Full next-to-eikonal quark propagator in the CGC and applications

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Outline of the talk

1. Introduction
2. Quark propagator through a shockwave at the next-to-eikonal accuracy
   - Eikonal limit: quark propagator in a pure $\mathcal{A}^-$ background
   - Subeikonal corrections: random walk in a pure $\mathcal{A}^-$ background
   - Subeikonal corrections: single $\mathcal{A}_\perp$ interaction
   - Subeikonal corrections: double $\mathcal{A}_\perp$ interaction
   - Full result
3. Forward quark-nucleus scattering
   - Unpolarized cross section
   - Quark helicity asymmetry
4. Summary and conclusions

Eikonal vs. Non-Eikonal

High-energy scatterings:

$$\text{projectile : dilute proton} \; - \; \text{target : dense nucleus (CGC)}$$
$$\text{color charge density} \; - \; \text{strong background field } \mathcal{A}^\mu(x)$$

Eikonal approximation - nucleus infinitely Lorentz contracted at high energies

* target localized in the longitudinal direction (around $x^+ = 0$)
* target represented by the leading component of the background field and other components are suppressed
* background field independent of x^- due to the Lorentz time dilation

Background field in the eikonal limit: $\mathcal{A}^\mu(x^-, x^+, \mathbf{x}) \approx \delta^{\mu -}\delta(x^+)\mathcal{A}^-(\mathbf{x})$

Strong hierarchy of components of $\mathcal{A}^\mu$ with respect to the boost factor γ:

$$\mathcal{A}^- = O(\gamma) \quad \gg \quad \mathcal{A}_\perp = O(1) \quad \gg \quad \mathcal{A}^+ = O(1/\gamma)$$

$\gamma \sim 1000$ for the LHC energies ($\sim$ a few TeV per nucleon)
$\gamma \sim 100$ for the highest RHIC and EIC energies (100 - 200 GeV per nucleon)
$\gamma \sim 10$ at intermediate RHIC and EIC energies ($\sim$ 50 GeV)

Beyond eikonal approximation - subeikonal corrections

* target has a finite width - Brownian motion of the parton within the medium
* interactions with $\mathcal{A}_\perp$ field cannot be neglected

Subeikonal corrections emerge $\sim \frac{L^+}{k^+}$ (L^+ - the width, k^+ - the energy)

Quark propagator - basics

Full quark propagator

$$S_F(x,y)_{\alpha\beta} = S_{0,F}(x,y)_{\alpha\beta} + \delta S_F(x,y)_{\alpha\beta}$$

free propagator + corrections due to interactions with the background field

Free quark propagator:

$$S_{0,F}(x,y)_{\alpha\beta} = (\mathbf{1})_{\alpha\beta} \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\, \frac{i(\not{k}+m)}{[k^2-m^2+i\epsilon]}$$

Corrections:

- **at the eikonal order**

$$\delta S_F\Big|^{\text{Eik}} \equiv \delta S_F\Big|^{\text{Eik}}_{\text{pure } \mathcal{A}^-}$$

- **at the next-to-eikonal order**

$$\delta S_F\Big|^{\text{NEik}} \equiv \delta S_F\Big|^{\text{NEik}}_{\text{pure } \mathcal{A}^-} + \delta S_F\Big|^{\text{NEik}}_{\text{single } \mathcal{A}_\perp} + \delta S_F\Big|^{\text{NEik}}_{\text{double } \mathcal{A}_\perp}$$

Brownian motion of the fast parton induces $\mathcal{A}_\perp$ field.

$$S_F(x,y)_{\alpha\beta}\Big|^{\text{Eik}} = S_{0,F}(x,y)_{\alpha\beta} + \delta S_F(x,y)_{\alpha\beta}\Big|^{\text{Eik}}_{\text{pure } \mathcal{A}^-}$$

In eikonal limit quark interacts with arbitrarily many soft $\mathcal{A}^-$ fields

$$S_F(x,y)_{\alpha\beta}\Big|^{\text{Eik}} = \mathbf{1}_{\alpha\beta}\,\delta^{(3)}(\underline{x}-\underline{y})\,\text{sgn}(x^- - y^-)\,\frac{\gamma^+}{4}$$
$$+\int\frac{d^3\underline{q}}{(2\pi)^3}\int\frac{d^3\underline{k}}{(2\pi)^3}\,2\pi\delta(q^+-k^+)e^{-ix\cdot\check{q}+iy\cdot\check{k}}\,\frac{(\check{\not{q}}+m)\gamma^+(\check{\not{k}}+m)}{(2k^+)^2}$$
$$\times\int d^2\mathbf{z}\,e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}\Big\{\theta(k^+)\theta(x^+-y^+)\,\mathcal{U}_F(x^+,y^+;\mathbf{z})_{\alpha\beta}$$
$$-\theta(-k^+)\theta(y^+-x^+)\,\mathcal{U}_F^\dagger(y^+,x^+;\mathbf{z})_{\alpha\beta}\Big\}$$

Medium contribution entangled in the Wilson lines:

$$\mathcal{U}_F(x^+,y^+;\mathbf{z}) \equiv \mathbf{1} + \sum_{N=1}^{+\infty}\frac{1}{N!}\,\mathcal{P}_+\left[-ig\int_{y^+}^{x^+} dz^+\,t\cdot\mathcal{A}^-(z^+,\mathbf{z})\right]^N$$

- background field $\mathcal{A}_a^-(z^+,\mathbf{z})$ has a finite support $[-L^+/2, L^+/2]$ - this is where the non-trivial medium contributions come from in the interval $[y^+, x^+]$
- if there is no support the propagator reduces to the Feynman propagator in vacuum

Subeikonal corrections: Brownian motion in a pure $\mathcal{A}^-$ background

Full next-to-eikonal quark propagator:

$$S_F\Big|^{\rm NEik} = \underbrace{S_F\Big|^{\rm Eik} + \delta S_F\Big|^{\rm NEik}_{\text{pure }\mathcal{A}^-}}_{S_F\big|_{\text{pure }\mathcal{A}^-}} + \delta S_F\Big|^{\rm NEik}_{\text{single }\mathcal{A}_\perp} + \delta S_F\Big|^{\rm NEik}_{\text{double }\mathcal{A}_\perp}$$

Quark propagator in pure $\mathcal{A}^-$ background field up to next-to-eikonal order for positive energy:

$$S_F(x,y)_{\alpha\beta}\Big|_{\text{pure }\mathcal{A}^-} = \int \frac{d^3\underline{q}}{(2\pi)^3}\int\frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+-k^+)\,\frac{\theta(k^+)}{(2k^+)^2}\, e^{-ix\cdot\check{q}+iy\cdot\check{k}}\,(\check{\not q}+m)\gamma^+(\check{\not k}+m)$$
$$\times\int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}\Big\{\mathcal{U}_F\Big(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{z}\Big)$$
$$-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{4k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\Big[\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\overleftrightarrow{\partial_{\mathbf{z}^j}}\,\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)\Big]$$
$$-\frac{i}{2k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\Big[\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\overleftarrow{\partial_{\mathbf{z}^j}}\,\overrightarrow{\partial_{\mathbf{z}^j}}\,\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)\Big]\Big\}$$

NEik corrections: Brownian motion with a drift

for the gluon propagator with subeikonal corrections see:
Altinoluk, Armesto, Beuf, Martinez, Salgado, JHEP **1407**, 068 (2014)
Altinoluk, Armesto, Beuf, Moscoso, JHEP **1601**, 114 (2016)

Subeikonal corrections: single $\mathcal{A}_\perp$ insertion

Full next-to-eikonal quark propagator:

$$S_F\Big|^{\text{NEik}} = S_F\Big|^{\text{Eik}} + \delta S_F\Big|^{\text{NEik}}_{\text{pure }\mathcal{A}^-} + \delta S_F\Big|^{\text{NEik}}_{\text{single }\mathcal{A}_\perp} + \delta S_F\Big|^{\text{NEik}}_{\text{double }\mathcal{A}_\perp}$$

Replace one $\gamma^+\mathcal{A}_a^-$ insertion in the Wilson line by $\gamma^j\mathcal{A}_j^a$

$$\delta S_F(x,y)\Big|^{\text{NEik}}_{\text{single }\mathcal{A}_\perp} = \int d^4z\; S_F(x,z)\Big|_{\text{pure }\mathcal{A}^-} [-ig\,\gamma^j\,t^a]\,\mathcal{A}_j^a(\underline{z}) \quad S_F(z,y)\Big|_{\text{pure }\mathcal{A}^-}$$

Subeikonal correction due to an interaction with $\mathcal{A}_\perp$:

$$\begin{aligned}\delta S_F(x,y)\Big|^{\text{NEik}}_{\text{single }\mathcal{A}_\perp} &= \int \frac{d^3\underline{q}}{(2\pi)^3}\int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+-k^+)\,\frac{\theta(k^+)}{(2k^+)^3}\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}} \\ &\times (\check{\not{q}}+m)\gamma^j\gamma^+\gamma^i\,(\check{\not{k}}+m)\int d^3\underline{z}\left[e^{-i\mathbf{z}\cdot\mathbf{q}}\,\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\right] \\ &\times \left[\overleftarrow{\partial_{\mathbf{z}^j}}\,[gt\cdot\mathcal{A}_i(\underline{z})] - [gt\cdot\mathcal{A}_j(\underline{z})]\,\overrightarrow{\partial_{\mathbf{z}^i}}\right]\left[\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)\, e^{i\mathbf{z}\cdot\mathbf{k}}\right]\end{aligned}$$

The external points x^+ and y^+ are enforced to be outside the support region $[-L^+/2, L^+/2]$

Subeikonal corrections: double $\mathcal{A}_\perp$ insertion

Full next-to-eikonal quark propagator:

$$S_F\Big|^{\text{NEik}} = S_F\Big|^{\text{Eik}} + \delta S_F\Big|^{\text{NEik}}_{\text{pure } \mathcal{A}^-} + \delta S_F\Big|^{\text{NEik}}_{\text{single } \mathcal{A}_\perp} + \delta S_F\Big|^{\text{NEik}}_{\text{double } \mathcal{A}_\perp}$$

Replace two $\gamma^+ \mathcal{A}_a^-$ insertions in the Wilson line by $\gamma^j \mathcal{A}_j^a$ and $\gamma^i \mathcal{A}_i^a$

$$\delta S_F(x,y)\Big|^{\text{NEik}}_{\text{double } \mathcal{A}_\perp} = \int d^4z \int d^4z' \; S_F(x,z')\Big|_{\text{pure } \mathcal{A}^-} [-ig\,\gamma^j\, t^b]\, \mathcal{A}_j^b(\underline{z}') \times S_F(z',z)\Big|_{\text{pure } \mathcal{A}^-} [-ig\,\gamma^i\, t^a]\, \mathcal{A}_i^a(\underline{z}) \;\; S_F(z,y)\Big|_{\text{pure } \mathcal{A}^-}$$

Subeikonal correction due to instantaneous double $\mathcal{A}_\perp$ interaction:

$$\delta S_F(x,y)\Big|^{\text{NEik}}_{\text{double } \mathcal{A}_\perp} = \int \frac{d^3\underline{q}}{(2\pi)^3} \int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+ - k^+)\, \frac{\theta(k^+)}{(2k^+)^3}\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}} \times (\check{\not{q}} + m)\gamma^j\gamma^+\gamma^i\,(\check{\not{k}} + m) \int d^3\underline{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})} \times (-i)\;\; \mathcal{U}_F\left(\frac{L^+}{2}, z^+; \mathbf{z}\right) [gt\cdot\mathcal{A}_j(\underline{z})]\,[gt\cdot\mathcal{A}_i(\underline{z})]\,\mathcal{U}_F\left(z^+, -\frac{L^+}{2}; \mathbf{z}\right)$$

Next-to-eikonal quark propagator - spinor structure

Spinor structure:

- $\mathcal{A}^-$ field associated with $(\check{\not{q}} + m)\gamma^+(\check{\not{k}} + m)$
- $\mathcal{A}_\perp$ field associated with $(\check{\not{q}} + m)\gamma^j\gamma^+\gamma^i\,(\check{\not{k}} + m)$

  - separate symmetric and anti-symmetric parts:

$$\gamma^j\gamma^+\gamma^i = \delta^{ij}\,\gamma^+ + \gamma^+\frac{[\gamma^i, \gamma^j]}{2}$$

(helicity independent + helicity dependent)

Helicity dependence:

$$[\gamma^i, \gamma^j] = -4i\epsilon^{ij}S^3$$

S^3 - helicity operator with properties:
$$S^3 u(\check{k}, h) = h u(\check{k}, h)$$
$$S^3 v(\check{k}, h) = -h v(\check{k}, h)$$

Quark propagator

$$S_F(x, y) = S_F(x, y)\Big|_{\mathrm{unpol.}} + S_F(x, y)\Big|_{\mathrm{h.\ dep.}}$$

Next-to-eikonal quark propagator - full result

Unpolarized part

$$S_F(x,y)\Big|_{\text{unpol.}} = \int \frac{d^3\underline{q}}{(2\pi)^3}\int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+-k^+)\,\frac{\theta(k^+)}{(2k^+)^2}\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}\,(\check{\not{q}}+m)\gamma^+(\check{\not{k}}+m)$$
$$\times \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}\Bigg\{\mathcal{U}_F\Big(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{z}\Big)$$
$$-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{4k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\,\Big[\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\overleftrightarrow{\mathcal{D}_{\mathbf{z}^j}}\,\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)\Big]$$
$$-\frac{i}{2k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\,\Big[\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\overleftarrow{\mathcal{D}_{\mathbf{z}^j}}\,\overrightarrow{\mathcal{D}_{\mathbf{z}^j}}\,\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)\Big]\Bigg\}$$

Helicity-dependent part

$$S_F(x,y)\Big|_{\text{h. dep.}} = \int \frac{d^3\underline{q}}{(2\pi)^3}\int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+-k^+)\,\frac{\theta(k^+)}{(2k^+)^3}\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}$$
$$\times(\check{\not{q}}+m)\gamma^+\frac{[\gamma^i,\gamma^j]}{4}(\check{\not{k}}+m)\int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}$$
$$\times\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\,\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\, gt\cdot\mathcal{F}_{ij}(\underline{z})\,\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)$$

The final result fully gauge invariant (due to covariant derivatives):

$$\overrightarrow{\mathcal{D}_{z^\mu}} \equiv \overrightarrow{\partial_{z^\mu}} + igt\cdot\mathcal{A}_\mu(\underline{z});\quad \overleftarrow{\mathcal{D}_{z^\mu}} \equiv \overrightarrow{\mathcal{D}_{z^\mu}}^{\dagger};\quad \overleftrightarrow{\mathcal{D}_{z^\mu}} \equiv \overrightarrow{\mathcal{D}_{z^\mu}} - \overleftarrow{\mathcal{D}_{z^\mu}}$$

Longitudinal chromo-magnetic field along the target associated with helicity:

$$\mathcal{F}^a_{ij}(\underline{z}) \equiv \partial_{\mathbf{z}^i}\mathcal{A}^a_j(\underline{z}) - \partial_{\mathbf{z}^j}\mathcal{A}^a_i(\underline{z}) - gf^{abc}\mathcal{A}^b_i(\underline{z})\mathcal{A}^c_j(\underline{z})$$

Scattering amplitude vs. quark propagator

- The simplest observable where subeikonal corrections matter: quark-target cross section
- Need for the relevant scattering amplitude

S-matrix element

Formal definition: $S_{q(\check{q},h',\beta)\leftarrow q(\check{k},h,\alpha)} = \langle 0|\hat{b}_{\text{out}}(\check{q},h,\beta)\hat{b}^{\dagger}_{\text{in}}(\check{k},h,\alpha)|0\rangle$

$$S_{q(\check{q},h',\beta)\leftarrow q(\check{k},h,\alpha)} = \lim_{x^+\to\infty}\lim_{y^+\to-\infty}\int d^2\mathbf{x}\int dx^-\int d^2\mathbf{y}\int dy^-\, e^{ix\cdot\check{q}-iy\cdot\check{k}} \times \bar{u}(\check{q},h')\gamma^+ S_F(x,y)_{\alpha\beta}\gamma^+ u(\check{k},h)$$

LSZ reduction: $S_{q(\check{q},h',\beta)\leftarrow q(\check{k},h,\alpha)} = (2k^+)2\pi\delta(q^+-k^+)i\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q})$

Quark-target scattering amplitude

$$\begin{aligned}
i\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q}) = \delta_{hh'}\int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}\Bigg\{&\mathcal{U}_F\left(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{z}\right) \\
&+\frac{\epsilon^{ij}h}{2k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\left[\mathcal{U}_F\left(\frac{L^+}{2},z^+;\mathbf{z}\right)\left(-igt\cdot\mathcal{F}_{ij}(\underline{z})\right)\mathcal{U}_F\left(z^+,-\frac{L^+}{2};\mathbf{z}\right)\right] \\
&-\frac{i}{2k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\left[\mathcal{U}_F\left(\frac{L^+}{2},z^+;\mathbf{z}\right)\overleftarrow{\mathcal{D}_{\mathbf{z}^j}}\,\overrightarrow{\mathcal{D}_{\mathbf{z}^j}}\mathcal{U}_F\left(z^+,-\frac{L^+}{2};\mathbf{z}\right)\right] \\
&-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{4k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\left[\mathcal{U}_F\left(\frac{L^+}{2},z^+;\mathbf{z}\right)\overleftrightarrow{\mathcal{D}_{\mathbf{z}^j}}\,\mathcal{U}_F\left(z^+,-\frac{L^+}{2};\mathbf{z}\right)\right]\Bigg\}_{\alpha\beta}
\end{aligned}$$

Unpolarized cross section

Differential cross section of the quark traversing the target is

$$\frac{d^2\sigma^{qA\to q+X}}{d^2\mathbf{q}} = \frac{1}{(2\pi)^2}\frac{1}{2N_c}\sum_{h,h'}\sum_{\alpha,\beta}\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q})^\dagger\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q})\Big|_{q^+=k^+}$$

Cross section averaged over the target $(\mathbf{z}-\mathbf{z}')\equiv\mathbf{r}$ and $(\mathbf{z}+\mathbf{z}')\equiv 2\mathbf{b}$

$$\left\langle\frac{d^2\sigma^{qA\to q+X}}{d^2\mathbf{q}}\right\rangle_A = \frac{1}{(2\pi)^2}\int d^2\mathbf{r}\, e^{-i(\mathbf{q}-\mathbf{k})\cdot\mathbf{r}}\Big\{1-\bar{P}(\mathbf{r}) + \Big(\bar{O}(\mathbf{r}) - \frac{(\mathbf{q}^j+\mathbf{k}^j)}{4k^+}\Big[\mathcal{O}^j_{(1)}(\mathbf{r})-\mathcal{O}^{\dagger j}_{(1)}(\mathbf{r})\Big] - \frac{i}{2k^+}\Big[\mathcal{O}_{(2)}(\mathbf{r})-\mathcal{O}^\dagger_{(2)}(\mathbf{r})\Big]\Big)\Big\}$$

Dipole operator:

$$d_F(\mathbf{r}) = 1-\bar{P}(\mathbf{r})+\bar{O}(\mathbf{r})$$

$$d_F(\mathbf{r}) = \frac{1}{N_c}\int d^2\mathbf{b}\Big\langle \mathrm{tr}\Big[\mathcal{U}^\dagger_F\Big(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{b}-\frac{\mathbf{r}}{2}\Big)\mathcal{U}_F\Big(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big]\Big\rangle_A$$

Decorated dipole operators:

$$\mathcal{O}^j_{(1)}(\mathbf{r}) = \frac{1}{N_c}\int d^2\mathbf{b}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\Big\langle \mathrm{tr}\Big[\mathcal{U}^\dagger_F\Big(\mathbf{b}-\frac{\mathbf{r}}{2}\Big) \times \mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\overleftrightarrow{\mathcal{D}}_{\mathbf{b}^j+\frac{\mathbf{r}^j}{2}}\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big]\Big\rangle_A$$

]

$$\mathcal{O}_{(2)}(\mathbf{r}) = \frac{1}{N_c}\int d^2\mathbf{b}\int_{-L^+/2}^{L^+/2} dz^+\Big\langle \mathrm{tr}\Big[\mathcal{U}^\dagger_F\Big(\mathbf{b}-\frac{\mathbf{r}}{2}\Big) \times \mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\overleftarrow{\mathcal{D}}_{\mathbf{b}^j+\frac{\mathbf{r}^j}{2}}\overrightarrow{\mathcal{D}}_{\mathbf{b}^j+\frac{\mathbf{r}^j}{2}}\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big]\Big\rangle_A.$$

Unpolarized cross section - symmetries

Anti-quark–target differential cross section:

$$\left\langle \frac{d^2\sigma^{\bar{q}A\to\bar{q}+X}}{d^2\mathbf{q}} \right\rangle_A = \frac{1}{(2\pi)^2}\int d^2\mathbf{r}\; e^{-i(\mathbf{q}-\mathbf{k})\cdot\mathbf{r}}\Big\{1-\bar{P}(\mathbf{r})$$
$$-\Big(\bar{O}(\mathbf{r}) - \frac{(\mathbf{q}^j+\mathbf{k}^j)}{4k^+}\Big[\mathcal{O}^j_{(1)}(\mathbf{r})-\mathcal{O}^{\dagger j}_{(1)}(\mathbf{r})\Big] - \frac{i}{2k^+}\Big[\mathcal{O}_{(2)}(\mathbf{r})-\mathcal{O}^\dagger_{(2)}(\mathbf{r})\Big]\Big)\Big\}$$

- **Signature transformation ($\mathcal{U}\to\mathcal{U}^\dagger$) and charge conjugation ($q\to\bar{q}$):**

  The symmetries conveniently studied by introducing Pomeron and Odderon

  $\bar{P}(\mathbf{r})$: Pomeron is **even** under both transformations
  $\bar{O}(\mathbf{r})$: Odderon is **odd** under both transformations

  * **Eikonal terms contain both Pomeron and Odderon**
  * **Next-to-eikonal corrections are of Odderon-type contributions**

- **Rotational symmetries due to target averaging and integration over b:**
  - $\mathcal{O}^j_{(1)}(\mathbf{r})$ behaves as a vector quantity
  - $\mathcal{O}_{(2)}(\mathbf{r})$ behaves as a scalar quantity

Quark helicity asymmetry

The difference between the cross sections for a quark of positive and negative helicity scattering on the nucleus target is:

$$\frac{d^2\Delta\sigma^{qA\to q+X}}{d^2\mathbf{q}} \equiv \frac{1}{(2\pi)^2}\frac{1}{2N_c}\sum_{h,h'}\sum_{\alpha,\beta}(2h)\,\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q})^\dagger\,\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q})\Big|_{q^+=k^+}$$

Quark helicity asymmetry averaged over the target

$$\left\langle\frac{d^2\Delta\sigma^{qA\to q+X}}{d^2\mathbf{q}}\right\rangle_A = \frac{1}{(2\pi)^2}\int d^2\mathbf{r}\, e^{-i(\mathbf{q}-\mathbf{k})\cdot\mathbf{r}}\frac{(-i)}{4k^+}\Big[O_{(3)}(\mathbf{r}) - O^\dagger_{(3)}(\mathbf{r})\Big]$$

New decorated dipole operator:

$$O_{(3)}(\mathbf{r}) = \frac{1}{N_c}\int d^2\mathbf{b}\int_{-L^+/2}^{L^+/2} dz^+\Big\langle \mathrm{Tr}\Big[\mathcal{U}_F^\dagger\Big(\mathbf{b}-\frac{\mathbf{r}}{2}\Big)$$
$$\times\,\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big\{\epsilon^{ij}\Big[gt\cdot\mathcal{F}_{ij}\Big(z^+,\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big]\Big\}\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big]\Big\rangle_A$$

The anti-quark helicity asymmetry is identical!

Symmetries:

- $O_{(3)}(\mathbf{r})$ behaves as a scalar quantity under rotations
- $O_{(3)}(\mathbf{r})$ is neither Pomeron nor Odderon
  (odd under signature transformation but even under charge conjugation)

Summary and conclusions

- Full form of the next-to-eikonal quark propagator through the background field derived
  - corrections due to random walk through the pure $\mathcal{A}^-$ field
  - corrections due to interaction with $\mathcal{A}_\perp$
- Gauge invariance guaranteed
- Next-to-eikonal corrections are the leading order for studying spin-sensitive phenomena
- Cross section of the quark traversing the target found obtained up to next-to-eikonal accuracy
- Quark and antiquark helicity asymmetries and their symmetries studied
- **Derived quark propagator is of a general form and therefore of a general use: applicable for different scattering processes**
  (calculation of DIS di-jet production in progress)