PMC_∞ Infinite-Order Scale-Setting method using the Principle of Maximum Conformality and preserving the Intrinsic Conformality (iCF)

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Outline

- The Renormalization Scale Setting in QED/QCD
- State of the art about renormalization in QCD
- The PMC_∞ scale setting procedure
- PMC_∞ and the Event Shape Variables
- Work in progress and future perspectives

Why The Scale Setting in QCD is a Key Issue?

- *To determine $\alpha_s(Q^2)$ to the highest precision;*
- *To make precision tests of the QCD;*
- *To eliminate the renormalization scale ambiguity and the scheme dependence in the observables;*
- *To reach the maximum sensitivity to the NP.*

The Renormalization Scale Problem in QED

- ***QED is perturbative ;***
- ***No ambiguity in the renormalization scale in QED;***
- ***The renormalization scale in QED is physical and set by the exchanged photon virtuality;***
- ***An infinite series of Vacuum Polarization diagrams is resummed;***
- ***The QED coupling is defined from physical observables (Gell Mann-Low scheme);***
- ***No scheme dependence is left;***
- ***Analyticity (space-like/time-like);***
- ***Exact number of active leptons is set;***
- ***Recover of a conformal-like series;***

QED: a Theoretical Constraint for QCD

QCD → ***Abelian Gauge Theory***

In the limit : ***Nc*** → ***0,***

at fixed $\alpha = C_F \alpha_s$, $n_l = n_F / C_F$

The scale setting procedure used in QCD must be consistent with the QED

Huet, S.J.Brodsky

The road to scale setting in QCD is paved with some misbeliefs

According to the Conventional practice :

- ***The renormalization scale is arbitrarily guessed or picked by judging its results «a posteriori»;***
- ***The renormalization scale is unique for each process;***
- ***The renormalization scale is a simple unphysical parameter;***
- ***The renormalization and factorization scales are equal;***

These assumptions are wrong either for QED and than for QCD!

Other methods such as PMS and FAC lead to incorrect and unphysical results violating important RG properties;

The Principle of Maximum Conformality is the principle underlying the BLM

S. J. Brodsky, G. P. Lepage and P. B. Mackenzie, Phys. Rev. D **28**, 228 (1983)

Observable in the initial parametrization

Stanley J. Brodsky, L.D.G.:
Phys. Rev. D 86, 085026 (2011)

$$\begin{aligned}\rho(Q^2) =& r_{0,0} + r_{1,0} a(Q) + [r_{2,0} + \beta_0 r_{2,1}] a(Q)^2 \\ &+ [r_{3,0} + \beta_1 r_{2,1} + 2\beta_0 r_{3,1} + \beta_0^2 r_{3,2}] a(Q)^3 \\ &+ [r_{4,0} + \beta_2 r_{2,1} + 2\beta_1 r_{3,1} + \frac{5}{2}\beta_1\beta_0 r_{3,2} + 3\beta_0 r_{4,1} \\ &+ 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}] a(Q)^4 + \mathcal{O}(a^5) \qquad (6)\end{aligned}$$

Mojaza, Matin and Brodsky, Stanley J. and Wu, Xing-Gang
Phys.Rev.Lett. 110 (2013) 192001

$r_{n,0}$ conformal coefficients

$$\rho(Q^2) = r_{0,0} + r_{1,0} a(Q_1) + r_{2,0} a(Q_2)^2 + r_{3,0} a(Q_3)^3 + r_{4,0} a(Q_4)^4 + \mathcal{O}(a^5) \ ,$$

Conformal-like expansion

$$\ln \frac{Q_k^2}{Q^2} = \frac{R_{k,1} + \Delta_k^{(1)}(a) R_{k,2} + \Delta_k^{(2)}(a) R_{k,3}}{1 + \Delta_k^{(1)}(a) R_{k,1} + \left(\Delta_k^{(1)}(a)\right)^2 (R_{k,2} - R_{k,1}^2) + \Delta_k^{(2)}(a) R_{k,1}^2} \ .$$

PMC scales

Features of the PMC

- All terms associated with the beta-function are included into the running coupling;
- PMC agrees with the QED in the Abelian limit;
- No scale ambiguities;
- Results are scheme independent ;
- The PMC scale sets the correct number of active flavors;
- Transitivity Property is preserved;
- No renormalon n! growth in pQCD associated with the beta function;
- Resulting series is identical to conformal series! (CSR - Crewther Relation ;)

PMC∞ - Results for Event Shape Variables distributions at NNLO

L.D.G. , Stanley J. Brodsky, Sheng-Quan Wang and Xing-Gang Wu

• *Phys.Rev.D* 102 (2020) 1, 014015

Thrust and C-Par distribution at NNLO: process: e+e- →3jets

$$T = \frac{\max_{\vec{n}} \sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|},$$

$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p_i}||\vec{p_j}| \sin^2 \theta_{ij}}{\left(\sum_i |\vec{p_i}|\right)^2},$$

Distributions from EERAD and Event2 codes by:

A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover and G. Heinrich, Phys. Rev. Lett. **99**, 132002 (2007).
A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover and G. Heinrich, JHEP **0712**, 094 (2007).
S. Weinzierl, JHEP **0906**, 041 (2009).
S. Weinzierl, Phys. Rev. Lett. **101**, 162001 (2008).

Strong Coupling from RunDec program :

K. G. Chetyrkin, J. H. Kuhn and M. Steinhauser, Comput. Phys. Commun. **133**, 43 (2000).

PMC$_\infty$ preserves the iCF: *the intrinsic Conformality*

$$\frac{1}{\sigma_{tot}}\frac{Od\sigma(\mu_0)}{dO} = \frac{\alpha_s(\mu_0)}{2\pi}\frac{Od\overline{A}_O(\mu_0)}{dO} + \left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^2 \frac{Od\overline{B}_O(\mu_0)}{dO} + \left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^3 \frac{Od\overline{C}_O(\mu_0)}{dO} + \mathcal{O}(\alpha_s^4)$$

Observable: Single variable distribution at NNLO calculated at the initial scale μ_0

$$A_O(\mu_0) = A_{Conf},$$

$$B_O(\mu_0) = B_{Conf} + \frac{1}{2}\beta_0 \ln\left(\frac{\mu_0^2}{\mu_I^2}\right) A_{Conf},$$

$$C_O(\mu_0) = C_{Conf} + \beta_0 \ln\left(\frac{\mu_0^2}{\mu_{II}^2}\right) B_{Conf} + \frac{1}{4}\left[\beta_1 + \beta_0^2 \ln\left(\frac{\mu_0^2}{\mu_I^2}\right)\right] \ln\left(\frac{\mu_0^2}{\mu_I^2}\right) A_{Conf} \quad (4)$$

iCF is an RG invariant parametrization : conformal coefficients and scales.

Implicit coefficients

Global change of scale

$$A_O(\mu_R) = A_{Conf},$$

$$B_O(\mu_R) = B_{Conf} + \frac{1}{2}\beta_0 \ln\left(\frac{\mu_R^2}{\mu_I^2}\right) A_{Conf},$$

$$C_O(\mu_R) = C_{Conf} + \beta_0 \ln\left(\frac{\mu_R^2}{\mu_{II}^2}\right) B_{Conf} + \\ + \frac{1}{4}\left[\beta_1 + \beta_0^2 \ln\left(\frac{\mu_R^2}{\mu_I^2}\right)\right] \ln\left(\frac{\mu_R^2}{\mu_I^2}\right) A_{Conf}$$

The scale dependence is explicit.

No redefinition of the conformal terms!
No initial scale dependence left.

The iCF is the most general RG invariant parametrization;
Other parametrizations can exist but are equivalent to iCF;

The *ordered scale invariance*

$$\sigma_I = \left\{ \left(\frac{\alpha_s(\mu_0)}{2\pi}\right) + \frac{1}{2}\beta_0 \ln\left(\frac{\mu_0^2}{\mu_I^2}\right)\left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^2 + \frac{1}{4}\left[\beta_1 + \beta_0^2 \ln\left(\frac{\mu_0^2}{\mu_I^2}\right)\right] \ln\left(\frac{\mu_0^2}{\mu_I^2}\right)\left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^3 \right\} A_{Conf}$$

$$\sigma_{II} = \left\{ \left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^2 + \beta_0 \ln\left(\frac{\mu_0^2}{\mu_{II}^2}\right)\left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^3 \right\} B_{Conf}$$

$$\sigma_{III} = \left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^3 C_{Conf} \qquad (6)$$

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s)\frac{\partial}{\partial \alpha_s}\right)\sigma_N = 0.$$

Conformal Subsets

If Aconf=0 the whole subset becomes null.

Each subset is scale invariant.

Any combination of conformal subsets is an invariant.

I can define a scale for each subset preserving the scale invariance.

→ if a theory has the property of the *ordered scale invariance* it preserves exactly the scale invariance of observables independently from the process, the kinematics and the starting order.

iCF underlies Scale Invariance

n limit splitted in $J_{/n}$ and $\bar{n}$ limit

$$
\begin{aligned}
\sigma_I &= \Bigg\{ \left(\frac{\alpha_s(\mu_0)}{2\pi}\right) + \frac{1}{2}\beta_0 \ln\left(\frac{\mu_0^2}{\mu_I^2}\right)\left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^2 \\
&+ \frac{1}{4}\left[\beta_1 + \beta_0^2 \ln\left(\frac{\mu_0^2}{\mu_I^2}\right)\right] \ln\left(\frac{\mu_0^2}{\mu_I^2}\right)\left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^3 + \dots \Bigg\} A_{Conf} \\
\sigma_{II} &= \left\{ \left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^2 + \beta_0 \ln\left(\frac{\mu_0^2}{\mu_{II}^2}\right)\left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^3 + \dots \right\} B_{Conf} \\
\sigma_{III} &= \left\{ \left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^3 + \dots \right\} C_{Conf}, \\
&\vdots \qquad \ddots \\
\sigma_n &= \left\{ \left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^n \right\} \mathcal{L}_{n\,Conf}, \qquad (7)
\end{aligned}
$$

$$
\begin{aligned}
\lim_{J_{/n}\to\infty} \sigma_I &\to \left(\frac{\alpha_s(\mu_I)|_{n-2}}{2\pi}\right) A_{Conf} \\
\lim_{J_{/n}\to\infty} \sigma_{II} &\to \left(\frac{\alpha_s(\mu_{II})|_{n-3}}{2\pi}\right)^2 B_{Conf} \\
\lim_{J_{/n}\to\infty} \sigma_{III} &\to \left(\frac{\alpha_s(\mu_{III})|_{n-4}}{2\pi}\right)^3 C_{Conf} \\
&\vdots \qquad \vdots \\
\lim_{J_{/n}\to\infty} \sigma_n &\equiv \left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^n \mathcal{L}_{n\,Conf} \qquad (9)
\end{aligned}
$$

β terms

$$
\begin{aligned}
\lim_{\bar{n}\to\infty} \sigma_I &\to \left(\frac{\alpha_s(\mu_I)}{2\pi}\right) A_{Conf} \\
\lim_{\bar{n}\to\infty} \sigma_{II} &\to \left(\frac{\alpha_s(\mu_{II})}{2\pi}\right)^2 B_{Conf} \\
\lim_{\bar{n}\to\infty} \sigma_{III} &\to \left(\frac{\alpha_s(\mu_{III})}{2\pi}\right)^3 C_{Conf} \\
&\vdots \qquad \vdots \\
\lim_{\bar{n}\to\infty} \sigma_n &\equiv \lim_{n\to\infty} \left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^n \mathcal{L}_{n\,Conf} \to \text{Conformal Limit} \qquad (10)
\end{aligned}
$$

Same conformal scales μ_N

iCF is sufficient and necessary for scale invariance either for convergent or asymptotic series.

$\lim_{n\to\infty} \alpha_s(\mu_0)^n \sim a^n$ with $a < 1$.

New «*How to*» method

$$B_O(N_f) = C_F\left[C_A B_O^{N_c} + C_F B_O^{C_F} + T_F N_f B_O^{N_f}\right] \quad (12)$$

Find the roots of the β terms and vary the number of flavors.

$$B_{Conf} = B_O\left(N_f \equiv \frac{33}{2}\right),$$

$$B_{\beta_0} \equiv \log\frac{\mu_0^2}{\mu_I^2} = 2\frac{B_O - B_{Conf}}{\beta_0 A_{Conf}}$$

$$C_O(N_f) = \frac{C_F}{4}\left\{N_c^2 C_O^{N_c^2} + C_O^{N_c^0} + \frac{1}{N_c^2} C_O^{\frac{1}{N_c^2}} + N_f N_c \cdot C_O^{N_f N_c} + \frac{N_f}{N_c} C_O^{N_f/N_c} + N_f^2 C_O^{N_f^2}\right\}.$$

$$C_{Conf} = C_O\left(N_f \equiv \frac{33}{2}\right) - \frac{1}{4}\overline{\beta}_1 B_{\beta_0} A_{Conf},$$

β0 killing value

$$\overline{\beta}_1 \equiv \beta_1(N_f = 33/2) = -107.$$

$$C_{\beta_0} \equiv \log\left(\frac{\mu_0^2}{\mu_{II}^2}\right) = \frac{1}{\beta_0 B_{Conf}}\left(C_O - C_{Conf} - \frac{1}{4}\beta_0^2 B_{\beta_0}^2 A_{Conf} - \frac{1}{4}\beta_1 B_{\beta_0} A_{Conf}\right),$$

PMC$_\infty$:NNLO event shape variables.

$$\frac{1}{\sigma_{tot}}\frac{Od\sigma(\mu_I,\mu_{II},\mu_0)}{dO} = \{\overline{\sigma}_I + \overline{\sigma}_{II} + \overline{\sigma}_{III} + \mathcal{O}(\alpha_s^4)\}, \quad (20)$$

$$\sigma_{tot} = \sigma_0\left(1 + \frac{\alpha_s(\mu_0)}{2\pi}A_{tot} + \left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^2 B_{tot} + O\left(\alpha_s^3\right)\right)$$

$\alpha_I \equiv \alpha_s(\mu_I)$, $\alpha_{II} \equiv \alpha_s(\tilde{\mu}_{II})$

$$\begin{aligned}
\overline{\sigma}_I &= A_{Conf}\frac{\alpha_I}{2\pi} \\
\overline{\sigma}_{II} &= (B_{Conf} + \eta A_{tot}A_{Conf})\left(\frac{\alpha_{II}}{2\pi}\right)^2 - \eta A_{tot}A_{Conf}\left(\frac{\alpha_0}{2\pi}\right)^2 \\
&\quad - A_{tot}A_{Conf}\frac{\alpha_0}{2\pi}\frac{\alpha_I}{2\pi} \\
\overline{\sigma}_{III} &= (C_{Conf} - A_{tot}B_{Conf} - (B_{tot} - A_{tot}^2)A_{Conf})\left(\frac{\alpha_0}{2\pi}\right)^3
\end{aligned} \quad (23)$$

Regularization terms do not introduce bias at NNLO. Responsible for 1.5% only of the errors .

$$\eta = 3.51$$

PMC$_\infty$ scales for Thrust and C-par

Red

Dashed Black

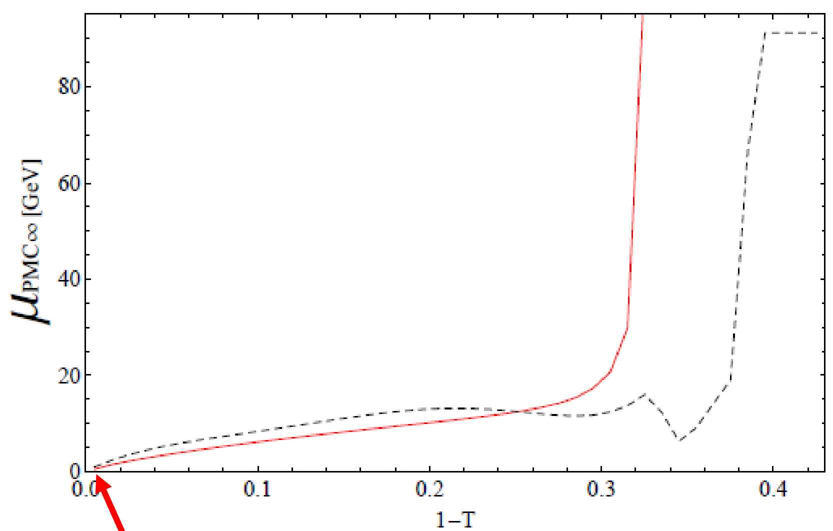
$$\eta = 3.51$$

$$\mu_I = \sqrt{s}\cdot e^{-\frac{1}{2}B_{\beta_0}}, \qquad (1-T)<0.33$$

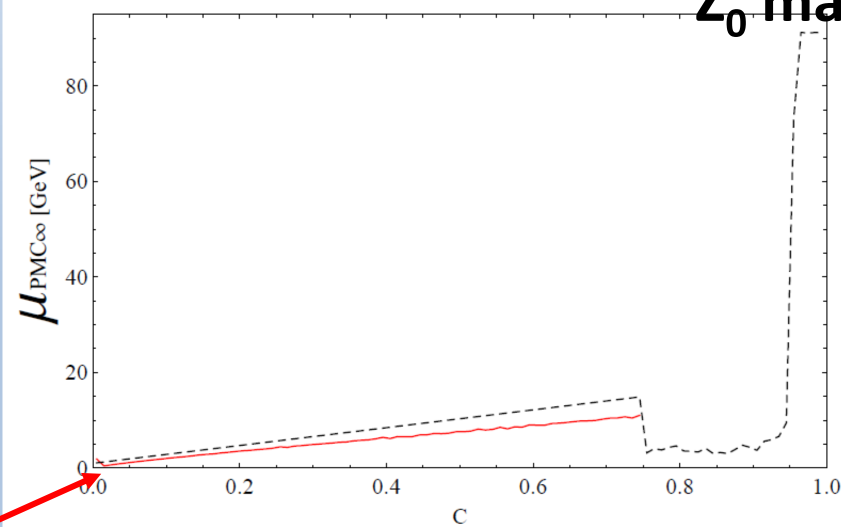
$$\tilde{\mu}_{II} = \begin{cases} \sqrt{s}\cdot e^{-\frac{1}{2}C_{\beta_0}\cdot\frac{B_{Conf}}{B_{Conf}+\eta\cdot A_{tot}A_{Conf}}}, & (1-T)<0.33 \\ \sqrt{s}\cdot e^{-\frac{1}{2}\left(\frac{C_1}{11B_1-\frac{2}{3}B_0}\right)}, & (1-T)>0.33 \end{cases}$$

The PMC$_\infty$ scales reflect the virtuality of quarks and gluons subprocesses

The PMC$_\infty$ scales are functions of the Event Shape Variable and of $\sqrt{s}$



Z_0 mass



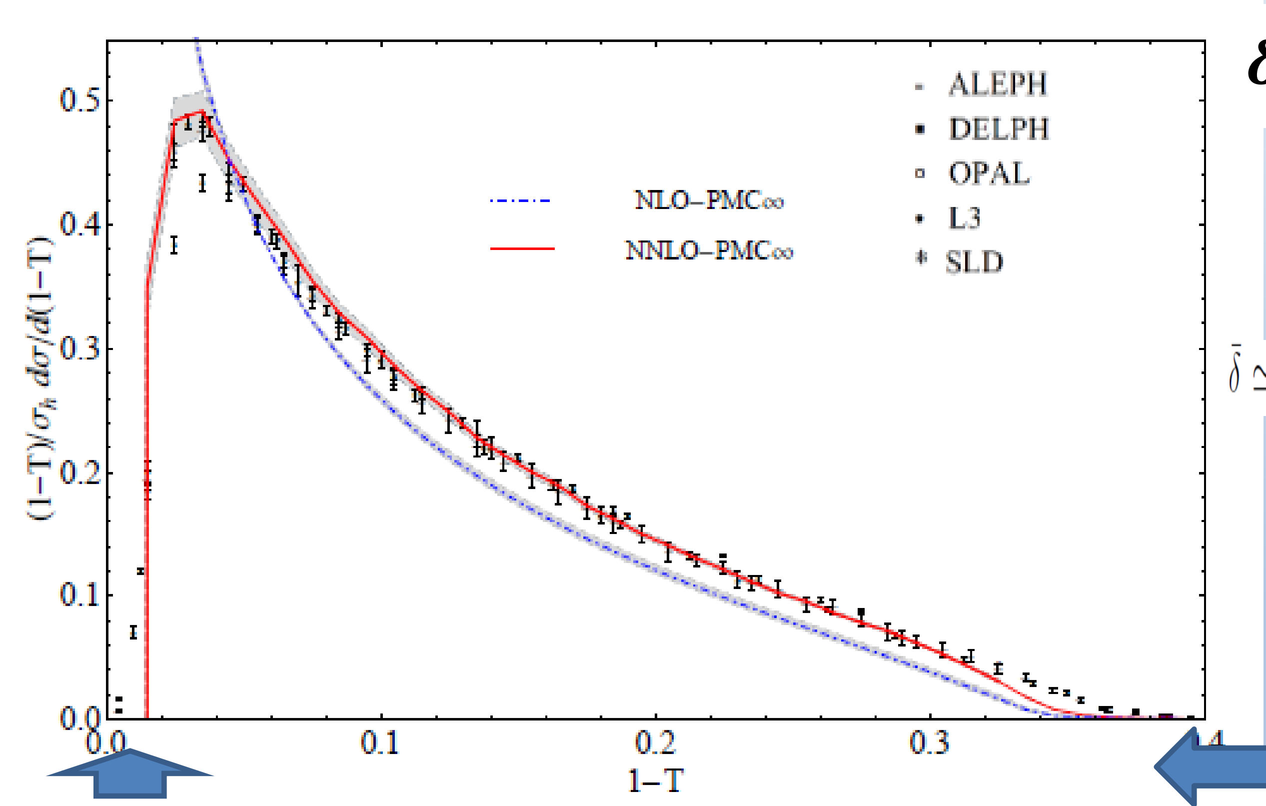
Correct physical behavior in the nonperturbative region (unlike other methods e.g. PMS and FAC)

PMC$_\infty$ Results for Thrust at NNLO

Errors:standard criteria

$$\delta = \left| \frac{\sigma(2M)) - \sigma(M/2)}{2\sigma(M)} \right|$$

M is the Z0 mass



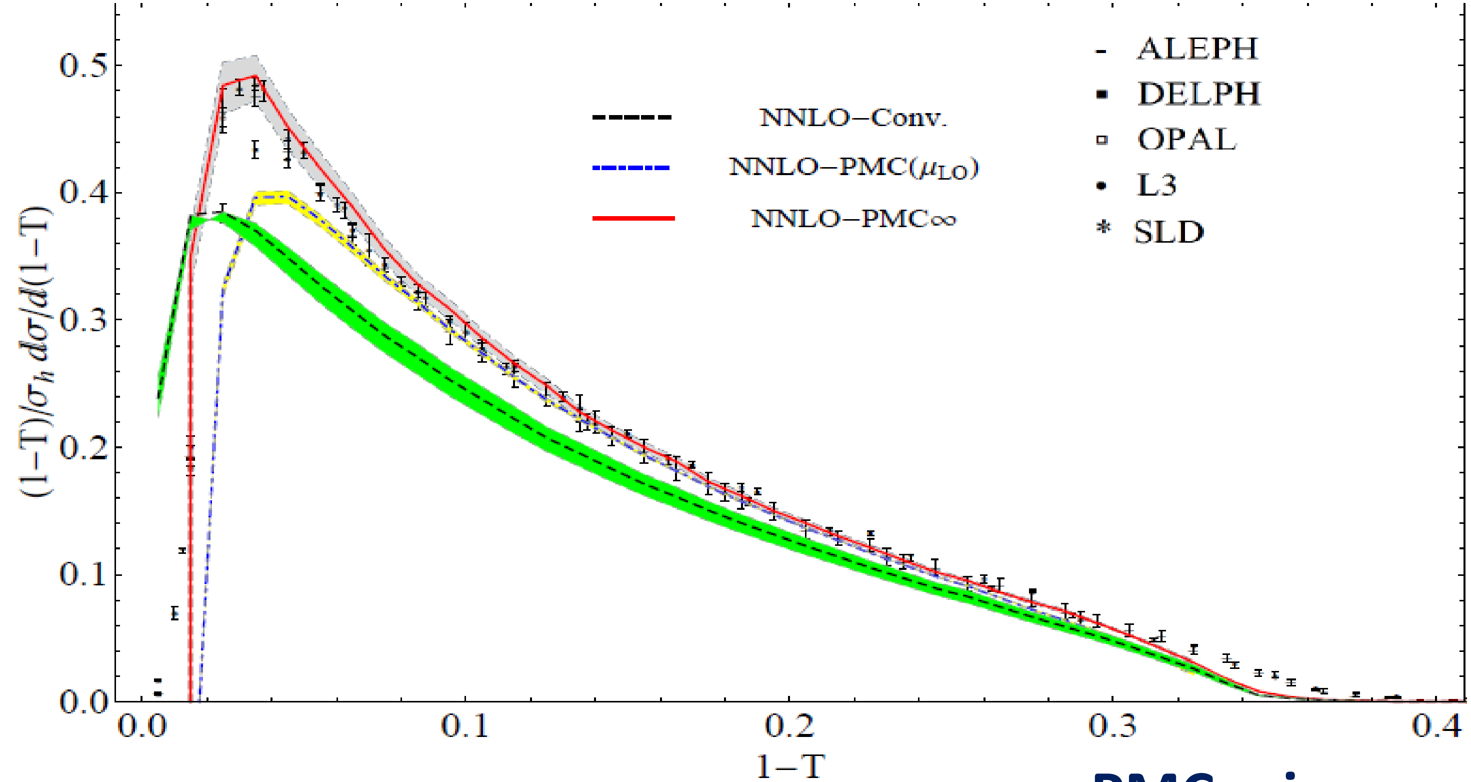
$\bar{\delta} \simeq 7.36\%$ to 1.95% from NLO to the NNLO accuracy

The PMC$_\infty$ improves the precision going to higher orders

NON perturbative region

iCF effect Multijet region

Comparison with Conv. Scale Sett.

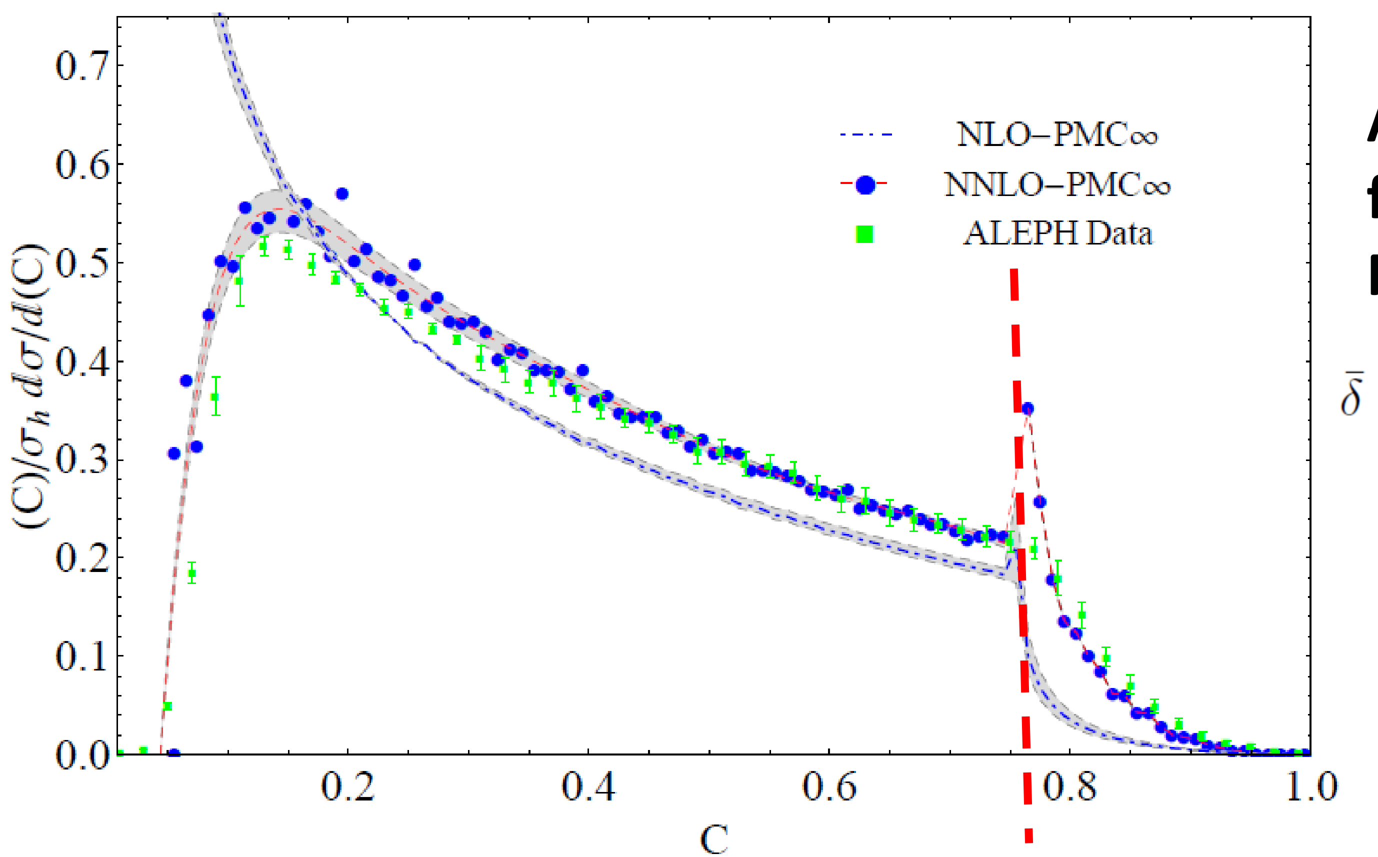


$\bar{\delta}[\%]$	Conv.	PMC(μ_{LO})	PMC$_\infty$
$0.10 < (1-T) < 0.33$	6.03	1.41	1.31
$0.21 < (1-T) < 0.33$	6.97	2.19	0.98
$0.33 < (1-T) < 0.42$	8.46	-	2.61
$0.00 < (1-T) < 0.33$	5.34	1.33	1.77
$0.00 < (1-T) < 0.42$	6.00	-	1.95

PMC∞ improves the precision of the pQCD predictions.

Errors:85% depends on the last unset scale

PMC∞ Results for C-parameter



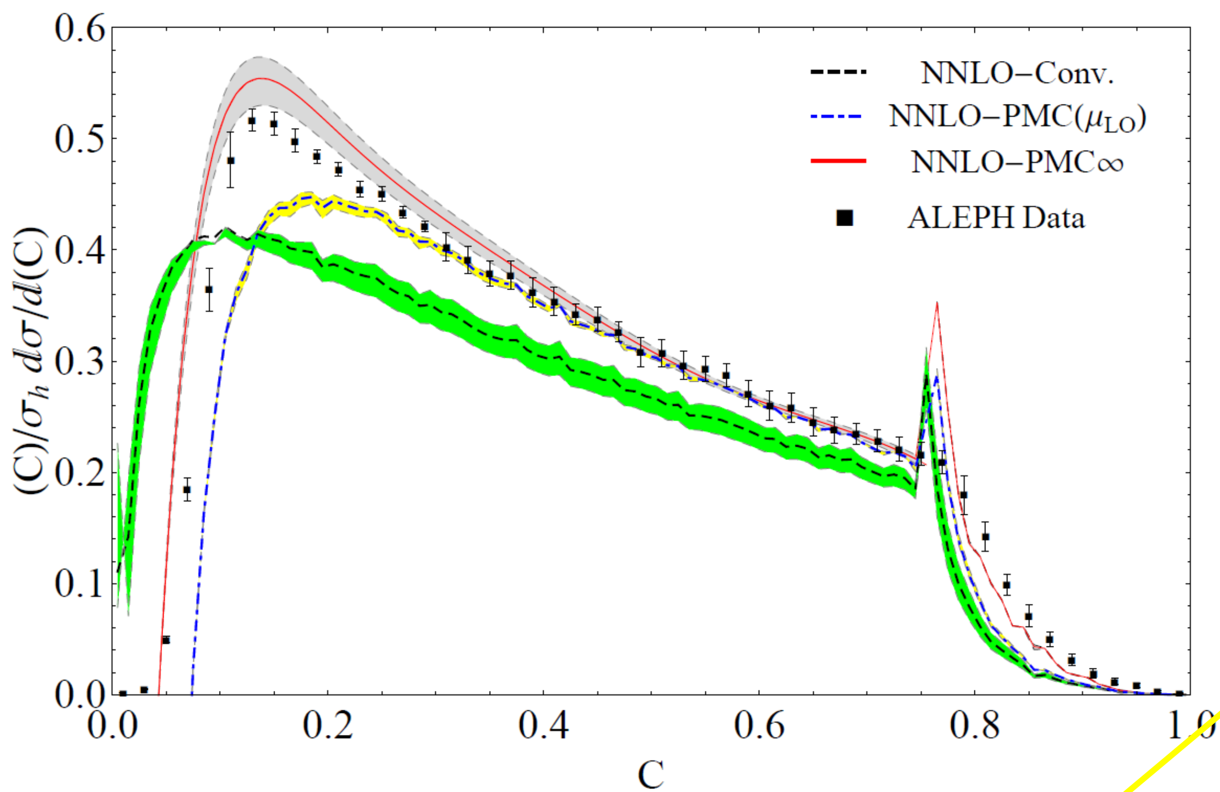
iCF effect

Average Errors from NLO to NNLO are

$\bar{\delta} \simeq 7.26\%$ → 2.43%

$$\mu_{II} = M_{Z0} e^{-\frac{1}{2}\left(\frac{C_1}{11B_1 - \frac{2}{3}B_0}\right)}$$

Comparison with Conv.Scale Setting



$\bar{\delta}$[%]	Conv.	PMC(μ_{LO})	PMC$_\infty$
$0.00 < (C) < 0.75$	4.77	0.85	2.43
$0.75 < (C) < 1.00$	11.51	3.68	2.42
$0.00 < (C) < 1.00$	6.47	1.55	2.43

The PMC(μ_{LO}) procedure leads to stable results. The last unknown scale can be fixed to the last known.

PMC∞ improves the results In the multijet region and the comparison with exp. data all over the spectra.

Conclusions

- The PMC_∞ is based on the PMC and preserves the iCF;
- The iCF underlies the scale invariance;
- We have shown «how to» easily apply PMC_∞ ;
- Event shape variables distributions results for T and C-par are in very good agreement with data in a wide range of values;
- The PMC_∞ improves the precision of the QCD predictions ;

Perspective in the short term

- Applcation of the PMC_∞ to other fundamental processes;
- Implementation with Large Log Resummation Techniques;

Thanks for your attention!