Resummation benchmarking for DY production at the LHC: current achievements and challenges to come

- **Caveats/disclaimers/context:**
  - The work presented here represents that of many theorists working on resummation calculations together with a few experimentalists. Several of them are present here of course and should help me answer any theoretical questions which might arise
  - This work is done in the context of the wider LHC precision EW working group, which itself is a sub-group of the LPCC SM WG.
  - Unlike many fixed-order perturbative calculations, a benchmarking of resummation has never been done (some attempts at Les Houches though)
  - This attempt began in earnest in late 2018 and has now produced enough interesting results that it is hoped to publish these within the next year.
  - Here a personal and partially historical overview as an experimentalist: apologies for any mistakes/misquotes and a huge thanks to all our theory colleagues who have borne with patience the brunt of all our questions over these two years

    A special thanks to Tom Cridge from whom I stole some slides reported at the last general LPCC SM meeting!

Codes taking part

- SCETlib [https://confluence.desy.de/display/scetlib]
- CuTe [https://cute.hepforge.org]

} SCET — Ebert et al. '17; Becher et al. '11,'20

- DYRes/DYTURBO [https://gitlab.cern.ch/DYdevel/DYTURBO]
- ReSolve [https://github.com/fkhorad/reSolve]

} qT-res. — Camarda et al., '19; Coradeschi, T.C., '17

- RadISH [https://arxiv.org/pdf/1705.09127.pdf]
- PB-TMD [https://arxiv.org/pdf/1906.00919.pdf]

} PB — Monni et al. '16,'17; Martinez et al. '20

- NangaParbat [https://github.com/vbertone/NangaParbat]
- arTeMiDe [https://teorica.fis.ucm.es/artemide/]

} TMD — Bacchetta et al., '19; Scimemi, Vladimirov, '17

Precision measurements in the EW sector at the LHC

The word precision has different meanings in different areas (note that mass measurements are a special case):

- It means sub-percent precision in DY and in some aspects of flavour physics in LHCb
- It means a few percent at best still for top physics
- It means 10-40% for Higgs physics (eg couplings), at least for quite a while

It is not a surprise therefore that DY measurements are the most demanding in terms of theoretical accuracy (far more than Higgs!).

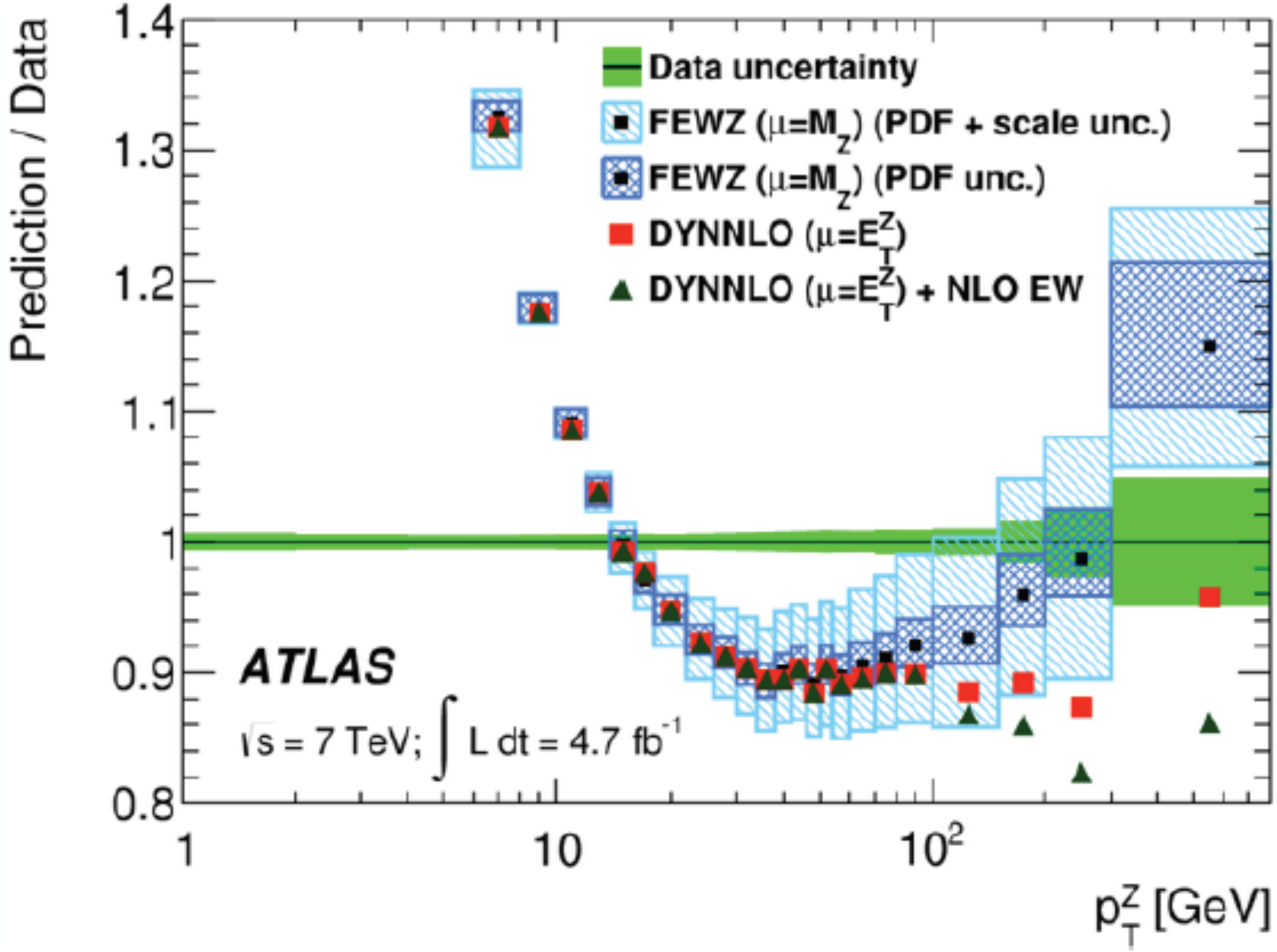
In a nutshell, there are two key difficulties we are confronted with:

a) The lack of a MC generator tool for DY production which would include N...NLO+N…NLL QCD (and EW/QED) calculations, perfectly matched and merged to PS, with a UE model reproducing the data

b) The complexity of dealing with a large number of sources of theoretical uncertainty which are not reliable at this level

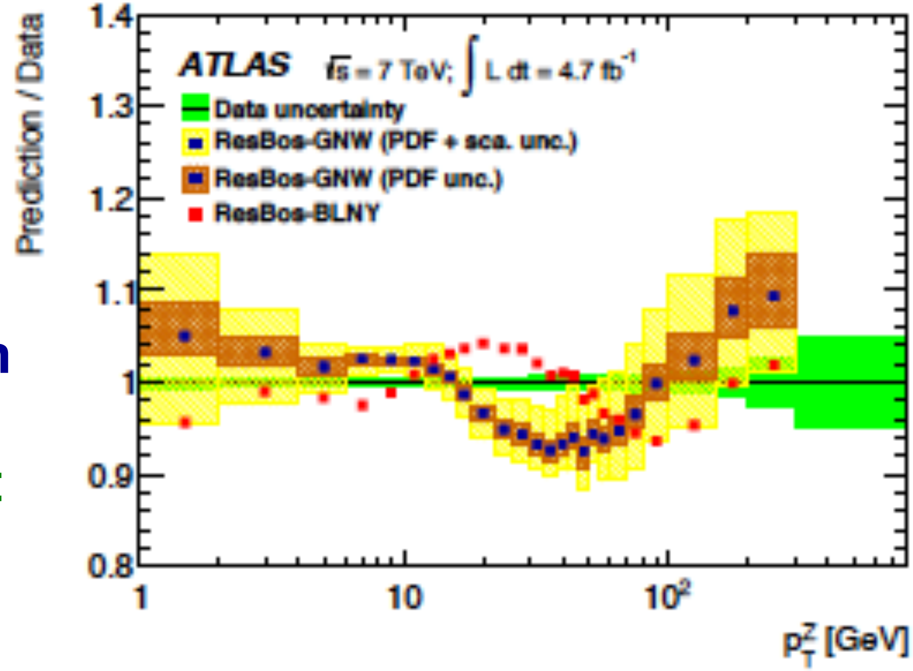
Very precise measurement of Z p_T has been posing problems to theory from early LHC data

ATLAS Z pT: NNLO / Data

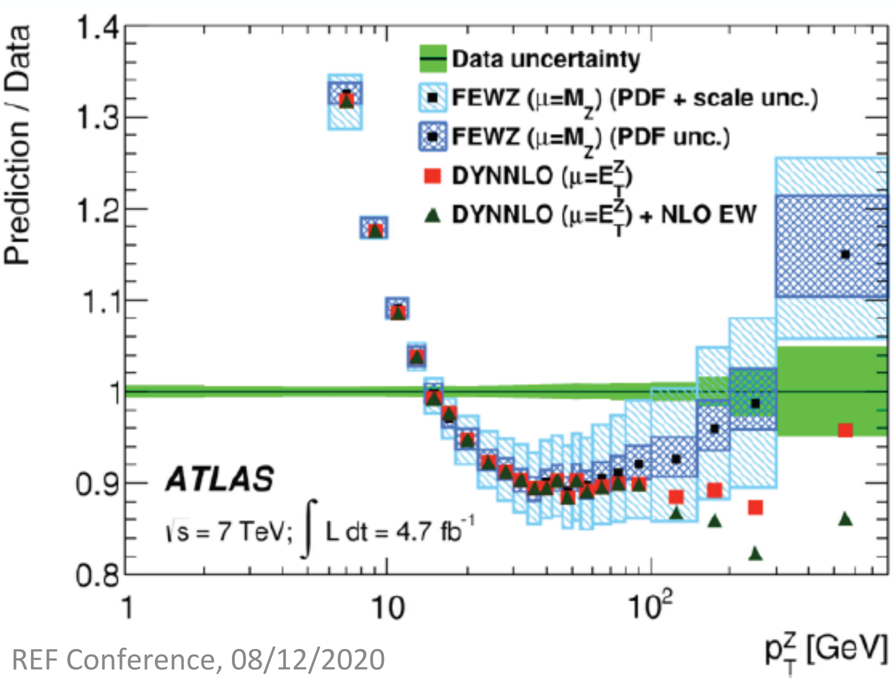


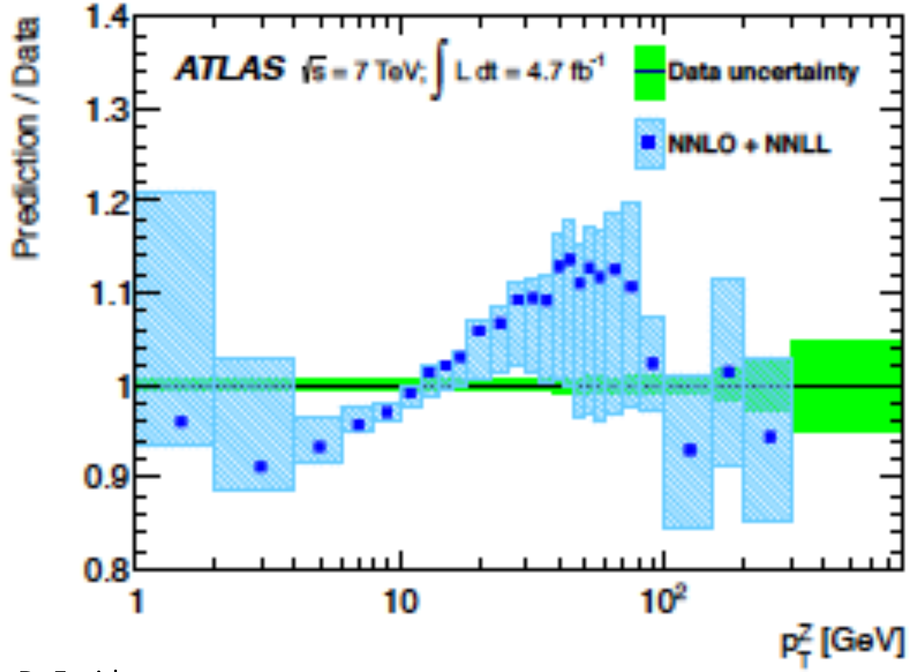
Very precise measurement of Z p_T poses problems to theory

- Shown also here are ResBos (top right) and resummation calculation by Banfi et al. (bottom right)
- Note: uncertainty on measurement at low p_T is ~ 0.5%, rising to 1.5% for p_T^Z ~ 150 GeV



ATLAS Z pT: NNLO / Data

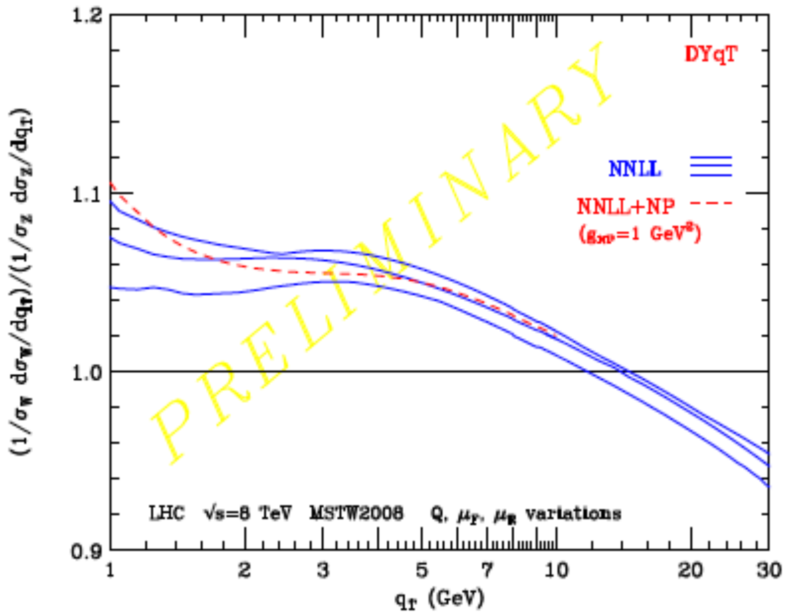




Cancellation of uncertainties in ratios (?)

- **Beware! Plot below assumes all three scales (renorm., fact. and resummation) are fully correlated between W and Z.**

W/Z ratio of observables: the q_T spectrum



DYqT resummed predictions for the ratio of W/Z normalized q_T spectra.

DYRES: a tool to be used at the LHC?
RESBOS discarded because of wrong predictions for angular coefficients

- The use of the W/Z ratio observables substantially reduces both the experimental and theoretical systematic uncertainties [Giele,Keller('97)].
- Resummed perturbative prediction for

$$\frac{\frac{1}{\sigma_W}\frac{d\sigma_W}{dq_T}}{\frac{1}{\sigma_Z}\frac{d\sigma_Z}{dq_T}}(\mu_R, \mu_F, Q)$$

  with the customary scale variation.
- NNLL perturbative uncertainty band very small: 2-5% for $1 < q_T < 2\,\text{GeV}$, 1.5-2% for $2 < q_T < 30\,\text{GeV}$.
- Non perturbative effects within 1% for $1.5 < q_T < 5\,\text{GeV}$ and negligible for $q_T > 5\,\text{GeV}$.

Very precise measurement of Z p_T has been posing problems to theory since early LHC data

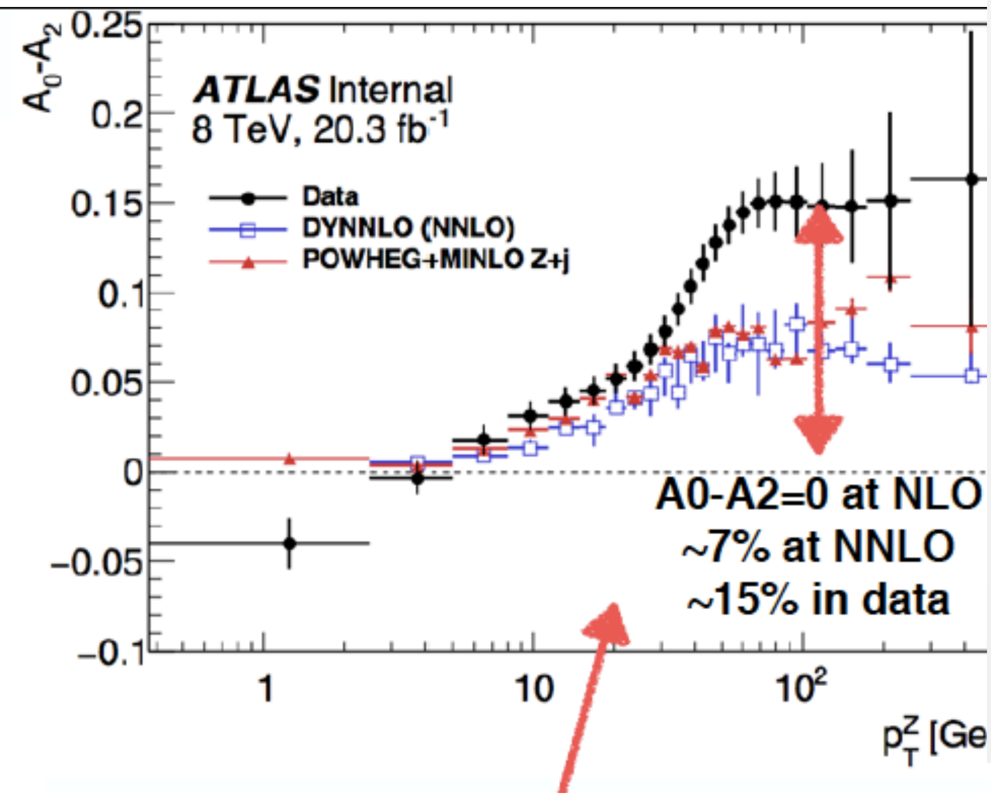
- ATLAS and CMS both have uncertainties which are far smaller than the theoretical ones and agree with each other to < 1%.
- However, ATLAS theory uncertainty estimates for FEWZ are smaller than those estimated by CMS, by a factor ~ 2. Why?? MC stats??

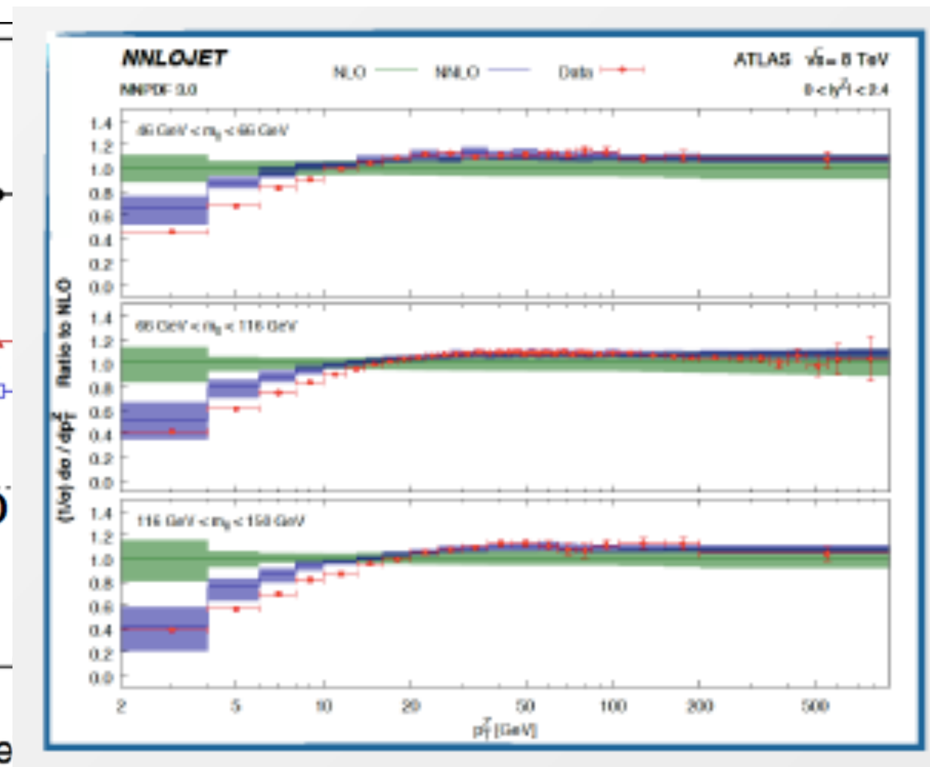
- Key point however is that PDF uncertainties are far smaller than the difference between data and theory at pT ~ 40 GeV which is well in perturbative regime
- This means that the data cannot be included in PDF fits because they will come out wrong.
- Why? Because PDF fits do not include theory scale uncertainties, they are not designed for this (yet). This has been a problem for jet physics results since a while and now it appears also for p_T^Z

- There are other more "hidden" uncertainties in PDF fits, related to assumptions such as that proven somewhat mistaken for the strange sea. PDF fit results can then suddenly move "out of their uncertainties".

Very precise measurement of Z p_T has been posing problems to theory from early LHC data

- NNLOJETS (using antenna subtraction) state-of-the-art and much needed for fixed-order part of DY qT distributions down to quite low qT values
- NNLOJETS also state-of-the art and much needed for certain combinations of angular coefficients (Lam-Tung relation)
- Full run-2 data will tell us whether V+jets@N3LO is required to match data!





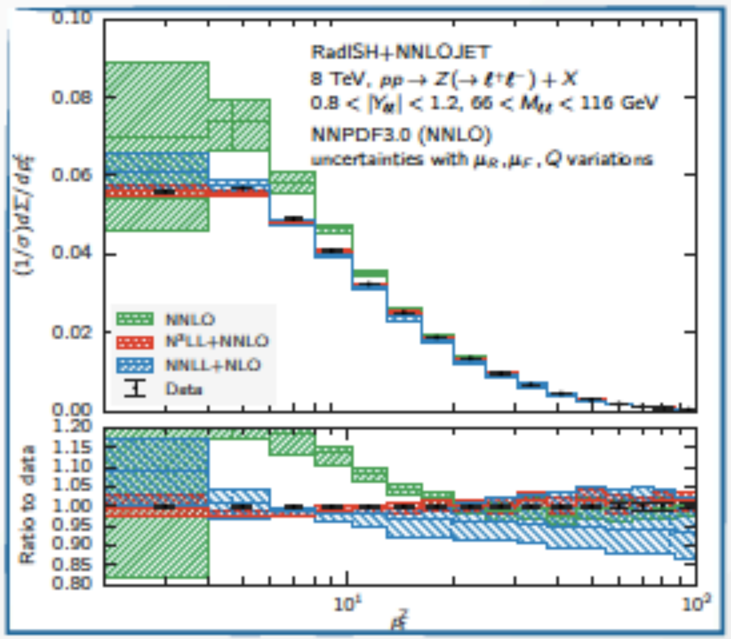
Very precise measurement of Z p_T has been posing problems to theory from early LHC data

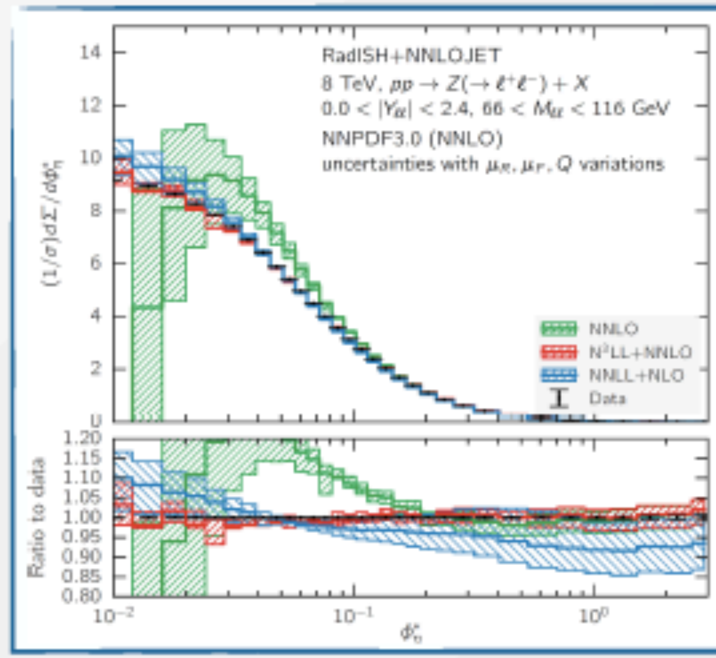
However, uncertainties below not really complete and perhaps optimistic

Results at N³LL+NNLO: 8 TeV (Z, p_t and φ^*)

[Bizon, Chen, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '18]

Data and fiducial cuts from **[ATLAS 1512.02192]** $p_t^{\ell^\pm} > 20\,\text{GeV}, \quad |\eta^{\ell^\pm}| < 2.4$





- **~7%-10% corrections** w.r.t. NNLL+NLO
- Scale uncertainties **below the 5% level**

Similar findings for the φ* angular observable

L. Rottoli

Benchmarking at level-1 (complete)

Benchmark settings: step 1

- ***Z/γ**** production at $\sqrt{s} = 13$ TeV,
- **Resummation only** (no matching to fixed order yet),
- A number of values of ***Q*** and ***y***:
  - we will mostly show results at $Q = M_Z$ and $y = 0$.
- Consider **all possible logarithmic orders**:
  - up to N^3LL.
- Favourite **Landau-pole regularisation** procedure:
  - b^*/k_T^* or "minimal prescription",
  - this is one of the main sources of (understood) differences at low q_T.
- Only **standard logs**:
  - no modified logs to enforce unitarity.
- **q_T distribution from 1 to 100 GeV**:
  - we are aware that for resummation breaks down well before,
  - benchmark exercise aimed at checking the consistency of codes/formalisms.

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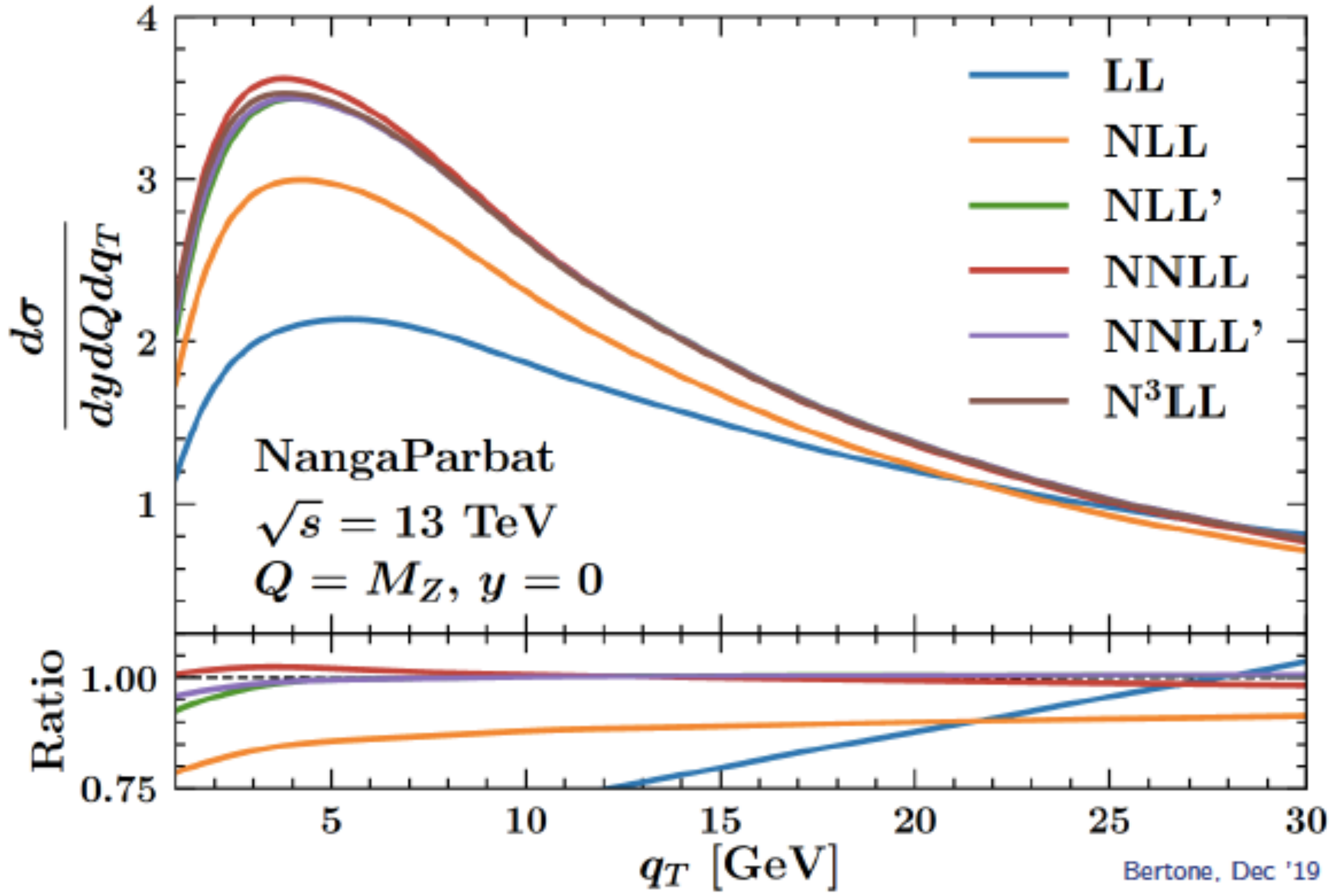
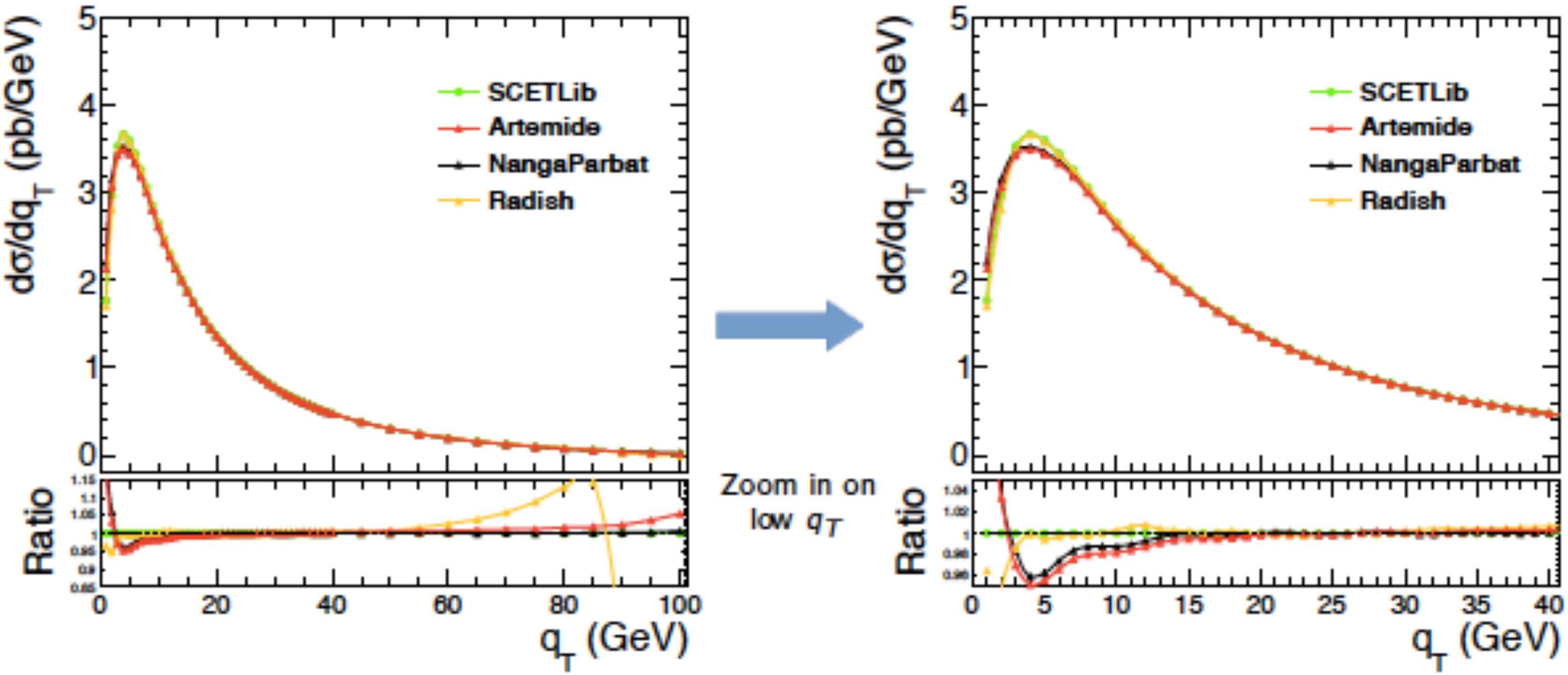
Benchmarking at level-1 (complete)

Perturbative Convergence



Benchmarking at level-1 (complete)

Level 1 - N3LL

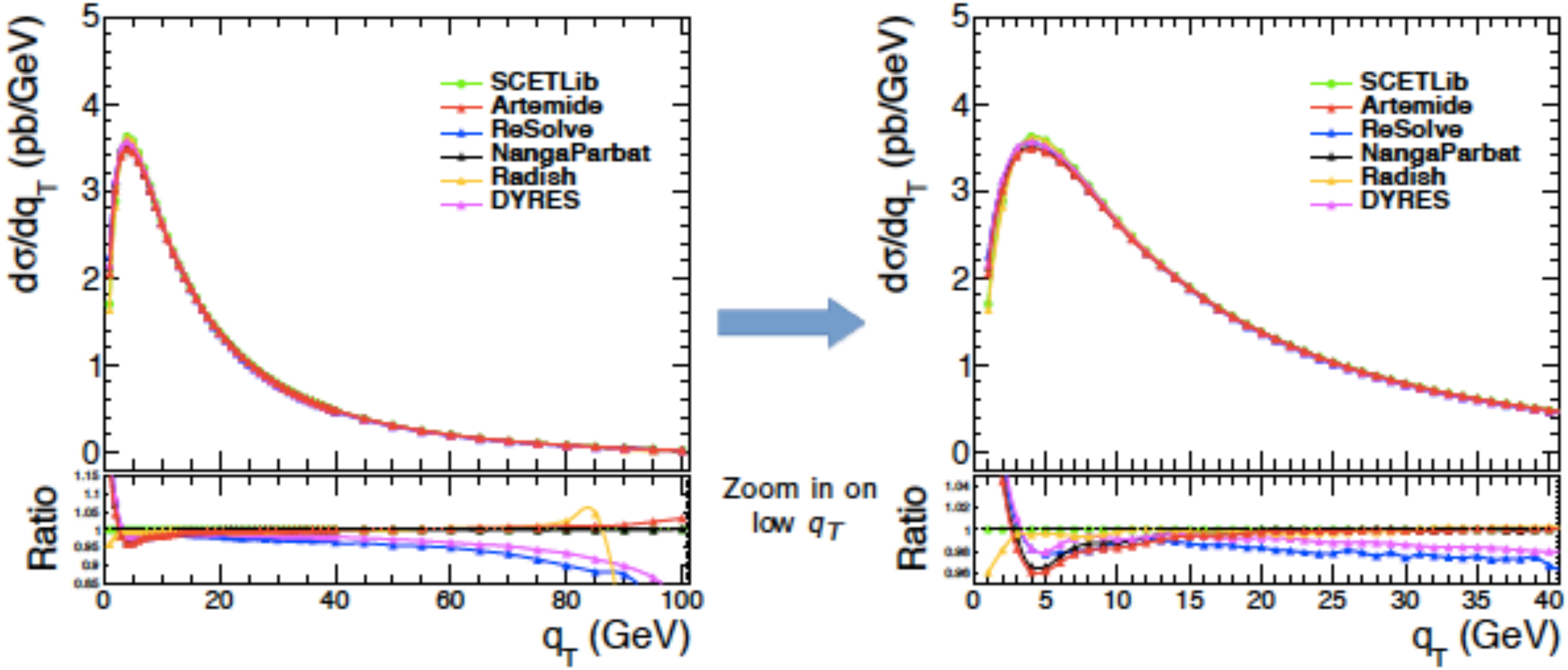


Highest order computed in benchmarking.

Remember, spectrum not physical at Level 1 (or 2) stage outside low q_T.

Benchmarking at level-1 (complete)

Level 1 - NNLL'



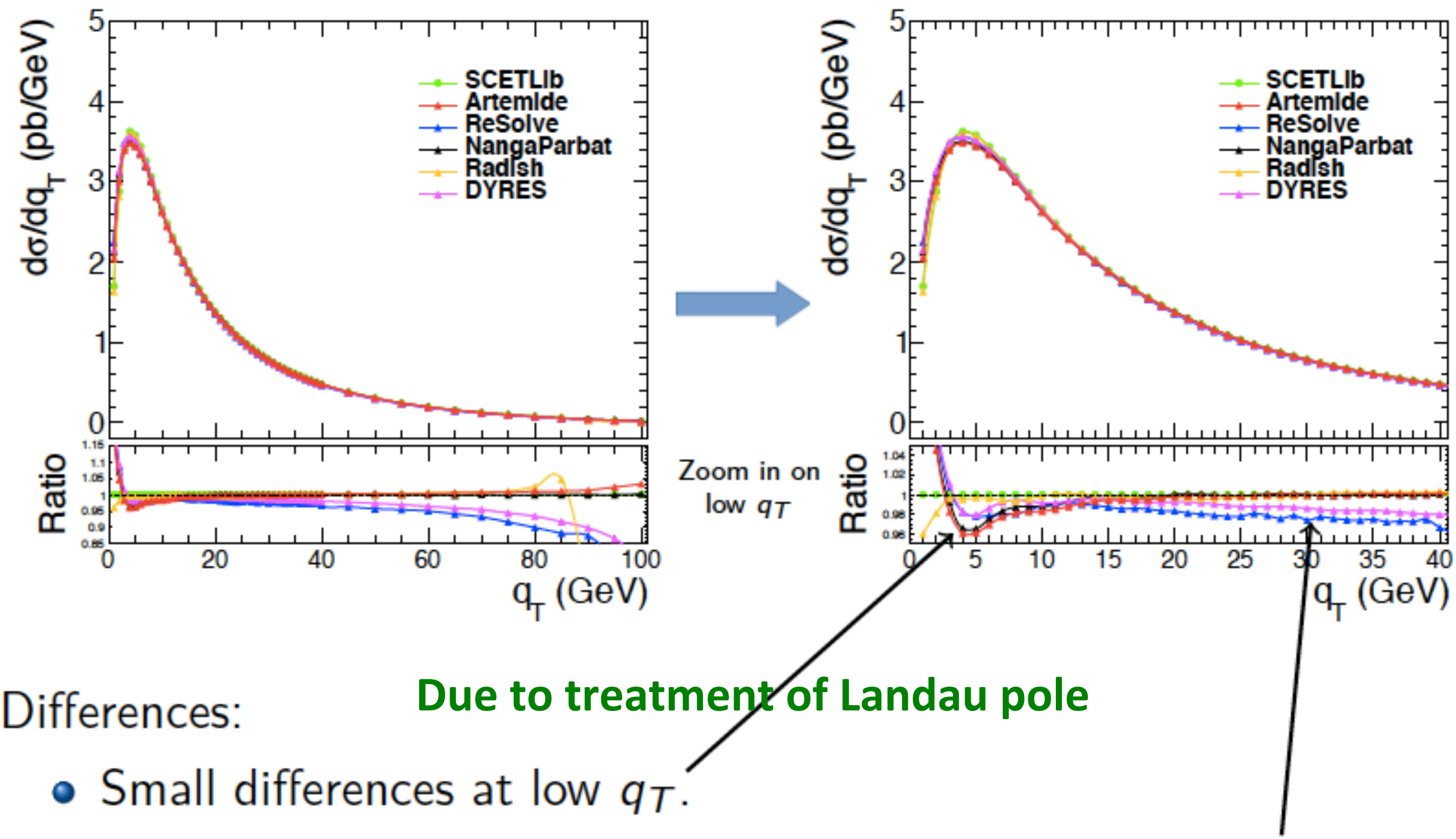
(see back-up slides for nice schematic view of difference between eg N3LL and NNLL')

Some codes only go up to NNLL' (log counting differences).

- **Two years ago, only one or two calculations to N3LL/NNLL'**
- **Now, we have six calculations at this formal accuracy and committed to pursue the benchmarking to its final goals**
  (perhaps seven with CUTE-MCFM, see talk by Tobias Neumann yesterday)

Benchmarking at level-1 (complete)

Level 1 - NNLL' differences



Differences:

- Small differences at low q_T.
- q_T resummation codes show differences at intermediate - high q_T.

Due to resummation scheme choice

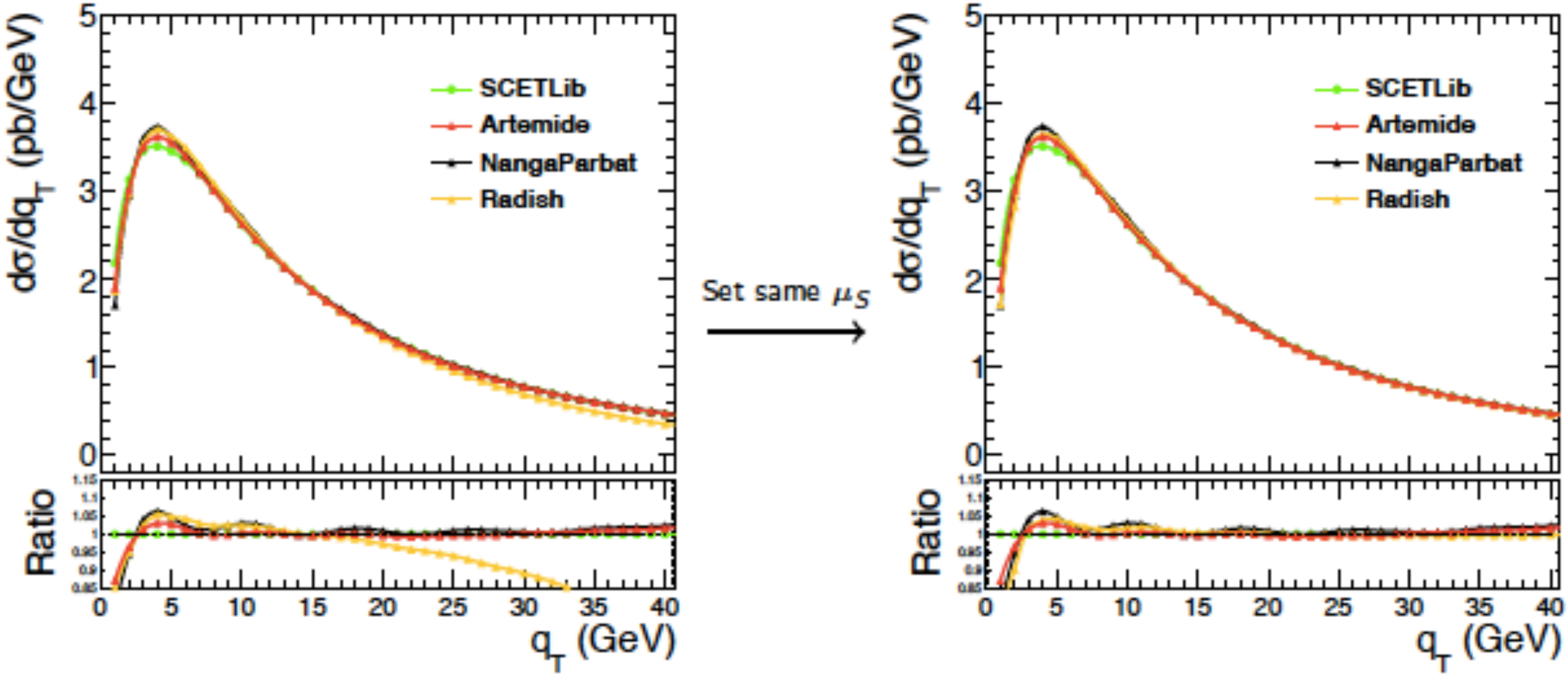
Benchmarking at level-2 (~ complete)

Level 2 Settings

- Still no non-perturbative S_{NP} factor included.
- Still no matching to finite piece - resummed piece only.
- Groups use their own *default settings* beyond this:
  - Different Landau pole regularisations, local vs global b^*, b_{lim} setting etc. $\Rightarrow$ Will affect low q_T.
  - **Nominal Modified Logs** now used $\log(Q^2 b^2) \Rightarrow \log(1 + Q^2 b^2)$, different groups have their own settings $\Rightarrow$ Will affect high q_T.
  - Choose own **scales** - e.g. resummation scale $\mu_S = Q/2, Q$, i.e. resumming $\log(1 + (m_Z)^2 b^2)$ or $\log(1 + (m_Z/2)^2 b^2)$ respectively. $\Rightarrow$ Will affect intermediate q_T most.
  - Potential inclusion of damping functions, profile scales, different modified logs, etc.

Benchmarking at level-2 (~ complete)

Level 2 - N3LL low q_T

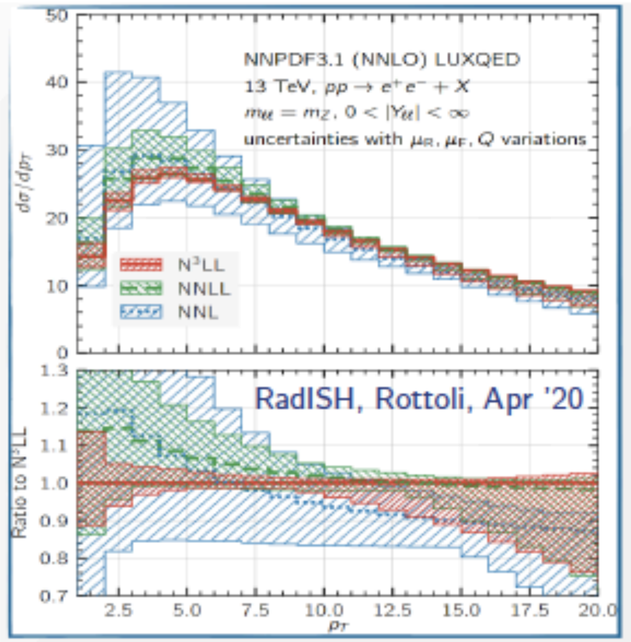


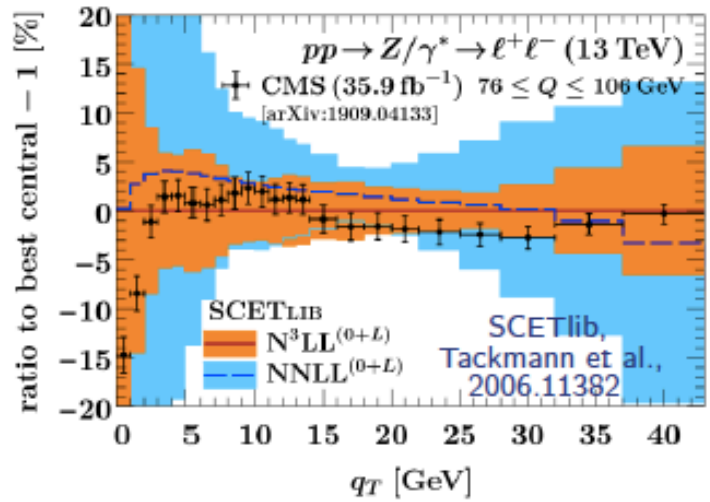
- Main difference is due to choice of central resummation scale $\mu_S = m_Z$ or $m_Z/2$, should be absorbed in matching with the finite piece (level 3).
- Differences in b^*, resummation scheme, etc as in level 1.

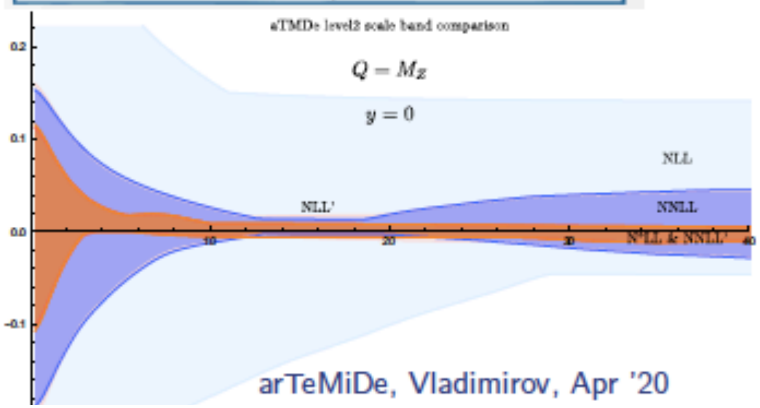
Benchmarking at level-2 (~ complete)

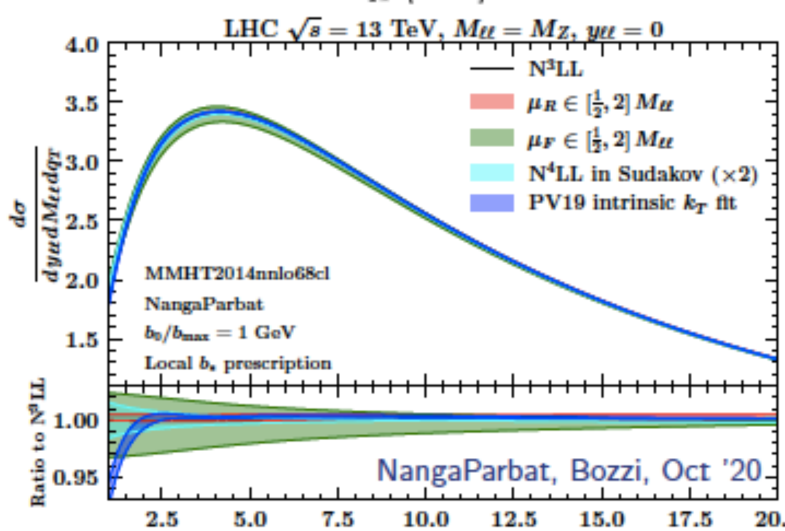
Level 2 with uncertainties

Not yet complete, an idea can be gained from separate results:







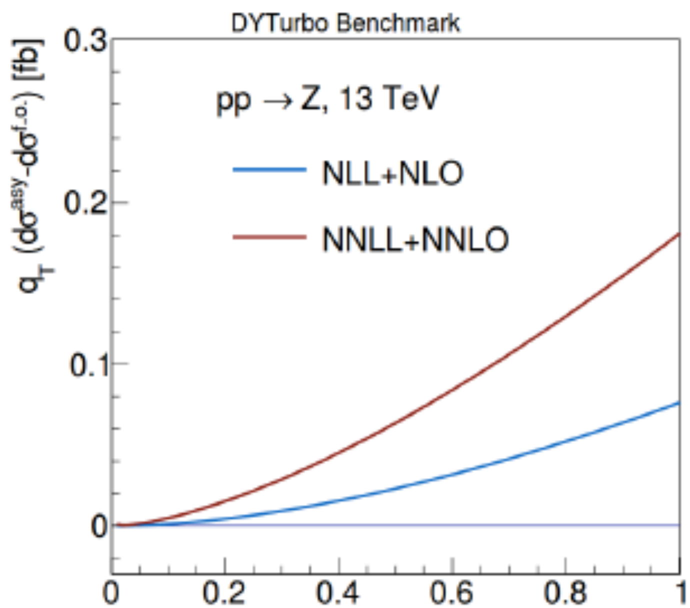


Clearly uncertainties vary widely between different calculations, quite some harmonisation required if feasible

Benchmarking at level-3 (first results)

Matching - Level 3

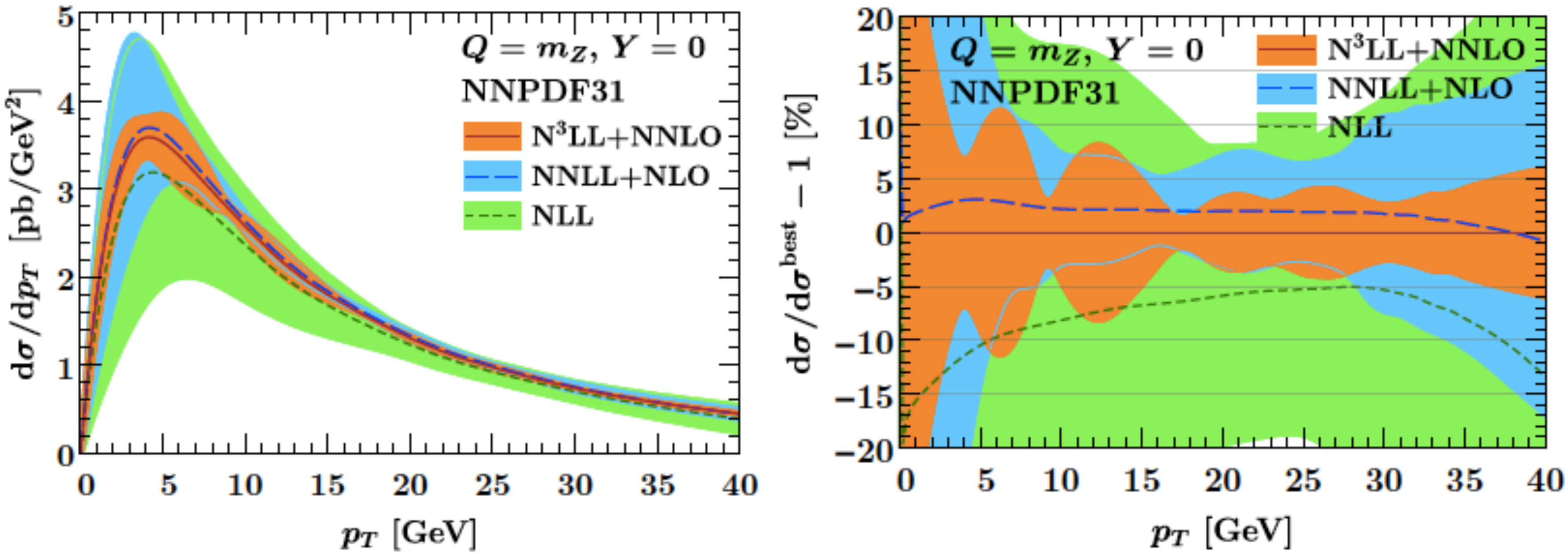
- Matched finite piece to be calculated at LO and NLO for $V+$ jet by DYTurbo to be used by the groups with their own resummed pieces and matching implementation.
- Enables Level 3 predictions where possible.
- Done for $Q = m_Z, y = 0$ point focused on.
- Renormalisation and factorisation scale variations up to factor of 2 provided.



Note: this is not state-of-the-art fixed order, but provides faster turnaround than through NNLOJET

Benchmarking at level-3 (first results)

SCETlib Level 3 with Variations (NNPDF31).



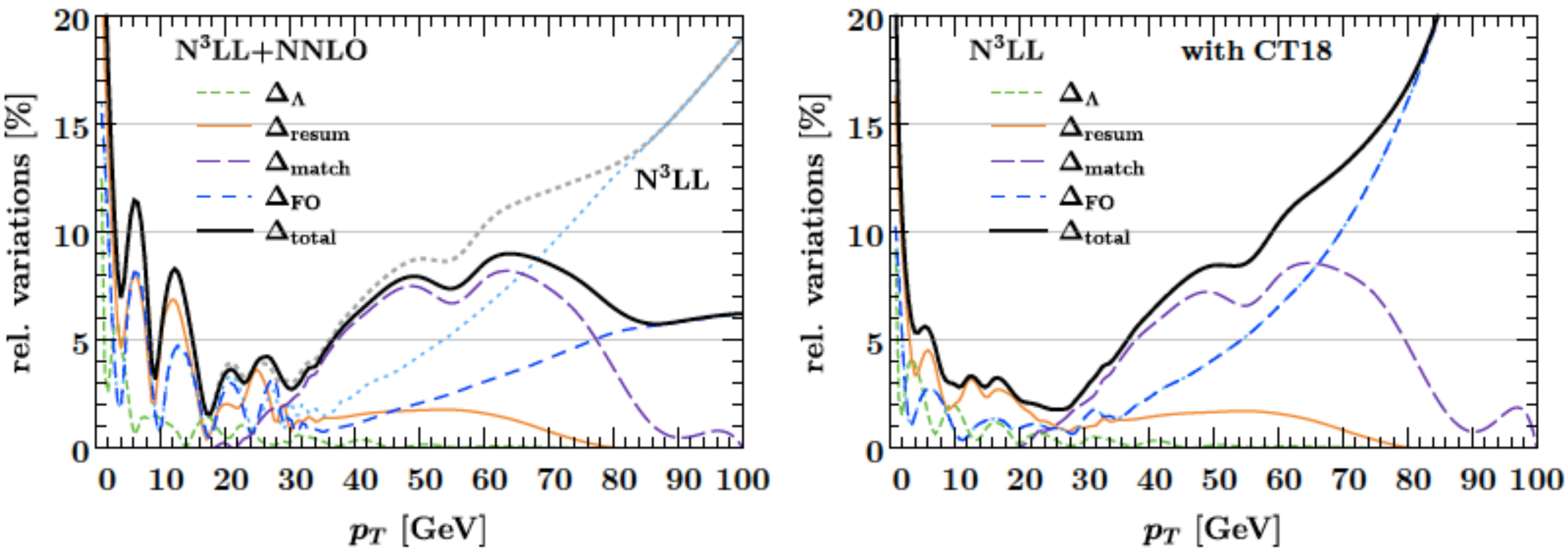
- Small kinks in PDF scale dependence (due to flavor thresholds) induces oscillations whenever a scale variation effectively changes the scale at which PDF is evaluated
  - Unfortunately, this seems to be exacerbated by our default choice of NNPDF31
  - Does not happen (at same level) with CT18

F. Tackmann

Benchmarking at level-3 (first results)

Breakdown of Variations: N^{3}LL+NNLO.

F. Tackmann



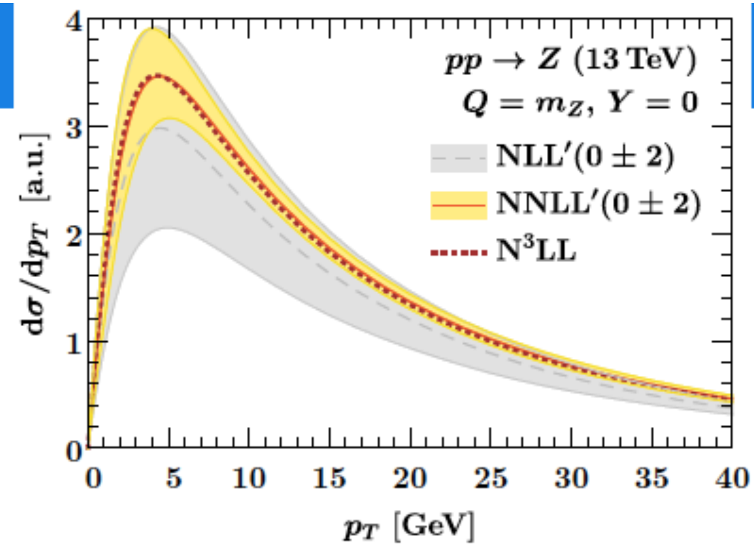
- **Hierarchy of sources is as expected but:**
  - **Matching variations are dominant and large between 30 and 60 GeV**
  - **Resummation variations are much larger at low qT than those obtained from eg TMDs. Without including NP physics, this seems much more credible to me ☺**

Theory nuisance parameters: a dream for experiments!?

$Z\ p_T$ Spectrum.

Orders

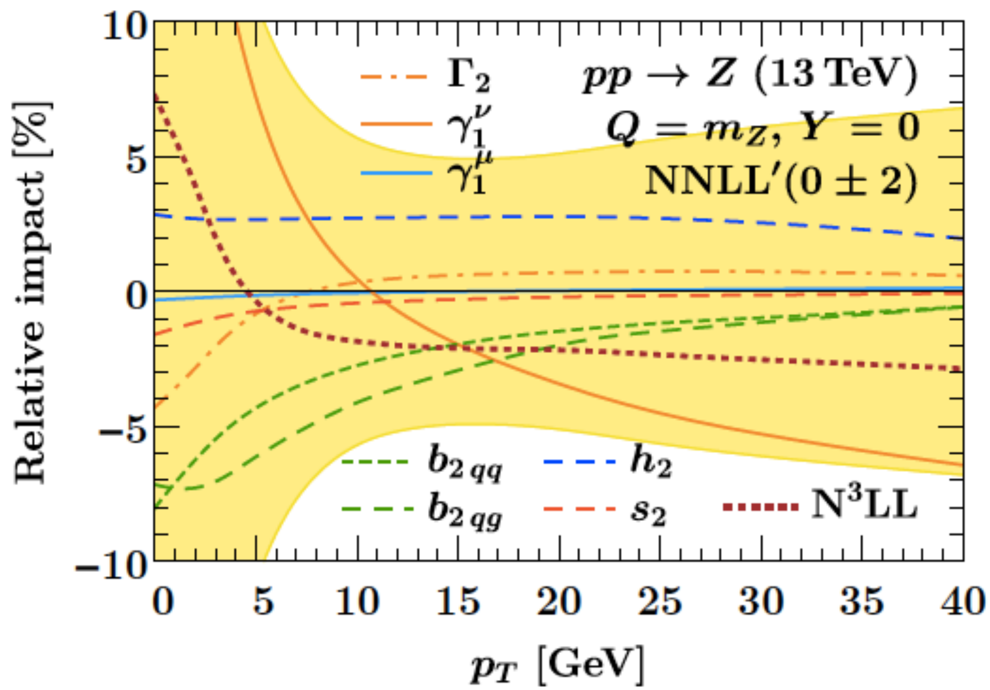
- $\mathrm{NLL}'(0 \pm 2)$ (level 2)
- $\mathrm{NLL}'(1 \pm 0.25)$ (level 1)
- $\mathrm{NNLL}'(0 \pm 2)$ (level 2)



Relative impact of different nuisance parameters

- h_2
- γ_1^μ
- b_2: $q \to q$, $g \to q$
- Γ_2
- γ_1^ν
- s_2

F. Tackmann



Frank Tackmann (DESY) | Theory Uncertainties and Correlations. | 2019-09-13 | 13 / 21

Theory nuisance parameters: a dream for experiments!?

One can verify the validity of "generic" choices of range of variations for unknown nuisance parameters

F. Tackmann

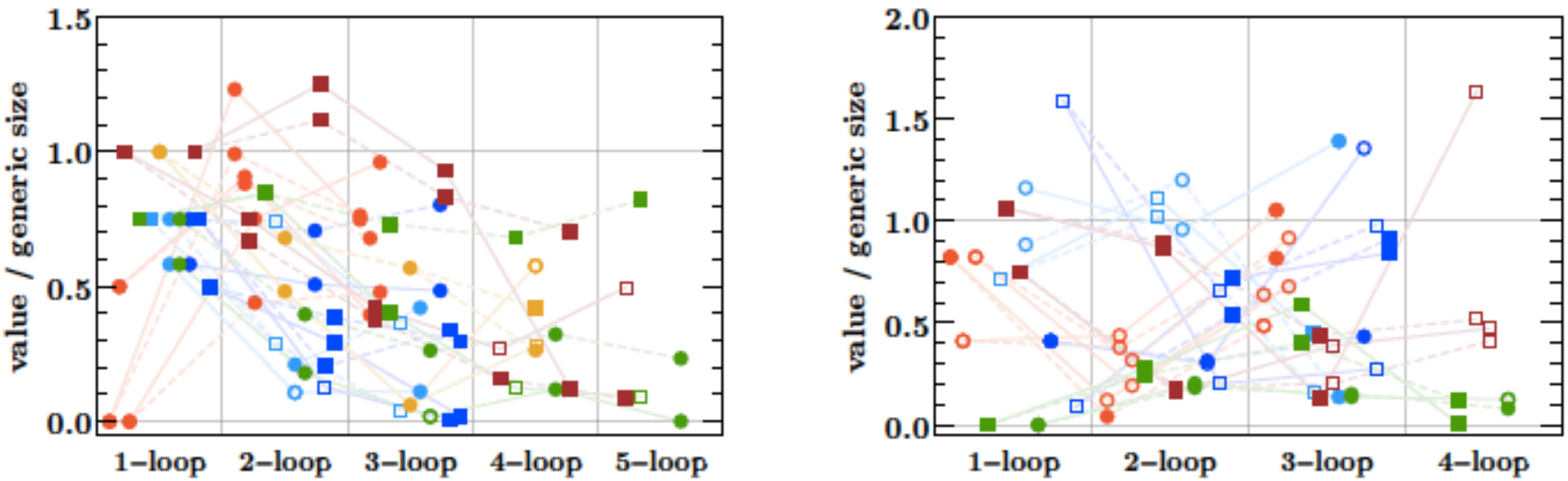


Figure 4: Known parameters for several anomalous dimensions (left) and boundary conditions (right). Parameters from the same perturbative series are connected by lines and are shown for $n_f = 3$ (dashed) and $n_f = 6$ (solid). Filled (open) markers represent positive (negative) sign. Each parameter is normalized to its expected generic size, which is obtained only from its overall color factor, loop order, leading N_c and n_f dependence.

- **It is beyond my competence to compare the uncertainty bands at eg NNLL' for nuisance parameters with those at eg N3LL for the scale variations in SCETLIB. They are of the same order at least ☺**
- **I wish to stress that as experimentalists we would dream of such a nuisance parameter approach for all feasible theoretical calculations for precision measurements at the LHC**
- **However, anomalous dimensions and cusps etc are not all universal ☹**

Fitting NP physics to data: a forlorn hope?

Level 3.5? - Non-perturbative factors:

- For description at low q_T need to include a non-perturbative contribution - S_{NP}, this is an exponential in b, x, Q_0 typically.
- Have both intrinsic transverse momentum dependence of initial states (boundary condition for TMDs) and non-perturbative contributions to evolution.
- This is modelled by fits to relevant data for low values of q_T/Q - e.g. Older Fermilab data, Tevatron data and new LHC DY data.
- NangaParbat - 9 parameter fit $(\lambda, N_1, \sigma, \alpha, N_{1B}, \sigma_B, \alpha_B, g_2, g_{2B})$.
- Can also fit along with SIDIS data, but need also high Q^2, **arTeMiDe** have used HERMES and COMPASS data as well as DY data.

  Scimemi et al., '19, '20
- Perhaps do a pseudodata fit?

Bacchetta et al., '19

$$f_{NP}(x, b_T, \zeta) = \left[\frac{1-\lambda}{1+g_1(x)\frac{b_T^2}{4}} + \lambda \exp\left(-g_{1B}(x)\frac{b_T^2}{4}\right)\right]$$

"intrinsic" NP contribution (x- and b_T-dependent)

Bertone, Mar '20

$$\times \exp\left[-\left(g_2 + g_{2B} b_T^2\right) \ln\left(\frac{\zeta}{Q_0^2}\right)\frac{b_T^2}{4}\right],$$

NP correction to pert. evolution (b_T-dependent)

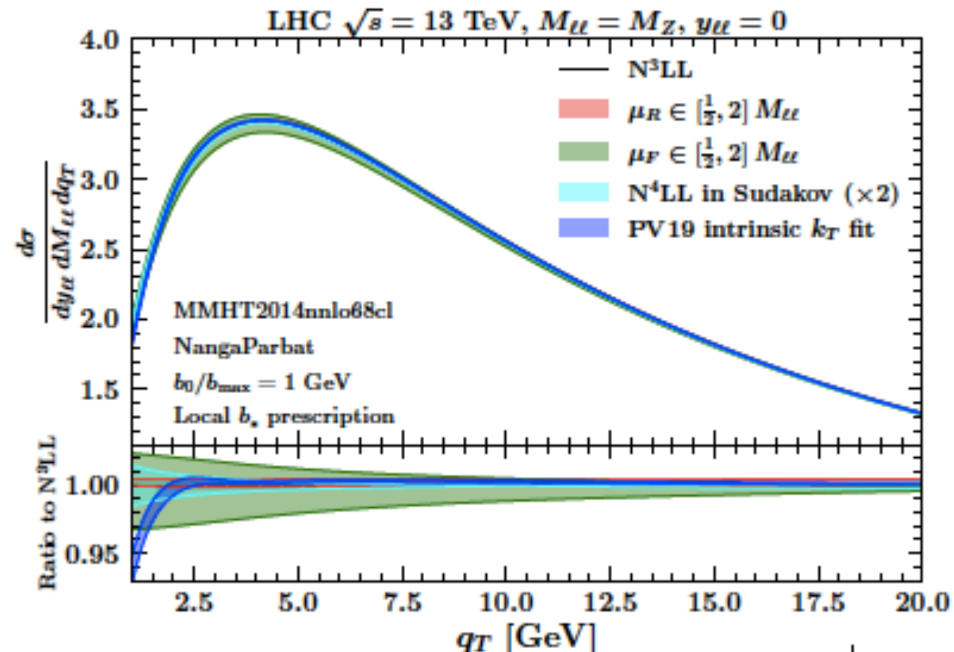
$$g_1(x) = \frac{N_1}{x\sigma}\exp\left[-\frac{1}{2\sigma^2}\ln^2\left(\frac{x}{\alpha}\right)\right],$$

$$g_{1B}(x) = \frac{N_{1B}}{x\sigma_B}\exp\left[-\frac{1}{2\sigma_B^2}\ln^2\left(\frac{x}{\alpha_B}\right)\right]$$

x-dep. width of TMDs

Fitting NP physics to data: a forlorn hope?

Non-perturbative fits

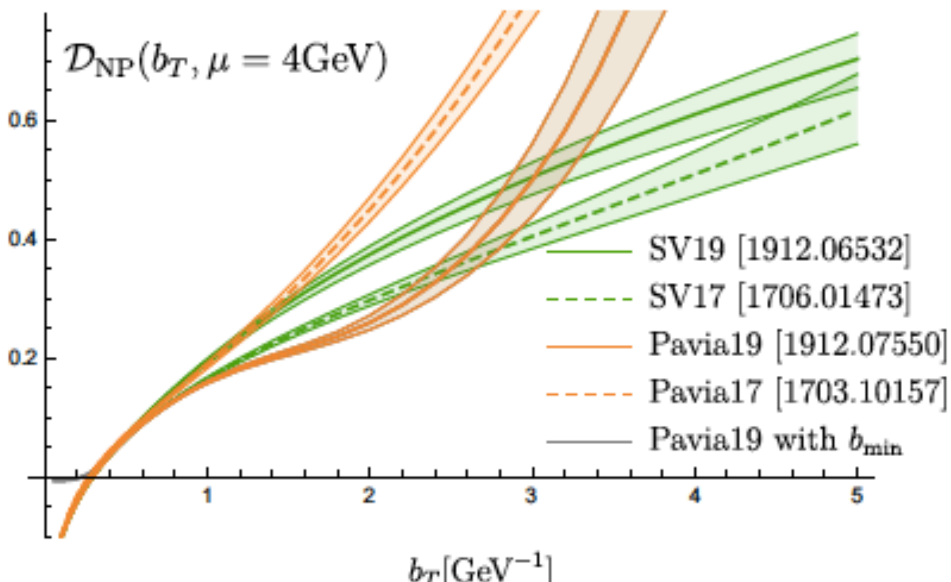


- Non-perturbative effects low q_T end of the spectrum.
- PV19 fit by `NangaParbat` shows potential sensitivity to these effects.

NangaParbat, Bozzi Oct '20
Bacchetta et al., 1912.07550

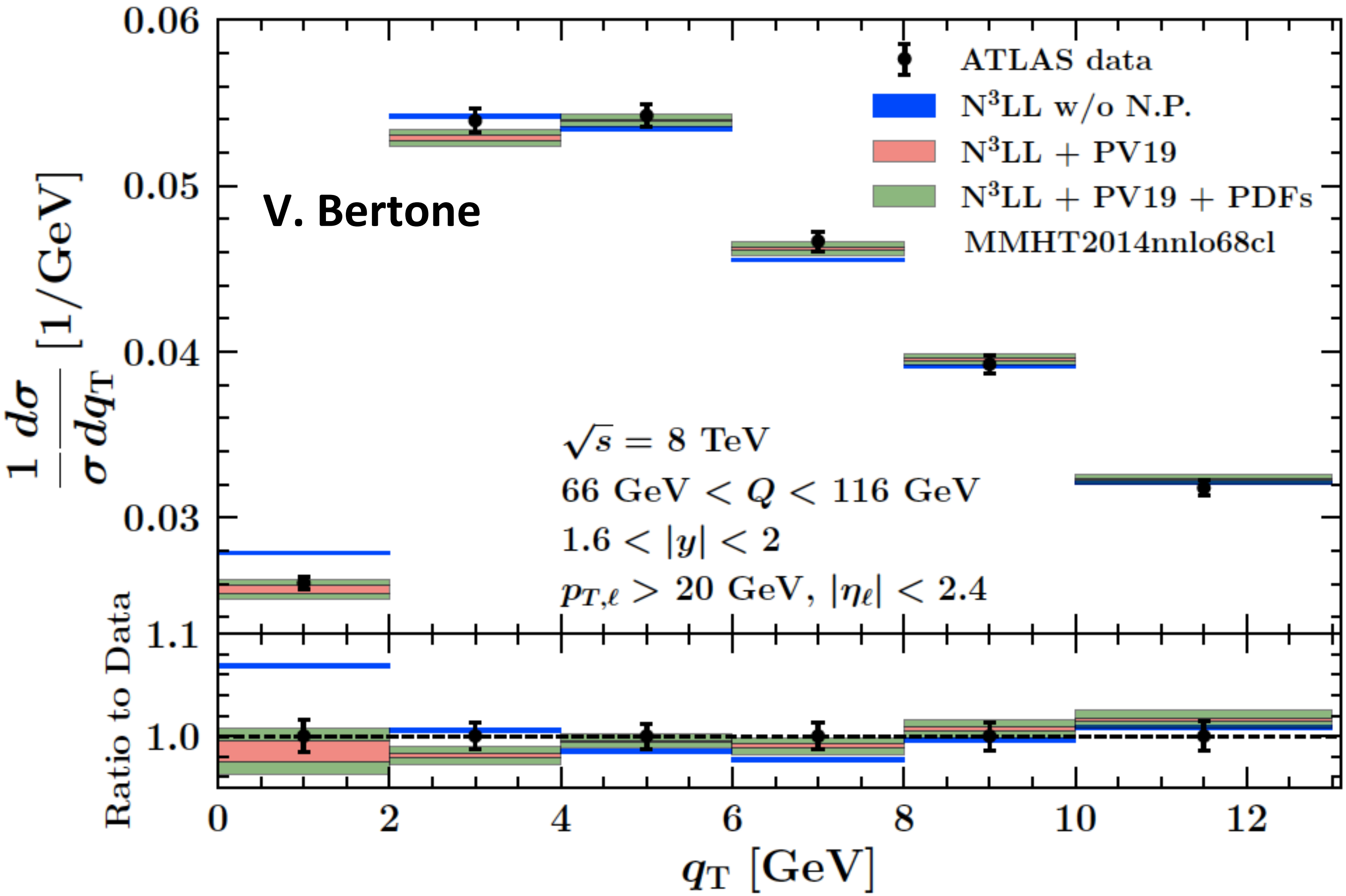
- SV19 fit by `arTeMiDe` results in non-perturbative factor $\mathcal{D}_{NP}$.
- Shows effect of inclusion of SIDIS data on top of Drell-Yan.

arTeMiDe, Vladimirov, 2003.02288



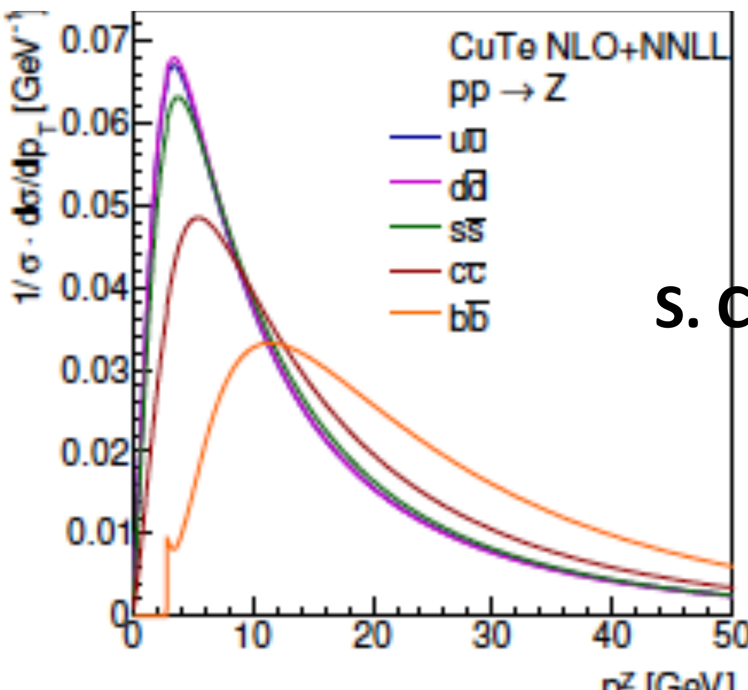
Fitting NP physics to data: a forlorn hope?

- PDF uncertainties at low qT are not negligible (example kindly provided by NangaParbat below (PDF unc. double-counted however)!
- As shown by Ignazio Scimemi yesterday, the spread between PDFs is not small

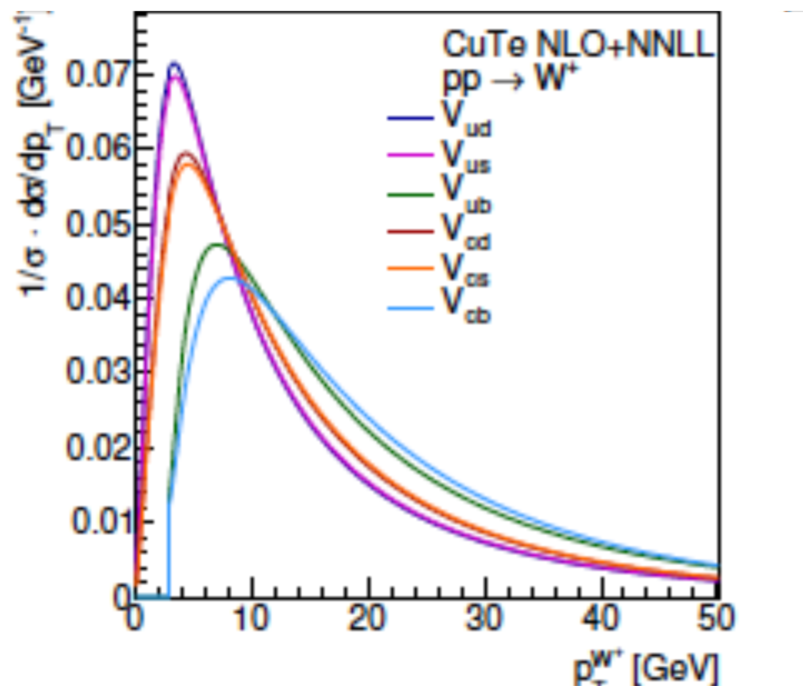


Fitting NP physics to data: a forlorn hope?

- PDF uncertainties discussed above need to be treated as is done most often by experiments, eg by adding in quadrature to the uncertainties of a chosen baseline PDF set, the spread given by the envelope of the central values of the three or four global PDF sets which should be considered (don't forget ABMP and please don't use PDF4LHC which is of no help at all in evaluating any of these issues!)
- There is however another source of uncertainties at low qT which cannot be swept under the carpet, namely that of the heavy quark mass thresholds. This is particularly important if one wishes to produce a ratio of pTW/pTZ with at least an attempt at decorrelating certain uncertainties



S. Camarda



Conclusions and next steps

- **Resummation benchmarking between many different calculations is converging for the first time and we have all learned a lot over the past two years**

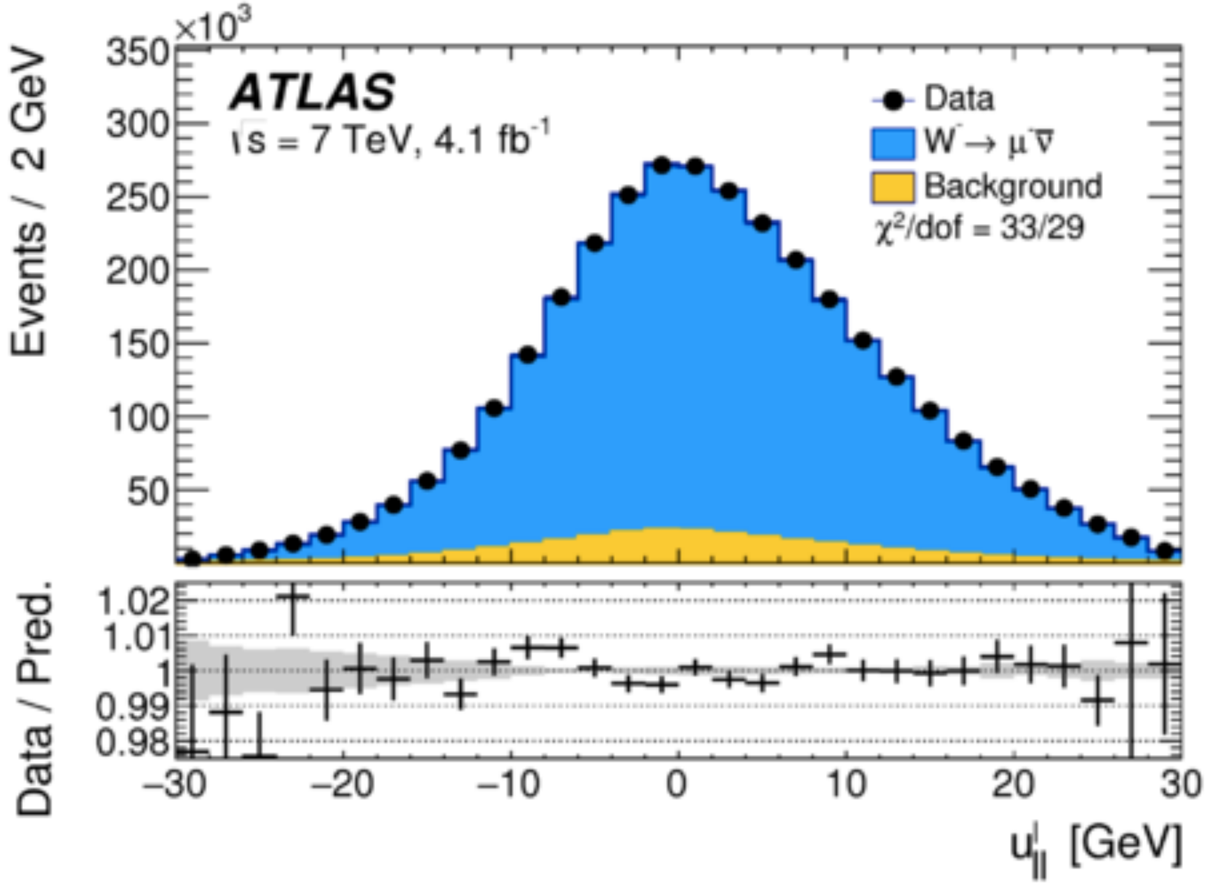
- **Over next year, need to converge on a publication (plus Yellow Report)**

- **Many issues will require further work and are actually being worked on in parallel (eg NP physics, heavy quark thresholds)**

- **For experimentalists, what is needed ideally for putting LHC precision EW physics on the map of electroweak fits with a proper treatment of uncertainties compared eg to the legacy from LEP is:**
  1) Predictions with nuisance parameter-style uncertainties for the qT spectrum of Z and W and more importantly even of their ratio
  2) A quantitative measurement of correlations between global PDF fits (work in progress with however some resistance from certain PDF groups)
  3) Probably N3LO+N3LL for DY, if possible in MC programme ☺

Back-up slides

Control of p_T^W modelling : $u_{||}^e$, $u_{||}^\mu$

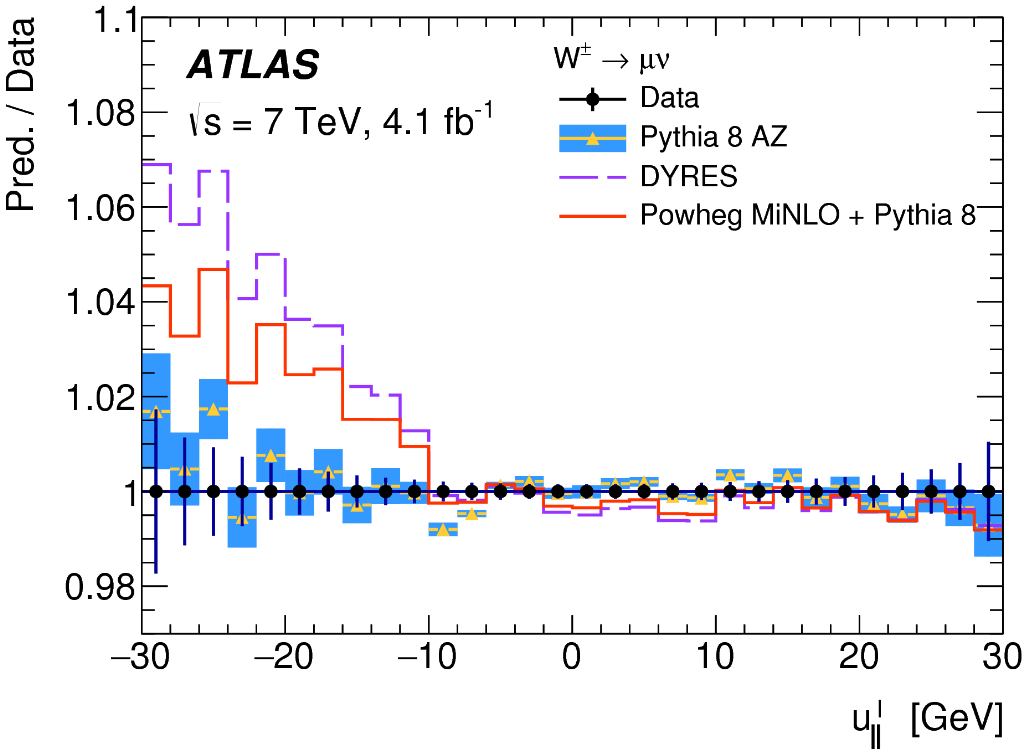


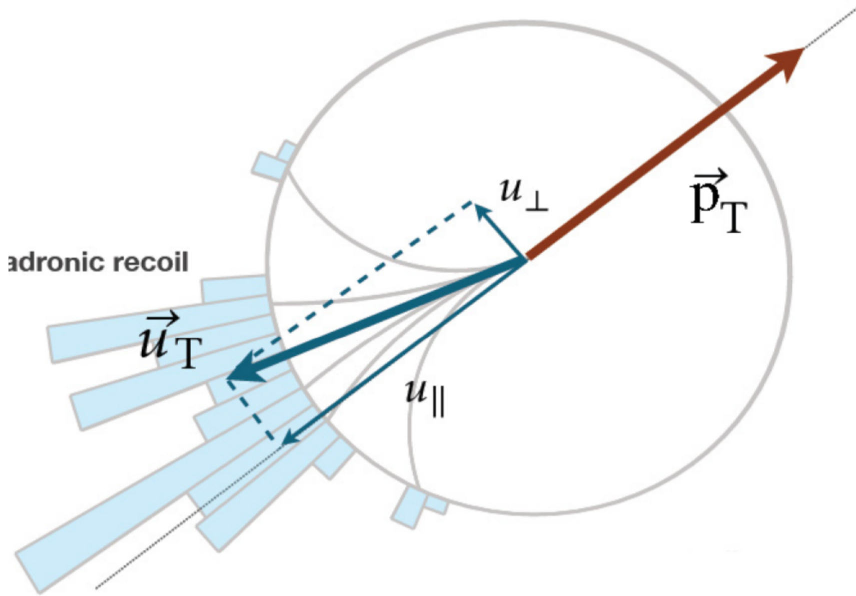
The region $u_{//}^{l} < -10$ GeV is sensitive to the physics modelling of the soft part of the p_T^W spectrum

With a total of e.g. ~ 0.8M W to $\mu\nu$ decays, one can constrain modelling uncertainties to ~ 10 MeV

Control of p_T^W modelling : $u_{||}^e$, $u_{||}^\mu$

- The $u_{||}^l$ distribution is very sensitive to the underlying p_T^W distribution, for $u_{||}^l < 0$. This feature can be exploited, even in a high pile-up environment to verify the accuracy of the baseline model, and to compare to alternative (more state-of-the-art?) models





Pythia 8 tuned to Z OK; DYRES, Powheg MiNLO disfavoured

Logarithmic counting

V. Bertone

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{res.}} \overset{\text{TMD}}{=} \sigma_0 H(Q) \int d^2 \mathrm{b}_T e^{i\mathrm{b}_T\cdot \mathrm{q}_T} F_1(x_1, \mathrm{b}_T, Q, Q^2) F_2(x_2, \mathrm{b}_T, Q, Q^2)$$

$$F_f(x, \mathrm{b}_T, \mu, \zeta) = \sum_j C_{f/j}(c, b_T; \mu_b, \zeta) \otimes f_j(x, \mu_b) \times \exp\left\{ K(b_T, \mu_b) \ln \frac{\sqrt{\zeta}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\}$$

Accuracy	γ_K	γ_F	K	$C_{f/j}$	H
LL	α_s	-	-	1	1
NLL	α_s^2	α_s	α_s	1	1
NLL'	α_s^2	α_s	α_s	α_s	α_s
N²LL	α_s^3	α_s^2	α_s^2	α_s	α_s
N²LL'	α_s^3	α_s^2	α_s^2	α_s^2	α_s^2
N³LL	α_s^4	α_s^3	α_s^3	α_s^2	α_s^2
N³LL'	α_s^4	α_s^3	α_s^3	α_s^3	α_s^3

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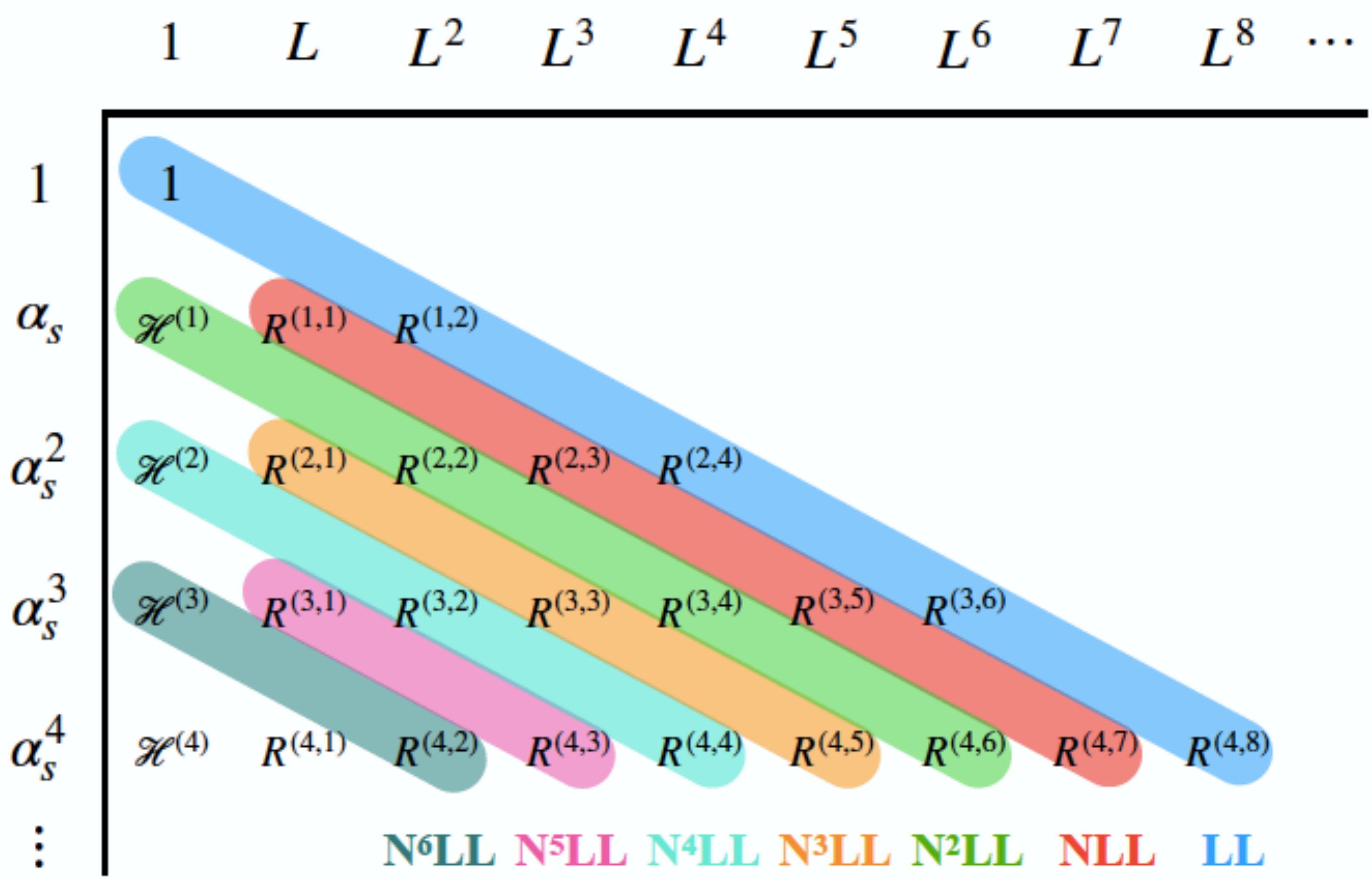
Logarithmic counting (1)

V. Bertone

$$\frac{d\sigma}{dq_T} \propto \left(1 + \sum_{m=1}^{\infty} \alpha_s^m \mathcal{H}^{(m)}\right)\left(1 + \sum_{n=1}^{\infty} \alpha_s^n \sum_{k=1}^{2n} R^{(n,k)} L^k\right)$$

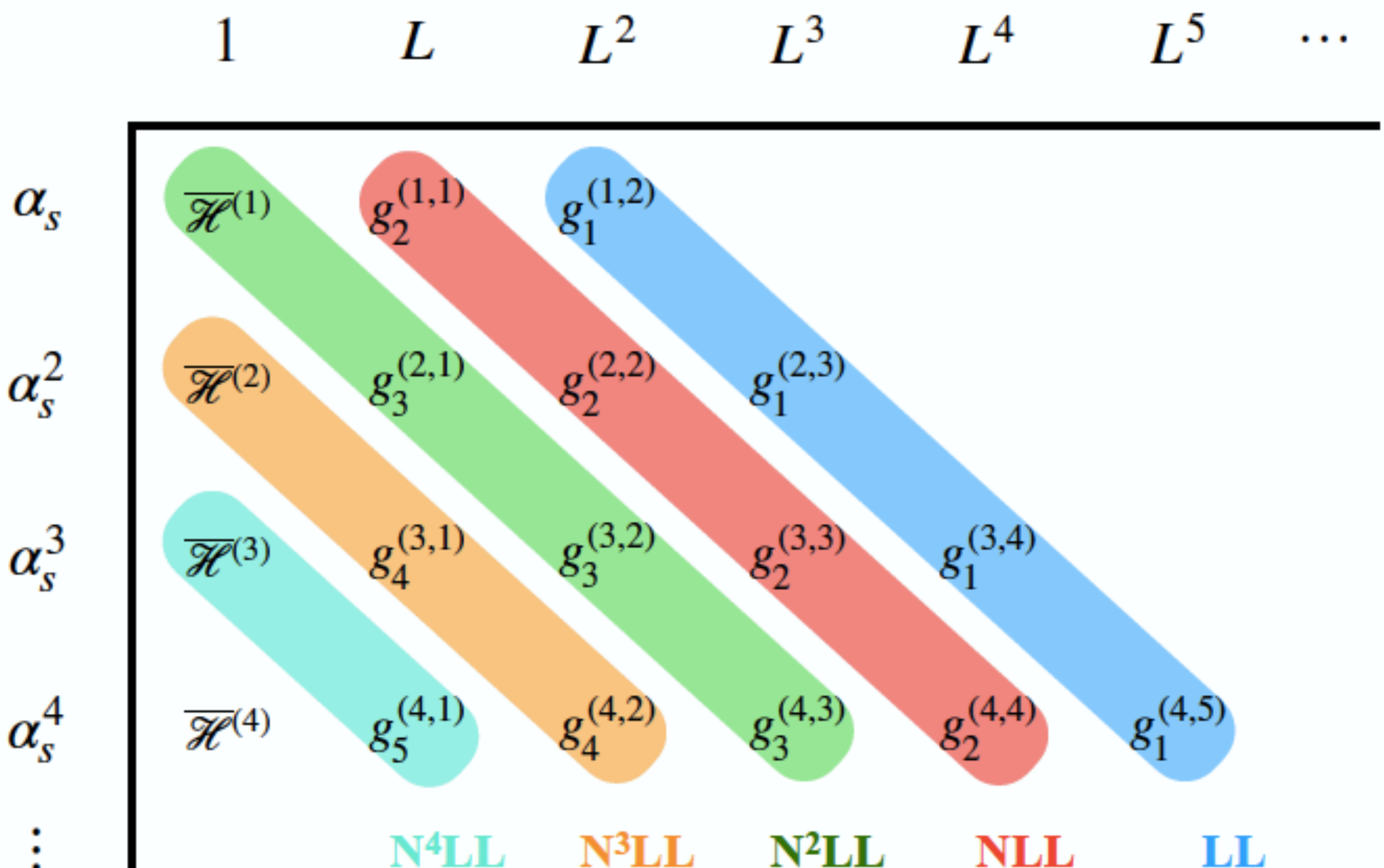
$\alpha_S L^2 \sim 1$



Logarithmic counting (2)

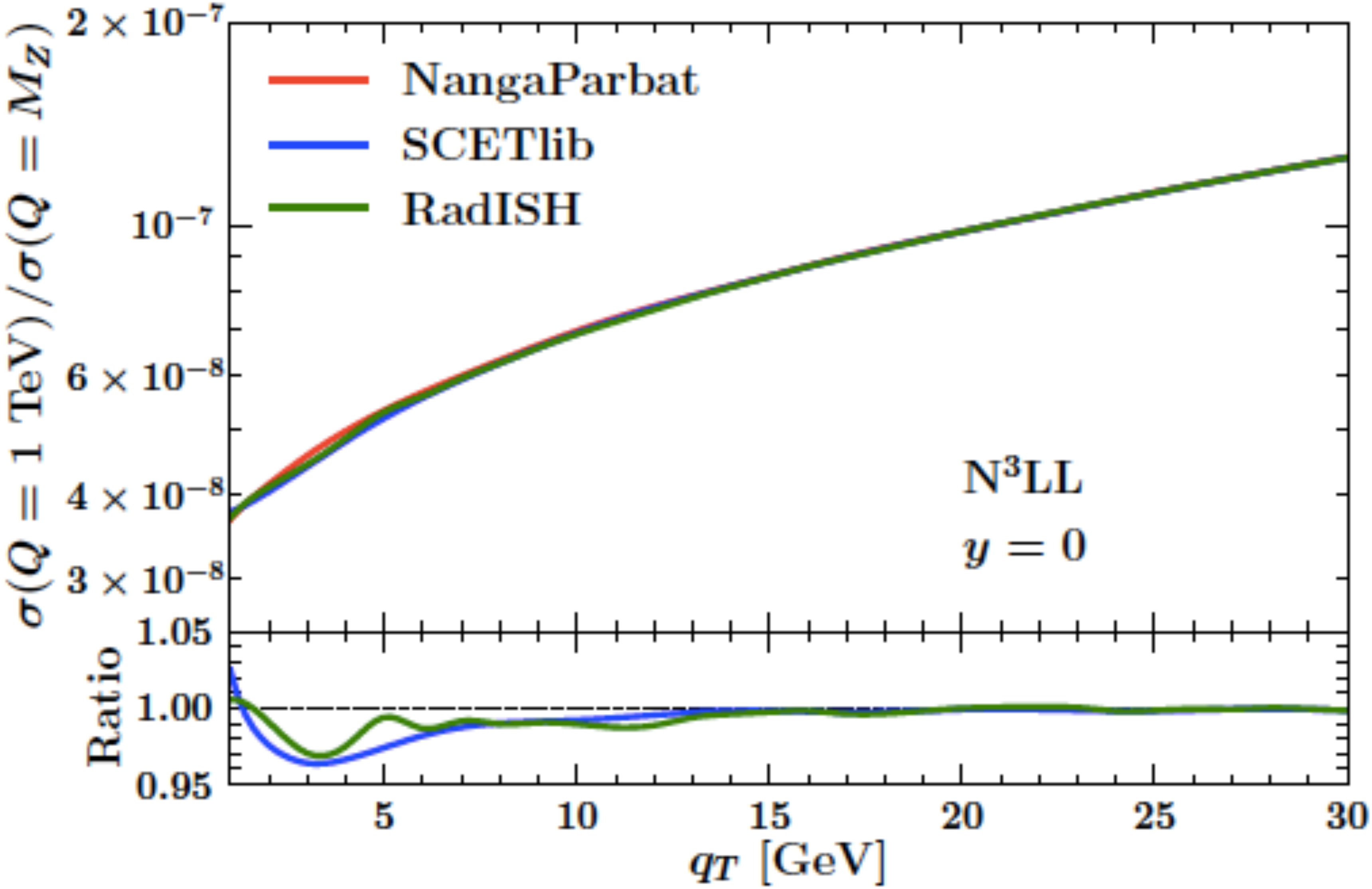
V. Bertone

$$\ln\left(\frac{d\sigma}{dq_T}\right) \propto \ln\left(1+\sum_{m=1}^{\infty}\alpha_s^m \mathcal{H}^{(m)}\right) + \sum_{n=1}^{\infty}\alpha_s^n \sum_{k=1}^{n+1} g^{(n,k)} L^k \qquad \alpha_S L \sim 1$$

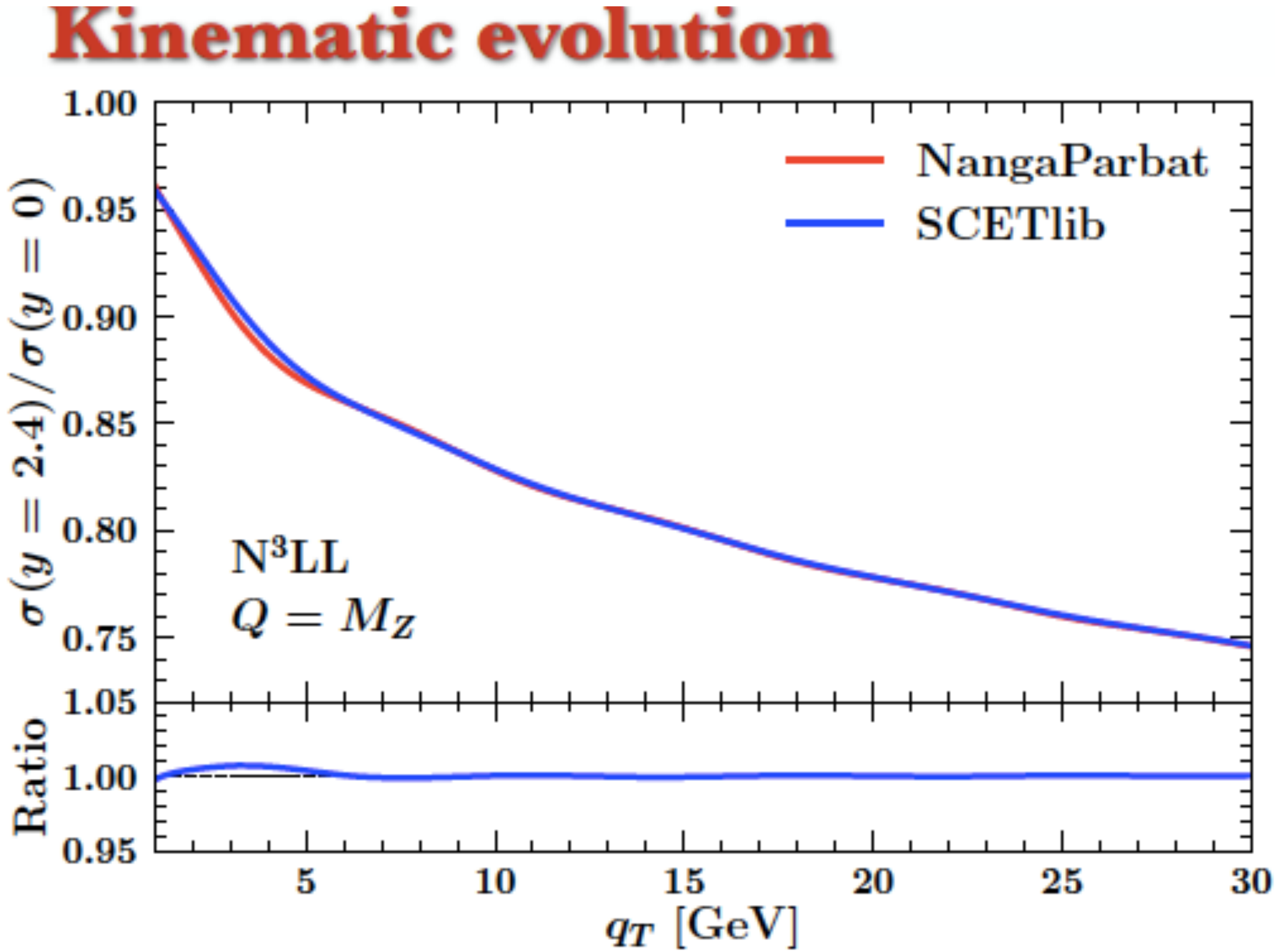


Benchmarking at level-1 (complete)

Kinematic evolution



Benchmarking at level-1 (complete)



Benchmarking at level-2 (~ complete)

Scales

T. Cridge

- As expected, more differences at Level 2 as codes using (largely with some exceptions) default settings.
- Several sources of theoretical uncertainties, many of which can be probed by scale variations.
- However there are different scales in different formalisms and implementations:
  - `reSolve`, `DYRes`, `RadISH` have factorisation (μ_F), renormalisation (μ_R) and resummation scale (μ_S) uncertainties all set around the hard scale:

$$\log(Q^2 b^2) = \log(\mu_S^2 b^2) + \log(Q^2/\mu_S^2)$$

  - Usually do a 9-point variation where μ_F and μ_R are varied together and μ_S separately, *envelope then taken as scale variation band*:

$$(\mu_R/Q, \mu_F/Q, \mu_S/Q) = (0.5, 0.5, 1), (0.5, 1, 1), (1, 0.5, 1), (1, 1, 1), (1, 2, 1), (2, 1, 1), (2, 2, 1), (1, 1, 0.5), (1, 1, 2).$$

Benchmarking at level-2 (~ complete)

Scales

T. Cridge

- In TMD factorisation you have 2 pairs scales $(\mu, \zeta) \to (\mu_0, \zeta_0)$ - initial and final scales for 2D evolution:
  - Ultimately 2 of these are varied:
    - $\Rightarrow$ 1 at high scale - related to renormalisation scale in CSS language.
    - $\Rightarrow$ 1 at low scale - related to PDFs which in TMDs are computed exactly at low scales here.
    - $\Rightarrow$ No resummation scale.
  - Differences in exact setup between `NangaParbat` and `arTeMiDe`.
- In `SCETlib` - again 2 starting scales (μ_i, ν_i) and 2 ending scales (μ, ν), can in theory be set separately for the Hard, Beam and Soft functions (although not ν for H), central choice:

$$\mu_H = Q, \qquad \mu_B = b_0/b, \qquad \nu_B = Q, \qquad \mu_S = \nu_S = b_0/b.$$

- "Profile scales" are used to switch μ_B, μ_S, ν_S between the resummed (b_0/b) at low q_T and fixed order (Q) at high q_T.
- 36 profile scale variations in relevant log ratios by a factor of 2 (not 4) $+$ fixed order scale Q varied by factor of 2.

Scale variations in SCETLIB

Matching to Fixed Order.

$$\mathrm{d}\sigma = \underbrace{\mathrm{d}\sigma^{(0)}(\mu_H, \mu_B, \nu_B, \mu_S, \nu_S)} + \underbrace{\left[\mathrm{d}\sigma^{\mathrm{FO}}(\mu_{\mathrm{FO}}) - \mathrm{d}\sigma^{(0)}(\mu_i, \nu_i \equiv \mu_{\mathrm{FO}})\right]}$$

$$\equiv \mathrm{d}\sigma^{\mathrm{resum}}(\mu_H, \mu_B, \nu_B, \mu_S, \nu_S) + \mathrm{d}\sigma^{\mathrm{nons}}(\mu_{\mathrm{FO}})$$

F. Tackmann

- σ^{resum} and σ^{nons} are *separately* scale independent (args show residual dep.)
  - In particular, σ^{nons} has no dependence (not even residual) on resum. scales
- Condition for $p_T \ll Q$: σ^{nons} must be power-suppressed by p_T/Q
  - $\mathrm{d}\sigma^{(0)}$ must exactly cancel all singular terms $\mathrm{d}\sigma^{\mathrm{FO}}$
- Condition for $p_T \sim Q$: Reproduce correct FO result $\mathrm{d}\sigma^{\mathrm{FO}}$
  - $\sigma^{\mathrm{resum}} - \mathrm{d}\sigma^{(0)}$ must vanish *exactly*
    (i.e. their difference must not introduce higher-order corrections, because in general they would be unphysical and can be arbitrarily large)
  - Guaranteed by profile scales $\mu_i, \nu_i \to \mu_{\mathrm{FO}}$ for $p_T \to Q$

⇒ Satisfying both requires consistent resummation and fixed orders
  - E.g. N^{3}LL+NLO or NLL+NNLO can only satisfy one or the other

Scale variations in SCETLIB

Profile Scales.

- Everything determined (only) by μ_i, ν_i choices: Use *profile scales*
  [for details see Lustermans, Michel, FT, Waalewijn, 1901.03331; Ebert, Michel, Stewart, FT 2006.11382]

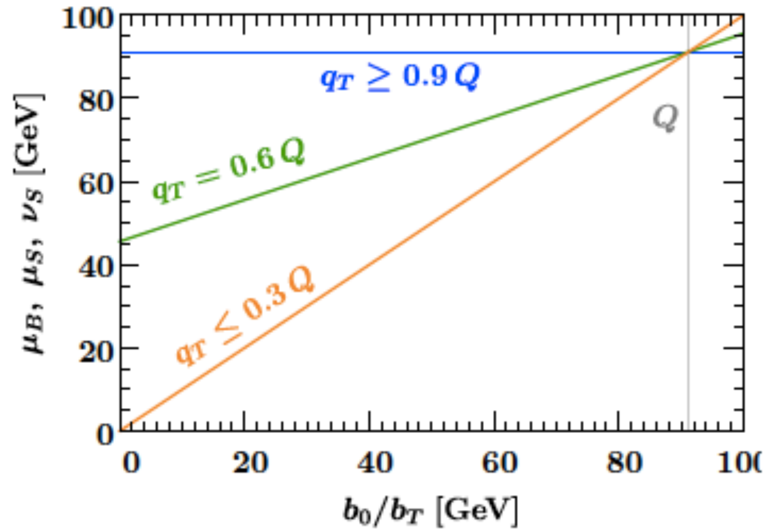
F. Tackmann

$$\mu_H = \nu_B = \mu_{\rm FO} = Q$$

$$\mu_B, \mu_S, \nu_S \equiv \mu_{\rm prof}(q_T, b_T) = \mu_{\rm FO} f_{\rm prof}\Big(\frac{q_T}{Q}, \frac{b_0}{b_T Q}\Big) \begin{cases} = b_0/b_T & q_T \ll Q \\ \to \mu_{\rm FO} & q_T \to Q \end{cases}$$

  - Key point: Resummation turn off for $q_T \to Q$ *does not* alter correct (canonical) resummation at $q_T \ll Q$
  - Plus nonpert. cutoff prescription for $b_0/b_T \lesssim 1\,\mathrm{GeV}$ (as at level 1) (freeze-out, local b^*, global b^*)

- Canonical (res. on) $\longrightarrow$ FO (res. off)
  - Transition driven by q_T/Q
    (b_T is just means to an end, we want to predict physical q_T spectrum not the b_T spectrum)
  - Transition points are chosen based on relative size of leading-power vs. nonsingular (power) corrections



Scale variations in SCETLIB

Scale Variations: Preliminaries.

F. Tackmann

Scales are (somewhat) arbitrary

- Scales are not physical parameters
- Residual dependence is formally of higher order and must be canceled at higher orders
  - Nonzero variation indicate (potential) uncertainty due to missing orders
- The reverse conclusion is not true
  - No (or small) variations *do not* imply no (or small) uncertainties

Things get worse in the context of resummation

- $\ln \mu_i$ dependence is always at least quadratic or higher-order polynomial
  - There are bound to be minima (and asymmetries) in variations
- There are multiple intertwined perturbative series in play
  - Different scales variations can probe the same higher-order uncertainty
  - Neither enveloping all variations nor adding all variations in quadrature necessarily makes sense

Scale variations in SCETLIB

Scale Variations.

Hoping that scale variations do give us sense of size of uncertainties

- "Resummation" Δ_{resum}: Max envelope of profile scale variations
  - 36 variations: chosen such that all possible scale ratios get probed and changed by factor 2 (but not 4) for $q_T \to 0$

F. Tackmann

- "Fixed-order" Δ_{FO}: Max envelope of μ_{FO} by factor of 2
  - Keeps all resummed scale ratios invariant
- "Matching" Δ_{match}: Max envelope of varying transition points
  - 4 variations: Start and end of transition up and down
- "Nonpert. cutoff" Δ_{Λ}: Max envelope of cutoff variation
  - 2 variations: Vary freezeout cutoff Λ up and down (not actually a scale variation)
- Rationale/interpretation for combination
  - Think of each as a (somewhat) independent "source" $\to$ add in quadrature
  - Within each: Arbitrary knobs all probing the same thing $\to$ take envelope

$$\Delta_{\mathrm{total}} = \sqrt{\Delta^2_{\mathrm{resum}} + \Delta^2_{\mathrm{FO}} + \Delta^2_{\mathrm{match}} + \Delta^2_{\Lambda}}$$

Nuisance parameters in SCETLIB

Theory Nuisance Parameters.

Perturbative series at leading power is determined to all orders by a coupled system of differential equations (RGEs)

→ Each resummation order only depends on a few semi-universal parameters

→ Unknown parameters at higher orders are the actual sources of perturbative theory uncertainty

	boundary conditions			anomalous dimensions			
order	h_n	s_n	b_n	γ_n^h	γ_n^s	Γ_n	β_n
LL	h_0	s_0	b_0	–	–	Γ_0	β_0
NLL′	h_1	s_1	b_1	γ_0^h	γ_0^s	Γ_1	β_1
NNLL′	h_2	s_2	b_2	γ_1^h	γ_1^s	Γ_2	β_2
N^{3}LL′	h_3	s_3	b_3	γ_2^h	γ_2^s	Γ_3	β_3
N^{4}LL′	h_4	s_4	b_4	γ_3^h	γ_3^s	Γ_4	β_4

- Basic Idea: Use them as theory nuisance parameters
  - ✓ Vary them independently to estimate the theory uncertainties
  - ✓ Impact of each independent nuisance parameter is fully correlated across all kinematic regions and processes
  - ✓ Impact of different nuisance parameters is fully uncorrelated

F. Tackmann

- Price to Pay: Calculation becomes quite a bit more complex
  - ▶ Implement complete next order in terms of arbitrary theory parameters