Helicity at Small x: Oscillations and LLA Corrections

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Background

Proton helicity can be decomposed into spin and orbital angular momentum (OAM) of quarks and gluons [Jaffe and Manohar, 1990]

$$\frac{1}{2} = S_q + S_G + L_q + L_G \tag{1}$$

where

$$S_q = \int_0^1 dx\, \Delta\Sigma(x, Q^2) = \int_0^1 dx \sum_f \left[\Delta q_f(x, Q^2) + \Delta \overline{q}_f(x, Q^2)\right]. \tag{2}$$

Experiments have measured S_q but can only include $0 < x_{\text{min}} \leq x \leq 1$.

Objective: Find the contribution to S_q coming from $\Delta\Sigma$ as $x \to 0$.

Polarized Dipole Amplitudes

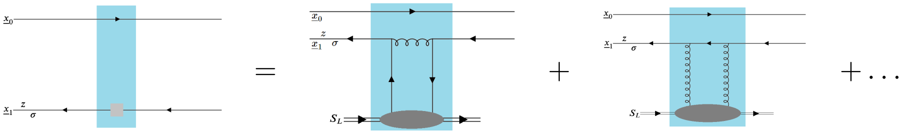
To learn about S_q, we consider two objects.

(i) Quark dipole amplitude:

$$Q(x_{10}^2, z) = \frac{zs}{2N_c} \left\langle \mathrm{tr}\left[V_{\underline{x}_0} V_{\underline{x}_1}^{\dagger}(\sigma)\right] + \mathrm{tr}\left[V_{\underline{x}_1}(\sigma) V_{\underline{x}_0}^{\dagger}\right] \right\rangle_{S_L}(z)$$
$$\sim \Delta\Sigma(x, Q^2) \tag{3}$$

(ii) Gluon dipole amplitude

The bracket averages over the polarized target proton's wave function.



Polarized Dipole Amplitudes

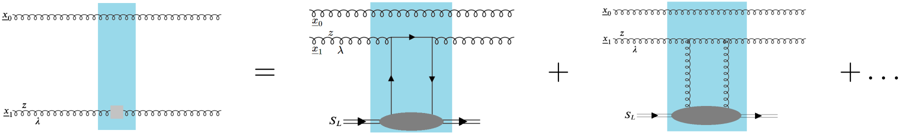
To learn about S_q, we consider two objects.

(i) Quark dipole amplitude

(ii) Gluon dipole amplitude:

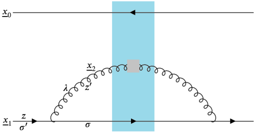
$$G(x_{10}^2, z) = \frac{zs}{2(N_c^2 - 1)} \left\langle \mathrm{Tr}\left[U_{\underline{x}_0} U_{\underline{x}_1}^{\dagger}(\lambda)\right] + \mathrm{Tr}\left[U_{\underline{x}_1}(\lambda) U_{\underline{x}_0}^{\dagger}\right] \right\rangle_{S_L}(z) \tag{4}$$

The bracket averages over the polarized target proton's wave function.



Evolution Equations

- The dipole amplitudes obey integral equations resulting from quark/gluon splitting outside the shockwave, e.g.



- To the first order in α_s, both $G(x_{10}^2, z)$ and $Q(x_{10}^2, z)$ evolve as

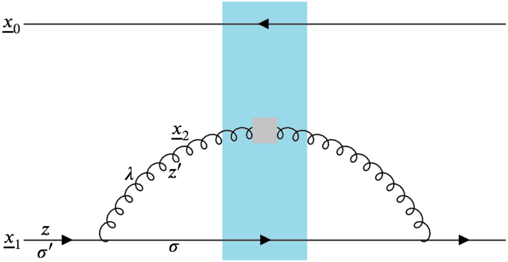
$$\delta Q(x_{10}^2, z) \sim \alpha_s \Bigg[\underbrace{\int \frac{dz'}{z'} \int \frac{dx_{21}^2}{x_{21}^2}}_{\text{DLA}} + \underbrace{\int dz' \int \frac{dx_{32}^2}{x_{32}^2}}_{\text{LLA}_\text{T}} + \underbrace{\int \frac{dz'}{z'} \int dx_{21}^2}_{\text{LLA}_\text{L}} + \ldots \Bigg]$$
$$\times \left(\text{dipole amplitudes:} G, Q\right). \tag{5}$$

- The LLA terms (resumming $\alpha_s \ln \frac{1}{x}$) are subleading to the DLA term (resumming $\alpha_s \ln^2 \frac{1}{x}$).

DLA Evolution Equations

- In a splitting in the limit $z' \ll z$, some regions in x_{21}^2-integral yield the DLA terms in the evolution equations.

$$Q(x_{10}^2, z) = Q^{(0)}(z) + \alpha_s \int \frac{dz'}{z'} \Bigg[\underbrace{A \int \frac{dx_{21}^2}{x_{21}^2}}_{\text{DLA}} + \underbrace{B \int dx_{21}^2}_{\text{LLA}_\text{L}} \Bigg] \text{ (dipole amps)} \tag{6}$$



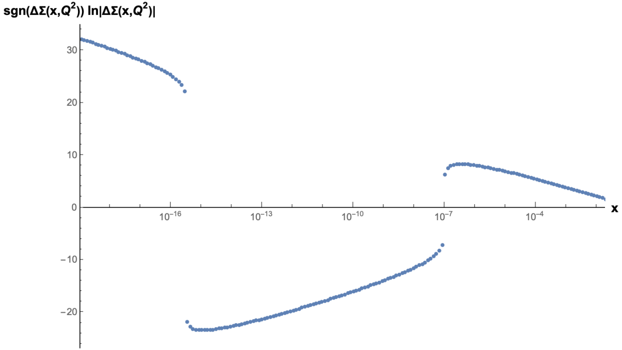
DLA Evolution Equations

- In the limit $N_c \gg 1$, the DLA equations have been solved analytically [Kovchegov et al, 2017], and the phenomenological implications are being studied.
- In the limit $N_c, N_f \gg 1$, the DLA equations are more complicated. [Kovchegov et al, 2016, 2019]

$$
\begin{aligned}
G(x_{10}^2, z) &= G^{(0)}(z) + \frac{\alpha_s N_c}{2\pi} \int_{\max\{\Lambda^2, 1/x_{10}^2\}/s}^{z} \frac{dz'}{z'} \int_{1/(z's)}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma(x_{10}^2, x_{21}^2, z') + 3G(x_{21}^2, z') \right] \\
&\quad - \frac{\alpha_s N_f}{2\pi} \int_{\Lambda^2/s}^{z} \frac{dz'}{z'} \int_{1/(z's)}^{x_{10}^2 z/z'} \frac{dx_{21}^2}{x_{21}^2} \, \overline{\Gamma}_{gen}(x_{10}^2, x_{21}^2, z') \\
Q(x_{10}^2, z) &= Q^{(0)}(z) + \frac{\alpha_s N_c}{2\pi} \int_{\Lambda^2/s}^{z} \frac{dz'}{z'} \int_{1/(z's)}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\frac{1}{2} G(x_{21}^2, z') + \frac{1}{2} \Gamma(x_{10}^2, x_{21}^2, z') + Q(x_{21}^2, z') - \overline{\Gamma}(x_{10}^2, x_{21}^2, z') \right] \\
&\quad + \frac{\alpha_s N_c}{4\pi} \int_{\Lambda^2/s}^{z} \frac{dz'}{z'} \int_{1/(z's)}^{x_{10}^2 z/z'} \frac{dx_{21}^2}{x_{21}^2} \, Q(x_{21}^2, z') \\
\Gamma(x_{10}^2, x_{21}^2, z') &= G^{(0)}(z) + \frac{\alpha_s N_c}{2\pi} \int_{\max\{\Lambda^2, 1/x_{10}^2\}/s}^{z'} \frac{dz''}{z''} \int_{1/(z''s)}^{\min\{x_{10}^2, x_{21}^2 z'/z''\}} \frac{dx_{32}^2}{x_{32}^2} \left[\Gamma(x_{10}^2, x_{32}^2, z'') + 3G(x_{32}^2, z'') \right] \\
&\quad - \frac{\alpha_s N_f}{2\pi} \int_{\Lambda^2/s}^{z'} \frac{dz''}{z''} \int_{1/(z''s)}^{x_{21}^2 z'/z''} \frac{dx_{32}^2}{x_{32}^2} \, \overline{\Gamma}_{gen}(x_{10}^2, x_{32}^2, z'') \\
\overline{\Gamma}(x_{10}^2, x_{21}^2, z') &= Q^{(0)}(z) + \frac{\alpha_s N_c}{4\pi} \int_{\Lambda^2/s}^{z'} \frac{dz''}{z''} \int_{1/(z''s)}^{x_{21}^2 z'/z''} \frac{dx_{32}^2}{x_{32}^2} \, Q(x_{32}^2, z'') \\
&\quad + \frac{\alpha_s N_c}{2\pi} \int_{\Lambda^2/s}^{z'} \frac{dz''}{z''} \int_{1/(z''s)}^{\min\{x_{10}^2, x_{21}^2 z'/z''\}} \frac{dx_{32}^2}{x_{32}^2} \left[\frac{1}{2} G(x_{32}^2, z'') + \frac{1}{2} \Gamma(x_{10}^2, x_{32}^2, z'') + Q(x_{32}^2, z'') - \overline{\Gamma}(x_{10}^2, x_{32}^2, z'') \right],
\end{aligned}
$$

DLA Numerical Solutions

- The DLA-equations at large $N_c \& N_f$ have been solved numerically for the asymptotic form of $\Delta\Sigma(x, Q^2)$ as $x \to 0$ [Kovchegov and Tawabutr, 2020].



Plot of $\mathrm{sgn}\left[\Delta\Sigma\left(x, Q^2\right)\right] \ln\left|\Delta\Sigma\left(x, Q^2\right)\right|$ vs x at fixed $Q^2 = 10\ \mathrm{GeV}^2$

DLA Numerical Solutions

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$$\Delta\Sigma(x, Q^2)\Big|_{N_c, N_f \gg 1} \sim \left(\frac{1}{x}\right)^{\alpha_h \sqrt{\frac{\alpha_s N_c}{2\pi}}} \cos\left[\omega\sqrt{\frac{\alpha_s N_c}{2\pi}} \ln\frac{1}{x} + \varphi\right], \quad (7)$$

where

$$\omega \approx \frac{0.220 N_f}{1 + 0.126 N_f}. \quad (8)$$

- The intercept, α_h, is roughly 2.3 with a weak dependence on N_f.
- In the large-N_c limit,

$$\Delta\Sigma(x, Q^2)\Big|_{N_c \gg 1} \sim \left(\frac{1}{x}\right)^{2.31\sqrt{\frac{\alpha_s N_c}{2\pi}}}. \quad (9)$$

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where

$$\omega \approx \frac{0.220 N_f}{1 + 0.126 N_f}. \quad (11)$$

- The oscillation period spans many units of $\ln\frac{1}{x}$.

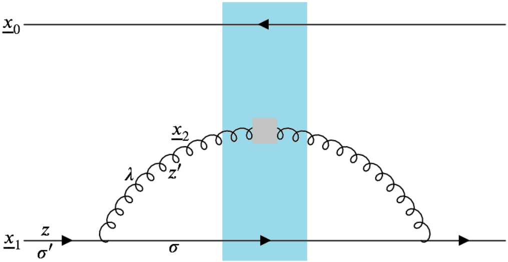
LLA Evolution Equations

- At DLA, a logarithmic factor comes from longitudinal integral, $\int \frac{dz'}{z'}$, and the other comes from transverse integral, $\int \frac{dx_{21}^2}{x_{21}^2}$.
- At LLA, single-logarithmic terms come from:
    - Terms with longitudinal logarithm only ($\mathrm{LLA_L}$): $\int \frac{dz'}{z'} \int dx_{21}^2$
    - Terms with transverse logarithm only ($\mathrm{LLA_T}$): $\int dz' \int \frac{dx_{21}^2}{x_{21}^2}$
    - The BK/JIMWLK evolution of unpolarized daughter dipoles
    - Running-coupling terms ($\ln \mu^2$ vs $\ln s$)
- The LLA equations are non-linear. The derivation is in progress.

LLA_L Terms

- The LLA_L terms come from the splitting in the $z' \ll z$ limit, but in the region where x_{21}^2-integral yields no additional logarithm.

$$Q(x_{10}^2, z) = Q^{(0)}(z) + \alpha_s \int \frac{dz'}{z'} \Bigg[\underbrace{A \int \frac{dx_{21}^2}{x_{21}^2}}_{\text{DLA}} + \underbrace{B \int dx_{21}^2}_{\text{LLA}_\text{L}} \Bigg] \text{(dipole amps)} \tag{12}$$

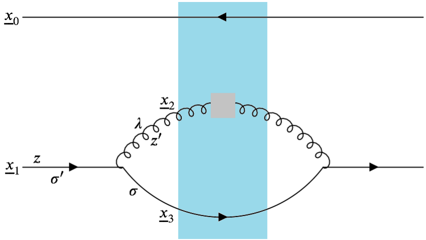


LLA$_T$ Terms

- In the limit $z' \sim z$, a parton splitting yields the LLA$_T$ terms.

$$Q(x_{10}^2, z) = Q^{(0)}(z) + \alpha_s \int dz' \, \Delta P(z'/z) \int \frac{dx_{32}^2}{x_{32}^2} \, \text{(dipole amps)} \qquad (13)$$

- Here, $\Delta P(z'/z)$ is the polarized DGLAP splitting function.



LLA$_T$ Terms

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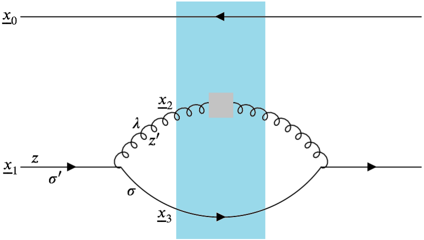
$$Q(x_{10}^2, z) = Q^{(0)}(z) + \alpha_s \int dz' \, \Delta P(z'/z) \int \frac{dx_{32}^2}{x_{32}^2} \, \text{(dipole amps)} \qquad (14)$$

- Since $z' \sim z - z' \sim z$, neither $\underline{x}_3$ nor $\underline{x}_2$ is necessarily close to $\underline{x}_1$.
- By momentum conservation, $z'\underline{x}_{21} + (z - z')\underline{x}_{31} = 0$.



LLA Evolution Equations – Next Steps

- Include the effect of running coupling
- Write systems of equations in the large-N_c and large-$N_c\&N_f$ limits
- Solve the resulting equations for asymptotic behavior of $\Delta\Sigma(x, Q^2)$ as $x \to 0$.

Conclusion

- Quark's helicity contribution follows evolution equations that contain leading DLA terms and subleading LLA terms.
- The DLA-order equations have been derived and numerically solved at large $N_c \& N_f$. The asymptotic form at small x shows an oscillation with $\ln \frac{1}{x}$. The period spans many units of rapidity.
- The LLA-order equations are in the process of derivation, in which one has to include the effect of running coupling and write the system of equations at large-N_c and large $N_c \& N_f$.
- Potential directions for future work
  - The complete evolution equations at LLA (in progress)
  - Analytic solution to DLA equation
  - More concrete implication to phenomenology

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