Gauge-invariant TMD factorization for Drell-Yan hadronic tensor at small x

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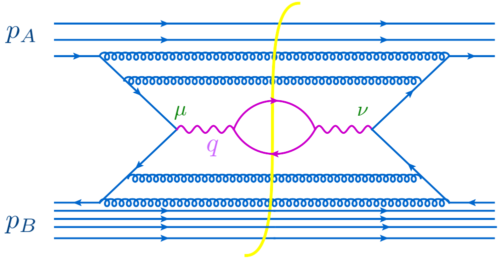
DY hadronic tensor

The hadronic tensor $W_{\mu\nu}$ is defined as

$$W_{\mu\nu}(p_A,p_B,q) \stackrel{\rm def}{=} \frac{1}{(2\pi)^4}\sum_X\int d^4x\, e^{-iqx}\langle p_A,p_B|J_\mu(x)|X\rangle\langle X|J_\nu(0)|p_A,p_B\rangle$$
$$= \frac{1}{(2\pi)^4}\int d^4x\, e^{-iqx}\langle p_A,p_B|J_\mu(x)J_\nu(0)|p_A,p_B\rangle$$



p_A, p_B = hadron momenta, q = the momentum of DY pair, $\sum_X$ = the sum over full set of “out” states and J_μ is the electromagnetic current.

For unpolarized hadrons, the hadronic tensor $W_{\mu\nu}$ is parametrized by 4 functions, for example in Collins-Soper frame

$$\begin{aligned} W_{\mu\nu} &= -\big(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\big)(W_T + W_{\Delta\Delta}) - 2X_\mu X_\nu W_{\Delta\Delta} \\ &\quad + Z_\mu Z_\nu (W_L - W_T - W_{\Delta\Delta}) - (X_\mu Z_\nu + X_\nu Z_\mu) W_\Delta \end{aligned}$$

where X, Z are unit vectors orthogonal to q and to each other

TMD representation for W_i

The hadronic tensor in the Sudakov region $q^2 \equiv Q^2 \gg q_\perp^2$ can be studied by TMD factorization. For example, functions W_T and $W_{\Delta\Delta}$ can be represented as

$$W_i = \sum_{\text{flavors}} e_f^2 \int d^2k_\perp \mathcal{D}^{(i)}_{f/A}(x_A, k_\perp)\mathcal{D}^{(i)}_{f/B}(x_B, q_\perp - k_\perp)C_i(q, k_\perp)$$
$$+ \text{ power corrections } + \text{ Y} - \text{terms} \tag{1}$$

- $\mathcal{D}_{f/A}(x_A, k_\perp)$ is the TMD density of a parton f in hadron A with fraction of momentum x_A and transverse momentum $k_\perp$,
- $\mathcal{D}_{f/B}(x, q_\perp - k_\perp)$ is a similar quantity for hadron B,
- $C_i(q, k)$ are determined by the cross section $\sigma(f\!f \to \mu^+\mu^-)$ of production of DY pair of invariant mass q^2 in the scattering of two partons.

TMD representation for W_i

There is, however, a problem with Eq. (1) for the functions W_L and W_Δ.

W_T and $W_{\Delta\Delta}$ are determined by leading-twist quark TMDs, but W_Δ and W_L start from terms $\sim \frac{q_\perp}{Q}$ and $\sim \frac{q_\perp^2}{Q^2}$ determined by quark-quark-gluon TMDs.

The power corrections $\sim \frac{q_\perp}{Q}$ were found more than two decades ago but there was no calculation of power corrections $\sim \frac{q_\perp^2}{Q^2}$ until recently.

Also, the leading-twist contribution is not EM gauge invariant.

Double functional integral for W

$$W(p_A,p_B,q) \ = \ \sum_X \int d^4x\ e^{-iqx}\langle p_A,p_B|J_\mu(x)|X\rangle\langle X|J_\nu(0)|p_A,p_B\rangle$$

$$= \lim_{\substack{t_f\to\infty \\ t_i\to-\infty}} \int d^4x\ e^{-iqx}\int^{\tilde{A}(t_f)=A(t_f)} D\tilde{A}_\mu DA_\mu \int^{\tilde{\psi}(t_f)=\psi(t_f)} D\tilde{\bar{\psi}}D\tilde{\psi}D\bar{\psi}D\psi\ \Psi^*_{p_A}(\vec{\tilde{A}}(t_i),\tilde{\psi}(t_i))$$

$$\times\ \Psi^*_{p_B}(\vec{\tilde{A}}(t_i),\tilde{\psi}(t_i))e^{-iS_{\rm QCD}(\tilde{A},\tilde{\psi})}e^{iS_{\rm QCD}(A,\psi)}J_\mu(x)J_\nu(y)\Psi_{p_A}(\vec{A}(t_i),\psi(t_i))\Psi_{p_B}(\vec{A}(t_i),\psi(t_i))$$

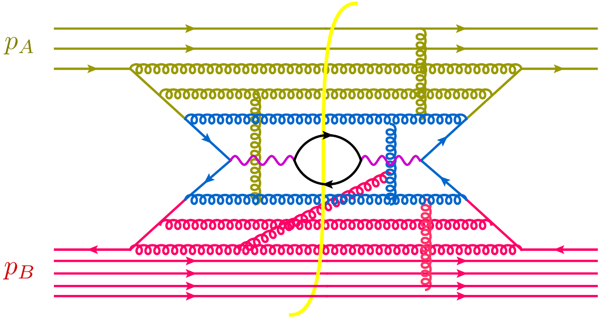
- "Left" A,ψ fields correspond to the amplitude $\langle X|J_\nu(0)|p_A,p_B\rangle$,
- "Right" fields $\tilde{A},\tilde{\psi}$ correspond to amplitude $\langle p_A,p_B|J_\mu(x)|X\rangle$

The boundary conditions $\tilde{A}(t_f)=A(t_f)$ and $\tilde{\psi}(t_f)=\psi(t_f)$ reflect the sum over intermediate states X.

Rapidity factorization for particle production

Sudakov variables:

$$p = \alpha p_1 + \beta p_2 + p_\perp, \qquad p_1 \simeq p_A, \;\; p_2 \simeq p_B, \;\; p_1^2 = p_2^2 = 0$$
$$x_* \equiv p_2 \cdot x = \sqrt{\frac{s}{2}} x^+, \qquad x_\bullet \equiv p_1 \cdot x = \sqrt{\frac{s}{2}} x^-$$



"Projectile″ fields : $|\beta| < \sigma_p$

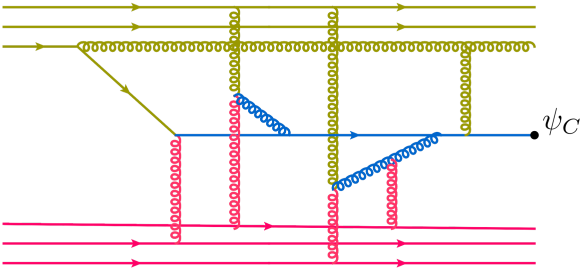
"Central" fields

"Target" fields : $|\alpha| < \sigma_t$

We integrate over "central" fields in the background of projectile and target fields.

ψ_C in the tree approximation

Tree approximation:
Projectile fields: $\beta = 0 \Rightarrow A(x_\bullet, x_\perp),\ \psi_A(x_\bullet, x_\perp)$
Target fields: $\alpha = 0 \Rightarrow B(x_*, x_\perp),\ \psi_B(x_\bullet, x_\perp)$



ψ_C = sum of tree diagrams in external $A, \tilde{A}, \psi_A, \tilde{\psi}_A$ and $B, \tilde{B}, \psi_B, \tilde{\psi}_B$ fields with sources

$J_\psi = (\not{P} + m)(\psi_A + \psi_B)$, $\qquad J_\nu = D^\mu F^{\mu\nu}(A + B)$

and

$\tilde{J}_\psi = (\not{P} + m)(\tilde{\psi}_A + \tilde{\psi}_B)$, $\qquad \tilde{J}_\nu = D^\mu F^{\mu\nu}(\tilde{A} + \tilde{B})$

The fields A, ψ and $\tilde{A}, \tilde{\psi}$ do not depend on $x_* \Rightarrow$
if they coincide at $x_* = \infty \Rightarrow$ they coincide everywhere.

Similarly,
B, ψ_b and $\tilde{B}, \tilde{\psi}_b$ do not depend on $x_\bullet \Rightarrow$
if they coincide at $x_\bullet = \infty$ they should be equal.

Since $\tilde{A} = A$ and $\tilde{B} = B$ the sources and background fields are the same to the left and to the right of the cut
$\Rightarrow$
ψ_C and C_μ are given by the sum of tree diagrams with *retarded* Green functions

(F. Gelis, R. Venugopalan)

Classical solution

The sum of diagrams with retarded Green functions $\Leftrightarrow$ solution of classical YM equations

$$(\not{P} + m_f)\psi^f \ = \ 0, \quad D^\nu F^a_{\mu\nu} \ = \ \sum_f g\bar{\psi}^f t^a \gamma_\mu \psi^f$$

$$\text{Boundary conditions :} \qquad \begin{array}{ll} A_\mu(x) \stackrel{x_*\to-\infty}{=} \bar{A}_\mu(x_\bullet, x_\perp), & \psi(x) \stackrel{x_*\to-\infty}{=} \psi_a(x_\bullet, x_\perp) \\ A_\mu(x) \stackrel{x_\bullet\to-\infty}{=} \bar{B}_\mu(x_*, x_\perp), & \psi(x) \stackrel{x_\bullet\to-\infty}{=} \psi_b(x_*, x_\perp) \end{array}$$

The projectile and target fields satisfy YM equations

$$\begin{aligned} (\not{P} + m_f)\psi^f_a \ &= \ 0, \quad D^\nu F^a_{\mu\nu} \ = \ g\bar{\psi}^f_a t^a \gamma_\mu \psi^f_a \\ (\not{P} + m_f)\psi^f_b \ &= \ 0, \quad D^\nu F^a_{\mu\nu} \ = \ g\bar{\psi}^f_b t^a \gamma_\mu \psi^f_b \end{aligned}$$

Projectile partons: $k = \alpha p_1 + k_\perp$, target partons: $k = \beta p_1 + k_\perp$ $\Rightarrow$ partons are *not* on the mass shell

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Projectile partons: $k=\alpha p_1+k_\perp$, target partons: $k=\beta p_1+k_\perp$ ⇒ partons are *not* on the mass shell

Method of solution:

- Start with $\psi_A+\psi_B$ and $\bar{A}_\mu+\bar{B}_\mu$ in the gauge $A_*=0$, $A_\bullet=0$
- Correct by computing Feynman diagrams (with retarded propagators) with sources $(\not{P}+m)(\psi_A+\psi_B)$ and $J_\nu=D^\mu F^{\mu\nu}(U+V)$

Classical fields in the leading order in $p_\perp^2/p_\parallel^2 \sim q_\perp^2/Q^2$

The solution of YM equations in general case (scattering of two "color glass condensates") is yet unsolved problem.

Fortunately, for our case of particle production with $\frac{q_\perp}{Q} \ll 1$ we can use this small parameter and construct the approximate solution.

At the tree level transverse momenta are $\sim q_\perp^2$ and longitudinal are $\sim Q^2 \Rightarrow$

$$\psi, A \ = \ \text{series in } \frac{q_\perp}{Q}: \qquad \psi = \psi^{(0)} + \psi^{(1)} + \ldots, \qquad A = A^{(0)} + A^{(1)} + \ldots$$

NB: After the expansion

$$\frac{1}{p^2 + i\epsilon p_0} \ = \ \frac{1}{p_\parallel^2 - p_\perp^2 + i\epsilon p_0} \ = \ \frac{1}{p_\parallel^2} - \frac{1}{p_\parallel^2 + i\epsilon p_0} p_\perp^2 \frac{1}{p_\parallel^2 + i\epsilon p_0} \ + \ldots$$

the dynamics in transverse space is trivial.

Fields are either at the point $x_\perp$ or at the point $0_\perp \Rightarrow$ TMDs

Expansion of quark fields

Expanding it in powers of $p_\perp^2/p_\parallel^2$ gives:

$$\Psi(x) \;=\; \Psi_1^{(0)} + \Psi_2^{(0)} + \Psi^{(1)} + \Psi^{(2)} + \ldots,$$

where

$$\begin{aligned}
\Psi_A^{(0)} &= \psi_A + \Xi_1, \quad \Xi_1 \;=\; -\frac{g\not{p}_2}{s}\gamma^i B_i \frac{1}{\alpha + i\epsilon}\psi_A,\\
\bar{\Psi}_A^{(0)} &= \bar{\psi}_A + \bar{\Xi}_1, \quad \bar{\Xi}_1 \;=\; -\big(\bar{\psi}_A \frac{1}{\alpha - i\epsilon}\big)\gamma^i B_i \frac{g\not{p}_2}{s},\\
\Psi_B^{(0)} &= \psi_B + \Xi_2, \quad \Xi_{1B} \;=\; -\frac{g\not{p}_1}{s}\gamma^i A_i \frac{1}{\beta + i\epsilon}\psi_B,\\
\bar{\Psi}_B^{(0)} &= \bar{\psi}_B + \bar{\Xi}_2, \quad \bar{\Xi}_2 \;=\; -\big(\bar{\psi}_B \frac{1}{\beta - i\epsilon}\big)\gamma^i A_i \frac{g\not{p}_1}{s}.
\end{aligned}$$

$\Psi^{(1)}$, $\Psi_B^{(2)}$,.. lead to next terms in series in $\frac{q_\perp^2}{Q^2}$

Leading-N_c power corrections

Power corrections are $\sim$ leading twist $\times\Big(\frac{q_\perp}{Q}$ or $\frac{q_\perp^2}{Q^2}\Big)\times\big(1+\frac{1}{N_c}+\frac{1}{N_c^2}\big)$.

(Pleasant) surprise: most of the terms not suppressed by $\frac{1}{N_c}$ are determined by the leading-twist TMDs due to QCD equations of motion

Leading twist:

$$\frac{1}{8\pi^3 s}\int dx_\bullet d^2x_\perp\ e^{-i\alpha x_\bullet+i(k,x)_\perp}\ \langle A|\hat{\bar\psi}_f(x_\bullet,x_\perp)\ \not{p}_2\hat\psi_f(0)|A\rangle\ =\ f_{1f}(\alpha,k_\perp^2)$$

Power correction:

$$\frac{1}{8\pi^3 s}\int dx_\bullet dx_\perp\ e^{-i\alpha_q x_\bullet+i(k,x)_\perp}$$

$$\times\ \langle A|\hat{\bar\psi}^f(x_\bullet,x_\perp)\ \not{p}_2[\hat U_i(x_\bullet,x_\perp)-i\gamma_5\hat{\tilde U}_i(x_\bullet,x_\perp)]\hat\psi^f(0)|A\rangle$$

$$=\ -k_i f_1(\alpha_q,k_\perp)+\alpha_q k_i\big[f_\perp(\alpha_q,k_\perp)+g^\perp(\alpha_q,k_\perp)\big],$$

(Mulders & Tangerman, 1996)

At small $\alpha_q\equiv x_A$ one can drop the second term

Result for $W_{\mu\nu}$ for unpolarized hadrons

Result:

$$W_{\mu\nu}(q) \ = \ W^1_{\mu\nu}(q) + W^2_{\mu\nu}(q)$$

The first, gauge-invariant, part is given by

$$W^1_{\mu\nu}(q) \ = \ W^{1F}_{\mu\nu}(q) + W^{1H}_{\mu\nu}(q),$$

$$W^{1F}_{\mu\nu}(q) \ = \ \sum_f e_f^2 W^{fF}_{\mu\nu}(q), \quad W^{fF}_{\mu\nu}(q) \ = \ \frac{1}{N_c}\int d^2k_\perp F^f(q,k_\perp)\mathcal{W}^F_{\mu\nu}(q,k_\perp),$$

$$W^{1H}_{\mu\nu}(q) \ = \ \sum_f e_f^2 W^{fH}_{\mu\nu}(q), \quad W^{fH}_{\mu\nu}(q) \ = \ \frac{1}{N_c}\int d^2k_\perp H^f(q,k_\perp)\mathcal{W}^H_{\mu\nu}(q,k_\perp)$$

where F^f and H^f are ($\alpha_q \equiv x_A$, $\beta_q \equiv x_B$)

$$F^f(q,k_\perp) \ = \ f_1^f(\alpha_q,k_\perp)\bar{f}_1^f(\beta_q,(q-k)_\perp) \ + \ f_1^f \leftrightarrow \bar{f}_1^f$$
$$H^f(q,k_\perp) \ = \ h^\perp_{1f}(\alpha_q,k_\perp)\bar{h}^\perp_{1f}(\beta_q,(q-k)_\perp) \ + \ h^\perp_{1f} \leftrightarrow \bar{h}^\perp_{1f}$$

and

Gauge-invariant structures $q^\mu W^F_{\mu\nu} = q^\mu W^H_{\mu\nu} = 0$

$$\mathcal{W}^F_{\mu\nu}(q,k_\perp)$$
$$= -g^\perp_{\mu\nu} + \frac{1}{Q_\parallel^2}(q^\parallel_\mu q^\perp_\nu + q^\parallel_\nu q^\perp_\mu) + \frac{q_\perp^2}{Q_\parallel^4} q^\parallel_\mu q^\parallel_\nu + \frac{\tilde q_\mu \tilde q_\nu}{Q_\parallel^2}[q_\perp^2 - 4(k,q-k)_\perp]$$
$$- \Big[\frac{\tilde q_\mu}{Q_\parallel^2}\Big(g^\perp_{\nu i} - \frac{q^\parallel_\nu q_i}{Q_\parallel^2}\Big)(q-2k)^i_\perp + \mu \leftrightarrow \nu\Big]$$

$$\tilde q \equiv \alpha_q p_1 - \beta_q p_2$$

$$m^2 \mathcal{W}^H_{\mu\nu}(q,k_\perp)$$
$$= -k^\perp_\mu (q-k)^\perp_\nu - k^\perp_\nu (q-k)^\perp_\mu - g^\perp_{\mu\nu}(k,q-k)_\perp + 2\frac{\tilde q_\mu \tilde q_\nu - q^\parallel_\mu q^\parallel_\nu}{Q_\parallel^4} k_\perp^2 (q-k)_\perp^2$$
$$- \Big(\frac{q^\parallel_\mu}{Q_\parallel^2}\big[k_\perp^2 (q-k)^\perp_\nu + k^\perp_\nu (q-k)^2_\perp\big] + \frac{\tilde q_\mu}{Q_\parallel^2}\big[k_\perp^2 (q-k)^\perp_\nu - k^\perp_\nu (q-k)^2_\perp\big] + \mu \leftrightarrow \nu\Big)$$
$$- \frac{\tilde q_\mu \tilde q_\nu + q^\parallel_\mu q^\parallel_\nu}{Q_\parallel^4}\big[q_\perp^2 - 2(k,q-k)_\perp\big](k,q-k)_\perp - \frac{q^\parallel_\mu \tilde q_\nu + \tilde q_\mu q^\parallel_\nu}{Q_\parallel^4}(2k-q,q)_\perp (k,q-k)_\perp$$

W^F part coincides with parton Reggeization result (Nefedov, Saleev, 2019)

Non-gauge-invariant corrections

$$
\begin{aligned}
W^2_{\mu\nu}(q) \;=\; & \frac{1}{N_c}\sum_f e_f^2 \frac{1}{Q^2}\int d^2k_\perp \Big[\frac{1}{m^2}H^f_A(q,k_\perp)\big\{[k^\perp_\mu (q-k)^\perp_\nu + \mu\leftrightarrow\nu](k,q-k)_\perp \\
& - k_\perp^2 (q-k)^\perp_\mu (q-k)^\perp_\nu - (q-k_\perp)^2 k^\perp_\mu k^\perp_\nu + g^\perp_{\mu\nu}(k,q-k)^2_\perp - g^\perp_{\mu\nu}k^2_\perp (q-k_\perp)^2\big]\big\} \\
& + \frac{N_c}{N_c^2-1}\big\{[k^\perp_\mu (q-k)^\perp_\nu + \mu\leftrightarrow\nu + g^\perp_{\mu\nu}(k,q-k)_\perp]J^f_1(q,k_\perp) \\
& \qquad - g^\perp_{\mu\nu}(k,q-k)_\perp J^f_2(q,k_\perp)\big\} \;+\; O\Big(\frac{1}{N_c^2}\Big)\Big] \;+\; O\Big(\frac{Q^4_\perp}{Q^4}\Big)
\end{aligned}
$$

H_A and $J_{1,2}$ terms involve twist-3 quark-quark-gluon TMDs which do not reduce to leading-twist distributions.

Non-gauge-invariant corrections

$$W^2_{\mu\nu}(q) = \frac{1}{N_c}\sum_f e_f^2 \frac{1}{Q^2}\int d^2k_\perp \Big[\frac{1}{m^2}H_A^f(q,k_\perp)\big\{[k_\mu^\perp(q-k)_\nu^\perp + \mu\leftrightarrow\nu](k,q-k)_\perp$$
$$- k_\perp^2(q-k)_\mu^\perp(q-k)_\nu^\perp - (q-k_\perp)^2 k_\mu^\perp k_\nu^\perp + g_{\mu\nu}^\perp(k,q-k)_\perp^2 - g_{\mu\nu}^\perp k_\perp^2(q-k_\perp)^2\big]\big\}$$
$$+ \frac{N_c}{N_c^2-1}\Big\{[k_\mu^\perp(q-k)_\nu^\perp + \mu\leftrightarrow\nu + g_{\mu\nu}^\perp(k,q-k)_\perp]J_1^f(q,k_\perp)$$
$$- g_{\mu\nu}^\perp(k,q-k)_\perp J_2^f(q,k_\perp)\Big\} + O\big(\frac{1}{N_c^2}\big)\Big] + O\big(\frac{Q_\perp^4}{Q^4}\big)$$

H_A and $J_{1,2}$ terms involve twist-3 quark-quark-gluon TMDs which do not reduce to leading-twist distributions.

Gauge invariance should be restored after adding sub-leading power corrections. For example,

$$q^\mu W^2_{\mu\nu}(q) \sim \frac{q_\nu^\perp q_\perp^2}{Q^2} \quad \text{and} \quad q^\mu \times \Big(\frac{q_\mu^\parallel q_\nu^\perp}{Q^2}\times\frac{q_\perp^2}{Q^2}\Big) = \frac{q_\nu^\perp q_\perp^2}{Q^2}$$

They are of the same order so one should expect that gauge invariance is restored after calculation of all such terms.

Estimate of power corrections

If $Q^2 \gg k_\perp^2 \gg m_N^2$ we can approximate

$$f_1(\alpha_z, k_\perp^2) \simeq \frac{f(\alpha_z)}{k_\perp^2}, \quad h_1^\perp(\alpha_z, k_\perp^2) \simeq \frac{m_N^2 h(\alpha_z)}{k_\perp^4}$$

For the total DY cross section

$$W_\mu^\mu(q) = -\frac{2}{N_c}\sum e_f^2 \int d^2k_\perp \Big\{\Big[1 - 2\frac{(k, q-k)_\perp}{Q^2}\Big] F^f(q, k_\perp) + 2\frac{k_\perp^2 (q-k)_\perp^2}{m_N^2 Q^2} H^f(q, k_\perp)$$

$$\simeq -\frac{2}{N_c}\sum e_f^2 \int d^2k_\perp \Big\{\Big[1 - 2\frac{(k, q-k)_\perp}{Q^2}\Big] \frac{F^f(\alpha_q, \beta_q)}{k_\perp^2 (q-k)_\perp^2} + \frac{2m^2}{Q^2}\frac{H^f(\alpha_q, \beta_q)}{k_\perp^2 (q-k)_\perp^2}\Big\}$$

$$\simeq -\frac{2}{N_c}\sum e_f^2 \int d^2k_\perp \Big[1 - 2\frac{(k, q-k)_\perp}{Q^2}\Big] \frac{F^f(\alpha_q, \beta_q)}{k_\perp^2 (q-k)_\perp^2}$$

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With logarithmic accuracy

$$W_\mu^\mu(q) = -\frac{4\pi}{N_c}\sum e_f^2 \Big[\frac{1}{q_\perp^2}\ln\frac{q_\perp^2}{m_N^2} + \frac{1}{Q^2}\ln\frac{Q^2}{q_\perp^2}\Big] \sum_f \big[f^f(\alpha_z)\bar{f}^f(\beta_z) + \bar{f}^f(\alpha_z) f^f(\beta_z)\big]$$

$\Rightarrow$ power corrections reach 10% level at $q_\perp \sim \frac{1}{4}Q$

Estimate of W_i at $Q^2 \gg k_\perp^2 \gg m_N^2$ with log accuracy

Neglecting H^f everywhere except $W_{\Delta\Delta}$ where it is a leading-twist term, we get

$$W_T(q) \simeq \frac{2\pi}{N_c}\Big[\frac{1}{q_\perp^2} - \frac{1}{2Q^2}\Big] \ln\frac{q_\perp^2}{m^2} \sum e_f^2 F^f(\alpha_q, \beta_q)$$

$$W_L(q) \simeq \frac{2\pi}{Q^2 N_c}\Big[\ln\frac{q_\perp^2}{m_N^2} + 2\ln\frac{Q^2}{q_\perp^2}\Big] \sum_f e_f^2 F^f(\alpha_q, \beta_q)$$

$$W_{\Delta\Delta} \simeq \frac{\pi}{Q^2 N_c}\ln\frac{q_\perp^2}{m^2}\sum_f e_f^2\Big[F^f(\alpha_q,\beta_q) + \frac{4m^2Q^2}{q_\perp^4}\Big(1 - \frac{q_\perp^2}{2Q^2}\Big)H^f(\alpha_q,\beta_q)\Big]$$

and $W_\Delta = 0$ if we use factorization models for TMDs $f(x, k_\perp) = \phi(x)\psi(k_\perp)$

Back-of-the-envelope estimates of angular coefficients

Take $s = 8$ TeV, $Q = 90$ GeV and $q_\perp = 20$ GeV where $x_A, x_B \sim 0.1$ and power corrections are small but sizable.

The differential cross section of DY process is parametrized as

$$\left(\frac{d\sigma}{d^4q}\right)^{-1}\frac{d\sigma}{d\Omega d^4q} = \frac{3}{4\pi(\lambda+3)}\left(1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right)$$

Estimates of angular coefficients

$$1-\lambda = 2\frac{W_L}{W_T+W_L} \simeq 2\frac{1+2\frac{\ln Q^2/q_\perp^2}{\ln q_\perp^2/m^2}}{\frac{Q^2}{q_\perp^2}-\frac{1}{2}+2\frac{\ln Q^2/q_\perp^2}{\ln q_\perp^2/m^2}} \simeq 0.19$$

$$\nu = \frac{2W_{\Delta\Delta}}{W_T+W_L} \simeq \frac{1}{\frac{Q^2}{q_\perp^2}-\frac{1}{2}+2\frac{\ln Q^2/q_\perp^2}{\ln q_\perp^2/m^2}} \simeq 0.05$$

$\mu = \frac{W_\Delta}{W_T+W_L}, = 0$ if we use factorization models for TMDs.

Approximately the same λ and ν values as in analysis of LHC data by Lambertsen and Vogelsang

Conclusions

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   - The resulting hadronic tensor for unpolarized hadrons is EM gauge-invariant and depends on two leading-twist TMDs: f_1 responsible for total DY cross section, and Boer-Mulders function $h_1^\perp$.
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Thank you for attention!