

Motivation

BFKL resummation

Heavy-quark pair hadroproduction

Theoretical set-up

Numerical analysis

Towards bound state

Flavor number schemes

Heavy quark FFs

Heavy-light mesons production

Conclusions and outlook

- ▶ Heavy flavor physics has long been considered as a perfect framework for testing perturbative QCD at colliders, due to the smallness of the running coupling.
- ▶ However, at modern colliders, heavy-flavor production enters a two-scale regime, called semi-hard.
- ▶ Semi-hard collision process, featuring the scale hierarchy

$$s \gg Q^2 \gg \Lambda_{\text{QCD}}^2, \quad Q^2 \text{ a hard scale,}$$

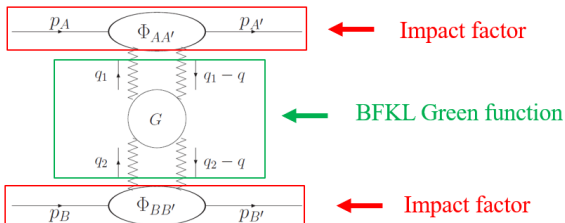
Regge kinematical region

$$\alpha_s(Q^2) \ln\left(\frac{s}{Q^2}\right) \sim 1 \implies \text{all-order resummation needed}$$

- ▶ The **Balitsky-Fadin-Kuraev-Lipatov (BFKL)** approach is the general framework for this resummation
 - Leading-logarithm-Approximation (LLA): $(\alpha_s \ln s)^n$
 - Next-to-leading-logarithm-Approximation (NLLA): $\alpha_s(\alpha_s \ln s)^n$

BFKL resummation

- ▶ Diffusion $A + B \rightarrow A' + B'$ in the **Regge kinematical region**
- ▶ Gluon Reggeization
- ▶ BFKL factorization for $\Im \mathcal{A}_{AB}^{A'B'}$: convolution of a **Green function** (process independent) with the **Impact factors** of the colliding particles (process dependent).

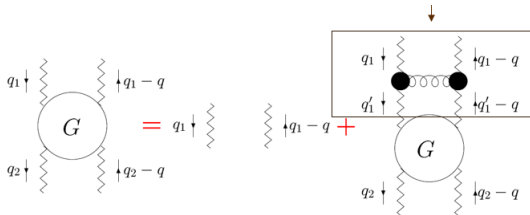


$$\begin{aligned}
 \Im \mathcal{A}_{AB}^{A'B'} &= \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2} q_1}{\vec{q}_1^2 (\vec{q}_1 - \vec{q})^2} \frac{d^{D-2} q_2}{\vec{q}_2^2 (\vec{q}_2 - \vec{q})^2} \\
 &\times \sum_{\nu} \Phi_{A'A}^{(R,\nu)}(\vec{q}_1, \vec{q}, s_0) \int \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_{\omega}^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}) \right] \Phi_{B'B}^{(R,\nu)}(-\vec{q}_2, \vec{q}, s_0)
 \end{aligned}$$

BFKL resummation

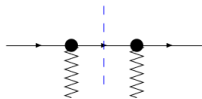
- ▶ $G_{\omega}^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q})$ -Mellin transform of the Green function for the Reggeon-Reggeon scattering

$$\omega G_{\omega}^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}) = \vec{q}_1^2 (\vec{q}_1 - \vec{q})^2 \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int \frac{d^{D-2} q'_1}{\vec{q}'_1{}^2 (\vec{q}'_1 - \vec{q})^2} \mathcal{K}^{(R)}(\vec{q}_1, \vec{q}'_1; \vec{q}) G_{\omega}^{(R)}(\vec{q}'_1, \vec{q}_2; \vec{q})$$



- ▶ $\Phi_{PP'}^{(R,\nu)}$ - LO impact factor in the t -channel color state (R, ν)

$$\Phi_{PP'}^{(R,\nu)} = \langle cc' | \hat{\mathcal{P}} | \nu \rangle \sum_{\{f\}} \int \frac{ds_{PR}}{2\pi} d\rho_f \Gamma_{\{f\}P}^c (\Gamma_{\{f\}P'}^{c'})^*$$



Pomeron channel

- ▶ **BFKL equation:** $\vec{q}^2 = 0$ and singlet color state representation

[Ya. Ya. Balitsky, V. S. Fadin, E.A Kuraev, L.N Lipatov (1975)]

$$\text{Redefinition : } G_\omega(\vec{q}_1, \vec{q}_2) \equiv \frac{G_\omega^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^2 \vec{q}_2^2}, \quad \mathcal{K}(\vec{q}_1, \vec{q}_2) \equiv \frac{\mathcal{K}^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^2 \vec{q}_2^2}$$

$$\omega G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2} q_r \mathcal{K}(\vec{q}_1, \vec{q}_r) G(\vec{q}_r, \vec{q}_2)$$

- ▶ **Elastic amplitude** factorization:

$$\Im \mathcal{A}_{AB}^{AB} = \frac{s}{(2\pi)^{D-2}} \int d^{D-2} q_1 d^{D-2} q_2 \\ \times \frac{\Phi_{AA}^{(0)}(\vec{q}_1, s_0)}{\vec{q}_1^2} \int \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^\omega G_\omega(\vec{q}_1, \vec{q}_2) \right] \frac{\Phi_{BB}^{(0)}(-\vec{q}_2, s_0)}{\vec{q}_2^2}$$

- ▶ **Optical Theorem:**

$$\sigma_{AB} = \frac{\Im \mathcal{A}_{AB}^{AB}}{s}$$

- ▶ Impact factor in the color singlet state:

$$\Phi_{PP}^{(0)} = \langle cc' | \hat{\mathcal{P}} | 0 \rangle \sum_{\{f\}} \int \frac{ds_{PR}}{2\pi} d\rho_f \Gamma_{\{f\}P}^c (\Gamma_{\{f\}P}^{c'})^*$$

Heavy flavor production

1) Inclusive heavy-quark pair hadroproduction

[A.D. Bolognino, F.G. Celiberto, M. F., D.Yu. Ivanov, A. Papa (2019)]

- ▶ Partonic process

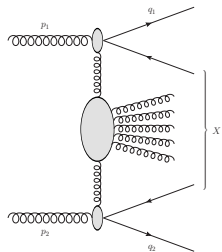
$$g(p_1) + g(p_2) \longrightarrow Q\text{-jet}(q_1) + Q\text{-jet}(q_2) + X$$

Q = charm, bottom (detected in the fragmentation regions)

- ▶ Partial NLLA resummation
- ▶ LO impact factor for the $g \rightarrow Q\text{-}\bar{Q}$ transition
- ▶ From the gluon-initiated process to the one initiated by proton-proton:

$$d\sigma_{pp} = f_{g_1}(x_1, \mu_{F_1}) f_{g_2}(x_2, \mu_{F_2}) d\sigma_{gg} dx_1 dx_2 ,$$

f_{g_i} , $i = 1, 2 \longrightarrow$ gluon collinear parton distribution functions

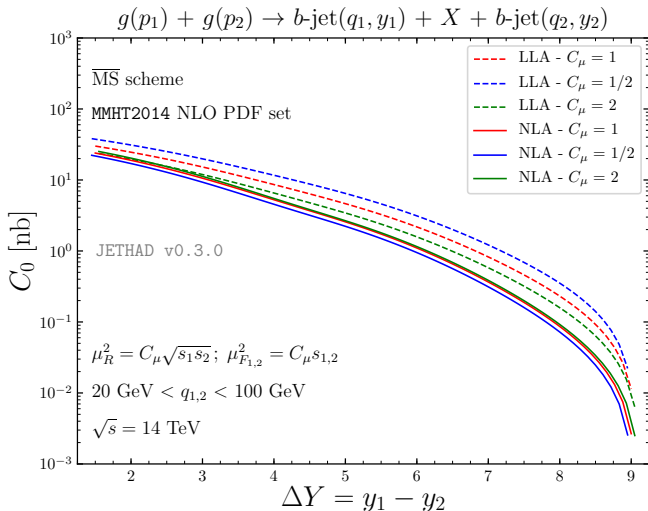


2) Inclusive heavy-quark pair photoproduction

[F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

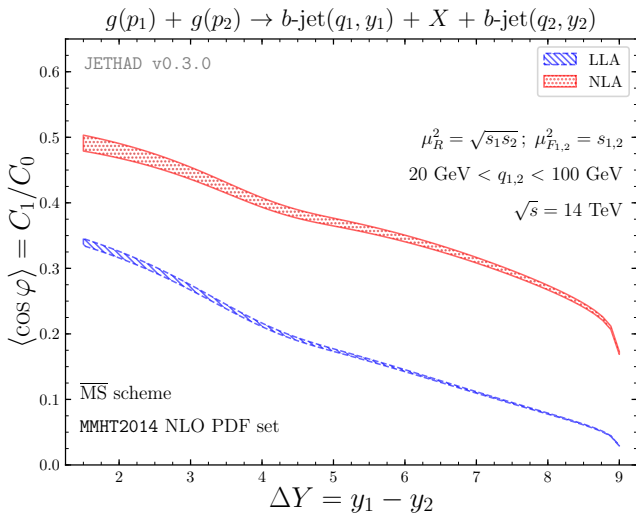
Heavy-quark pair hadroproduction: Observables

- ▶ C_0
- ▶ b-jet ($m_b = 4.8$ GeV), $\sqrt{s} = 14$ TeV, $q_{\min} = 20$ GeV, $q_{\max} = 100$ GeV



Heavy-quark pair hadroproduction: Observables

- ▶ C_1/C_0
- ▶ b-jet ($m_b = 4.8$ GeV), $\sqrt{s} = 14$ TeV, $q_{\min} = 20$ GeV, $q_{\max} = 100$ GeV



Towards bound states: Flavor number schemes

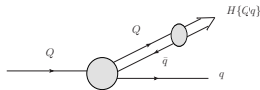
- ▶ The mass of light quarks ($q = u, d, s$) is always set to zero. They are always present in the initial state.
- ▶ The presence in the initial state and the way one must treat the mass of an heavy-quark ($Q = c, b, t$) depends on kinematical conditions.
- ▶ **Zero-mass variable flavor number scheme**
 - $m_Q = 0$
 - Heavy quark is present in the initial state above a fixed threshold.
 - Powers of $m_Q^2/p_{T,HQ}^2$ missed by the scheme
 - It is appropriate in region of high $p_{T,HQ}^2 \gg m_Q^2$
- ▶ **Fixed flavor number scheme**
 - $m_Q \neq 0$
 - Heavy quark is present only in the final state
 - Logarithms of $p_{T,HQ}^2/m_Q^2$ missed by the scheme
 - It is appropriate in regions of moderate $p_{T,HQ}^2$
- ▶ **General-mass variable flavor number schemes**
 - It is a matching between the previous schemes
 - There is some arbitrariness in the combination

Towards bound states: Heavy quark FFs

To describe the heavy-quark hadronization, many models of heavy-quark fragmentation functions (FFs) has been proposed.

- ▶ Peterson FF [C. Peterson, D. Schlatter, I. Schmitt, P. M. Zerwas (1983)]

$$D_Q^H(z) = N \frac{z(1-z)^2}{[(1-z)^2 + \epsilon_H z]^2}$$



$\epsilon_H = (m_q^2)/(m_Q^2)$. N is a normalization.

- ▶ Colangelo-Nason FF [G. Colangelo, P. Nason (1992)]

$$D_Q^H(z) = N(1-z)^\alpha z^\beta;$$

α, β are parameters. N is a normalization.

- ▶ Bowler FF [M.G. Bowler (1984)]

- String hadronization model

$$D_Q^H(z) = \frac{N}{z^{1+bm_Q^2}} (1-z)^\beta e^{(-bm_{H,\perp}^2)/z}$$

b, β are parameters.

$m_Q = 0 \rightarrow$ Lund FF [B. Andersson, G. Gustafson, B. Soderberg (1983)]

Towards bound state: Heavy quark FFs

- ▶ Kartvelishvili FF [V.G. Kartvelishvili, A.K. Likehoded, V.A. Petrov (1978)]
 - Based on Gribov-Lipatov Reciprocity ($D_Q^H(z) \leftrightarrow f_H^Q(z)$) [V. N. Gribov and L. N. Lipatov (1971)]

$$D_Q^H(z) = (\alpha + 1)(\alpha + 2)z^\alpha(1 - z)$$

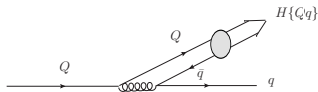
- ▶ Collins-Spiller FF [P. D. B. Collins, T. P. Spiller (1985)]
- ▶ HQEFT FFs [E. Braaten, K. Cheung, S. Fleming (1994)]
 - Calculated using an effective field theory based on spin flavor symmetry (arising from the fact that $m_c, m_b, m_t \gg \Lambda_{QCD}$)

$${}^1S_0 \longrightarrow D_{Q \rightarrow P}(z) = \frac{a(y)}{r} + b(y) + \mathcal{O}(r), \quad {}^3S_1 \longrightarrow D_{Q \rightarrow V}(z) = \frac{a^*(y)}{r} + b^*(y) + \mathcal{O}(r)$$
$$r = (m_H - m_Q)/(m_H), \quad y = (1 - (1 - r)z)/(rz)$$

- ▶ Suzuki FFs [M. Suzuki (1986)]

$${}^1S_0 \longrightarrow D_{Q \rightarrow P}(z)$$

$${}^3S_1 \longrightarrow D_{Q \rightarrow V}(z)$$



Heavy-light mesons production

- ▶ Process

$$p_1 + p_2 \longrightarrow H_1 + X + H_2$$

- ▶ Heavy quark pair LO impact factor

$$\frac{d\Phi_{gg}^{\{Q\bar{Q}\}}(\vec{k}, \vec{q}, z)}{d^2\vec{q}dz} = \frac{\alpha_s^2 \sqrt{N_c^2 - 1}}{2\pi N_c} \left[\left(m^2 (R + \bar{R})^2 + (z^2 + \bar{z}^2) (\vec{P} + \vec{\bar{P}})^2 \right) - \frac{N_c^2}{N_c^2 - 1} \left(2m^2 R\bar{R} + (z^2 + \bar{z}^2) 2\vec{P} \cdot \vec{\bar{P}} \right) \right],$$

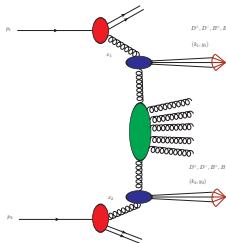
- ▶ Cross section factorization

$$d\sigma_{p,\bar{p}}^{H_1, H_2} = D_{Q_1}^{H_1}(\alpha_1) \otimes D_{Q_2}^{H_2}(\alpha_2) \otimes d\sigma_{p,\bar{p}}^{Q_1, Q_2}$$

- ▶ Heavy meson LO impact factor

$$\frac{d\Phi_{gg}^{\{H\bar{Q}\}}}{dz_h d^2\vec{h}}(\vec{k}, \vec{h}, z_h) = \int_{z_h}^1 \frac{dz}{z\alpha^2} \mathcal{H}(\vec{k}, \frac{\vec{h}}{\alpha}, z) D_Q^H(\alpha)$$

- ▶ Numerical analysis in progress...



Conclusion

- ▶ Inclusive processes with jets and/or identified hadrons in the final state featuring large rapidity separation are a promising testfield in the search of BFKL dynamics in current and future colliders.
- ▶ Among them, the photo- or hadro-production of a pair of heavy-quarks/mesons is an interesting new possibility.
- ▶ Theoretical predictions, including a relevant part of the energy resummation in the NLLA, are available for both photo- and hadro-production cases.

Future projects

- ▶ Inclusion of **subleading corrections** from the impact factors, needed to produce full-NLLA predictions.
- ▶ Production of **quarkonia state**.
- ▶ Single forward heavy-flavored jet production, both in the LLA and in the NLLA, via the introduction of the **small- x transverse-momentum-dependent gluon distribution (UGD)**.

Thank you for the attention

Backup

JETHAD, BFKL inspired but for HEP purposes!

It is a Fortran2008-Python3 hybrid library by Cosenza collaboration

- ▶ Main features:
 1. Modularity
 2. Extensive use of structures and dynamic memory
 3. Smart management of final-state phase-space integration
- ▶ Developed software:
 1. BFKL tools (BFKL kernel and Impact factors)
 2. UGD modular package
- ▶ External interfaces:
 1. LHAPDF and native FF parametrizations
 2. CUBA multi-dim integrators
 3. QUADPACK one-dim integrators
 4. CERNLIB (multi-dim integrators, special functions, MINUIT, etc.)

Semi-hard sector

Total inclusive and exclusive processes

- ▶ $A = A' = \text{quark}, B = B' = \text{gluon}$ [M. Ciafaloni and G. Rodrigo (2000)]
[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
- ▶ $A = \gamma_L^*, A' = V_L$, with $V_L = \rho, \omega, \phi$ (forward)
[D.Yu. Ivanov, M.I. Kotsky, A.Papa (2004)]
- ▶ $A = A' = \gamma^*$ (forward) [J. Bartels, S. Gieseke, C.F. Qiao (2001)]
[J. Bartels, S. Gieseke, A. Kyrieleis (2002)]
[J. Bartels, D. Colferai, S. Gieseke, A. Kyrieleis (2002)]
[V.S. Fadin, D.Yu. Ivanov, M.I. Kotsky (2003)]
[J. Bartels, A. Kyrieleis (2004)]
[I. Balitsky, G.A. Chirilli (2013)] [G.A. Chirilli, Yu.V. Kovchegov (2014)]

Partially inclusive processes

- ▶ Mueller-Navelet jet production
 1. NLO jet vertex [J. Bartels, D. Colferai, G.P. Vacca (2003)]
[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, A. Perri (2011)]
[D.Yu. Ivanov, A. Papa (2012)]
[D. Colferai, A. Niccoli (2015)]
 2. Azimuthal correlations (full NLA) [B. Ducloué, L. Szymanowski, S. Wallon (2013,2014)]
[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]
[F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]
 3. Compatible with CMS (7 TeV)

- ▶ Hadron - hadron production
 1. NLO hadron vertex [D.Yu. Ivanov, A. Papa (2012)]
 2. Azimuthal correlations (full NLA) [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa (2016,2017)]
- ▶ Hadron - jet production (full NLA) [A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]
- ▶ Three / four jet production (partial NLA)
 - [F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2016)]
 - [F. Caporale, F.G. Celiberto, G. Chachamis, A. Sabio Vera (2016)]
 - [F. Caporale, F.G. Celiberto, G. Chachamis, D.G. Gomez, A. Sabio Vera (2016,2017)]
- ▶ J/Ψ - jet production (partial NLA) [R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]
- ▶ Drell-Yan pair - jet (partial NLA) [K. Golec-Biernat, L. Motyka, T. Stebel (2018)]
- ▶ Higgs - jet (partial NLA) [F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2020)]
- ▶ Heavy-quark pair photoproduction (partial NLA) [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]
- ▶ Heavy-quark pair hadroproduction (partial NLA) [A.D. Bolognino, F.G. Celiberto, M.F, D.Yu. Ivanov, A. Papa (2019)]
- ▶ Heavy-light meson pair hadroproduction (partial NLA) [A.D. Bolognino, F.G. Celiberto, M.F, D.Yu. Ivanov, A. Papa (in preparation)]