Rapidity anomalous dimension: theory and practice

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Resummation, Evolution, Factorization

Edinburgh, 2020

Collins-Soper kernel is a fundamental universal function
that measures properties of QCD vacuum

Many names – single object

- Collins-Soper kernel; K [Collins,Soper 1981]
- Rapidity anomalous dimension (RAD); $\mathcal{D}$, $\gamma_\nu^{f_\perp}$, γ_ζ
- Collinear anomaly; $F_{q\bar{q}}$
- ...

$$\mathcal{D} = -\frac{1}{2}K = \frac{1}{2}F_{q\bar{q}} = -\frac{1}{2}\gamma_\nu^{f_\perp} = -\frac{1}{2}\gamma_\zeta$$

Outline

- Rapidity divergences and their renomalization
- Extractions of CS-kernel
- Non-perturbative definition

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Collins-Soper kernel is the evolution kernel for TMD distributions

$$\mu^2 \frac{d}{d\mu^2} F_{f\leftarrow h}(x,b;\mu,\zeta) = \frac{\gamma_F^f(\mu,\zeta)}{2} F_{f\leftarrow h}(x,b;\mu,\zeta)$$
$$\zeta \frac{d}{d\zeta} F_{f\leftarrow h}(x,b;\mu,\zeta) = -\mathcal{D}^f(b,\mu) F_{f\leftarrow h}(x,b;\mu,\zeta)$$

Some properties

- RAD depends only on the color-representation (quark/gluon) I will assume quark everywhere.
- RAD is non-perturbative
- RAD evolves as

$$\mu \frac{d}{d\mu}\mathcal{D}(b,\mu) = \Gamma_{\text{cusp}}(\mu)$$

- RAD is a property of operator

$$O_{\mathrm{TMD}} = \bar{q}(\lambda n + b)[\lambda n + b, -\infty n + b]...[-\infty n, 0]q(0)$$

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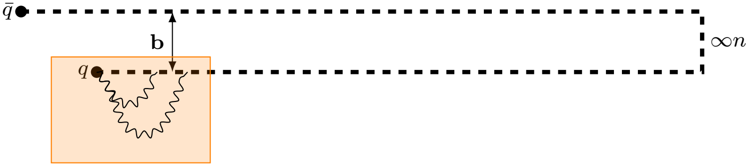

UV divergence

- Local (number)
- Anomalous dimension of quark field in LC gauge

$$\mu^2 \frac{d}{d\mu^2} F_{f\leftarrow h}(x, b; \mu, \zeta) = \frac{\gamma_F^f(\mu, \zeta)}{2} F_{f\leftarrow h}(x, b; \mu, \zeta)$$

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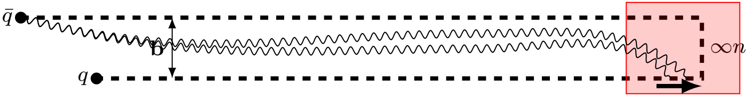


Rapidity divergence

- Non-Local (depends on b)
- Not regularized by dim.reg.
- Rapidity anomalous dimension

$$\zeta \frac{d}{d\zeta} F_{f\leftarrow h}(x, b; \mu, \zeta) = -\mathcal{D}^f(b, \mu) F_{f\leftarrow h}(x, b; \mu, \zeta)$$

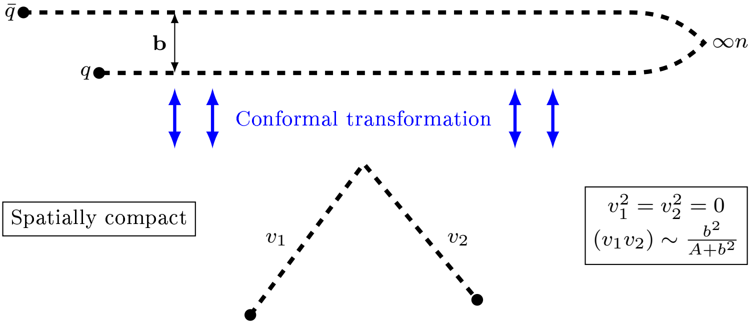
$$O_{\text{TMD}} = \bar{q}(\lambda n + b)[\lambda n + b, -\infty n + b]...[-\infty n, 0]q(0)$$

In non-singular gauges infinity is a single point

Rapidity divergence

- = Anomalous dimension of a distant cusp
- Distance $b \leftrightarrow$ angle

$$O_{\text{TMD}} = \bar{q}(\lambda n + b)[\lambda n + b, -\infty n + b]...[-\infty n, 0]q(0)$$



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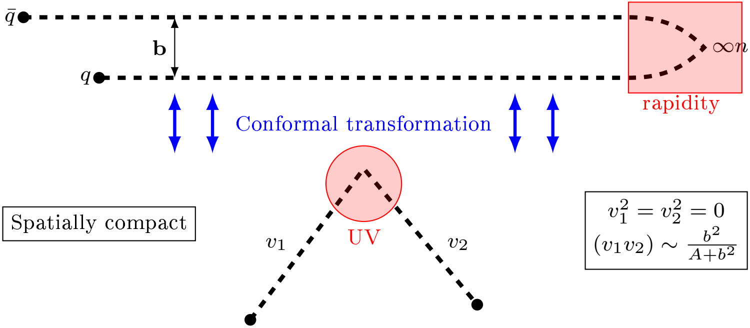

$$\boxed{2\mathcal{D}(\mu, b) = \gamma_S((v_1 v_2), \mu)}$$

RAD $\leftrightarrow$ SAD correspondance

Exact in conformal field theories

In QCD:

$$2\mathcal{D}\Big(\mu, b; \epsilon = \frac{\beta(\alpha_s)}{\alpha_s}\Big) = \gamma_S((v_1 v_2), \mu)$$

Consequences

- Rapidity divergence is **multiplicatively renormalizable**, by factor R

$$\mathcal{D}(b, \mu) = \frac{1}{2} R^{-1}(b, \mu; \nu) \frac{d}{d \ln \nu} R(b, \mu; \nu)$$

- Same RAD for all TMDs of twist-2 and twist-3 (same soft-factor at sub-leading power)
- ...
- N-loop RAD + (N+1)-loop SAD ⇒ (N+1)-loop RAD   checked at N^{3}LO
- Absence of odd-color structures in SAD

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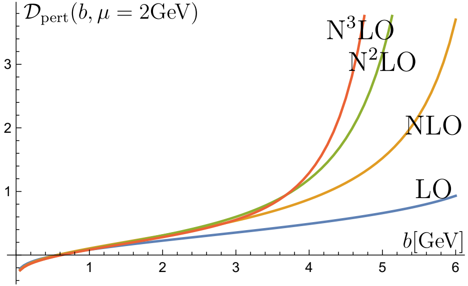
Rapidity anomalous dimension in perturbation theory

- 1-loop

$$\mathcal{D} = -\frac{\Gamma_0}{2\beta_0}\ln\left(1-\beta_0 a_s(\mu)\ln\left(\frac{\mu^2\mathbf{b}^2}{4e^{-2\gamma}}\right)\right) + a_s \ldots$$

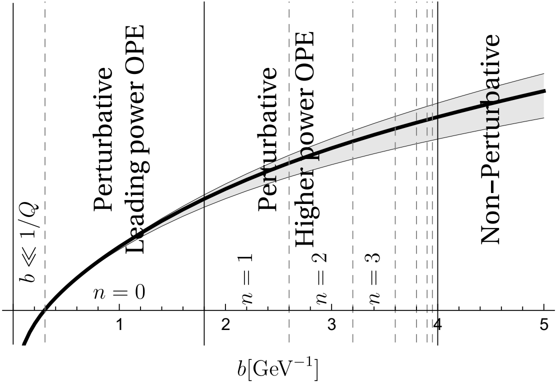
- 2-loop [Echevarria,et al,1511.05590][many others]
- 3-loop [Li,Zhu,1604.01404]
- SAD/RAD correspondence [AV,1610.05791]



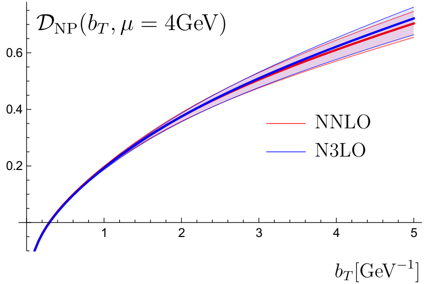
Rapidity anomalous dimension is not an ordinary anomalous dimension

- RAD is non-perturbative function at large-b
- IR Renormalons at $n = 1, 2, 3...$ [Korchemsky,Tafat,2001;Scimemi,AV;2016]

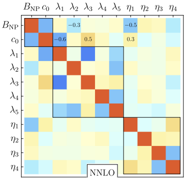
$$\mathcal{D}(b,\mu) = \mathcal{D}_{\rm pert}(\mu, b) + b^2 g_K + b^4 ... \tag{1}$$



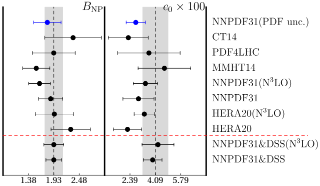
Extracted rapidity anomalous dimension



$$\mathcal{D}_{\text{NP}}(b,\mu) = \mathcal{D}_{\text{perp}}(b^*,\mu) + c_0 b b^*, \qquad b^* = b/\sqrt{1 + b^2/B_{NP}^2}$$



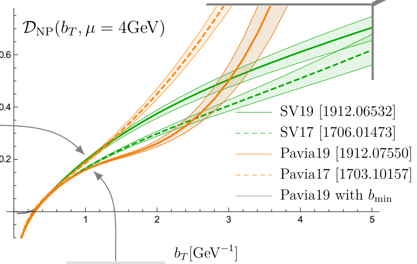
- Extracted from **joined fit** of DY + SIDIS ($2 < Q < 150$GeV)
- Mild correlation with TMDs
- Stable with respect to collinear input



Comparing different extractions

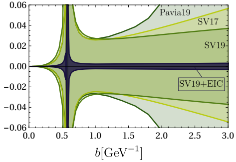
- SV19 $\sim b$
- SV17 $\sim b^2$
- Pavia19 $\sim b^4$
- Pavia17 $\sim b^2$

SIDIS+DY



only DY

Expected impact of EIC

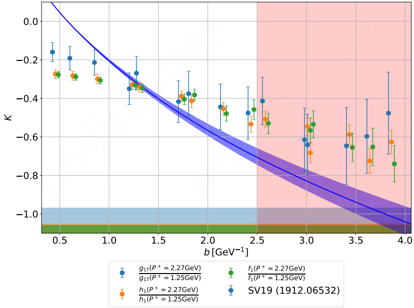


Measurements of CS-kernel on lattice

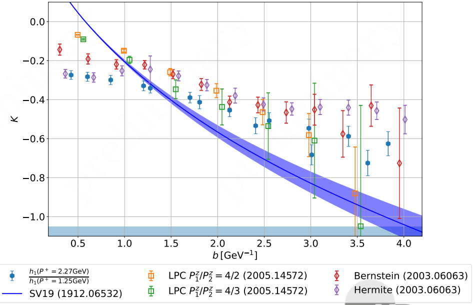
based on [AV,Schafer,2002.07527] see also [Ebert,Stewart,Zhao,1811.00026]

Results by Regensburg lattice group

Different matrix elements



Comparison with [Shanahan, et al, & Zhang, et al]



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Many questions!

- If RAD is “an independent non-perturbative function” how to define in quantum field theory?
- Are there ways to compute the large-b for RAD?
- What it measures/means?

$\Longrightarrow$ [AV,2003.02288], Phys.Rev.Lett. 125 (2020) 19

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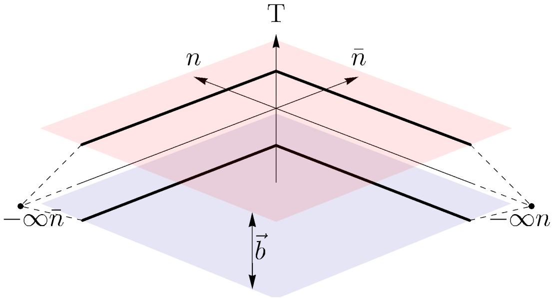
Defining rapidity anomalous dimension

- RAD appears in TMD soft factor

$$S(b,\mu) = \frac{\text{Tr}}{N_c}\langle 0|W_C|0\rangle Z_S^2(\mu)$$

- In some proper regularization of rap.div.($\varrho \to 0$)

$$S(b,\mu) = \exp(2\mathcal{D}(b,\mu)\ln\varrho + B(b,\mu) + ...)$$



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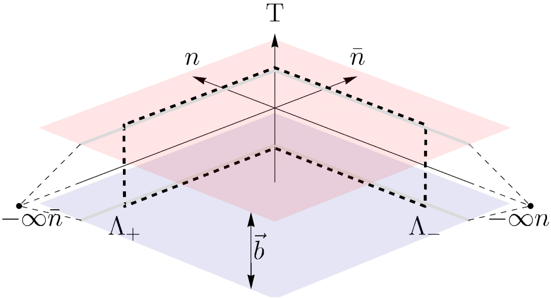


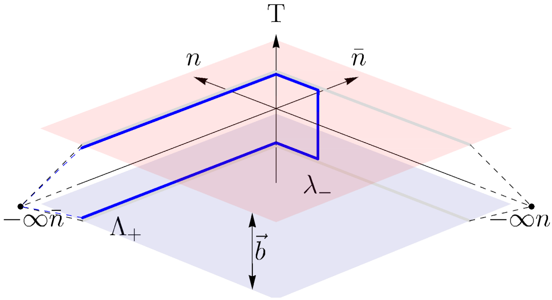
Suitable regulator

- Defined on the operator level
- Preserve gauge invariance
- Do not introduce extra divergences (nor mix with other divergences)

All "convenient" regulators fail...

It can be done by geometric deformation of WL's

$$\varrho = (\Lambda_+ \Lambda_-)^{-1}$$



Only one Λ is needed

$$S(b,\mu)=\frac{\text{Tr}}{N_c}\langle 0|W_{C'}(\Lambda_+\lambda_-)|0\rangle Z_S^2(\mu)=\exp\left(-2\mathcal{D}(b,\mu)\ln(\Lambda_+\lambda_-)+B(\mu,b)+O\left(\frac{1}{\Lambda_+\lambda_-}\right)\right)$$

$$\mathcal{D}(b,\mu)=\frac{1}{2}\lim_{\Lambda_+\to\infty}\frac{d\ln S_{C'}(b,\mu)}{d\ln\lambda_-}$$

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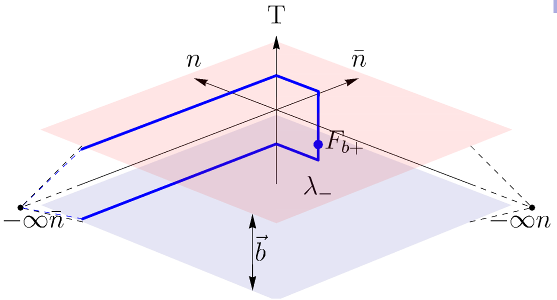

$$\mathcal{D}(b,\mu) = \lambda_- \frac{ig}{2} \frac{\mathrm{Tr} \int_0^1 d\beta \langle 0 | F_{b+}(-\lambda_- n + b\beta) W_{C'} | 0 \rangle}{\mathrm{Tr} \langle 0 | W_{C'} | 0 \rangle} + Z_{\mathcal{D}}(\mu)$$

- λ_- independent!
- Additive renormalization $\frac{dZ_{\mathcal{D}}}{d\ln\mu} = \Gamma_{cusp}(\mu)$

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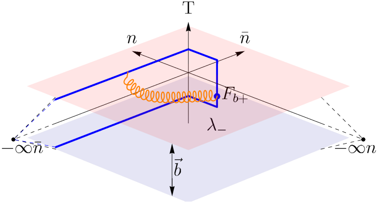
Perturbation

$$\text{LO-diagram} = -4g^2 C_F \frac{\Gamma(2-\epsilon)}{4\pi^{d/2}} \lambda_- \int_0^1 d\beta \int_{-\infty}^0 d\alpha \frac{b^2\beta}{[-2\alpha\lambda_- - \beta^2 b^2 + i0]^{2-\epsilon}}$$



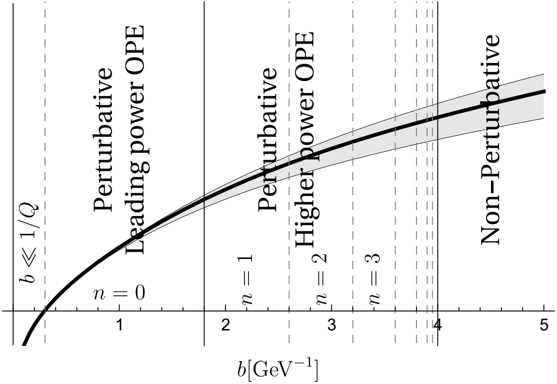
$$\mathcal{D}_{\text{LO}} = -2C_F a_s \left[\Gamma(-\epsilon)\left(\frac{\mathbf{b}^2\mu^2}{4e^{-\gamma_E}}\right)^\epsilon + \frac{1}{\epsilon}\right]$$

- Exactly coincides with ordinary computation (as function of ϵ)
- Renormalization is indeed additive
- Structure of integrals here and in SF is different
- Some two-loop checks also made:
  - Cancellation of rapidity divergences
  - N_f and $1/N_c$ terms (coincide at arbitrary ϵ!)

Small-b expansion

$$\mathcal{D}(b,\mu) = \mathcal{D}_0(b,\mu) + b^2\mathcal{D}_2(b) + b^4\mathcal{D}_4(b) + \ldots$$

- $\mathcal{D}_0$ starts from 1-loop, known up to 3-loop
- $\mathcal{D}_{2,4,..}$ starts from tree-orders, unknown



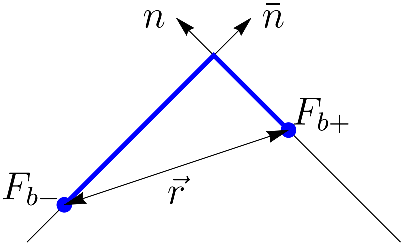
b^2-terms

$$\mathcal{D}_2(b) = \frac{1}{2}\int_0^\infty d\mathbf{r}^2 \frac{\varphi_1(\mathbf{r}^2,0,0)}{\mathbf{r}^2} + O(\alpha_s)$$

$$\Phi_{\mu\nu}(x,y) = g^2\langle 0|F_{\mu x}(x)[x,0][0,y]F_{\nu y}(y)|0\rangle$$

$$\Phi_{\mu\nu}(x,y) = \left(g_{\mu\nu} - \frac{y_\mu x_\nu}{(xy)}\right)\varphi_1(r^2,x^2,y^2) \qquad (16)$$
$$+\frac{(x_\mu(xy)-y_\mu x^2)(y_\nu(xy)-x_\nu y^2)}{(xy)((xy)^2-x^2y^2)}\varphi_2(r^2,x^2,y^2),$$

where $r^2 = (x-y)^2$. At $x^2 = y^2 = 0$, φ_2 vanishes



- Model-independent expression for power correction to CS kernel
- I have also checked it by direct computation of TMD SF in the back-ground (and extraction of rapidity divergence in δ-regulator)
- Gluon$= \frac{C_A}{C_F}$Quark
- Intriguing: This operator is a conf.trans. of gluon TMD operator...

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Numerical estimation

$$\mathcal{D}_2(b) = \frac{1}{2}\int_0^\infty d\mathbf{r}^2 \frac{\varphi_1(\mathbf{r}^2,0,0)}{\mathbf{r}^2} + O(\alpha_s)$$

$$\lim_{\mathbf{r}^2\to 0} \frac{\varphi_1(\mathbf{r}^2,0,0)}{\mathbf{r}^2} = \frac{\pi^2}{36}G_2$$

Assuming that φ_1 has some effective radius $\sim \Lambda_{QCD}^{-1}$

$$\mathcal{D}_2 \sim \frac{\pi^2}{72}\frac{G_2}{\Lambda_{QCD}^2} \simeq (1.-5.)\times 10^{-2}\mathrm{GeV}^{-2} \qquad (2)$$

	Pavia17	SV19	SV17	Pavia19	BLNY(03/14)
$\mathcal{D}_2 \times 10^2$	2.8 ± 0.5	2.9 ± 0.6	$0.7^{+1.2}_{-0.7}$	0.9 ± 0.2	$20 - 35$

The power correction(s) is notably small numerically.
Must be so, since it is (they are) vacuum-dominated parameters

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Some results from Stochastic Vacuum Model

- At large-b linear asymptotic

$$\lim_{\mathbf{b}^2\to\infty}\mathcal{D}(b)=\sqrt{\mathbf{b}^2}\underbrace{\int_0^\infty d\mathbf{y}^2 2\sqrt{y^2}\Delta(\mathbf{y}^2)}_{c_\infty},$$

- Lattice computations [Bali,Brambilla,Vairo,97; Meggiolaro,98]

$$c_\infty \simeq 0.01-0.4\text{GeV} \qquad \text{compare to} \qquad c_\infty^{\text{SV19}} \simeq 0.06\pm0.01\text{GeV}$$

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- In SVM the confining potential is [Brambilla,Vairo,hep-ph/9606344]

$$V(\boldsymbol{b}) = 2\int_0^{\boldsymbol{b}} d\boldsymbol{y}(\boldsymbol{b}-\boldsymbol{y})\int_0^\infty d\boldsymbol{r}\Delta(\sqrt{\boldsymbol{r}^2+\boldsymbol{y}^2}) + \int_0^{\boldsymbol{b}} d\boldsymbol{y}\boldsymbol{y}\int_0^\infty d\boldsymbol{r}\Delta_1(\sqrt{\boldsymbol{r}^2+\boldsymbol{y}^2}).$$

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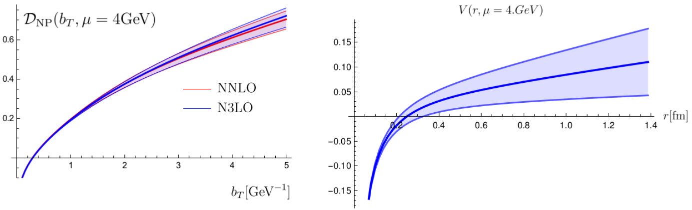

$$V(r) = r\frac{\pi}{4}\mathcal{D}''(0) + \frac{\mathcal{D}'(0)}{2} + \frac{r^2}{2}\int_r^\infty \frac{dx}{x^2}\frac{\mathcal{D}'(x)}{\sqrt{x^2-r^2}} + \dots$$

Conclusion

Collins-Soper kernel is the way to study QCD vacuum with hadron-collider experiments

Self-contained definition of Collins-Soper kernel

- Definition without reference to any process/distribution
- A way to power-corrections, non-perturbative modeling and interpretations
- A peculiar quantum-field-theoretical construction: anomalous dimension as a matrix element
- Casimir scaling at power-correction level

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