

Transverse momentum dependent cross-section factorization for dijet and heavy-meson pair processes in DIS

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U N I V E R S I D A D
C O M P L U T E N S E
M A D R I D



Outline

Dijet production

- EIC coverage
- Cross-section factorization
- New TMD Soft Function
- Consistency AD check

Heavy meson pair production

- Cross-section factorization
- Soft Function AD up to three-loops

Based on the work recently published (accepted)

Rafael F. del Castillo, Miguel G. Echevarría, Yiannis Makris & Ignazio Scimemi

<https://arxiv.org/abs/2008.07531v2>

Introduction

- Gluon transverse momentum dependent distributions (TMDs) are difficult to access due to the lack of clean processes where the factorization of the cross-section holds and incoming gluons constitute the dominant effect.

For example: Higgs production

Gutierrez-Reyez, Leal-Gómez, Scimemi, Vladimirov, 2019

- We consider two processes which are presently attracting increasing attention

$$\ell + h \rightarrow \ell' + J_1 + J_2 + X$$

Dijet

$$\ell + h \rightarrow \ell' + H + \bar{H} + X$$

Heavy-meson

Dominguez, Xiao, Yuan, 2013

Boer, Brodsky, Mulders, Pisano, 2011

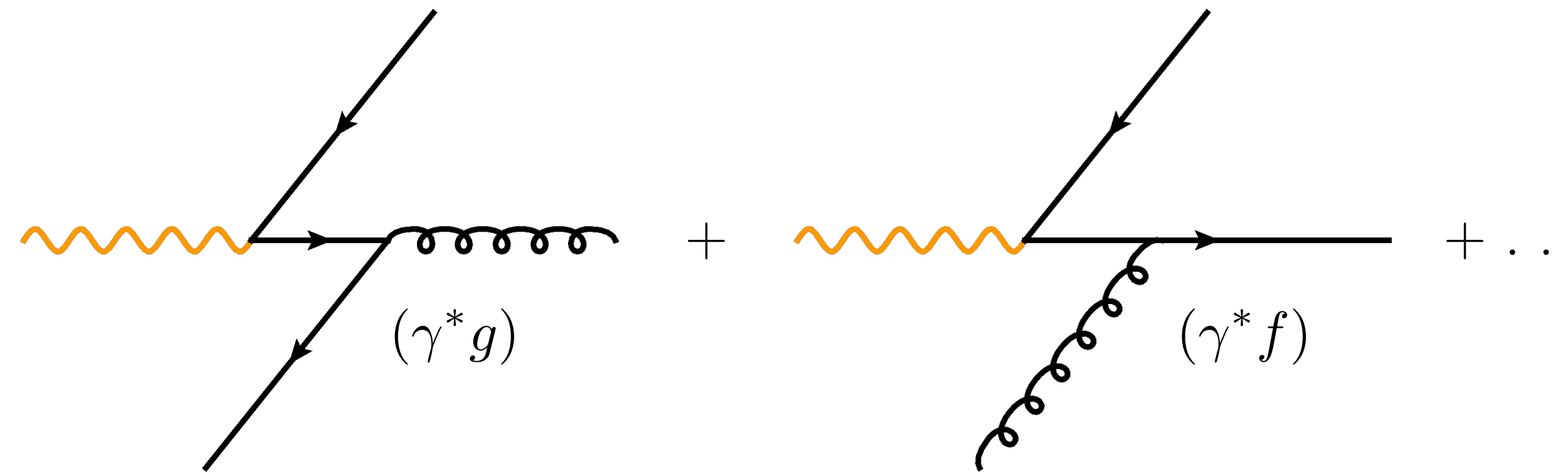
Zhang, 2017

Working in the Breit frame

Dijet production

$$\ell + h \rightarrow \ell' + J_1 + J_2 + X$$

dijet LO process:



- Sensitive of polarized and polarized and unpolarized TMDPDFs
- Experimental observation should be possible in the future EIC Page, Chu, Aschenauer, 2020
- Jets here described have $p_T \in [4, 20]$ GeV and are found in the central rapidity region
- Factorization within SCET

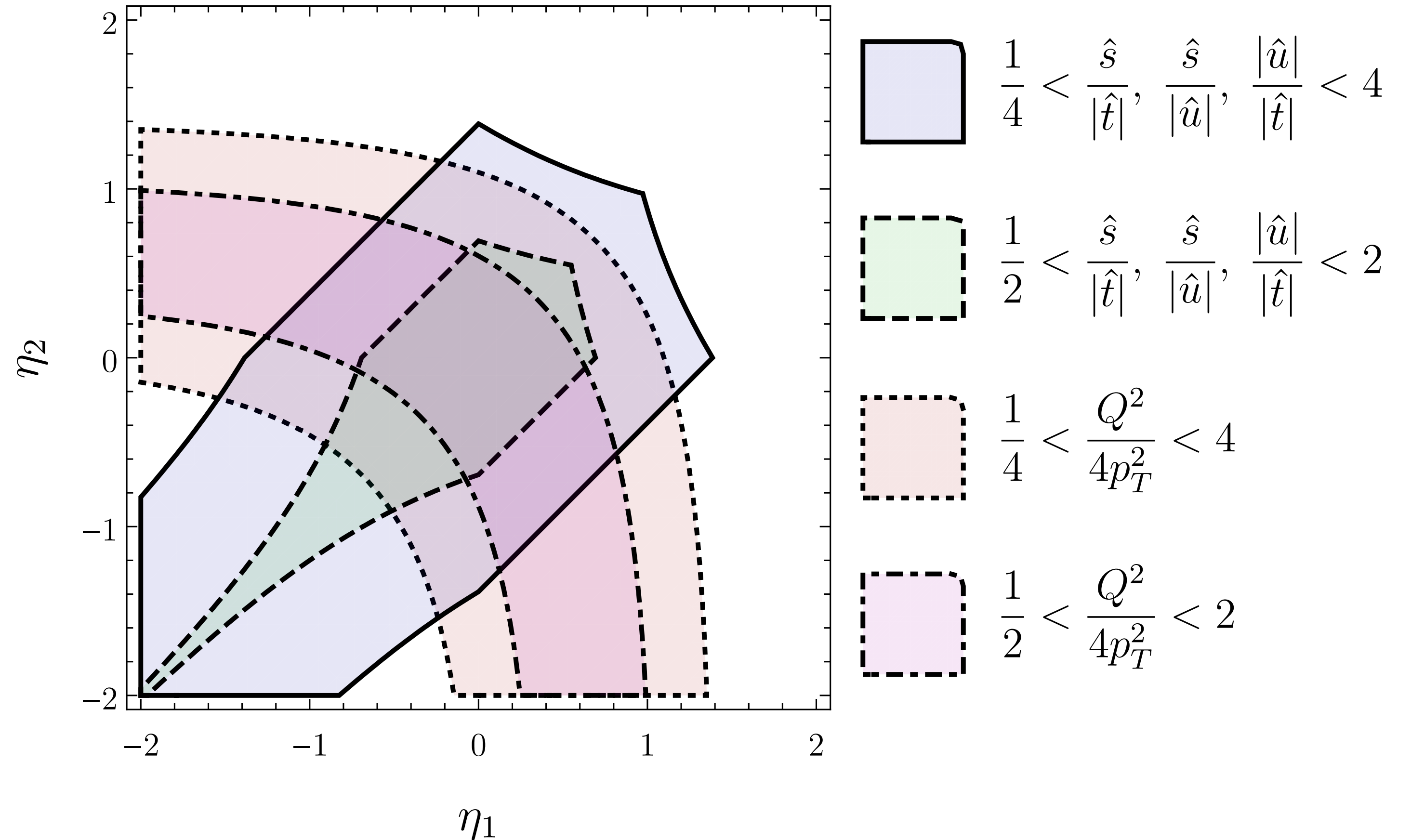
Kinematic region

Dijet production

$$\mathbf{r}_T = \mathbf{p}_{1T} + \mathbf{p}_{2T}$$

$$p_T = \frac{|\mathbf{p}_{1T}| + |\mathbf{p}_{2T}|}{2}$$

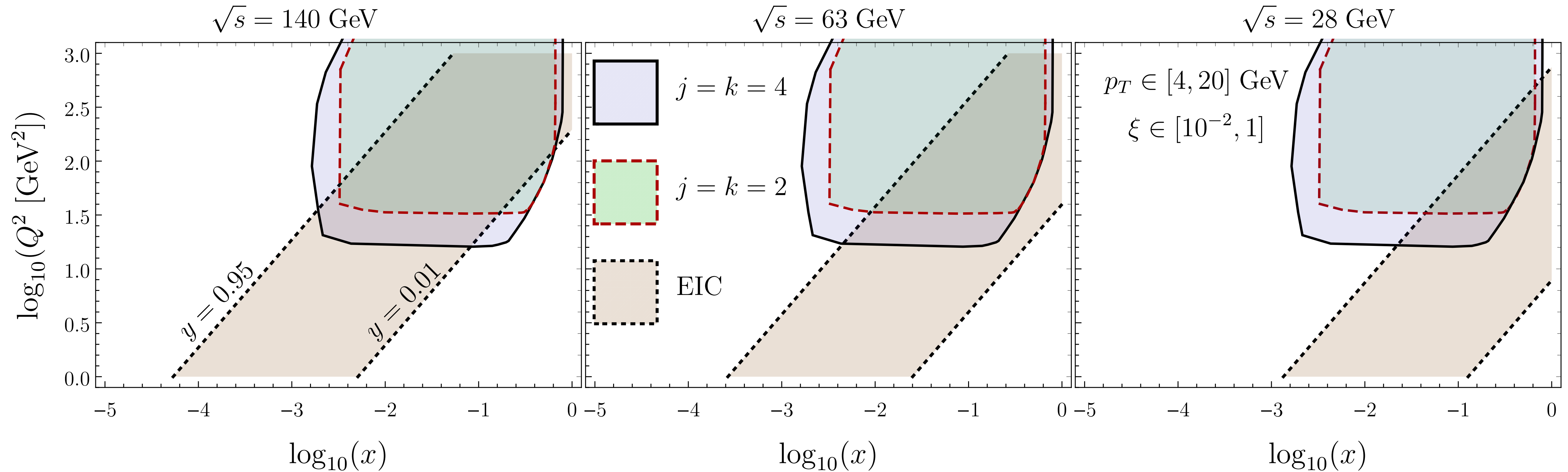
$$|\mathbf{r}_T| \ll p_T$$



Factorization holds for $|\mathbf{r}_T| \ll p_T$ and for the central rapidity region

Kinematic region vs EIC coverage

Dijet production



$$\frac{1}{j} < \frac{\hat{s}}{|\hat{t}|}, \frac{\hat{s}}{|\hat{u}|}, \frac{|\hat{u}|}{|\hat{t}|} < j \quad \frac{1}{k} < \frac{Q^2}{4p_T^2} < k$$

Overlapping increases with higher beam energies

Cross-section factorization

Dijet production

$$\frac{d\sigma}{dx d\eta_1 d\eta_2 dp_T d\mathbf{r}_T}$$

- x parton momentum fraction
- η_i jet pseudorapidity
- p_T transverse momentum
- \mathbf{r}_T transverse momentum imbalance

We measure over

$$\begin{aligned}
 (\gamma^* g) \quad \frac{d\sigma(\gamma^* g)}{dx d\eta_1 d\eta_2 dp_T d\mathbf{r}_T} &= \sum_f H_{\gamma^* g \rightarrow f \bar{f}}^{\mu\nu}(\hat{s}, \hat{t}, \hat{u}, \mu) \int \frac{d^2\mathbf{b}}{(2\pi)^2} \exp(i\mathbf{b} \cdot \mathbf{r}_T) F_{g, \mu\nu}(\xi, \mathbf{b}, \mu, \zeta_1) \\
 &\times S_{\gamma g}(\mathbf{b}, \eta_1, \eta_2, \mu, \zeta_2) \left(C_f(\mathbf{b}, R, \mu) J_f(p_T, R, \mu) \right) \left(C_{\bar{f}}(\mathbf{b}, R, \mu) J_{\bar{f}}(p_T, R, \mu) \right)
 \end{aligned}$$

$$\begin{aligned}
 (\gamma^* f) \quad \frac{d\sigma^U(\gamma^* f)}{dx d\eta_1 d\eta_2 dp_T d\mathbf{r}_T} &= \sigma_0^{fU} \sum_f H_{\gamma^* f \rightarrow g \bar{f}}^U(\hat{s}, \hat{t}, \hat{u}, \mu) \int \frac{d^2\mathbf{b}}{(2\pi)^2} \exp(i\mathbf{b} \cdot \mathbf{r}_T) F_f(\xi, \mathbf{b}, \mu, \zeta_1) \\
 &\times S_{\gamma f}(\mathbf{b}, \zeta_2, \mu) \left(C_g(\mathbf{b}, R, \mu) J_g(p_T, R, \mu) \right) \left(C_{\bar{f}}(\mathbf{b}, R, \mu) J_{\bar{f}}(p_T, R, \mu) \right)
 \end{aligned}$$

New soft function

n - incoming beam direction

v_1 - jet 1 direction

v_2 - jet 2 direction

Soft
function

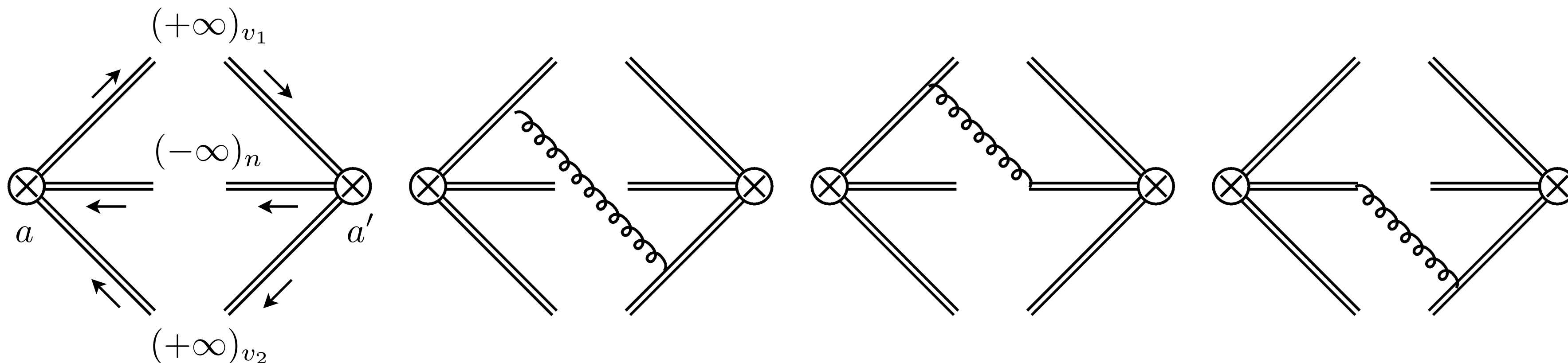
$$\hat{S}_{\gamma g}(\mathbf{b}) = \frac{1}{C_F C_A} \langle 0 | \mathcal{S}_n^\dagger(\mathbf{b}, -\infty)_{ca'} \text{Tr} \left[S_{v_2}(+\infty, \mathbf{b}) T^{a'} S_{v_1}^\dagger(+\infty, \mathbf{b}) \right. \\ \left. \times S_{v_1}(+\infty, 0) T^a S_{v_2}^\dagger(+\infty, 0) \right] \mathcal{S}_n(0, -\infty)_{ac} | 0 \rangle$$

$$\hat{S}_{\gamma f} = \hat{S}_{\gamma g}(n \leftrightarrow v_2)$$

Wilson
lines

$$S_v(+\infty, \xi) = P \exp \left[-ig \int_0^{+\infty} d\lambda v \cdot A(\lambda v + \xi) \right] \quad S_v^\dagger(+\infty, \xi) = P \exp \left[ig \int_0^{+\infty} d\lambda \bar{v} \cdot A(\lambda \bar{v} + \xi) \right]$$

$$S_n(+\infty, \xi) = \lim_{\delta^+ \rightarrow 0} P \exp \left[-ig \int_0^{+\infty} d\lambda n \cdot A(\lambda n + \xi) e^{-\delta^+ \lambda} \right] \quad \delta - \text{regulator !!!}$$



Echevarría, Scimemi, Vladimirov, 2016

+ virtual diagrams
at one-loop order...

New soft function

Finite result

(γ^*g) - channel

$$\hat{S}_{\gamma g}^{\text{finite}}(\mathbf{b}) = 1 + a_s \left\{ C_A \left[\ln(B \mu^2 e^{2\gamma_E}) \left(\ln(B \mu^2 e^{2\gamma_E}) + 4 \ln\left(\frac{\sqrt{2} \delta^+}{\mu}\right) + 2 \ln(2A_n) \right) - \ln^2(-A_b) \right. \right. \\ \left. \left. - \frac{\pi^2}{6} - 2\text{Li}_2(1 + A_b) \right] + C_F \left[\frac{\pi^2}{3} + 2 \ln^2\left(\frac{B \mu^2 e^{2\gamma_E}}{-A_b}\right) + 4\text{Li}_2(1 + A_b) \right] \right\}$$

with... $A_b = \frac{(v_1 \cdot v_2)}{2 (v_1 \cdot \hat{b}) (v_2 \cdot \hat{b})} = -\frac{\hat{s}}{4 p_T^2 c_b^2}$ $A_n = \frac{(v_1 \cdot v_2)}{2 (v_1 \cdot n) (v_2 \cdot n)}$

Zero-bin subtraction

- We need to subtract the zero-bin from the TMD beam function
- The zero-bin corresponds to the two-direction back-to-back soft function (the one used in Drell-Yan or SIDIS). Here, we use the subtraction as done in Echevarría, Idilbi, Scimemi, 2013.
- We can reorganize the zero-bin to obtain rapidity divergence-free function as expressed in the cross-section factorization
- This leads to the universal TMDPDF and a rapidity divergence-free new TMD soft function and the introduction of the scale ζ

$$B_i^{\text{un.}}(\xi, \mathbf{b}, \mu, k^- / \delta^-) \longrightarrow F_i(\xi, \mathbf{b}, \mu, \zeta_1)$$

$$\hat{S}_{\gamma i}(\mathbf{b}, \mu, \sqrt{A_n} \delta^+) \longrightarrow S_{\gamma i}(\mathbf{b}, \mu, \zeta_2)$$

Zero-bin subtraction

Due to the rapidity divergencies structure of the two-direction soft function (zero-bin) can be split as

$$S(\mathbf{b}, \mu, \sqrt{\delta^+ \delta^-}) = S^{\frac{1}{2}}(\mathbf{b}, \mu, \delta^+ \nu) S^{\frac{1}{2}}(\mathbf{b}, \mu, \delta^- / \nu)$$

ν arbitrary positive number

$$\left(S_i^{\text{bare}}(\mathbf{b}, \delta) \right)^{\frac{1}{2}} = 1 + a_s C_i \left\{ -\frac{2}{\epsilon^2} + \frac{4}{\epsilon} \ln \left(\frac{\sqrt{2} \delta}{\mu} \right) + \ln(B\mu^2 e^{2\gamma_E}) \left[4 \ln \left(\frac{\sqrt{2} \delta}{\mu} \right) + \ln(B\mu^2 e^{2\gamma_E}) \right] + \frac{\pi^2}{6} \right\}$$

This way we defined the rapidity divergence-free objects

Universal TMDPDF

Rapidity divergence-free new soft function

$$F_i(\xi, \mathbf{b}, \mu, \zeta_1) = \frac{B_i^{\text{un.}}(\xi, \mathbf{b}, \mu, k^- / \delta^-)}{S^{\frac{1}{2}}(\mathbf{b}, \mu, \delta^- / \nu)} \Bigg|_{\sqrt{2} k^- / \nu \rightarrow \sqrt{\zeta_1}} \quad S_{\gamma i}(\mathbf{b}, \mu, \zeta_2) = \frac{\hat{S}_{\gamma i}(\mathbf{b}, \mu, \sqrt{A_n} \delta^+)}{S^{\frac{1}{2}}(\mathbf{b}, \mu, \delta^+ \nu)} \Bigg|_{\nu / \sqrt{2 A_n} \rightarrow \sqrt{\zeta_2}}$$

ζ scale associated with the δ - regulator and zero-bin split

Zero-bin subtraction

ζ scale

The scale can be removed from the final result by introducing the constrain

$$\zeta_1 \zeta_2 = \frac{(k^-)^2}{A_n} = \frac{\hat{u} \hat{t}}{\hat{s}}$$

In the Breit-frame this leads to

$$\zeta_1 \zeta_2 = p_T^2$$

Notice that

ζ_1 has square mass dimension

ζ_2 is dimensionless



$\zeta_1 = p_T^2$ natural way of
 $\zeta_2 = 1$ choosing the scale

Procedure totally analogous to the one used in Drell-Yan or SIDIS

This allows to use ζ - prescription for TMDPDF and SF evolution (work in progress)

Zero-bin subtraction

Subtracted soft function, finite result

(γ^*g) - channel

$$S_{\gamma g}(\mathbf{b}, \mu, \zeta_2) = 1 + a_s \left\{ C_F \left[\frac{\pi^2}{3} + 2 \ln^2 \left(\frac{B\mu^2 e^{2\gamma_E}}{-A_b} \right) + 4 \text{Li}_2(1 + A_b) \right] \right. \\ \left. + C_A \left[-2 \ln(B\mu^2 e^{2\gamma_E}) \ln \zeta_2 - \ln^2(-A_b) - \frac{\pi^2}{3} - 2 \text{Li}_2(1 + A_b) \right] \right\} + \mathcal{O}(a_s^2)$$

with... $A_b = \frac{(v_1 \cdot v_2)}{2 (v_1 \cdot \hat{b}) (v_2 \cdot \hat{b})} = -\frac{\hat{s}}{4 p_T^2 c_b^2}$

Consistency check

Dijet-production

$$\frac{d}{d \ln \mu} G(\mu) = \gamma_G(\mu) G(\mu)$$

($\gamma^* g$)-channel	$\gamma_{H_{\gamma g}} + \gamma_{S_{\gamma g}} + \gamma_{F_g} + 2\gamma_{J_f} + \gamma_{c_1} + \gamma_{c_2} + \gamma_\alpha = 0$
($\gamma^* f$)-channel	$\gamma_{H_{\gamma f}} + \gamma_{S_{\gamma f}} + \gamma_{F_f} + \gamma_{J_f} + \gamma_{J_g} + \gamma_{c_f} + \gamma_{c_g} + \gamma_\alpha = 0$

The sum of all anomalous dimensions should cancel for each channel

$$\gamma_{S_{\gamma g}}^{[1]} = 4 \left\{ -C_A \ln \zeta_2 + 2C_F \left[\ln(B\mu^2 e^{2\gamma_E}) - \ln \hat{s} + \ln p_T^2 + \ln(4c_b^2) \right] \right\},$$

$$\gamma_{S_{\gamma f}}^{[1]} = 4 \left\{ (C_F + C_A) \left[\ln(B\mu^2 e^{2\gamma_E}) - \ln \hat{s} + \ln p_T^2 + \ln(4c_b^2) \right] + (C_F - C_A) \left[\ln \left(\frac{\hat{t}}{\hat{u}} \right) - \kappa(v_f) \right] - C_F \ln \zeta_2 \right\}$$

$$\gamma_{F_i}^{[1]} = 4C_i \left[-\ln \left(\frac{\zeta_1}{\mu^2} \right) + \gamma_i \right],$$

$$\gamma_{c_g}^{[1]} = 4C_A \left[-\ln \left(B\mu^2 e^{2\gamma_E} \right) + \ln R^2 - \ln(4c_b^2) + \kappa(v_g) \right]$$

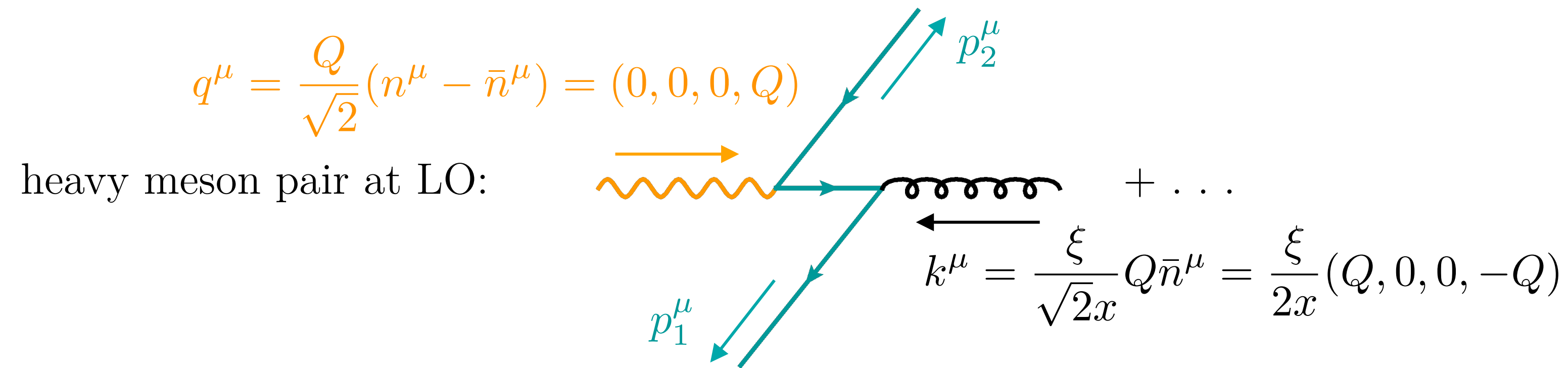
$$\gamma_{c_i}^{[1]} = 4C_F \left[-\ln \left(B\mu^2 e^{2\gamma_E} \right) + \ln R^2 - \ln(4c_b^2) + \kappa(v_i) \right]$$

$$\kappa(v_f) = -\kappa(v_{\bar{f}}) = -\kappa(v_g) = i\pi \text{sign}(c_b)$$

They cancel !!!

Heavy-meson pair production

$$\ell + h \rightarrow \ell' + H + \bar{H} + X$$



- Experimentally more challenging
- Observation of charmed mesons could be possible

Arratia, Furltova, Hobbs, Olness, Nguyen et al. 2020

Li, Liu, Vitev, 2020

Chudakov, Higinbotham, Hyde, Furltov, Furltova, Nguyen, 2016

Cross-section factorization

Heavy meson pair production

$$\frac{d\sigma(\gamma^* g)}{dx d\eta_H d\eta_{\bar{H}} dp_T d\mathbf{r}_T}$$

We measure over

- x parton momentum fraction
- $\eta_H, \eta_{\bar{H}}$ heavy meson pseudorapidity
- p_T transverse momentum
- \mathbf{r}_T transverse momentum imbalance

$$\begin{aligned} \frac{d\sigma(\gamma^* g)}{dx d\eta_H d\eta_{\bar{H}} dp_T d\mathbf{r}_T} &= H_{\gamma^* g \rightarrow Q\bar{Q}}^{\mu\nu}(\hat{s}, \hat{t}, \hat{u}, \mu) \int \frac{d\mathbf{b}}{(2\pi)^2} \exp(i\mathbf{b} \cdot \mathbf{r}_T) F_{g, \mu\nu}(\xi, \mathbf{b}, \mu, \zeta_1) \\ &\times S_{\gamma g}(\mathbf{b}, \mu, \zeta_2) \boxed{J_{Q \rightarrow H}(\mathbf{b}, p_T, m_Q, \mu) J_{\bar{Q} \rightarrow \bar{H}}(\mathbf{b}, p_T, m_Q, \mu)} \end{aligned}$$

Fickinger, Fleming, Kim, Mereghetti, 2016

Region sensitive to TMD $|\mathbf{r}_T| \ll p_T^{H, \bar{H}}$
 Factorization for highly boosted heavy mesons $p_T^H \gg m_H$
 We have a new scale m_Q

Refactorization of heavy-quark fragmentation

Heavy meson pair production

We use heavy-quark jet function to describe the fragmentation of heavy mesons from heavy quarks. In the limit $r_T \ll p_T$ there are two scales that need to be resummed

$$\mu_+ = m_Q, \quad \text{and} \quad \mu_{\mathcal{J}} = m_Q \frac{r_T}{p_T}$$

To do this we use **bHQET (boosted heavy quark effective theory)** to factorize the jet function into a hard matching coefficient and a TMD matrix element

$$J_{Q \rightarrow H}(\mathbf{b}, p_T, m_Q, \mu) = H_+(m_Q, \mu) \mathcal{J}_{Q \rightarrow H}\left(\mathbf{b}, \frac{m_Q}{p_T}, \mu\right)$$

Appears for the first time

Known up to two-loops

Refactorization of heavy-quark fragmentation

Heavy meson pair production

Up to one loop order we find

$$H_+(m_Q, \mu) = 1 + \frac{\alpha_s C_F}{4\pi} \left\{ \ln \left(\frac{\mu^2}{m_Q^2} \right) + \ln^2 \left(\frac{\mu^2}{m_Q^2} \right) + 8 + \frac{\pi^2}{6} \right\}$$

$$\gamma_+ = \frac{\alpha_s C_F}{\pi} \left\{ \frac{1}{2} - \ln \left(\frac{m_Q^2}{\mu^2} \right) \right\}$$

$$\mathcal{J}_{Q \rightarrow Q}^{\text{bare}} \left(\mathbf{b}, \frac{m_Q}{p_T} \right) = 1 + \frac{\alpha_s C_F}{\pi} \left\{ -\frac{1}{2\epsilon^2} + \frac{1}{2\epsilon} \left[1 - 2 \ln \mathcal{R} \right] + \ln \mathcal{R} - \ln^2 \mathcal{R} - \frac{5\pi^2}{24} \right\}$$

$$\gamma_{\mathcal{J}} = \frac{\alpha_s C_F}{\pi} \{ 1 - 2 \ln \mathcal{R} \}$$

with... $\mathcal{R} = -\frac{i p_T \mu e^{\gamma_E} (\mathbf{v} \cdot \mathbf{b})}{m_Q |\mathbf{v}|}$

We have separated scales and can now perform resummation

Anomalous dimension are consistent $\gamma_{\mathcal{J}} + \gamma_+ = \gamma_J + \gamma_{\mathcal{C}_f}$



Consistent !!!

Connection to the fragmentation shape function

Heavy meson pair production

We can see that the shape function is related to the bHQET jet function

$$\mathcal{J}_{Q \rightarrow H}(\mathbf{b}) = \frac{m_H}{\sqrt{2} p_H^-} \tilde{S}_{Q \rightarrow H} \left(\tau \rightarrow \frac{\mathbf{v} \cdot \mathbf{b}}{\sqrt{2}} \right)$$

- We can check our NLO calculation (finite terms)
- We can get the jet function AD up to two loops

Fickinger, Fleming, Kim, Mereghetti, 2016

This sum is known up to three-loops...

$$\gamma_{S_{\gamma g}} = - \left(\gamma_{H_{\gamma g}} + \gamma_{F_g} + \gamma_{\alpha} + \gamma_{\mathcal{J}}(\mathbf{v}_1) + \gamma_{\mathcal{J}}(\mathbf{v}_2) + 2\gamma_+ \right)$$

Soft function

anomalous dimension

$$\gamma_{S_{\gamma g}} = \gamma_{\text{cusp}} \left[2C_F \ln \left(\frac{B\mu^2 e^{2\gamma_E}}{-A_b} \right) - C_A \ln \zeta_2 \right] + \delta\gamma_{S_{\gamma g}}$$

$$\delta\gamma_{S_{\gamma g}}^{[1]} = 0$$

$$\delta\gamma_{S_{\gamma g}}^{[2]} = C_F \left[C_A \left(\frac{1616}{27} - \frac{22}{9} \pi^2 - 56 \zeta_3 \right) + n_f T_F \left(-\frac{448}{27} + \frac{8}{9} \pi^2 \right) \right]$$

$$\delta\gamma_{S_{\gamma g}}^{[3]} = \dots$$

Conclusion

- We have found that factorization for the dijet and heavy meson production is consistent
- Both cases can be potentially observed in the future EIC
- We have been able to compute the new TMD Soft Function up to NLO and its anomalous dimension up to three-loops
- Rapidity structure of this new SF allows us to use the ζ -prescription
- The presence of the new SF makes the gluon TMDPDF extraction non-trivial
- We are working on codes for the phenomenology of these processes

Thank you for listening!