Structure functions for the spin-1 deuteron

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7th workshop on Resummation, Evolution, Factorization 2020
(Online) University of Edinburgh, United Kingdom, December 7-11, 2020
https://indico.ph.ed.ac.uk/event/63/

Recent papers: (1) SK and Qin-Tao Song, PRD 94 (2016) 054022.
(2) W. Cosyn, Yu-Bing Dong, SK, M. Sargsian, PRD 95 (2017) 074036.
(3) SK and Qin-Tao Song, PRD 101 (2020) 054011 & 094013.
(4) SK and Qin-Tao Song, arXiv:2011.08583.
(5) A. Arbuzov *et al*. (NICA project), arXiv:2011.15005.

December 9, 2020

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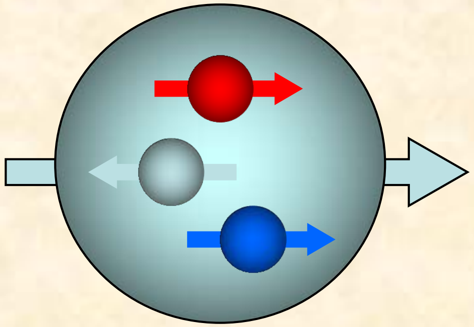
Note on my notations

Gluon transversity: $\Delta_T g$

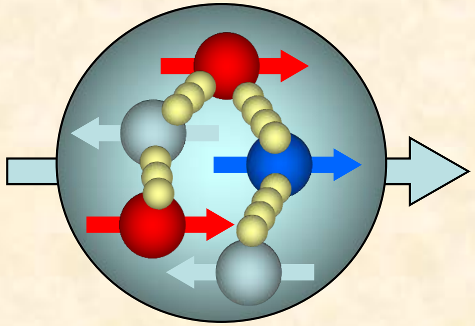
Tensor-polarized gluon distribution: $\delta_T g$

Nucleon spin

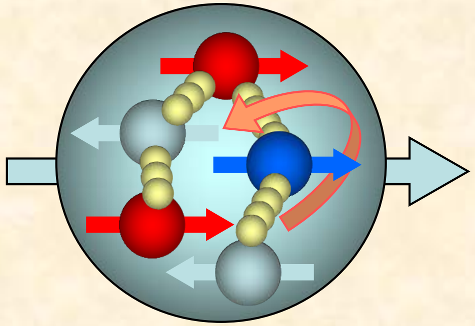
Almost none of nucleon spin is carried by quarks! → **Nucleon spin crisis!?**



Naïve Quark Model



Sea-quarks and gluons?

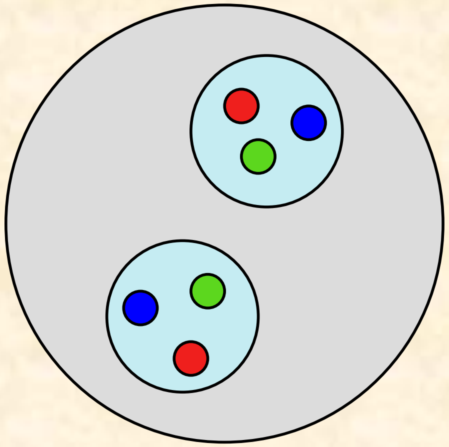


Orbital angular momenta ?

"old" standard model

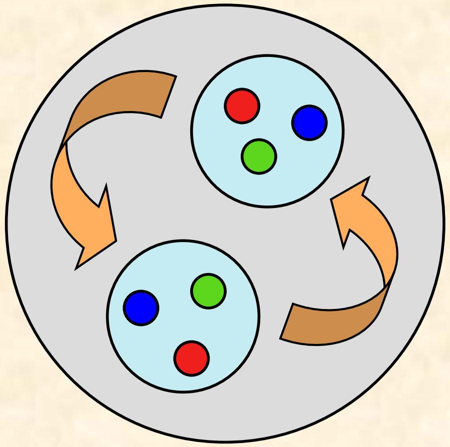
Tensor structure b_1 (*e.g.* deuteron)

Tensor-structure crisis!?



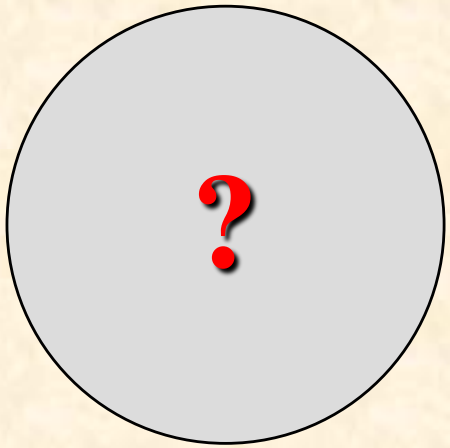
only S wave

$b_1 = 0$



S + D waves

standard model $b_1 \neq 0$

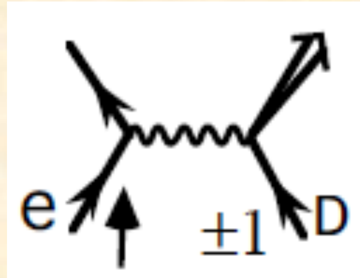


$b_1^{\text{experiment}} \neq b_1^{\text{"standard model"}}$

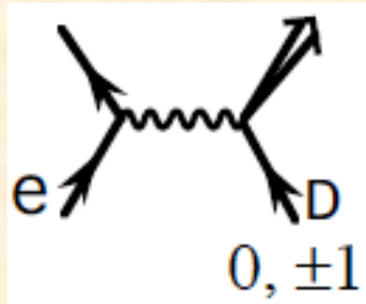
Structure Functions

$$F_1 \propto \langle d\sigma \rangle$$

$$g_1 \propto d\sigma(\uparrow,+1) - d\sigma(\uparrow,-1)$$



$$b_1 \propto d\sigma(0) - \frac{d\sigma(+1) + d\sigma(-1)}{2}$$



note: $\sigma(0) - \frac{\sigma(+1) + \sigma(-1)}{2} = 3\langle \sigma \rangle - \frac{3}{2}[\sigma(+1) + \sigma(-1)]$

Parton Model

$$F_1 = \frac{1}{2}\sum_i e_i^2 \left(q_i + \bar{q}_i\right) \qquad q_i = \frac{1}{3}\left(q_i^{+1} + q_i^{0} + q_i^{-1}\right)$$

$$g_1 = \frac{1}{2}\sum_i e_i^2 \left(\Delta q_i + \Delta \bar{q}_i\right) \qquad \Delta q_i = q_{i\uparrow}^{+1} - q_{i\downarrow}^{+1}$$

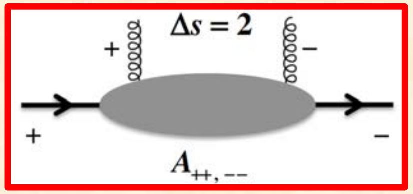
$$\left[q_{\uparrow}^{H}\left(x,Q^2\right)\right]$$

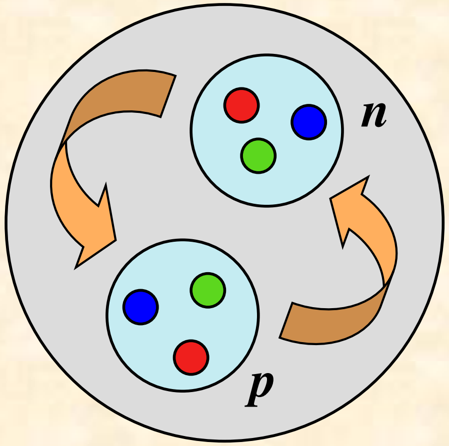
$$b_1 = \frac{1}{2}\sum_i e_i^2 \left(\delta_T q_i + \delta_T \bar{q}_i\right) \qquad \delta_T q_i = q_i^0 - \frac{q_i^{+1} + q_i^{-1}}{2}$$

Gluon transversity $\Delta_T g$

Helicity amplitude $A(\Lambda_i, \lambda_i, \Lambda_f, \lambda_f)$, conservation $\Lambda_i - \lambda_i = \Lambda_f - \lambda_f$

Gluon transversity in deuteron: $\Delta_T g(x) \sim A(+1+1,\ -1-1)$, ~~$A\left(+\frac{1}{2}+1,\ -\frac{1}{2}-1\right)$~~ not possible for nucleon

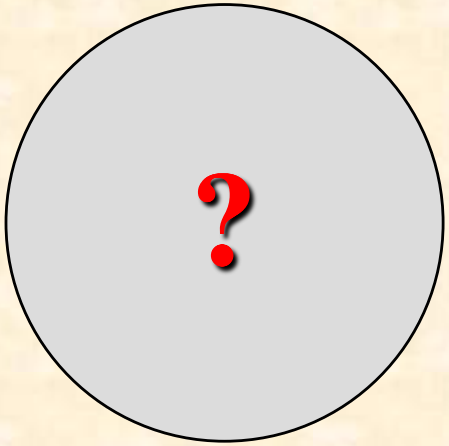




S + D waves

Note: Gluon transversity does not exist for spin-1/2 nucleons.

$b_1\ (\delta_T q,\ \delta_T g) \neq 0 \ \Leftrightarrow\ \text{still } \Delta_T g = 0$



What would be the mechanism(s) for creating $\Delta_T g \neq 0$?

Tensor-polarized structure functions of spin-1 hadrons

S. Kumano, Phys. Rev. D82 (2010) 017501;

S. Kumano and Qin-Tao Song, PRD 94 (2016) 054022;

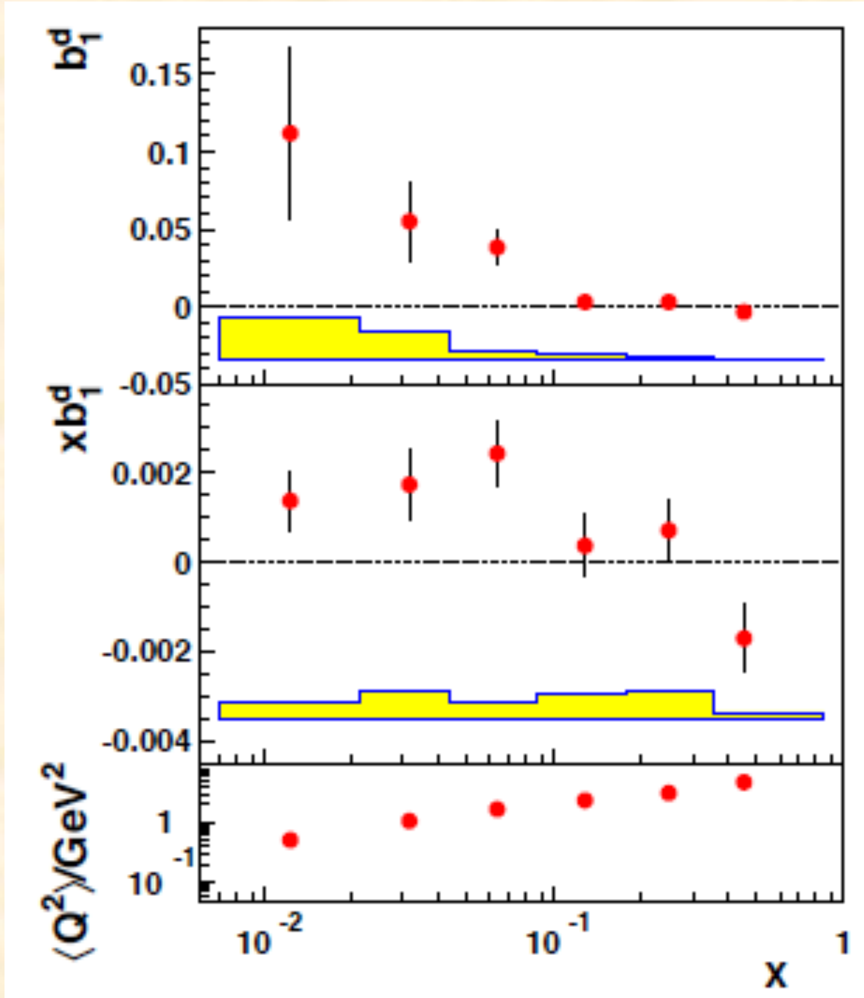
W. Cosyn, Yu-Bing Dong, S. Kumano, and M. Sargsian, PRD 95 (2017) 074036.

HERMES results on b_1

A. Airapetian *et al.* (HERMES), PRL 95 (2005) 242001.

27.6 GeV/c $\quad \rightleftarrows, 0$

positron → ● **deuteron**



b_1 measurement in the kinematical region

$$0.01 < x < 0.45, \quad 0.5 \text{ GeV}^2 < Q^2 < 5 \text{ GeV}^2$$

b_1 sum in the restricted Q^2 range $Q^2 > 1$ GeV^2

$$\int_{0.02}^{0.85} dx\, b_1(x) = [0.35 \pm 0.10(\text{stat}) \pm 0.18(\text{sys})] \times 10^{-2}$$

at $Q^2 = 5$ GeV^2

$$\int dx\, b_1^D(x) = \lim_{t \to 0} -\frac{5}{12}\frac{t}{M^2} F_Q(t) + \underline{\sum_i e_i^2 \int dx\, \delta_T \bar{q}_i(x)} = 0 \ ?$$

b_1 sum rule: F. E. Close and SK, PRD 42 (1990) 2377.

$$\int \frac{dx}{x}\left[F_2^p(x) - F_2^n(x)\right] = \frac{1}{3}\int dx\left[u_v - d_v\right] + \underline{\frac{2}{3}\int dx\left[\bar{u} - \bar{d}\right]} \neq 1/3$$

Drell-Yan experiments probe these antiquark distributions.

Standard model prediction for b_1 of deuteron

Convolution model: $A_{hH,hH}(x,Q^2) = \varepsilon_{h'}^{*\mu} W_{\mu\nu}^{H'H} \varepsilon_h^{\nu} = \int \frac{dy}{y} \sum_s f_s^H(y) \hat{A}_{hs,hs}(x/y, Q^2)$

$$b_1 = A_{+0,+0} - \frac{A_{++,++} + A_{+-,+-}}{2}, \quad \hat{A}_{+\uparrow,+\uparrow} = F_1 - g_1, \quad \hat{A}_{+\downarrow,+\downarrow} = F_1 + g_1$$

Nucleon momentum distribution: $f^H(y) \equiv f_\uparrow^H(y) + f_\downarrow^H(y) = \int d^3p \, y \, |\phi^H(\vec{p})|^2 \delta\left(y - \frac{E - p_z}{M_N}\right)$

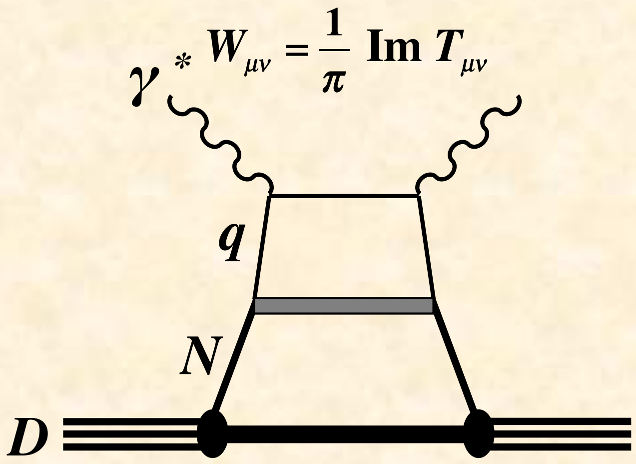
D-state admixture: $\phi^H(\vec{p}) = \phi_{\ell=0}^H(\vec{p}) + \phi_{\ell=2}^H(\vec{p})$

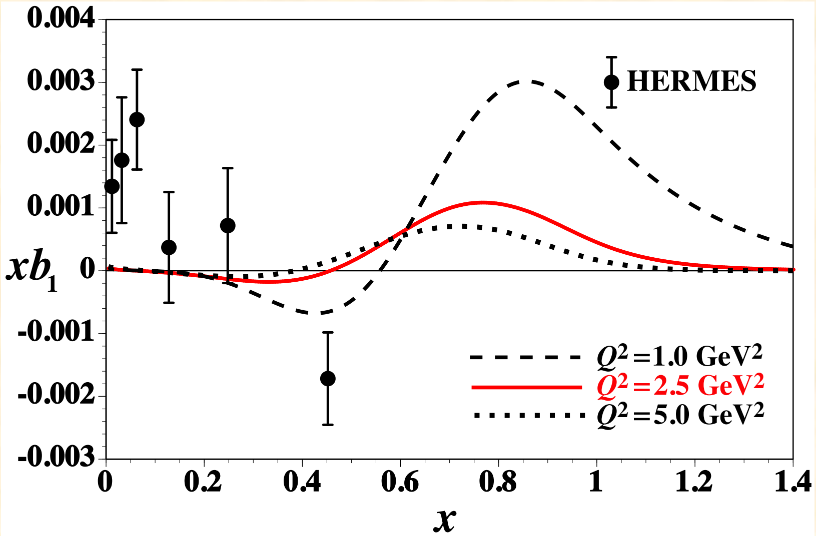
$$b_1(x) = \int \frac{dy}{y} \delta_T f(y) F_1^N(x/y, Q^2), \qquad y = \frac{Mp\cdot q}{M_N P\cdot q} \simeq \frac{2p^-}{P^-}$$

$$\delta_T f(y) = f^0(y) - \frac{f^+(y) + f^-(y)}{2}$$

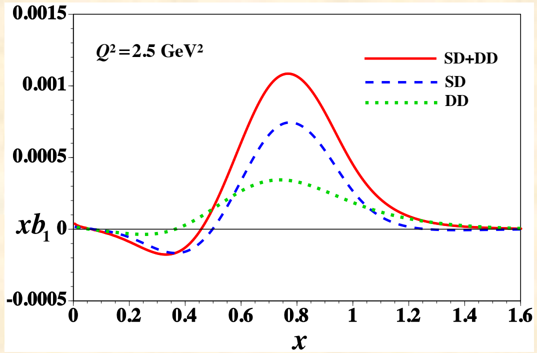
$$= \int d^3p \, y \left[-\frac{3}{4\sqrt{2}\pi} \phi_0(p)\phi_2(p) + \frac{3}{16\pi} |\phi_2(p)|^2 \right] (3\cos^2\theta - 1) \delta\left(y - \frac{p\cdot q}{M_N \nu}\right)$$

S-D term D-D term



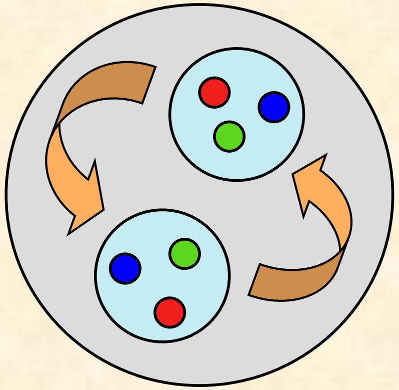


W. Cosyn, Yu-Bing Dong, S. Kumano, M. Sargsian, Phys. Rev. D 95 (2017) 074036.



$|b_1(\text{theory})| \ll |b_1(\text{HERMES})|$ at $x < 0.5$

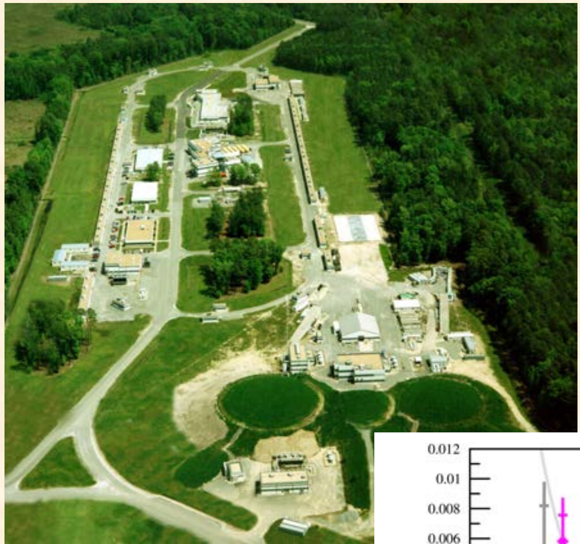
Standard model of the deuteron



S + D waves

Standard convolution model does not work for the deuteron tensor structure!?

Experimental possibilities



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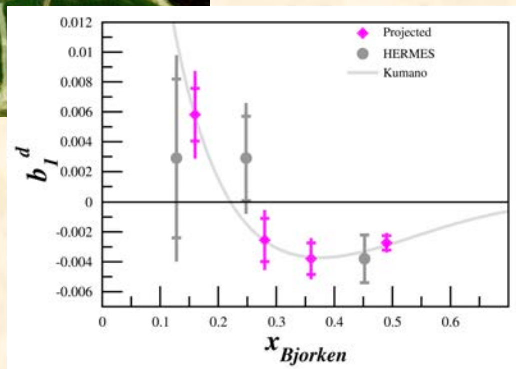
Approved experiment! (PR12-11-110)



E1039 experiment

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NICA

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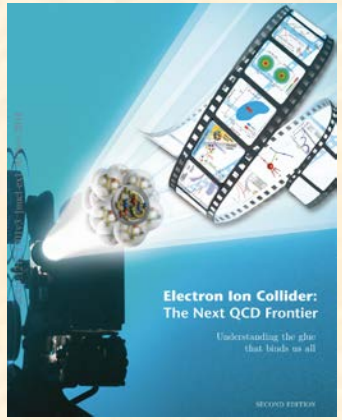
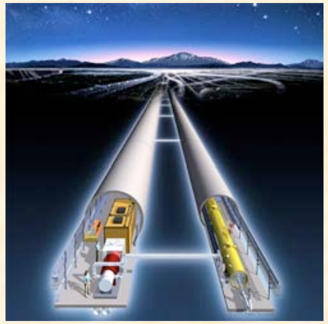
EIC/EicC





Linear Collider?
(with fixed target)

Possibilities: Spin-1 projects are possible in principle at other hadron facilities.

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Experimental possibility at Fermilab

Polarized fixed-target experiments at the Main Injector

Fermilab-E1039 (SpinQuest)

Drell-Yan experiment with a polarized proton target

Co-Spokespersons: A. Klein, X. Jiang, Los Alamos National Laboratory

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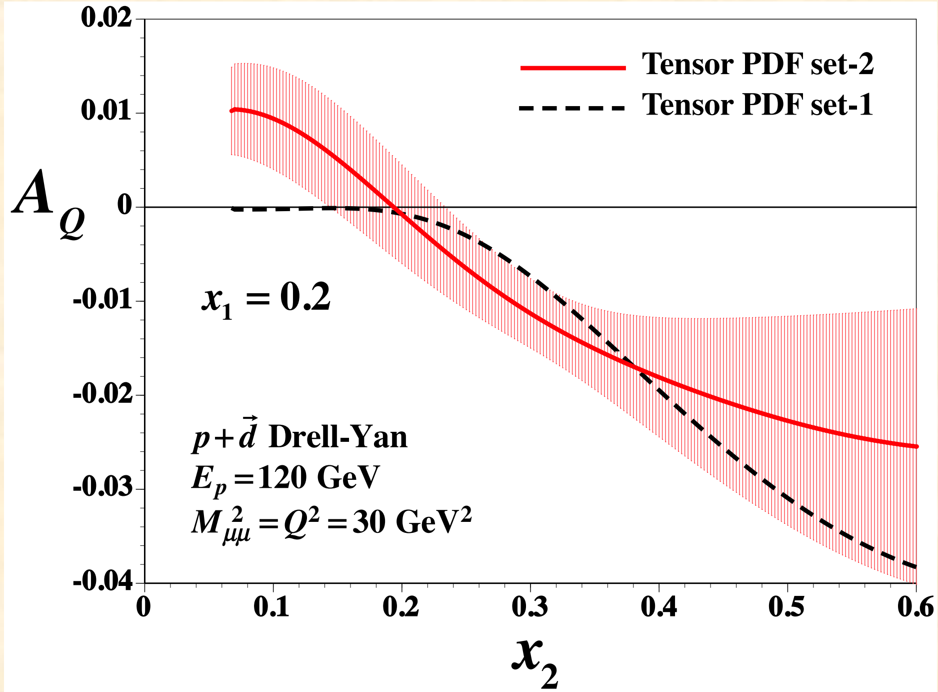
D. Crabb, D. Day, D. Keller, O. Rondon
University of Virginia, Charlottesville, VA 22904

Tensor-polarized spin asymmetry

$$A_Q = \frac{\sum_a e_a^2 \left[q_a(x_A)\, \delta_T \bar{q}_a(x_B) + \bar{q}_a(x_A)\, \delta_T q_a(x_B) \right]}{\sum_a e_a^2 \left[q_a(x_A)\, \bar{q}_a(x_B) + \bar{q}_a(x_A)\, q_a(x_B) \right]}$$

$$\simeq \frac{\sum_a e_a^2 q_a(x_1)\, \delta_T \bar{q}_a(x_2)}{2\sum_a e_a^2 q_a(x_1)\, \bar{q}_a(x_2)} \quad \text{at large } x_F = x_1 - x_2$$



S. Kumano and Qin-Tao Song, Phys. Rev. D94 (2016) 054022.

Possible studies on gluon transversity

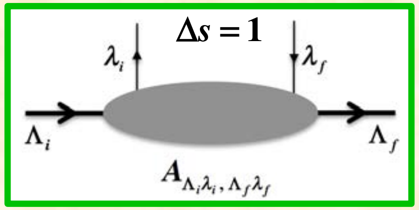
S. Kumano and Qin-Tao Song, Phys. Rev. D 101 (2020) 054011 & 094013;
A. Arbuzov et al. (NICA project), arXiv:2011.15005, submitted to Progress in Nuclear and Particle Physics.

Gluon transversity $\Delta_T g$

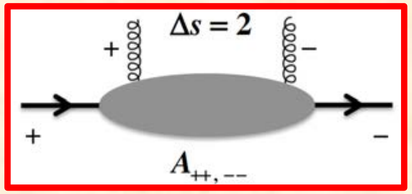
Helicity amplitude $A(\Lambda_i,\lambda_i,\ \Lambda_f,\lambda_f)$, conservation $\Lambda_i-\lambda_i=\Lambda_f-\lambda_f$

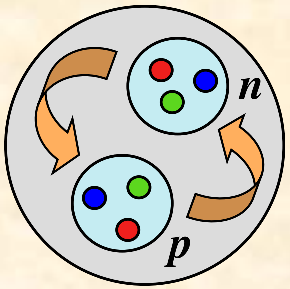
Longitudinally-polarized quark in nucleon: $\Delta q(x)\sim A\left(+\frac{1}{2}+\frac{1}{2},\ +\frac{1}{2}+\frac{1}{2}\right)-A\left(+\frac{1}{2}-\frac{1}{2},\ +\frac{1}{2}-\frac{1}{2}\right)$

Quark transversity in nucleon: $\Delta_T q(x)\sim A\left(+\frac{1}{2}+\frac{1}{2},\ -\frac{1}{2}-\frac{1}{2}\right)$, $\lambda_i=+\frac{1}{2}\to\lambda_f=-\frac{1}{2}$ quark spin flip ($\Delta s=1$)



Gluon transversity in deuteron: $\Delta_T g(x)\sim A(+1+1,\ -1-1)$, ~~$A\left(+\frac{1}{2}+1,\ -\frac{1}{2}-1\right)$~~ not possible for nucleon





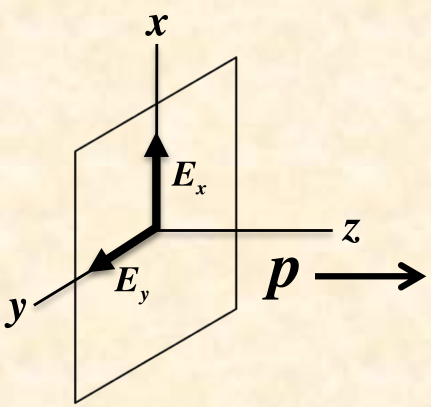
S + D waves

Note: Gluon transversity does not exist for spin-1/2 nucleons.

$b_1\ (\delta_T q,\ \delta_T g)\neq 0\ \Leftrightarrow$ still $\Delta_T g=0$

What would be the mechanism(s) for creating $\Delta_T g\neq 0$?

Gluon transversity distribution in deuteron



Linear-polarization difference: $d\sigma(E_x - E_y) \propto \Delta_T g$

$$\Delta_T g(x) = \int \frac{d\xi^-}{2\pi} x p^+ e^{ixp^+\xi^-} \left\langle pE_x \middle| A^x(0)A^x(\xi) - A^y(0)A^y(\xi) \middle| pE_x \right\rangle_{\xi^+=\vec{\xi}_T=0}$$

$$= g_{\hat{x}/\hat{x}} - g_{\hat{y}/\hat{x}}$$

$g_{\hat{y}/\hat{x}}$ = gluon distribution with the gluon linear polarization ε_y in the deuteron linear polarization E_x

Polarization vectors $\vec{E}_x = \vec{\varepsilon}_x = (1, 0, 0)$, $\vec{E}_y = \vec{\varepsilon}_y = (0, 1, 0)$

Confusing situation of gluon transversity

(no consensus even on its notation: publication # ≈ different notation #)

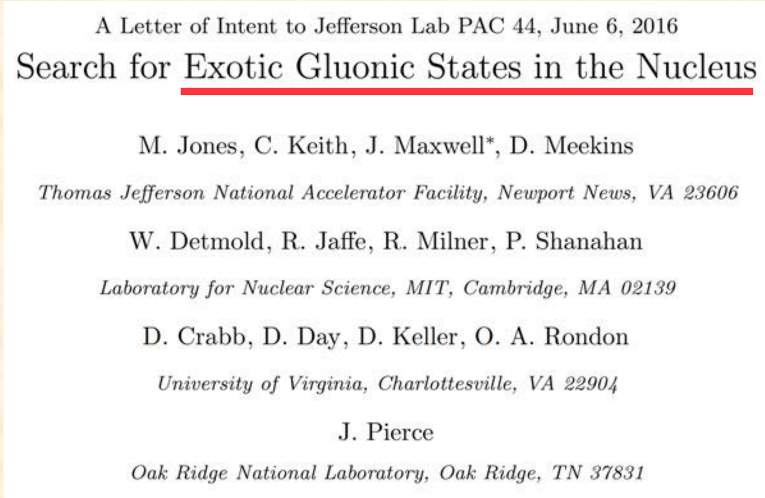
$\Delta_2 G(x) = g_{\hat{x}/\hat{x}}(x) - g_{\hat{y}/\hat{x}}(x)$	[13, 44],
$a(x) = g_{\hat{x}/\hat{x}}(x) - g_{\hat{y}/\hat{x}}(x)$	[23, 25],
$\Delta_L g(x) = g_{\hat{x}/\hat{x}}(x) - g_{\hat{y}/\hat{x}}(x)$	[19],
$\delta G(x) = -g_{\hat{x}/\hat{x}}(x) + g_{\hat{y}/\hat{x}}(x)$	[26, 45],
$h_{1TT,g}(x) = -g_{\hat{x}/\hat{x}}(x) + g_{\hat{y}/\hat{x}}(x)$	[36, 38, 46],
$\Delta_T g(x) = g_{\hat{x}/\hat{x}}(x) - g_{\hat{y}/\hat{x}}(x)$	[47], this work,

→ One can imagine how premature this field is!

Letter of Intent at Jefferson Lab (middle 2020's)

Jefferson Lab,
Electron accelerator ~12 GeV

LoI, arXiv:1803.11206



A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016

Search for Exotic Gluonic States in the Nucleus

M. Jones, C. Keith, J. Maxwell*, D. Meekins

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D. Crabb, D. Day, D. Keller, O. A. Rondon

University of Virginia, Charlottesville, VA 22904

J. Pierce

Oak Ridge National Laboratory, Oak Ridge, TN 37831

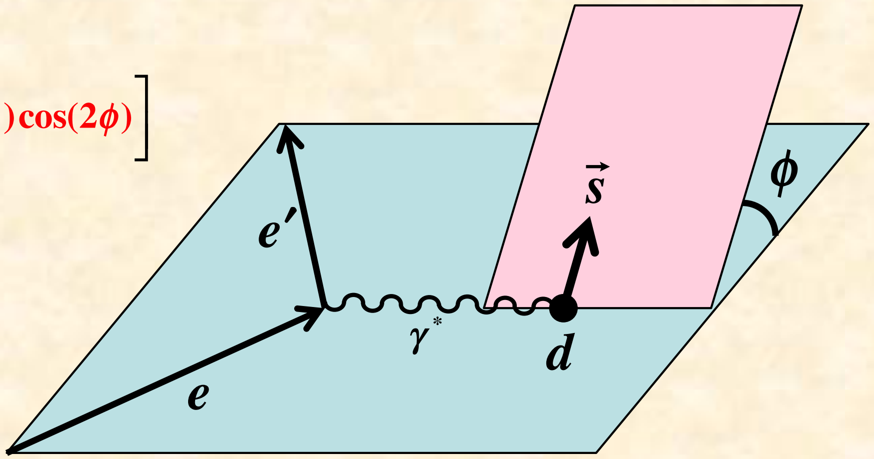
For development of polarized deuteron target,
see D. Keller, D. Crabb, D. Day
Nucl. Inst. Meth. Phys. Res. A981 (2020) 164504.

Electron scattering with polarized-deuteron target

$$\left.\frac{d\sigma}{dx\,dy\,d\phi}\right|_{Q^2 \gg M^2} = \frac{e^4 ME}{4\pi^2 Q^4}\left[xy^2 F_1(x,Q^2) + (1-y)F_2(x,Q^2) - \frac{1}{2}x(1-y)\Delta(x,Q^2)\cos(2\phi)\right]$$

$$\Delta(x,Q^2) = \frac{\alpha_s}{2\pi}\sum_q e_q^2 x^2 \int_x^1 \frac{dy}{y^3}\Delta_T g(y,Q^2)$$

By looking at the deuteron-polarization angle ϕ,
the quark transversty $\Delta_T g$ can be measured.



Proton-deuteron Drell-Yan cross section

Drell-Yan cross section

$$\frac{d\sigma_{pd\to\mu^+\mu^-X}(E_x-E_y)}{d\tau\, dq_T^2\, d\phi\, dy} = \frac{\alpha^2\alpha_s C_F q_T^2}{6\pi s^3}\cos(2\phi)\int_{\min(x_a)}^{1} dx_a \frac{1}{(x_a x_b)^2(x_a-x_1)(\tau-x_a x_2)^2}\sum_q e_q^2 x_a\left[q_A(x_a)+\bar{q}_A(x_a)\right]x_b \Delta_T g_B(x_b)$$

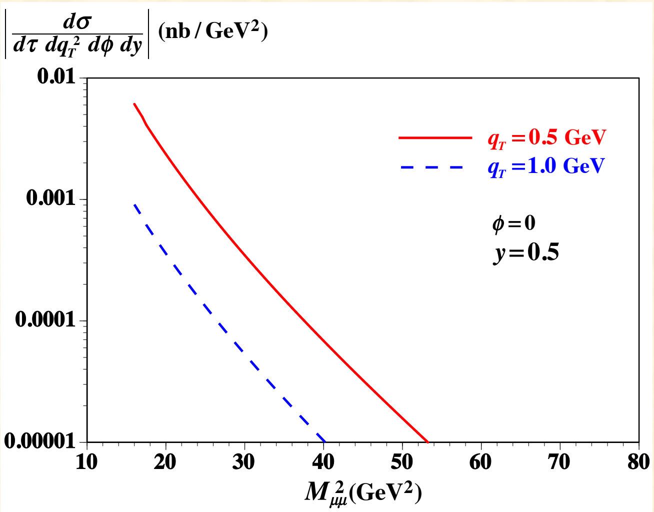
$$C_F=\frac{N_c^2-1}{2N_c},\quad \min(x_a)=\frac{x_1-\tau}{1-x_2},\quad x_b=\frac{x_a x_2-\tau}{x_a-\tau}$$

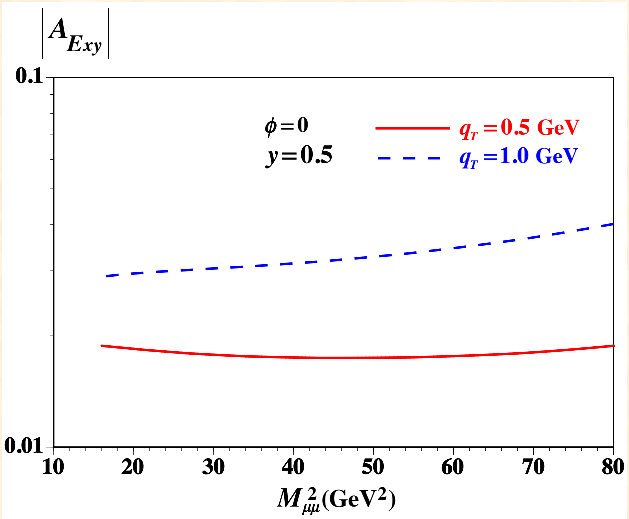
= (unpolarized PDFs of proton) * (gluon transversity distribution in the deuteron)

- Consider the Fermilab-E1039 experiment with the proton beam of $p = 120$ GeV
- No available $\Delta_T g$, so we may tentatively assume $\Delta_T g = \Delta g_p + \Delta g_n$ $\left(\text{or } \frac{\Delta g_p+\Delta g_n}{2},\ \frac{\Delta g_p+\Delta g_n}{4}\right)$
- CTEQ14 for $q(x)+\bar{q}(x)$, NNPDFpol1.1 for $\Delta g(x)$

Cross section: Dimuon mass squred ($M_{\mu\mu}^2 = Q^2$) dependence

Spin asymmetry: $A_{E_{xy}} = \dfrac{\frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy}(E_x)-\frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy}(E_y)}{\frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy}(E_x)+\frac{d\sigma_{pd\to\mu^+\mu^-X}}{d\tau\, dq_T^2\, d\phi\, dy}(E_y)}$





New proposal at Fermilab-PAC in 2021 (D. Keller) !

Experimental possibility at Fermilab in 2020's

Polarized fixed-target experiments
at the Main Injector,
Proton beam = 120 GeV

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Fermilab-E1039

Drell-Yan experiment with a polarized proton target

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Fermilab experimentalists are interested in the gluon transversity by replacing the E1039 proton target for the deuteron one. (Spokesperson of E1039: D. Keller) However, there was no theoretical formalism until our work.

The Transverse Structure of the Deuteron with Drell-Yan

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New proposal is being written for a Fermilab-PAC in 2021.

Nuclotron-based Ion Collider fAcility (NICA)

SPD (Spin Physics Detector for physics with polarized beams)
MPD (MultiPurpose Detector for heavy ion physics)

$\vec{p}+\vec{p}: \quad \sqrt{s_{pp}} = 12 \sim 27$ GeV

$\vec{d}+\vec{d}: \quad \sqrt{s_{NN}} = 4 \sim 14$ GeV

$\vec{p}+\vec{d}$ is also possilbe.

On the physics potential to study the gluon content of proton and deuteron at NICA SPD, A. Arbuzov *et al.*, to be submitted to Progress in Nuclear and Particle Physics.

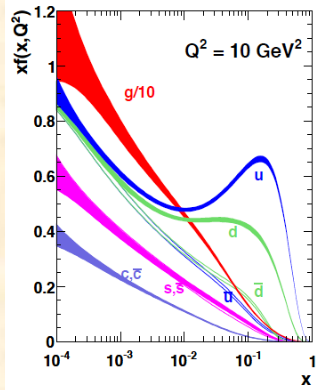
Unique opportunity in high-energy spin physics, especially on the deuteron spin physics.
→ Theoretical formalisms need to be developed.

It is a timely project in 2020's in competition with JLab, Fermilab, and EIC (possibly also J-PARC, GSI-FAIR, EicC).

TMDs for spin-1 hadrons

S. Kumano and Qin-Tao Song,
arXiv:2011.08583

GTMD and Wigner distribution for various structure functions



Form factor ← $\int dx\, d^2k_T$

PDF (Parton Distribution Function) ← $\int d^2k_T,\ \Delta \rightarrow 0$

GTMD $W(x, \vec{k}_T, \Delta) \xrightarrow[\Delta^+ \rightarrow 0]{}$ **Wigner** $W(x, \vec{k}_T, \vec{r}_T)$

$\Delta = p' - p$

$\int d^2k_T$ →

GPD (Generalized Parton Distribution)

$\xrightarrow[s \leftrightarrow t]{}$ **GDA (Generalized Distribution Amplitude)**

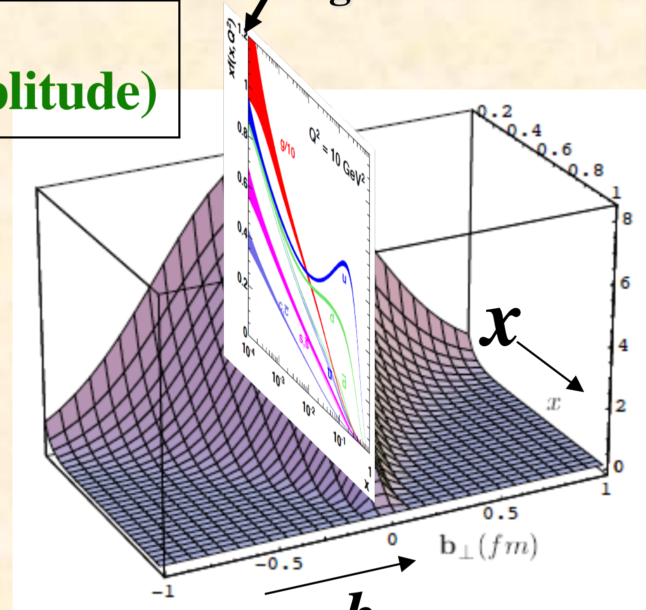
By the two-photon process $\gamma^* \gamma \rightarrow h\bar{h}$.

3D world

$\Delta \rightarrow 0$

TMD (Transverse Momentum Dependent) parton distribution

e.g. HERA studies



TMDs for spin-1 hadrons

Twist-2 TMDs

Quark / Hadron	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5$ / σ^{i+})	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					$[h_1^\perp]$
L			g_{1L}		$[h_{1L}^\perp]$	
T		$f_{1T}^\perp$	g_{1T}		$[h_1], [h_{1T}^\perp]$	
LL	f_{1LL}					$[h_{1LL}^\perp]$
LT	f_{1LT}			g_{1LT}		$[h_{1LT}], [h_{1LT}^\perp]$
TT	f_{1TT}			g_{1TT}		$[h_{1TT}], [h_{1TT}^\perp]$

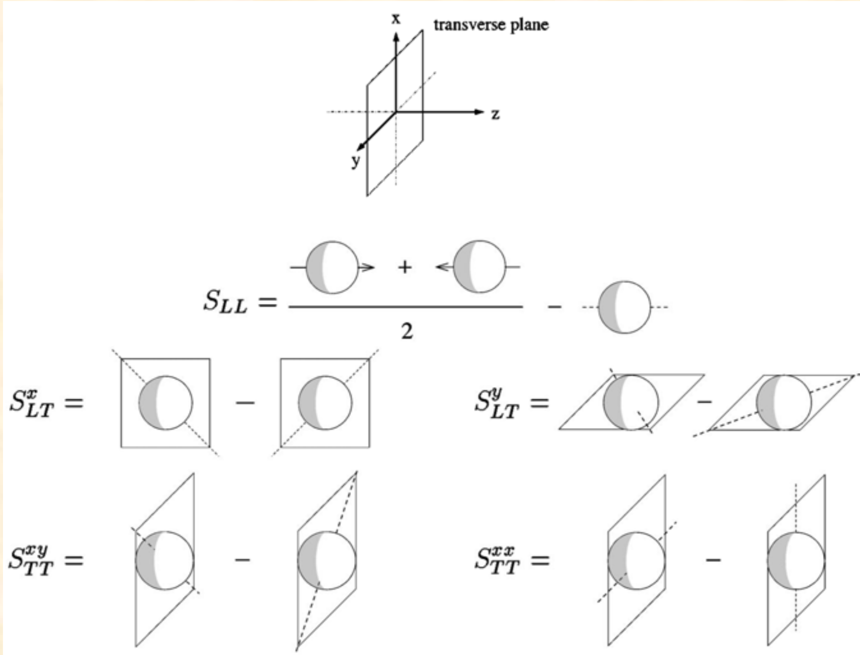
Spin-1/2 nucleon

Bacchetta-Mulders, PRD 62 (2000) 114004.

Twist-2 collinear PDFs

$[\cdots]$= chiral odd

Quark / Hadron	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5$ / σ^{i+})	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					
L			g_{1L} (g_1)			
T					$[h_1]$	
LL	f_{1LL} (b_1)					
LT						
TT						



TMD correlation functions for spin-1 hadrons

Spin vector: $S^\mu = S_L \frac{P^+}{M}\bar{n}^\mu - S_L \frac{M}{2P^+} n^\mu + S_T^\mu$

Tensor: $T^{\mu\nu} = \frac{1}{2}\left[\frac{4}{3}S_{LL}\frac{(P^+)^2}{M^2}\bar{n}^\mu\bar{n}^\nu + \frac{P^+}{M}\bar{n}^{\{\mu}S_{LT}^{\nu\}} - \frac{2}{3}S_{LL}\left(\bar{n}^{\{\mu}n^{\nu\}} - g_T^{\mu\nu}\right) + S_{TT}^{\mu\nu} - \frac{M}{2P^+}n^{\{\mu}S_{LT}^{\nu\}} + \frac{1}{3}S_{LL}\frac{M^2}{(P^+)^2}n^\mu n^\nu\right]$

Tensor part (twist-2): Bacchetta, Mulders, PRD 62 (2000) 114004

$$\Phi(k,P,T) = \left(\frac{A_{13}}{M}I + \frac{A_{14}}{M^2}\not{P} + \frac{A_{15}}{M^2}\not{k} + \frac{A_{16}}{M^3}\sigma_{\rho\sigma}P^\rho k^\sigma\right)k_\mu k_\nu T^{\mu\nu} + \left[A_{17}\gamma_\nu + \left(\frac{A_{18}}{M}P^\rho + \frac{A_{19}}{M}k^\rho\right)\sigma_{\nu\rho} + \frac{A_{20}}{M^2}\varepsilon_{\tau\nu\rho\sigma}P^\rho k^\sigma\gamma^\tau\gamma_5\right]k_\mu T^{\mu\nu}$$

Tensor part (twist-2, 3, 4): n^μ dependent terms are added for up to twist 4.

[For the spin-1/2 nucleon: Goeke, Metzand, Schlegel, PLB 618 (2005) ,90; Metz, Schweitzer, Teckentrup, PLB 680 (2009) 141.]

Kumano-Song-2020, for the details see KEK-TH-2258

$$\Phi(k,P,T\,|\,n) = \left(\frac{A_{13}}{M}I + \frac{A_{14}}{M^2}\not{P} + \frac{A_{15}}{M^2}\not{k} + \frac{A_{16}}{M^3}\sigma_{\rho\sigma}P^\rho k^\sigma\right)k_\mu k_\nu T^{\mu\nu} + \left[A_{17}\gamma_\nu + \left(\frac{A_{18}}{M}P^\rho + \frac{A_{19}}{M}k^\rho\right)\sigma_{\nu\rho} + \frac{A_{20}}{M^2}\varepsilon_{\tau\nu\rho\sigma}P^\rho k^\sigma\gamma^\tau\gamma_5\right]k_\mu T^{\mu\nu}$$

Bacchetta -Mulders

$$+\left(\frac{B_{21}M}{P\cdot n}k_\mu + \frac{B_{22}M^3}{(P\cdot n)^2}n_\mu\right)n_\nu T^{\mu\nu} + i\gamma_5\varepsilon_{\mu\tau\rho\sigma}P^\rho\left(\frac{B_{23}}{(P\cdot n)M}k^\tau n^\sigma k_\nu + \frac{B_{24}M}{(P\cdot n)^2}k^\tau n^\sigma n_\nu\right)T^{\mu\nu}$$

$$+\left[\frac{B_{25}}{P\cdot n}\not{n}k_\mu k_\nu + \left(\frac{B_{26}M^2}{(P\cdot n)^2}\not{n} + \frac{B_{28}}{P\cdot n}\not{P} + \frac{B_{30}}{P\cdot n}\not{k}\right)k_\mu n_\nu + \left(\frac{B_{27}M^4}{(P\cdot n)^3}\not{n} + \frac{B_{29}M^2}{(P\cdot n)^2}\not{P} + \frac{B_{31}M^2}{(P\cdot n)^2}\not{k}\right)n_\mu n_\nu + \frac{B_{32}M^2}{P\cdot n}\gamma_\mu n_\nu\right]T^{\mu\nu}$$

$$-\left[\varepsilon_{\mu\tau\rho\sigma}\gamma^\tau P^\rho\left(\frac{B_{34}}{P\cdot n}n^\sigma k_\nu + \frac{B_{33}}{P\cdot n}k^\sigma n_\nu + \frac{B_{35}M^2}{(P\cdot n)^2}n^\sigma n_\nu\right) + \varepsilon_{\lambda\tau\rho\sigma}k^\lambda\gamma^\tau P^\rho n^\sigma\left(\frac{B_{36}}{P\cdot nM^2}k_\mu k_\nu + \frac{B_{37}}{(P\cdot n)^2}k_\mu n_\nu + \frac{B_{38}M^2}{(P\cdot n)^3}n_\mu n_\nu\right)\right]\gamma_5 T^{\mu\nu}$$

$$+\varepsilon_{\mu\tau\rho\sigma}k^\tau P^\rho n^\sigma\left(\frac{B_{39}}{(P\cdot n)^2}k_\nu + \frac{B_{40}M^2}{(P\cdot n)^3}n_\nu\right)\not{n}\gamma_5 T^{\mu\nu}$$

$$+\sigma_{\rho\sigma}\left[P^\rho k^\sigma\left(\frac{B_{41}}{(P\cdot n)M}k_\mu n_\nu + \frac{B_{42}M}{(P\cdot n)^2}n_\mu n_\nu\right) + P^\rho n^\sigma\left(\frac{B_{43}}{(P\cdot n)M}k_\mu k_\nu + \frac{B_{44}M}{(P\cdot n)^2}k_\mu n_\nu + \frac{B_{45}M^3}{(P\cdot n)^3}n_\mu n_\nu\right)\right]T^{\mu\nu}$$

$$+\sigma_{\rho\sigma}\left[k^\rho n^\sigma\left(\frac{B_{46}}{(P\cdot n)M}k_\mu k_\nu + \frac{B_{47}M}{(P\cdot n)^2}k_\mu n_\nu + \frac{B_{48}M^3}{(P\cdot n)^3}n_\mu n_\nu\right)\right]T^{\mu\nu} + \sigma_{\mu\sigma}\left[n^\sigma\left(\frac{B_{49}M}{P\cdot n}k_\nu + \frac{B_{50}M^3}{(P\cdot n)^2}n_\nu\right) + \left(\frac{B_{51}M}{P\cdot n}P^\sigma + \frac{B_{52}M}{P\cdot n}k^\sigma\right)n_\nu\right]T^{\mu\nu}$$

New terms in our paper

From this correlation function, new tensor-polarized TMDs are defined in twist-3 and 4 in addition to twist-2 ones.

Terms associated with $n = \frac{1}{\sqrt{2}}(1,\ 0,\ 0,\ -1)$

Twist-3,4 TMDs for spin-1 hadrons

New TMDs and PDFs!

Kumano-Song, arXiv:2011.08583

Twist-3 TMDs

Quark / Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$f^\perp$ $[e]$			$g^\perp$		$[h]$
L		$f_L^\perp$ $[e_L]$	$g_L^\perp$		$[h_L]$	
T		$f_T, f_T^\perp$ $[e_T, e_T^\perp]$	$g_T, g_T^\perp$		$[h_T], [h_T^\perp]$	
LL	$f_{LL}^\perp$ $[e_{LL}]$			$g_{LL}^\perp$		$[h_{LL}]$
LT	$f_{LT}, f_{LT}^\perp$ $[e_{LT}, e_{LT}^\perp]$			$g_{LT}, g_{LT}^\perp$		$[h_{LT}], [h_{LT}^\perp]$
TT	$f_{TT}, f_{TT}^\perp$ $[e_{TT}, e_{TT}^\perp]$			$g_{TT}, g_{TT}^\perp$		$[h_{TT}], [h_{TT}^\perp]$

Twist-4 TMDs

Quark / Hadron	γ^-		$\gamma^-\gamma_5$		σ^{i-}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_3					$[h_3^\perp]$
L			g_{3L}		$[h_{3L}^\perp]$	
T		$f_{3T}^\perp$	g_{3T}		$[h_{3T}], [h_{3T}^\perp]$	
LL	f_{3LL}					$[h_{3LL}^\perp]$
LT	f_{3LT}			g_{3LT}		$[h_{3LT}], [h_{3LT}^\perp]$
TT	f_{3TT}			g_{3TT}		$[h_{3TT}], [h_{3TT}^\perp]$

Twist-3 collinear PDFs

$[\cdots]$= chiral odd

Quark / Hadron	$\gamma^i, 1, i\gamma_5$		$\gamma^+\gamma_5$		σ^{ij}, σ^{-+}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$[e]$					
L					$[h_L]$	
T			g_T			
LL	$[e_{LL}]$					
LT	f_{LT}					
TT						

Twist-4 collinear PDFs

$[\cdots]$= chiral odd

Quark / Hadron	γ^-		$\gamma^-\gamma_5$		σ^{i-}	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_3					
L			g_{3L}			
T					$[h_{3T}]$	
LL	f_{3LL}					
LT						
TT						

Sum rules for TMDs of spin-1 hadrons

Twist-2 TMDs

Quark / Hadron	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-even	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-odd
U	f_1					$[h_1^\perp]$
L			g_{1L}		$[h_{1L}^\perp]$	
T		$f_{1T}^\perp$	g_{1T}		$[h_1], [h_{1T}^\perp]$	
LL	f_{1LL}					$[h_{1LL}^\perp]$
LT	f_{1LT}			g_{1LT}		$[h_{1LT}], [h_{1LT}^\perp]$
TT	f_{1TT}			g_{1TT}		$[h_{1TT}], [h_{1TT}^\perp]$

Time-reversal invariance in colliear corrlation functions (PDFs)

$$\int d^2k_T \Phi_{\text{T-odd}}(x,k_T^2) = 0$$

New sum rules for the TMDs of spin-1 hadrons

$$\int d^2k_T\, h_{1LT}(x,k_T^2) = 0, \qquad \int d^2k_T\, g_{LT}(x,k_T^2) = 0,$$

$$\int d^2k_T\, h_{LL}(x,k_T^2) = 0, \qquad \int d^2k_T\, h_{3LT}(x,k_T^2) = 0$$

For example, in the twist-4

$$\int d^2k_T\, h_{3LT}(x,k_T^2) \equiv \int d^2k_T \left[h'_{3LT}(x,k_T^2) - \frac{k_T^2}{2M^2} h_{3LT}^\perp(x,k_T^2) \right] = 0$$

$$\Phi^{[\sigma^{i-}]} = \frac{M^2}{P^{+2}} \Big[h_{3LL}^\perp(x,k_T^2) S_{LL} \frac{k_T^i}{M} + h'_{3LT}(x,k_T^2) S_{LT}^i - h_{3LT}^\perp(x,k_T^2) \frac{k_T^i S_{LT}\cdot k_T}{M^2} - h'_{3TT}(x,k_T^2) \frac{S_{TT}^{ij} k_{Tj}}{M} + h_{3TT}^\perp(x,k_T^2) \frac{k_T\cdot S_{TT}\cdot k_T}{M^2} \frac{k_T^i}{M} \Big]$$

Twist-3 TMDs

Quark / Hadron	$\gamma^i, 1, i\gamma_5$ T-even	$\gamma^i, 1, i\gamma_5$ T-odd	$\gamma^+\gamma_5$ T-even	$\gamma^+\gamma_5$ T-odd	σ^{ij}, σ^{-+} T-even	σ^{ij}, σ^{-+} T-odd
U	$f^\perp$ $[e]$			$g^\perp$		$[h]$
L		$f_L^\perp$ $[e_L]$	$g_L^\perp$		$[h_L]$	
T		$f_T, f_T^\perp$ $[e_T, e_T^\perp]$	$g_T, g_T^\perp$		$[h_T], [h_T^\perp]$	
LL	$f_{LL}^\perp$ $[e_{LL}]$			$g_{LL}^\perp$		$[h_{LL}]$
LT	$f_{LT}, f_{LT}^\perp$ $[e_{LT}, e_{LT}^\perp]$			$g_{LT}, g_{LT}^\perp$		$[h_{LT}], [h_{LT}^\perp]$
TT	$f_{TT}, f_{TT}^\perp$ $[e_{TT}, e_{TT}^\perp]$			$g_{TT}, g_{TT}^\perp$		$[h_{TT}], [h_{TT}^\perp]$

Twist-4 TMDs

Quark / Hadron	γ^- T-even	γ^- T-odd	$\gamma^-\gamma_5$ T-even	$\gamma^-\gamma_5$ T-odd	σ^{i-} T-even	σ^{i-} T-odd
U	f_3					$[h_3^\perp]$
L			g_{3L}		$[h_{3L}^\perp]$	
T		$f_{3T}^\perp$	g_{3T}		$[h_{3T}], [h_{3T}^\perp]$	
LL	f_{3LL}					$[h_{3LL}^\perp]$
LT	f_{3LT}			g_{3LT}		$[h_{3LT}], [h_{3LT}^\perp]$
TT	f_{3TT}			g_{3TT}		$[h_{3TT}], [h_{3TT}^\perp]$

New fragmentations for spin-1 hadrons

Corresponding fragmentation functions exist for the spin-1 haddrons

simply by changing function names and kinematical variables.

TMD distribution functions: $f,\ g,\ h,\ e\ ;\quad x,\ k_T,\ S,\ T,\ M,\ n,\ \gamma^+,\ \sigma^{i+}$

$$\Downarrow$$

TMD fragmentation functions: $D,\ G,\ H,\ E\ ;\quad z,\ k_T,\ S_h,\ T_h,\ M_h,\ \bar{n},\ \gamma^-,\ \sigma^{i-}$

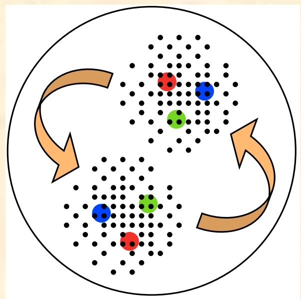
Summary

Spin-1 structure functions of the deuteron (new spin structure)

- **tensor structure in quark-gluon degrees of freedom**
- **gluon transversity**
- **new signature beyond "standard" hadron physics?**
- **experiments: JLab (approved), Fermilab (to be proposed), ... ,**
  NICA (in progress), EIC, EicC, ...
- **TMDs: interdisciplinary field of physics**
  ***e.g.* Color Aharonov-Bohm effect, Color entanglement**

We proposed new TMDs and PDFs in twist 3 and 4.

standard model



?

New exotic mechanisms in b_1 ($\delta_T q$, $\delta_T g$) and $\Delta_T g$

The End

The End