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Resummation of non-global observables

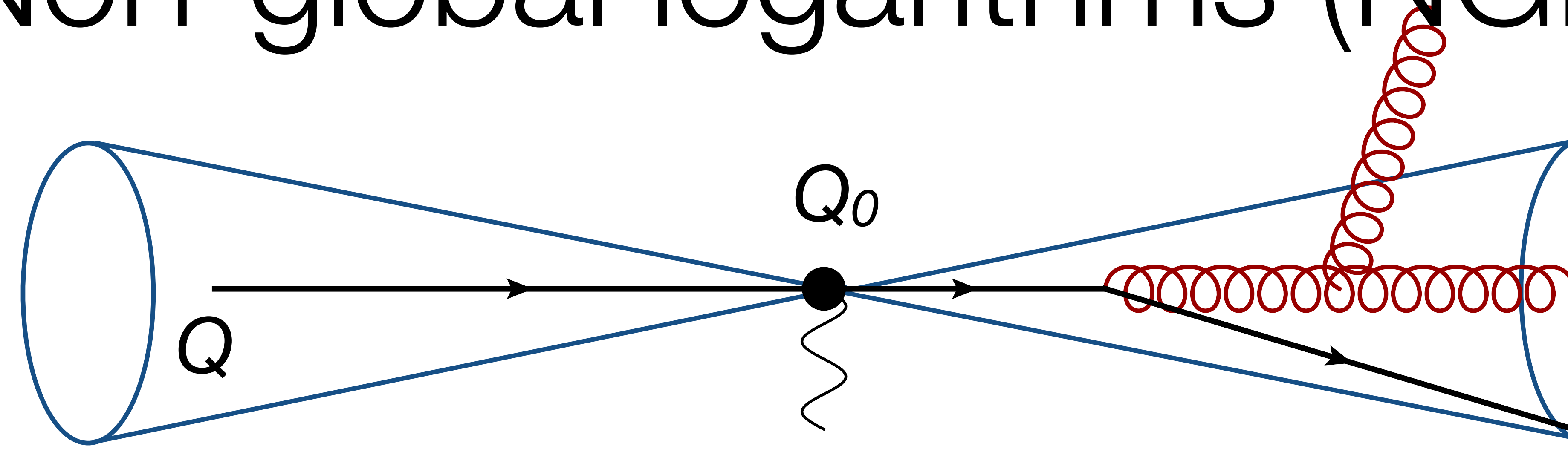
Thomas Becher
University of Bern

Resummation, **E**volution, **F**actorization 2020,
Higgs Centre Workshop, December 7-11, 2020

Outline

- **R**esummation
  - Top production with central jet veto
  - Towards higher logarithmic accuracy
- **E**volution
  - RG evolution for NGLs
  - Implementation as a parton shower: `ngl_resum`
- **F**actorization
  - Global vs. non-global observables
  - Soft radiation and multi-Wilson-line operators

Non-global logarithms (NGLs)



\`\`rapidity slice” *aka* \`\`gap between jets” *aka* \`\`interjet energy flow”

Dasgupta, Salam ’02: soft gluons from emissions inside the jets lead to complicated pattern of logs $\alpha_s^n \ln^m(Q/Q_0)$, $m \leq n$: single-log \`\`NLL” effect

- Even leading NGLs do not simply exponentiate!
- At large-N_c logs can be obtained with parton shower Dasgupta, Salam ‘02 or by solving a non-linear integral equation Banfi, Marchesini, Smye ’02
- Some first finite-N_c results Hatta, Ueda ’13, ’20 + Hagiwara ‘15 based on Weigert ’03. + a lot of ongoing work: Nagy, Soper ’07,…; Plätzer, Sjödahl ‘12; + Thorén ’18, De Angelis, Forshaw, Holguin, Plätzer ’19, … ; Hoeche Reichelt ’20; Hamilton, Medves, Salam, Scyboz, Soyez ’20; …

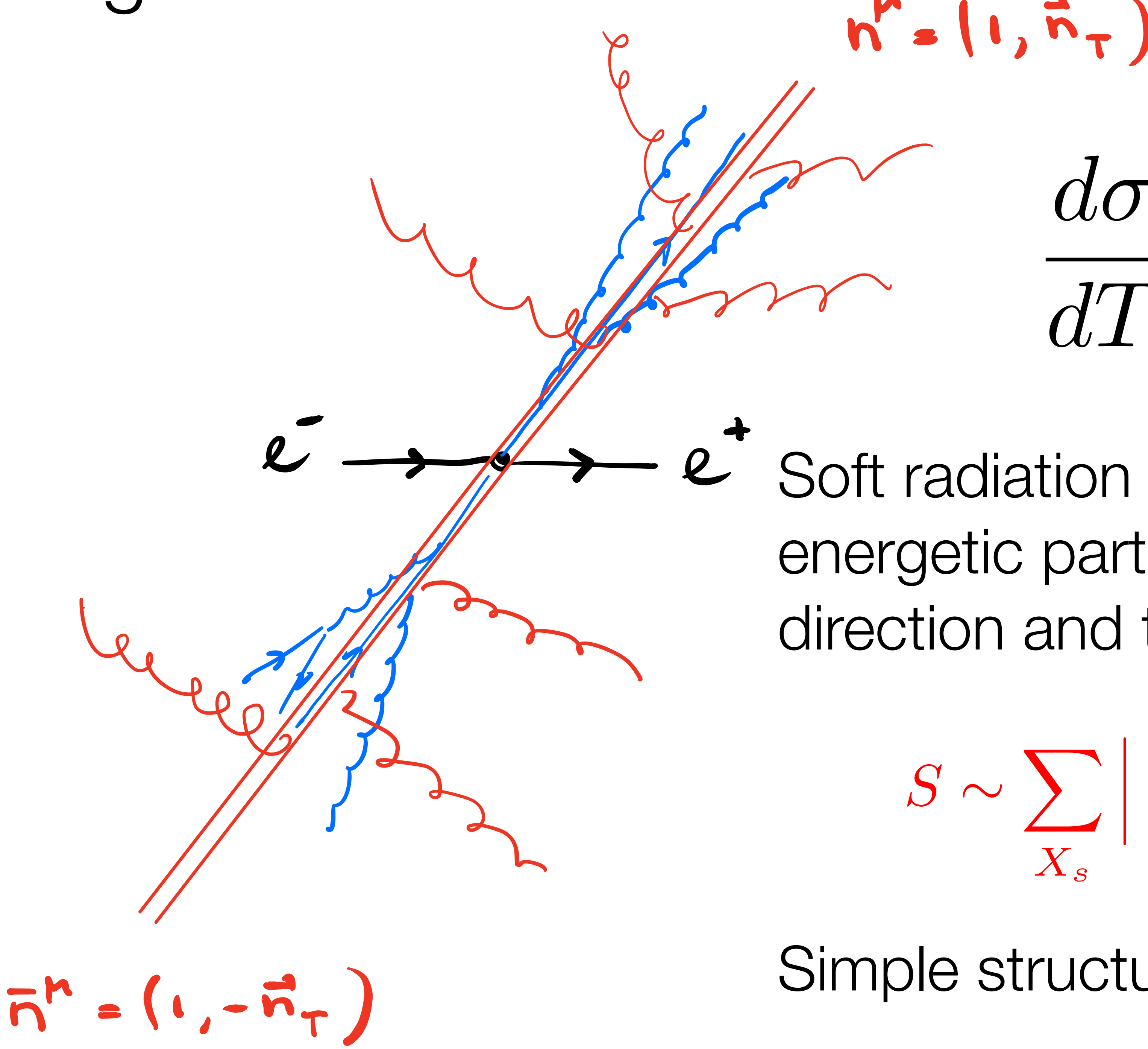
Resummation

Evolution

Factorization

Soft radiation in global observables

e.g. thrust $T \sim 1$



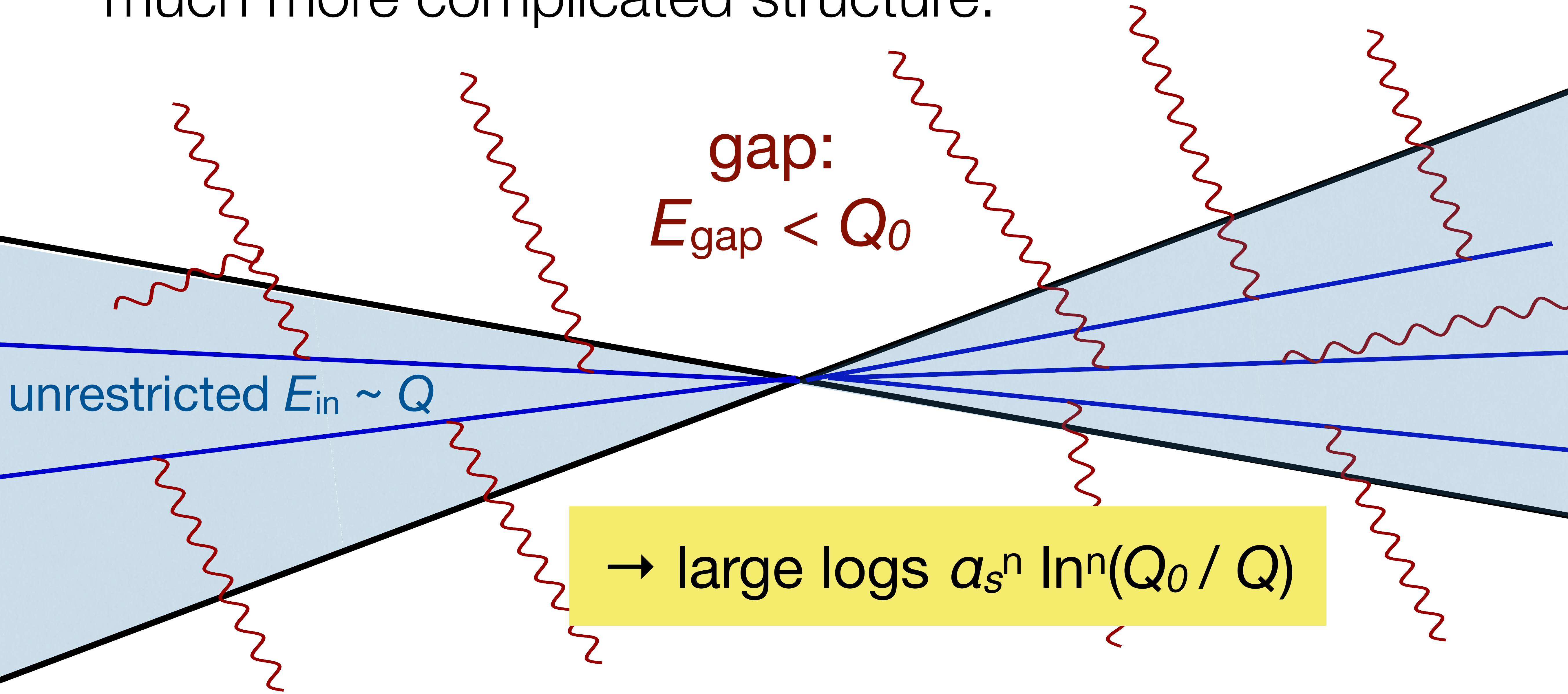
$$\frac{d\sigma}{dT} = H \cdot J \otimes S$$

Soft radiation does not resolve individual energetic partons. Sensitive only to direction and total charge of the jets

$$S \sim \sum_{X_s} \Big| \langle X_s | \, \boldsymbol{S}(n) \boldsymbol{S}(\bar{n}) \, |0\rangle \Big|^2$$

Simple structure → N³LL resummation

Soft radiation in non-global observables has a much more complicated structure:



Hard partons (quarks and gluons) inside jets act as sources: soft radiation pattern depends on color-charges and directions of all hard partons!

Factorization for gap between jets in e^+e^-

TB, Neubert, Rothen, Shao ’15 ’16, see also Caron-Huot ‘15

Hard function
m hard partons along
fixed directions $\{n_1, \ldots, n_m\}$
$\boldsymbol{\mathcal{H}}_m \propto |\mathcal{M}_m\rangle\langle\mathcal{M}_m|$

Soft function
squared amplitude
with *m* Wilson lines

$$\sigma(Q, Q_0) = \sum_{m=2}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \mu) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q_0, \mu) \rangle$$

color trace

integration over directions

Soft emissions in process with m energetic particles are obtained from the matrix elements of the operator

$$\boldsymbol{S}_1(n_1)\,\boldsymbol{S}_2(n_2)\;\ldots\;\boldsymbol{S}_m(n_m)|\mathcal{M}_m(\{\underline{p}\})\rangle$$

soft Wilson lines along the directions of the energetic particles / jets (color matrices)

hard scattering amplitude with *m* particles (vector in color space)

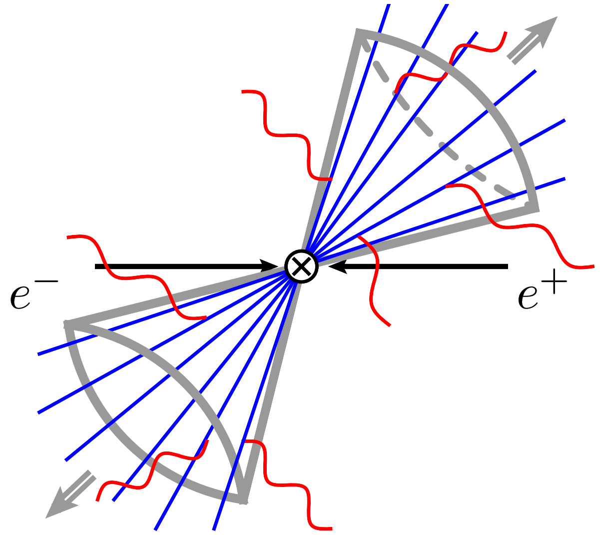
To get the amplitudes with additional soft partons, one takes the matrix element of the multi-Wilson-line operators:

$$\langle X_s|\,\boldsymbol{S}_1(n_1)\;\ldots\;\boldsymbol{S}_m(n_m)\,|0\rangle$$

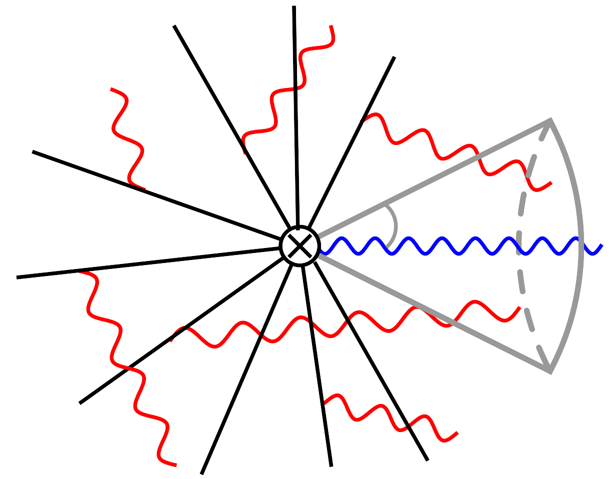
$$\sigma(Q,Q_0) = \sum_{m=2}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\mu) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\},Q_0,\mu) \rangle$$

Factorization theorem:

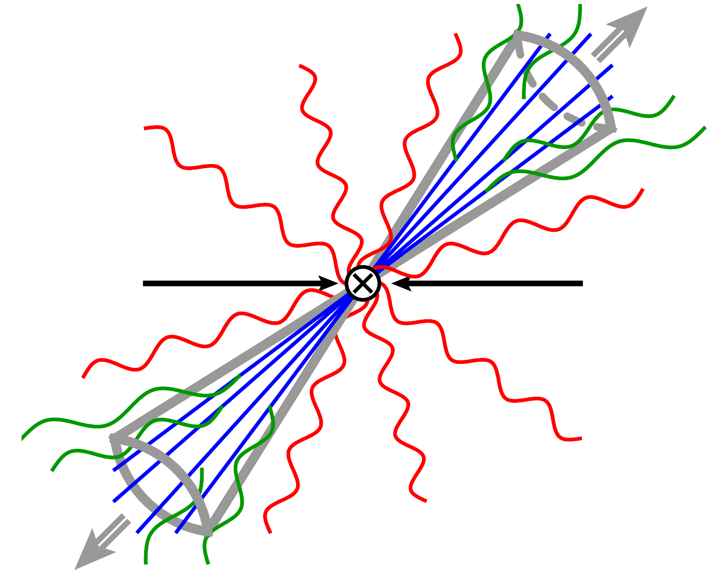
- Separates contributions from scales Q and Q_0
- Valid in the soft limit $Q_0/Q \to 0$ up to power corrections
- Operator definitions of ingredients
- Provides a natural way to perform resummation via renormalization group (RG) evolution
- Not limited to leading logarithms or leading color



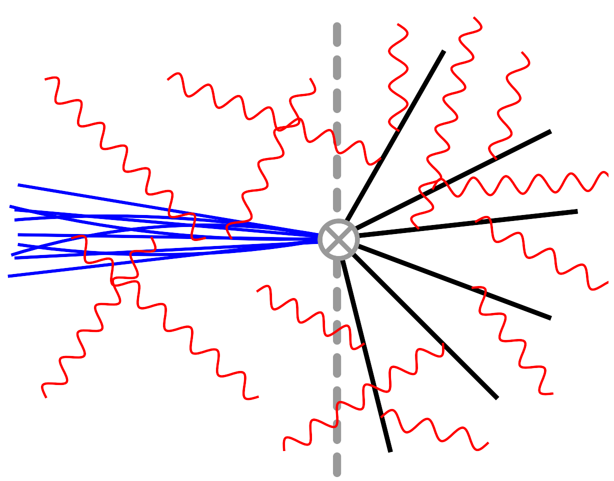
1.) 2.) cone jets, gaps between jets, 7.) for tops



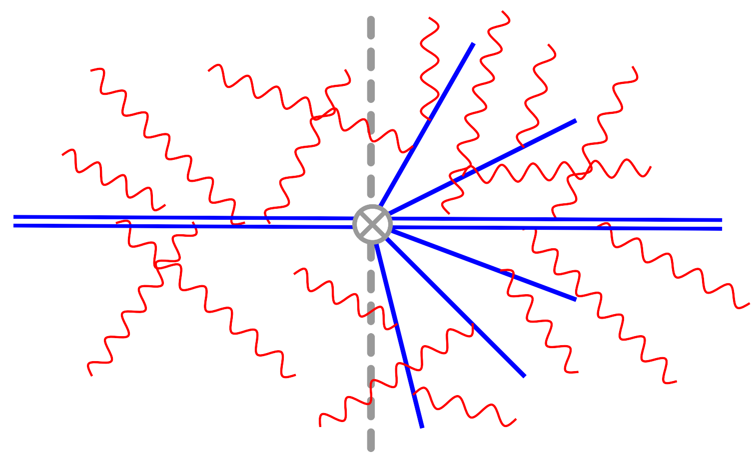
5.) isolation cones



1.) narrow jets; 6.) Z+jet q_T



3.) light-jet mass
4.) narrow broadening
8.) single-hadron q_T



3.) hemisphere soft function

1.) 2.) TB, Neubert, Rothen, Shao ’15 ’16

3.) TB, Pecjak, Shao ’16

4.) TB, Rahn, Shao ’17

5.) Balsiger, TB, Shao, ’18

6.) Chien Shao Wu ’19

7.) Balsiger, TB, Ferroglia ’20

8.) Kang, Shao, Zhao ’20

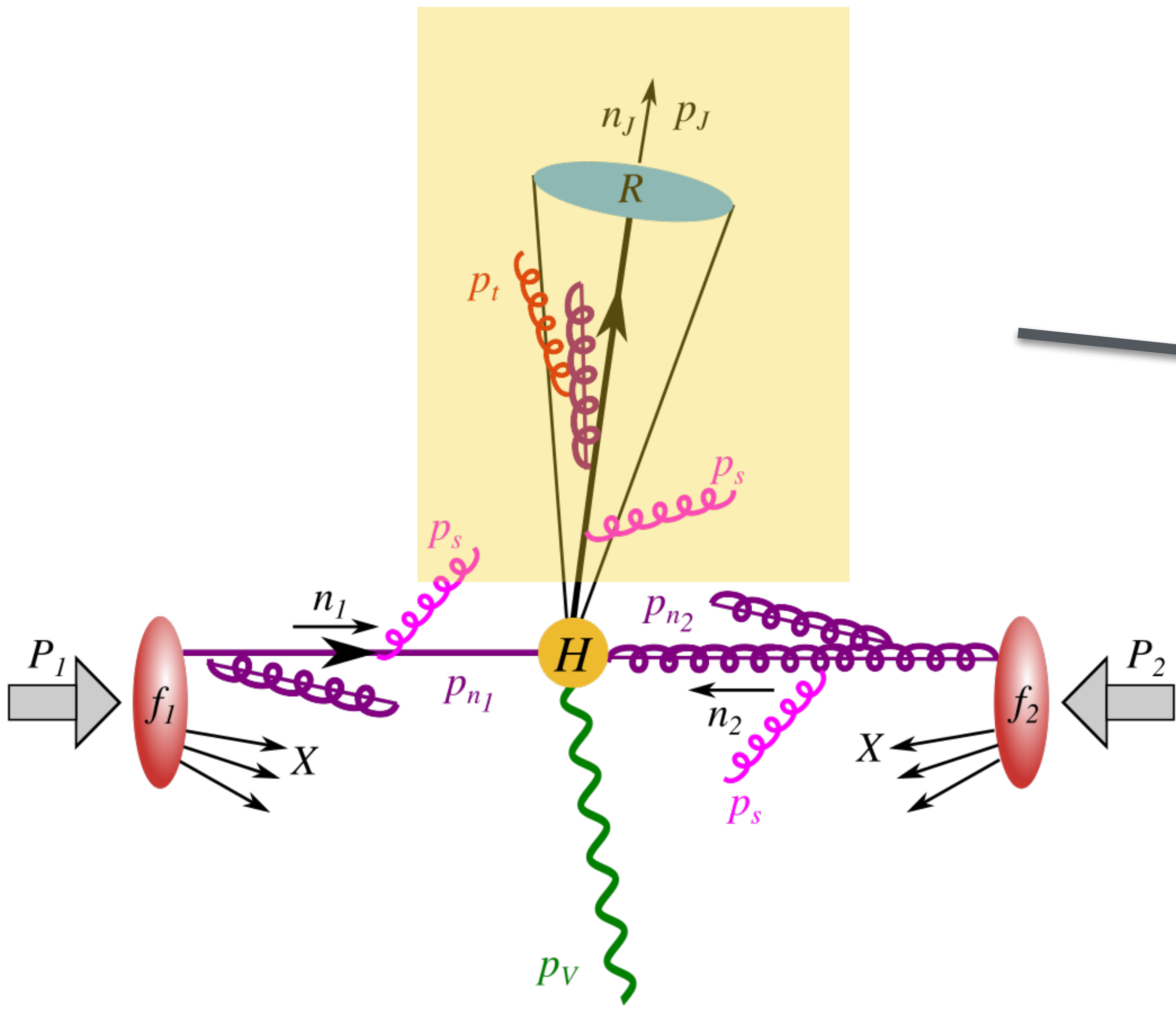
Effective field theory for (non-global) jet observables!

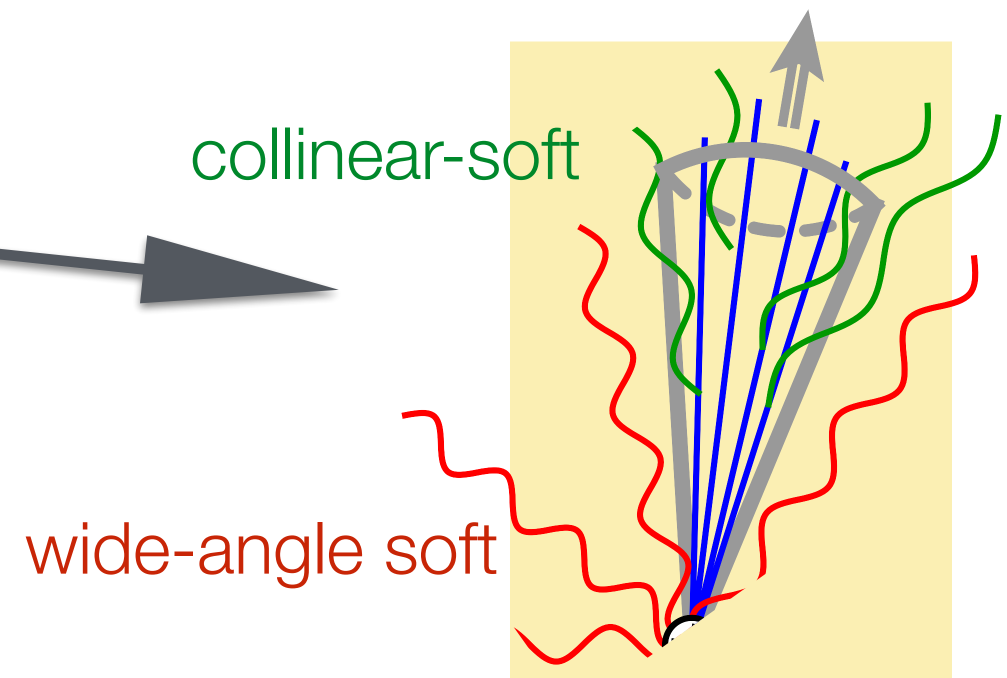
Transverse momentum & NGLs

- As for global observables, one encounters rapidity logarithms in TMD processes. Same structure as in the global case, because it is tied to collinear physics
  - Resum using Collinear Anomaly or Rapidity RG formalisms
- Factorization theorems for several transverse observables
  - Narrow broadening TB, Rahn, Shao ’17
  - q_T resummation for $pp \to Z + \text{jet}$ Chien, Shao and Wu ’19
    - Note: No NGLs in azimuthal decorrelation for WTA axis Chien, Rahn, Schrijnder van Velzen, Shao, Waalewijn and Wu ‘20
  - Single hadron q_T distribution Kang, Shao, Zhao ’20

q_T resummation for $pp \to Z + \text{jet}$

Chien, Shao and Wu '19





$$\mathcal{J}^k(p_J^2, \vec{x}_T, \epsilon) \to \sum_{m=1}^{\infty} \langle \boldsymbol{\mathcal{J}}^k_m(\{\underline{n_J}\}, R\,p_J, \epsilon) \otimes \boldsymbol{\mathcal{U}}^k_m(\{\underline{n_J}\}, R\,\vec{x}_T, \epsilon) \rangle$$

Interesting also in the context of heavy-ion collisions.

Resummation

Evolution

Factorization

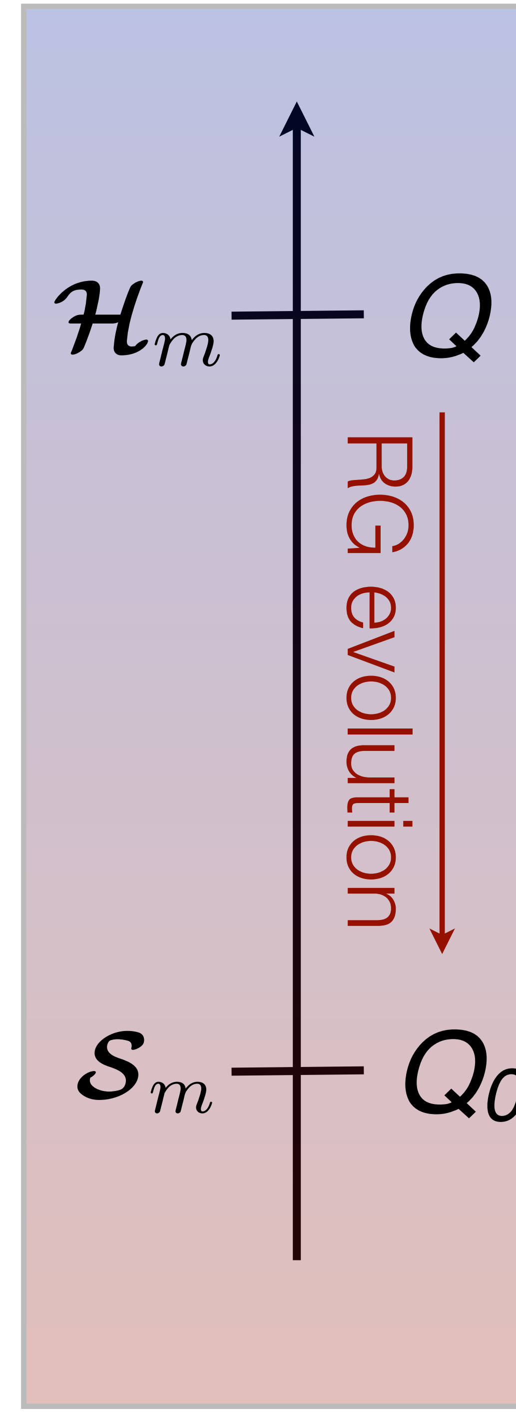
Resummation by RG evolution

Wilson coefficients fulfill RG equations

$$\frac{d}{d\ln\mu}\boldsymbol{\mathcal{H}}_m(Q,\mu) = -\sum_{l=2}^{m}\boldsymbol{\mathcal{H}}_l(Q,\mu)\,\boldsymbol{\Gamma}^H_{lm}(Q,\mu)$$

1. Compute $\mathcal{H}_m$ at a characteristic high scale $\mu_h \sim Q$
2. Evolve $\mathcal{H}_m$ to the scale of low energy physics $\mu_s \sim Q_0$
3. Evaluate $\mathcal{S}_m$ at low scale $\mu_s \sim Q_0$

Avoids large logarithms $\alpha_s^n \ln^n(Q/Q_0)$ of scale ratios which spoil convergence of perturbation theory.



Ingredients for *k* jets at NLL

1. LO hard function $\mathcal{H}_k$ at $\mu_h \sim Q$.

Squared tree-level amplitude with color information, c.f. density matrix of Nagy and Soper '07, …

Hard partons must be inside jet

$$\boldsymbol{\mathcal{H}}_k(\{\underline{n}\}, Q, \mu) = \frac{1}{2Q^2} \sum_{\text{spins}} \prod_{i=1}^{k} \int \frac{dE_i \, E_i^{d-3}}{(2\pi)^{d-2}} \, |\mathcal{M}_k(\{\underline{p}\})\rangle\langle\mathcal{M}_k(\{\underline{p}\})| (2\pi)^d \, \delta\Big(Q - \sum_{i=1}^{k} E_i\Big) \, \delta^{(d-1)}(\vec{p}_{\text{tot}}) \, \Theta_{\text{in}}\big(\{\underline{p}\}\big)$$

- $\mathcal{H}_{\text{m}}$ with more than $m > k$ suppressed by $(\alpha_s)^{m\text{-}k}$
- No large logs at $\mu_h \sim Q$

3. LO soft functions are trivial $\boldsymbol{\mathcal{S}}_m = \mathbf{1}$ and no large logs at $\mu_s \sim Q_0$.

2.) 1-loop anomalous dimension, ...

$$\Gamma^{(1)} = \begin{pmatrix} \boldsymbol{V}_k & \boldsymbol{R}_k & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_{k+1} & \boldsymbol{R}_{k+1} & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_{k+2} & \boldsymbol{R}_{k+2} & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_{k+3} & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Notation:

$\boldsymbol{\mathcal{H}}_m \propto |\mathcal{M}_m\rangle\langle\mathcal{M}_m|$

$\boldsymbol{T}_{i,L}$: acts on $|\mathcal{M}_m\rangle$

$\boldsymbol{T}_{i,R}$: acts on $\langle\mathcal{M}_m|$

$$\boldsymbol{V}_m = 2\sum_{(ij)}\int\frac{d\Omega(n_k)}{4\pi}\left(\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L}+\boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R}\right)W_{ij}^k$$

$$-\,2\,i\pi\sum_{(ij)}\left(\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L}-\boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R}\right)\Pi_{ij}$$

$$\boldsymbol{R}_m = -\,4\sum_{(ij)}\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,R}\,W_{ij}^{m+1}\,\Theta_{\rm in}(n_{m+1})$$

$$W_{ij}^k = \frac{n_i\cdot n_j}{n_i\cdot n_k\, n_j\cdot n_k}$$

product of eikonal factors

Glauber phase, superleading logs absent in large N_c limit!

2.) … 1-loop RG equation

Due to form of $\boldsymbol{\Gamma}^{(1)}$, RG simplifies to

$$\frac{d}{dt}\,\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t)\,\boldsymbol{V}_m + \boldsymbol{\mathcal{H}}_{m-1}(t)\,\boldsymbol{R}_{m-1}$$

Equivalent to the parton-shower equation

$$\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t_0)\,e^{(t-t_0)\boldsymbol{V}_m} + \int_{t_0}^{t} dt'\,\boldsymbol{\mathcal{H}}_{m-1}(t')\,\boldsymbol{R}_{m-1}\,e^{(t-t')\boldsymbol{V}_m}$$

Have traded RG scale μ for the shower time

$$t = \int_{\alpha(\mu)}^{\alpha(Q)} \frac{d\alpha}{\beta(\alpha)}\,\frac{\alpha}{4\pi}$$

2.) … and its iterative solution

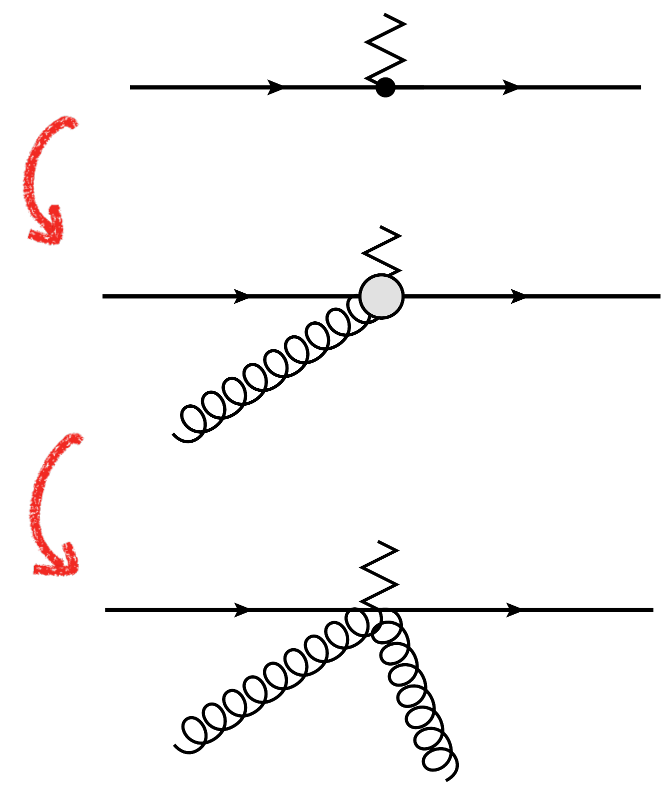
Parton shower to generate higher multiplicities

$$\boldsymbol{\mathcal{H}}_k(t) = \boldsymbol{\mathcal{H}}_k(0)\, e^{t\boldsymbol{V}_k}$$

$$\boldsymbol{\mathcal{H}}_{k+1}(t) = \int_0^t dt'\, \boldsymbol{\mathcal{H}}_k(t')\, \boldsymbol{R}_k\, e^{(t-t')\boldsymbol{V}_{k+1}}$$

$$\boldsymbol{\mathcal{H}}_{k+2}(t) = \int_0^t dt'\, \boldsymbol{\mathcal{H}}_{k+1}(t')\, \boldsymbol{R}_{k+1}\, e^{(t-t')\boldsymbol{V}_{k+2}}$$

$$\boldsymbol{\mathcal{H}}_{k+3}(t) = \dots$$



MC integrate over t, Ω

cross section

$$\sigma_{\mathrm{LL}} = \left\langle \boldsymbol{\mathcal{H}}_k(t) + \int \frac{d\Omega_1}{4\pi} \boldsymbol{\mathcal{H}}_{k+1}(t) + \int \frac{d\Omega_1}{4\pi} \int \frac{d\Omega_2}{4\pi} \boldsymbol{\mathcal{H}}_{k+2}(t) + \dots \right\rangle$$

The parton shower emerging from the RG at NLL accuracy in the large-N_c is equivalent to shower introduced by Dasgupta and Salam ‘02

Have a convenient and flexible implementation of the parton shower

- Initial hard parton configuration from LHEF event file produced by tree-level generator
- Python library `ngl_resum` reads event information and performs shower, fills histograms.

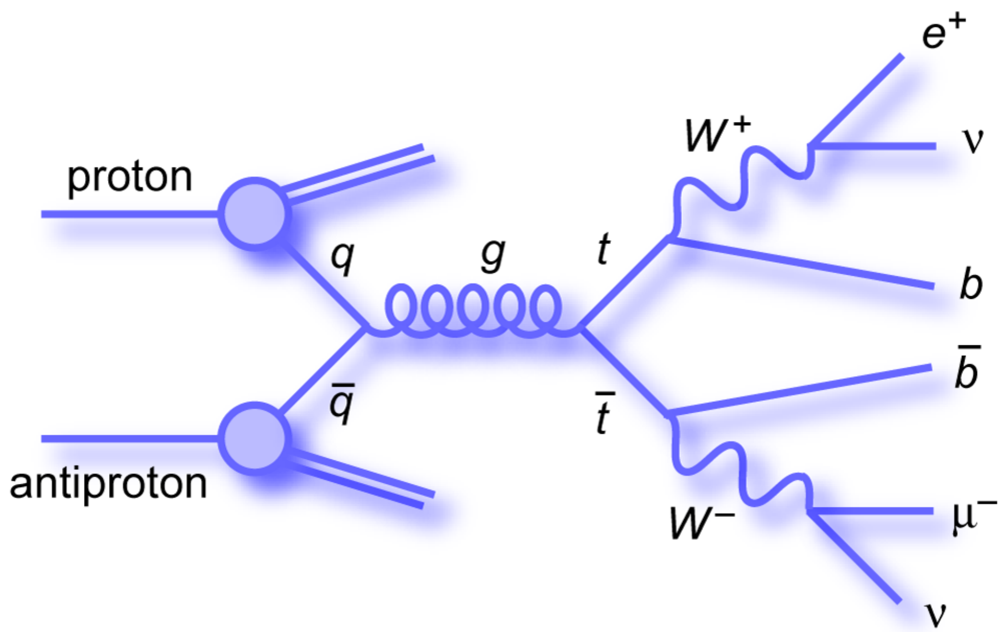
Will now illustrate this in an application in top production. TB, Balsiger, Ferroglia, 2006.00014

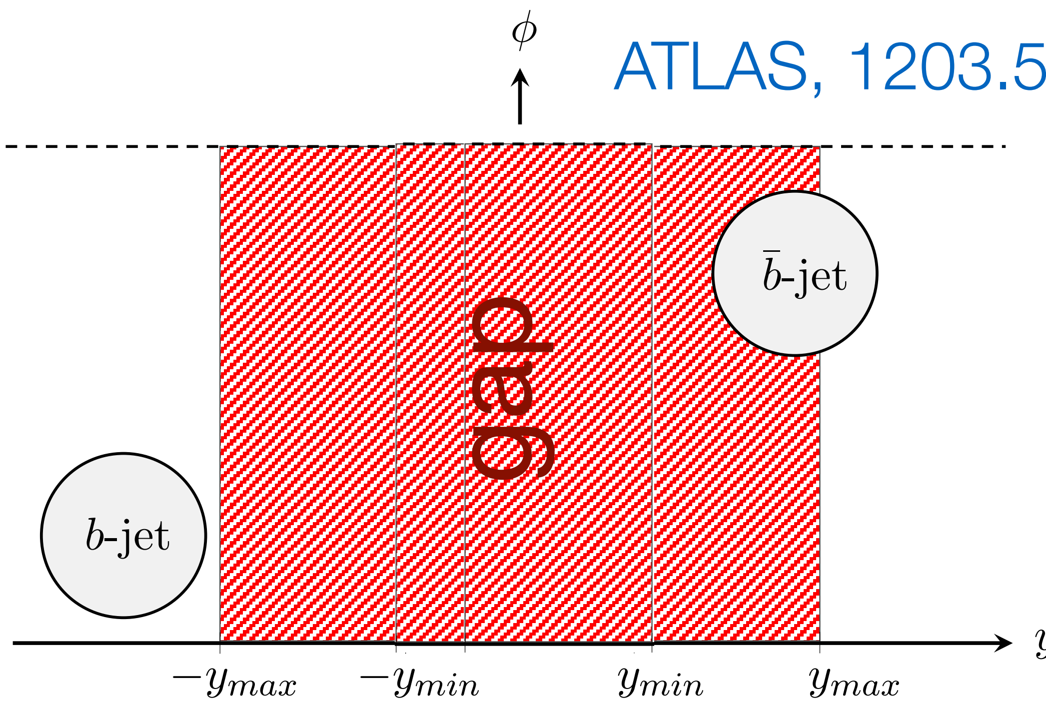
Resummation

Evolution

Factorization

Gaps in top-production

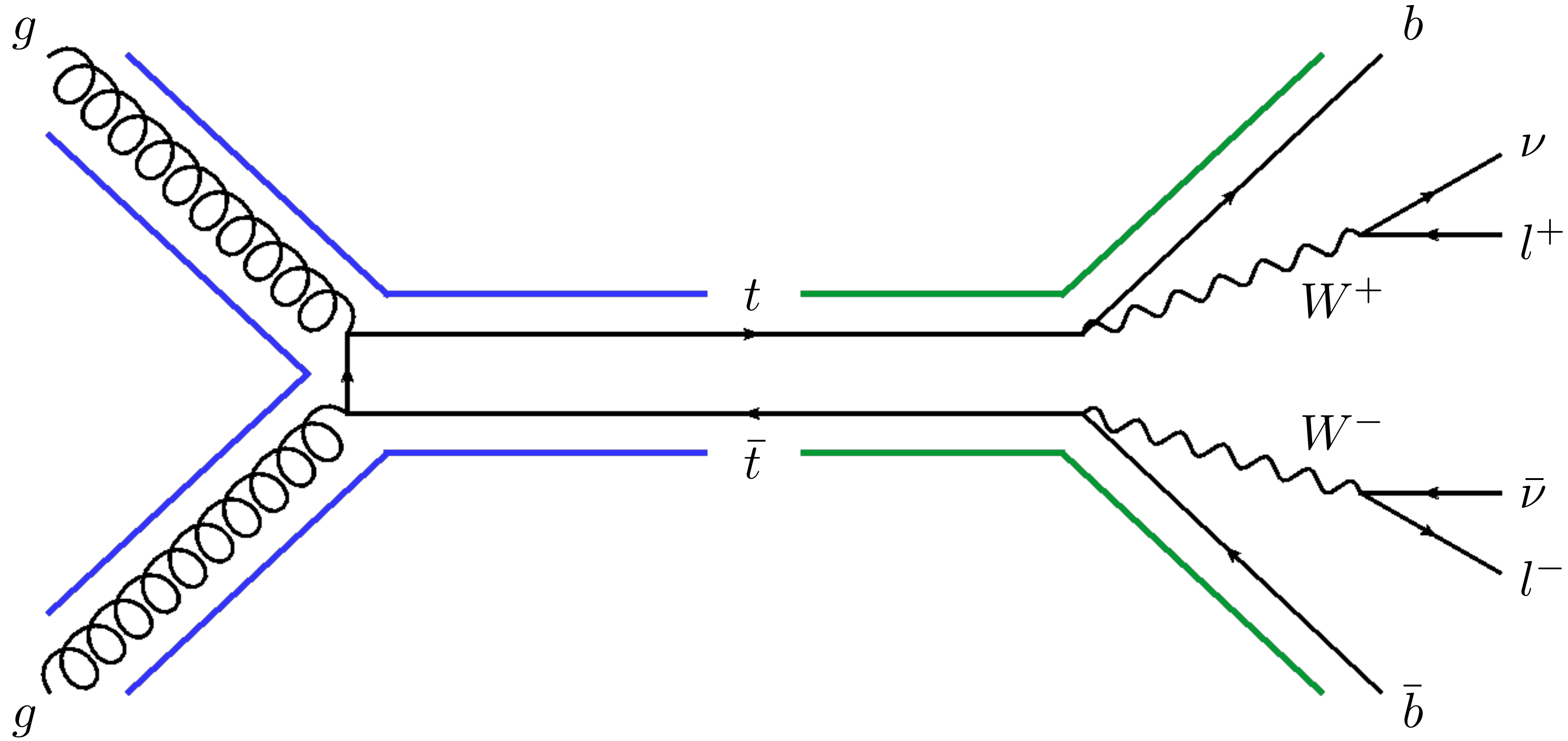




Measure top production with a veto $p_T^J < Q_0$ on jet activity in rapidity range $|y| < y_{\max}$. Define

$$R(Q_0) = \sigma_{t\bar{t}}^{\text{veto}}(Q_0)/\sigma_{t\bar{t}}^{\text{tot}}$$

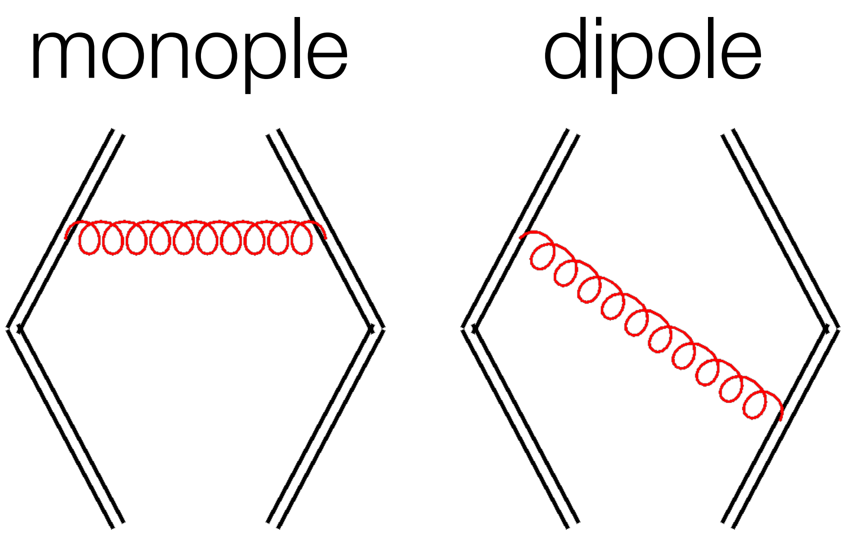
Gap fraction $R(Q_0)$ measures soft radiation from top + initial state, as well as final state radiation from the b-quarks.

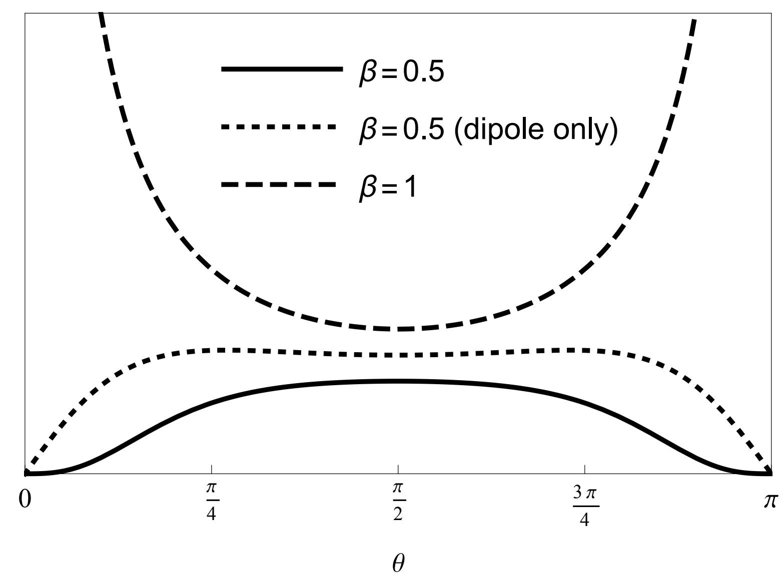


- LHEF from MG5_aMC@NLO
- Large N_c: radiation from color dipoles
  - massive, massless and mixed dipoles
- Narrow width approximation
  - Radiation from top production
  - times radiation from decays

NGLs with massive quarks

TB, Balsiger, Ferroglia, [2006.00014]()





Usual eikonal structure, but with time-like vectors. Not only dipole, but also monopole contributions

- Absorb monopoles into dipole terms
- Generate radiation in dipole rest frame, after Householder transformations.

Mass suppresses radiation, especially at large rapidity ("dead cone effect").

pypi.org

ngl-resum 0.1.0

```
pip install ngl-resum
```

Latest version

Released: Oct 19, 2020

Resummation of non-global logarithms at leading logarithmic accuracy

Navigation

- Project description
- Release history
- Download files

Project links

- Homepage

Meta

License: MIT License

Author: Marcel Balsiger

Requires: Python >=3.6

Maintainers

Project description

ngl_resum

This package is used to resum non-global logarithms at leading logarithmic accuracy.

If you use this code, please cite doi:10.1007/JHEP09(2020)029

Documentation and Examples

We suggest using the following notebooks in Binder:

The package ngl_resum: launch binder

Example of the resummation of non-global logarithms of a rapidity gap, starting from a single color dipole: launch binder

Example of the resummation of non-global logarithms of a rapidity gap, starting from a Les Houches Event File: launch binder

We also invite you to download the example codes from https://github.com/MarcelBalsiger/ngl_resum/tree/master/examples.

Jupyter notebooks with documentation on Binder:

notebooks.gesis.org

jupyter resummator_dipole (unsaved changes)

Visit repo | Copy Binder link

File Edit View Insert Cell Kernel Widgets Help | Not Trusted | Python 3

Run | Markdown | Download | **Memory:** 115 / 8192 MB

GitHub | Binder

This command takes the stub of `outside(self, v)` from our instance `outsideRegion` and replaces it with the function `_outside(self, v)` from above. Now, our `outsideRegion` is properly set up and its `outside(self,v)` -method returns True, if `v` lands in the red region of the picture from above. To illustrate its behaviour, let us check two FourVectors, one that is outside and one that is inside:

In [10]:
```
fvOut = ngl.FourVector(1,0,1,0) # light-like FourVector along y-axis - is outside!
fvIn = ngl.FourVector(1,0,0,1)  # light-like FourVector along z-axis - is inside!
```

In [11]:
```
outsideRegion.outside(fvOut)
```

Out[11]: `True`

In [12]:
```
outsideRegion.outside(fvIn)
```

Out[12]: `False`

We have now collected all the required ingredients of our shower and we resum the non-global logarithms of this specific problem.

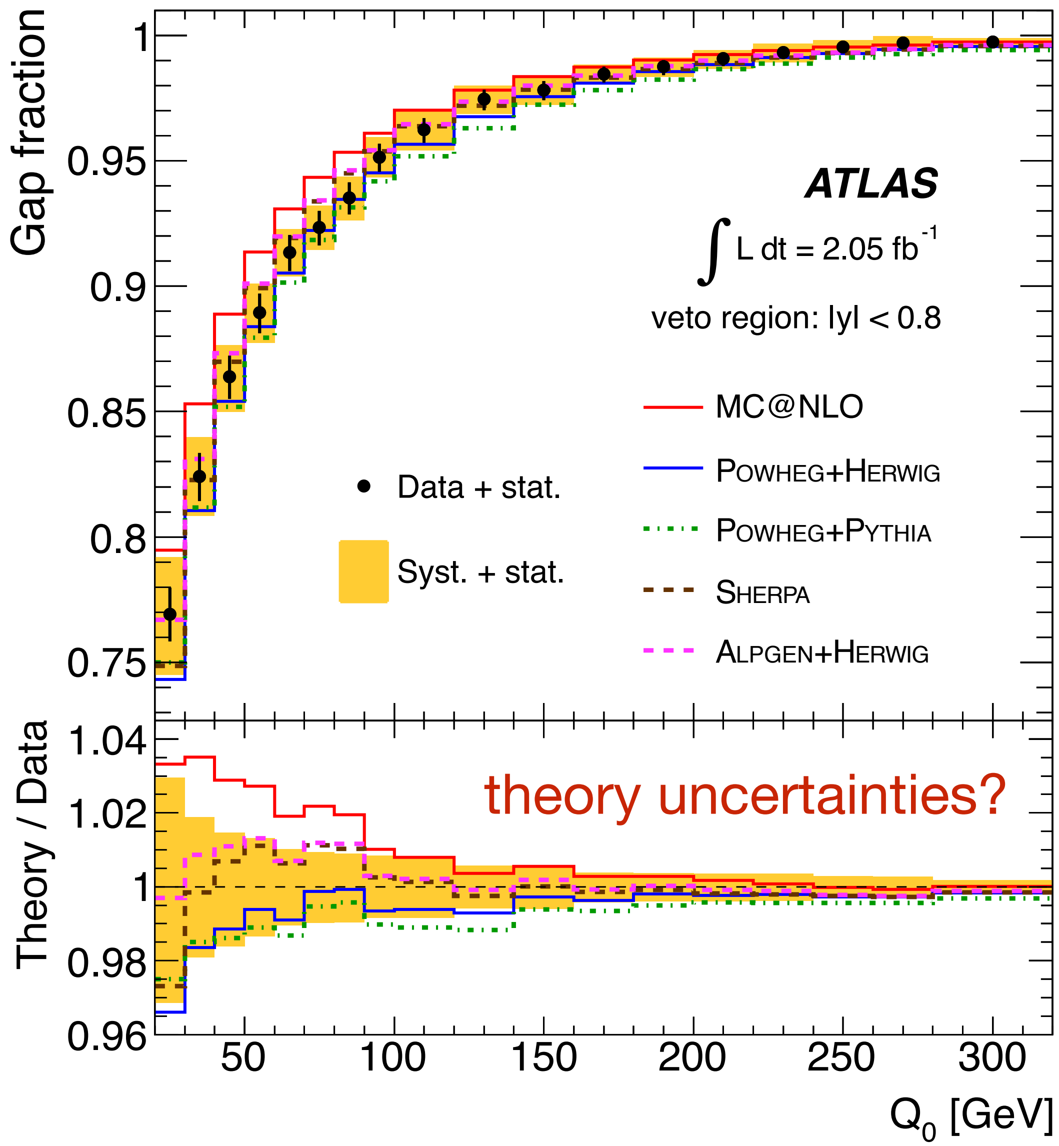
Showering the dipole

To set up the showering procedure of this problem, we now need to instantiate the `Shower` class of ngl_resum with all the required ingrdients:

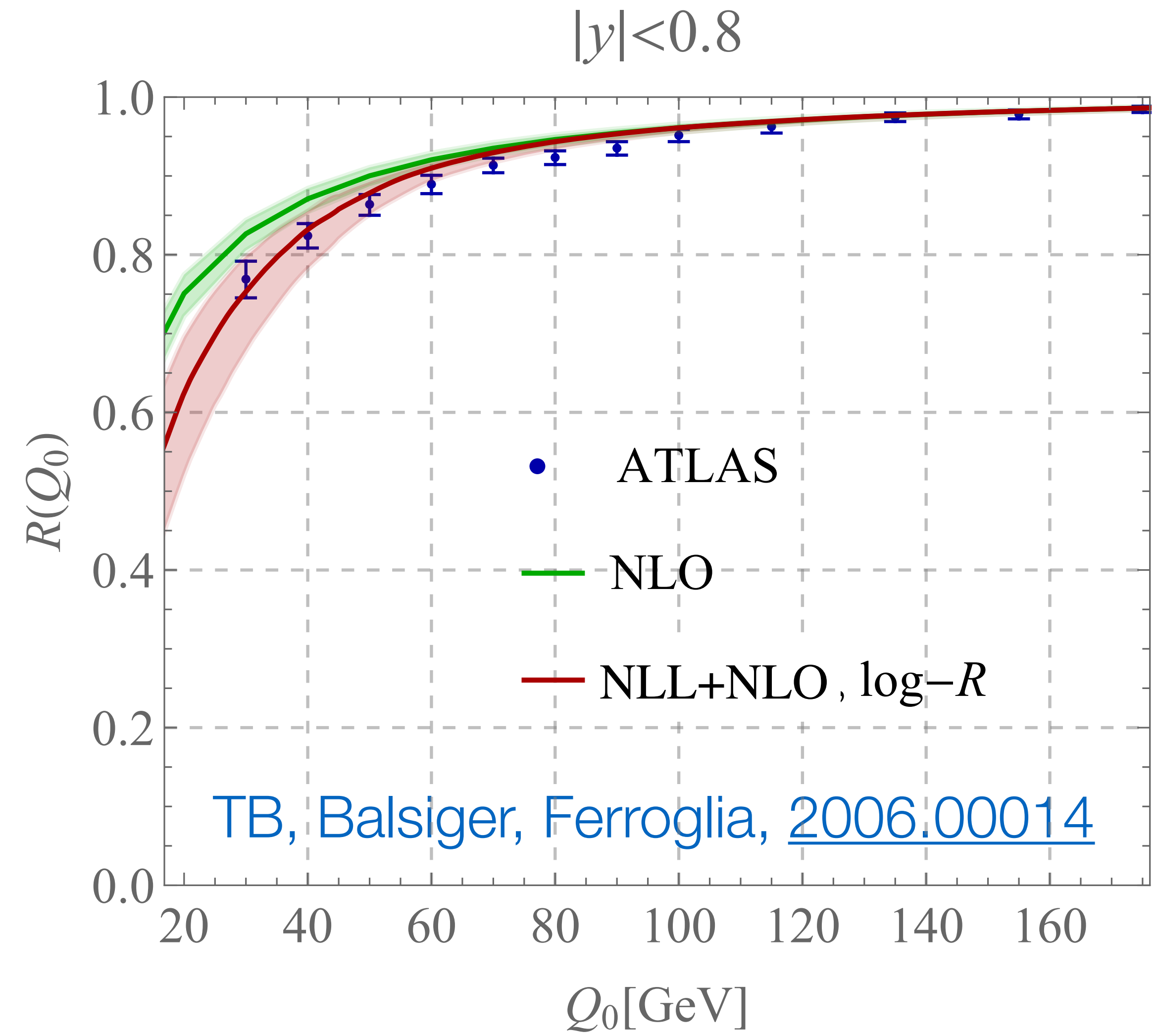
In [13]:
```
shower = ngl.Shower(ev,outsideRegion,nsh,nbins,tmax,showerCutoff)
```

That's it! Now what we are left to do is to actually shower the event. To do this, we just have to invoke the `shower()` -

Gaps in top-production



ATLAS, 1203.5015



- Large scale uncertainties and matching scheme dependence (R vs. log-R, profile functions)
- Agreement gets worse for larger gaps due to collinear logs

Ingredients for NNLL

1. One-loop matching corrections

- Hard functions

TB, Neubert, Rothen, Shao '15

$$\boldsymbol{\mathcal{H}}_2 = \sigma_0 \left(\boldsymbol{\mathcal{H}}_2^{(0)} + \frac{\alpha_s}{4\pi} \boldsymbol{\mathcal{H}}_2^{(1)} + \cdots \right), \qquad \boldsymbol{\mathcal{H}}_3 = \sigma_0 \left(\frac{\alpha_s}{4\pi} \boldsymbol{\mathcal{H}}_3^{(1)} + \cdots \right)$$

- Soft functions

$$\boldsymbol{\mathcal{S}}_m = \mathbf{1} + \frac{\alpha_s}{4\pi} \boldsymbol{\mathcal{S}}_m^{(1)} + \cdots$$

2. Two-loop anomalous dimension

$$\boldsymbol{\Gamma}^{(2)} = \begin{pmatrix} \boldsymbol{v}_2 & \boldsymbol{r}_2 & \boldsymbol{d}_2 & 0 & \dots \\ 0 & \boldsymbol{v}_3 & \boldsymbol{r}_3 & \boldsymbol{d}_3 & \dots \\ 0 & 0 & \boldsymbol{v}_4 & \boldsymbol{r}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{v}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$\boldsymbol{d}_m$: ``cluster'' of two unordered emissions

$\boldsymbol{r}_m$: real-virtual

$\boldsymbol{v}_m$: double-virtual

see Caron-Huot '15

work in progress TB, Rauh, Shao, Xu

Ingredients for NLL′

Balsiger, TB, Shao 1901.09038

1. One-loop matching corrections

- Hard functions

$$\boldsymbol{\mathcal{H}}_2 = \sigma_0 \left(\boldsymbol{\mathcal{H}}_2^{(0)} + \frac{\alpha_s}{4\pi} \boldsymbol{\mathcal{H}}_2^{(1)} + \cdots \right), \qquad \boldsymbol{\mathcal{H}}_3 = \sigma_0 \left(\frac{\alpha_s}{4\pi} \boldsymbol{\mathcal{H}}_3^{(1)} + \cdots \right)$$

- Soft functions

$$\boldsymbol{\mathcal{S}}_m = \mathbf{1} + \frac{\alpha_s}{4\pi} \boldsymbol{\mathcal{S}}_m^{(1)} + \cdots$$

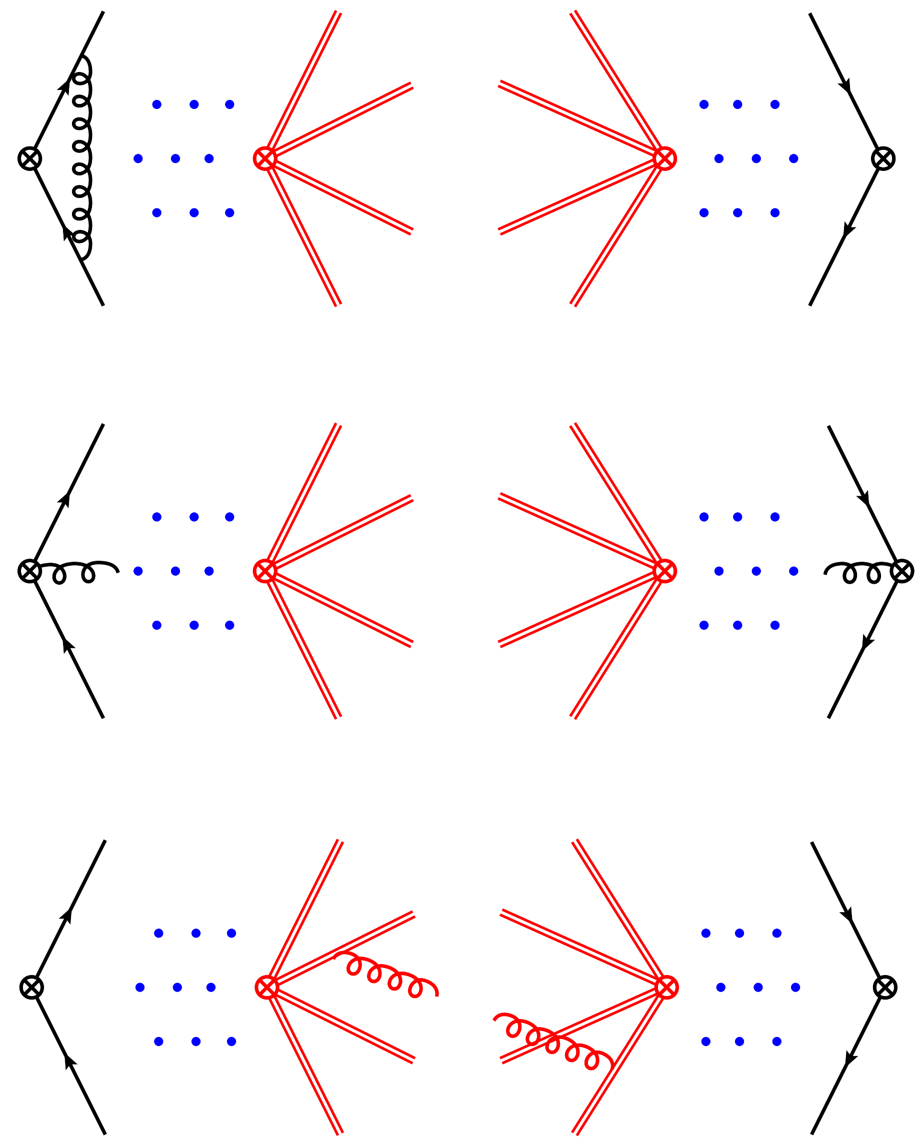
2. Two-loop anomalous dimension

$$\boldsymbol{\Gamma}^{(2)} = \begin{pmatrix} \boldsymbol{v}_2 & \boldsymbol{r}_2 & \boldsymbol{d}_2 & 0 & \ldots \\ 0 & \boldsymbol{v}_3 & \boldsymbol{r}_3 & \boldsymbol{d}_3 & \ldots \\ 0 & 0 & \boldsymbol{v}_4 & \boldsymbol{r}_4 & \ldots \\ 0 & 0 & 0 & \boldsymbol{v}_5 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

not yet implemented

Resummation at NLL′

Balsiger, TB, Shao 1901.09038



$$\sim \quad \boldsymbol{\mathcal{H}}_2^{(1)} \otimes \boldsymbol{U}_{2m} \,\hat{\otimes}\, \boldsymbol{\mathcal{S}}_m^{(0)}$$

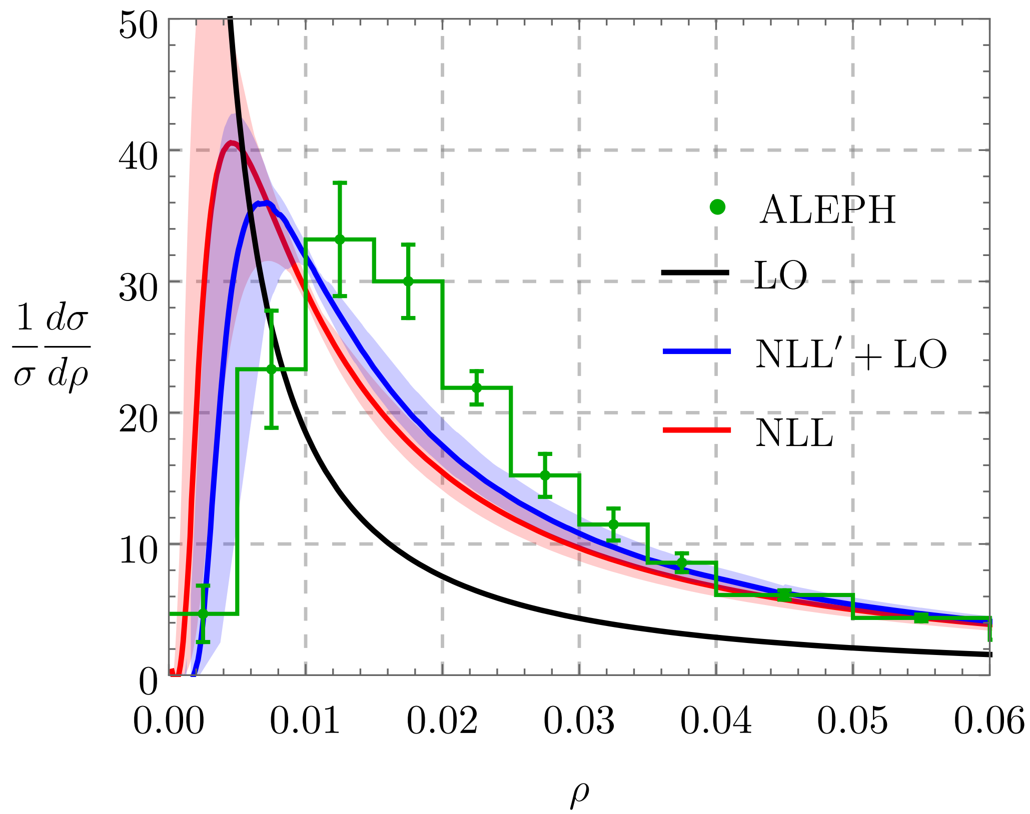
$$\sim \quad \boldsymbol{\mathcal{H}}_3^{(1)} \otimes \boldsymbol{U}_{3m} \,\hat{\otimes}\, \boldsymbol{\mathcal{S}}_m^{(0)}$$

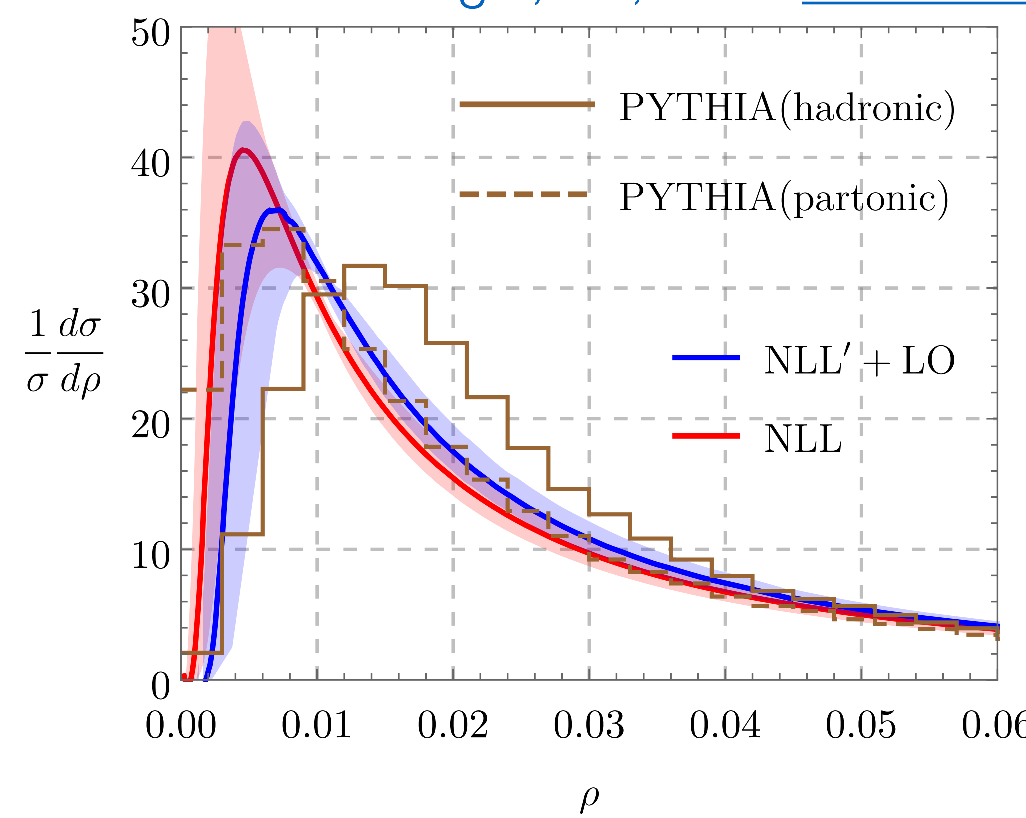
$$\sim \quad \boldsymbol{\mathcal{H}}_2^{(0)} \otimes \boldsymbol{U}_{2m} \,\hat{\otimes}\, \boldsymbol{\mathcal{S}}_m^{(1)}$$

- Implemented $O(\alpha_s)$ matching corrections: a systematically improved parton shower!
- Will need to add two-loop evolution for full NNLL accuracy.

NLL$'$ results for jet mass $M_J^2 = \rho Q^2$

Balsiger, TB, Shao 1901.09038





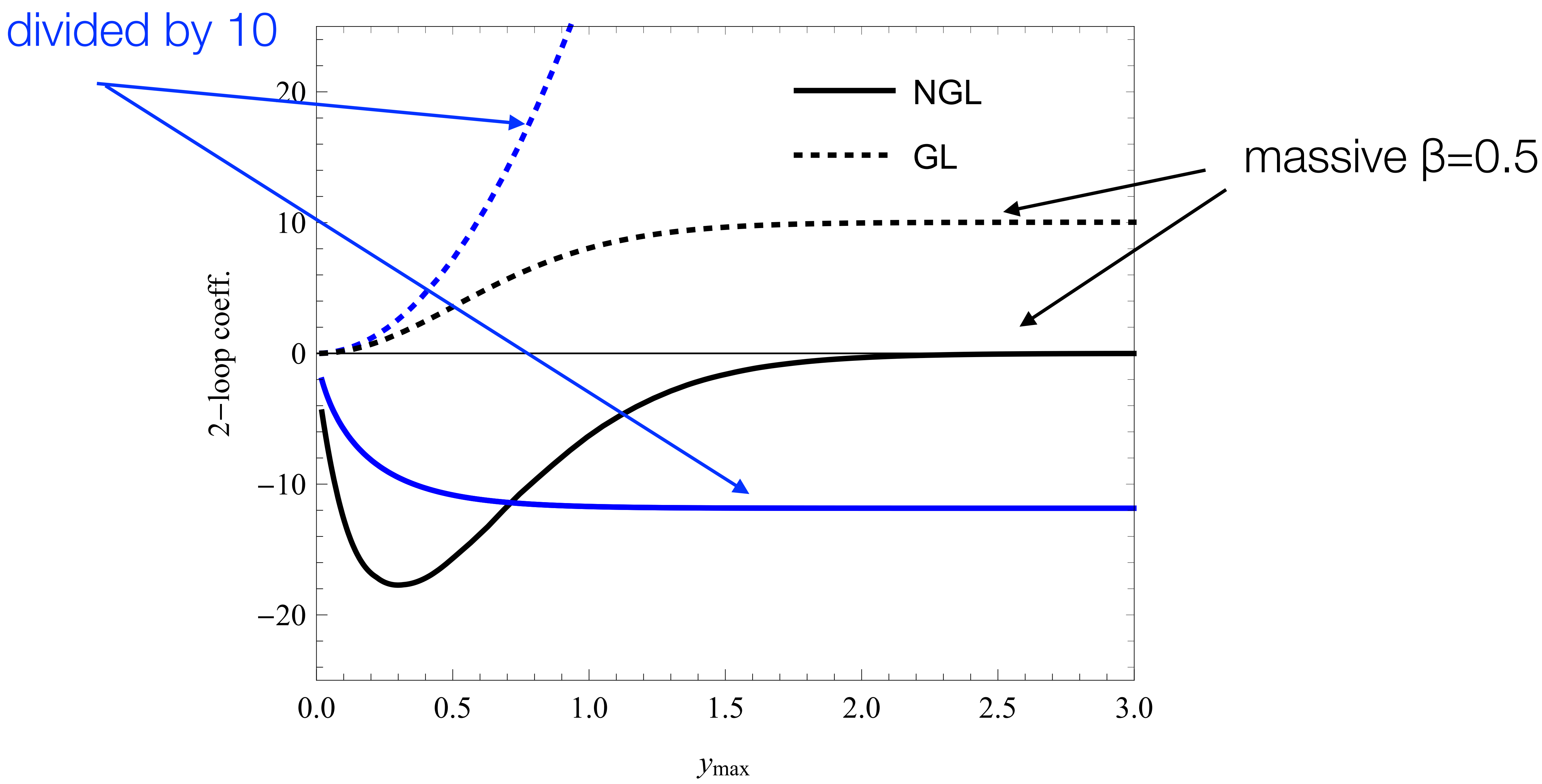
- Jet mass is a double logarithmic variable. Double logs can be subtracted and resummed analytically
- Exp. result from combining ALEPH light- and heavy-jet mass data
- Peak at $\rho \approx 0.006$ corresponds to $\mu_s \approx$ 0.5 GeV. Non-perturbative effects are important and shift the peak, see PYTHIA
- Partonic PYTHIA is close to NLL$'$

Summary

- Factorization theorems for a wide variety of non-global observables, including observables sensitive to small q_T
- Flexible parton shower framework for NLL resummation of non-global observables
  - ngl_resum public Python library
  - used to analyze top production with jet veto
- First NLL$'$ results, ongoing work on anomalous dimension for NNLL

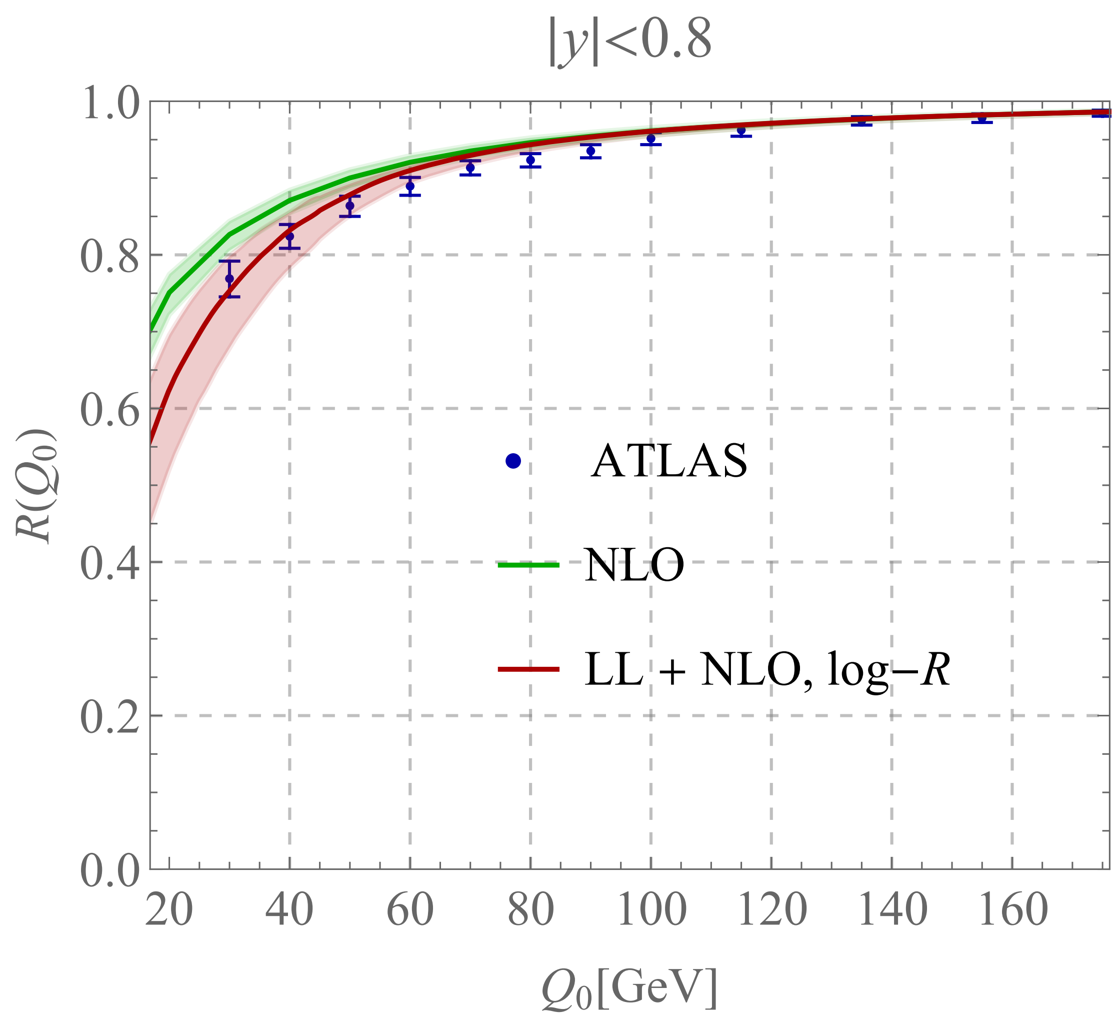
Extra slides

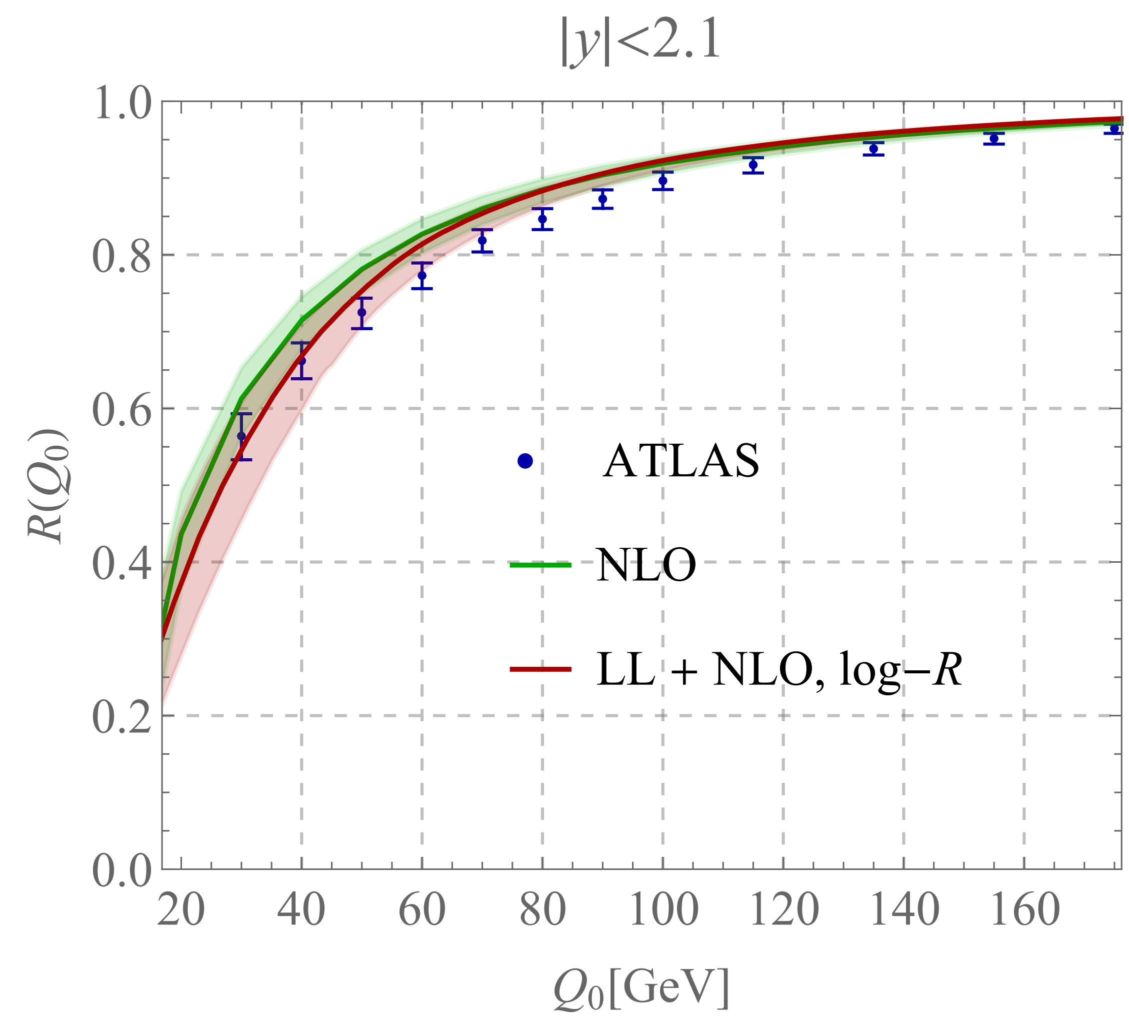
Massive vs. massless



Results are for a centered, back-to-back dipole.

Result for different gap sizes

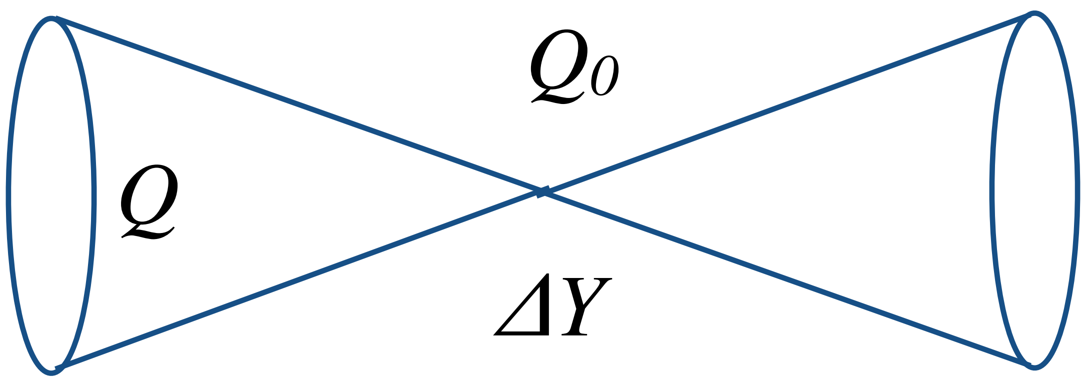


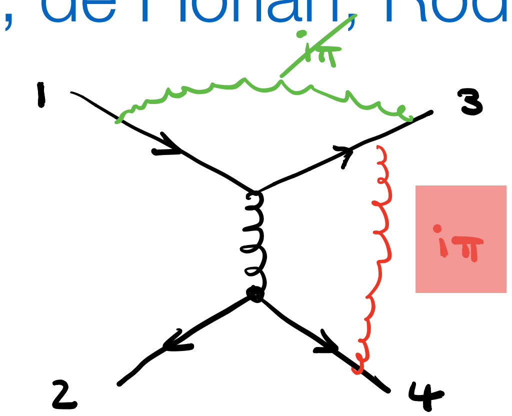


Note: additive *R*-matching fails for large gap, unphysical results at small Q_0.

Super-leading logarithms (SLLs)

Forshaw, Kyrieleis, Seymour ’06 ’08, Catani, de Florian, Rodrigo ’11, …





In proton-proton collisions Glauber (aka Coulomb) phases can spoil the cancellation of collinear singularities in (seemingly) soft obervables.

- After first few orders, gap between jets becomes double logarithmic $(\alpha_s L_Q)^3 \times (\alpha_s L_Q^2)^m$! For octet exchange

$$S_O^{(4)} = \left(\frac{\alpha_s}{4\pi}\right)^4 L_Q^5 \, \Delta Y \, \pi^2 \, \frac{8}{15} \left(3N_c^2 - 4\right) \, \sigma_0 \,,$$

$$S_O^{(5)} = \left(\frac{\alpha_s}{4\pi}\right)^5 L_Q^7 \, \Delta Y \, \pi^2 \, \frac{4}{315} \, N_c \left(-27N_c^2 + 44\right) \, \sigma_0$$

$$L_Q = \ln \frac{Q_0^2}{Q^2}$$

- $1/N_c^2$ color suppressed effect