Obtaining stable NLO corrections in High-Energy Factorization using Modified Multi-Regge Kinematics[1]

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Introduction

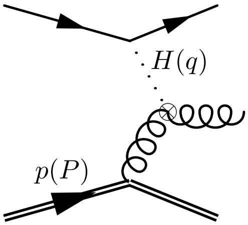
The program of *High-Energy* or k_T*-factorization* have lost it's popularity in ealy 2000s, due to discovery of large NLO corrections to BFKL kernel [Fadin, Lipatov; Camici, Ciafaloni 98'], pointing towards some physics problems of the formalism, and due to the advent of "NLO-revolution" in Collinear-Parton Model calculations.
At present the interest is renewed for several reasons:

- It is unlikely, that fully-differential NNLO calculations for general colored final-states will be available in the near future.
- HEF provides more realistic kinematics than CPM already at LO. Lots of successful pheno. studies at LO (e.g. talk by A. van Hameren on multijet production or V.Saleev on Drell-Yan).
- Problems in some NLO CPM calculations: e.g. $\sqrt{S}-$behaviour of η_c total cross-section at NLO [Lansberg, Ozcelik, 2020]
- A lot has been done to understand "collinear" problems of NLO BFKL calculations ("Duality approach", "kinematic constraint approach", "hybrid DGLAP-BFKL evolution", etc.).
- Recent successful applications of collinearly-improved NLL BFKL resummation to resolve long-standing tensions with small-x_B DIS data, e.g. [R.D.Ball,*et.al.*,2018; `xFitter`, 2018].
- ...

Test-process: Higgs-induced DIS

For the **exploratory** NLO calculation we pick a process:

$$\mathcal{O}(q) + p(P) \to X,$$



where operator $\mathcal{O}(x) = -\frac{1}{2}\mathrm{tr}\left[G_{\mu\nu}(x)G^{\mu\nu}(x)\right]$, can be understood as effective Higgs-Gluon coupling in the limit $m_t \to \infty$. See also the talk by M.Hentschinski on H-production in HEF.

- QCD corrections for the inclusive "structure function" are known in CPM up to $O(\alpha_s^3)$ [Soar *et.al.*, 2009] (see also talk by S.Jaskiewicz)
- One can study p_T**-spectrum of the leading jet**: two-scale (Q^2, p_T) process **already at LO in HEF**
- CFs have a weird LP small-z behaviour: $\alpha_s^k \ln^{2k}(1/z)/z$ starting at NNLO [Hautmann, 2002]

CMS of momenta P and q ($x_B = Q^2/(2Pq)$, $Q^2 = -q^2$):

$$P_- = \sqrt{\frac{Q^2}{x_B(1-x_B)}},\ P_+ = \mathbf{P}_T = 0,$$

$$q_+ = \frac{Q^2}{x_B P_-},\ q_- = -x_B P_-,\ \mathbf{q}_T = 0,$$

where $k_\pm = n_\pm \cdot k = k^0 \pm k^3$, $n_\pm^2 = 0$, $n_+ n_- = 2$.

Leading-twist HEF

Structure-function in CPM ($n_F = 0$ **from now on!**):

$$F_{\mathcal{O}}(x_B, Q^2) = \frac{\pi\lambda_{\mathcal{O}}^2}{4} \int\limits_{x_B}^{1} \frac{dz}{z} \frac{x_B}{z} f_g\left(\frac{x_B}{z}, \mu_F\right) C(z, Q^2, a_s, \mu_F, \mu^2) + O\left(\left(\Lambda_{QCD}^2/Q^2\right)^{\nu}\right),$$

Leading-twist HEF hypothesis[Collins, Ellis 91'; Catani, Hautmann 94']:

$$C(z) = \int \frac{d^2\mathbf{q}_{T1}}{\pi} \int\limits_{x_B}^{x_B/z} \frac{dx_1}{x_1} \mathcal{C}\left(\frac{z x_1}{x_B}, \mathbf{q}_{T1}, a_s, \mu_F, \mu, \mu_Y\right) H\left(\frac{x_B}{x_1}, \mathbf{q}_{T1}, Q^2, a_s, \mu, \mu_Y\right),$$

$\Rightarrow \Phi_g = \mathcal{C} \otimes_z f_g$ – unintegrated PDF

- **Proven** at LL and NLL w.r.t. $\ln 1/z$ at leading power in $z \ll 1$.
- **Violated** by multi-Reggeon exchanges at NNLL and beyond [B-JIMWLK; Fadin, Lipatov, 2017; ...] (see also talks by G.Falcioni, and G.Ridgway)
- **pheno. assumption:** most important effects are one-Reggeon exchange and collinear/running-coupling corrections to it. All other effects – subleading at $Q^2/x_B \to \infty$? (see the talk by R.Boussarie).

A lot of successful LO phenomenology is done under this assumption (see e.g. talk by V.Saleev on Drell-Yan), so it is reasonable to push it to NLO

Leading Order

LO subprocess in HEF:

$$\mathcal{O}(q) + R_-(q_1) \to g(q+q_1),$$

where R_- – Reggeized gluon with $q_1^\mu = \frac{q_1^-}{2} n_+^\mu + q_{T1}^\mu$, $q_1^- = x_1 P_-$.
Structure function at LO:

$$F_{\mathcal{O}}^{(\mathrm{LO\ PRA})}(x_B, Q^2) = \frac{\pi\lambda_{\mathcal{O}}^2}{4} \times \int\limits_0^\infty d\mathbf{q}_{T1}^2 \Phi_g(x_1, \mathbf{q}_{T1}), \boxed{x_1 = \frac{Q^2 + \mathbf{q}_{T1}^2}{Q^2} x_B,}$$

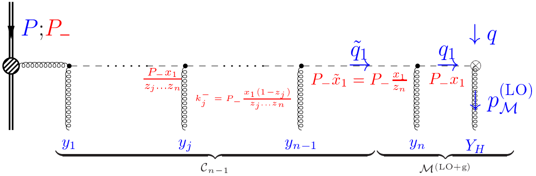
where $\mathbf{p}_{Tg} = \mathbf{q}_{T1}$, *so one can study* p_T*-spectrum of a gluon (jet) at LO.*
Gluon (jet) rapidity

$$Y_H = \frac{1}{2} \ln\left(\frac{Q^2(1-x_B)}{\mathbf{q}_{T1}^2 x_B}\right).$$

We put $\pi\lambda_{\mathcal{O}}^2/4 = 1$ so that at LO CPM:

$$F_{\mathcal{O}}^{(\mathrm{LO\ CPM})}(x_B, Q^2) = x_B f_g(x_B, \mu^2 = Q^2).$$

MRK LO evolution equation



Is the LO BFKL-equation with real emissions ordered in physical rapidity $\boxed{y_j = \ln(k_j^+/k_j^-)/2}$: $\mathcal{C}(x, \mathbf{q}_T, \mu_Y) = \mathcal{C}_0(x, \mathbf{q}_T) + \sum\limits_{n=1}^{\infty} \mathcal{C}_n(x, \mathbf{q}_T, \mu_Y)$,
with $\mathcal{C}_0 = \pi\delta(x-1)\delta(\mathbf{q}_T)$, $\mathcal{C}_1(x, \mathbf{q}_T, \mu_Y) = \frac{\alpha_s C_A}{\pi} \frac{1}{\mathbf{q}_T^2} \theta(\Delta(|\mathbf{q}_T|, \mu_Y) - x)$
where $\Delta(q_T, \mu) = \mu/(\mu + q_T)$ (=cutoff in KMRW UPDF), $\boxed{\mu_Y = q_1^- e^{Y_\mu}}$ – **rapidity scale**, and (ω_g – one-loop gluon Regge trajectory)

$$\mathcal{C}_n(x, \mathbf{q}_T, \mu_Y) = \int\limits_x^1 \frac{dz}{z(1-z)} \left\{ \frac{\alpha_s C_A}{\pi} \int \frac{d^{D-2}\mathbf{k}_T}{\pi(2\pi)^{-2\epsilon}} \frac{1}{\mathbf{k}_T^2} \right. \leftarrow \text{ real emission}$$

$$\times \mathcal{C}_{n-1}\left(\frac{x}{z}, \mathbf{q}_T + \mathbf{k}_T, \frac{|\mathbf{k}_T|}{1-z}\right) \theta\left(\Delta(|\mathbf{k}_T|, \mu_Y) - z\right) \leftarrow y - \text{ordering}$$

$$\left. + 2\omega_g(\mathbf{q}_T^2) \mathcal{C}_{n-1}\left(\frac{x}{z}, \mathbf{q}_T, \mu_Y \frac{x(1-z)}{z(z-x)}\right) \theta\left(\Delta(|\mathbf{q}_T|, \mu_Y) - z\right) \right\} \leftarrow \text{ virt.part}$$

Doubly-logarithmic UPDF

One has to solve the **dimensionally-regularized** evolution equation to subtract **collinear divergences.** To demonstrate how it works let's skip all $O(z)$-corrections in MRK-equation and go to $(N, \mathbf{x}_T)$-space:

$$\mathcal{C}(N, \mathbf{x}_T) = 1 + \frac{\hat{\alpha}_s}{N} \frac{\Gamma(1-\epsilon)(\mu^2)^{\epsilon}}{(-\epsilon)\pi^{-\epsilon}} \int d^{2-2\epsilon}\mathbf{y}_T \, \mathcal{C}(N, \mathbf{y}_T) \times \left[(\mathbf{x}_T^2)^{\epsilon} \delta(\mathbf{x}_T - \mathbf{y}_T) - \frac{\epsilon\Gamma(1-\epsilon)}{\pi^{1-\epsilon}((\mathbf{x}_T - \mathbf{y}_T)^2)^{1-2\epsilon}} \right],$$

where $\hat{\alpha}_s = \alpha_s C_A (\mu^2)^{-\epsilon}/\pi$, then we solve it iteratively and collinear divergences at each order organize into:

$$Z_{\text{coll.}} = \exp\left[-\frac{1}{\epsilon} \int_0^{\hat{\alpha}_s S_\epsilon} \frac{d\alpha}{\alpha} \gamma_N(\alpha) \right], \; \gamma_N(\alpha) = \gamma_1(N)\alpha + \gamma_2(N)\alpha^2 + \ldots,$$

where $S_\epsilon = \exp[\epsilon(-\gamma_E + \ln 4\pi)]$ for $\overline{MS}$-scheme and [Jaroszewicz 82', Catani, Hautmann, 94']:

$$\gamma_1 = \frac{1}{N}, \gamma_2 = \gamma_3 = 0, \gamma_4 = \frac{2\zeta_3}{N^4}, \gamma_5 = \frac{2\zeta_5}{N^5}, \ldots$$

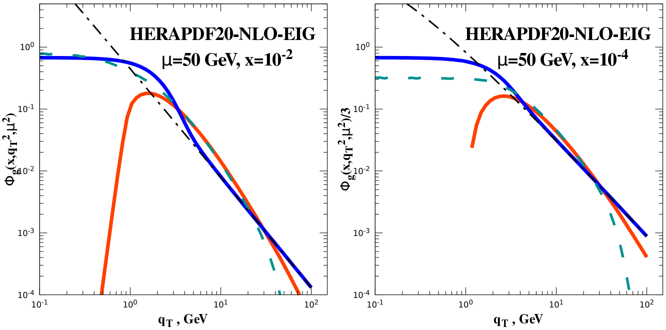
and poles in N correspond to $\ln^k(1/z)/z$ in the DGLAP $P_{gg}(z)$.

Doubly-logarithmic UPDF

In *doubly-logarithmic appriximation* (corrections to which start at $O(\alpha_s^3)$!), the finite part of $\mathcal{C}$ can be expressed as:

$$\mathcal{C}^{(\text{ren.})}(N, \mathbf{x}_T, \mu) \underset{\text{DLA}}{\simeq} \exp\left[-\hat{\alpha}_s(\mu)\frac{\ln(\mu^2\bar{\mathbf{x}}_T^2)}{N}\right] \times F_{NP}(\mathbf{x}_T),$$

where $\bar{\mathbf{x}}_T = \mathbf{x}_T e^{\gamma_E}/2$.
To improve convergence of Fourier-transform to $\mathbf{q}_T$-space we add a non-perturbative factor: $F_{NP} = e^{-\Lambda^2\mathbf{x}_T^2}$. *It has no effect on $q_T \gg \Lambda$ or cross-sections.* Some numerical results:



EFT-framework

The gauge-invariant EFT for Multi-Regge processes in QCD, which includes *Reggeized gluons* [Lipatov; 1995] and *Reggeized quarks* [Lipatov, Vyazovsky; 2001] has been introduced as a systematic tool to *compute and resum* the higher-order corrections in QCD, enhanced by $\ln(s/(-t))(\sim \ln(1/z))$, with the arbitrary $N^k LL$ accuracy at leading power in $(-t)/s$ (or z). Advantages:

- Gauge invariance (even with regularization of RDs!).
- **Possibility to work in covariant gauges.**
- *Provides foundation for HEF:* one-Reggeon exchange contribution is well-defined and gauge-invariant to all orders in α_s.

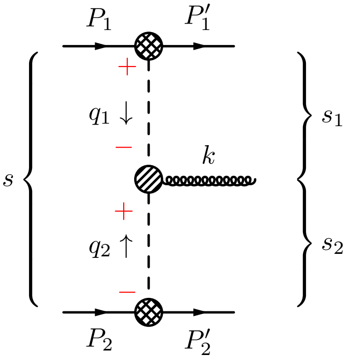
Parton Reggeization Approach (PRA): *gauge-invariant amplitudes with off-shell(Reggeized) initial-state partons from Lipatov's EFT should be used as short-distance parts in k_T-factorization calculations.*

Phenomenological applications to Dijet production [M.N., Saleev, Shipilova, 2012], $B\bar{B}$-correlations [Karpishkov, M.N., Saleev, 2017], J/ψ pair production [He, Kniehl, M.N., Saleev, 2019] and many more...

Thanks to Andreas van Hameren and his `KaTie` code, the problem of tree-level calculations is solved for most of the practical purposes.

Multi-Regge Kinematics.

Consider the $2 \to 2 + n$ scattering in Multi-Regge(MRK) or Quasi-Multi Regge(QMRK) kinematics.



Double Regge limit (MRK):

$$s_1 \gg -q_1^2 \simeq \mathbf{q}_{T1}^2,\ s_2 \gg -q_2^2 \simeq \mathbf{q}_{T2}^2,$$

momentum fractions $z_1 = q_1^+/P_1^+$, $z_2 = q_2^-/P_2^-$.

Intuition: t-channel diagrams dominate.

Properties of MRK:

- $y(P_1') \to +\infty$, $y(P_2') \to -\infty$, $y(k)$ – finite,
- $\boxed{z_1 \sim z_2 \sim z \ll 1}$, $|\mathbf{k}_T| \ll \sqrt{s}$,
- $q_1^+ \sim |\mathbf{q}_{T1}| \sim O(z) \gg q_1^- \sim O(z^2)$, $q_2^- \sim |\mathbf{q}_{T2}| \sim O(z) \gg q_2^+ \sim O(z^2)$.

Reggeon fields

Let's introduce **gauge-invariant** Reggeon fields $R_\pm(x) = T^a R^a_\pm(x)$ subjet to kinematic constraints ($\Leftrightarrow$ (Q)MRK, $\partial_\pm = n^\mu_\pm \partial_\mu = 2\frac{\partial}{\partial x^\mp}$):

$$\partial_- R_+ = \partial_+ R_- = 0 \Rightarrow$$

R_+ carries $(k_+, \mathbf{k}_T)$ and R_- carries $(k_-, \mathbf{k}_T)$.

Effective action [Lipatov, 1995]:

$$S = \int d^4x(-2R^a_+ \partial^2_\perp R^a_-) + \sum_{\text{rap. ints.}} \int d^2\mathbf{x}_T \left\{ \int \frac{dx_+ dx_-}{2} L_{\text{QCD}}(x, A_\mu, \psi) \right.$$
$$\left. + \int \frac{dx_+}{2} \operatorname{tr}\left[R^a_-(x_+, \mathbf{x}_T)\mathcal{T}_+[x, A_\mu]\right] + \int \frac{dx_-}{2} \operatorname{tr}\left[R^a_+(x_-, \mathbf{x}_T)\mathcal{T}_-[x, A_\mu]\right] \right\},$$

what are the interaction operators $\mathcal{T}_\pm$?

Infinite light-like Wilson lines

Constraints we have:

- At leading power in energy, partons highly separated in rapidity perceive each-other as infinite light-like Wilson lines [Mueller, Nikolaev, Zakharov, 1990s; ...; Caron-Huot, 2013; ...],
- Hermiticity [Lipatov, 1997; Bondarenko, Zubkov, 2018]
- $R_\pm \to g$ transition is given by "non-sense" polarization $n^\mu_\mp$.

$$\Rightarrow \mathcal{T}_\pm[x, A_\mu] = \frac{i}{g_s}\partial_\perp^2 \left(W_\infty[x_\pm, \mathbf{x}_T, A_\mu] - W^\dagger_\infty[x_\pm, \mathbf{x}_T, A_\mu]\right),$$

Where:

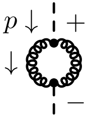
$$\begin{aligned} W_{x_\mp}[x_\pm, \mathbf{x}_T, A_\pm] &= P\exp\left[\frac{-ig_s}{2}\int\limits_{-\infty}^{x_\mp} dx'_\mp A_\pm(x_\pm, x'_\mp, \mathbf{x}_T)\right] \\ &= \left(1 + ig_s\partial_\pm^{-1}A_\pm\right)^{-1}, \end{aligned}$$

and $\partial_\pm^{-1} \to -i/(k^\pm + i\varepsilon)$ in the Feynman rules.
After IBP trick:

$$\boxed{S_{\text{int.}} = \int dx \frac{i}{g_s}\,\text{tr}\left[R_+(x)\partial_\perp^2\partial_-\left(W_{x_+}[A_-] - W^\dagger_{x_+}[A_-]\right) + (+\leftrightarrow -)\right],}$$

Rapidity divergences and regularization

$$\Pi^{(1)}_{ab} = q\downarrow$$



$$= g_s^2 C_A \delta_{ab} \int \frac{d^d q}{(2\pi)^D} \frac{\left(\mathbf{p}_T^2 (n_+ n_-)\right)^2}{q^2 (p-q)^2 q^+ q^-}$$

Cutoff in rapidity [Lipatov, 1995] ($q^\pm = \sqrt{q^2 + \mathbf{q}_T^2} e^{\pm y}$, $p^+ = p^- = 0$):

$$\int \frac{dq^+ dq^-}{q^+ q^-} = \int\limits_{y_1}^{y_2} dy \int \frac{dq^2}{q^2 + \mathbf{q}_T^2},$$

$$\Pi^{(1)}_{ab} \sim \delta_{ab} \mathbf{p}_T^2 \times \underbrace{C_A g_s^2 \int \frac{\mathbf{p}_T^2 d^{D-2}\mathbf{q}_T}{\mathbf{q}_T^2 (\mathbf{p}_T - \mathbf{q}_T)^2}}_{\omega^{(1)}(\mathbf{p}_T^2)} \times (y_2 - y_1) + \text{finite terms}$$

Tilted Wilson-line regularization [Hentschinski, Sabio Vera, Chachamis *et. al.*, 2012-2013]:

$$\boxed{\tilde{n}^\pm = n^\pm + r \cdot n^\mp,\ \tilde{k}^\pm = k^\pm + r \cdot k^\mp,\ r \to 0,}$$

+ modified kinematics [M.N.,2019]: $\boxed{\tilde{\partial}_+ R_- = \tilde{\partial}_- R_+ = 0.}$

Rapidity divergences at one loop

Only log-divergence $\sim \ln r$ (Blue cells in the table) is related with Reggeization of particles in t-channel.
Integrals which **do not** have log-divergence may still contain the power-dependence on r:

- $r^{-\epsilon} \to 0$ for $r \to 0$ and $\epsilon < 0$.
- $r^{+\epsilon} \to \infty$ for $r \to 0$ and $\epsilon < 0$ – **weak-power divergence** (Pink cells in the table)
- $r^{-1+\epsilon} \to \infty$ – **power divergence.** (Red)

(# LC prop.) \ (# quadr. prop.)	1	2	3	4
1	$A_{[-]}$	$B_{[-]}$	$C_{[-]}$	...
2	$A_{[+-]}$	$B_{[+-]}$	$C_{[+-]}$	...
3	...	...	...	...

The **weak-power** and **power-divergences** cancel between Feynman diagrams describing one region in rapidity, so only log-divergences are left.

Triangle integrals, logarithmic RD

Result for $Q^2 = 0$:

$$C_{[-]}(t_1, 0, q^-) = \frac{1}{q^- t_1}\left(\frac{\mu^2}{t_1}\right)^{\epsilon}\frac{1}{\epsilon}\left[\ln r + i\pi - \ln\frac{|q_-|^2}{t_1} - \psi(1+\epsilon) - \psi(1) + 2\psi(-\epsilon)\right] + O(r^{1/2}),$$

coincides with the result of [G. Chachamis, *et. al.*, 2012].

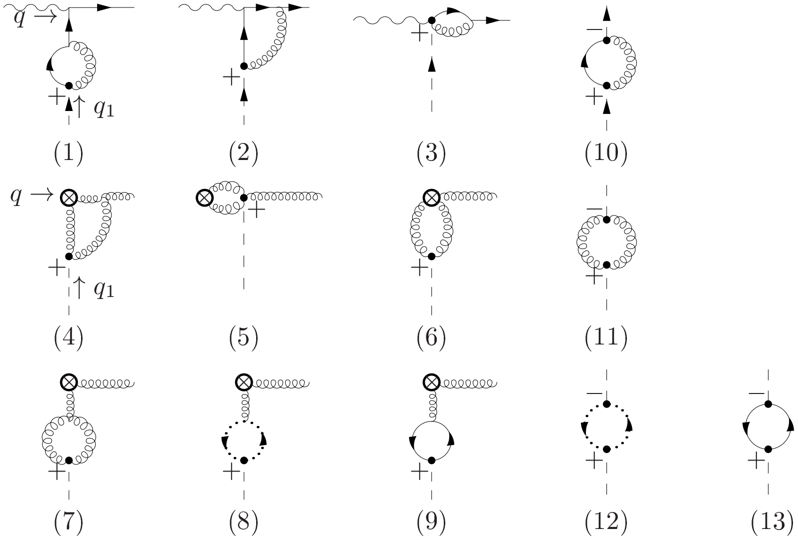
Result for $Q^2 \neq 0$ [M.N., 2019]:

$$C_{[-]}(t_1, Q^2, q_-) = C_{[-]}(t_1, 0, q_-) + \left(\frac{\mu^2}{t_1}\right)^{\epsilon}\frac{I(Q^2/t_1)}{q_- t_1} - \frac{1}{t_1}\underbrace{\Delta B_{[-]}(Q^2, q_-)}_{\propto r^{-\epsilon}},$$

where

$$I(X) = -\frac{2X^{-\epsilon}}{\epsilon^2} - \frac{2}{\epsilon}\int\limits_0^X \frac{(1-x^{-\epsilon})dx}{1-x} = -\frac{2X^{-\epsilon}}{\epsilon^2} + 2\left[-\mathrm{Li}_2(1-X) + \frac{\pi^2}{6}\right] + O(\epsilon).$$

$R\mathcal{O}g$-vertex (diags. 4-9)



ROg-vertex

The one-loop correction is proportional to the Born vertex:

$$G^{(0)}_{+\mu} = \frac{i}{2}\left((Q^2 - t_1)n^-_\mu - 2q_-(q_1)_\mu\right),$$

with the coefficient

$$C\left[G^{(0)}_{+}\right] = \frac{\bar{\alpha}_s}{4\pi}\frac{1}{2}\left\{\frac{B(t_1)}{(d-2)(d-1)(Q^2-t_1)^2}\right.$$

$$\times\left[C_A\left((d-2)(5d-4)Q^4 - 2(d(7d-24)+16)Q^2 t_1\right.\right.$$

$$\left.\left.+(d-2)(5d-4){t_1}^2\right) - 2(d-2)^2 n_F (Q^2-t_1)^2\right]$$

$$-\frac{2C_A(d-4)Q^2 B(Q^2)}{(d-2)(Q^2-t_1)^2}\left[(d-4)Q^2 - (d-2)t_1\right]$$

$$\left.-2C_A\left[q_-\left(t_1 C_{[-]}(t_1, Q^2, q_-) + B_{[-]}(q) - B_{[-]}(q+q_1)\right) + (t_1 - Q^2)C(t_1, Q^2)\right]\right\},$$

ϵ-expansion of the one-loop coefficient function:

$$H^{(\mathrm{NLO}),\,\mathcal{O}}_{\text{virt. unsubtr.}} = 2\mathrm{Re}\left(C\left[G^{(0)}\right]\right) = \frac{\bar{\alpha}_s}{2\pi}\left\{-\frac{C_A}{\epsilon^2} + \frac{1}{\epsilon}\left[\beta_0 - C_A(1+L_1)\right] - C_A\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{t_1}\right)\ln\bar{r}\right.$$

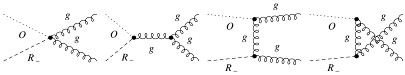
$$\left.+C_A\left[2\mathrm{Li}_2\left(1 - \frac{Q^2}{t_1}\right) + \frac{L_2^2}{2} - L_2 - \frac{1}{2}L_1(L_1+2) + \frac{\pi^2}{6} - \frac{2}{3}\right] + \beta_0\left[\frac{10}{6} + L_1 + L_2\right] + O(r,\epsilon)\right\},$$

where $L_1 = \ln(\mu^2/Q^2)$, $L_2 = \ln(Q^2/t_1)$, $t_1 = \mathbf{q}^2_{T1}$ and $\bar{r} = rQ^2/q_+^2$.

NLO real-emission amplitude

Given by diagrams (with some combined vertices):

$O \; R_- \to g \; g$



Convenient parametrization for kinematics ($k_{1,2}$– final-state gluons):

$$\mathbf{k}_{T1},\ \mathbf{k}_{T2},\ \hat{z} = \frac{k_1^-}{q_1^- + q_-}.$$

Singular limits ($f = \overline{|\mathcal{M}_{\rm NLO}|^2}/\overline{|\mathcal{M}_{\rm LO}|^2}/(4C_A g_s^2)$):

- Final-state collinear limit $\mathbf{k}_1 \parallel \mathbf{k}_2$ ($\hat{s} = (k_1+k_2)^2$):

$$f_{\rm coll.} = \frac{1}{\hat{s}} p_{gg}(\hat{z}),\ p_{gg}(z) = \frac{1-z}{z} + \frac{z}{1-z} + z(1-z),$$

- Soft limits ($k_1^0 \to 0$ or $k_2^0 \to 0$):

$$f_{\rm soft.}^{(\hat{z}\to 0)} = \frac{\hat{z}^2(\mathbf{k}_{T1}+\mathbf{k}_{T2})^2}{\mathbf{k}_{T1}^2(\mathbf{k}_{T1}(1-\hat{z}) - \mathbf{k}_{T2}\hat{z})^2},\ f_{\rm soft.}^{(\hat{z}\to 1)} = \left[\hat{z}\to 1-\hat{z},\ \mathbf{k}_{T1}\leftrightarrow\mathbf{k}_{T2}\right].$$

Phase-space slicing

Following classic two-cutoff PS-slicing method [Harris, Owens, 2001] we define:

- Soft region ($\delta_s \ll 1$):

$$\frac{k_1^+ + k_1^-}{q_+} < \delta_s \text{ or } \frac{k_2^+ + k_2^-}{q_+} < \delta_s.$$

- Hard-Collinear region ($\delta_c \ll \delta_s$):

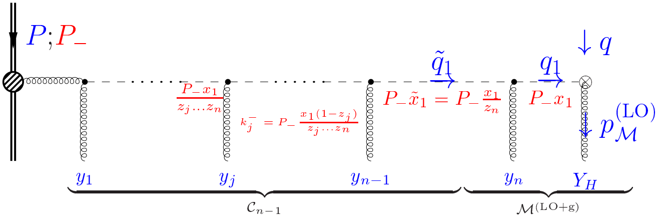
$$\Delta\phi_{1,2}^2 + \Delta y_{1,2}^2 < \delta_c \text{ and NOT soft.}$$

Soft region condition:

$$\mathbf{k}_{T1}^2 < \mathbf{k}_{T2}^2 \hat{z}(\delta_s - e^{-2Y_H}\hat{z}) \text{ and } [\hat{z} \to 1 - \hat{z},\ \mathbf{k}_{T1} \leftrightarrow \mathbf{k}_{T2}]$$

Integrals over soft and collinear regions are done analytically by standard methods.

Double-counting subtraction: MRK



We have to subtract double-counting with the evolution of $\mathcal{C}$ **in all phase-space, including soft region** (see backup).

One iteration of MRK evolution eqn. $\otimes$ LO gives ($w =$ UPDF$\times$ ME^2):

$$w^{(\mathrm{MRK})}_{\mathrm{sub.}\ \hat{t}} = \Phi_g(x_1^{(\hat{t},\ \mathrm{MRK})}, \mathbf{k}_{T1} + \mathbf{k}_{T2}) \frac{\alpha_s C_A}{\pi} \frac{1}{\mathbf{k}_{T2}^2} \theta\left(\frac{(1-\hat{z})^2}{\hat{z}^2} - \frac{\mathbf{k}_{T2}^2}{\mathbf{k}_{T1}^2} \right),$$

with $x_1^{(\hat{t},\ \mathrm{MRK})} = \frac{x_B}{Q^2}\left(Q^2 + \frac{\mathbf{k}_{T1}^2}{\hat{z}}\right)$, for $\hat{z} \to 0$ Regge limit. Also the subtraction for $\hat{z} \to 1$ Regge limit has to be included, which is obtained by:

$$\hat{z} \to 1 - \hat{z}, \mathbf{k}_{T1} \leftrightarrow \mathbf{k}_{T2}.$$

Subtractions in the virtual part

Consider the process:

$$\mathcal{O}(q) + g(P) \to g(k_2, Y_H) + g(k_1, y_1),$$

Regge limit of the one-loop ampl. is predicted by the EFT (**checked!**):

$$\frac{2\mathrm{Re}\left(\mathcal{M}_{1-\mathrm{loop}}\mathcal{M}^*_{\mathrm{tree}}\right)}{|\mathcal{M}_{\mathrm{tree}}|^2} = H^{(\mathrm{NLO}),\ \mathcal{O}}_{\mathrm{virt.\ unsubtr.}}(\mathbf{q}^2_{T1}, Y_H, \ln r) + H^{(\mathrm{NLO}),\ g}_{\mathrm{virt.\ unsubtr.}}(\mathbf{q}^2_{T1}, y_1, \ln r) - 2\Pi^{(1)}(\mathbf{q}^2_{T1}, \ln r),$$

where $\Pi^{(1)}$ is one-loop Reggeon self-energy. Subtracting the evolution contribution:

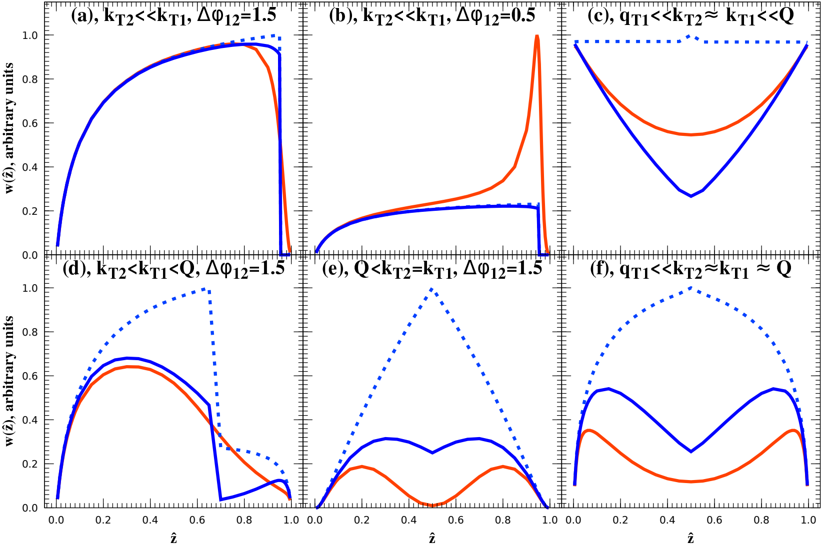
$$(Y_H - y_1) \times 2\omega_g(\mathbf{q}^2_{T1}),$$

and rearranging terms, one gets:

$$\boxed{H^{(\mathrm{NLO}),\ \mathcal{O}}_{\mathrm{virt.\ subtr.}}(\mathbf{q}^2_{T1}) = H^{(\mathrm{NLO}),\ \mathcal{O}}_{\mathrm{virt.\ unsubtr.}}(\mathbf{q}^2_{T1}, Y_H, \ln r) - \Pi^{(1)}(\mathbf{q}^2_{T1}, \ln r) - 2Y_H\omega_g(\mathbf{q}^2_{T1}),}$$

$$H^{(\mathrm{NLO}),\ g}_{\mathrm{virt.\ subtr.}}(\mathbf{q}^2_{T1}) = H^{(\mathrm{NLO}),\ g}_{\mathrm{virt.\ unsubtr.}}(\mathbf{q}^2_{T1}, y_1, \ln r) - \Pi^{(1)}(\mathbf{q}^2_{T1}, \ln r) + 2y_1\omega_g(\mathbf{q}^2_{T1}).$$

Double-counting subtraction: MRK vs. MMRK



Red – exact w-function, dashed – MRK approx. , solid – MMRK approx.

Double-Counting subtraction: Modified-MRK approximation

The main problem of MRK-subtraction is, that it very poorly reproduces an exact ME in the DGLAP limit:

$$\mathbf{q}_{T1}^2 \ll \mathbf{k}_{T1}^2 \simeq \mathbf{k}_{T2}^2 \ll Q^2.$$

To overcome this, we add a "propagator factor" to the subtraction term

$$w_{\text{sub. } \hat{t}}^{(\text{MMRK})} = \frac{\alpha_s(\mu) C_A}{\pi} \Phi_g(x_1^{(\hat{t},\ \text{MRK})}, \mathbf{k}_{T1} + \mathbf{k}_{T2}) \times \frac{1}{\mathbf{k}_{T2}^2} \left(1 + \frac{\hat{z}\mathbf{k}_{T2}^2}{(1-\hat{z})\,\mathbf{k}_{T1}^2}\right)^{-2} \theta\left(\frac{(1-\hat{z})^2}{\hat{z}^2} - \frac{\mathbf{k}_{T2}^2}{\mathbf{k}_{T1}^2}\right),$$

Inspirations:

- TMD splitting functions (talk by L.Keersmaekers)
- **High-Energy-Jets** approach [J. Andersen *et.al.*, 2009].
- Recent developments in the **Kinematic Constraint** approach [M. Deak, *et al.*, 2019].

UPDF Evolution, MMRK approximation

Adding the same "propagator factor" to the kernel we obtain the MMRK evolution for UPDF:

$$
\begin{aligned}
\mathcal{C}_n(x, \mathbf{q}_T, \mu_Y, \mu_S) \quad = \quad & \int\limits_x^1 \frac{dz}{z(1-z)} \left\{ \frac{\alpha_s C_A}{\pi} \int \frac{d^{D-2}\mathbf{k}_T}{\pi(2\pi)^{-2\epsilon}} \frac{1}{\mathbf{k}_T^2} \left(1 + \frac{z\mathbf{k}_T^2}{(1-z)\mu_S^2}\right)^{-2} \right. \\
& \times \; \mathcal{C}_{n-1}\left(\frac{x}{z}, \mathbf{q}_T + \mathbf{k}_T, \frac{|\mathbf{k}_T|}{1-z}, |\mathbf{q}_T + \mathbf{k}_T|\right) \theta\left(\Delta(|\mathbf{k}_T|, \mu_Y) - z\right) \\
& + \; 2\omega_g(\mathbf{q}_T^2)\mathcal{C}_{n-1}\left(\frac{x}{z}, \mathbf{q}_T, \mu_Y \frac{x(1-z)}{z(z-x)}, |\mathbf{q}_T|\right) \theta\left(\Delta(|\mathbf{q}_T|, \mu_Y) - z\right) \Bigg\},
\end{aligned}
$$

where $\mu_S^2 = Q^2 + \mathbf{q}_{T1}^2$ for DIS kinematics.

Analytic part together (with standard μ_Y-choice: $Y_\mu = Y_H$)

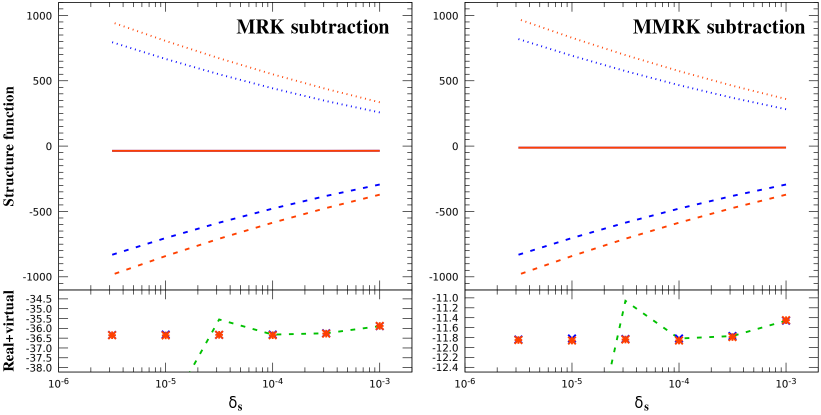
Taking together the **loop part**, **soft and collinear integrals** and **(M)MRK-soft subtraction terms** we find, that all IR divergences cancel and only remaining UV divergence is related with $\mathcal{O}(x)$-renormalization: $Z_{\mathcal{O}} = 1 + \frac{\alpha_s}{4\pi}\frac{\beta_0}{\epsilon} + \dots$ The finite part of the answer is:

$$H^{(\text{NLO}),\ Y_\mu = Y_H}_{\text{analyt.}} = \frac{\bar{\alpha}_s C_A}{2\pi}\left[\frac{67}{6} - \frac{\pi^2}{2} + \frac{11}{3}\ln\left(\frac{\mu^2}{\mathbf{q}_{T1}^2}\right) + 2\text{Li}_2\left(1 - \frac{Q^2}{\mathbf{q}_{T1}^2}\right)\right.$$
$$-\frac{11}{6}\ln\delta_c - 2\ln\delta_c\ln\delta_s + 2\ln\delta_c\ln(1+\xi)$$
$$\left.+2\ln\xi\ln(1+\xi) - 2\ln^2(1+\xi) + O(\epsilon)\right].$$

where $\xi = e^{-2Y_H}$ with $Y_H = \frac{1}{2}\ln\left(\frac{Q^2(1-x_B)}{\mathbf{q}_{T1}^2 x_B}\right)$ – *rapidity of the jet in the LO hard subprocess.*

Terms $\propto Y_H$ have cancelled, only terms suppressed as e^{-2Y_H} are left.

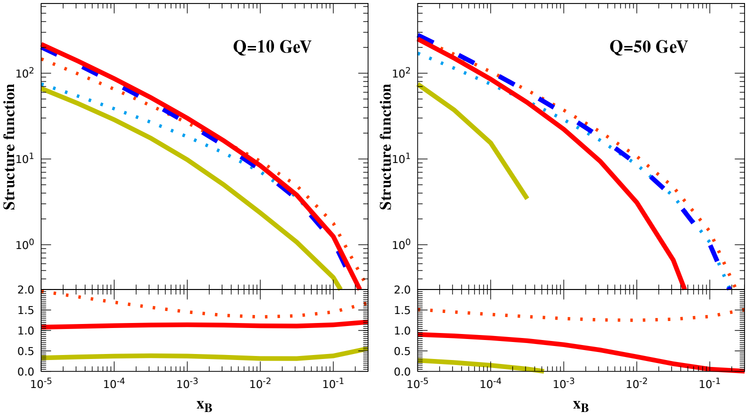
Numerical results: cancellation of δ_s and δ_c-dependence



Red – $\delta_c = 3 \times 10^{-3}\delta_s$, blue – $\delta_c = 10^{-4}\delta_s$. Dashed – analytic part, dotted – numerical part, solid – sum; green-dashed line – double precision.

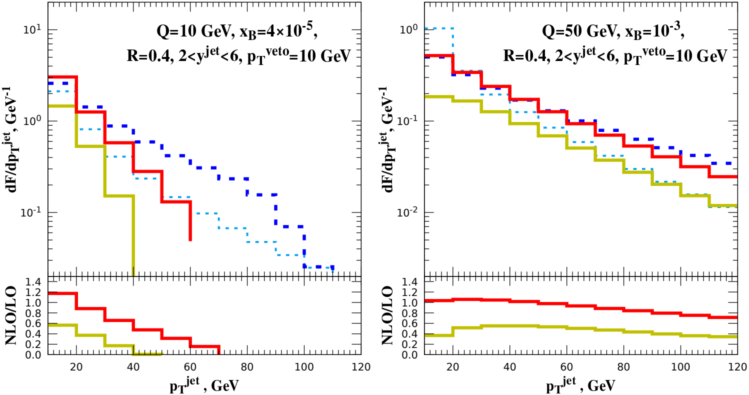
MRK-subtracted NLO converges very quickly, but NLO correction **is negative and > LO! For MMRK – NLO correction is smaller!**

Numerical results: inclusive structure function



Dotted lines: blue – LO CPM, orange – NLO CPM; dashed line – LO PRA with DL UPDF; solid lines: yellow – NLO PRA with MRK subtraction, red – NLO PRA with MMRK subtraction.

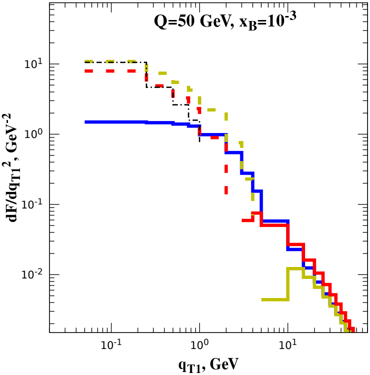
Numerical results: jet p_T-spectrum



Thin dashed line – LO CPM, thick dashed line – LO PRA with DL UPDF. Solid lines: yellow – NLO PRA with MRK subtraction, red – NLO PRA with MMRK subtraction.

Distribution of NLO correction over $\mathbf{q}_{T1}$

Why NLO correction to inclusive SF is larger at higher Q^2?



Dashed lines – negative contribution, solid lines – positive contributions. Blue solid line – LO.

Small-$\mathbf{q}_{T1}$ asymptotics of analytic part of the NLO correction (black dash-dotted line):

$$H^{(\mathrm{NLO})}_{\mathbf{q}_{T1}\to 0} = \frac{\bar{\alpha}_s C_A}{2\pi}\left[\frac{11}{3}\ln\left(\frac{\mu^2}{\mathbf{q}_{T1}^2}\right) - \ln^2\left(\frac{Q^2}{\mathbf{q}_{T1}^2}\right)\right]$$

The doubly-logarithmic corrections $\sim \ln^2(Q^2/\mathbf{q}_{T1}^2)$ can be further factorized into UPDF.

Analytic part with μ_Y-dependence

The $\mu_Y^{(\mathrm{st.})} = (Q^2 + \mathbf{q}_{T1}^2)/|\mathbf{q}_{T1}|$ corresponds to $Y_\mu = Y_H$. Introducing: $\mu_Y^2 \to (\mu_Y^{(\mathrm{st.})})^2 \xi_\mu$ one obtains:

$$H_{\text{analyt.}}^{(\mathrm{NLO})} = \frac{\bar{\alpha}_s C_A}{2\pi}\left[\frac{67}{6} - \frac{\pi^2}{2} + \frac{11}{3}\ln\left(\frac{\mu^2}{\mathbf{q}_{T1}^2}\right) + 2\mathrm{Li}_2\left(1 - \frac{Q^2}{\mathbf{q}_{T1}^2}\right)\right.$$
$$+2\ln(1+\xi)\left(\ln\xi - \ln(1+\xi)\right) + \ln\delta_c\left(2\ln(1+\xi) - \frac{11}{6} - 2\ln\delta_s\right)$$
$$-\frac{1}{2}\ln\xi_\mu\left(4\ln\delta_s + \ln\xi_\mu - 4\ln(\xi_\mu+\xi)\right)$$
$$\left.-\ln^2(\xi_\mu+\xi) - 2\mathrm{Li}_2(-\xi) - 2\mathrm{Li}_2\left(\frac{\xi}{\xi_\mu+\xi}\right) + O(\epsilon)\right].$$

With the choice $\xi_\mu = (\mathbf{q}_{T1}^2/Q^2)^{\sqrt{2}}$ the $\ln^2(\mathbf{q}_{T1}^2/Q^2)$ is removed, so the optimal scale-choice for μ_Y is:

$$\mu_Y^2 = \min\left(\left(\frac{\mathbf{q}_{T1}^2}{Q^2}\right)^{\sqrt{2}}, 1\right)\frac{(Q^2+\mathbf{q}_{T1}^2)^2}{\mathbf{q}_{T1}^2}.$$

Future plans

- Get "the number" out of full MMRK evolution equation. Add Reggeized quark contributions.
- Formalism by [van Hameren, 2017] is useful to automatize NLO loop corretions and is fully compatible with our scheme
- Reproduce the physical $e + p$ DIS for the phenomenological cross-check
- Another cross-checks: inclusive jet, prompt-photon production
- Interesting target: Drell-Yan, especially angular distributions of leptons and Lam-Tung-relation violating effects (NNLO in CPM!)
- D-meson production at NLO as a cross-check for heavy-flavor production
- Challenging task: heavy quarkonium production in NRQCD-factorization at NLO
- ...

Thank you for your attention!

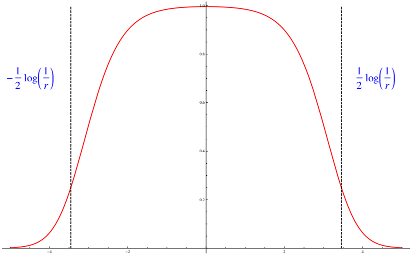
Backup: rapidity divergences in real corrections

Constraint $\tilde{\partial}_+ R_- = \tilde{\partial}_- R_+ = 0$. Lipatov's vertex ($k = q_1 - q_2$, $k^2 = 0$):

$$\Gamma_{+\mu-} = -(\tilde{n}_+ \tilde{n}_-) \left((q_1 + q_2)_\mu + q_1^2 \frac{\tilde{n}_\mu^-}{\tilde{q}_2^-} + q_2^2 \frac{\tilde{n}_\mu^+}{\tilde{q}_1^+} \right) + 2 \left(\tilde{q}_1^+ \tilde{n}_\mu^- + \tilde{q}_2^- \tilde{n}_\mu^+ \right),$$

without modified constraint, the Slavnov-Taylor identity $\boxed{k^\mu \Gamma_{+\mu-} = 0}$ is violated by terms $O(r)$.

The square of regularized LV:



$$\Gamma_{+\mu-}\Gamma_{+\nu-}P^{\mu\nu} = \frac{16\mathbf{q}_{T1}^2 \mathbf{q}_{T2}^2}{\mathbf{k}_T^2} f(y),$$

$$\longleftarrow f(y) = \frac{1}{(re^{-y} + e^{y})^2 (re^{y} + e^{-y})^2},$$

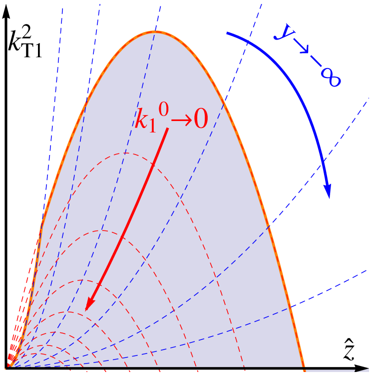
$$\int\limits_{-\infty}^{+\infty} dy\ f(y) = -1 - \ln r + O(r)$$

Backup: double-counting in the soft region

MRK subtraction term is **IR-divergent in the soft region**. ⇒ We have to subtract integral of **soft asymptotics of subtraction term over soft region** from the integral of **exact amplitude in the soft region.** E.g. for $\hat{z} \to 0$ limit:

$$\mathcal{J}_{\text{MRK-soft}} = \int\limits_0^{\hat{z}_m} \frac{d\hat{z}}{\hat{z}} \int\limits_0^{\mathbf{k}_{T2}^2 \hat{z}^2} \frac{d\mathbf{k}_{T1}^2}{(\mathbf{k}_{T1}^2)^{1+\epsilon}} + \int\limits_{\hat{z}_m}^{e^{2Y_H}\delta_s} \frac{d\hat{z}}{\hat{z}} \int\limits_0^{\mathbf{k}_{T2}^2 \hat{z}(\delta_s - e^{-2Y_H}\hat{z})} \frac{d\mathbf{k}_{T1}^2}{(\mathbf{k}_{T1}^2)^{1+\epsilon}},$$

where $z_m = \delta_s/(1+e^{-2Y_H})$.



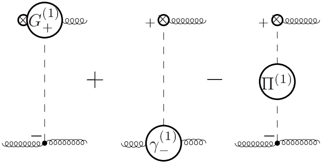
Backup: comparison with QCD

Regge limit of the one-loop amplitude for the process:

$$\mathcal{O}(q) + g(P) \to g(k_2, Y_H) + g(k_1, y_1),$$

has been reproduced as the sum of:

- One-Reggeon contribution (*negative signature*, Re+Im parts @ 1 loop):



- Two-Reggeon contribution (*positive signature*, only Im part @ 1 loop):