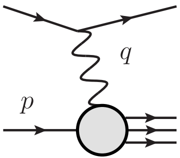
NLP factorization and endpoint divergences in DIS

Sebastian Jaskiewicz

Resummation, Evolution, Factorization 2020
December 8th, 2020
Edinburgh

[2008.04943] with Martin Beneke, Mathias Garny, Robert Szafron, Leonardo Vernazza and Jian Wang

Motivations



Deep inelastic scattering (DIS) at threshold contains a hierarchy of scales: $Q^2 \gg P_X^2 \sim Q^2(1-x)$

$$x = \frac{Q^2}{2p \cdot q} \to 1 \quad (\text{large } N \text{ in Mellin space})$$

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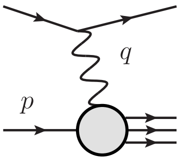

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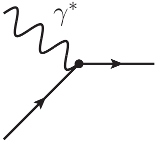

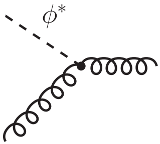

DIS is well understood at leading power. The coefficient function known to N^3LL

$$C(Q^2) \sim \exp\left[g_1 \ln(N) + \ldots\right] + \mathcal{O}(N^{-1}\ln^n(N))$$

via traditional resummation techniques

[S. Moch, J.A.M. Vermaseren, A. Vogt, hep-ph/0506288]

and equivalent results obtained in SCET using RG equations directly in momentum space

[T. Becher, M. Neubert, B. D. Pecjak, hep-ph/0607228]

$$P_{qq/gg}^{(n-1)} \sim \frac{A^{(n)}}{(1-x)_+} + B^{(n)}\ln(1-x) + \ldots$$

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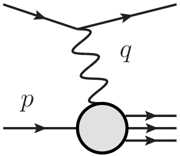

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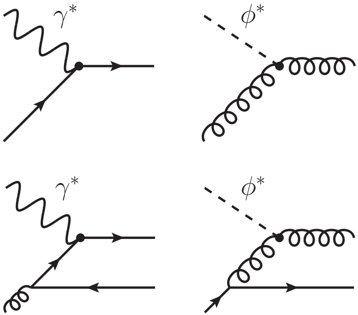

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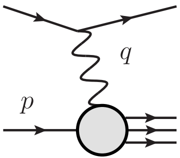

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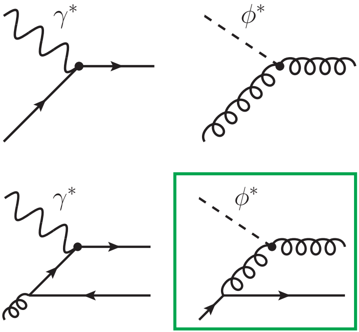

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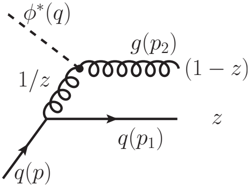
Off-diagonal Deep Inelastic Scattering (DIS)



Off-diagonal DIS at threshold: $x = Q^2/2p\cdot q \to 1$

$$q(p) + \phi^*(q) \to X(p_X)$$

gives access to

$$P_{gq}^{\rm LL}(N) = \frac{1}{N}\frac{\alpha_s C_F}{\pi}\,\mathcal{B}_0(a), \qquad a = \frac{\alpha_s}{\pi}(C_F - C_A)\ln^2 N\,,$$

where

$$\mathcal{B}_0(x) = \sum_{n=0}^{\infty}\frac{B_n}{(n!)^2}x^n$$

with Bernoulli numbers $B_0 = 1$, $B_1 = -1/2$, ...

[A. Vogt, 1005.1606] [A.A. Almasy, G. Soar A. Vogt, 1012.3352] [A. Vogt, C. H. Kom, N. A. Lo Presti, G. Soar, A. A. Almasy, S. Moch, J. A. M. Vermaseren, K. Yeats, 1212.2932]

The resummed coefficient function is

$$C_{\phi,q}^{\rm LL}(N,\alpha_s) = \frac{1}{2N\ln N}\,\frac{C_F}{C_F - C_A}\Big\{\exp\big[2C_A\alpha_s\ln^2 N\big]\,\mathcal{B}_0(a) - \exp\big[2C_F\alpha_s\ln^2 N\big]\Big\}$$

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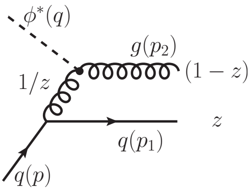

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Partonic structure function:

$$W_{\phi,i=q} = \frac{1}{8\pi Q^2}\int d^4x\, e^{iq\cdot x}\,\langle i(p)|\,[G^A_{\mu\nu}G^{\mu\nu A}](x)\,[G^B_{\rho\sigma}G^{\rho\sigma B}](0)\,|i(p)\rangle$$

At lowest order

$$q(p) + \phi^*(q) \rightarrow q(p_1) + g(p_2)$$

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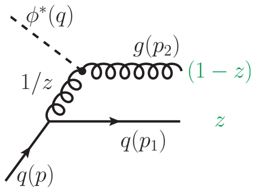

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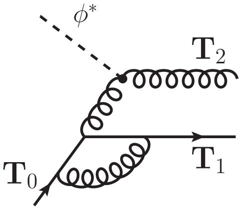
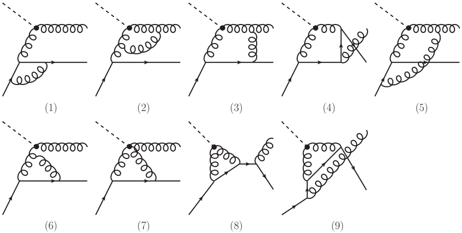
Parametrise with momentum fraction z:

$$W_{\phi,q}\big|_{q\phi^*\to qg} = \int_0^1 dz\left(\frac{\mu^2}{s_{qg}z\bar{z}}\right)^{\epsilon}\mathcal{P}_{qg}(s_{qg},z) \qquad z \equiv \frac{n_- p_1}{n_- p_1 + n_- p_2}$$

$$\mathcal{P}_{qg}(s_{qg},z) \equiv \frac{e^{\gamma_E\epsilon}Q^2}{16\pi^2\Gamma(1-\epsilon)}\frac{|\mathcal{M}_{q\phi^*\to qg}|^2}{|\mathcal{M}_0|^2} \qquad \mathcal{P}_{qg}(s_{qg},z)\big|_{\mathrm{tree}} = \frac{\alpha_s C_F}{2\pi}\frac{\bar{z}^2}{z}$$

Momentum distribution function





$$\mathcal{P}_{qg}(s_{qg}, z)|_{1-\text{loop}} = \mathcal{P}_{qg}(s_{qg}, z)|_{\text{tree}} \frac{\alpha_s}{\pi} \frac{1}{\epsilon^2} \left\{ \mathbf{T}_1 \cdot \mathbf{T}_0 \left(\frac{\mu^2}{zQ^2}\right)^\epsilon + \mathbf{T}_2 \cdot \mathbf{T}_0 \left(\frac{\mu^2}{\bar{z}Q^2}\right)^\epsilon \right.$$

$$\left. + \mathbf{T}_1 \cdot \mathbf{T}_2 \left[\left(\frac{\mu^2}{Q^2}\right)^\epsilon - \left(\frac{\mu^2}{zQ^2}\right)^\epsilon + \left(\frac{\mu^2}{zs_{qg}}\right)^\epsilon \right] \right\}$$

Colour operator notation [S. Catani, hep-ph/9802439]

$$\mathbf{T}_1 \cdot \mathbf{T}_0 = C_A/2 - C_F, \quad \mathbf{T}_2 \cdot \mathbf{T}_0 = \mathbf{T}_1 \cdot \mathbf{T}_2 = -C_A/2$$

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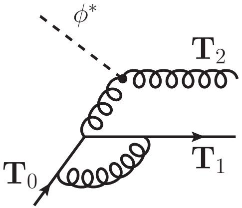

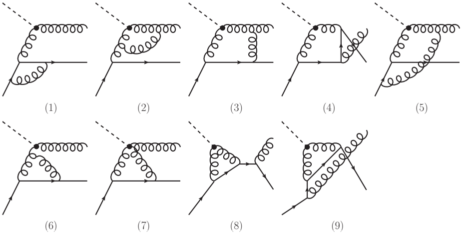

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1712.04416, 1808.04742]

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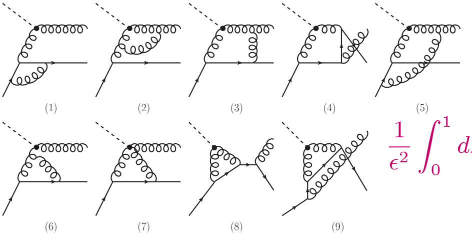

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$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\left(\epsilon\ln z - \frac{\epsilon^2}{2!}\ln^2 z + \frac{\epsilon^2}{3!}\ln^3 z + \cdots\right)$$
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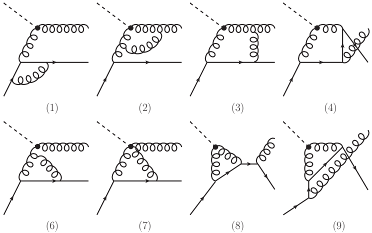

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We must keep the quantities dimensionally regularized!

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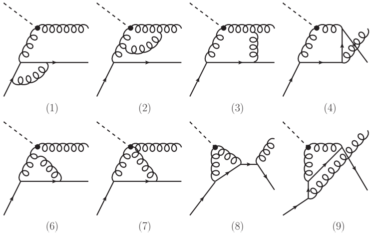

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Similarly to the conjectured Soft Quark Sudakov in [I. Moult, I.W. Stewart, G. Vita, H.X. Zhu, 1910.14038] we exponentiate the one-hard-loop result

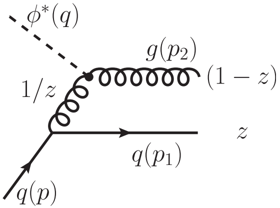
$$\mathcal{P}_{qg}(s_{qg}, z) = \frac{\alpha_s C_F}{2\pi}\frac{1}{z}\,\exp\left[\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(-C_A\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} + (C_A - C_F)\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}\right)\right]$$

The EFT perspective

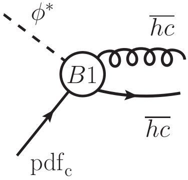
DIS factorization formula involves the scales:

- hard, $p^2 = Q^2$
- anti-hardcollinear, $p^2 = Q^2\lambda^2 = Q^2/N$
- collinear, $p^2 = \Lambda^2$
- softcollinear, $p^2 = \Lambda^2\lambda^2 = \Lambda^2/N$

where $\lambda = \sqrt{1-x}$. [T. Becher, M. Neubert, B. D. Pecjak, hep-ph/0607228]



Operator matching →



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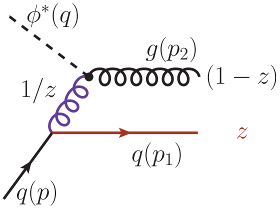
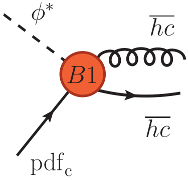


If the quark is soft $z \to 0$,



the matching coefficient contains a $1/z$ divergence.

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Endpoint divergence points to a new scale in the problem.
$\rightarrow$ Refactorization required

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New power counting parameter z: $1 \gg z \gg \lambda$

Name	$(n_+l\,,\, l_\perp\,,\, n_-l)$	virtuality l^2
hard $[h]$	$Q(1\,,\,1\,,\,1)$	Q^2
z-hardcollinear $[z-hc]$	$Q(1\,,\,\sqrt{z}\,,\,z)$	$z\,Q^2$
z-anti-hardcollinear $[z-\overline{hc}]$	$Q(z\,,\,\sqrt{z}\,,\,1)$	$z\,Q^2$
z-soft $[z-s]$	$Q(z\,,\,z\,,\,z)$	$z^2\,Q^2$
z-anti-softcollinear $[z-\overline{sc}]$	$Q(\lambda^2\,,\,\sqrt{z}\,\lambda\,,\,z)$	$z\,\lambda^2\,Q^2$

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Expansion-by-regions method → Large $\ln(z)$ from hard and z-hardcollinear regions.

$$\int d^d x\, T\left\{J^{A0}, \mathcal{L}^{(1)}_{\xi q_{z-\overline{sc}}}(x)\right\} = D^{B1}(zQ^2, \mu^2)\, J^{B1}$$

Refactorization

Endpoint divergence points to a new scale in the problem.
→ Refactorization required
→ Then solve RGEs

First step matching

$$\left[C^{A0}\left(zQ^2,\mu^2\right)\right]_{\text{bare}} = C^{A0}\left(Q^2,Q^2\right)\exp\left[-\frac{\alpha_s C_A}{2\pi}\frac{1}{\epsilon^2}\left(\frac{Q^2}{\mu^2}\right)^{-\epsilon}\right]$$

Second step matching

$$\left[D^{B1}\left(zQ^2,\mu^2\right)\right]_{\text{bare}} = D^{B1}\left(zQ^2,zQ^2\right)\exp\left[-\frac{\alpha_s}{2\pi}\left(C_F-C_A\right)\frac{1}{\epsilon^2}\left(\frac{zQ^2}{\mu^2}\right)^{-\epsilon}\right]$$

Final step in the Soft Sudakov derivation: combination of these terms gives the exponentiated $\mathcal{P}_{qg}$. ← used as input earlier.
[M. Beneke, M. Garny, **SJ**, R. Szafron, L. Vernazza, J. Wang, 2008.04943]

Consistency relations

- We know that an observable must be a finite quantity.
- Imposing the constraint allows us to infer structure of partonic objects.

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The hadronic tensor is given by

$$W = \sum_i W_{\phi,i}\, f_i \,,$$

related to their finite counterparts through

$$\tilde{f}_k = Z_{ki} f_i, \qquad W_{\phi,i} = \tilde{C}_{\phi,k} Z_{ki} \,,$$

such that

$$W_{\phi,i} f_i = \tilde{C}_{\phi,k} \tilde{f}_k \,.$$

The splitting kernels are given by

$$P_{ij} = -\gamma_{ij} = \frac{dZ_{ik}}{d\ln\mu} (Z^{-1})_{kj} \,.$$

Consistency relations

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- Imposing the constraint allows us to infer structure of partonic objects.

Focusing on the quark initiated NLP contribution

$$\sum_i (W_{\phi,i} f_i)^{NLP} = \left(W_{\phi,q}^{NLP} U_{qq}^{LP} + W_{\phi,g}^{LP} U_{gq}^{NLP} \right) f_q(\Lambda)$$

where U_{ij} are the evolution factors

$$f_i(\mu) = U_{ij}(\mu) f_j(\Lambda)$$

The general expansion for the cross section is

$$\sum_i (W_{\phi,i} f_i)^{NLP} = f_q(\Lambda) \times \frac{1}{N} \sum_{n=1} \left(\frac{\alpha_s}{4\pi} \right)^n \frac{1}{\epsilon^{2n-1}} \sum_{k=0}^{n} \sum_{j=0}^{n} c_{kj}^{(n)}(\epsilon) \left(\frac{\mu^{2n} N^j}{Q^{2k} \Lambda^{2(n-k)}} \right)^\epsilon$$

The scaling of the regions: hard (Q^2), anti-hardcollinear (Q^2/N), collinear (Λ^2), softcollinear (Λ^2/N)

Consistency relations

- We know that an observable must be a finite quantity.
- Imposing the constraint allows us to infer structure of partonic objects.

Invoking pole cancellation → $(n+1)^2$ coefficients $c_{kj}^{(n)}$ determined, up to three unknowns.
Use initial conditions:

$$c_{n0}^{(n)} = 0 \qquad , c_{00}^{(n)} = 0 \qquad \text{for all } n\,.$$

and the third initial condition, $c_{n1}^{(n)}$, is the discussed exponentiated $\mathcal{P}_{qg}$.

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A closed form solution from all order algebraic relations for $\tilde{C}_{\phi,q}^{\mathrm{NLP,LL}}$ in agreement with [A. Vogt, 1005.1606]. Arrive at identical splitting kernels:

$$P_{gq}^{\mathrm{LL}}(N) = \frac{1}{N}\frac{\alpha_s C_F}{\pi}\,\mathcal{B}_0(a), \qquad a = \frac{\alpha_s}{\pi}(C_F - C_A)\ln^2 N\,,$$

$$\mathcal{B}_0(x) = \sum_{n=0}^{\infty}\frac{B_n}{(n!)^2}x^n$$

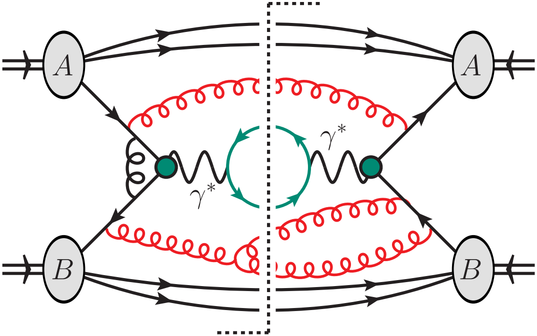
with Bernoulli numbers $B_0 = 1$, $B_1 = -1/2$, $\ldots$.

Outlook: Ubiquitous divergent convolutions at NLP

$$A(p_A) + B(p_B) \to \gamma^*(Q^2)[\to \ell(l_1)\bar{\ell}(l_2)\,] + X(p_X)$$

Threshold limit:

$$z = \frac{Q^2}{\hat{s}} \to 1$$



Schematic form for production cross-sections near threshold, $z \to 1$:

$$\hat{\sigma}(z) = \sum_{n=0}^{\infty} \alpha_s^n \left[c_n \delta(1-z) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-z)}{1-z} \right]_+ + \quad d_{nm} \ln^m(1-z) \right) + \ldots \right]$$

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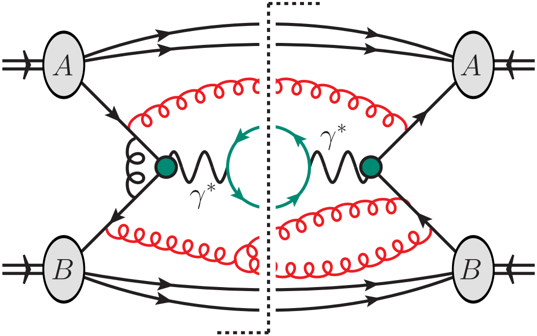

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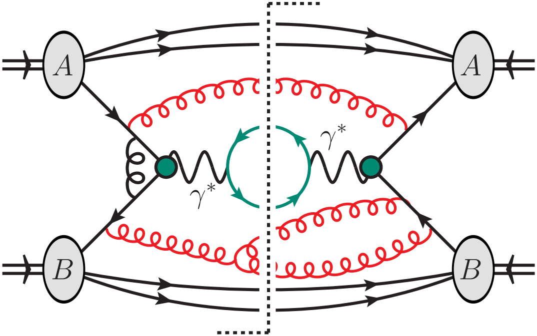

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Factorization of partonic cross sections

First let us compare leading power and *next*-to-leading power cross-sections schematically:

$$\frac{d\sigma_{\rm DY}}{dQ^2} \sim \sum_{a,b} \int_0^1 dx_a dx_b\, f_{a/A}(x_a) f_{b/B}(x_b) \left(\hat{\sigma}_{ab}^{\rm LP}(z) \; + \; \hat{\sigma}_{ab}^{\rm NLP}(z) \; + \ldots \right) + \mathcal{O}\left(\Lambda/Q\right)$$

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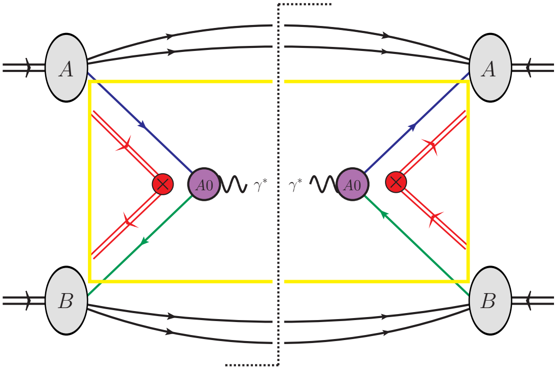

$$\hat{\sigma}^{\rm LP}(z) = Q\, H(Q^2)\, S_{\rm DY}(\Omega)$$

[G. P. Korchemsky G. Marchesini, 1993] [S. Moch, A. Vogt, hep-ph/0508265]

[T. Becher, M. Neubert, G. Xu, 0710.0680]

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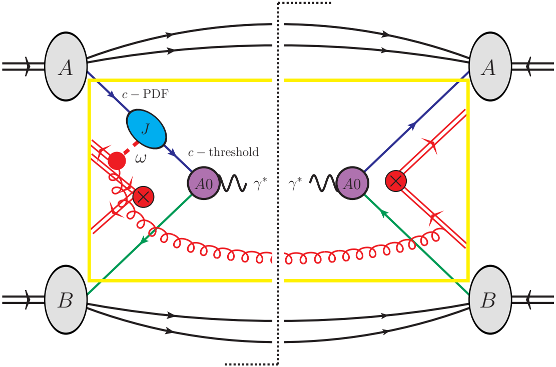


$$\hat{\sigma}^{\rm NLP}(z) = \sum_{\rm terms} \left[C \otimes J \otimes \bar{J}\right]^2 \otimes S$$

[M. Beneke, A. Broggio, M. Garny, **SJ**, R. Szafron, L. Vernazza, J. Wang, 1809.10631]

[M. Beneke, A. Broggio, **SJ**, L. Vernazza, 1912.01585]

Divergent convolutions

A term in the factorization formula

$$\int d\omega\, J_1^{(1)}(x_a n_+ p_A;\omega)\, \widetilde{S}_{2\xi}^{(1)}(\Omega,\omega)$$

Calculated one loop collinear function [M.Beneke, A.Broggio, **SJ**, L.Vernazza, 1912.01585]

$$J_1^{(1)}(x_a\, n_+ p_A;\,\omega) \quad = \quad \frac{\alpha_s}{4\pi}\frac{1}{(x_a\, n_+ p_A)}\left(\frac{(x_a\, n_+ p_A)\,\omega}{\mu^2}\right)^{-\epsilon} \frac{e^{\epsilon\,\gamma_E}\,\Gamma[1+\epsilon]\Gamma[1-\epsilon]^2}{(-1+\epsilon)(1+\epsilon)\Gamma[2-2\epsilon]}$$
$$\times\left(C_F\left(-\frac{4}{\epsilon}+3+8\epsilon+\epsilon^2\right)-C_A\left(-5+8\epsilon+\epsilon^2\right)\right)$$

$$S_{2\xi}\,(\Omega,\omega) = \frac{\alpha_s\, C_F}{2\pi}\frac{\mu^{2\epsilon}\, e^{\epsilon\gamma_E}}{\Gamma[1-\epsilon]}\,\frac{1}{\omega^{1+\epsilon}}\,\frac{1}{(\Omega-\omega)^{\epsilon}}\theta(\omega)\theta(\Omega-\omega)\, +\, \mathcal{O}(\alpha_s^2)$$

First $d\omega$ convolution integral in d-dimensions, and ϵ expansion *after*.

Divergent convolutions

A term in the factorization formula

$$\int d\omega\, J_1^{(1)}(x_a n_+ p_A;\omega)\, \widetilde{S}_{2\xi}^{(1)}(\Omega,\omega)$$

The factorization formula contains unrenormalized objects. Convolution in d -dimensions reproduces the NNLO result:

[M.Beneke, A.Broggio, **SJ**, L.Vernazza, 1912.01585]

$$\Delta^{(2)}_{\mathrm{NLP-coll}} = \frac{\alpha_s^2}{(4\pi)^2}\Bigg(C_A C_F \left(\frac{20}{\epsilon} - 60\log(1-z) + 8 + \mathcal{O}(\epsilon^1)\right)$$
$$+C_F^2 \left(\frac{-16}{\epsilon^2} - \frac{20}{\epsilon} + \frac{48}{\epsilon}\log(1-z) + 60\log(1-z) - 72\log^2(1-z) + \mathcal{O}(\epsilon^1)\right)\Bigg)$$

In agreement with equation (4.22) of [D. Bonocore, E. Laenen, L. Magnea, S. Melville, L. Vernazza, C. White, 1503.05156].

Note that result is valid beyond LL. Can we obtain a resummed result?

Divergent convolutions

A term in the factorization formula

$$\int d\omega\, J_1^{(1)}(x_a n_+ p_A;\omega)\, \widetilde{S}_{2\xi}^{(1)}(\Omega,\omega)$$

For resummation, expand in ϵ first. We find a problem! At two loops:

[M.Beneke, A.Broggio, **SJ**, L.Vernazza, 1912.01585]

$$J_1^{(1)}(x_a n_+ p_A;\omega) \sim \alpha_s \log(\omega)$$

and

$$S_{2\xi}(\Omega,\omega) \sim \alpha_s\, \delta(\omega) + \mathcal{O}(\alpha^2)$$

The convolution $d\omega$ integral is now divergent. This prohibits the application of standard RG methods.

The $d\omega$ divergent convolution here $\rightarrow$ the $(z \rightarrow 0)$ endpoint divergence in the dz integration in DIS.

For LL resummation, tree level collinear function is needed.

Conclusions: Resummations at NLP in SCET

Subleading power resummed thrust spectrum for $H \to gg$ (LL)

[I. Moult, I. Stewart, G. Vita, H. Zhu, 1804.04665]

Drell-Yan and Higgs production at threshold (LL)

[M. Beneke, A.Broggio, M. Garny, **SJ**, R. Szafron, L. Vernazza, J.Wang, 1809.10631]

[M. Beneke, M. Garny, **SJ**, R. Szafron, L. Vernazza, J.Wang, 1910.12685]

Resummation of rapidity logarithms: the EE correlator in N=4 SYM (LL)

[I. Moult, G. Vita, K. Yan, 1912.02188]

Factorization at Subleading Power and Endpoint Divergences in SCET (LL, NLL)

[Z. L. Liu, M.Neubert, 1912.08818]

[Z. L. Liu, B. Mecaj, M.Neubert, X.Wang, 2009.04456, 2009.06779]

Drell-Yan q_T Resummation of Fiducial Power Corrections at N^{3}LL

[M. Ebert, J. Michel, I. Stewart, F. Tackmann, 2006.11382] See M. Ebert talk on Monday

Power-enhanced QED corrections to $B_q \to \mu^+\mu^-$ (LL)

[M. Beneke, C. Bobeth , R. Szafron, 1908.07011]

Violation of KSZ theorem in SCET

[M. Beneke, M. Garny, R. Szafron, J.Wang, 1907.05463]

Resummation of double logarithms in loop-induced processes with EFT

[J.Wang, 1912.09920]

Summary

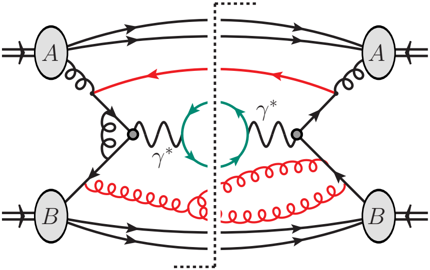
- Divergence in the convolution integral takes the considerations outside the standard SCET paradigm.
- New modes appear due to endpoint divergences
- We require a consistent refactorization of the operator to truly separate the scales.
- Resummation can still be performed in d-dimensions.
- Interesting conceptual challenges ahead. Important to understand from the point of view of gauge theories, as well as for delivering precise theoretical predictions.

Thank you

Auxiliary slides

Off-diagonal Drell-Yan process

As we have seen, divergent convolutions are appear already at leading logarithmic accuracy in the off-diagonal channels. In addition to DIS, we have $g\bar{q}$-channel of the Drell-Yan Process.



DIS modes

Name	$(n_+l\,,\,l_\perp\,,\,n_-l)$	virtuality l^2
hard $[h]$	$Q(1\,,\,1\,,\,1)$	Q^2
z-hardcollinear $[z-hc]$	$Q(1\,,\,\sqrt{z}\,,\,z)$	$z\,Q^2$
z-anti-hardcollinear $[z-\overline{hc}]$	$Q(z\,,\,\sqrt{z}\,,\,1)$	$z\,Q^2$
z-soft $[z-s]$	$Q(z\,,\,z\,,\,z)$	$z^2\,Q^2$
z-anti-softcollinear $[z-\overline{sc}]$	$Q(\lambda^2\,,\,\sqrt{z}\,\lambda\,,\,z)$	$z\,\lambda^2\,Q^2$
hardcollinear $[hc]$	$Q(1\,,\,\lambda\,,\,\lambda^2)$	$\lambda^2\,Q^2$
anti-hardcollinear $[\overline{hc}]$	$Q(\lambda^2\,,\,\lambda\,,\,1)$	$\lambda^2\,Q^2$
soft $[s]$	$Q(\lambda^2\,,\,\lambda^2\,,\,\lambda^2)$	$\lambda^4\,Q^2$
collinear $[c]$	$Q(1\,,\,\eta\,,\,\eta^2)$	$\eta^2\,Q^2$
softcollinear $[sc]$	$Q(\lambda^2\,,\,\lambda\,\eta\,,\,\eta^2)$	$\lambda^2\,\eta^2\,Q^2$

Table: Scaling of the momentum modes relevant for DIS.

Thank you