symmetries (identities)

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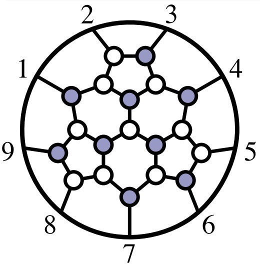

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On-Shell Physics: planar $\mathcal{N}=4$

- on-shell diagrams
  - bi-colored, **un**directed, **planar**
- physical symmetries
  - trivial symmetries (identities)

$\Leftrightarrow$

Grassmannian Geometry

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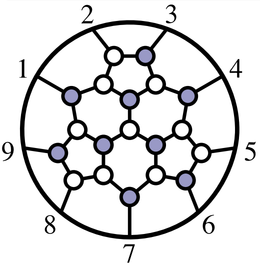

On-Shell Physics/Grassmannian Geometry Correspondence

$$f_\Gamma \equiv \prod_i \Big(\sum_{h_i, q_i} \int d^3\mathrm{LIPS}_i \Big) \prod_v \mathcal{A}_v \equiv \int \Omega_C \; \delta(C, p, h)$$

On-Shell Physics: planar $\mathcal{N}=4$

- on-shell diagrams
  - bi-colored, **un**directed, **planar**
- physical symmetries
  - trivial symmetries (identities)

$\Leftrightarrow$

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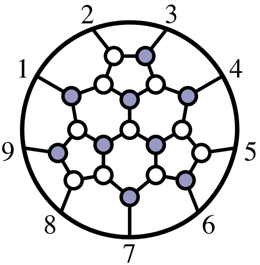

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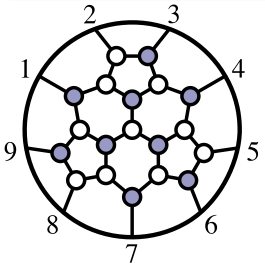

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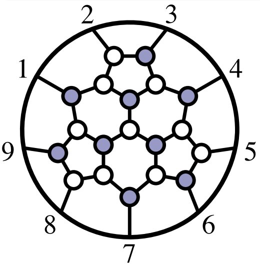

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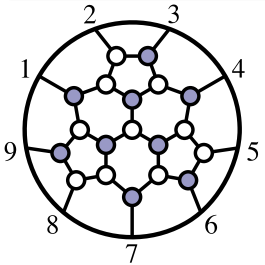

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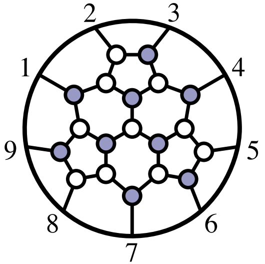

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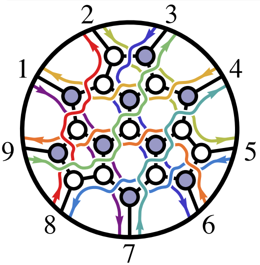

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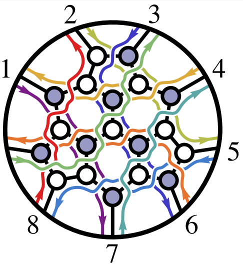

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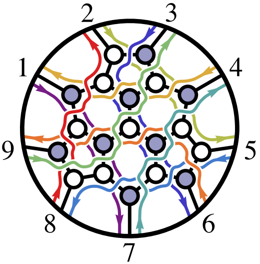

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- $\{$strata $C\in G(k,n)$, volume-form $\Omega_C\}$
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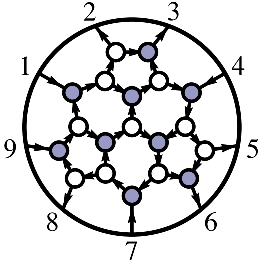

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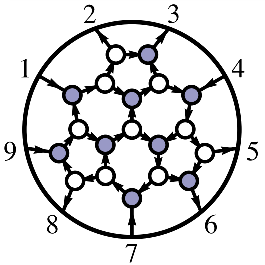

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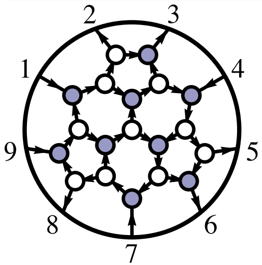

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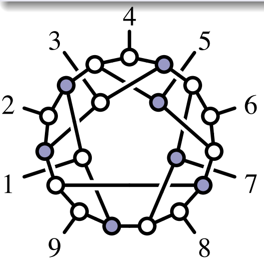

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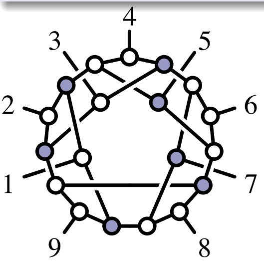

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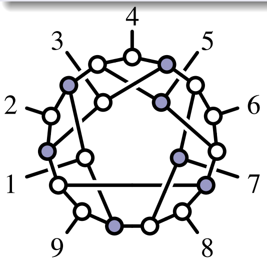

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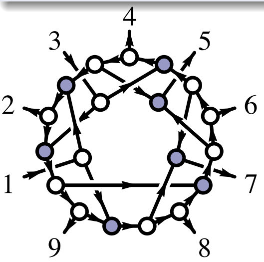


$$C^\perp \equiv \begin{pmatrix} \alpha_1 & 1 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_3 & 1 & 0 & \alpha_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_5 & 1 & \alpha_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & \alpha_7 & \alpha_8 \\ 0 & 0 & 0 & 0 & 0 & \alpha_9 & 1 & \alpha_{10} & 0 \\ \alpha_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_{12} \\ 0 & \alpha_{13} & \alpha_{14} & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Omega_C \equiv \left(\frac{d\alpha_1}{\alpha_1}\wedge\cdots\wedge\frac{d\alpha_{14}}{\alpha_{14}}\right)\times\big(1+\alpha_2\alpha_4\alpha_{13}(\alpha_8+\alpha_7\alpha_{12})\big)^{\mathcal{N}-4}$$

On-Shell Physics/Grassmannian Geometry Correspondence

$$f_\Gamma \equiv \prod_i \Big(\sum_{h_i, q_i} \int d^3\mathrm{LIPS}_i \Big) \prod_v \mathcal{A}_v \equiv \int \Omega_C \; \delta(C, p, h)$$

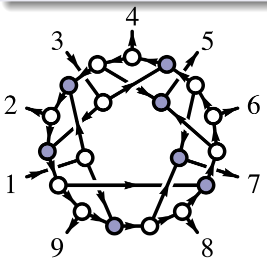
On-Shell Physics: non-planar $\mathcal{N}<4$

- on-shell diagrams
  - bi-colored, **directed** , **non**-planar
- physical symmetries: **?**
  - trivial symmetries (identities)

$\Longleftrightarrow$

Grassmannian Geometry

- $\{$strata $C \in G(k,n)$, volume-form $\Omega_C\}$
  - cluster variety(?), $\big(\prod_i \frac{d\alpha_i}{\alpha_i}\big) \times \mathcal{J}^{\mathcal{N}-4}$
- volume-preserving diffeomorphisms
  - cluster coordinate mutations, …



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Organization and Outline

1. A New Class of Physical Observables: On-Shell Functions
   - Beyond (Mere) Scattering Amplitudes: On-Shell Functions
   - Physically Observable Data Describing Massless Particles in 4d
   - Basic Building Blocks: S-Matrices for Three Massless Particles
2. Building-Up the Grassmannian Correspondence: On-Shell Varieties
   - *Grassmannian* Representations of On-Shell Functions
   - Iterative Construction of Grassmannian 'On-Shell' Varieties
   - Characteristics of Grassmannian Representations
3. The Classification of On-Shell (Cluster) Varieties
   - Warm-Up: Classifying On-Shell Functions of $G(2,n)$
   - Exploration: the Stratification of On-Shell Varieties in $G(3,6)$
4. More to Explore: 'Color-Dressed' On-Shell Functions
   - Color-Dressed On-Shell Diagrams in sYM
   - Application: All Two-Loop '$G(2,n)$' Amplitudes in sYM

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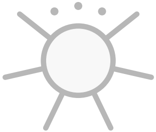

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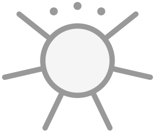

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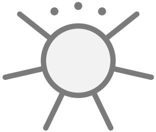

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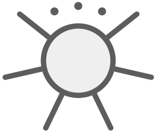

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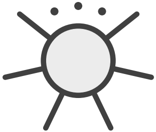

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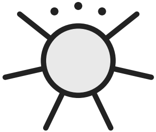

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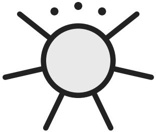

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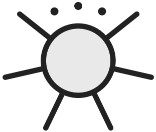

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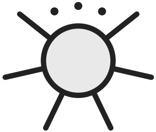

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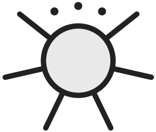

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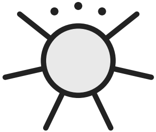

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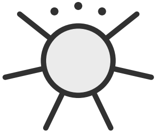

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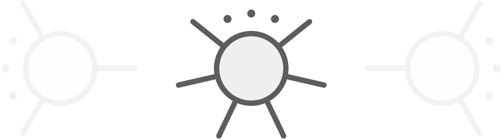

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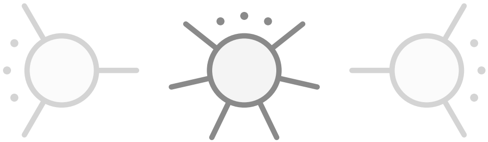

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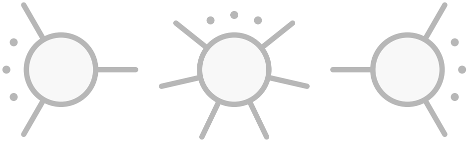

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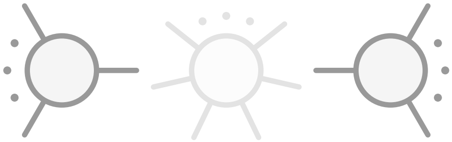

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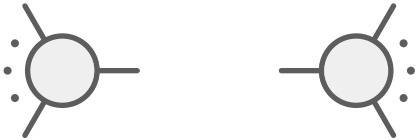

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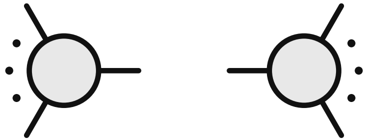

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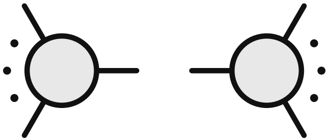

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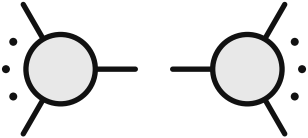

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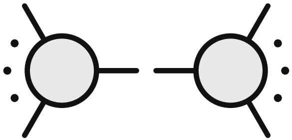

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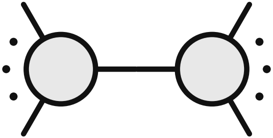

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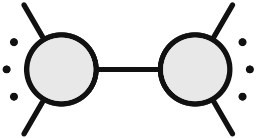

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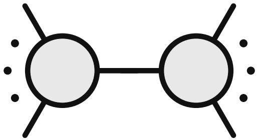

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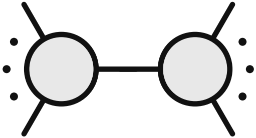

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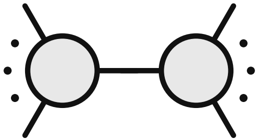

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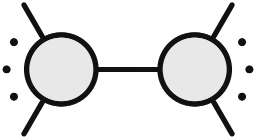

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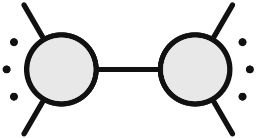

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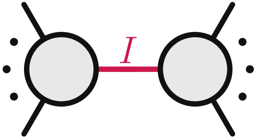

Internal Particles:

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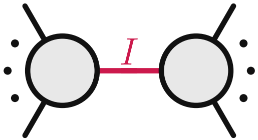

Internal Particles: locality dictates that we multiply each amplitude,

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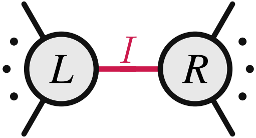

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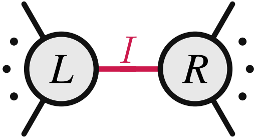

Internal Particles: locality dictates that we multiply each amplitude, and unitarity dictates that we marginalize over unobserved states

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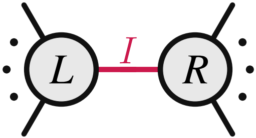

Internal Particles: locality dictates that we multiply each amplitude, and unitarity dictates that we marginalize over unobserved states—integrating over the Lorentz-invariant phase space ("LIPS") for each particle I,

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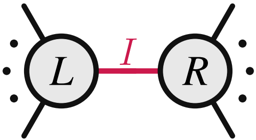

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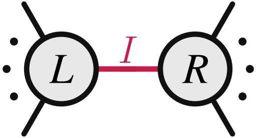

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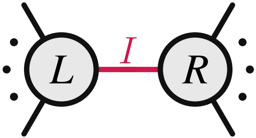

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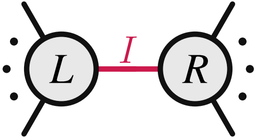

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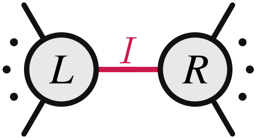

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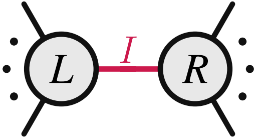

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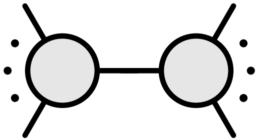

On-Shell Functions:

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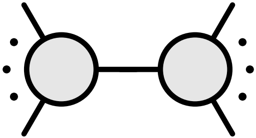

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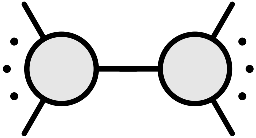

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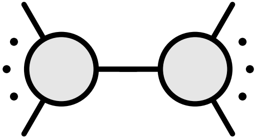

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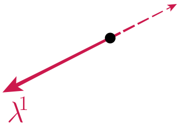

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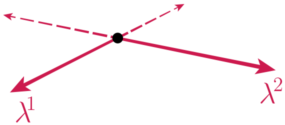

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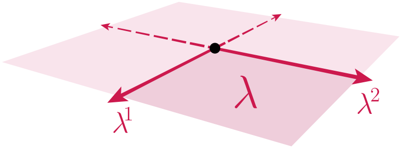

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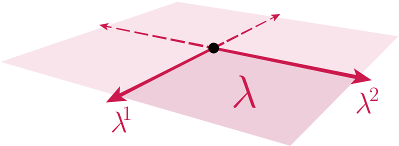

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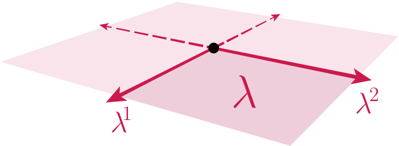

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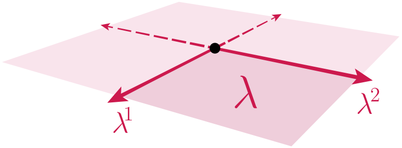

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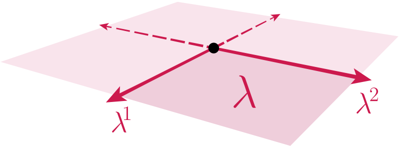

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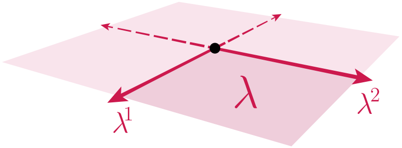

- **Momentum conservation**:

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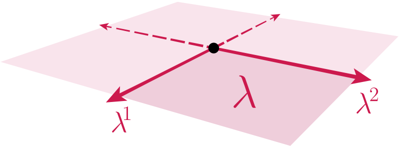

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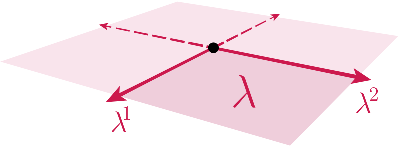

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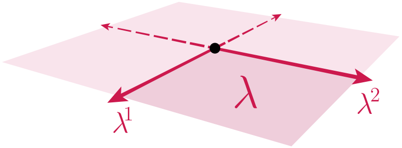

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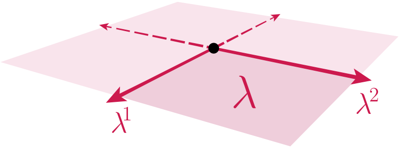

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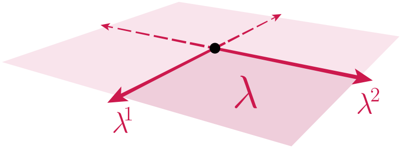

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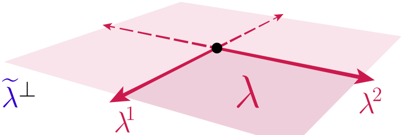

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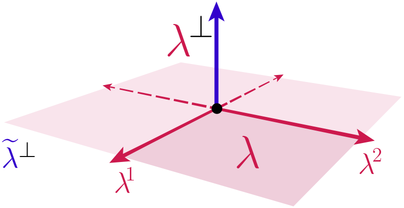

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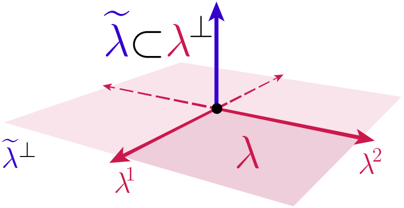

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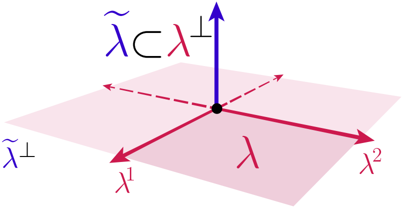

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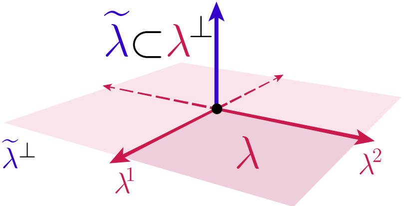

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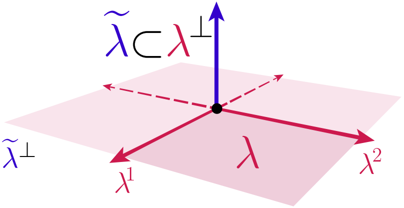

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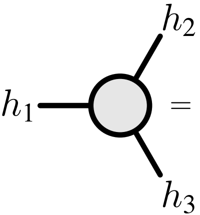
Building Blocks: the S-Matrix for Three Massless Particles

Momentum conservation and Poincaré-invariance **uniquely** fix the kinematic dependence of the amplitude for three massless particles (to all loop orders!).



$$= f(\lambda_1\widetilde{\lambda}_1, \lambda_2\widetilde{\lambda}_2, \lambda_3\widetilde{\lambda}_3)\delta^{2\times 2}(\lambda\cdot\widetilde{\lambda})$$

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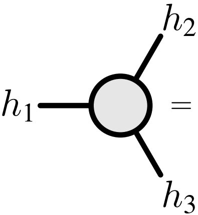

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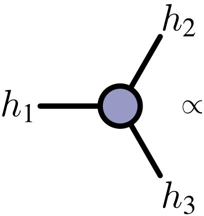

$$\propto \quad \langle 12\rangle^{h_3-h_1-h_2}\langle 23\rangle^{h_1-h_2-h_3}\langle 31\rangle^{h_2-h_3-h_1}$$

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$$\lambda^{\perp}\equiv\big(\langle 23\rangle\,\langle 31\rangle\,\langle 12\rangle\big)\supset\widetilde{\lambda}$$

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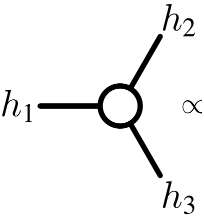

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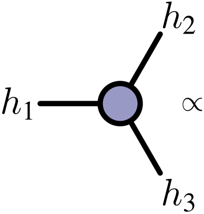

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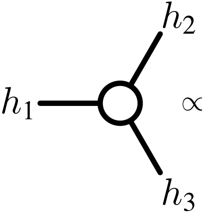

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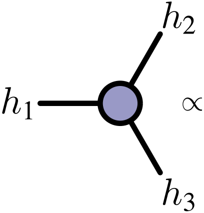

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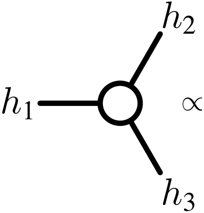

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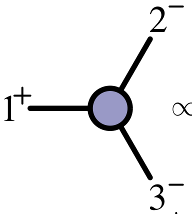

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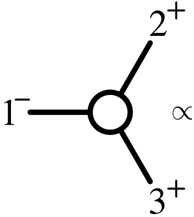

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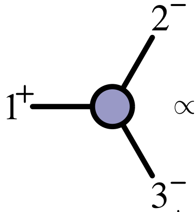
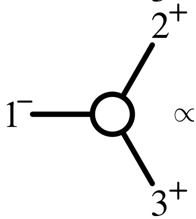


$$\propto \quad \frac{\langle 2\,3\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,1\rangle}\,\delta^{2\times 2}(\lambda\cdot\widetilde{\lambda})$$



$$\propto \quad \frac{[2\,3]^4}{[1\,2]\,[2\,3]\,[3\,1]}\,\delta^{2\times 2}(\lambda\cdot\widetilde{\lambda})$$

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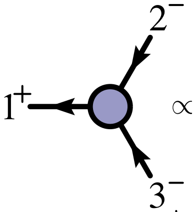
Momentum conservation and Poincaré-invariance **uniquely** fix the kinematic dependence of the amplitude for three massless particles (to all loop orders!).



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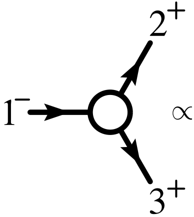

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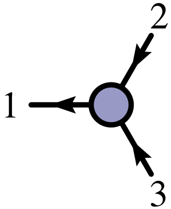

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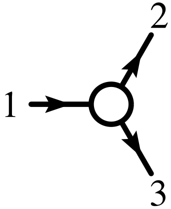

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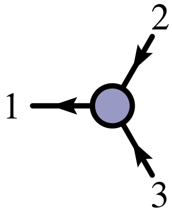

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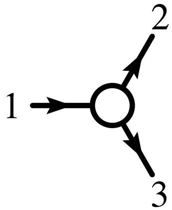

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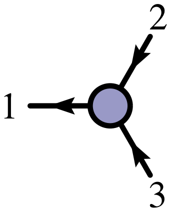

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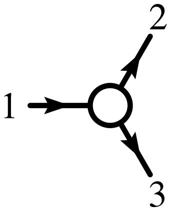

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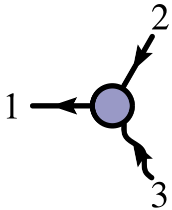
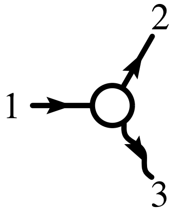


$$\propto \quad \frac{\langle 3\,1\rangle\langle 2\,3\rangle^3}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,1\rangle}\,\delta^{2\times 2}\big(\lambda\cdot\widetilde{\lambda}\big) \equiv \mathcal{A}_3\big(+\tfrac{1}{2},-\tfrac{1}{2},-\big)$$



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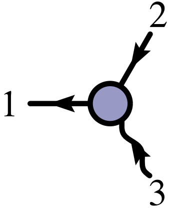

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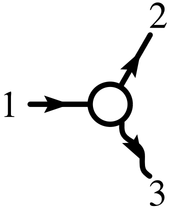

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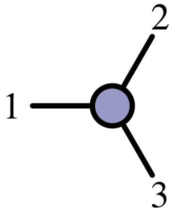
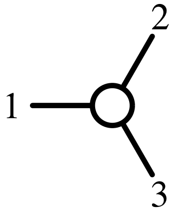


$$\propto \quad \frac{\delta^{2\times4}\big(\lambda\cdot\widetilde{\eta}\big)}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,1\rangle}\;\delta^{2\times2}\big(\lambda\cdot\widetilde{\lambda}\big)\equiv\mathcal{A}_3^{(2)}$$



$$\propto \quad \frac{\delta^{1\times4}\big(\widetilde{\lambda}^{\perp}\cdot\widetilde{\eta}\big)}{[1\,2]\,[2\,3]\,[3\,1]}\;\delta^{2\times2}\big(\lambda\cdot\widetilde{\lambda}\big)\equiv\mathcal{A}_3^{(1)}$$

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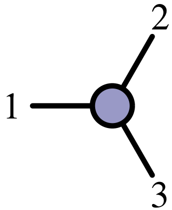

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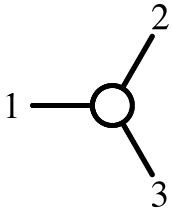

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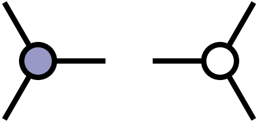

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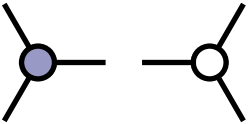

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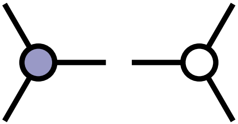

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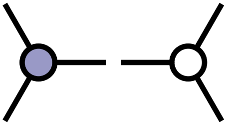

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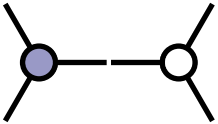

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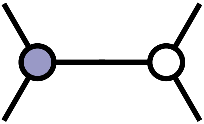

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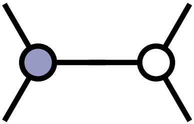

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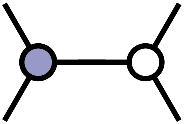

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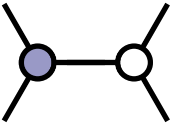

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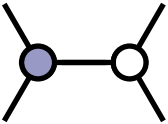

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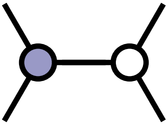

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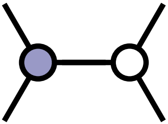

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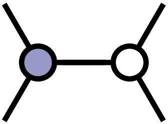

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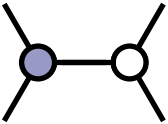

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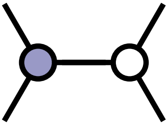

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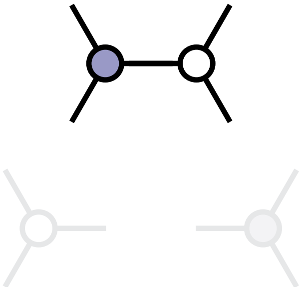

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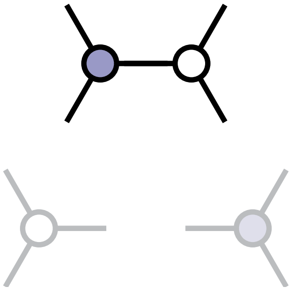

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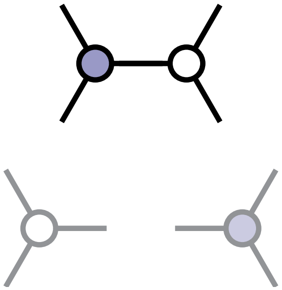

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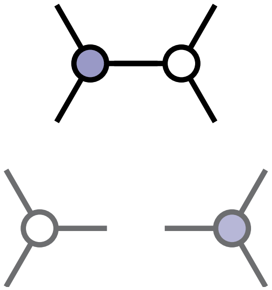

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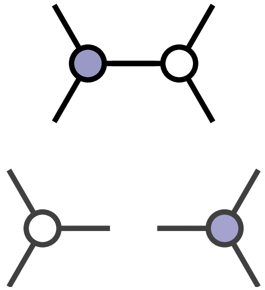

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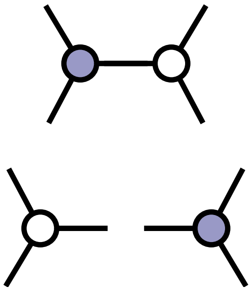

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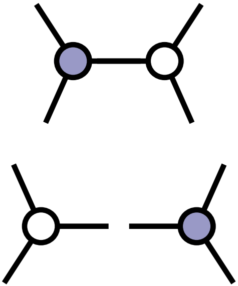

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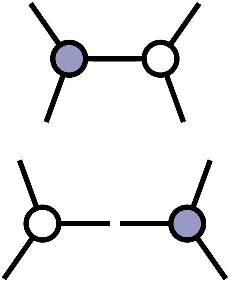

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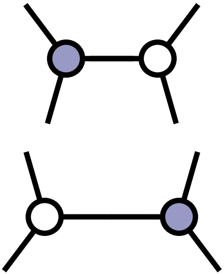

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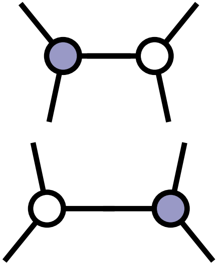

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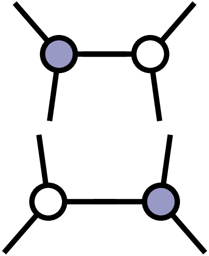

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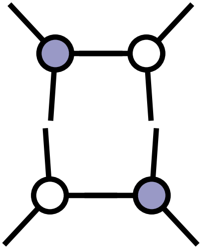

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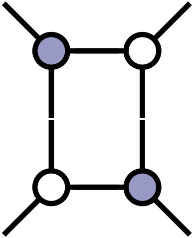

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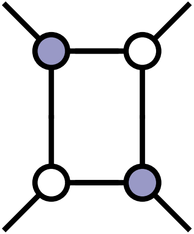

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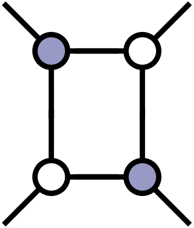

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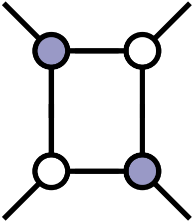

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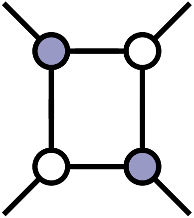

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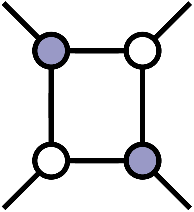

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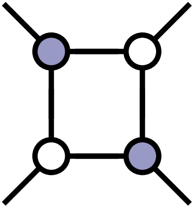

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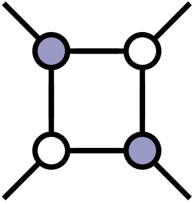

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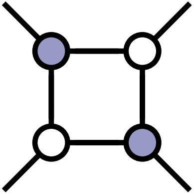

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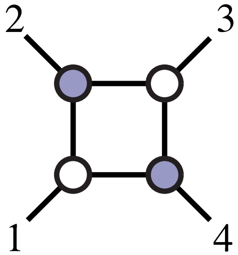

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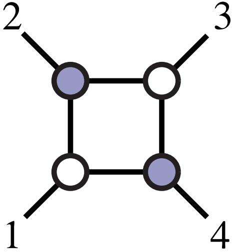

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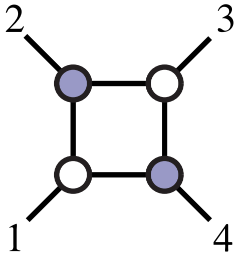

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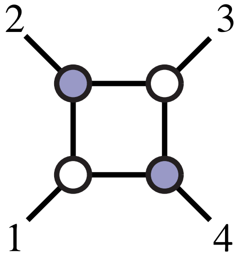

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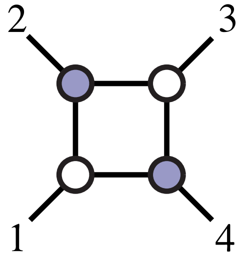

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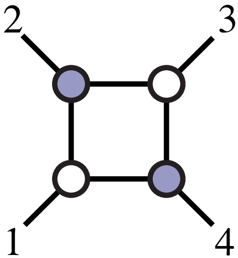

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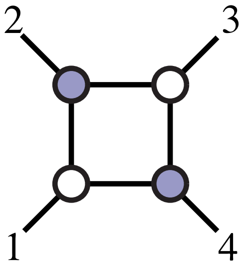

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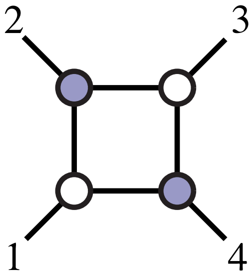

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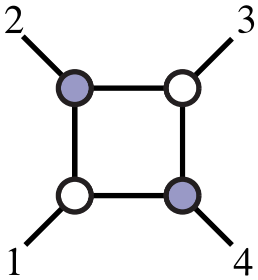

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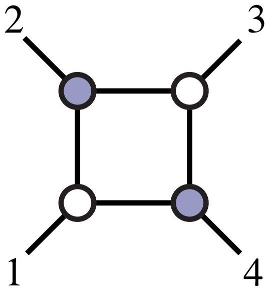

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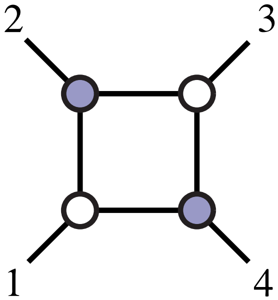

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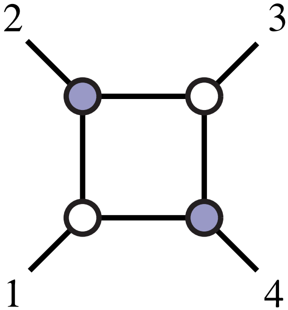

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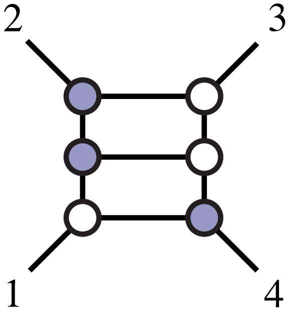

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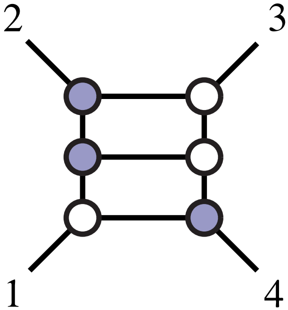

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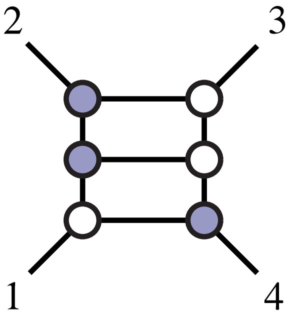

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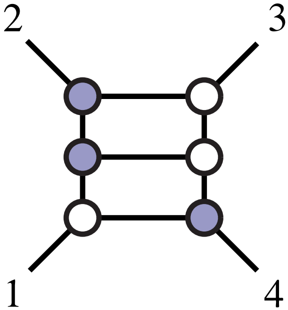

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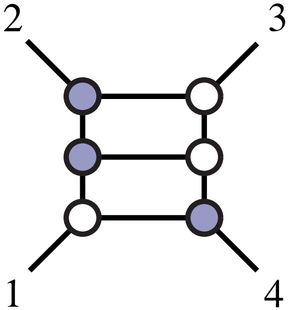

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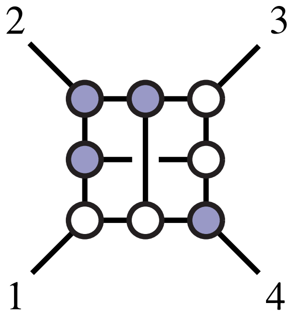

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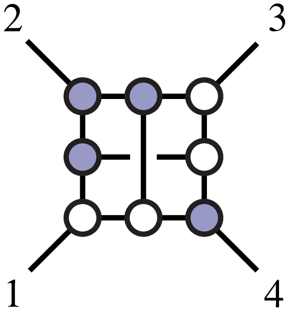

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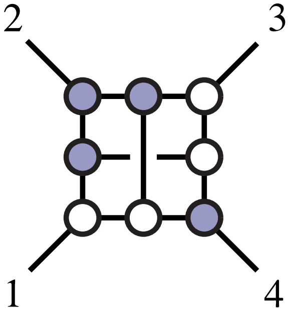

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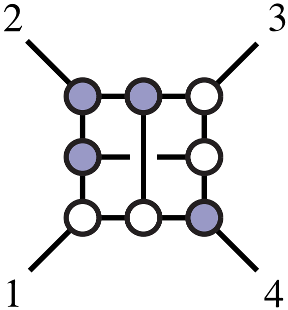

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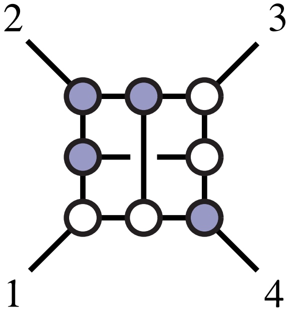

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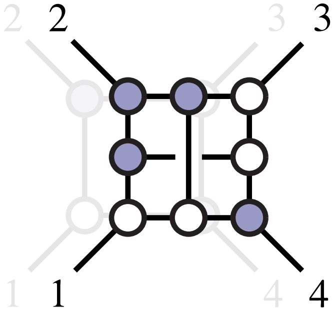

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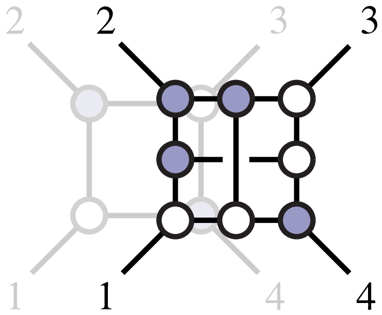

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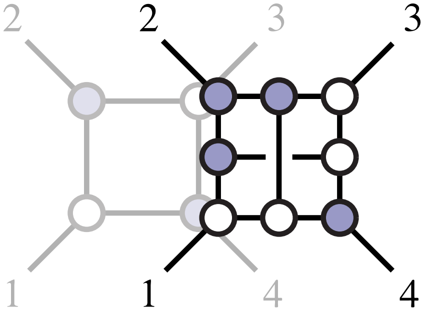

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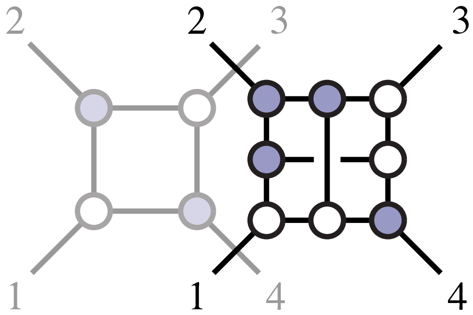

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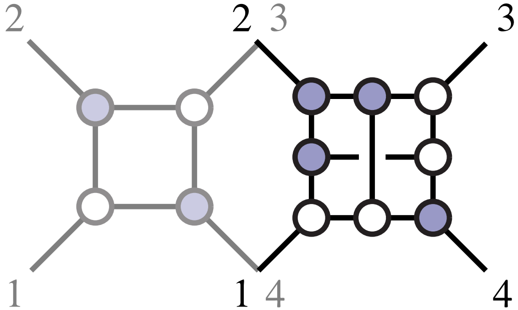

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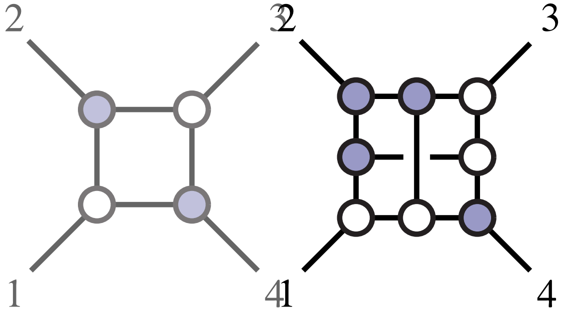

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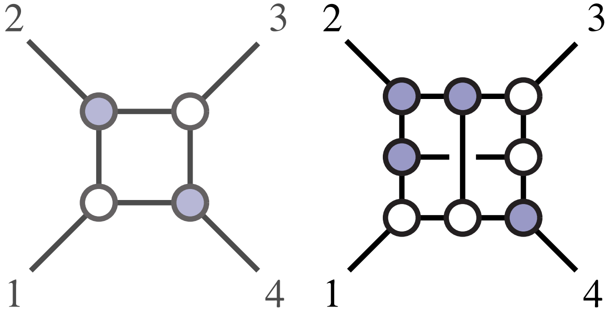

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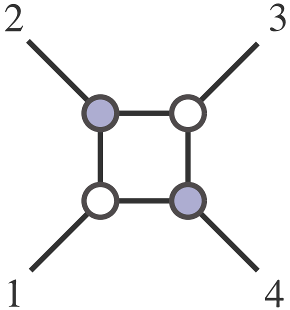

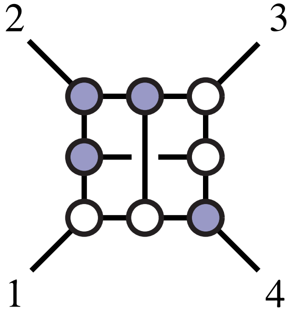

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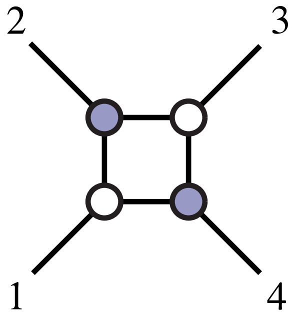

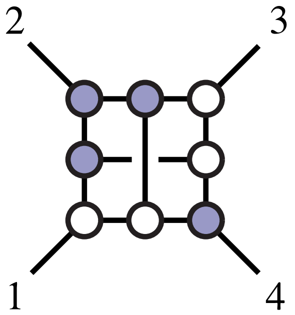

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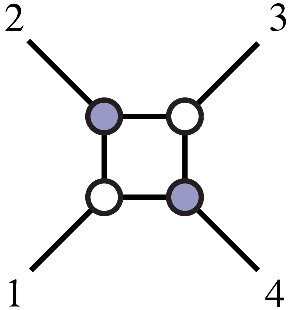

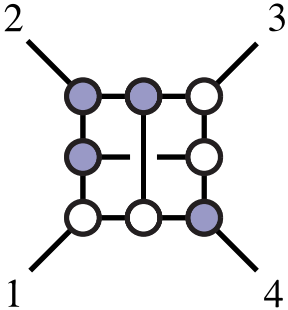

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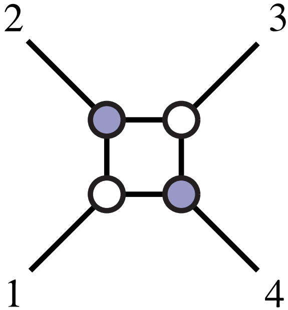

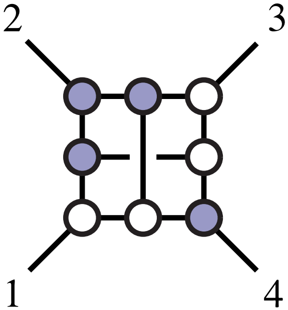

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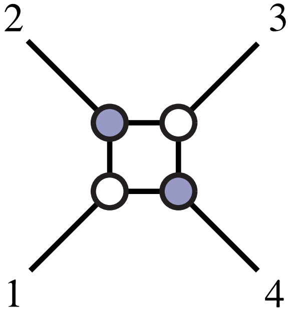

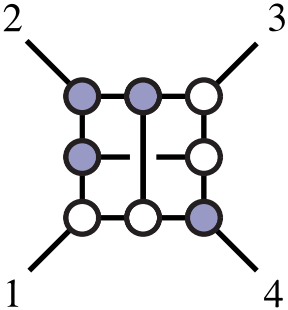

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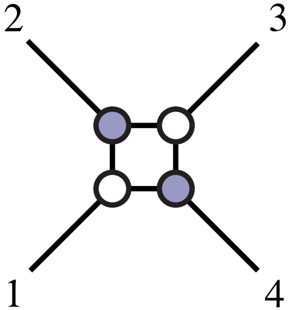

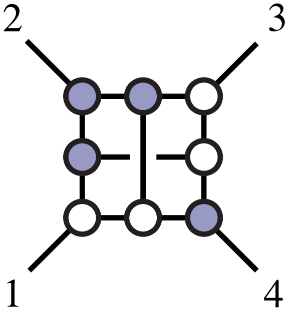

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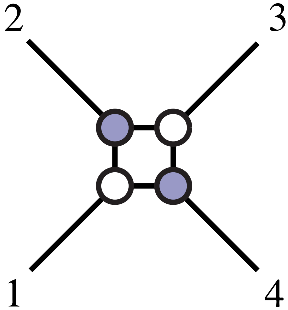

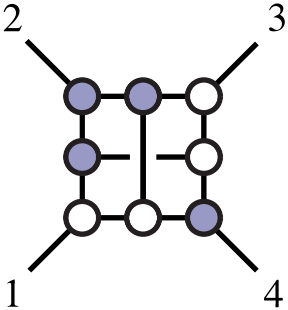

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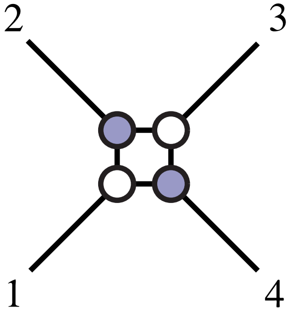

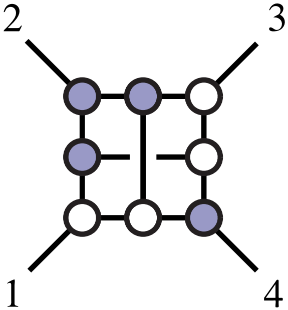

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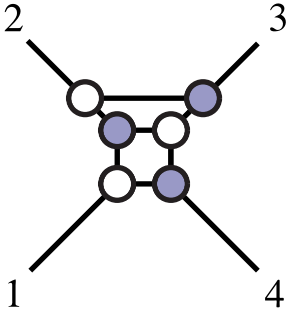

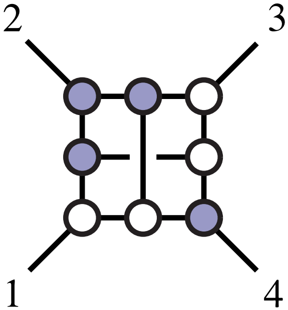

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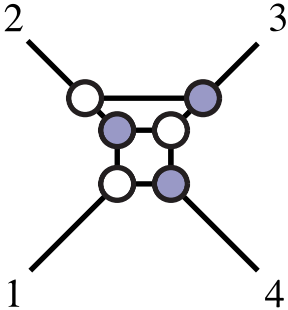

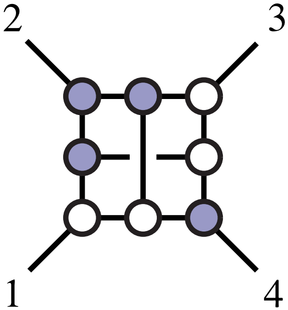

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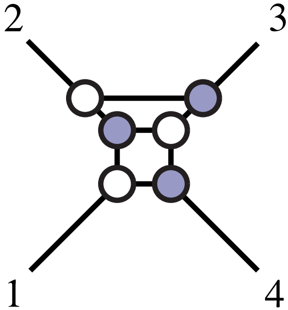

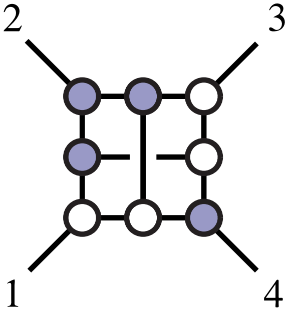

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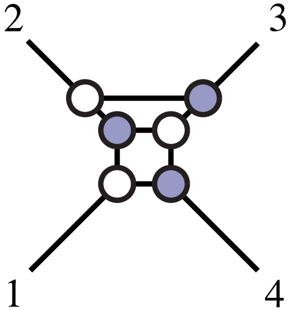

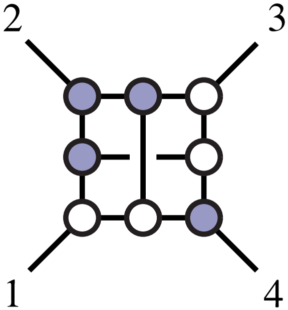

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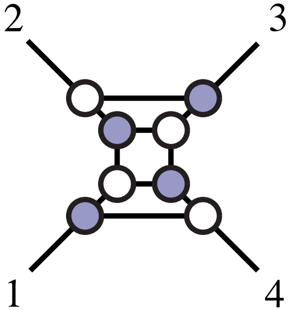

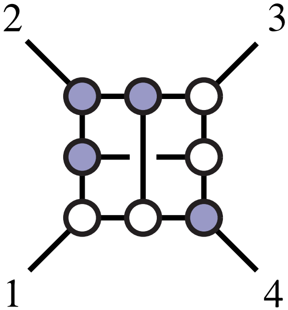

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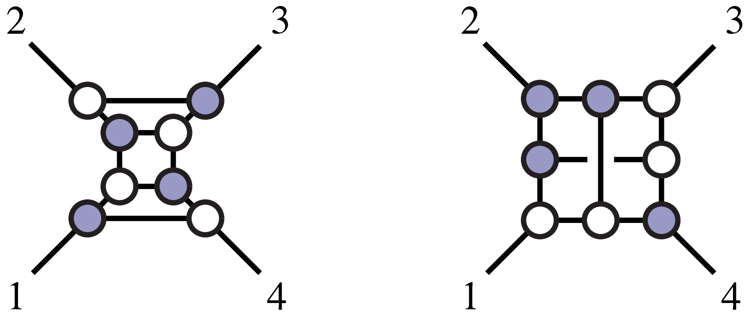

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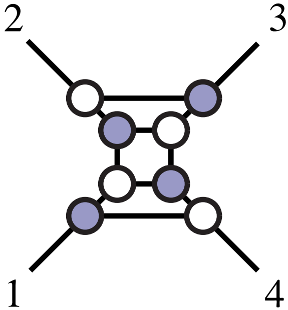

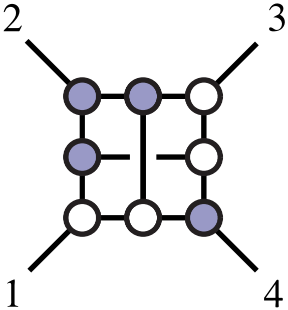

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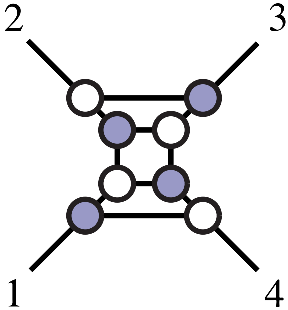

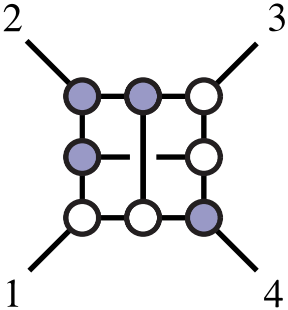

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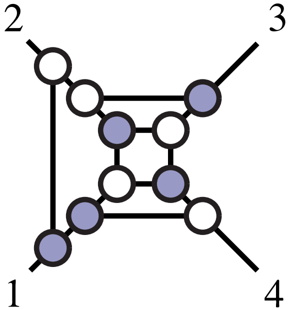

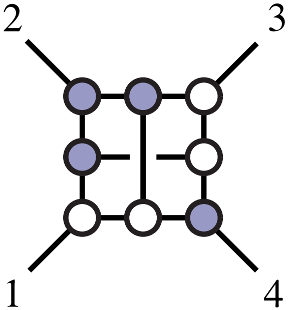

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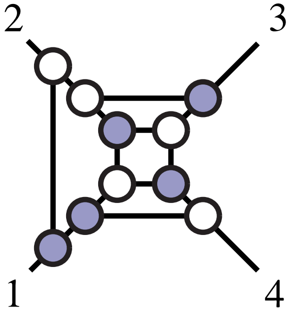

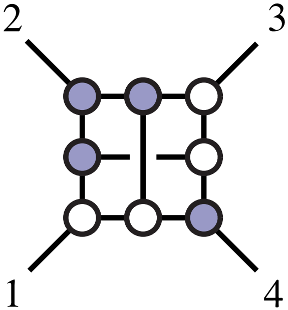

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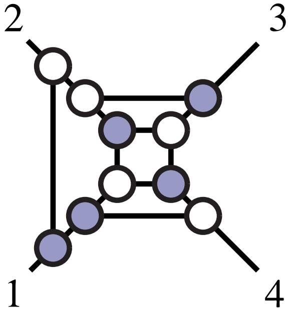

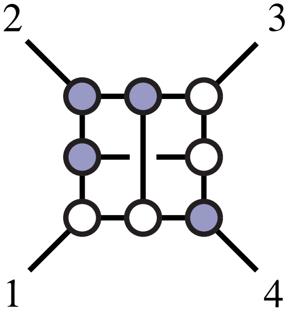

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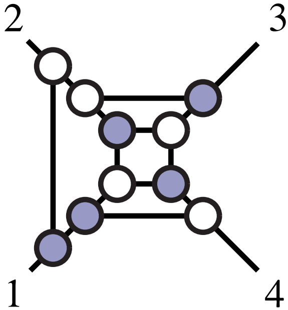

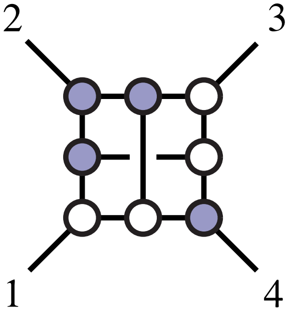

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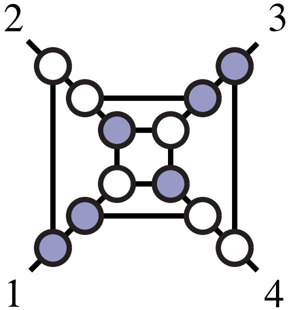

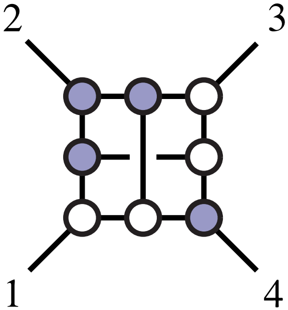

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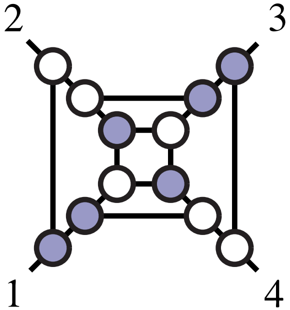

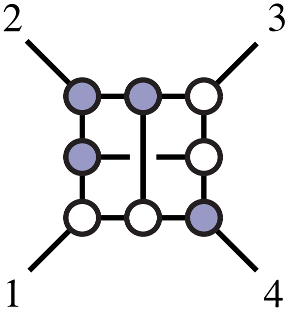

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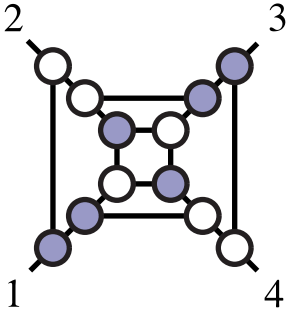

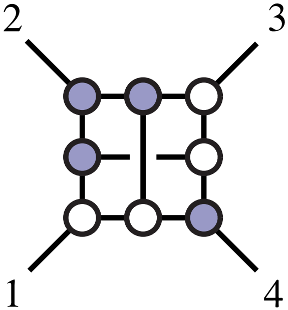

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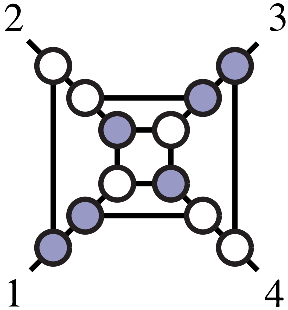

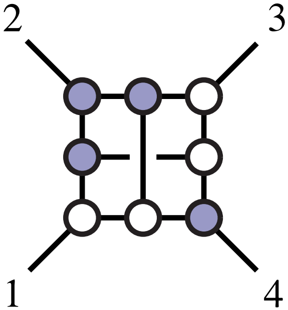

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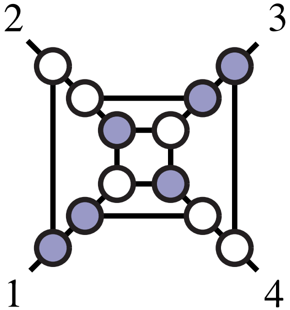

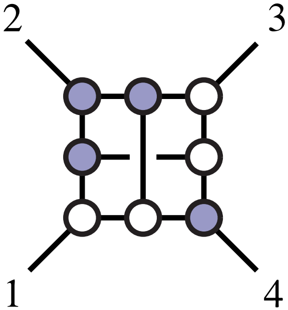

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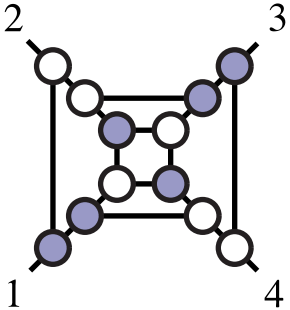

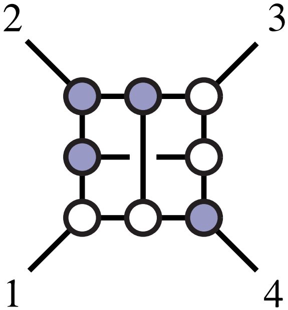

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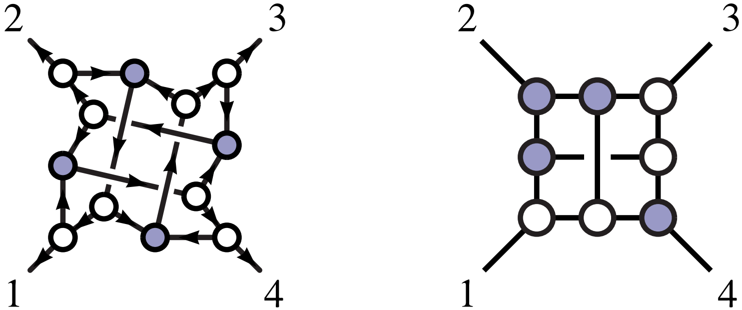

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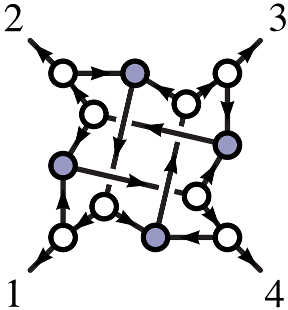

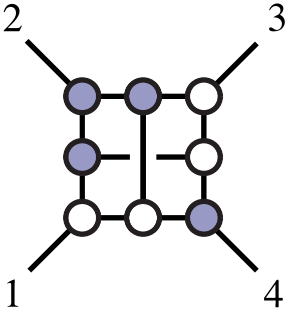

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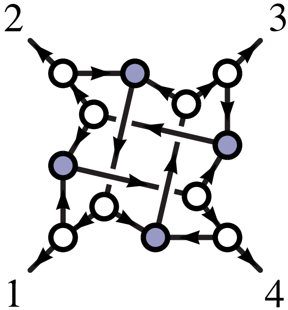

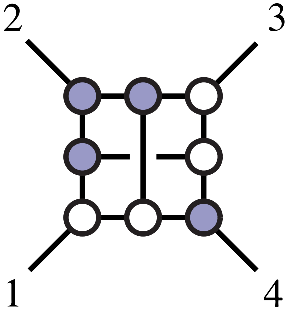

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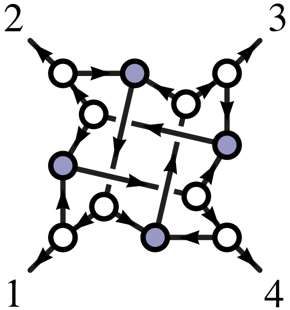

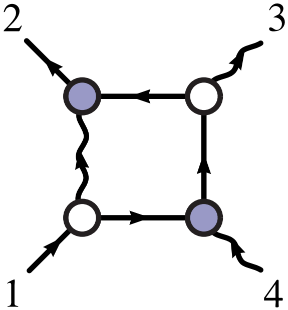

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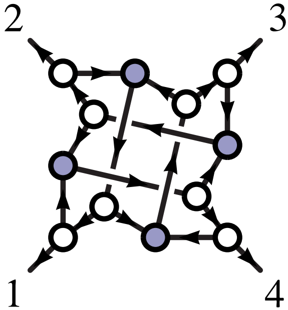

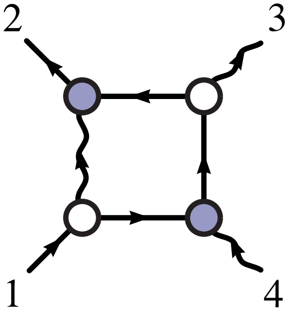

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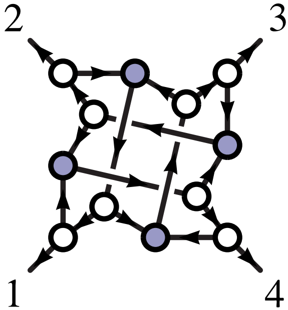

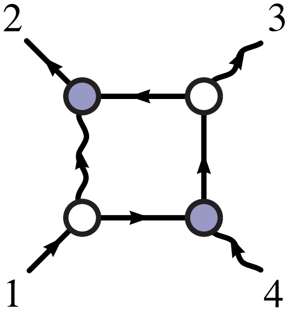

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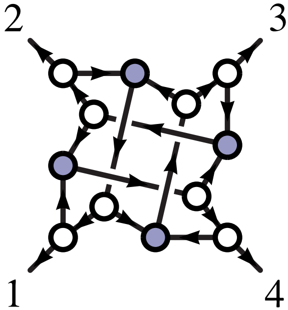

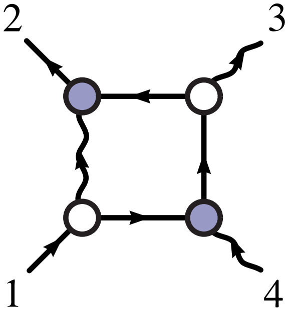

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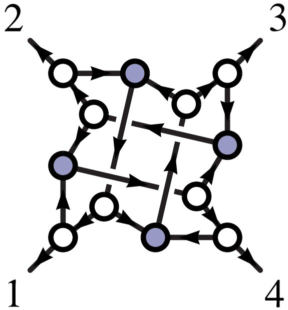

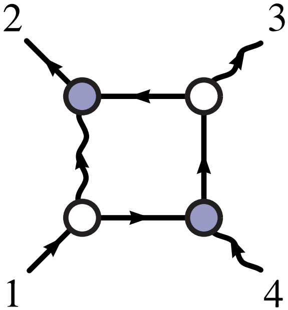

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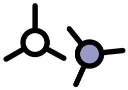

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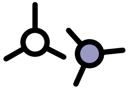

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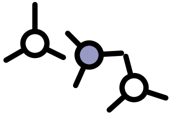

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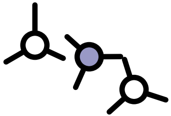

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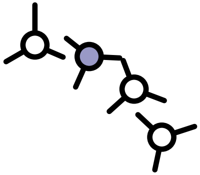

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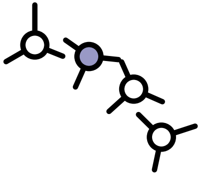

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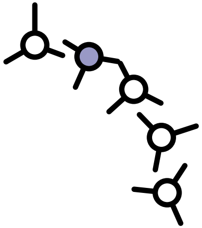

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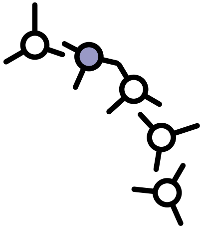

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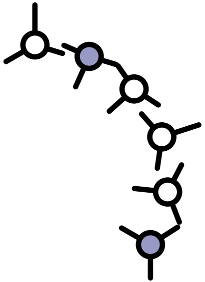

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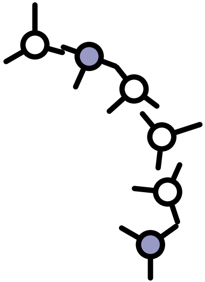

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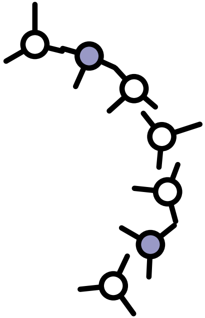

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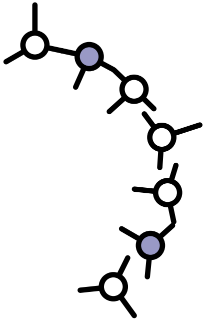

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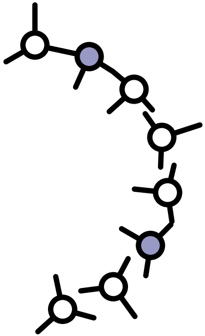

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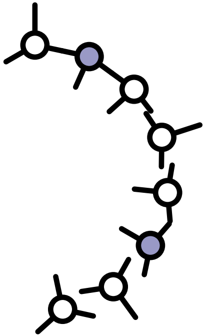

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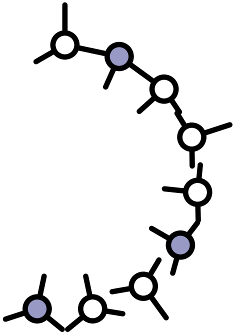

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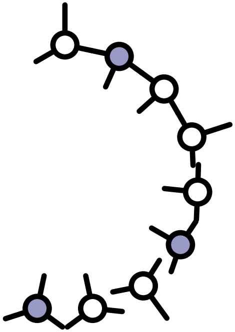

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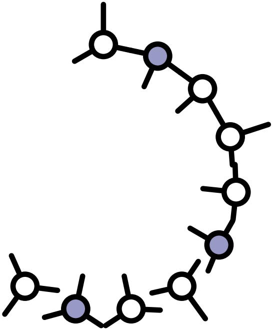

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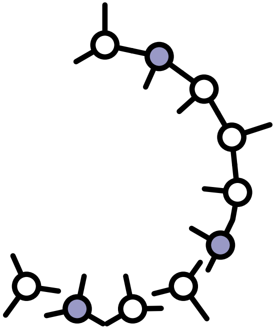

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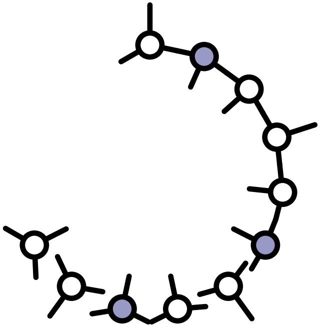

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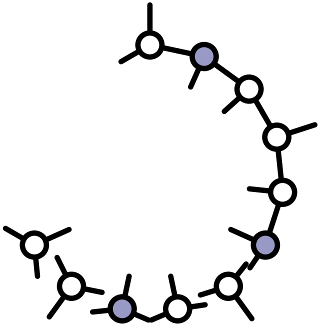

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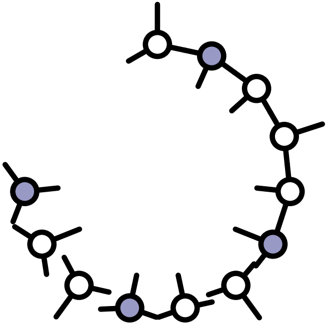

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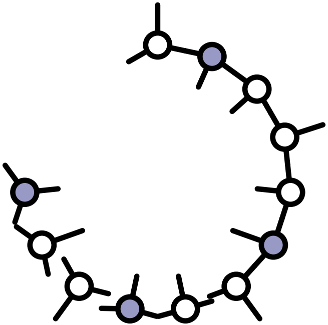

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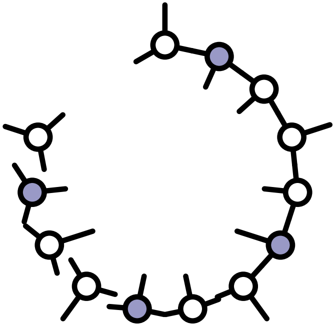

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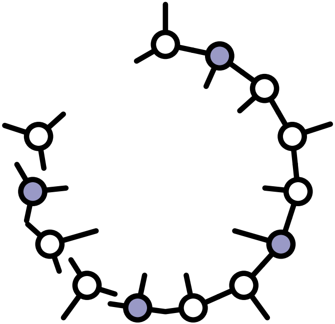

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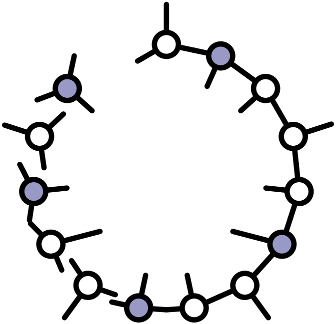

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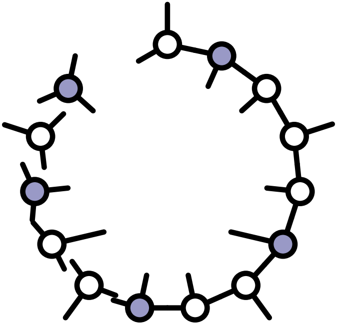

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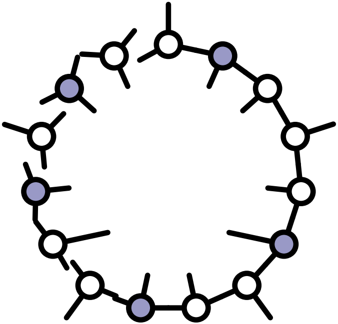

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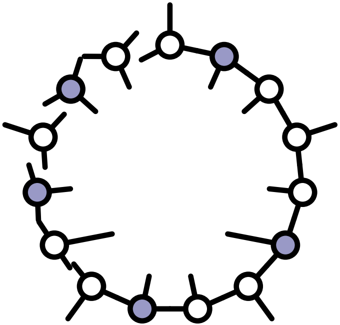

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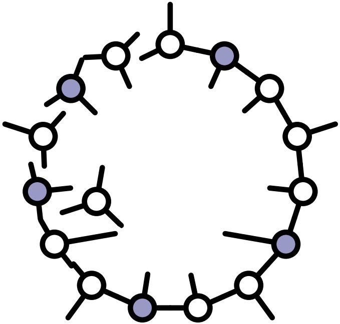

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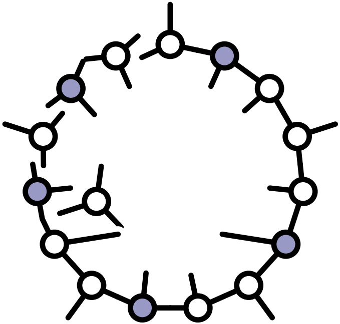

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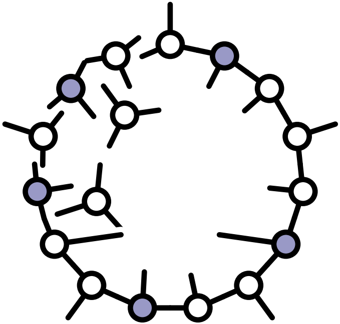

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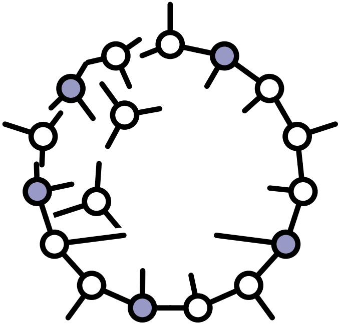

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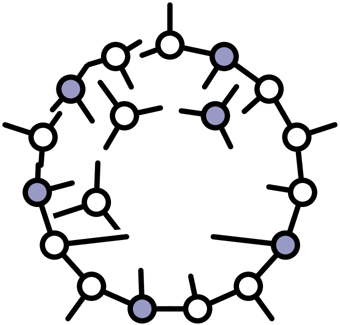

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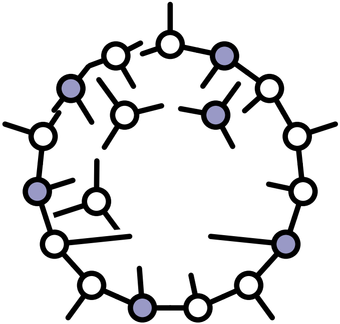

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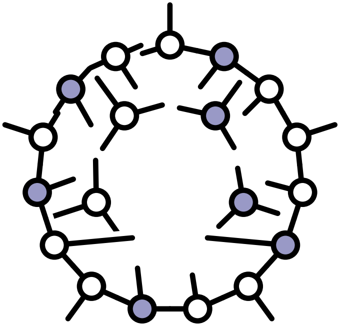

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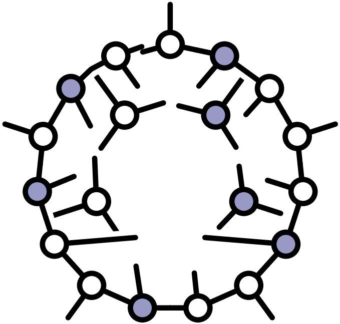

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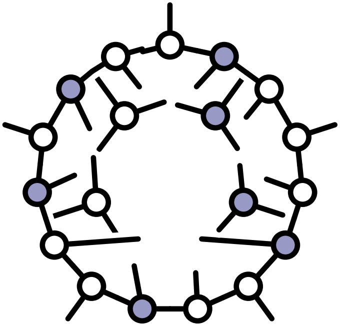

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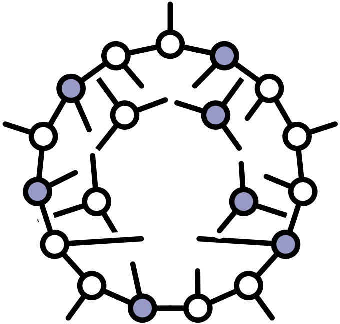

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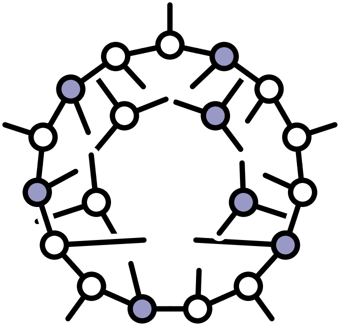

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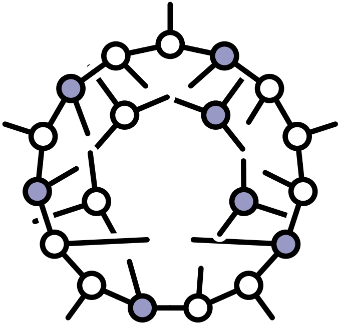

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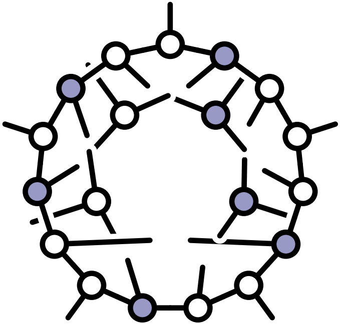

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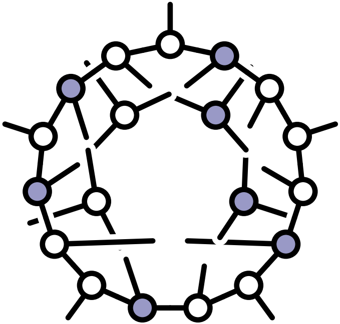

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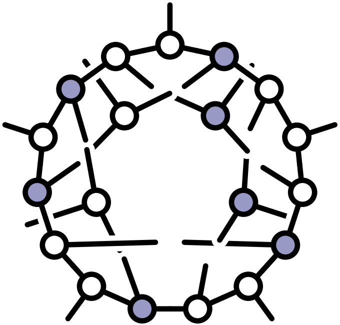

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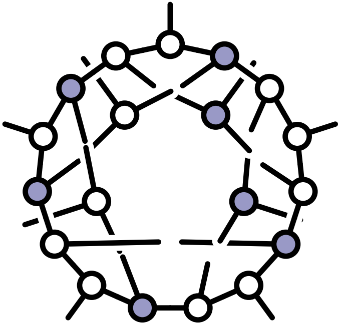

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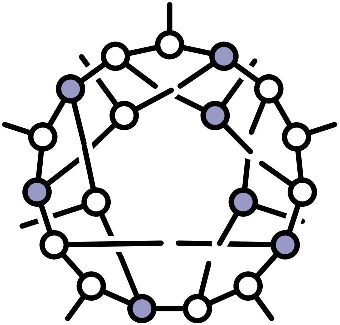

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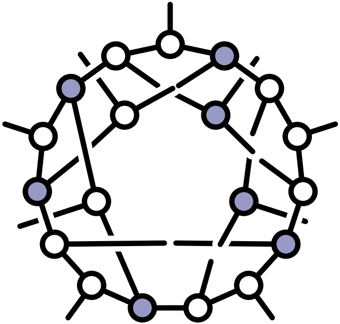

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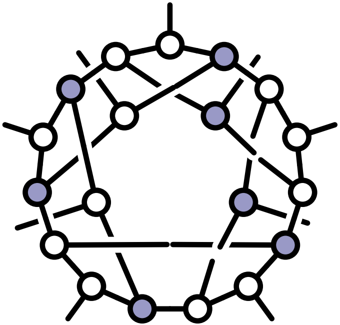

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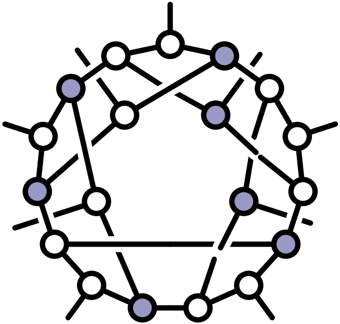

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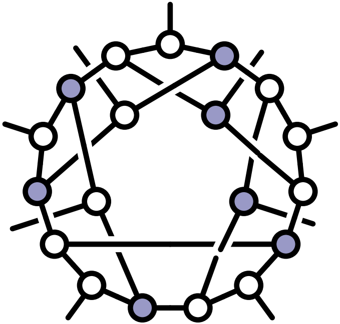

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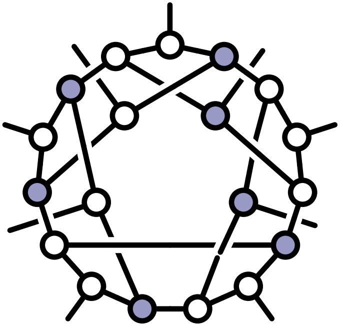

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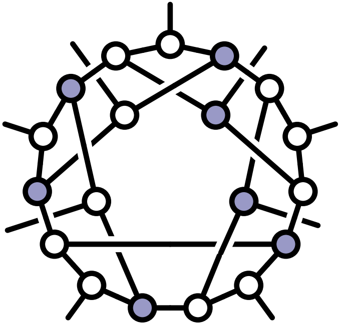

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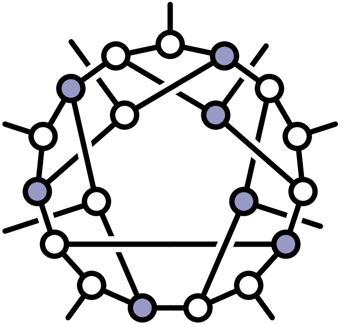

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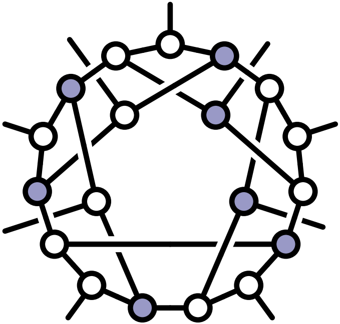

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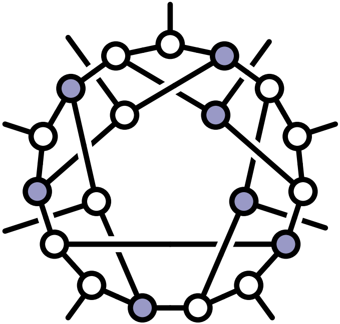

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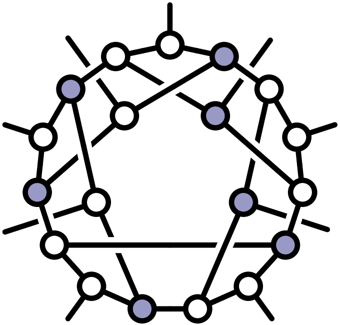

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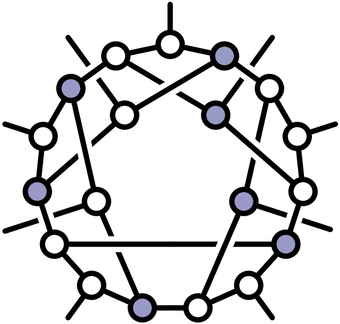

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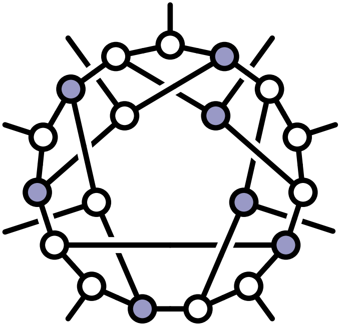

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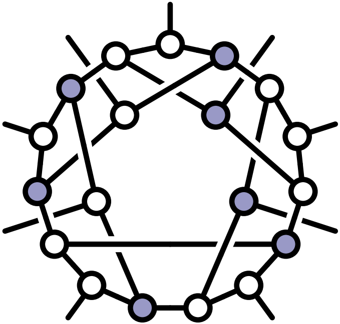

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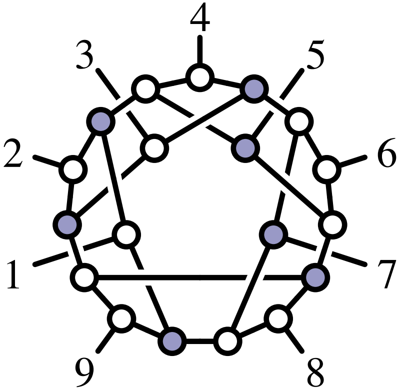

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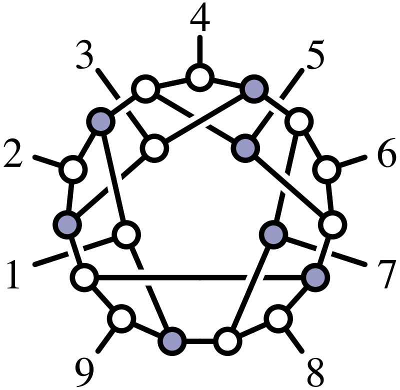

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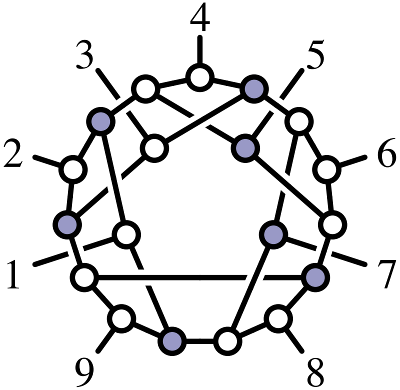

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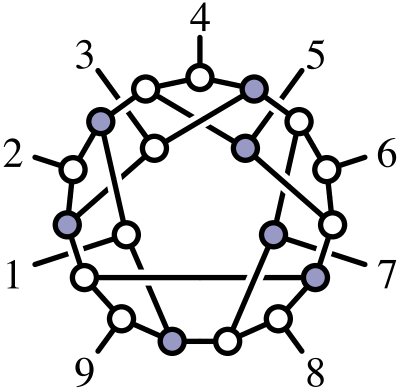

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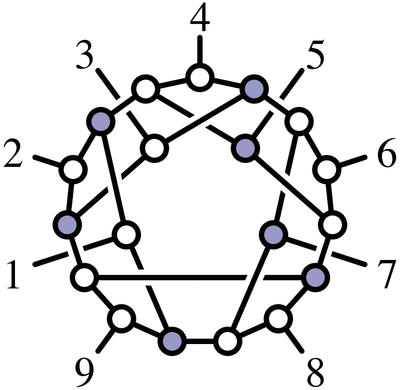

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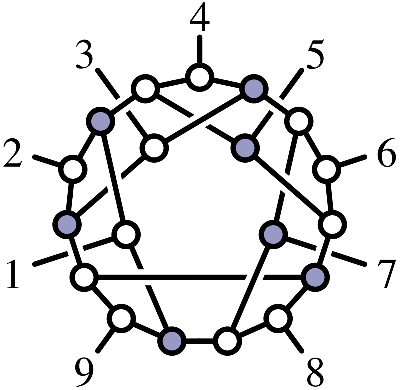

$$= \frac{(\langle 91\rangle\langle 23\rangle\langle 46\rangle - \langle 16\rangle\langle 34\rangle\langle 29\rangle)^2 \quad \delta^{2\times 4}(\lambda\cdot\widetilde{\eta})\delta^{2\times 2}(\lambda\cdot\widetilde{\lambda})}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 67\rangle\langle 78\rangle\langle 81\rangle\langle 14\rangle\langle 42\rangle\langle 29\rangle\langle 96\rangle\langle 63\rangle\langle 39\rangle\langle 91\rangle}$$

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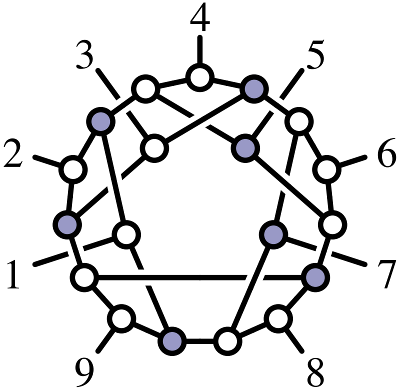

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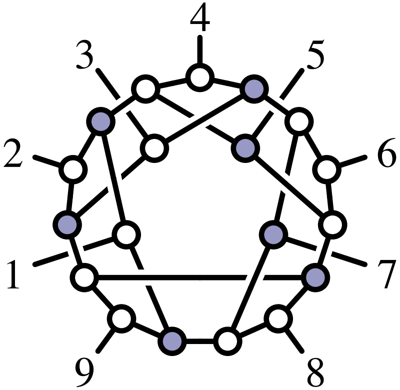

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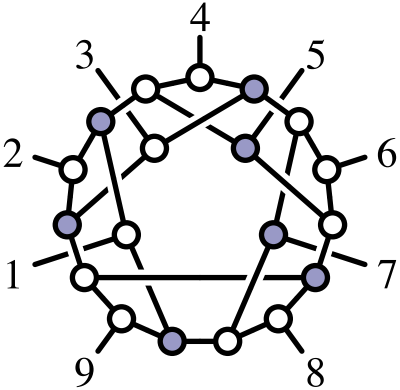

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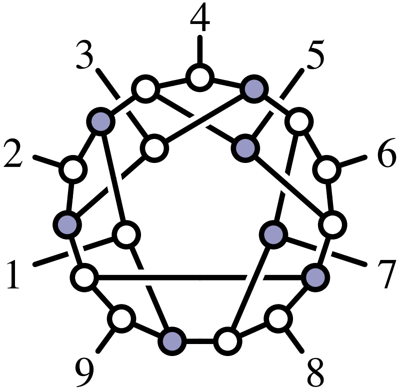

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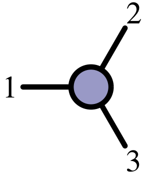

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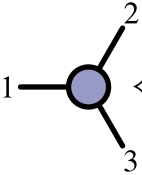

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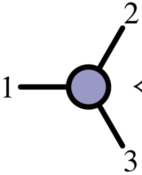

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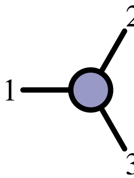

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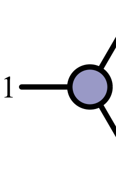

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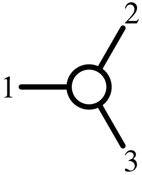

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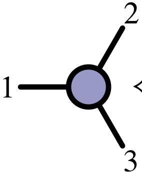

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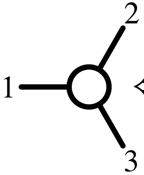

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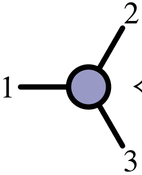

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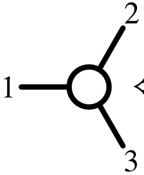

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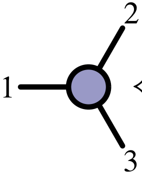

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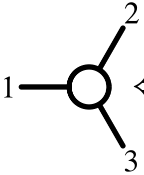

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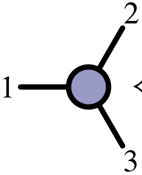

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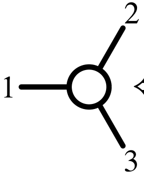

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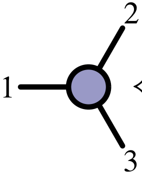

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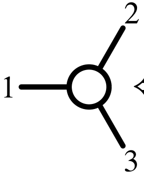

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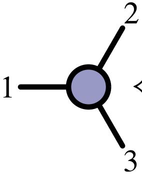

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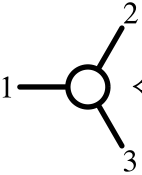

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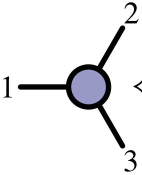

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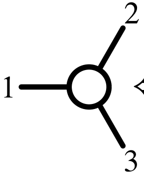

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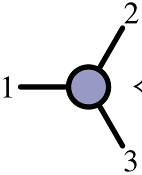

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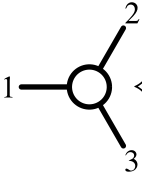

$$\Leftrightarrow W\equiv\begin{pmatrix}w_1^1 & w_2^1 & w_3^1\end{pmatrix}$$

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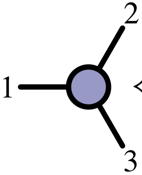
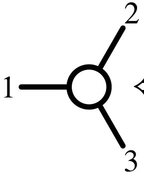


$$\Leftrightarrow B\equiv\begin{pmatrix}1&0&b_3^1\\0&1&b_3^2\end{pmatrix}$$



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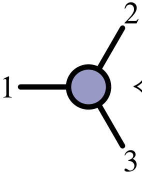
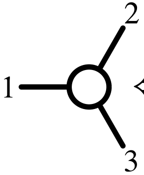


$$\Leftrightarrow B\equiv\begin{pmatrix} b_1^1 & 1 & 0\\ b_1^2 & 0 & 1\end{pmatrix}$$



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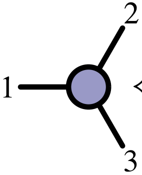

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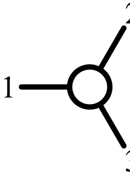

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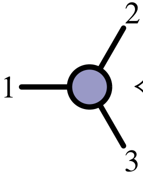

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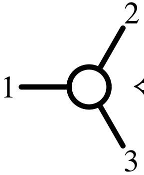

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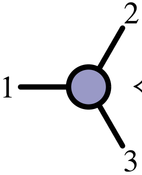

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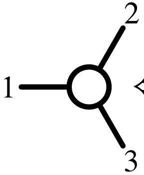

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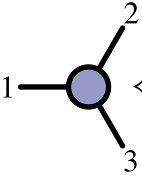

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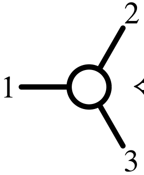

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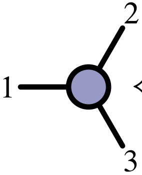

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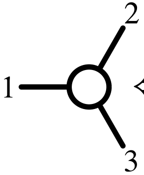

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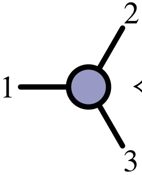

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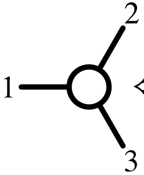

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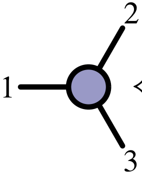

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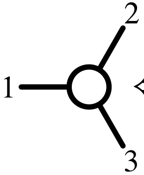

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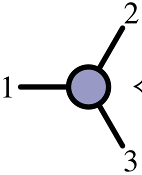

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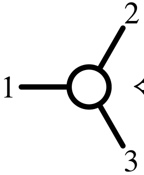

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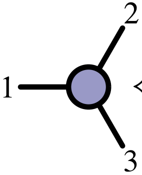

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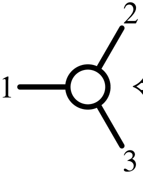

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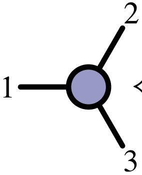

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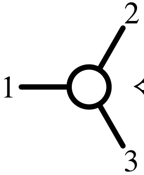

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$$\mathcal{A}_3^{(2)}=\frac{\delta^{2\times4}(\lambda\cdot\widetilde{\eta})}{\langle12\rangle\langle23\rangle\langle31\rangle}\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})\equiv\int\frac{d^{2\times3}B}{\mathrm{vol}(GL_2)}\frac{\delta^{2\times4}(B\cdot\widetilde{\eta})}{(12)(23)(31)}\delta^{2\times2}(B\cdot\widetilde{\lambda})\underbrace{\delta^{1\times2}(\lambda\cdot B^{\perp})}_{B\mapsto B^*=\lambda}$$

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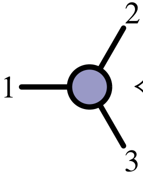
Grassmannian Representations of Three-Point Amplitudes

In order to **linearize** momentum conservation at each three-particle vertex, (and to specify *which* of the solutions to three-particle kinematics to use) we introduce **auxiliary** $B\in G(2,3)$ and $W\in G(1,3)$ for each vertex:



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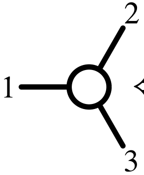

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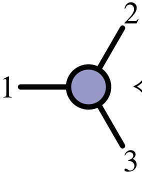

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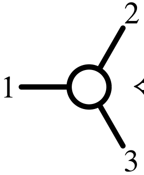

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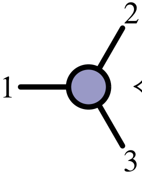

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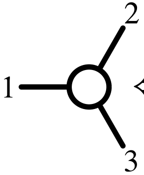

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Constructing the Correspondence: Amalgamations & Bridges

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Direct/Outer Products

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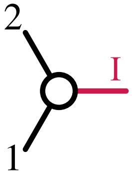

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$$\begin{array}{ccc} 1 & 2 & \mathrm{I} \\ \end{array}$$
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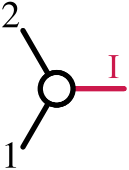

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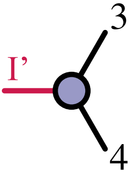

$$\begin{array}{ccc} \mathrm{I'} & 3 & 4 \\ \hline \end{array}$$
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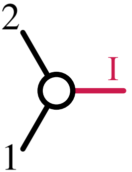

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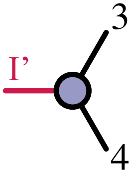

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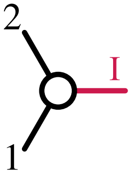

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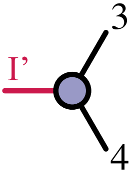

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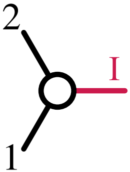

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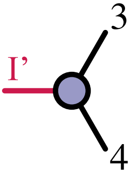

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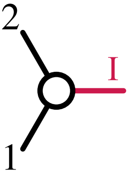

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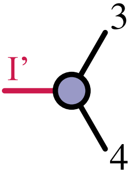

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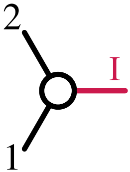

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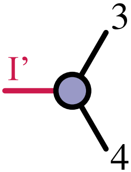

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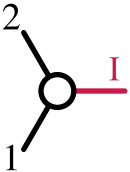

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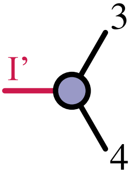

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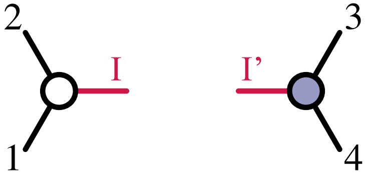

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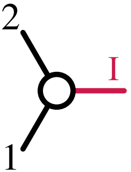

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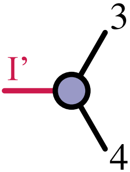

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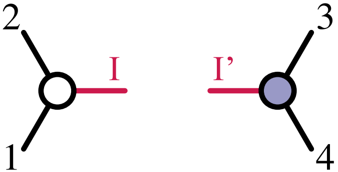

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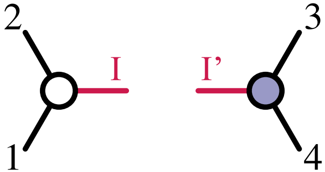

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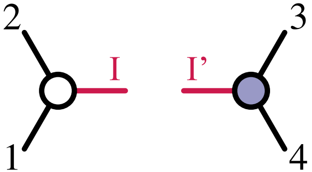

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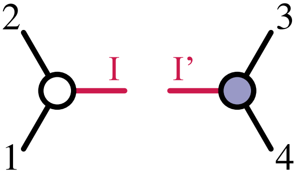

$$\begin{array}{ccc}1&2&\mathrm{I}\\ \hline\end{array}\qquad\begin{array}{ccc}\mathrm{I'}&3&4\\ \hline\end{array}$$

$$\begin{pmatrix}1&w_2&w_{\mathrm{I}}\end{pmatrix}\qquad\begin{pmatrix}1&0&b_4^1\\0&1&b_4^2\end{pmatrix}$$

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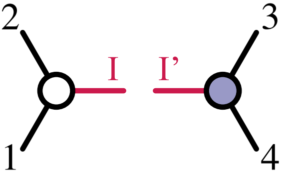

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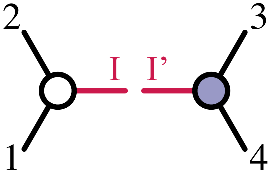

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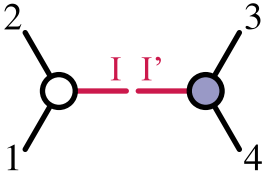

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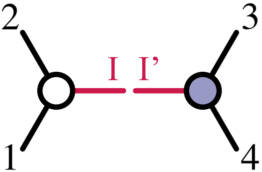

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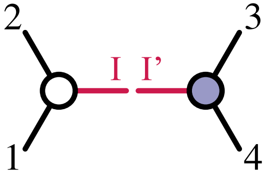

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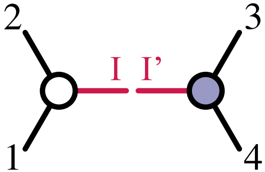

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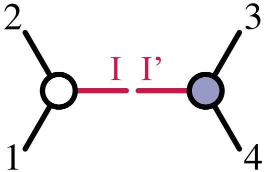

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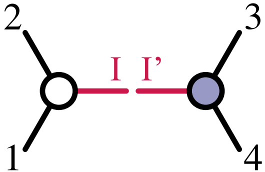

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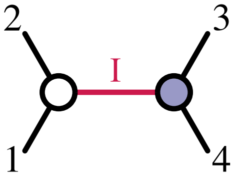

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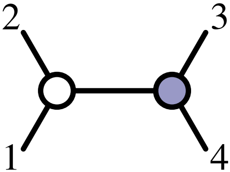

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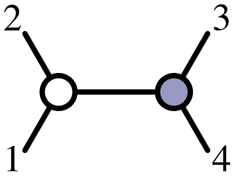

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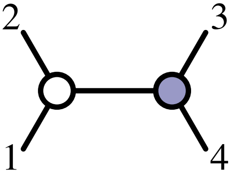

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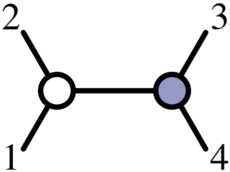

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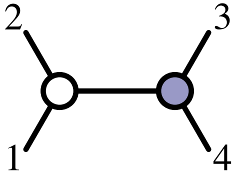

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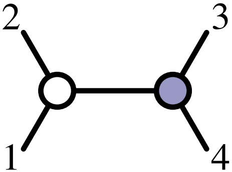

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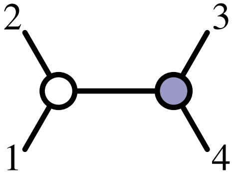

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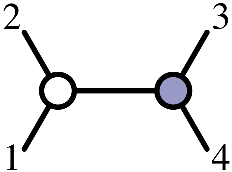

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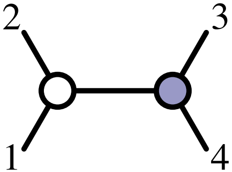

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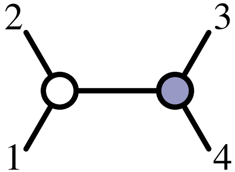

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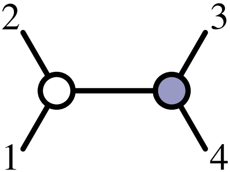

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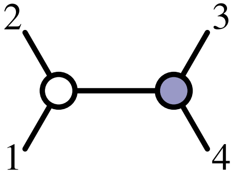

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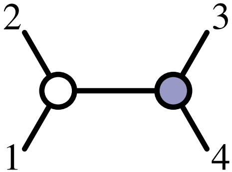

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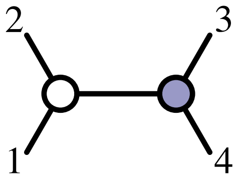

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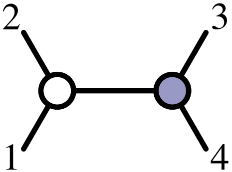

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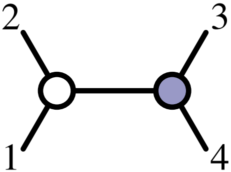

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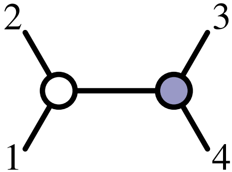

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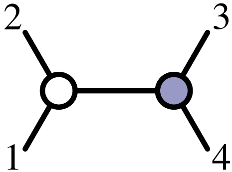

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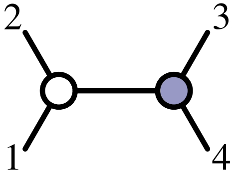

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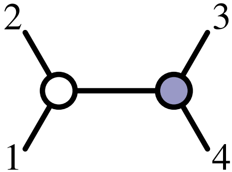

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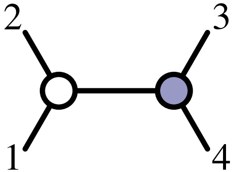

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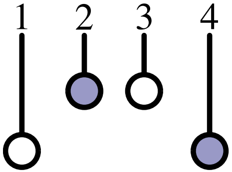

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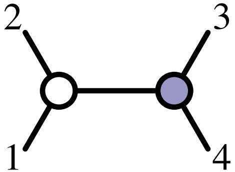

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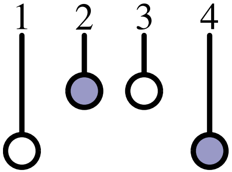

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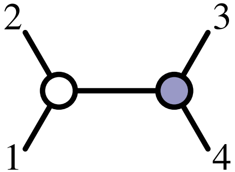

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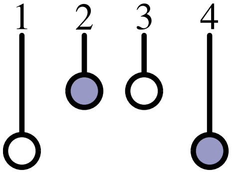

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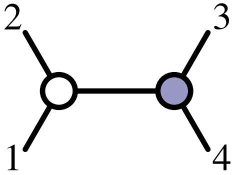

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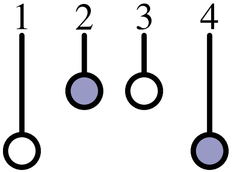

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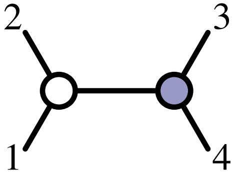

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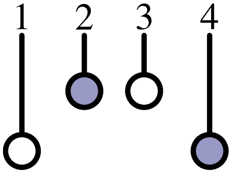

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$$(f_1,f_2)\mapsto f_1\times f_2$$
$$(C_1,C_2)\mapsto C_1\oplus C_2\subset G(k_1{+}k_2,n_1{+}n_2)$$
$$(\Omega_1,\Omega_2)\mapsto\Omega_1\wedge\Omega_2\qquad(d_1,d_2)\mapsto d_1{+}d_2$$

Amalgamation: Gluing Legs (A,B)

$$f\mapsto f'\qquad c_i\mapsto c_i\cap(c_A{+}c_B)^{\perp}$$
$$C\mapsto C/(c_A{+}c_B)\subset G(k{-}1,n{-}2)$$
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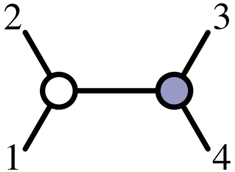

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$$f\mapsto f'\qquad c_B\mapsto c_B{+}\alpha\,c_A$$
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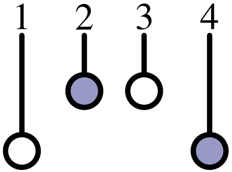

$$C\equiv\begin{array}{c}\begin{array}{cccc}1&2&3&4\end{array}\\\left(\begin{array}{cccc}1&w_2&0&b_4^1\\0&0&1&b_4^2\end{array}\right)\end{array}$$

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$$f_\Gamma\equiv\int\Omega_C\;\delta^{k\times2}(C\cdot\widetilde{\lambda})\,\delta^{2\times(n-k)}(\lambda\cdot C^{\perp})$$

$$f_\Gamma\equiv\delta^2(\widetilde{\lambda}_1)\delta^2(\lambda_2)\delta^2(\widetilde{\lambda}_3)\delta^2(\lambda_4)$$

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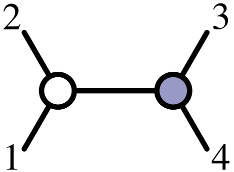

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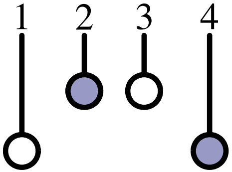

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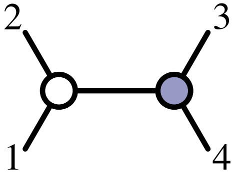

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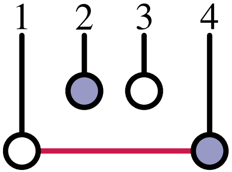

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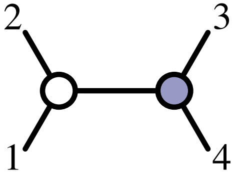

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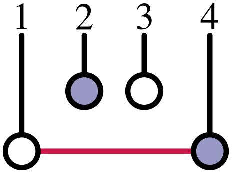

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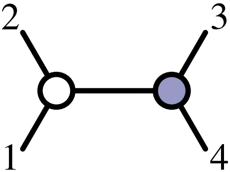

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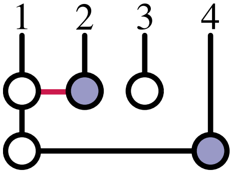

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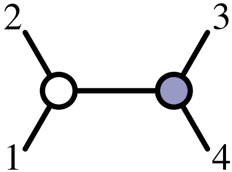

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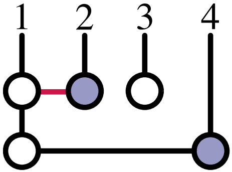

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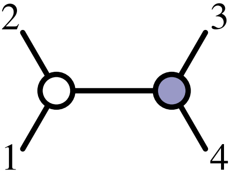

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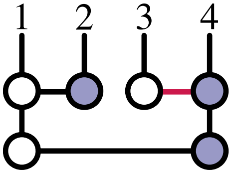

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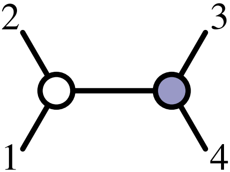

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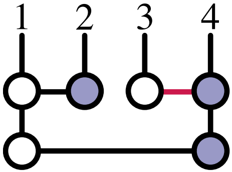

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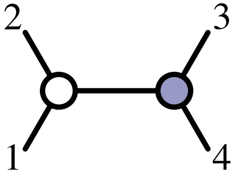

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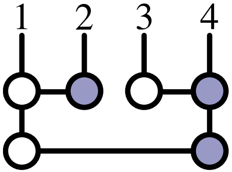

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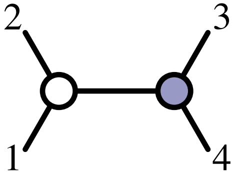

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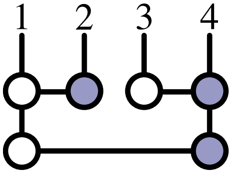

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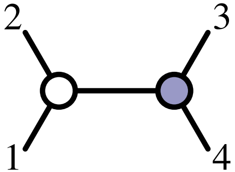

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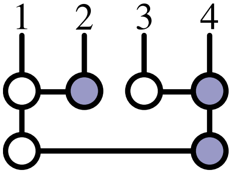

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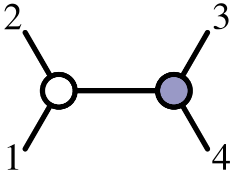

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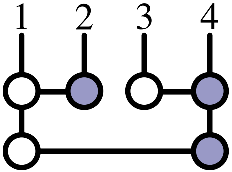
$$\Omega \mapsto \Omega \wedge d\alpha/\alpha \qquad d \mapsto d+1$$



$$C \equiv \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{pmatrix} 1 & w_2 & 0 & b_4^1 \\ 0 & 0 & 1 & b_4^2 \end{pmatrix} \end{array}$$

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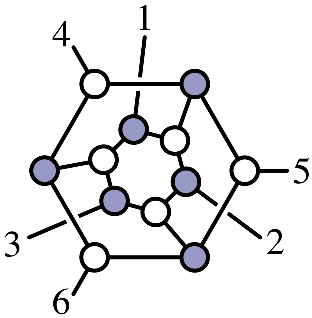

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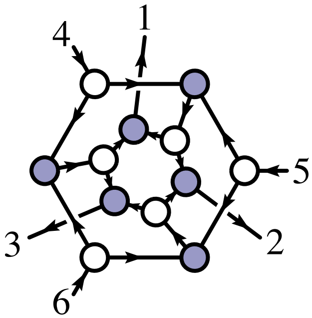

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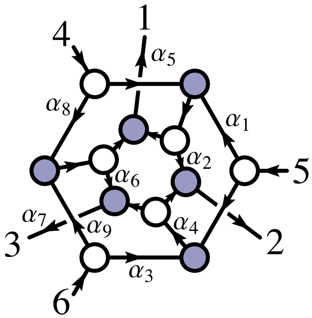

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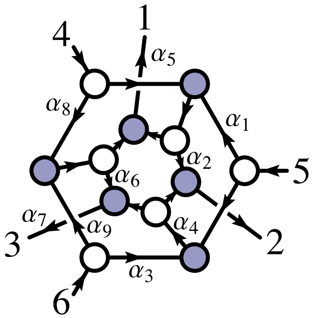

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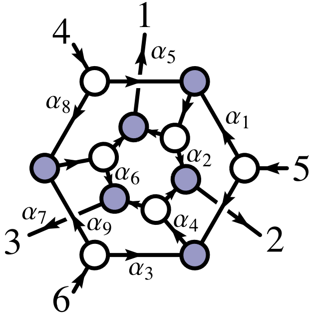

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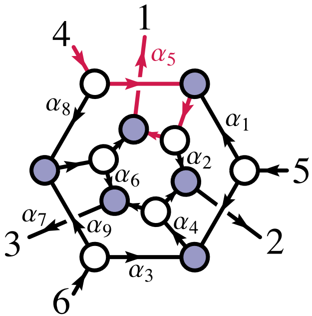

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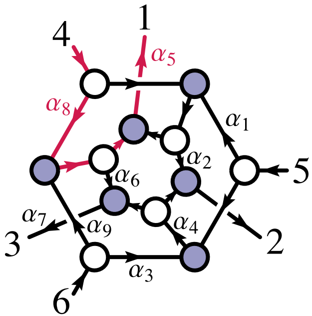

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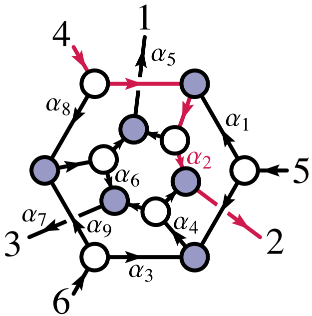

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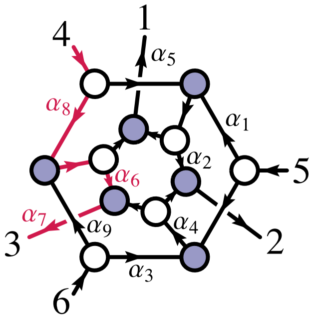

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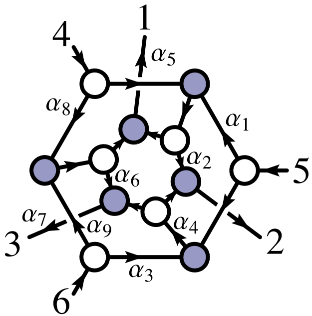

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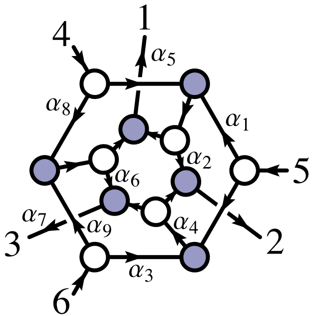

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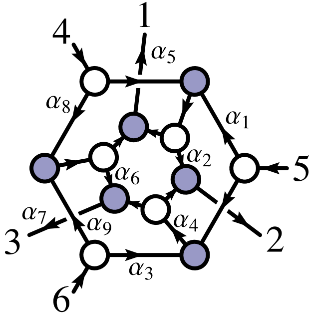

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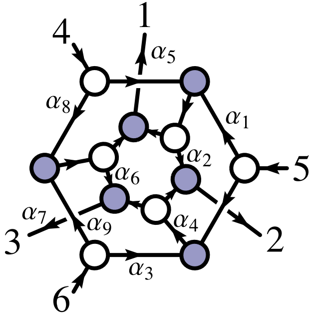

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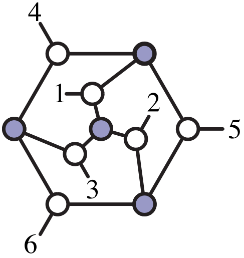

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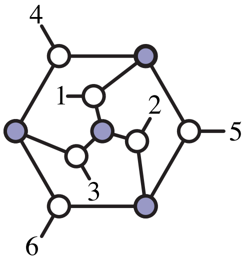

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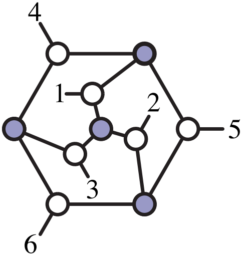

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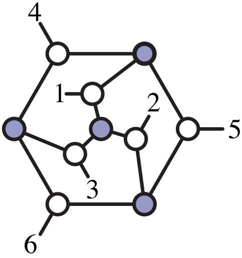

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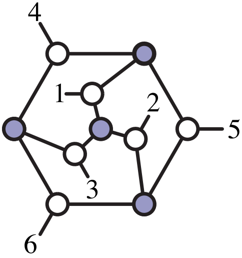

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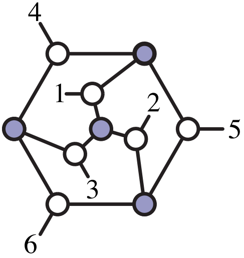

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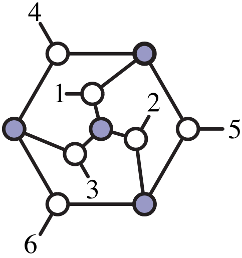

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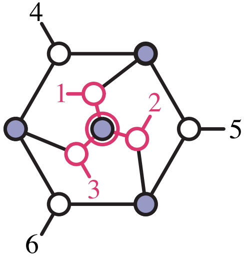

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Application: Classifying On-Shell Functions for $k=2$ (MHV)

For $k=2$ and $\widehat{n}_\delta=0$, reduced diagrams correspond to *top-dimensional* varieties.

A simple exercise shows that for any such reduced diagram:

- $n_B = (n-2)$
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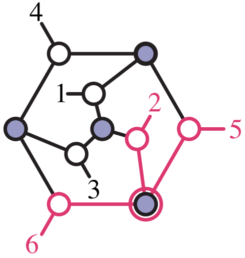

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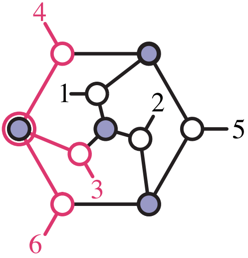

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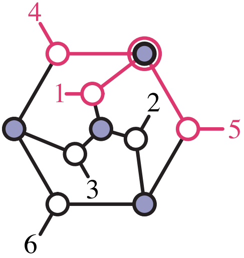

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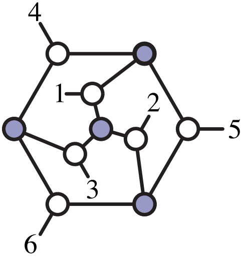

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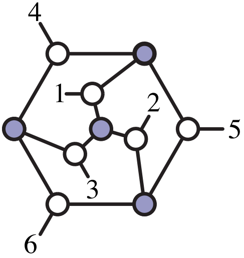

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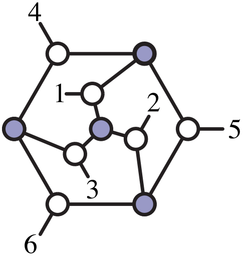

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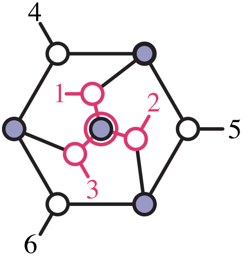

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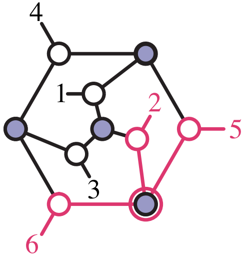

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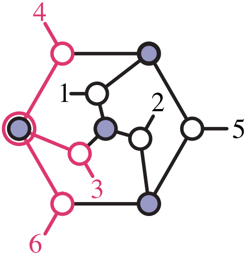

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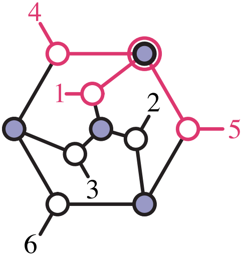

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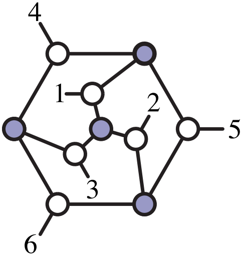

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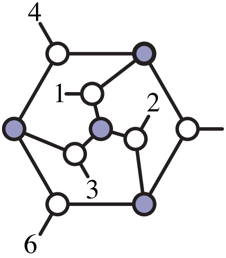

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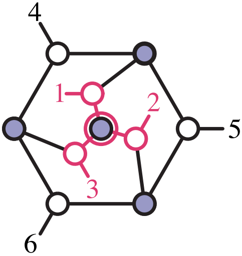

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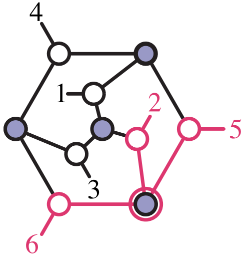

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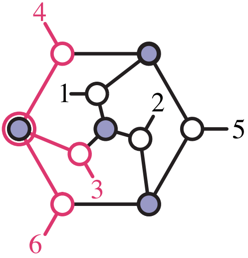

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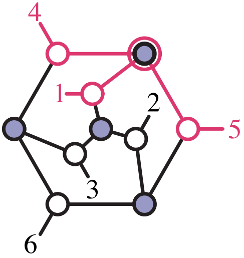

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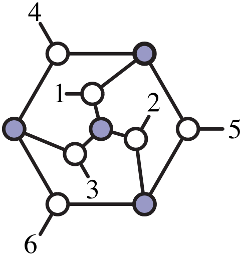

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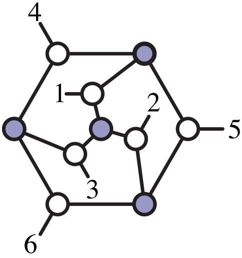

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Application: Classifying On-Shell Functions for $k\!=\!2$ (MHV)

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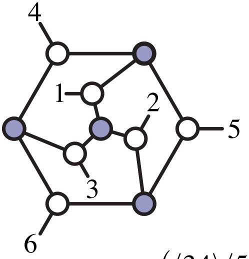

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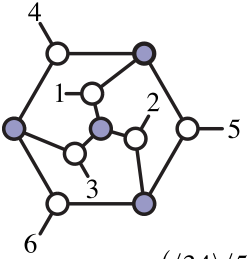

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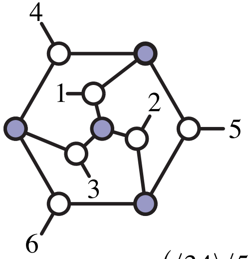

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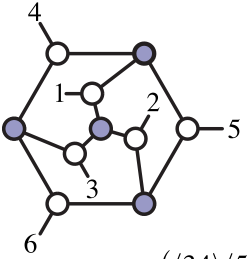

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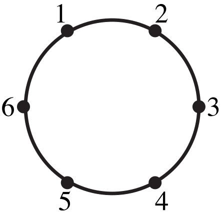

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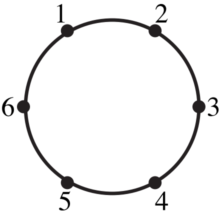

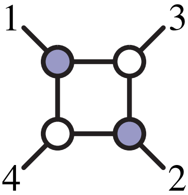

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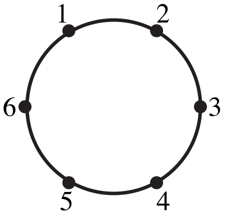

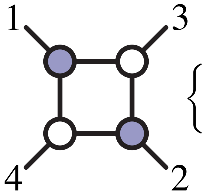

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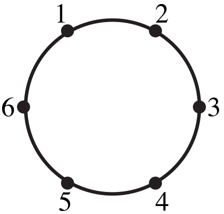

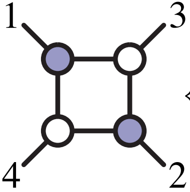

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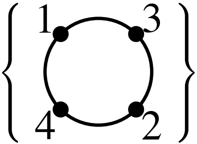

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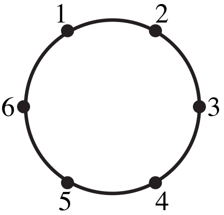

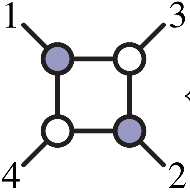

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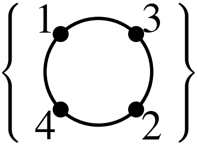

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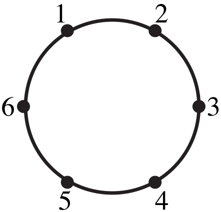

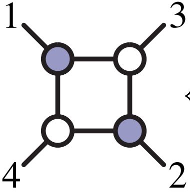

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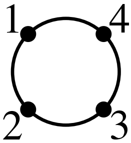

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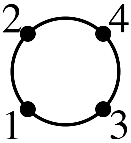

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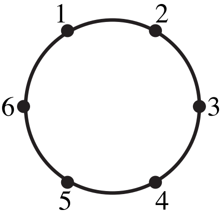

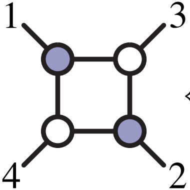

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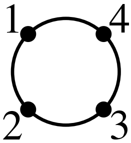

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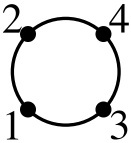

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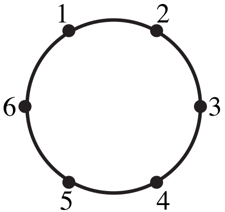

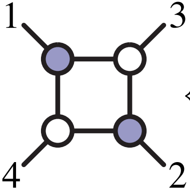

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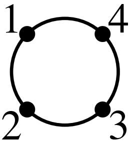

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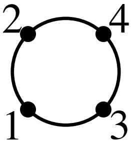

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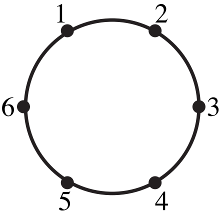

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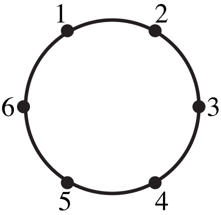

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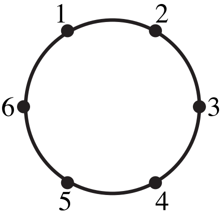

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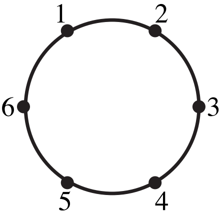

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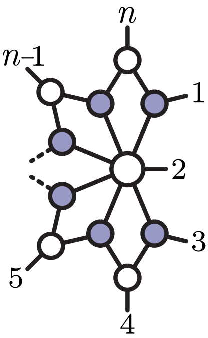

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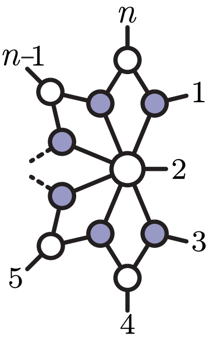

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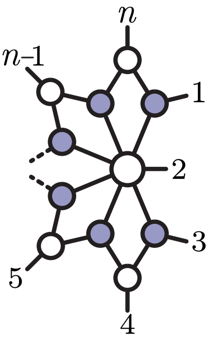

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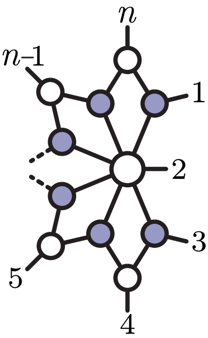

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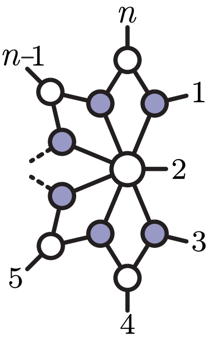

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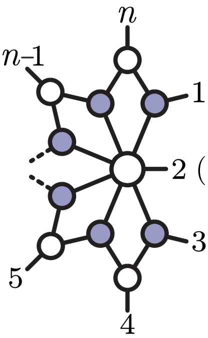

$$(-1)\frac{\left\{\begin{matrix}(2 & n & 1)\\(2 & 3 & 4)\\(2 & 4 & 5)\\ & \vdots & \\(2 & n-1 & n)\end{matrix}\right\}}{\mathrm{PT}(1,2,\ldots,n)} = \frac{\left\{\begin{matrix}(2 & 1 & n)\\(2 & 3 & 4)\\(2 & 4 & 5)\\ & \vdots & \\(2 & n-1 & n)\end{matrix}\right\}}{\mathrm{PT}(1,n,2,\ldots,n-1)+\ldots+\mathrm{PT}(1,3,\ldots,n,2)}$$

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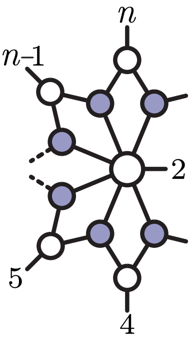

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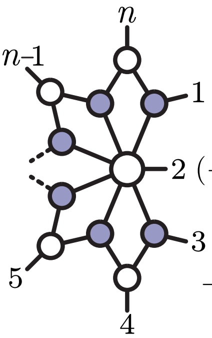

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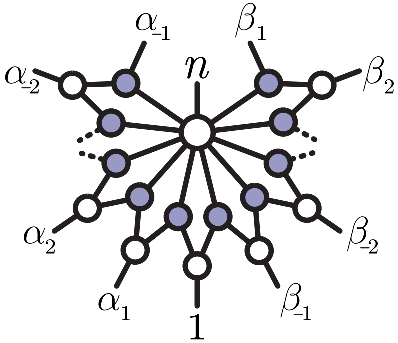

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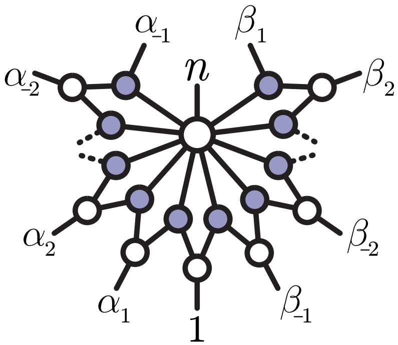

$$\left\{\begin{array}{c} (1\;\alpha_1\;n) \\ (\alpha_1\;\alpha_2\;n) \\ \vdots \\ (\alpha_{-2}\;\alpha_{-1}\;n) \\ (n\;\beta_1\;\beta_2) \\ \vdots \\ (n\;\beta_{-2}\;\beta_{-1}) \\ (n\;\beta_{-1}\;1) \end{array}\right\}$$

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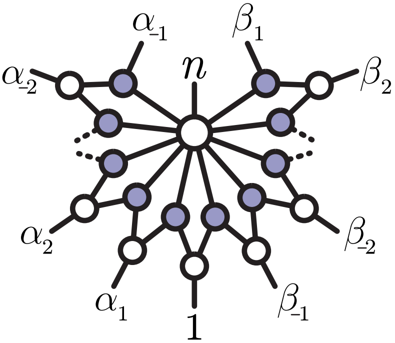

$$\left\{\begin{pmatrix} (\,1 & \alpha_1 & n\,) \\ (\alpha_1 & \alpha_2 & n\,) \\ & \vdots & \\ (\alpha_{-2} & \alpha_{-1} & n\,) \\ (\,n & \beta_1 & \beta_2) \\ & \vdots & \\ (\,n & \beta_{-2} & \beta_{-1}) \\ (\,n & \beta_{-1} & 1\,) \end{pmatrix}\right\} \qquad \left\{\begin{pmatrix} (\,1 & \alpha_1 & n\,) \\ (\alpha_1 & \alpha_2 & n\,) \\ & \vdots & \\ (\alpha_{-2} & \alpha_{-1} & n\,) \\ (\,n & \beta_2 & \beta_1) \\ & \vdots & \\ (\,n & \beta_{-1} & \beta_{-2}) \\ (\,n & 1 & \beta_{-1}) \end{pmatrix}\right\}$$

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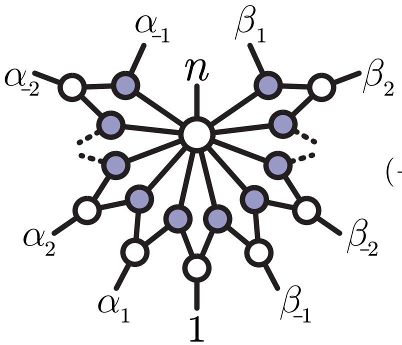

$$(-1)^{n_\beta}\left\{\begin{matrix}(1\;\alpha_1\;n)\\(\alpha_1\;\alpha_2\;n)\\\vdots\\(\alpha_{-2}\;\alpha_{-1}\;n)\\(n\;\beta_1\;\beta_2)\\\vdots\\(n\;\beta_{-2}\;\beta_{-1})\\(n\;\beta_{-1}\;1)\end{matrix}\right\}=\left\{\begin{matrix}(1\;\alpha_1\;n)\\(\alpha_1\;\alpha_2\;n)\\\vdots\\(\alpha_{-2}\;\alpha_{-1}\;n)\\(n\;\beta_2\;\beta_1)\\\vdots\\(n\;\beta_{-1}\;\beta_{-2})\\(n\;1\;\beta_{-1})\end{matrix}\right\}$$

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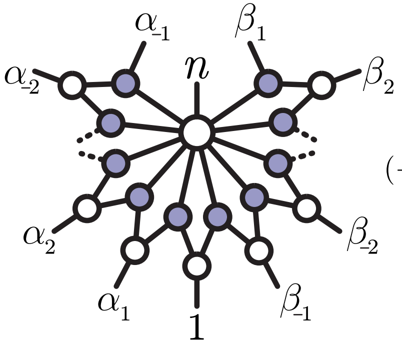

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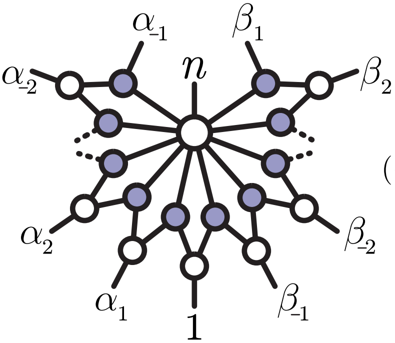

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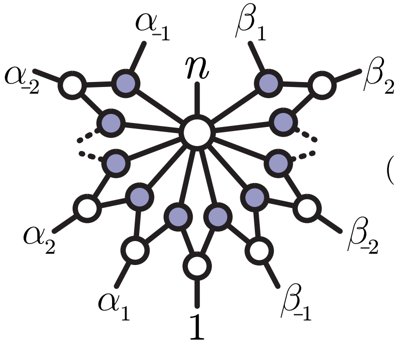

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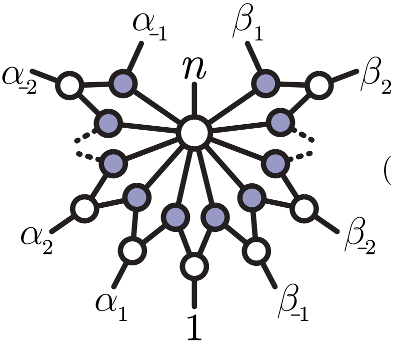

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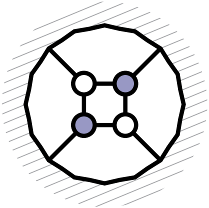

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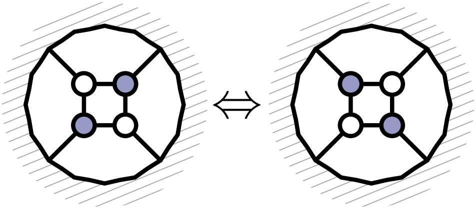

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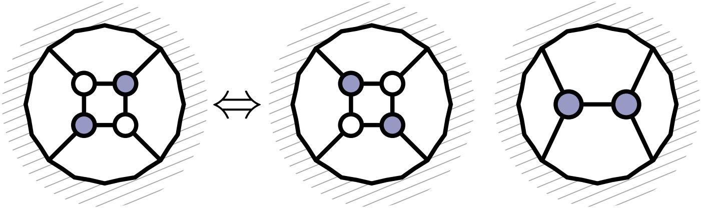

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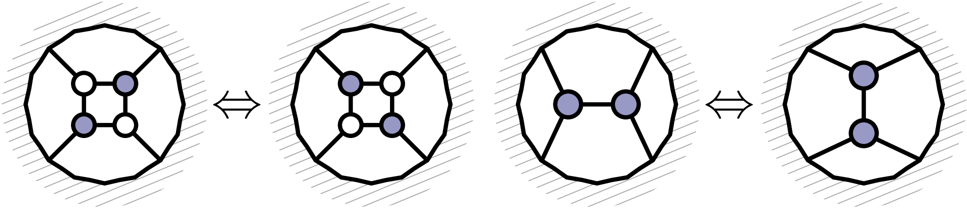

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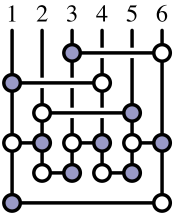

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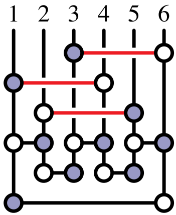

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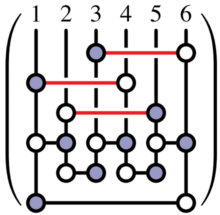
∂ 

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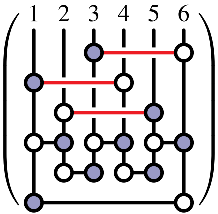
∂ 

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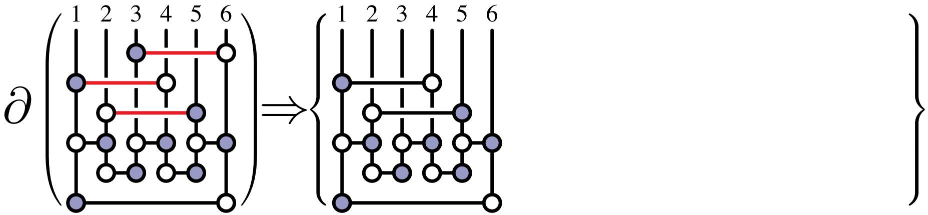

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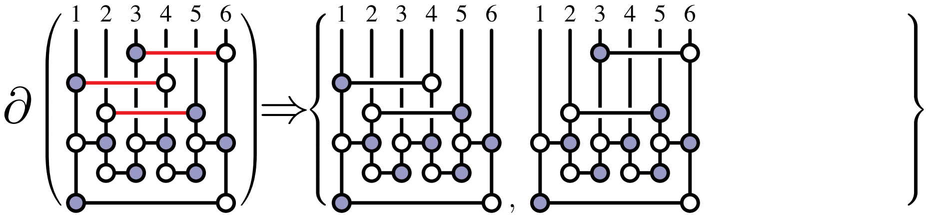

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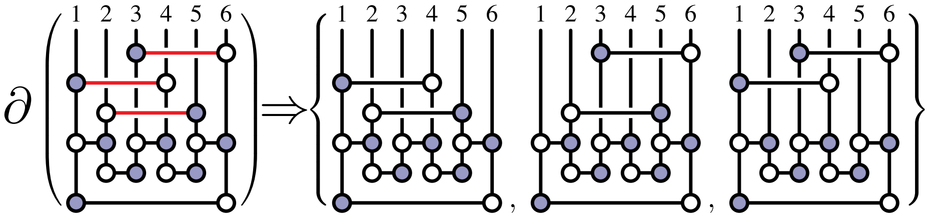

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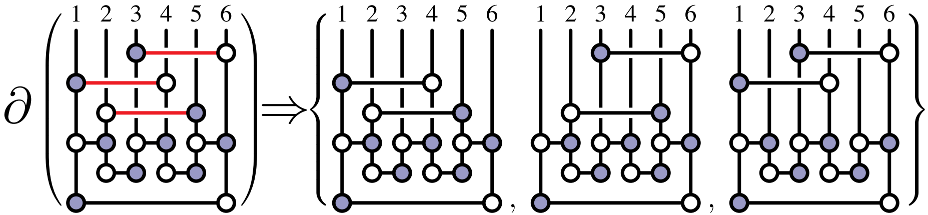

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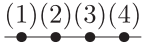

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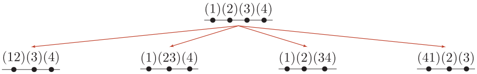

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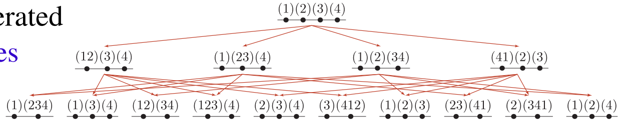

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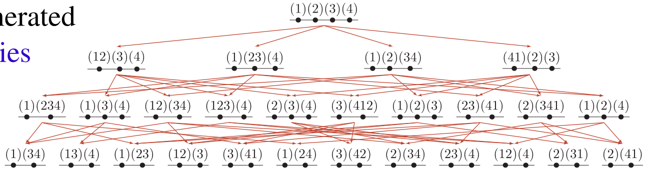

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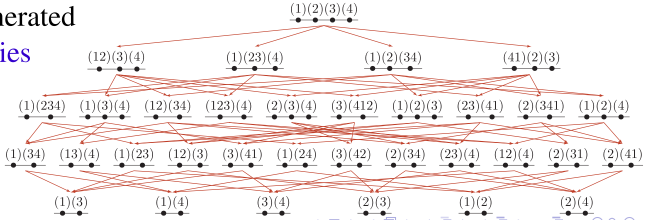

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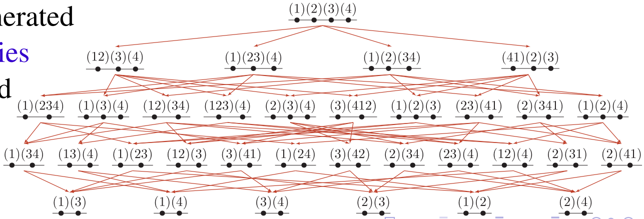

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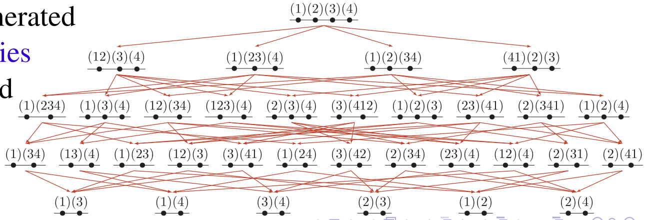

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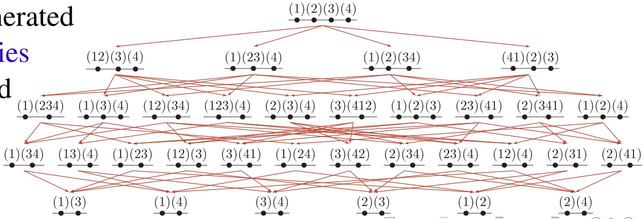

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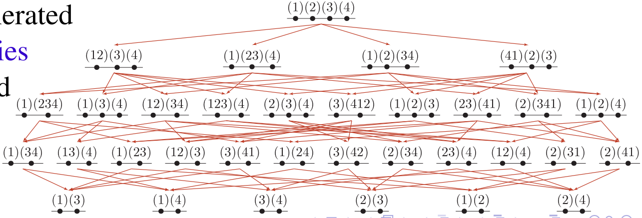

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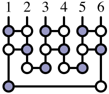

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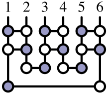

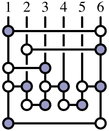

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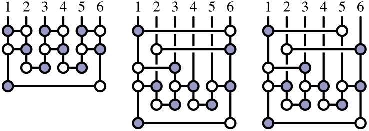

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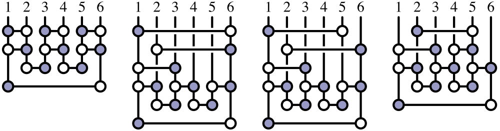

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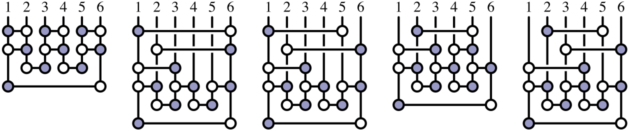

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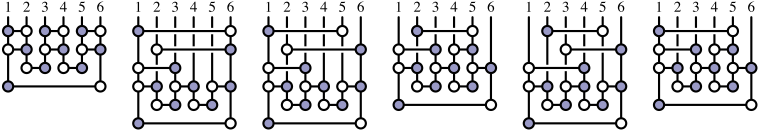

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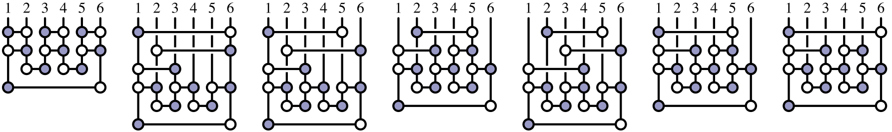

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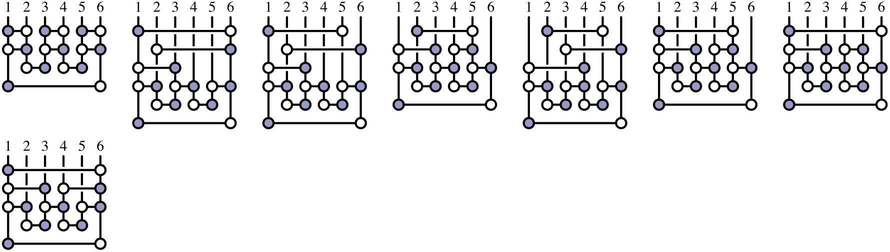

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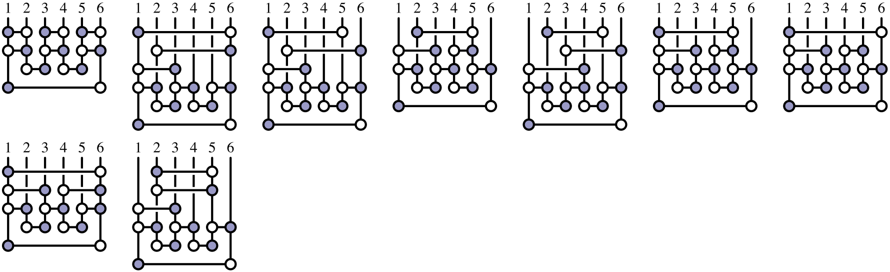

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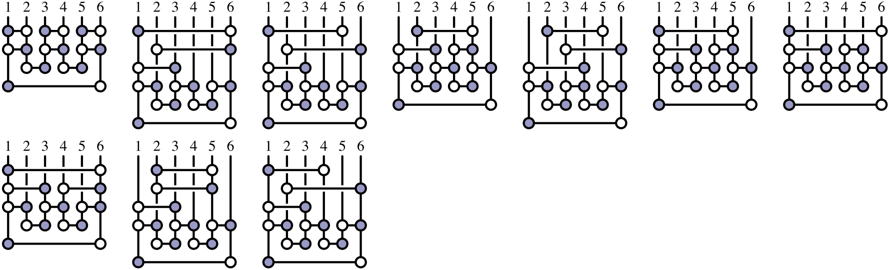

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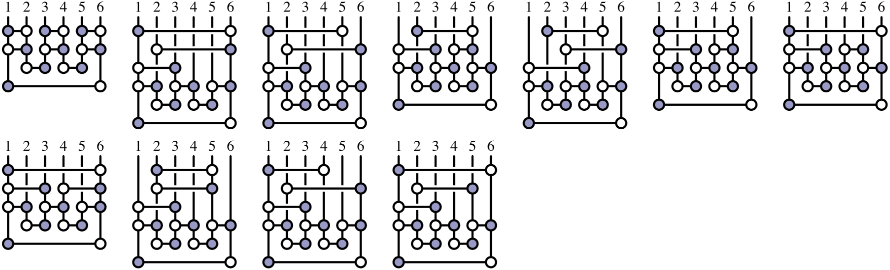

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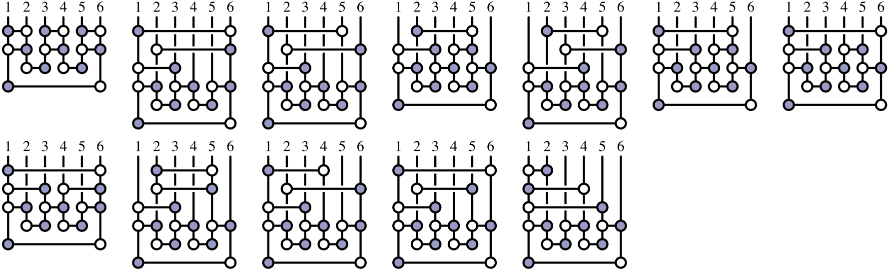

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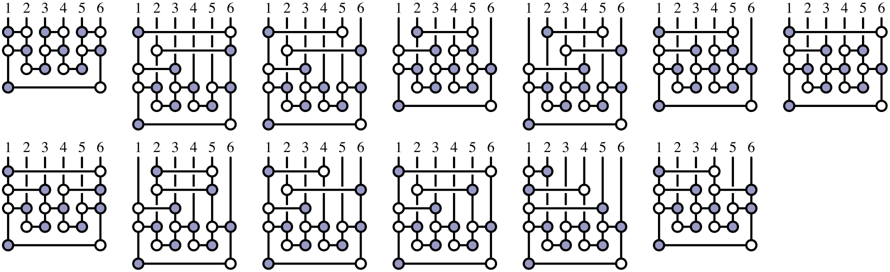

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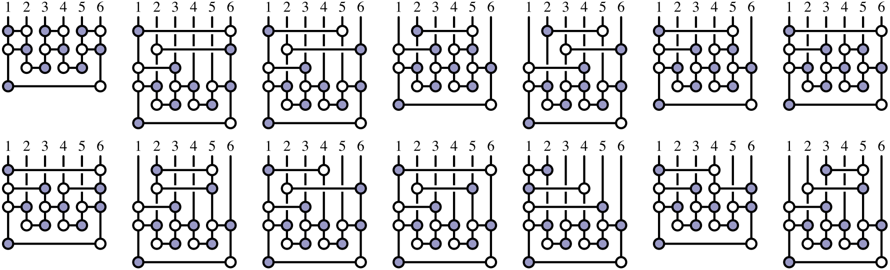

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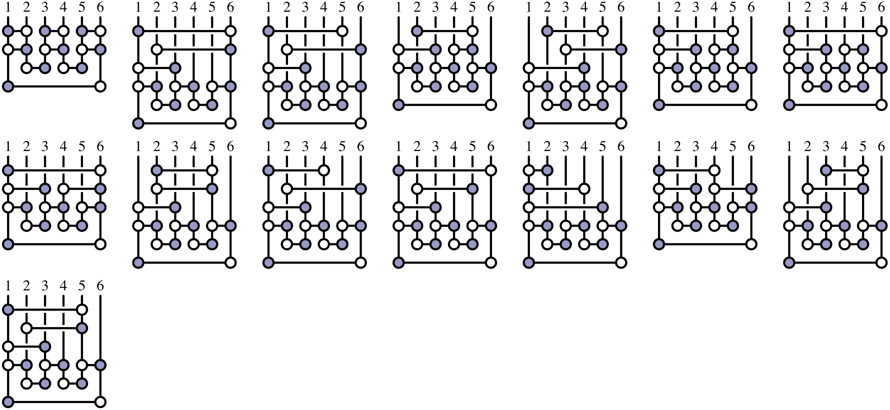

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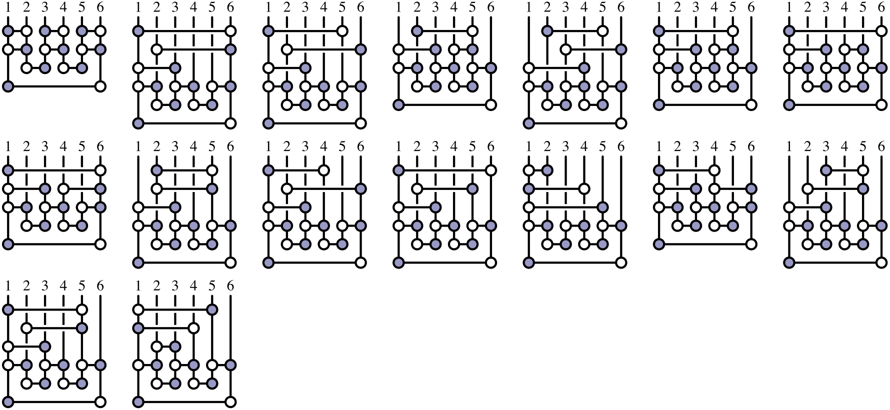

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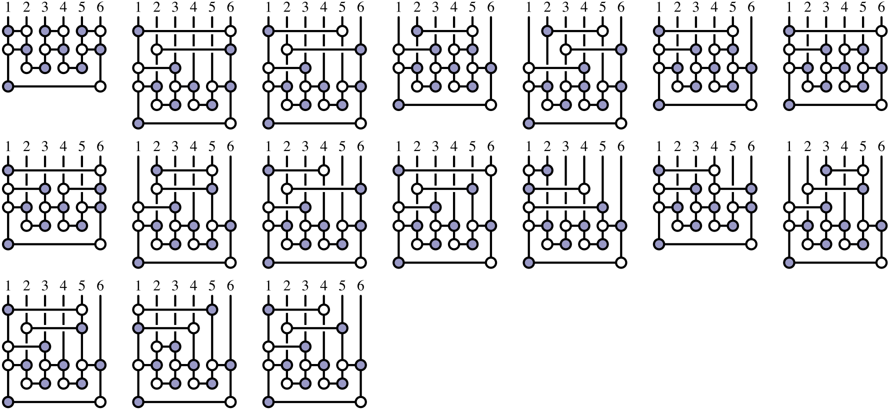

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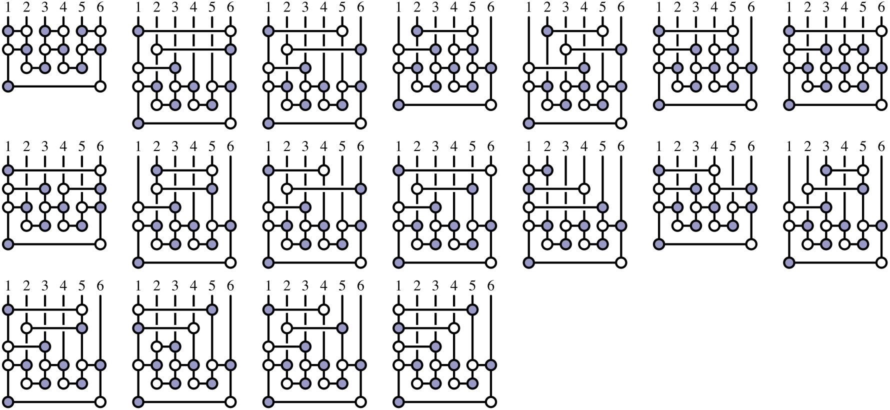

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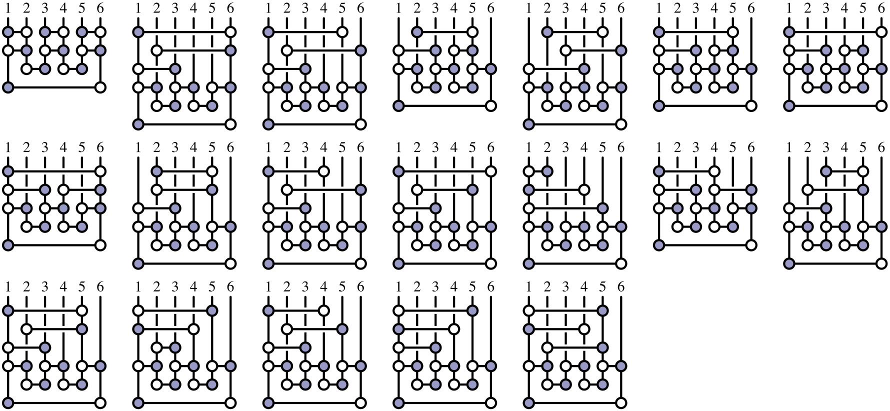

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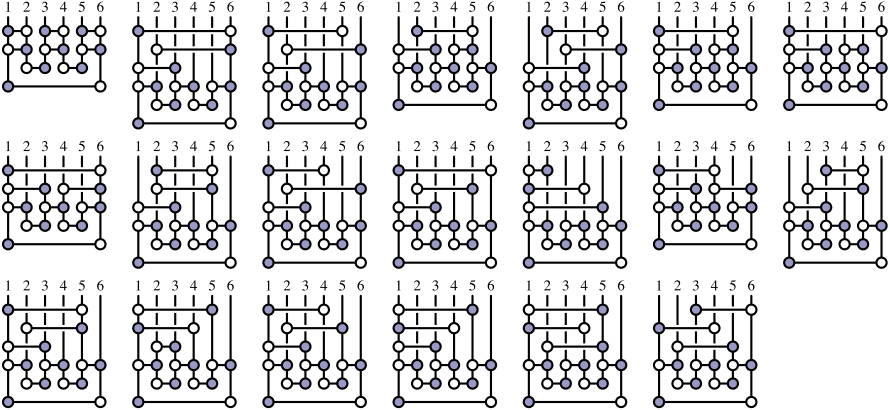

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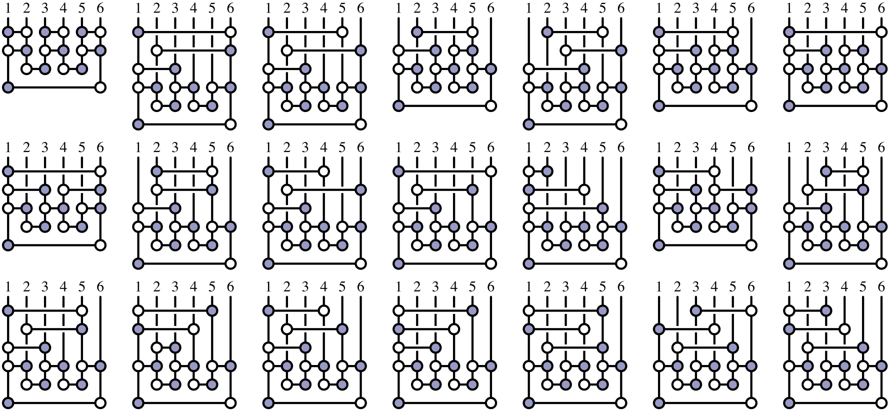

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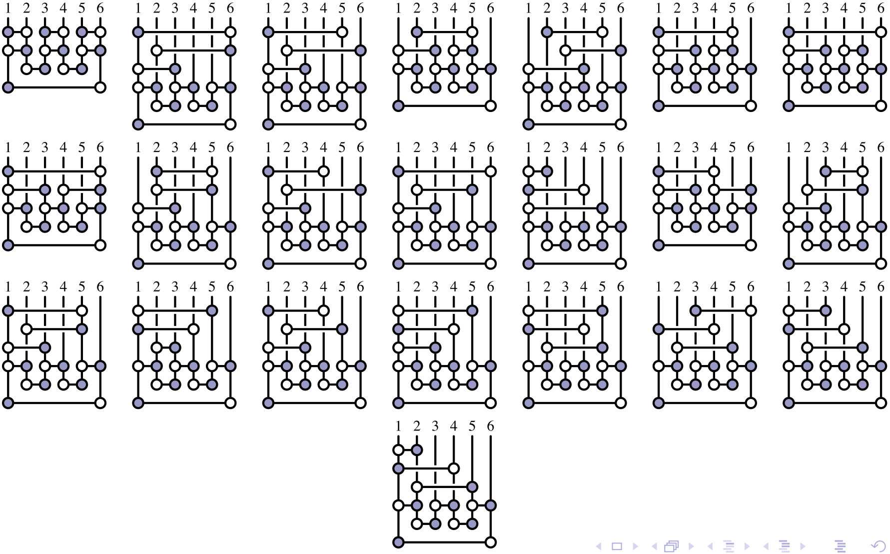

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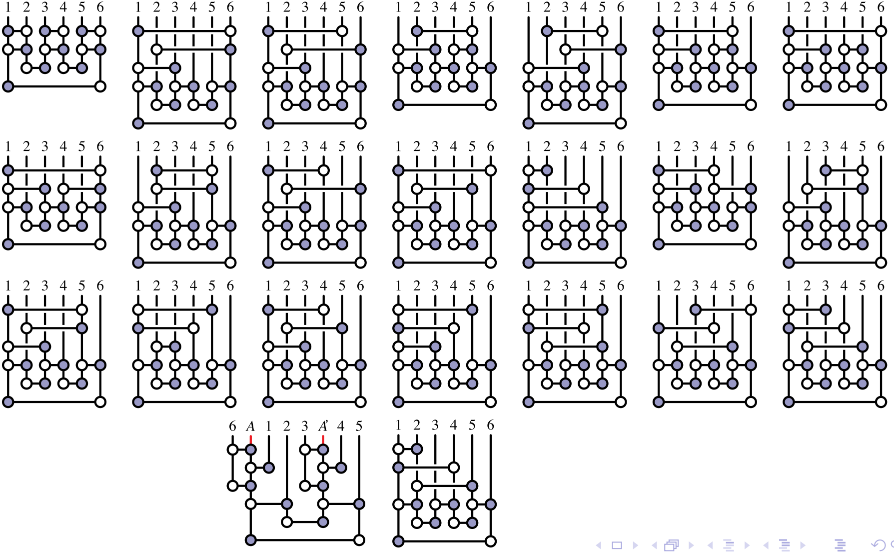

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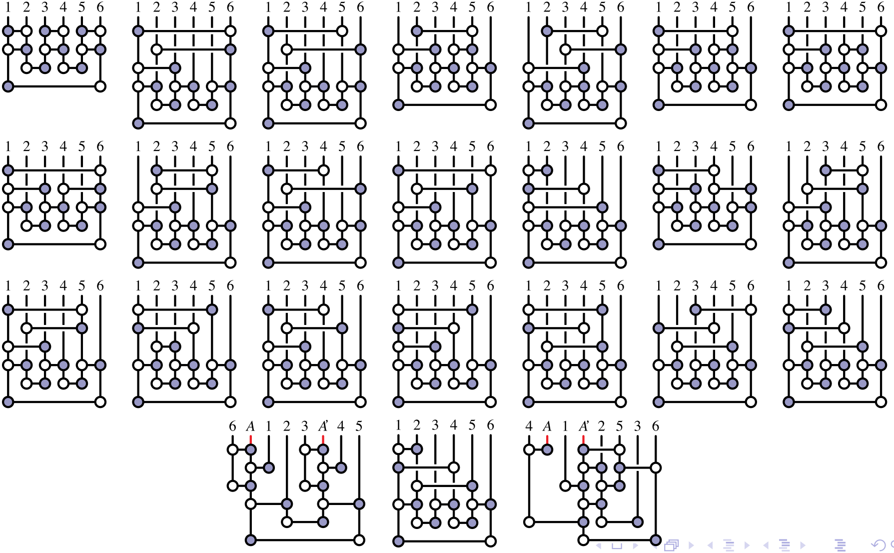

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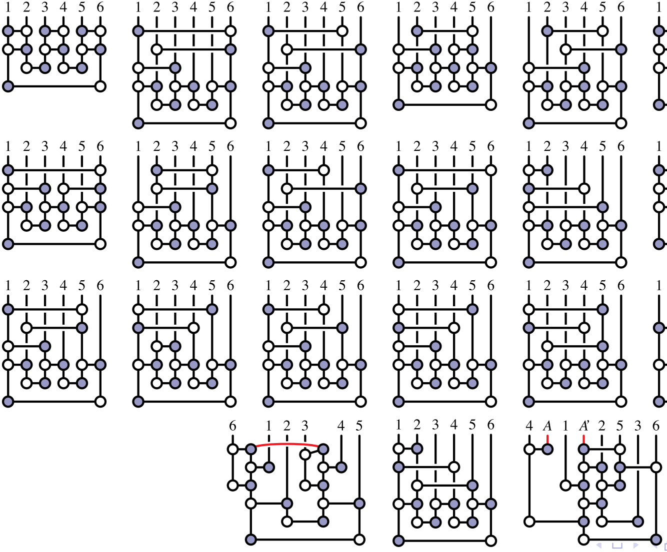

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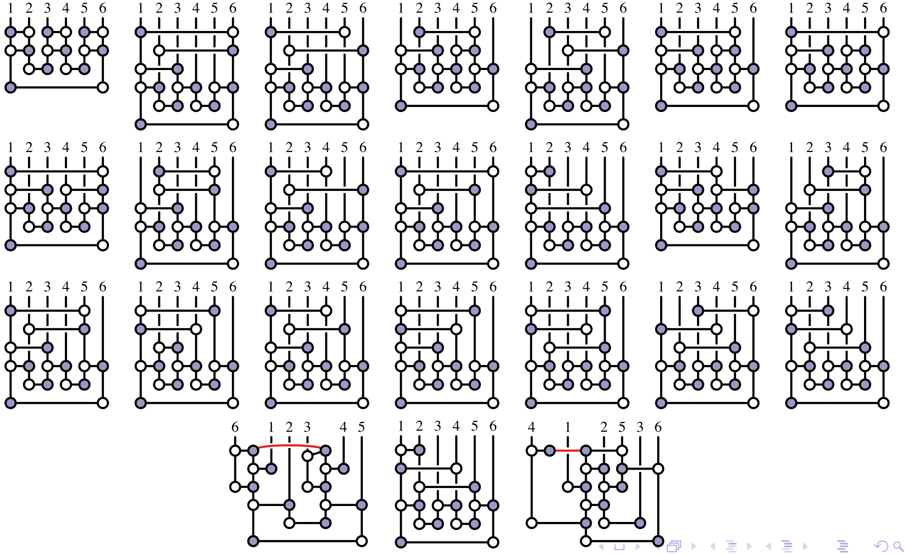

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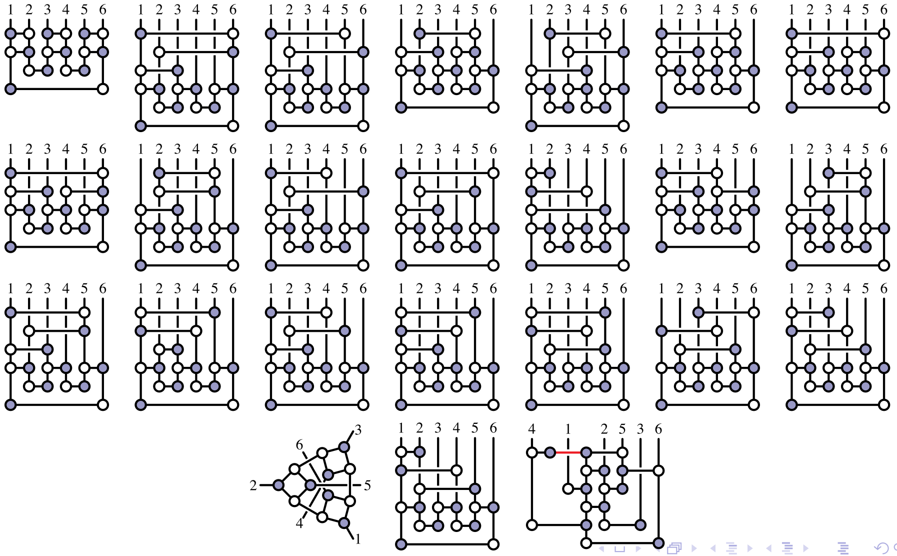

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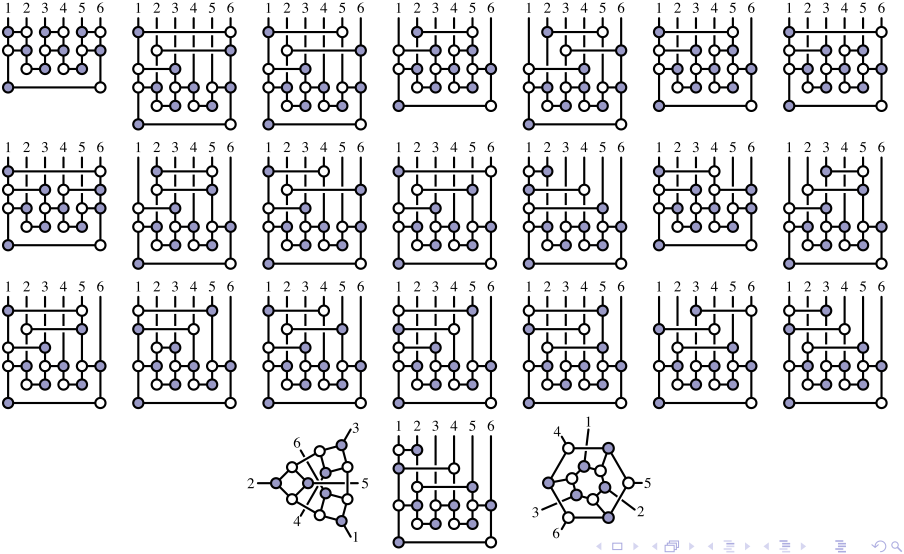

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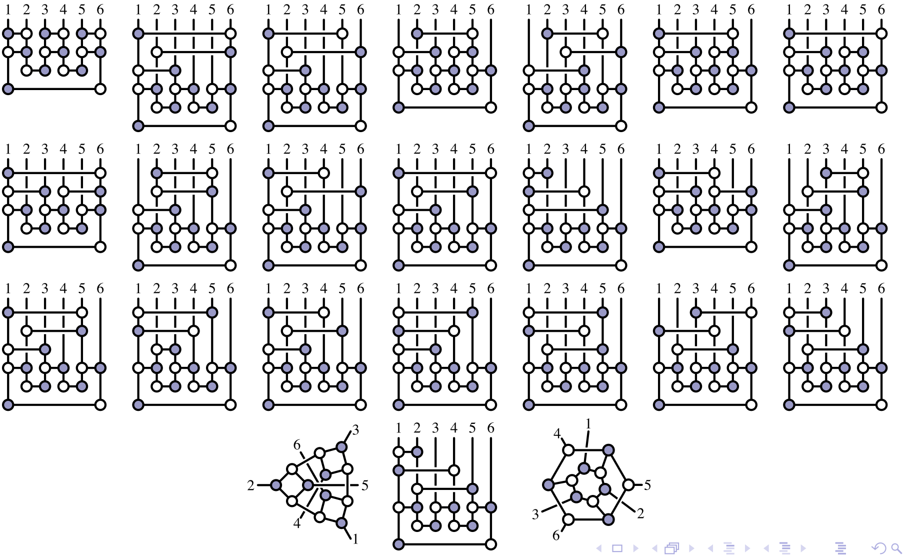
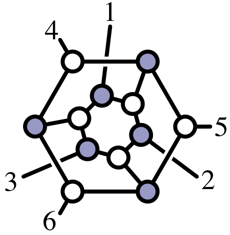


Novel Boundary Structures for On-Shell Varieties in $G(3,6)$

There is one especially interesting on-shell variety—associated with the graph:



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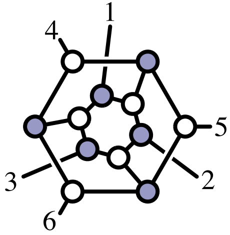

It has twelve removable edges

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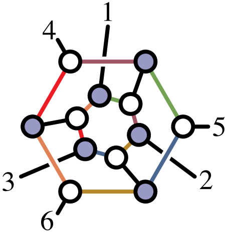

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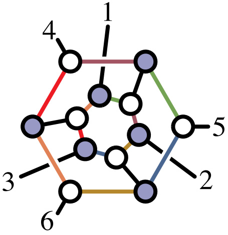

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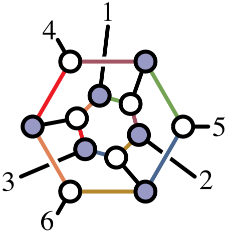

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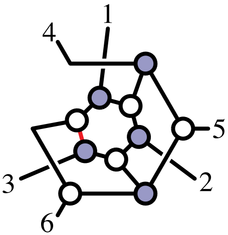

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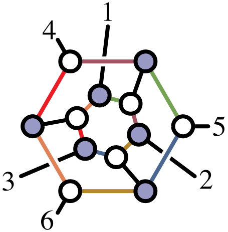

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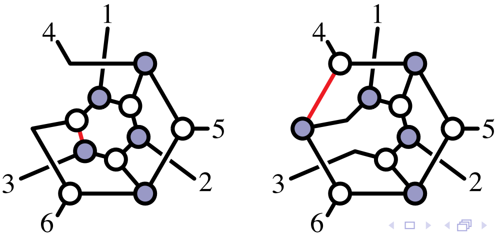

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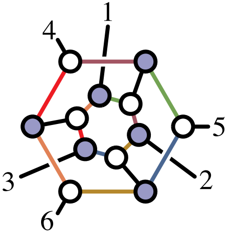

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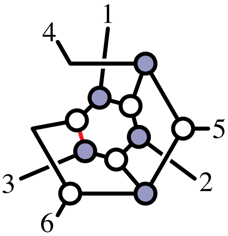

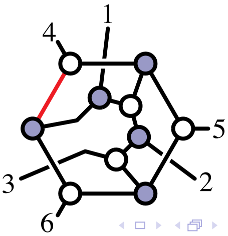

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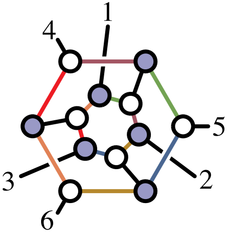
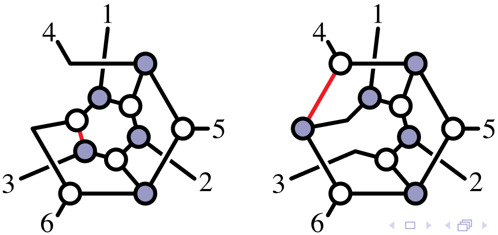


It has twelve removable edges, but only six (non-isomorphic) boundaries(!)



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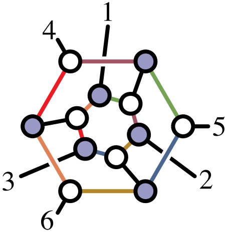

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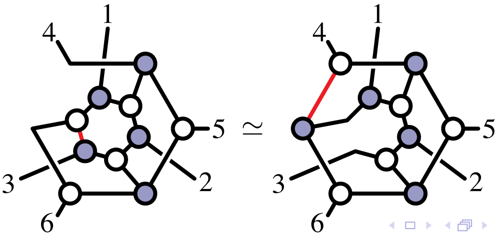

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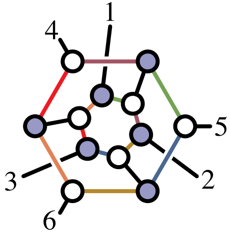
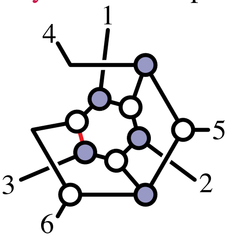
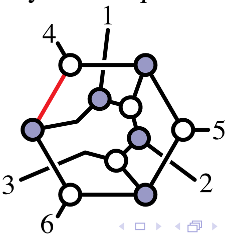


It has twelve removable edges, but only six (non-isomorphic) boundaries(!)

They are oppositely oriented: separated by **three** square moves.



$\simeq$



Enumeration of (8-dim) ‘Leading Singularities’ of $G(3,6)$

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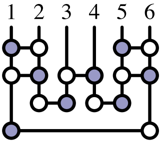
$$f_1 \equiv \oint\limits_{(123)=0} \Omega_1 = \left.\frac{\delta^{3\times 4}\big(C^*\!\cdot\widetilde{\eta}\big)\,\delta^{2\times 2}\big(\lambda\cdot\widetilde{\lambda}\big)}{(234)(345)(456)(561)(612)}\right|_{C^*} \qquad C^* \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & [56] & [64] & [45] \end{pmatrix}$$

$$= \frac{\delta^{3\times 4}\big(C^*\!\cdot\widetilde{\eta}\big)\,\delta^{2\times 2}\big(\lambda\cdot\widetilde{\lambda}\big)}{\langle 23\rangle\,[56]\,\langle 3|4{+}5|6]\,s_{456}\,\langle 1|5{+}6|4]\,\langle 12\rangle\,[45]}$$



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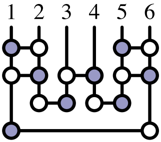

$$f_2 \equiv \oint_{(123)=0} \Omega_2 = \left.\frac{(235)\,\delta^{3\times4}\big(C^*\!\cdot\widetilde{\eta}\big)\delta^{2\times2}\big(\lambda\cdot\widetilde{\lambda}\big)}{(136)(156)(234)(245)(256)(345)}\right|_{C^*} \qquad C^* \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & [56] & [64] & [45] \end{pmatrix}$$

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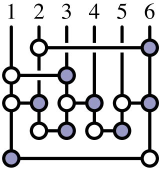

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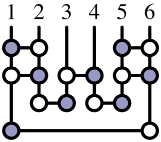

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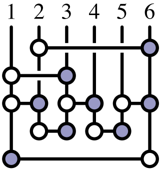
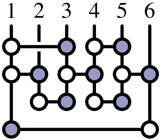


$$f_3 \equiv \oint_{(123)=0} \Omega_4 = \left.\frac{(145)\,\delta^{3\times4}(C^*\cdot\widetilde{\eta})\,\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{(124)(136)(156)(245)(345)(456)}\right|_{C^*} \qquad C^* \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & [56] & [64] & [45] \end{pmatrix}$$

$$= \frac{\langle1|4{+}5|6]\;\delta^{3\times4}(C^*\cdot\widetilde{\eta})\,\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{\langle12\rangle\,[56]\,\langle13\rangle\,[45]\,\langle1|5{+}6|4]\,\langle2|4{+}5|6]\,\langle3|4{+}5|6]\,s_{456}}$$



Enumeration of (8-dim) ‘Leading Singularities’ of $G(3,6)$

$$f_4 \equiv \oint_{(123)=0} \Omega_5 = \left.\frac{(135)\,\delta^{3\times4}\big(C^*\!\cdot\widetilde{\eta}\big)\delta^{2\times2}\big(\lambda\cdot\widetilde{\lambda}\big)}{(124)(145)(156)(236)(345)(356)}\right|_{C^*} \qquad C^* \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & [56] & [64] & [45] \end{pmatrix}$$

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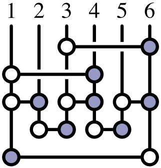

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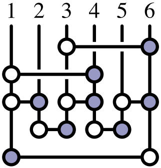

$$f_5 \equiv \oint_{(123)=0} \Omega_9 = \left.\frac{(125)\,\delta^{3\times4}(C^*\!\cdot\!\widetilde{\eta})\delta^{2\times2}(\lambda\!\cdot\!\widetilde{\lambda})}{(134)(156)(245)(256)(16(25)\bigcap(34))}\right|_{C^*} \qquad C^* \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & [56] & [64] & [45] \end{pmatrix}$$

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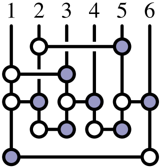

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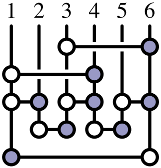
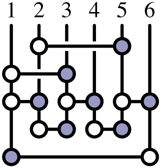


$$f_5 \equiv \oint_{(123)=0} \Omega_9 = \left.\frac{(125)\,\delta^{3\times4}(C^{*}\!\cdot\widetilde{\eta})\,\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{(134)(156)(245)(256)(16(25)\bigcap(34))}\right|_{C^{*}} \qquad C^{*} \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & [56] & [64] & [45] \end{pmatrix}$$

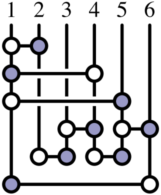
$$= \frac{\langle 12\rangle\,[64]\;\delta^{3\times4}(C^{*}\!\cdot\widetilde{\eta})\,\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{\langle 13\rangle\,[56]\,\langle 1|5{+}6|4]\langle 2|4{+}5|6]\langle 2|5{+}6|4]\big(\langle 23\rangle\,[56]\,\langle 1|5{+}6|4]-\langle 12\rangle\,[45]\,\langle 3|4{+}5|6]\big)}$$



$$C^{*} \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & [56] & [64] & [45] \end{pmatrix}$$

$$f_6 \equiv \oint_{(123)=0} \Omega_{12} = \left.\frac{(134)^2(456)\,\delta^{3\times4}(C^{*}\!\cdot\widetilde{\eta})\,\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{(124)(145)(146)(156)(234)(345)(346)(356)}\right|_{C^{*}}$$

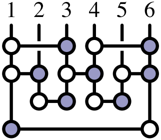
$$= \frac{\langle 13\rangle^2 s_{456}\,\delta^{3\times4}(C^{*}\!\cdot\widetilde{\eta})\,\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{\langle 12\rangle\langle 1|4{+}5|6]\langle 1|4{+}6|5]\langle 1|5{+}6|4]\langle 23\rangle\langle 3|4{+}5|6]\langle 3|4{+}6|5]\langle 3|5{+}6|4]}$$



Enumeration of (8-dim) 'Leading Singularities' of $G(3,6)$

$$f_7 \equiv \oint_{(123)=0} \Omega_{13} = \left.\frac{(145)^2\,\delta^{3\times 4}\big(C^*\!\cdot\widetilde{\eta}\big)\,\delta^{2\times 2}\big(\lambda\cdot\widetilde{\lambda}\big)}{(125)(134)(146)(156)(245)(345)(456)}\right|_{C^*} \qquad C^* \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & [56] & [64] & [45] \end{pmatrix}$$

$$= \frac{\langle 1|4{+}5|6]^2\,\delta^{3\times 4}\big(C^*\!\cdot\widetilde{\eta}\big)\,\delta^{2\times 2}\big(\lambda\cdot\widetilde{\lambda}\big)}{\langle 12\rangle\,[64]\,\langle 13\rangle\,[56]\,\langle 1|4{+}6|5]\langle 1|5{+}6|4]\langle 2|4{+}5|6]\langle 3|4{+}5|6]\,s_{456}}$$



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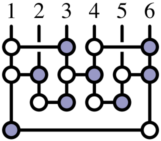
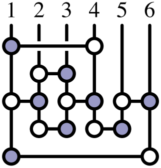


$$f_8 \equiv \oint_{(14(23)\cap(56))=0} \Omega_{16} = \int \frac{d\alpha_1}{\alpha_1}\wedge\cdots\wedge\frac{d\alpha_8}{\alpha_8}\;\delta^{3\times4}\big(C(\alpha)\cdot\widetilde{\eta}\big)\delta^{3\times2}\big(C(\alpha)\cdot\widetilde{\lambda}\big)\delta^{2\times3}\big(\lambda\cdot C^{\perp}(\alpha)\big)$$

$$C(\alpha) \equiv \begin{pmatrix} 1 & \alpha_6 & \alpha_6\,\alpha_7 & 0 & 0 & \alpha_1 \\ 0 & 1 & \alpha_5{+}\alpha_7 & 0 & \alpha_2 & \alpha_2\,\alpha_4 \\ \alpha_8 & 0 & 0 & 1 & \alpha_3 & \alpha_3\,\alpha_4 \end{pmatrix}$$



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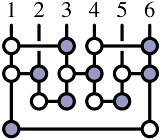

$$f_8 \equiv \oint\limits_{(14(23)\cap(56))=0} \Omega_{16} = \int \frac{d\alpha_1}{\alpha_1}\wedge\cdots\wedge\frac{d\alpha_8}{\alpha_8}\;\delta^{3\times4}(C(\alpha)\cdot\widetilde{\eta})\delta^{3\times2}(C(\alpha)\cdot\widetilde{\lambda})\delta^{2\times3}(\lambda\cdot C^{\perp}(\alpha))$$

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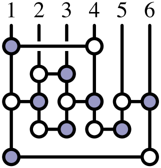

$$f_9 \equiv \oint\limits_{(14(23)\cap(56))=0} \Omega_{18} = \int \frac{d\alpha_1}{\alpha_1}\wedge\cdots\wedge\frac{d\alpha_8}{\alpha_8}\;\delta^{3\times4}(C(\alpha)\cdot\widetilde{\eta})\delta^{3\times2}(C(\alpha)\cdot\widetilde{\lambda})\delta^{2\times3}(\lambda\cdot C^{\perp}(\alpha))$$

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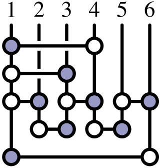

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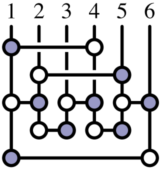
$$C(\alpha) \equiv \begin{pmatrix} \alpha_6\,\alpha_8 & \alpha_1 & 1 & \alpha_6 & \alpha_1\,\alpha_7 & 0 \\ \alpha_8 & 0 & 0 & 1 & \alpha_5 & \alpha_4 \\ \alpha_3 & \alpha_2 & 0 & 0 & \alpha_2\,\alpha_7 & 1 \end{pmatrix}$$



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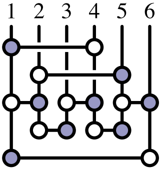

The (Lie-Algebra) '*Coloring*' of On-Shell Diagrams

In Yang-Mills theory, states are labelled by (non-kinematic) 'colors' c_a;
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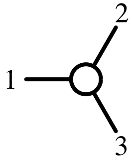

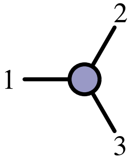

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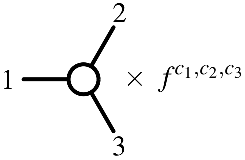

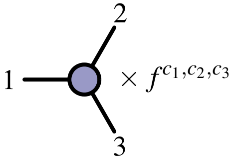

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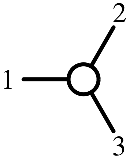

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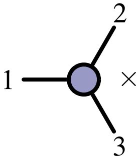

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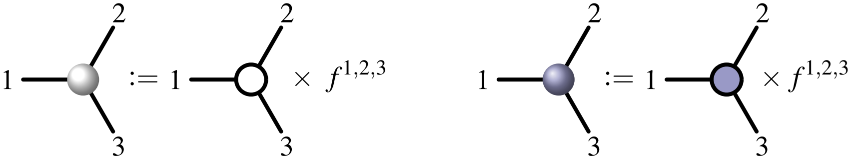

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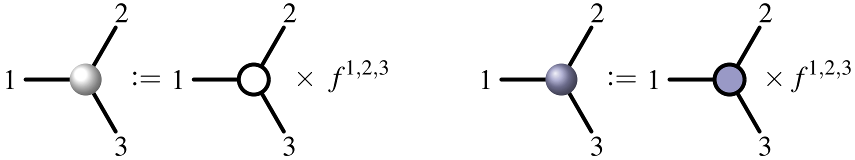

Consistency implies that these coupling constants obey Jacobi relations(!)

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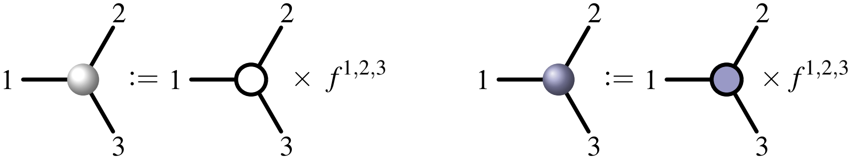

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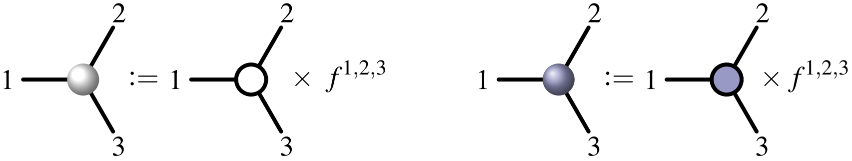

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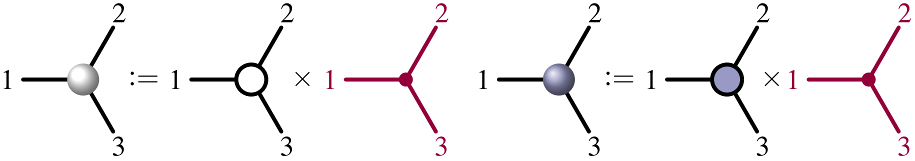

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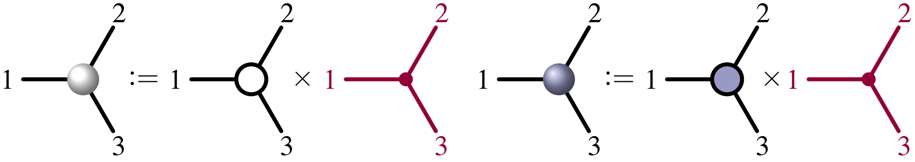

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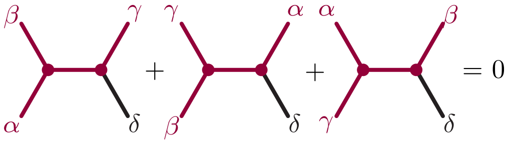

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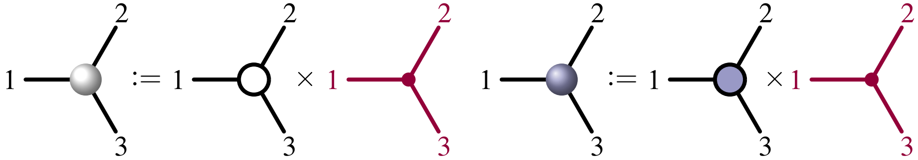

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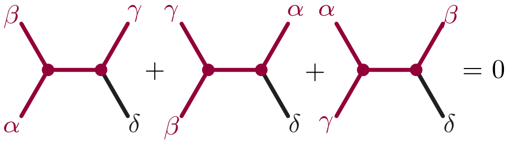

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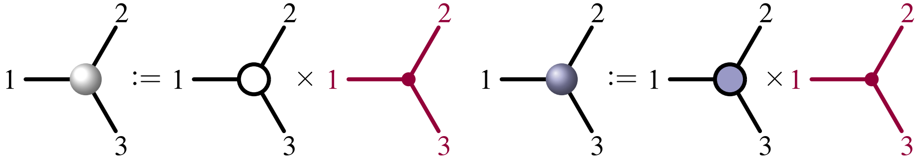

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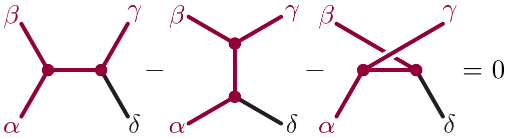

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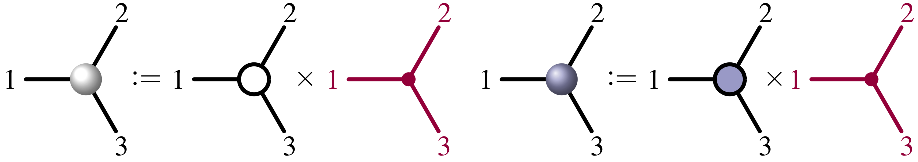

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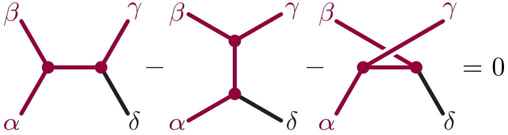

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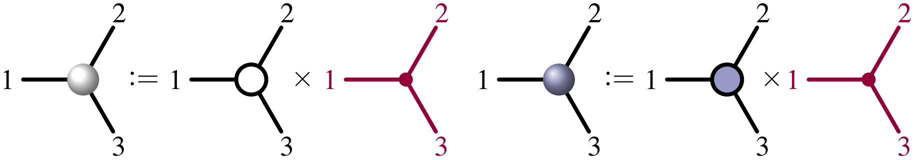

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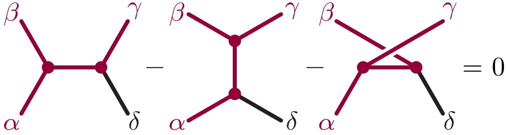

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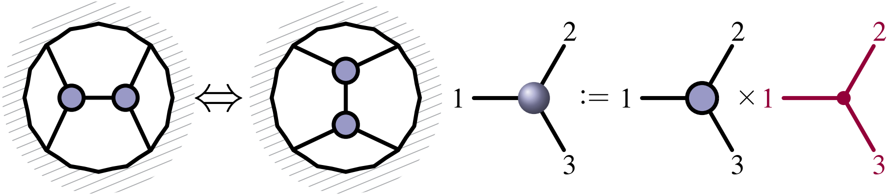

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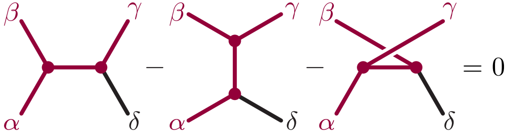

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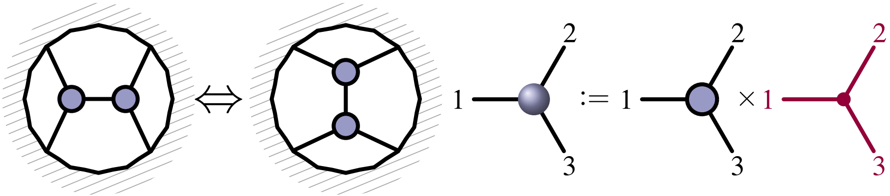

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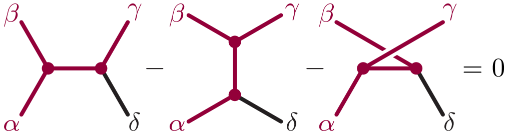

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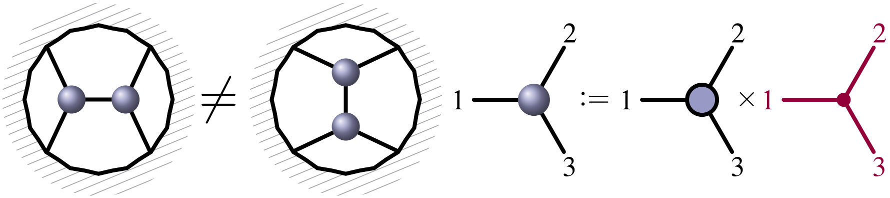

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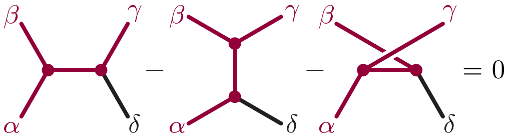

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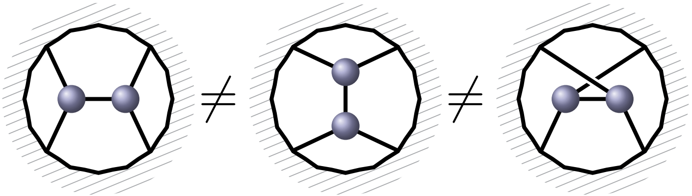

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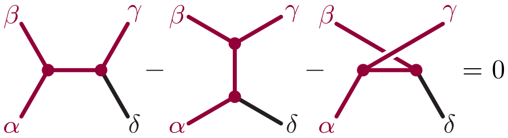

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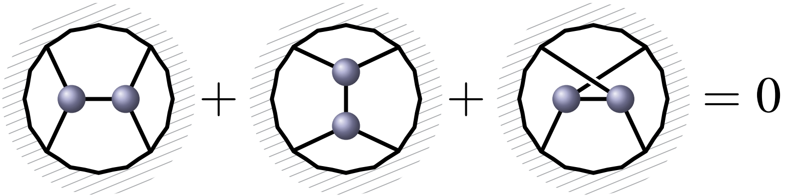

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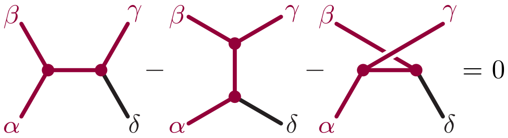

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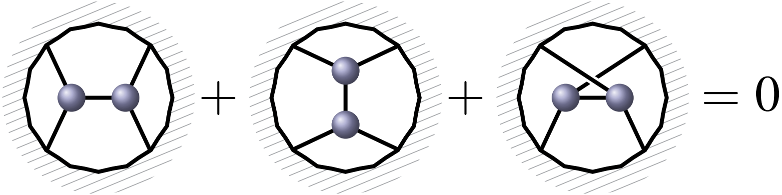

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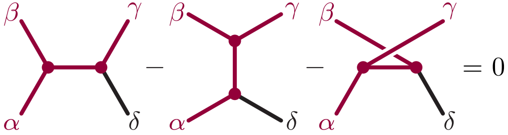

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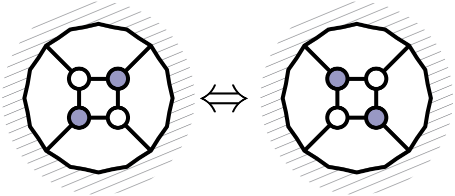

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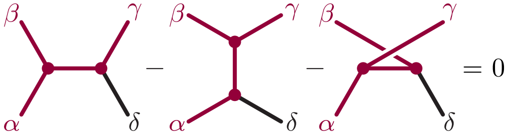

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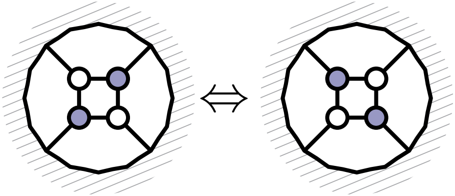

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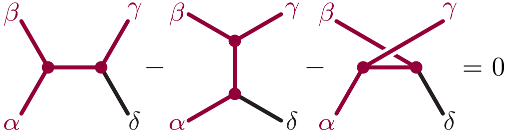

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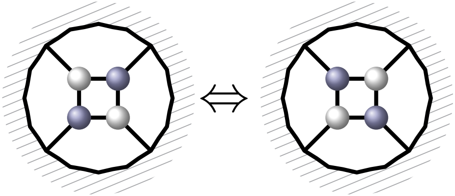

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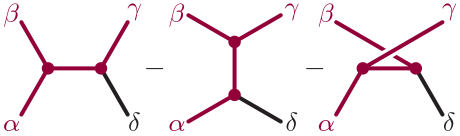

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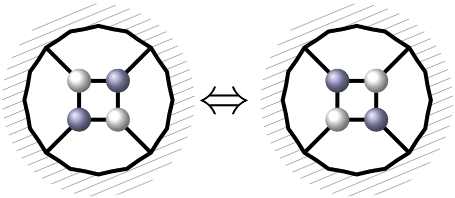

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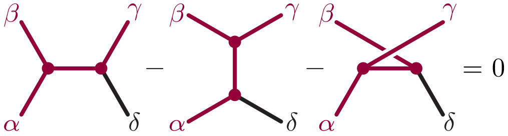

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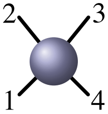

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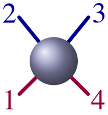

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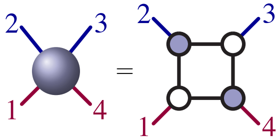

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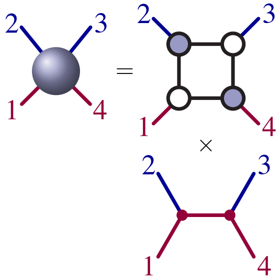

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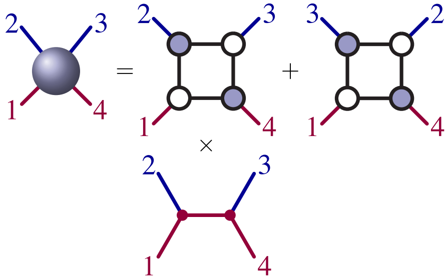

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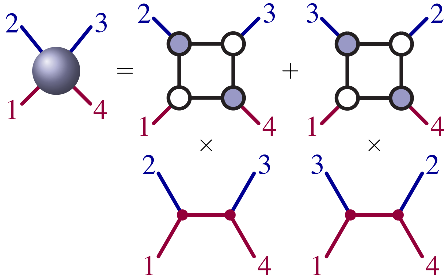

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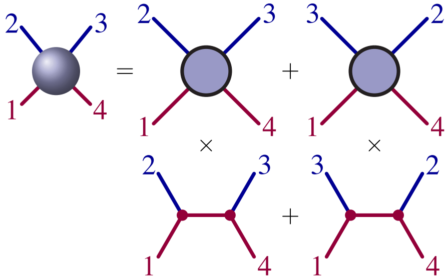

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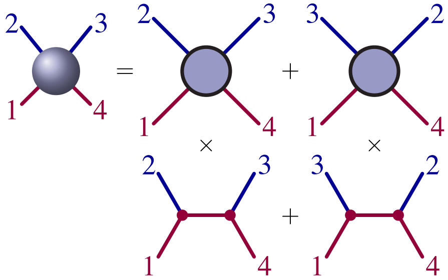

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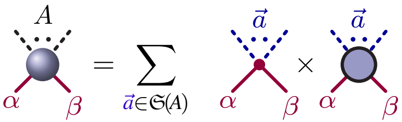

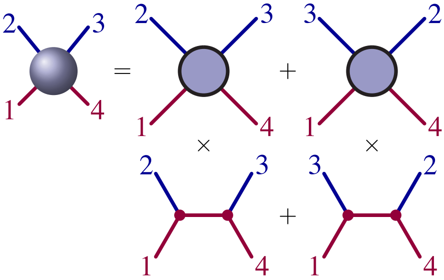

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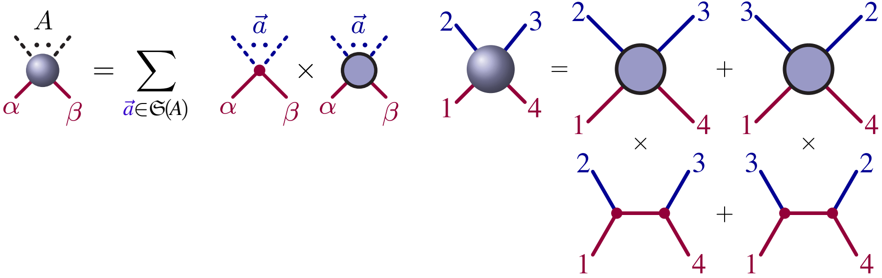

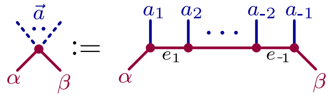

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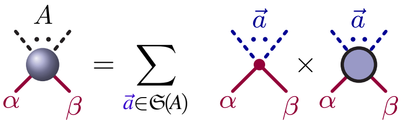

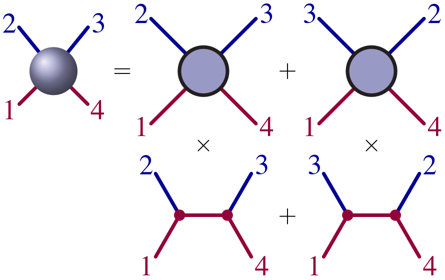

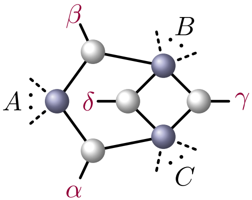

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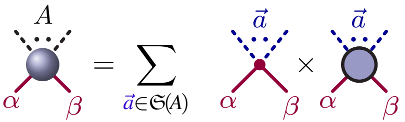

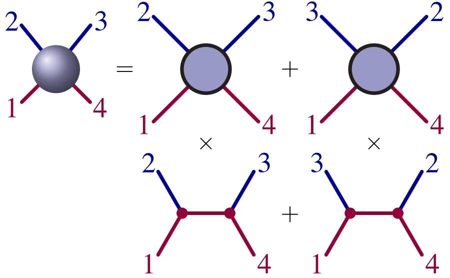

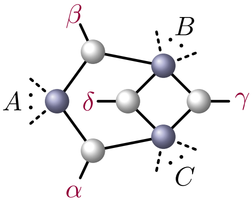

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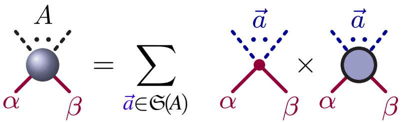

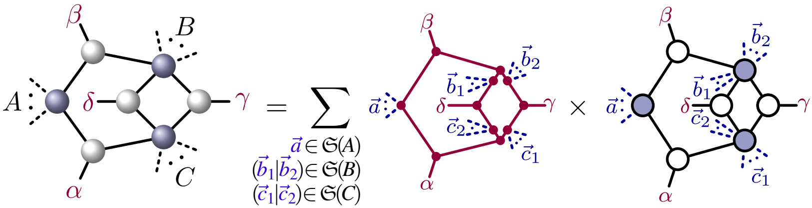

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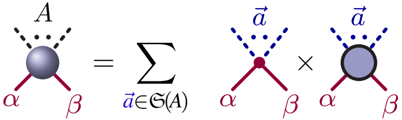

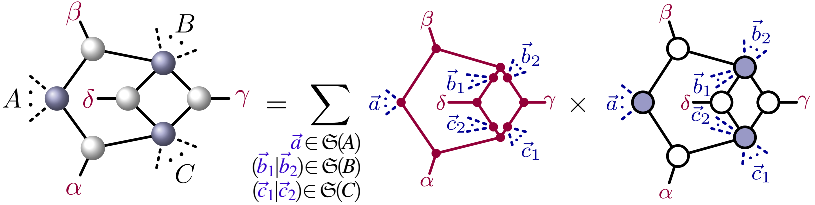

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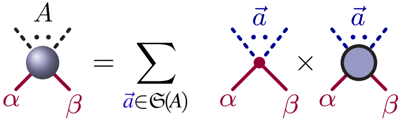

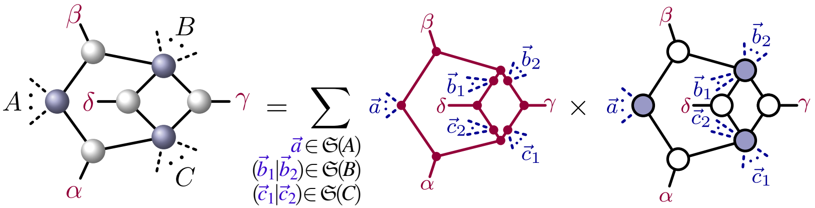

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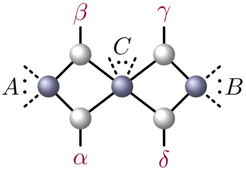

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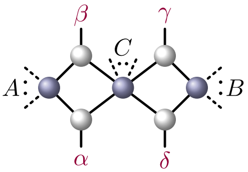

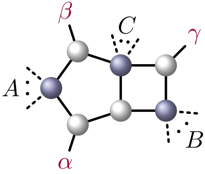

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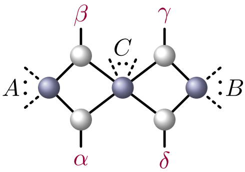

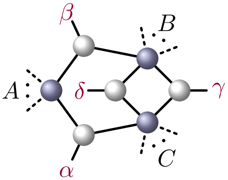

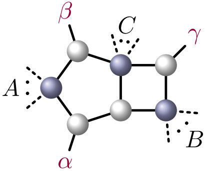

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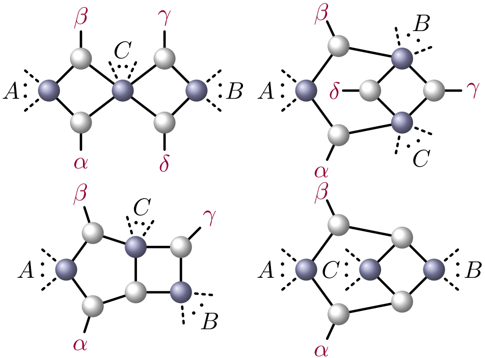

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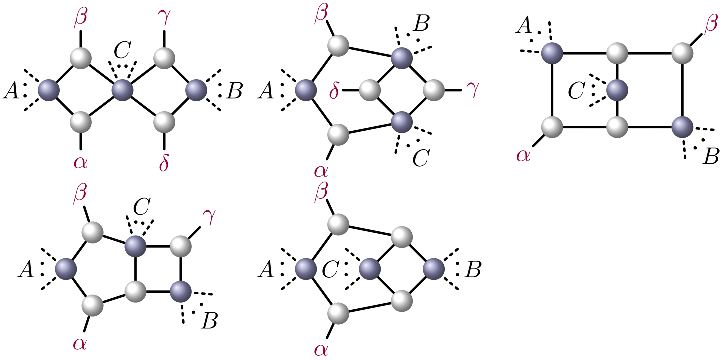

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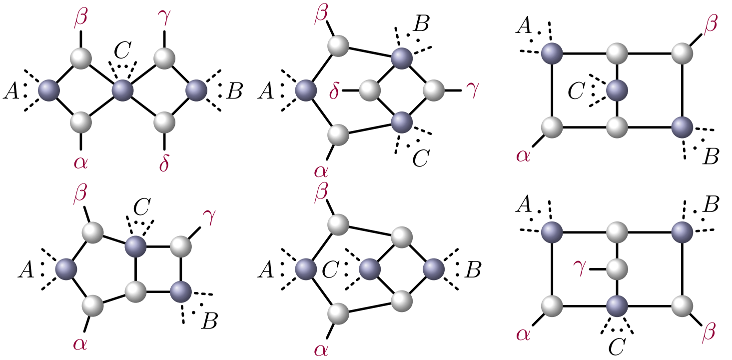

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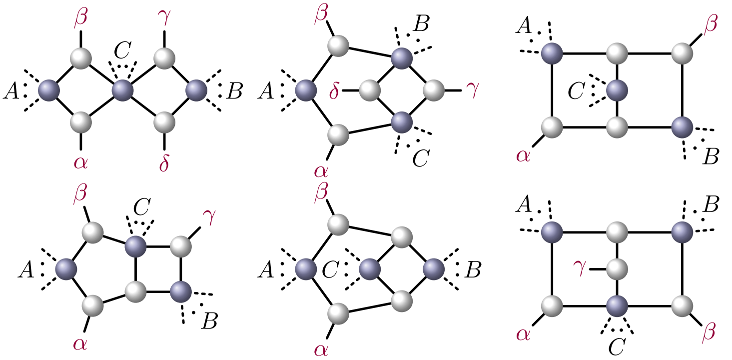

Matching each of these 'cuts' (including degenerations) and the ensuring the absence of any other boundaries suffices to determine all-multiplicity MHV amplitudes at two loops!

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$$f_\Gamma \equiv \prod_i \Big(\sum_{h_i, q_i} \int d^3\mathrm{LIPS}_i \Big) \prod_v \mathcal{A}_v \equiv \int \Omega_C \; \delta(C, p, h)$$

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On-Shell Physics

- on-shell diagrams
- physical symmetries
  - trivial symmetries (identities)

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Grassmannian Geometry

- $\{$strata $C \in G(k,n)$, volume-form $\Omega_C\}$
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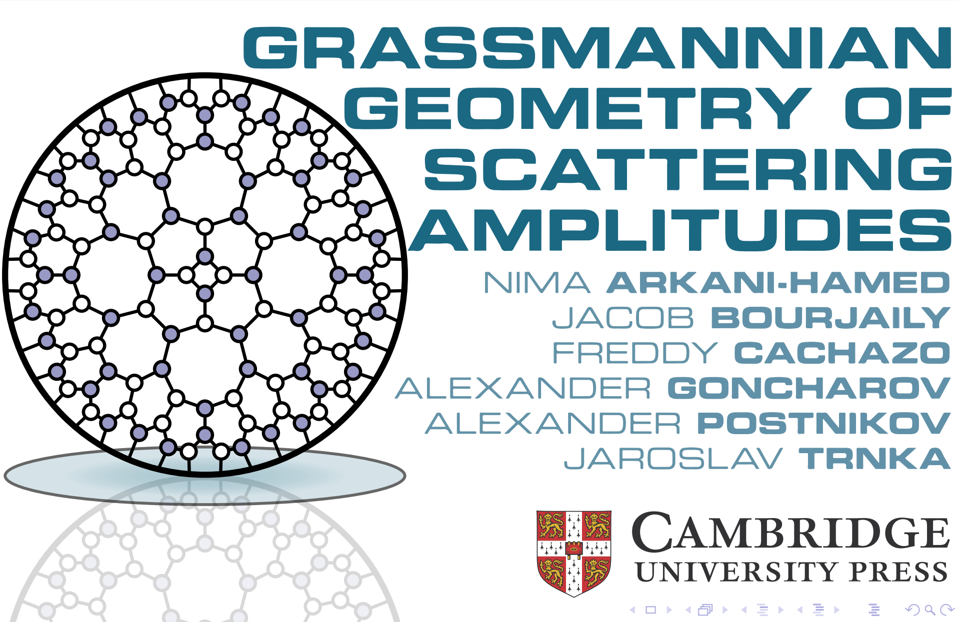
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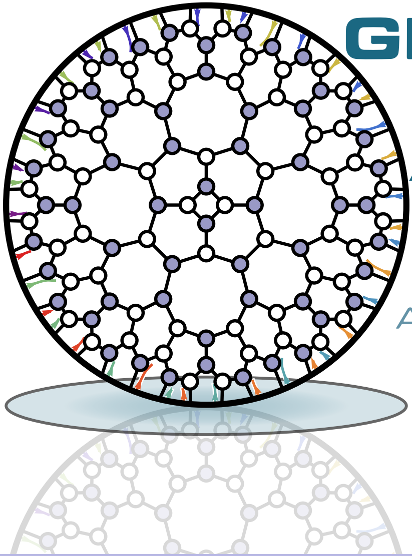

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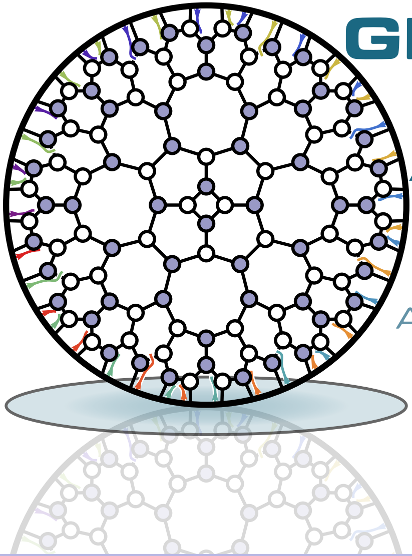

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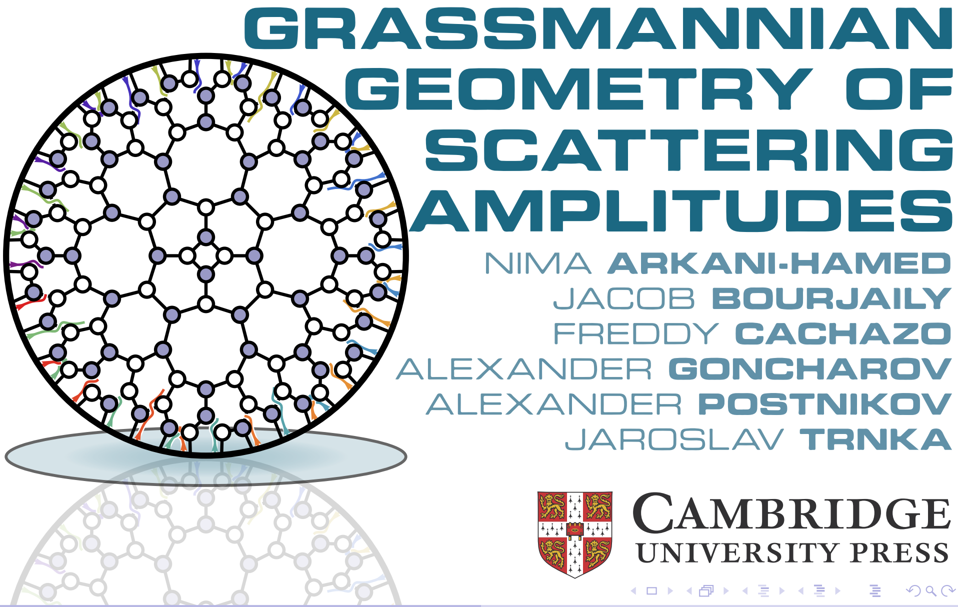
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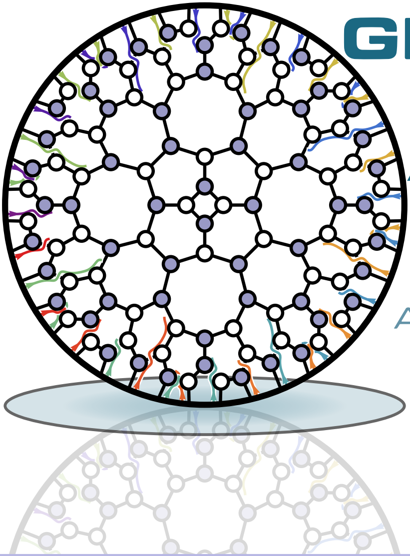

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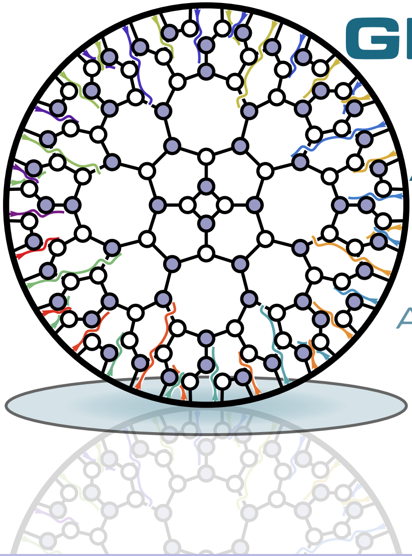

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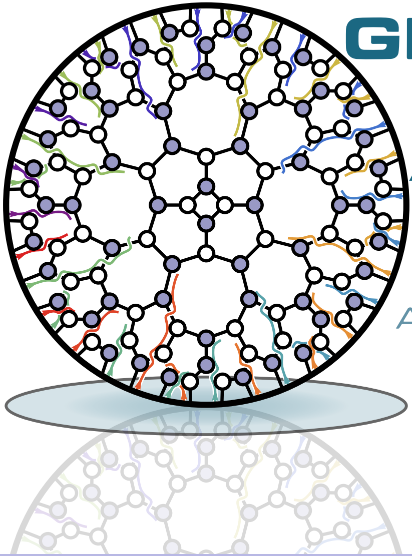

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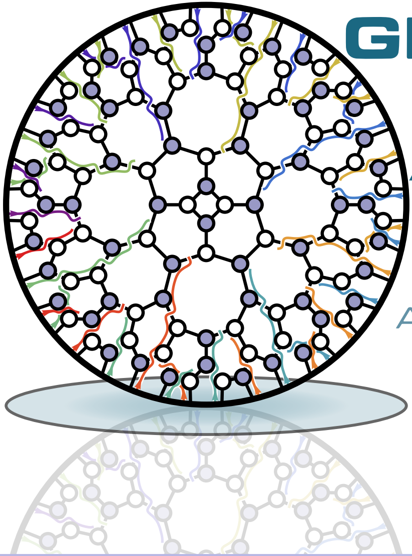

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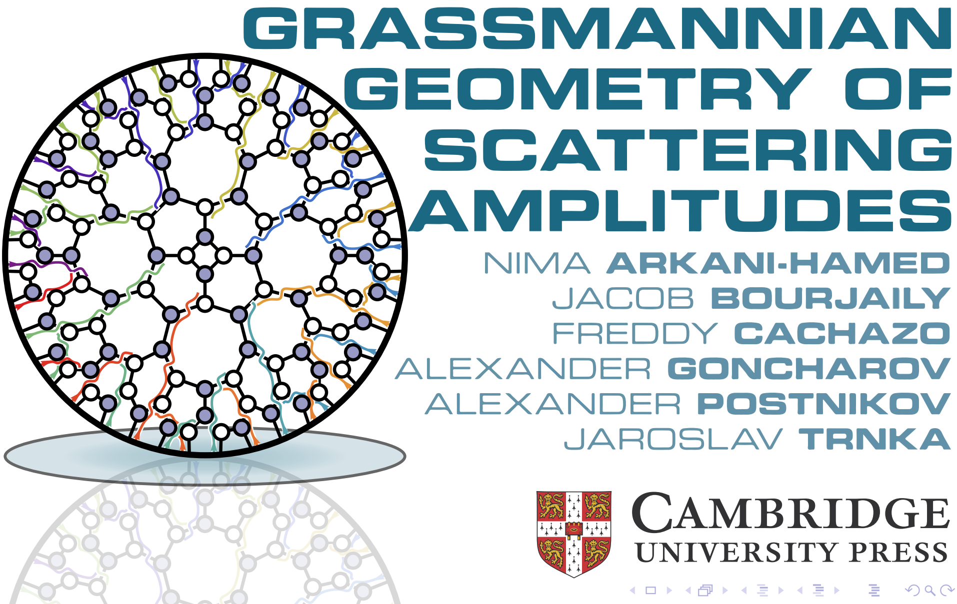
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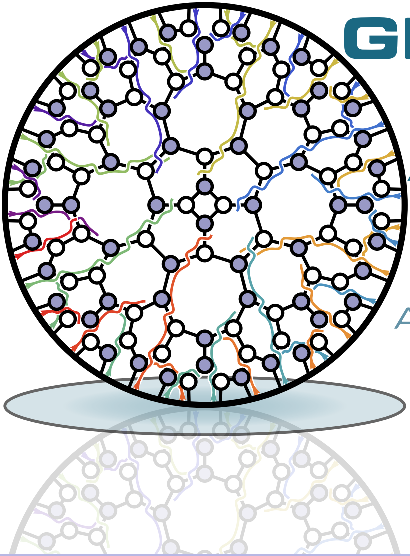

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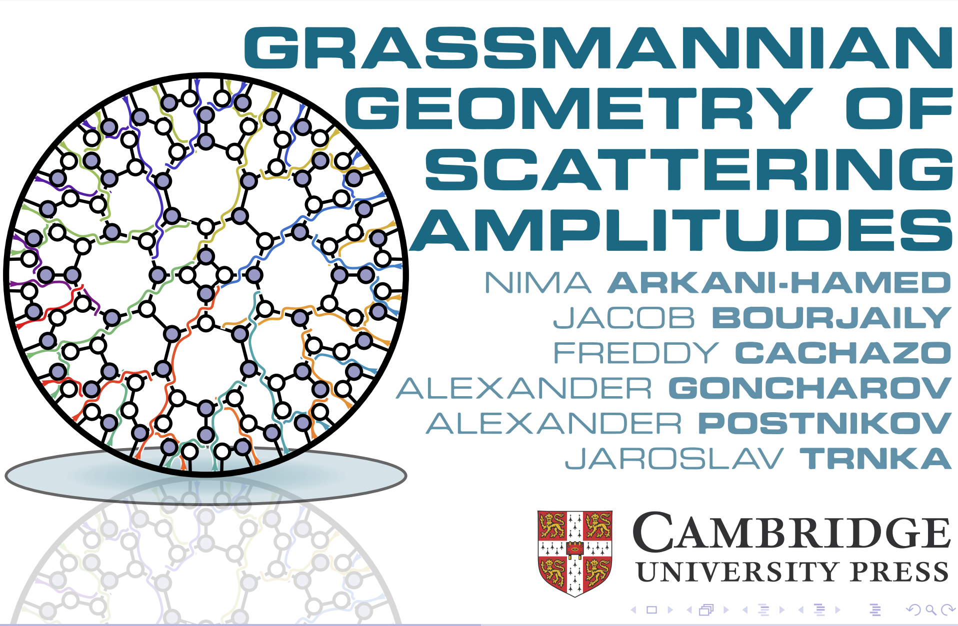
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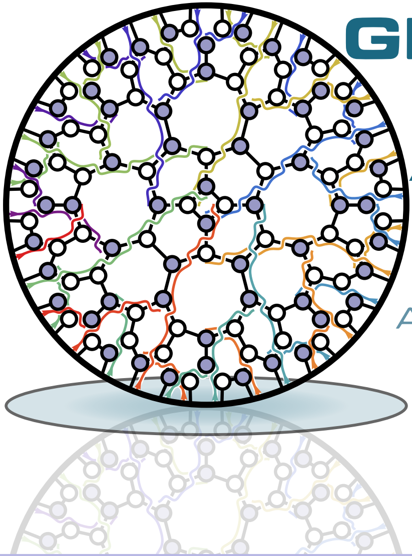

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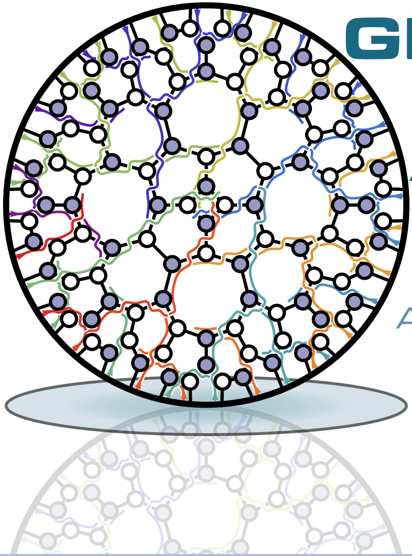

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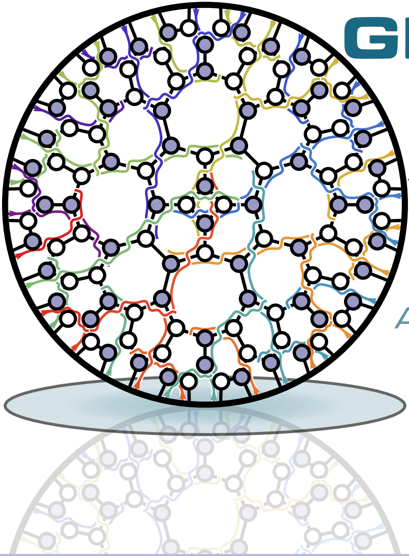

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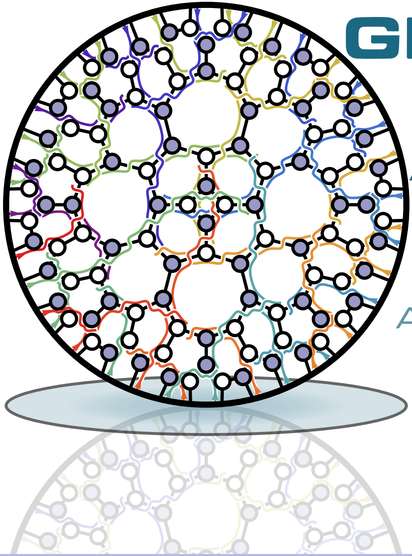

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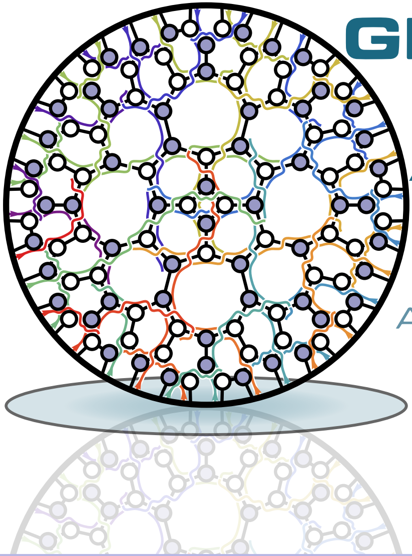

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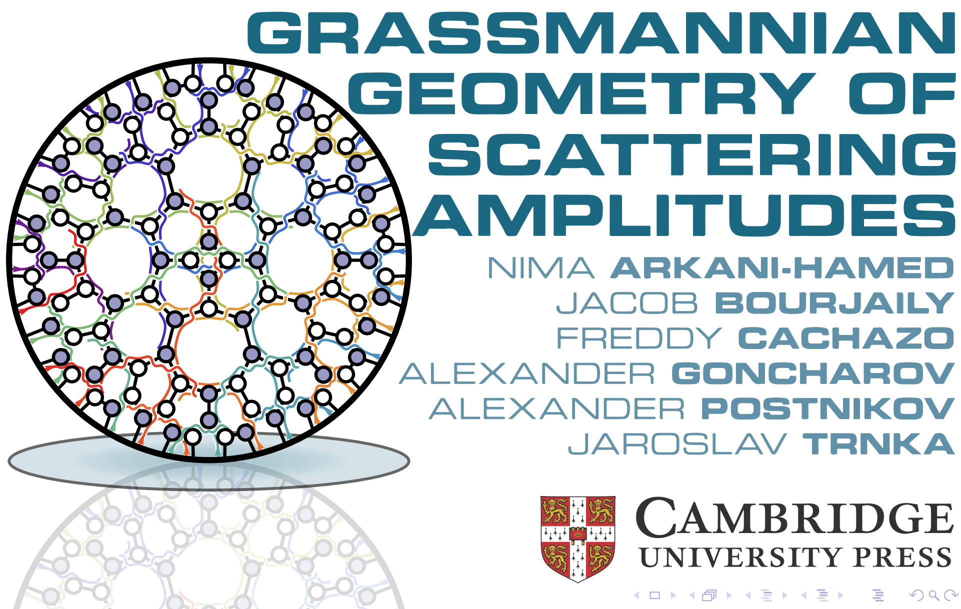
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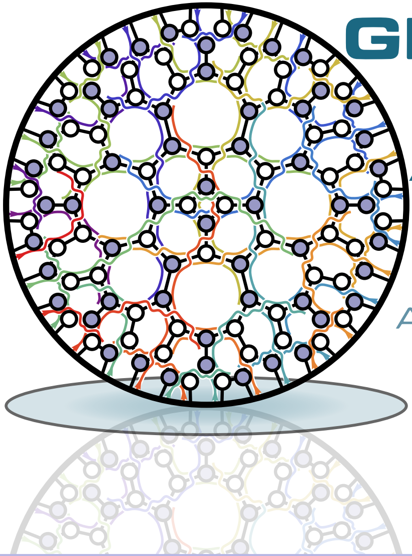

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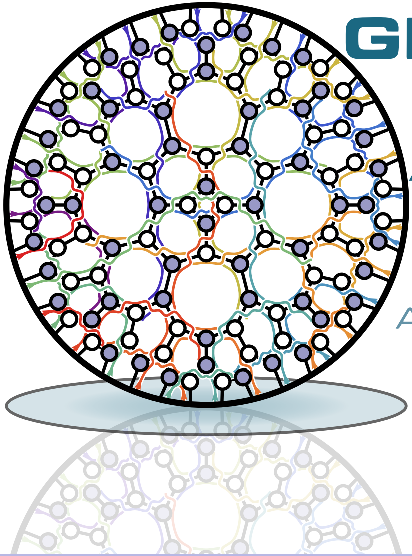

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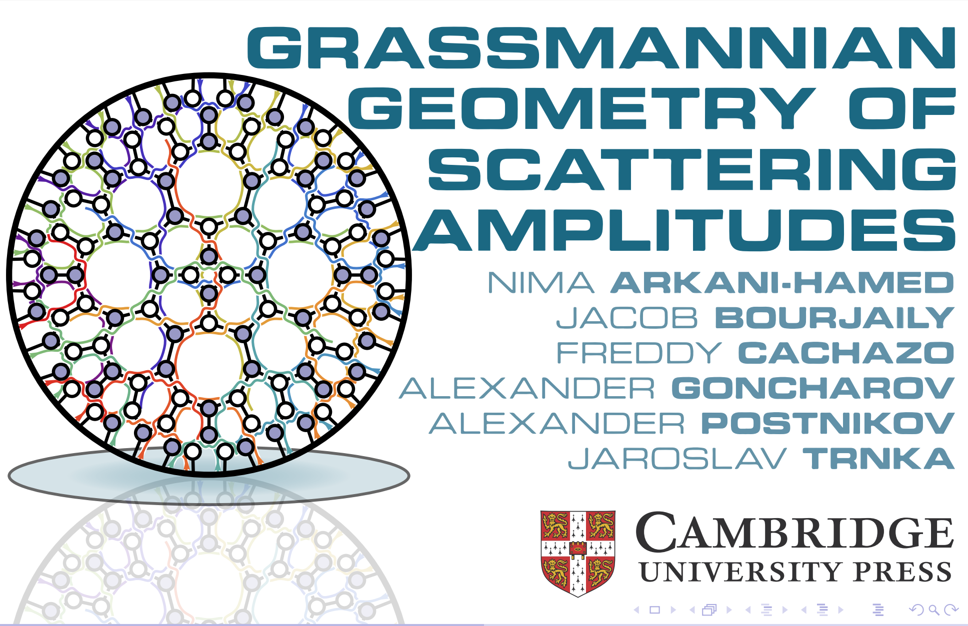
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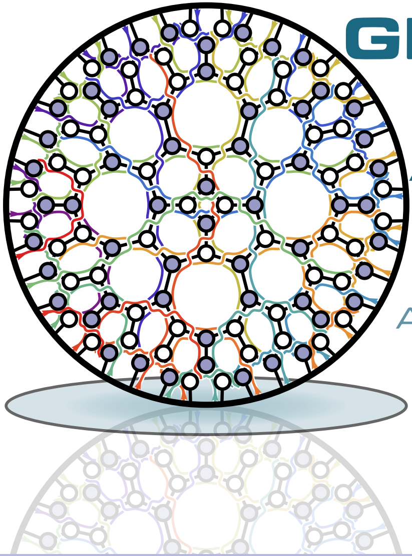

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JACOB **BOURJAILY**
FREDDY **CACHAZO**
ALEXANDER **GONCHAROV**
ALEXANDER **POSTNIKOV**
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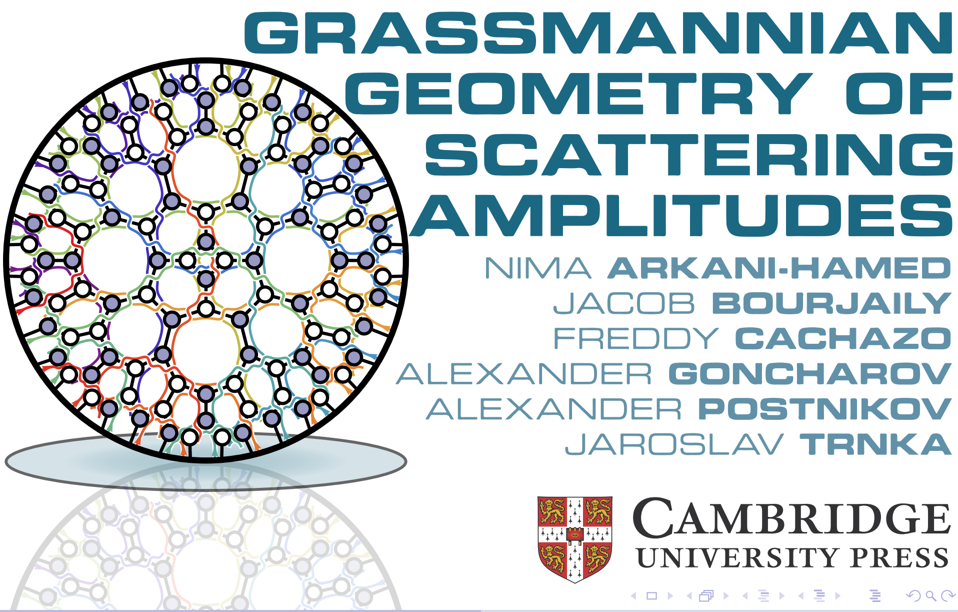
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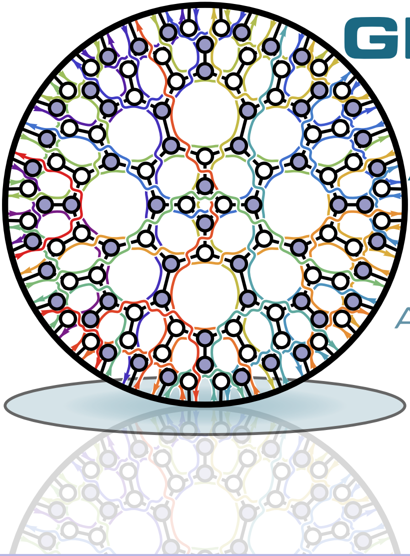

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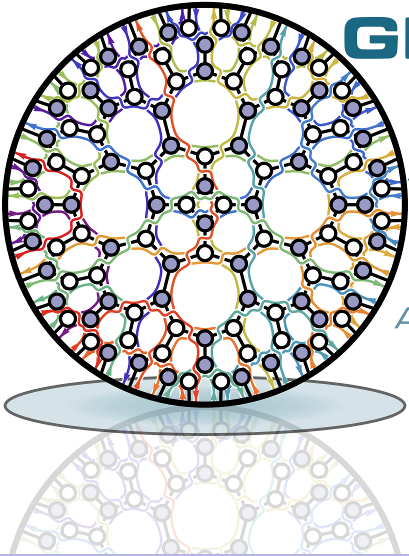

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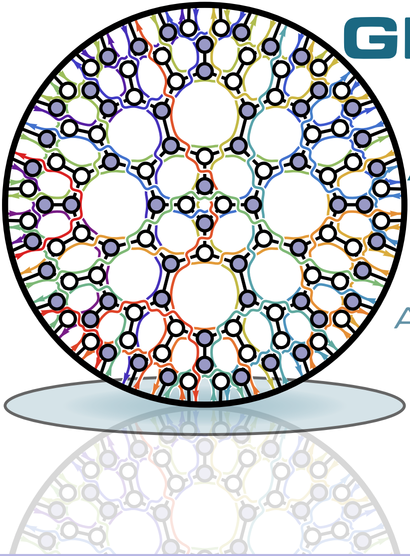

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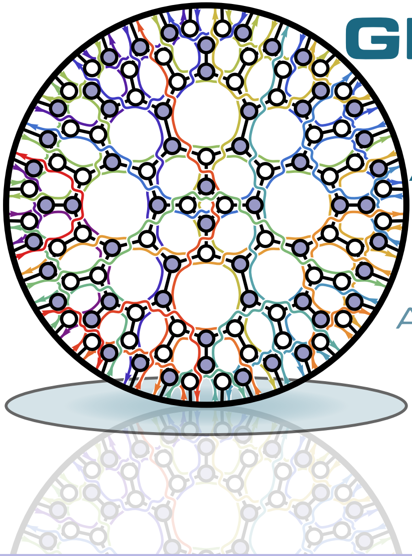

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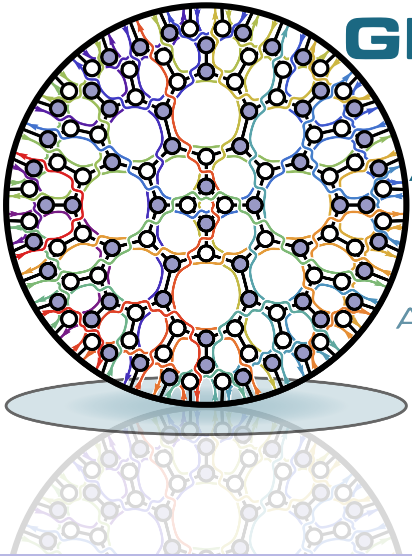

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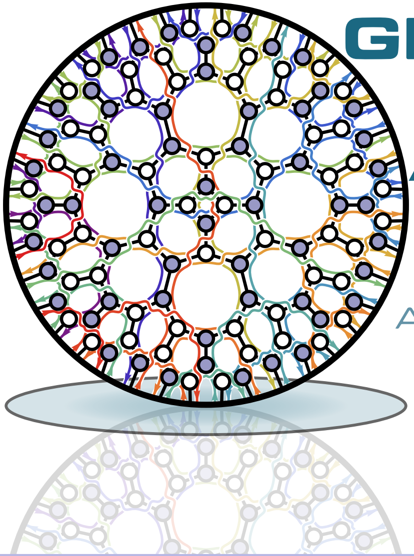

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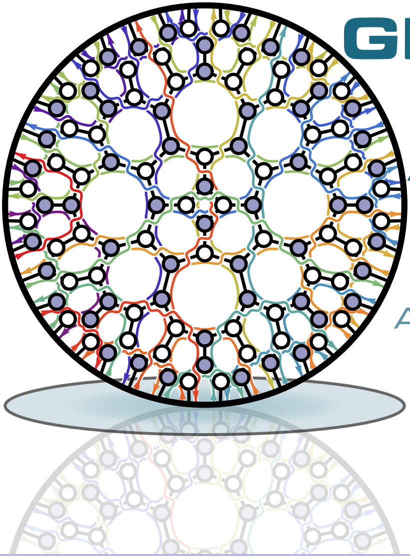

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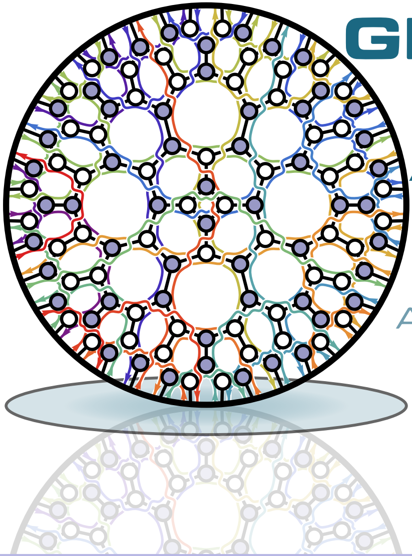

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