The true unintegrated PDF for inclusive DIS?

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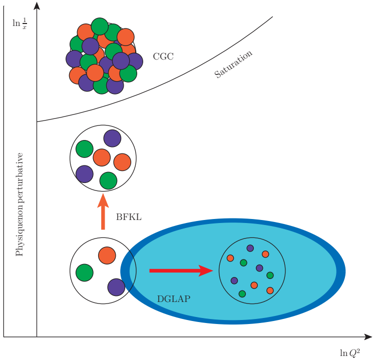
REF 2020

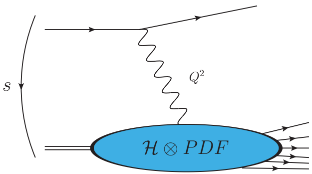
In collaboration with Yacine Mehtar-Tani

QCD at moderate x

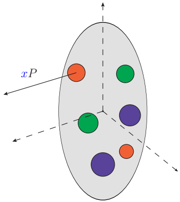
$$Q^2 \sim s$$

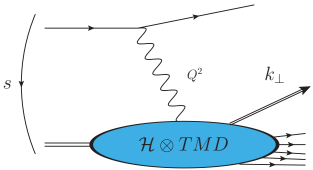


Parton Distributions



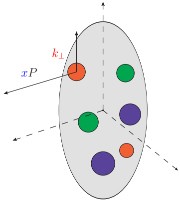
Parton Distribution Fonction (PDF)





Transverse Momentum Dependent

distributions (TMD)



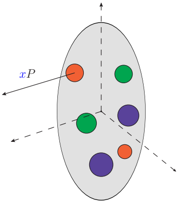
Operator definition for parton distributions

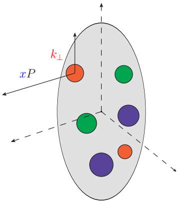
Parton distribution function

$$\mathcal{F}(x) \propto \int dz^+ e^{ixP^- z^+} \left\langle P \left| F^{-i}(z^+) \left[z^+, 0^+\right] F^{-i}(0) \left[0^+, z^+\right] \right| P \right\rangle$$

Transverse Momentum Dependent distribution

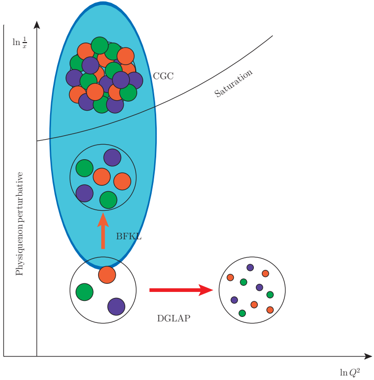
$$\mathcal{F}(x, k_\perp) \propto \int d^4z \delta(z^-) e^{ixP^- z^+ + i(k_\perp \cdot z_\perp)} \left\langle P \left| F^{-i}(z) \mathcal{U}_{z,0} F^{-i}(0) \mathcal{U}_{0,z} \right| P \right\rangle$$



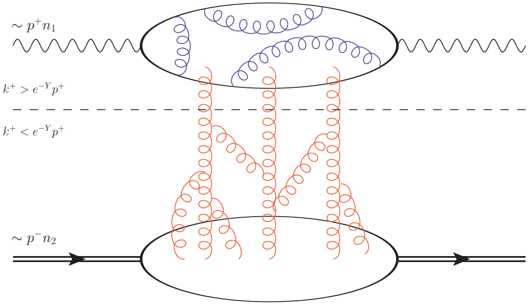


QCD at small x

$$Q^2 \ll s$$



Rapidity separation

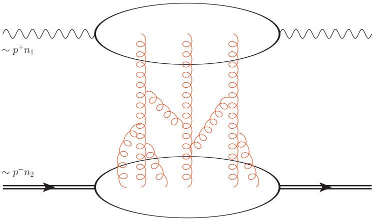


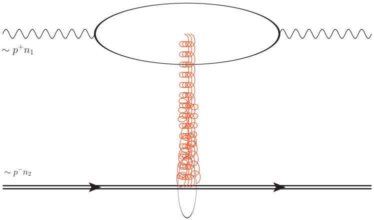
Let us split the gluonic field between "fast" and "slow" gluons

$$\mathcal{A}^{\mu a}(k^+, k^-, \boldsymbol{k}) = A^{\mu a}_{Y_c}(|k^+| > e^{-Y_c} p^+, k^-, \boldsymbol{k}) + a^{\mu a}_{Y_c}(|k^+| < e^{-Y_c} p^+, k^-, \boldsymbol{k})$$

$e^{-Y_c} \ll 1$

Large longitudinal boost to the projectile frame





$$a^+(x^+, x^-, \boldsymbol{x}) \longrightarrow \frac{1}{\Lambda} a^+\left(\Lambda x^+, \frac{x^-}{\Lambda}, \boldsymbol{x}\right)$$

$$a^-(x^+, x^-, \boldsymbol{x}) \longrightarrow \Lambda a^-\left(\Lambda x^+, \frac{x^-}{\Lambda}, \boldsymbol{x}\right)$$

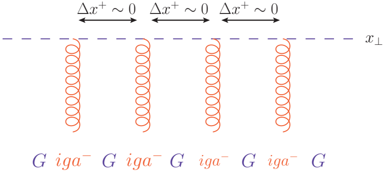
$$a^k(x^+, x^-, \boldsymbol{x}) \qquad \Lambda \sim \sqrt{\frac{s}{m_t^2}} \qquad a^k\left(\Lambda x^+, \frac{x^-}{\Lambda}, \boldsymbol{x}\right)$$

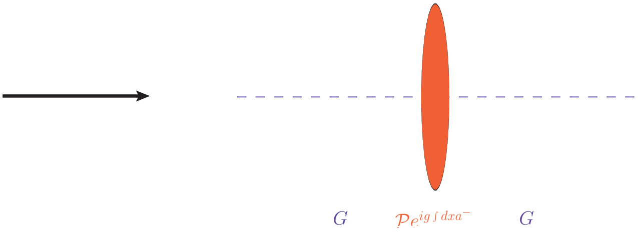
$$a^\mu(x) \to a^-(x)\, n_2^\mu = \delta(x^+)\, \mathbf{a}(\boldsymbol{x})\, n_2^\mu + O\left(\sqrt{\frac{m_t^2}{s}}\right)$$

Shock wave approximation

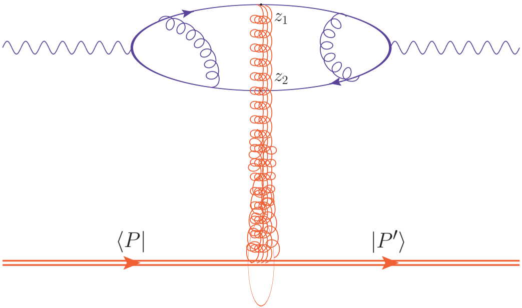
Effective Feynman rules in the slow background field

The interactions with the background field can be exponentiated





Factorized picture



Factorized amplitude

$$\mathcal{A}^{Y_c} = \int d^2 z_1 d^2 z_2 \, \Phi^{Y_c}(z_1, z_2) \, \langle P' | [\mathrm{Tr}(U_{z_1}^{Y_c} U_{z_2}^{Y_c \dagger}) - N_c] | P \rangle$$

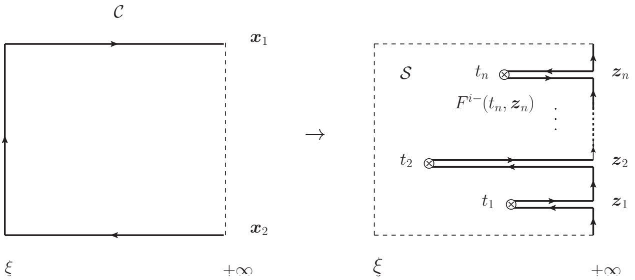
Dipole operator $\mathcal{U}_{ij}^{Y_c} = \frac{1}{N_c} \mathrm{Tr}(U_{z_i}^{Y_c} U_{z_j}^{Y_c \dagger}) - 1$

The operator satisfies the B-JIMWLK equation for Y_c

The Wilson line $\leftrightarrow$ parton distribution equivalence

Most general equivalence: use the Non-Abelian Stokes theorem

[RB, Mehtar-Tani]



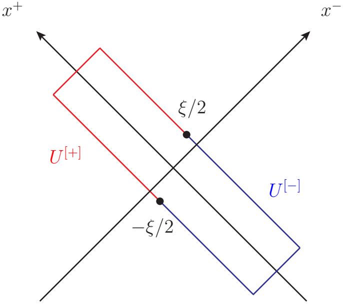
$$\mathcal{P}\exp\left[\oint_{\mathcal{C}} dx_\mu A^\mu(x)\right] = \mathcal{P}\exp\left[\int_{\mathcal{S}} d\sigma_{\mu\nu}\, U F^{\mu\nu} U^\dagger\right]$$

$$U_{x_{1\perp}} U^\dagger_{x_{2\perp}} = [\hat{x}_{1\perp}, \hat{x}_{2\perp}]$$

An example: the dipole operator as a TMD distribution

[Hatta, Xiao, Yuan]

$$\int \frac{\mathrm{d}^2 \boldsymbol{b}}{(2\pi)^2} \frac{\left\langle P \left| \mathrm{tr}\left(U_{\boldsymbol{b}+\frac{\boldsymbol{r}}{2}} U^{\dagger}_{\boldsymbol{b}-\frac{\boldsymbol{r}}{2}} \right) - N_c \right| P \right\rangle}{\langle P | P \rangle} = \alpha_s \int \mathrm{d}^2 \boldsymbol{k} \frac{\mathrm{e}^{i(\boldsymbol{k}\cdot\boldsymbol{r})}}{\boldsymbol{k}^2} f^D(x=0, \boldsymbol{k})$$

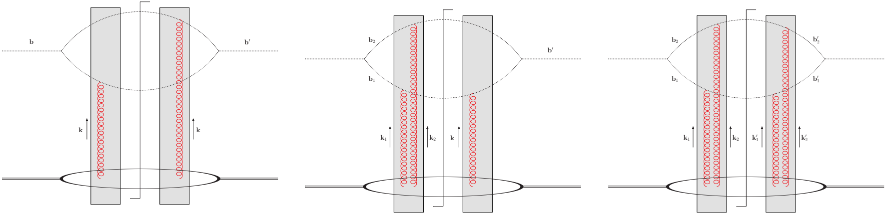


$$f^D(x, \boldsymbol{k}) \equiv \frac{1}{P^-} \int \frac{\mathrm{d}\xi^+}{2\pi} \int \frac{\mathrm{d}^2 \boldsymbol{\xi}}{(2\pi)^2} \mathrm{e}^{ixP^-\xi^+ - i(\boldsymbol{k}\cdot\boldsymbol{\xi})} \times \left\langle P \left| \mathrm{tr} F^{i-}(\xi) \mathcal{U}^{[-]}_{\xi,0} F^{i-}(0) \mathcal{U}^{[+]}_{0,\xi} \right| P \right\rangle_{\xi^-=0}$$

Inclusive low x cross section

Inclusive low x cross section = TMD cross section

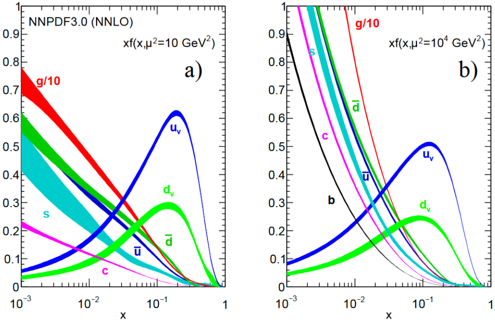
[Altinoluk, RB, Kotko], [Altinoluk, RB]



$$\sigma = \mathcal{H}_2^{ij}(\boldsymbol{k}) \otimes f_2^{ij}(x=0, \boldsymbol{k})$$
$$+ \mathcal{H}_3^{ijk}(\boldsymbol{k}, \boldsymbol{k}_1) \otimes f_3^{ijk}(x=0, x_1=0, \boldsymbol{k}, \boldsymbol{k}_1)$$
$$+ \mathcal{H}_4^{ijkl}(\boldsymbol{k}, \boldsymbol{k}_1, \boldsymbol{k}_1') \otimes f_4^{ijkl}(x=0, x_1=0, x_1'=0, \boldsymbol{k}, \boldsymbol{k}_1, \boldsymbol{k}_1')$$

All distributions are evaluated in the strict $x = 0$ limit

All distributions are evaluated in the strict $x = 0$ limit



[NNLO NNPDF3.0 global analysis, taken from PDG2018]

Will be a problem if we expand in twists

All distributions are evaluated in the strict $x = 0$ limit
Unless...?

The B-JIMWLK evolution equation for the Wilson lines allows to give the distribution an energy, thus x, dependence. BUT:

- B-JIMWLK is meant to resum the large logarithm $\ln(1/x_{\mathrm{Bj}}) \neq \ln(1/x)$
- $f(0, \boldsymbol{k}) \ln(x) \neq f(x, \boldsymbol{k})$
- Beyond the leading genuine twist, more than one x is required.
- In fully inclusive observables, there is no x to chose from.

[Bialas, Navelet, Peschanski, 2001]
The dipole description of DIS is incompatible with an x dependence.
No cop out for DIS

Inclusive DIS beyond the eikonal limit

A consistency test for small x physics

Previous works beyond $x = 0$

- First capture of some of the subleading terms [Altinoluk, Armesto, Beuf, Martinez, Moscoso, Salgado]
- Applications: see Pedro Agostini's and Alina Czajka's talks on Wednesday
- Fully consistent expansion [Chirilli]
- Capture of the spin terms [Cougoulic, Kovchegov, Pitonyak, Santiago, Sievert, Tawabutr]
- Inclusion of a single non-eikonal scattering [Jalilian-Marian]

[RB, Mehtar-Tani]

Bjorken limit

$$s \sim Q^2$$

$$f(x, k_\perp = 0) + O(Q^{-2})$$

Regge limit

$$s \gg Q^2$$

$$f(x = 0, k_\perp) + O(x_{\mathrm{Bj}})$$

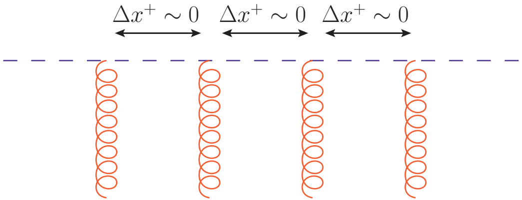
Interpolation?

$$s \gtrsim Q^2$$

$$f(x, k_\perp) + O(x_{\mathrm{Bj}} Q^{-2})$$

Basic observation: in both limits, $k^+ \simeq 0$ for t-channel gluons

Effective propagator in the shock wave approximation

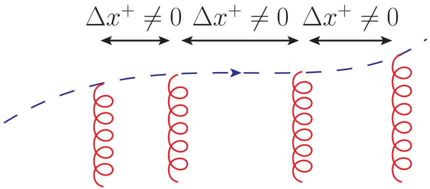


Wilson line:

$$\left[\frac{i\partial}{\partial x^+} + g a^-(x)\right][x^+, x_0^+]_{\mathbf{x}} = i\delta(x^+ - x_0^+)\delta^2(\mathbf{x} - \mathbf{x}_0)$$

$$a^-(x) = \delta(x^+)a(\mathbf{x})$$

Effective propagator beyond the shock wave approximation



Beyond the Wilson line:

$$\left[\frac{i\partial}{\partial x^+} + \frac{\boldsymbol{\partial}_{\boldsymbol{x}}^2}{2p^+} + g a^-(x)\right] \mathcal{G}_{p^+}(x^+, x_0^+; \boldsymbol{x}, \boldsymbol{x}_0) = i\delta(x^+ - x_0^+)\delta^2(\boldsymbol{x} - \boldsymbol{x}_0)$$

$$a^-(x) = a(x^+, \boldsymbol{x})$$

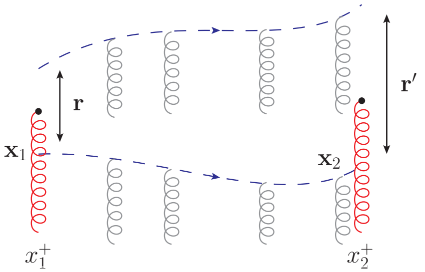
DIS cross section beyond the shock wave approximation

$$\sigma_{\mathrm{DIS}} \propto \mathrm{Re} \int \frac{\mathrm{d}z}{2\pi} \int \mathrm{d}^2\boldsymbol{r}\, \mathrm{d}\boldsymbol{r}' \sum_{\lambda,\lambda'} \varphi_\lambda(z,\boldsymbol{r}) \varphi^*_{\lambda'}(z,\boldsymbol{r}')$$

$$\times \int \mathrm{d}x_2^+ \mathrm{d}x_1^+ \mathrm{e}^{iq^-(x_2^+ - x_1^+)} \int \mathrm{d}^2\boldsymbol{x}_2\, \mathrm{d}^2\boldsymbol{x}_1 \frac{\langle P|\mathcal{O}(x_2^+, x_1^+, \boldsymbol{x}_2, \boldsymbol{x}_1, \boldsymbol{r}, \boldsymbol{r}')|}{\langle P|P\rangle}$$

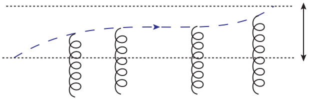
- Standard wave functions encountered in semi-classical small x QCD
- But no decoupling of light cone times, at the cost of a much more complicated operator

The operator for inclusive DIS



$$\mathcal{O}(x_2^+, x_1^+, \boldsymbol{x}_2, \boldsymbol{x}_1, \boldsymbol{r}, \boldsymbol{r}')$$
$$= \mathrm{tr}\left\{ (\boldsymbol{x}_2|\mathcal{G}_{zq^+}(x_2^+, x_1^+)|\boldsymbol{x}_1) \left[A^-(x_1^+, \boldsymbol{x}_1 + \boldsymbol{r}) - A^-(x_1^+, \boldsymbol{x}_1)\right] \right.$$
$$\left. \times (\boldsymbol{x}_1 + \boldsymbol{r}|\mathcal{G}_{-\bar{z}q^+}(x_1^+, x_2^+)|\boldsymbol{x}_2 + \boldsymbol{r}') \left[A^-(x_2^+, \boldsymbol{x}_2 + \boldsymbol{r}') - A^-(x_2^+, \boldsymbol{x}_2)\right] \right\},$$

Classical expansion of the operator



Typical transverse recoil of a fast parton in DIS:

$$\Delta x^2 \sim x_{\mathrm{Bj}}/Q^2$$

x_{Bj}-suppressed in the Regge limit

$1/Q^2$-suppressed in in the Bjorken limit

$$\mathcal{O}(x_2^-, x_1^-, \boldsymbol{x}_2, \boldsymbol{x}_1, \boldsymbol{r}, \boldsymbol{r}')$$
$$\simeq (\boldsymbol{x}_2|\mathcal{G}^{(0)}_{zq^+}(x_2^+, x_1^+)|\boldsymbol{x}_1)(\boldsymbol{x}_1 + \boldsymbol{r}|\mathcal{G}^{(0)}_{-\bar{z}q^+}(x_1^+, x_2^+)|\boldsymbol{x}_2 + \boldsymbol{r}')$$
$$\times \operatorname{tr}\left\{[x_2^+, x_1^+]_{\frac{\boldsymbol{x}_1+\boldsymbol{x}_2}{2}}\left[A^-(x_1^+, \boldsymbol{x}_1 + \boldsymbol{r}) - A^-(x_1^+, \boldsymbol{x}_1)\right]\right.$$
$$\left.\times [x_1^+, x_2^+]_{\frac{\boldsymbol{x}_1+\boldsymbol{x}_2}{2}+\frac{\boldsymbol{r}+\boldsymbol{r}'}{2}}\left[A^-(x_2^+, \boldsymbol{x}_2 + \boldsymbol{r}') - A^-(x_2^+, \boldsymbol{x}_2)\right]\right\}.$$

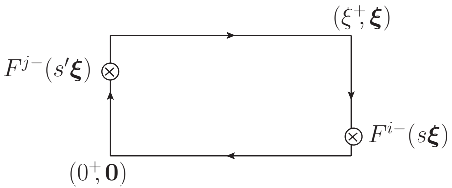
Classical approximation of the DIS cross section

$$\sigma \propto \mathrm{Re} \int \frac{\mathrm{d}z}{2\pi} \int \mathrm{d}^2\boldsymbol{k}\,\mathrm{d}^2\boldsymbol{\ell} \int \frac{\mathrm{d}x}{2\pi}\, \delta\left(x - x_{\mathrm{Bj}} \frac{\boldsymbol{\ell}^2 + z\bar{z}Q^2}{z\bar{z}Q^2}\right)$$
$$\times \left(\partial^i \phi\right)(z, \boldsymbol{\ell} + \boldsymbol{k}/2) \left(\partial^j \phi^*\right)(z, \boldsymbol{\ell} - \boldsymbol{k}/2)\, xG^{ij}(x, \boldsymbol{k})$$

- Standard wave functions
- x-dependent unintegrated distribution

Worth noting: G **is completely different from the expected** $f^D(x \neq 0, \boldsymbol{k})$.

The x-dependent unintegrated PDF



$$xG^{ij}(x, \boldsymbol{k}) \equiv \int \frac{\mathrm{d}\xi^+ \mathrm{d}^2\boldsymbol{\xi}}{(2\pi)^3 P^-} \mathrm{e}^{ixP^-\xi^+ - i(\boldsymbol{k}\cdot\boldsymbol{\xi})} \int_0^1 \mathrm{d}s \int_0^1 \mathrm{d}s' \times \langle P|\mathrm{tr}\left[0^+, \xi^+\right]_0 F^{i-}\left(\xi^+, s\boldsymbol{\xi}\right)\left[\xi^+, 0^+\right]_{\boldsymbol{\xi}} F^{j-}\left(0^+, s'\boldsymbol{\xi}\right)|P\rangle$$

The x-dependent unintegrated PDF

Spans the gluon PDF

$$\int \mathrm{d}^2\boldsymbol{k}\left[xG^{ii}(x,\boldsymbol{k})\right] = xg(x)$$

Spans the dipole

$$\int \mathrm{d}^2\boldsymbol{k}\,\mathrm{e}^{i(\boldsymbol{k}\cdot\boldsymbol{r})}\left[x\boldsymbol{r}^i\boldsymbol{r}^j G^{ij}(x,\boldsymbol{k})\right]_{x=0}$$
$$= \frac{2}{\alpha_s}\int \frac{\mathrm{d}^2\boldsymbol{v}}{(2\pi)^2}\mathrm{Re}\frac{\langle P|N_c - \mathrm{tr}(U_{\boldsymbol{v}}U_{\boldsymbol{r}}^\dagger)|P\rangle}{\langle P|P\rangle}$$

Provides the interpolation between the leading twist term in the Bjorken limit and the eikonal term in the Regge limit.

What is the point?

Diagnosing semi-classical small x physics

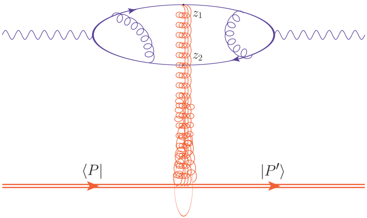
Semi-classical small x cross sections tend to become negative at NLL

Many *ad hoc* modifications were proposed

- Modifications of the evolution kernel
  [Beuf], [Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos]
- Better choice of scale, threshold resummation
  [Liu, Kang, Liu]
- Non-local factorization
  [Iancu, Mueller, Triantafyllopoulos]
- Better choice of evolution variable
  [Ducloué, Iancu, Mueller, Soyez, Triantafyllopoulos]

But never addressed the elephant in the room: the actual scheme itself.

Contribution from a collinear quark
$(z \to 0, |\boldsymbol{p}| \gg |\boldsymbol{k}|)$



General result

$$\int \frac{\mathrm{d}^2\boldsymbol{p}}{\boldsymbol{p}^2} \int_{x_{\mathrm{Bj}}}^{1} \mathrm{d}y \mathcal{P}_{qg}(y) \left[xg(x)\right]_{x=x_{\mathrm{Bj}}/y}$$

Result from the Regge limit

$$\left[xg(x)\right]_{x=0} \int \frac{\mathrm{d}^2\boldsymbol{p}}{\boldsymbol{p}^2} \int_{0}^{1} \mathrm{d}y \mathcal{P}_{qg}(y)$$

Implicit assumption of semi-classical small x physics

$$\lim_{x_{\rm Bj}\to 0}\int_{x_{\rm Bj}}^{1} {\rm d}y\mathcal{P}_{(q,g)g}(y)\left[xg(x)\right]_{x=x_{\rm Bj}/y} = \left[xg(x)\right]_{x=0}\int_{0}^{1} {\rm d}y\mathcal{P}_{(q,g)g}(y)$$

Problems

- The intercept of the PDF?
  $xg(x)$ is not a constant at small x
- The integral of the splitting function
  How does the integral of $\mathcal{P}_{gg}$ behave?

Neglecting x in the distribution is the origin of the problematic handling of collinear logarithms

Conclusions

Conclusion

Where do we stand?

Bad news

- Semi-classical small x physics has, at its core, issues with collinear logarithms
- The problem can be traced down to the very starting point

Good news

- We now have a minimal correction of semi-classical small x which solves the problem from first principles
- Wave functions, and thus hard parts, are not modified by the scheme

Conclusion

Prospects

What needs to be done now

- Compute the evolution equation for the x-dependent DIS uPDF
- For consistency, add non-pure gauge transverse gluons
- Check if the x-dependent DIS uPDF appears in another process

What could be done now

- Compute the DIS uPDF on the lattice?
- Check the existence and energy dependence of a saturation scale?