Transverse momentum dependent splitting functions in Parton Branching based evolution equations

REF workshop 2020

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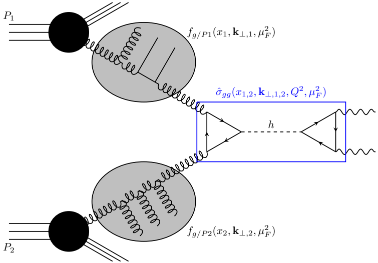
Universiteit Antwerpen

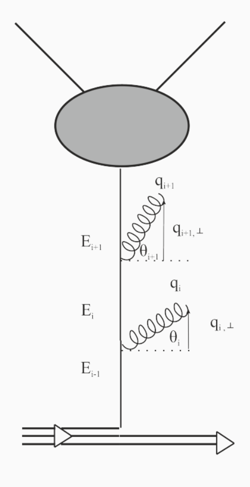
Motivation



- Collinear factorization is commonly used
- Some classes of processes require more general scheme
- Factorization in partonic cross-section and transverse momentum dependent PDFs (TMDs)

- TMDs much less known than PDFs at present $\rightarrow$ future experimental programs
- TMDs from Parton Branching (PB) method: Can be used in Monte Carlo events generators

Angular ordering



- Angular ordering in initial state radiation:

  $\Theta_{i+1} > \Theta_i \to \bar{q}_{\perp,i+1} > z_i \bar{q}_{\perp,i}$

  with $\bar{q}_{\perp,i} = \frac{q_{\perp,i}}{1-z_i}$ rescaled transverse momentum of emitted parton

  In limit z$\longrightarrow$1, this gives: $\bar{q}_{\perp,i+1} > \bar{q}_{\perp,i}$

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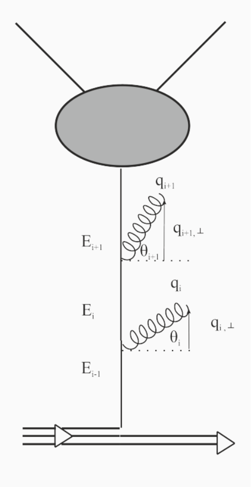

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  $\rightarrow$ Associate evolution scale $\mu = \bar{q}_{\perp,i}$

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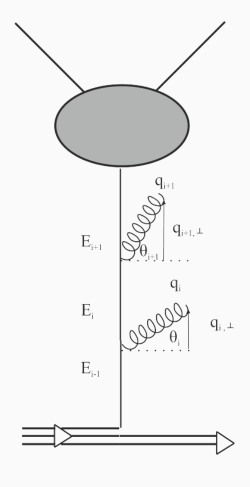

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  Widely used concept, examples:

  - PDFs: Catani-Marchesini-Webber (CMW)
  - Event generator: HERWIG
  - To obtain TMDs: PB

Iterative evolution equations

- $P^R_{ab}(z)$: (real emission part of) Splitting functions: Probabilty that a branching will happen
  b: incoming parton, a: outgoing parton, z momentum fraction of parton a to b
- Sudakov form factor:
  $\Delta_a(\mu^2) = \text{exp}\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\, z\, P^R_{ba}(z)\right)$
  Interpretation: probability of an evolution without any resolvable branchings

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Iterative form of the PB evolution equation: [Hautmann, Jung, Lelek, Radescu, Zlebcik JHEP 01 (2018) 070, 1708.03279]

$$\tilde{A}_a(x, \mathbf{k}_\perp, \mu^2) = \Delta_a(\mu^2)\tilde{A}_a(x, \mathbf{k}_\perp, \mu_0^2) + \sum_b \int \frac{d^2\mu'_\perp}{\pi\mu'^2}\Theta(\mu^2 - \mu'^2)\Theta(\mu'^2 - \mu_0^2)\frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \times$$
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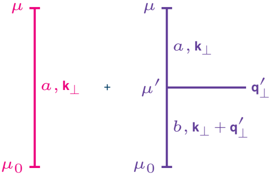

$$\mathbf{q}'_\perp = (1 - z')\mu'$$

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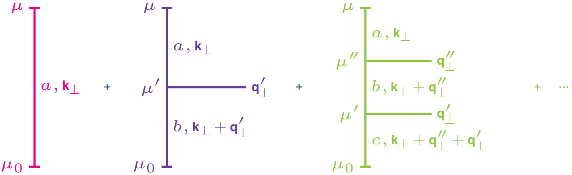

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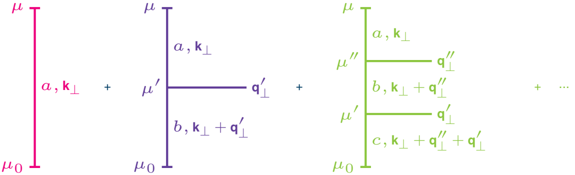

$\mathbf{q}'_\perp = (1 - z')\mu'$
$\mathbf{q}''_\perp = (1 - z'')\mu''$

PB calculates $\mathbf{k}_\perp$ from every branching:
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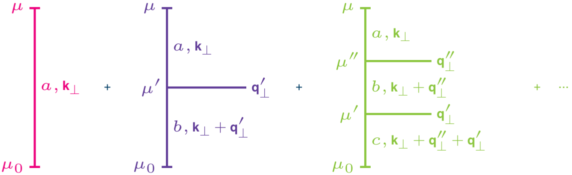

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Can be solved with MC methods.

Parton Branching equations

Parton branching equations for TMDs:

$$\tilde{\mathcal{A}}_a(x, \mathbf{k}_\perp, \mu^2) = \Delta_a(\mu^2)\tilde{\mathcal{A}}_a(x, \mathbf{k}_\perp, \mu_0^2) + \sum_b \int \frac{d^2\mu'_\perp}{\pi\mu'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \Theta(\mu^2 - \mu'^2)\Theta(\mu'^2 - \mu_0^2) \times$$
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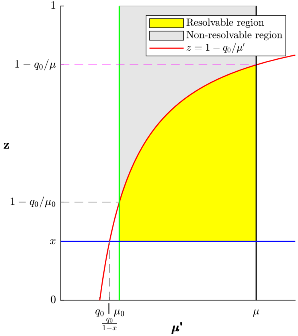
  q_0 smallest emitted transverse momentum

[Hautmann, Keersmaekers, Lelek, van Kampen NuclPhysB (2019) 114795,1908.08524



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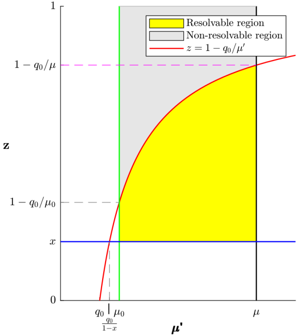

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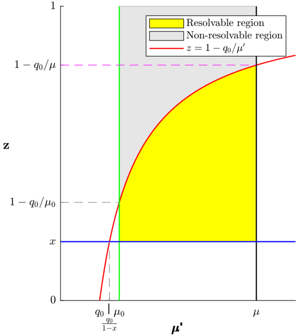
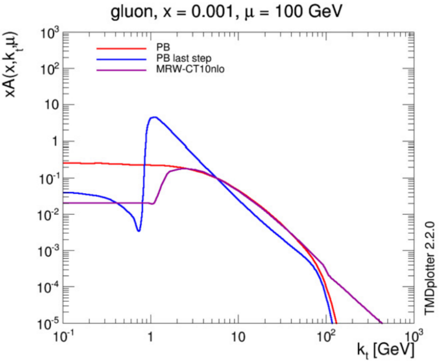
- Implicit in $P_{ab}^R(z)$ and $\Delta_a(\mu^2)$: $\alpha_s(q_\perp)$
- $f_a(x,\mu^2) = \int \frac{d^2\mathbf{k}_\perp}{\pi}\mathcal{A}_a(x,\mathbf{k}_\perp,\mu^2)$
  at LO $\rightarrow$ Catani-Marchesini-Webber
  $\alpha_s(\mu)$ and $z_M = 1 \rightarrow$ DGLAP



Effects of multiple branchings



PB last step is a toy model where
$\mathbf{k}_\perp = \mathbf{k}_{\perp,0} - \mathbf{q}_{\perp,n}$ ($q_\perp$ from last branching)
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Figure from [NuclPhysB (2019) 114795,1908.08524]

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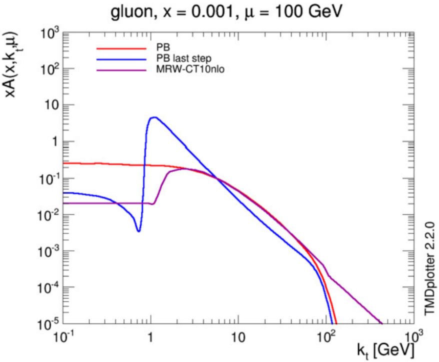

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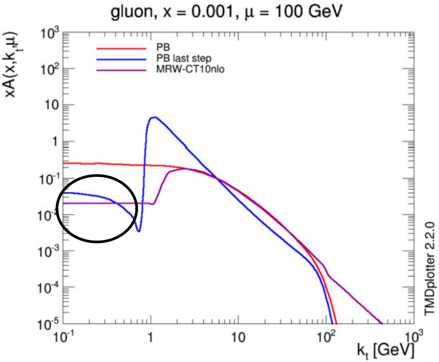

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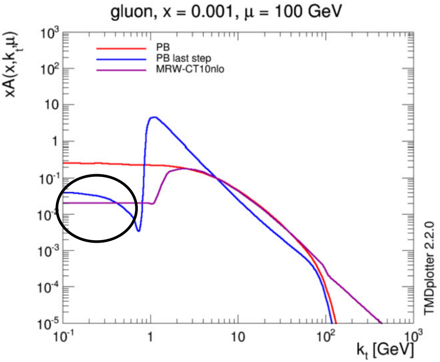

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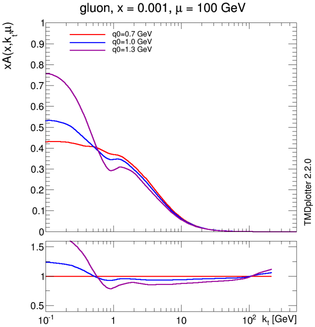
  with $q_s^2 = \sigma^2/2$. Here q_s=0.5GeV
- Gaussian function is clearly visible in "PB last step"$\rightarrow$ partons that had no branching
- Very large bump visible around minimal emitted $q_\perp$: q_0=1 GeV for "PB last step"
- Many branchings smear out this bumps

Effects of q_0 and q_s



q_s=0.5 GeV

$$z_M = 1 - q_0/\mu'$$

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- q_0 minimal emitted $q_\perp$. When q_0 is larger, resolvable region is smaller $\rightarrow$ Less branchings $\rightarrow$ More partons with intrinsic $k_\perp$
- Bumps matching between $\Delta_a(\mu)\tilde{\mathcal{A}}_0$ and evolution. Many branchings smooth out bumps

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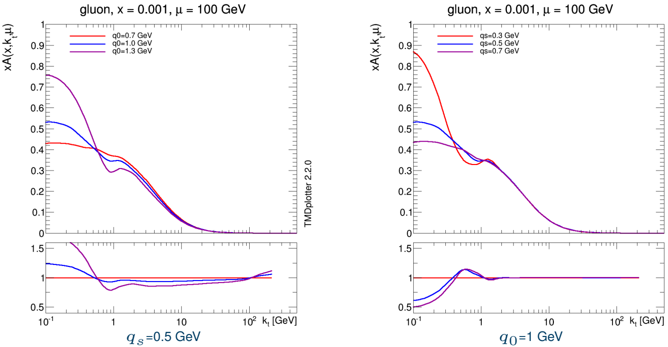

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$\tilde{\mathcal{A}}_0 \equiv \tilde{\mathcal{A}}_a(x, \mathbf{k}_{\perp,0}, \mu_0^2) = f_a(x, \mu_0^2) \times \frac{1}{q_s\sqrt{\pi}} \exp\left(-\frac{\mathbf{k}_{\perp,0}^2}{q_s^2}\right)$

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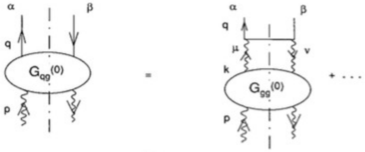
- Bumps matching between $\Delta_a(\mu)\tilde{\mathcal{A}}_0$ and evolution. Many branchings smooth out bumps
- When q_s closer to q_0 $\rightarrow$ smoother
- q_s only affects small $k_\perp$-region when $\mu \gg q_s$
- Best value of q_0, q_s yet to be determined (fits with dyn. z_M). Choices of q_0=1 GeV, q_s=0.5 GeV seem to give good results [Eur.Phys.J.C 80 (2020) 7, 598]

TMD Splitting functions

- Concept from high-energy factorization [Catani, Hautmann NPB427 (1994) 475524, hep-ph/9405388]
- Goal of TMD Splitting Functions:
  - Resummation in $\alpha_s \ln \frac{1}{x}$
  - Exact kinematics in both $k_\perp$ and x



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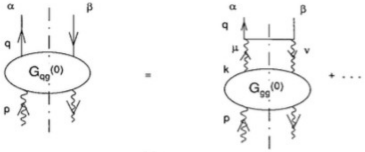

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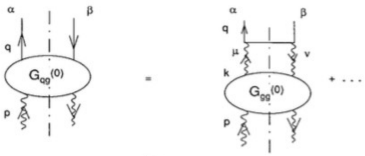

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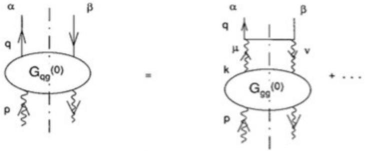

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- PB uses DGLAP splitting functions, but those are not valid for small-x
- In PB $k_\perp$ is known at every branching

$\Rightarrow$ Goal of this work: extend PB by including TMD splitting functions

Evolution equations with TMD Splitting functions

$P_{ab}(z) \rightarrow \tilde{P}_{ab}(z, \mathbf{k}_\perp, \mathbf{q}_\perp)$

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Sudakov form factor: probability of an evolution without any resolvable branchings

- Sudakov should depend on $\mathbf{k}_\perp$
- Should sum over all possible splittings $\rightarrow$ integrate over all angles

$$\Delta_a(\mu^2) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\, z\, P_{ba}^R(z)\right) \rightarrow$$

$$\Delta_a(\mu^2, \mathbf{k}_\perp^2) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\, z\, \frac{1}{\pi} \int_0^{\pi} d\phi\, \tilde{P}_{ba}^R\left(z, \mathbf{k}_\perp, (1-z)\mu_\perp{}'\right)\right)$$

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$$\tilde{\mathcal{A}}_a(x, \mathbf{k}_\perp, \mu^2) = \Delta_a(\mu^2, \mathbf{k}_\perp)\tilde{\mathcal{A}}_a(x, \mathbf{k}_\perp, \mu_0^2) + \sum_b \int \frac{d^2\mu'_\perp}{\pi\mu'^2} \frac{\Delta_a(\mu^2, \mathbf{k}_\perp)}{\Delta_a(\mu'^2, \mathbf{k}_\perp)} \Theta(\mu^2 - \mu'^2)\Theta(\mu'^2 - \mu_0^2) \times$$
$$\times \int_x^{z_M} dz \tilde{P}_{ab}^R(z, \mathbf{k}_\perp + (1-z)\mu'_\perp, (1-z)\mu'_\perp)\tilde{\mathcal{A}}_b(\frac{x}{z}, \mathbf{k}_\perp + (1-z)\mu'_\perp, \mu'^2)$$

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- In PB MC code, the scale should be generated according to the Sudakov form factor: Generate $R = \Delta_a(\mu_i^2)/\Delta_a(\mu_{i-1}^2) \rightarrow$ find $\mu_i^2 = \Delta_a^{-1}(R\Delta_a(\mu_{i-1}^2))$
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- In the normal PB code a table is calculated $\to$ interpolate to find μ_i
  With $k_\perp$-dependent Sudakov form factor, additional dimension $\to$ extensive calculation
- Used VETO-algorithm instead

VETO Algorithm [hep-ph/0603175]

$$\Delta_a(\mu^2, \mathbf{k}_\perp) = \exp\left(-\int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} f_a(\mu'^2, \mathbf{k}_\perp)\right) \text{ with}$$

$$f_a(\mu^2, \mathbf{k}_\perp) = \sum_b \int_0^{z_M} dz\, z\, \frac{1}{\pi} \int_0^{\pi} d\phi P_{ba}^R(z, \mathbf{k}_\perp, \mu'_\perp)$$

Find better function $g_a(\mu^2) \geq f_a(\mu^2, \mathbf{k}_\perp)$ for all μ.

1. Start with $j = 0$, $\mu^2_{j=0} = \mu^2_{i-1}$
2. j=j+1. Select $\mu_j^2 > \mu_{j-1}^2$ according to $R_1 = \exp\left(-\int_{\mu_{j-1}^2}^{\mu_j^2} \frac{d\mu'^2}{\mu'^2} g_a(\mu'^2)\right)$
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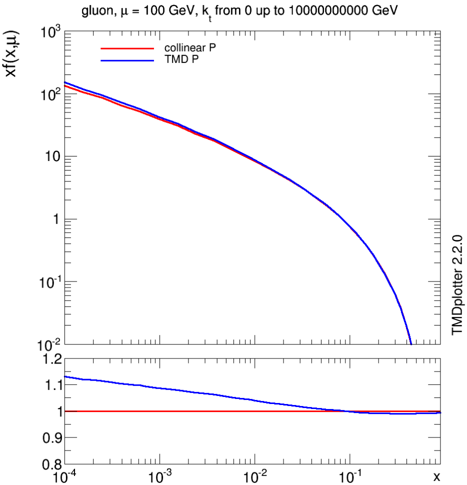
Usually g is chosen to have an known inverse function. We chose
$g_a(\mu^2) = \sum_b \int_0^{z_M} dz\, z\, (P^R_{ba}(z) + h_{ba}(z))$. No known inverse, but:

- Close to $f_a \rightarrow$ efficient
- No $k_\perp$-dependence $\rightarrow$ table

Integrated TMDs

We studied effects of TMD Splitting functions on the evolution.
No fits has been done yet:

- Not yet ready for phenomenology



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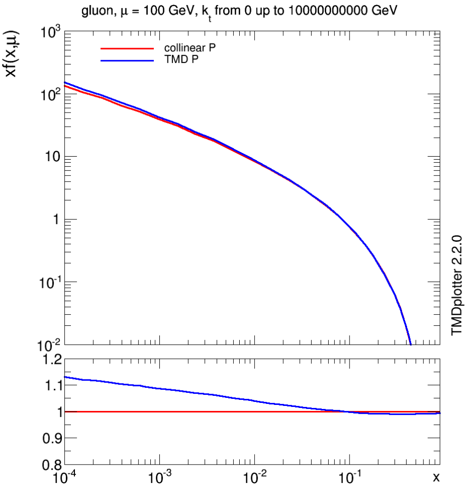

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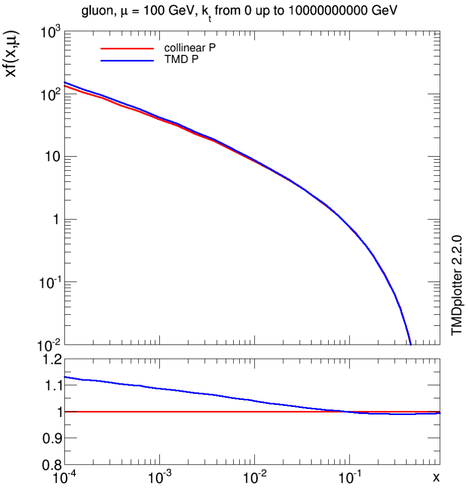

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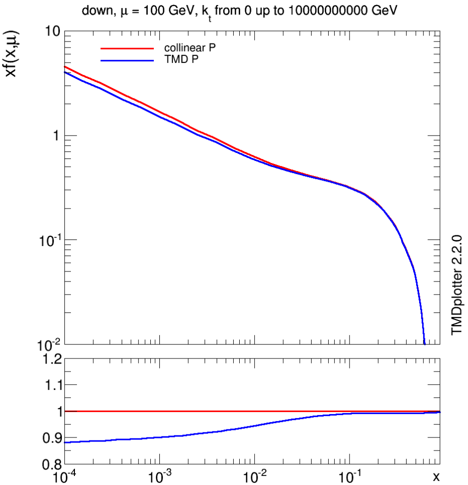

TMDs vs $\mathbf{k}_\perp$

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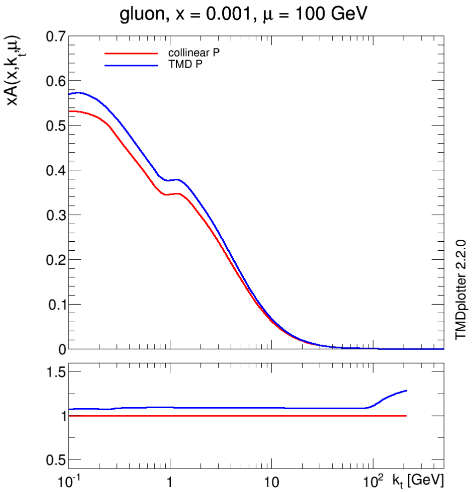

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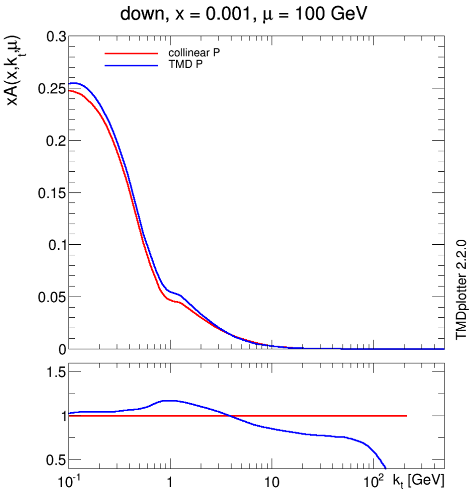

TMDs vs $\mathbf{k}_\perp$

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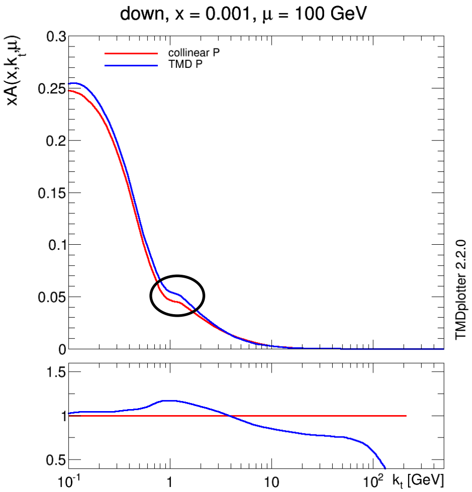
- Whole $\mathbf{k}_\perp$-region is affected
- With TMD P, bumps in distribution are also visible
- Effects from q_0 and q_s in TMD P are similar as in collinear case



Summary and outlook

- I presented a parton branching algorithm for space-like parton evolution with $k_\perp$-dependent splitting functions
- The splitting functions are a (positive-definite) $k_T \neq 0$ continuation of the LO DGLAP splitting functions originally obtained from high-energy factorization
- $k_\perp$-dependent splittings affect both real emission and Sudakov form factors
- They have been implemented in the PB-TMD Monte Carlo code uPDFevolv using the veto algorithm
- New code is working and produces both collinear and TMD parton distributions - paper in preparation
- Ready to do phenomenology:
  - Perform fits to DIS and DY data to determine nonperturbative TMDs
  - Use them to make PB-TMD predictions for LHC and EIC processes including for the first time the effects of TMD splittings