

Impact factors: *Current status and future*

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Outline

- 1 Introduction
- 2 Impact factors definition
- 3 The impact factors map
- 4 Case study: Higgs IF
- 5 Conclusions

Introduction

- hadronic scattering processes with a hard scale $Q^2 \gg \Lambda_{\text{QCD}}^2$ described within pQCD
- in high-energy limit $s \gg Q^2$: $\Rightarrow \alpha_s(Q) \ln s/Q^2 \sim 1$ need to be resummed

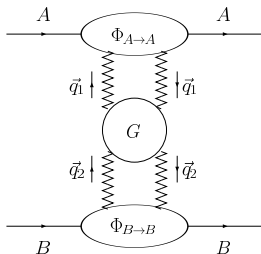
- BFKL resummation:**

leading logarithmic approximation (LLA): $\alpha_s^n (\ln s)^n$

next-to-leading logarithmic approximation (NLA): $\alpha_s^{n+1} (\ln s)^n$

[Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

[V.S. Fadin, L.N. Lipatov, D. Ciafaloni, G. Giamci (1998)]



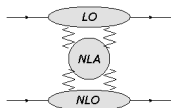
BFKL factorization:

$$\sigma_{AB}(s) = \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \int \frac{d^2 q_2}{2\pi \vec{q}_2^2} \int \frac{d^2 q_1}{2\pi \vec{q}_1^2} \left(\frac{s}{s_0} \right)^\omega$$

$$\times \Phi_A(\vec{q}_1, s_0) \quad G_\omega(\vec{q}_1, \vec{q}_2) \quad \Phi_B(-\vec{q}_2, s_0)$$

- Green's function** is process-independent
 - \rightarrow determined through the **BFKL equation**
- Impact factors** are process-dependent
 - \rightarrow known in the NLA just for limited cases.

- Partial inclusion of NLA effects: by using the two impact factors in the leading-order (LO), along with the NLA BFKL Green's function:



- Full NLA:

(mn-jets)[B. Ducloué, L. Szymanowski, S. Wallon (2014)
 [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)
 [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A.Papa (2015)]

(di-hadron)[F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A.Papa (2016,2017)]

(hadron-jet)[A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A.M, A.Papa (2018)]

- Partial NLA:

(four-jet)[F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2016)
 [F. Caporale, F.G. Celiberto, G. Chachamis, A. Sabio Vera (2016)]

(multi-jet)[F. Caporale, F.G. Celiberto, G. Chachamis, D.G. Gomez, A. Sabio Vera (2016,2017)]

(J/ψ -jet production)[R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]

(Drell-Yan pair -jet)[K. Golec-Biernat, L. Motyka, T. Stebel (2018)]

(photoproduction)[F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa,(2018)]

[A.D. Bolognino, F.G. Celiberto, M. Fucilla, D.Yu. Ivanov, B. Murdaca, A. Papa,(2019)]

(hadroproduction) [A.D. Bolognino, F.G. Celiberto, M. Fucilla, D.Yu. Ivanov, B. Murdaca, A. Papa, (2019).]

(Higgs-jet) [F.G Celiberto's talk]

● why do we need NLO Impact factors:

NLO corrections generally found to be large → need to calculate them

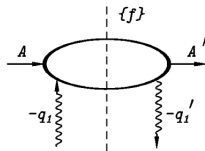
- Reduce scale uncertainties due to (μ_R, μ_F, s_0) .
- Understand: the main physical effects of the radiative corrections in BFKL.
- Check the validation of the Reggeization beyond LLA.
- Match realistic cases (jet with more than one parton).
- Complete and open the door for more phenomenological studies beyond LLA.

● tools and methods:

- Lipatov's high energy effective action. [L.N. Lipatov,(1995)]
- Standard (Feynman diagrams), Reggeized gluone is an external gluon with the "nonsense" polarization.
- Operator product expansion "OPE" .

Impact factor definition

- Impact factors:** account for the coupling of the Pomeron to the hadrons.



Universal property: $\Phi(k, q)|_{k \rightarrow 0}^{k-q \rightarrow 0} \rightarrow 0,$

which guarantees the infra-red finiteness of the BFKL amplitudes.

LLA

$$\Phi_{AA'}(\vec{q}) = \sum_{\{f\}} \int \frac{dM_{AR}}{2\pi} \Gamma_{\{f\}A}^{(0)c} [\Gamma_{\{f\}A'}^{(0)c'}]^* d\rho_f$$

- Phase space**

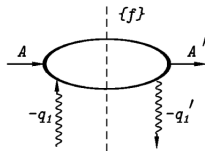
[V.S. Fadin ,R. Fiore (1998)]

- particle-Reggeon squared **invariant mass**
- Effective vertices**

$$d\rho_f = (2\pi)^D \delta^{(D)} \left(P_A - q_1 - \sum_{\{f\}} l_f \right) \prod_{\{f\}} \frac{d^{D-1} l_f}{2\epsilon_f (2\pi)^{D-1}}$$

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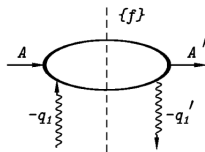
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- Phase space
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$$M_{AR} = (P_A - q_1)^2 = (P_{A'} - q_1')^2$$

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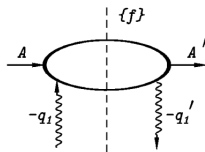
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Evaluated in the LLA or Born approximation

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Remark: In the LLA impact factors do not depend on s_0

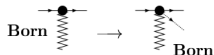
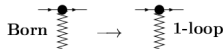
● In the NLA, modification needed..

● Higher order corrections to

$$\Gamma_{AA'} \rightarrow \Gamma_{AA'}^{(0)} + \Gamma_{AA'}^{(1)}$$

● Extra gluons:

- fragmentation region
- central region



NLA

$$\Phi_{AA'}(\vec{q}, s_0) = \sum_{\{f\}} \int \frac{dM_{AR} d\rho_f}{2\pi} \Gamma_{\{f\}A}^c [\Gamma_{\{f\}A'}^{c'}]^* \theta(M_\Lambda - M_{AR})$$

$$- \frac{1}{2} \int \frac{d^{D-2}q'}{\vec{q}'^2 (\vec{q}' - \vec{q})^2} \Phi_{A'A}^{(B)}(\vec{q}', \vec{q}) \mathcal{K}_r^{(B)}(\vec{q}', \vec{q}_R) \ln \left(\frac{s_\Lambda^2}{(\vec{q}' - \vec{q}_R) s_0} \right)$$

● Two more remarks:

- ✓ s_0 -dependence of the BFKL amplitude is canceled.
- ✓ NLO impact factors for colorless particles are IR finite.

[V.S. Fadin, R. Fior (1998)]

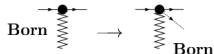
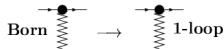
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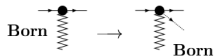
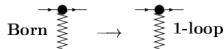
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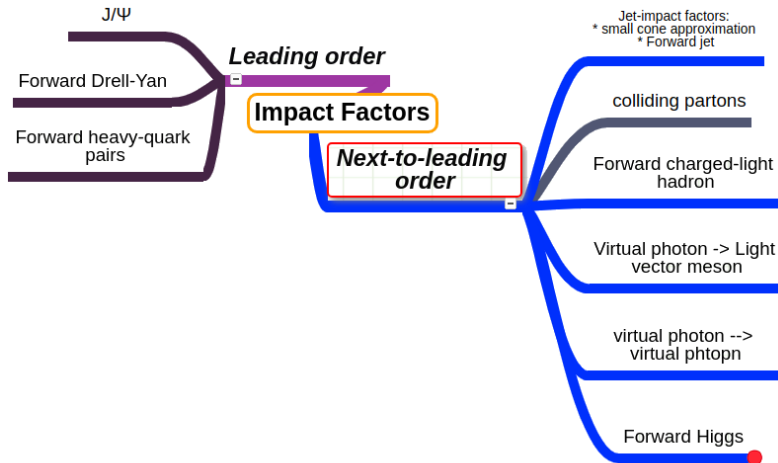
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Impact factors map

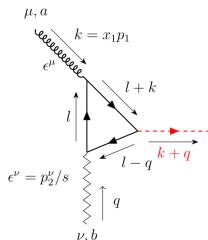


Review of Higgs IF calculation

- Leading-order
 - Calculating $\mathcal{M}(gg^* \rightarrow H)$
 - Use QCD collinear factorization

$$f_g \otimes [\text{Higgs vertex}]$$

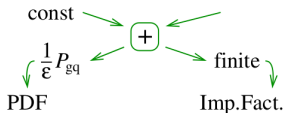
- $\int P.S^{(1)} \rightarrow 2\pi\delta(s_{gR} - M_H^2)$
- Simple result projected into $(\nu, n = 0)$ -space.



$$\frac{dV_{gg \rightarrow H}^{(0)}(\nu, n=0)}{dx_1 d^2 \vec{P}_H} = \frac{\alpha_s^2}{v^2} \frac{|\mathcal{F}(|q_\perp^2|)|^2}{128\pi^3 \sqrt{2(N_c^2 - 1)}} (\vec{P}_H^2)^{i\nu-1/2} f_g(x_1).$$

NLO:

part of **virtual** **real**



NL contribution

● **large top quark mass limit** : $m_t \rightarrow \infty$

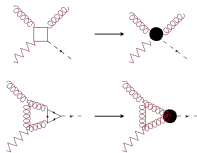
- features of HEFT

$$\mathcal{L}_{eff} = -\frac{A}{4} h G_{\mu\nu}^a G_a^{\mu\nu}, \quad A = \frac{\alpha_s}{3\pi v} \left(1 + \frac{11}{4} \frac{\alpha_s}{\pi} \right)$$

- Reduces calculations by **one loop order**.
- Turns a two-scale problem into two one-scale problems

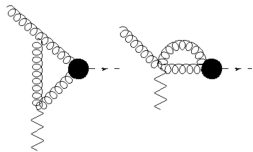
▷ **NLA recipe...**

- ★ Pick a regularization scheme (dimensional regularization for us)
- ★ Get the tree-level result.
- ★ Calculate 1-loop diagrams as a series in ϵ
- ★ Perform the ultraviolet renormalization.
- ★ Calculate the real emission diagrams, extract singularities that appear in soft/collinear regions of phase space.
- ★ Absorb initial-state collinear singularities into PDFs
- ★ Get a finite result



Virtual correction

- non-zero contributions to $gg^* \rightarrow H$ at one-loop in the HEFT



- Born in $(d = 4 - 2\epsilon)$ dimensions:

$$|\mathcal{M}^{(B)}|^2 = \frac{\alpha_s^2}{72\pi^2} \frac{|q_\perp^2|}{v^2} \frac{1}{(1-\epsilon)}$$

- tensor integrals appearing in the amplitudes so to express the results in terms

- triangle diagram

$$\mu^{2\epsilon} \int \frac{d^d z}{(2\pi)^d} \frac{1}{(k-z)^2 z^2 (q+z)^2} = \frac{-i}{(4\pi)^2} \frac{G(\epsilon)}{\epsilon^2} \frac{(\mu^2/q_\perp^2)^\epsilon}{2k \cdot q} \left[1 + \left(\frac{M_H^2}{q_\perp^2} \right)^{-\epsilon} \right],$$

- bubble diagram

$$\mu^{2\epsilon} \int \frac{d^d z}{(2\pi)^d} \frac{1}{z^2 (P_H + z)^2} = \frac{i}{16\pi^2} \frac{G(\epsilon)}{\epsilon} \frac{(-\mu^2/P_H^2)^\epsilon}{(1-2\epsilon)}$$

$$G(\epsilon) = (4\pi)^\epsilon \frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}$$

- Taking into our account the QCD charge renormalization, LO dependence on α_S gives the counterterm

$$V_{g \rightarrow H}^{(0)}(\vec{q}^2) \rightarrow V_{g \rightarrow H}^{(0)}(\vec{q}^2) \left[1 - \frac{\alpha_s}{2\pi} G(\epsilon) \left(\frac{\mu^2}{\mu_R^2} \right)^\epsilon \frac{2\beta_0}{\epsilon} \right],$$

- Using the NLO IF definition and $\omega(-\vec{q}^2)$ in the 1-loop approximation

$$\omega(-\vec{q}^2) = \frac{g^2 C_A}{2} \mu^{2\epsilon} \int \frac{d^{d-2}k}{(2\pi)^{d-1}} \frac{(-\vec{q}^2)}{\vec{k}^2 (\vec{q} - \vec{k})^2} = -\frac{\alpha_s}{2\pi} G(\epsilon) \frac{C_A}{2} \left(\frac{\mu^2}{q_\perp^2} \right)^\epsilon$$

- result

$$V_{g \rightarrow H}^{(1)virt}(\vec{q}^2, s_0) = V_{g \rightarrow H}^{(0)}(\vec{q}^2) \left(\frac{\alpha_s}{2\pi} \right) G(\epsilon) \left(\frac{\mu^2}{q_\perp^2} \right)^\epsilon \left\{ \frac{-2C_A}{\epsilon^2} + \frac{C_A}{\epsilon} \left(\ln \frac{M_H^2}{q_\perp^2} - \ln \frac{s_0}{q^2} - 2\beta_0 \right) + C_A \left(\frac{2\beta_0}{C_A} \ln \frac{q_\perp^2}{\mu_R^2} - \frac{5}{3} \right) + \dots \right\}$$

- apart from the notation, this coincides with what has been computed two years ago

[Bo-Wen; Yuan, Feng (2018), Physics Letters B, 782(), 28-33.]

Real correction

- imply the calculation of $2 \rightarrow 2$ tree-level amplitudes and integration over **part** of the phase space in d -dimensions

$$PS = \frac{1}{8\pi} \left(\frac{4\pi}{M_H^2} \right)^\epsilon \frac{(1-z)^{1-2\epsilon}}{\Gamma(1-\epsilon)} \int_0^1 \omega^{-\epsilon} (1-\omega)^{-\epsilon} d\omega, \quad z = \frac{M_H^2}{s}, \quad \omega = \frac{1 + \cos(\theta)}{2}$$

- When we combine matrix elements and phase space, get terms of the following form:

$$(1-z)^{1-2\epsilon} (1-\omega)^{-\epsilon}$$

- The integrals over ω can be done in terms of $\Gamma(\epsilon)$, while the soft singularities as can be extracted using $[\]_+$:

$$(1-z)^{1-2\epsilon} = \left[\frac{1}{1-z} \right]_+ - 2\epsilon \left[\frac{\ln(1-z)}{1-z} \right]_+ - \frac{1}{2\epsilon} \delta(1-z)$$

- up to the following contribution to IF

$$\simeq \left\{ \frac{6}{\epsilon^2} \delta(1-z) + \frac{18z(z^2 - z + 2)}{\epsilon} + 12 \left[\frac{\ln(1-z)}{1-z} \right]_+ + \dots \right\}$$

- remaining initial-state collinear singularities \rightarrow PDFs.

Conclusions

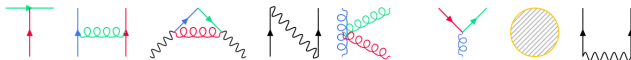
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- It is important to have an independent calculation of NL IF for forward Higgs.

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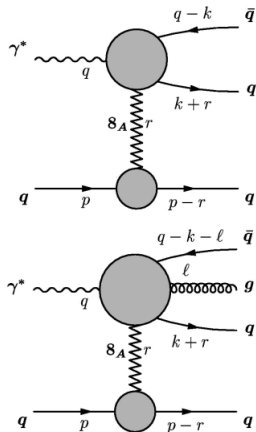


BACKUP slides

A brief history of NLO γ^* impact factor









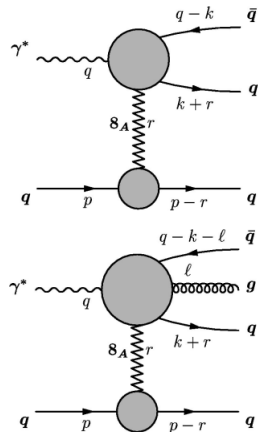
- one-loop correction $\Gamma_{q\bar{q}}^c$.
- real corrections: $\gamma^* + q \rightarrow (q\bar{q}g) + q$.
- Combining real and virtual corrections.
- possibility for simplification.
- 1st numerical results for the real corrections γ_L^* .
- Wilson lines:



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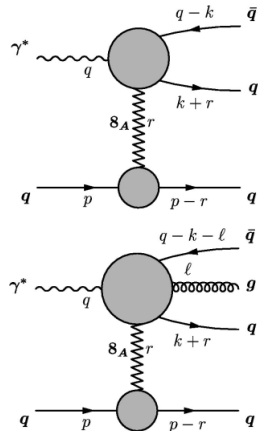
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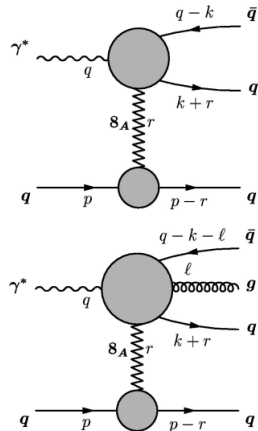
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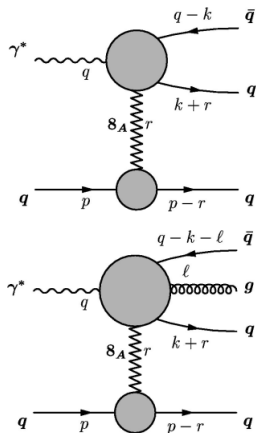
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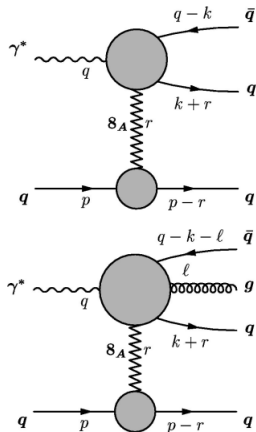


A brief history of NLO γ^* impact factor



2011

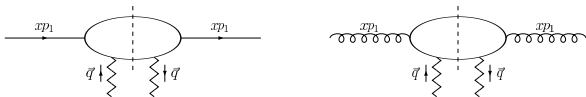
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Yesterday

Impact factors for colliding parton

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
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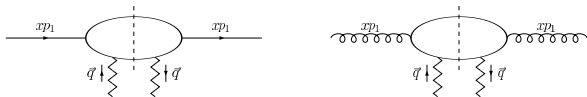
- ★ LLA \rightarrow leading-order (LO) impact factor \rightarrow one-particle intermediate state.
- ★ NLA \rightarrow next-to-LO (NLO) impact factor:
 - virtual corrections \rightarrow one-particle intermediate state
 - real particle production \rightarrow two-particle intermediate state

- ✓ Check the so-called "bootstrap" conditions.
- ✓ Expected infrared divergences observed.

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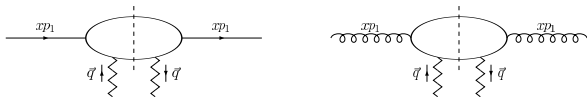
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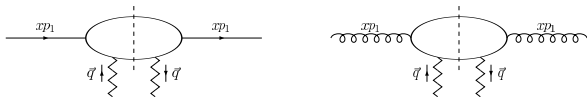
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- ★ NLA \rightarrow next-to-LO (NLO) impact factor:
virtual corrections \rightarrow one-particle intermediate state
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- ✓ Check the so-called "bootstrap" conditions.
- ✓ Expected infrared divergences observed.

Yesterday

Impact factors for colliding parton

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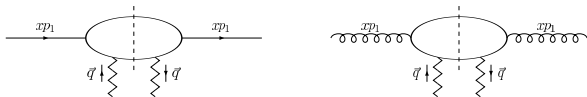
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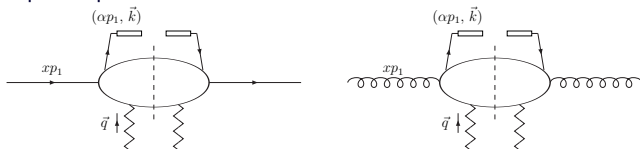
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Forward jet / Identified hadron impact factors

- To allow one parton to produce jet (or hadron), open one of the integrations over the phase space of the intermediate state.



- use QCD collinear factorization:

$$\sum_{a=q,\bar{q}} f_a \otimes (\text{quark jet vertex}) + f_g \otimes (\text{gluon jet vertex}).$$

[J. Bartels, D. Colferai, G.P. Vacca (2003)]

[F. Caporale, D.Yu. Ivanov, B. Murdaca, A.Papa, A. Pini (2011)]

[D.Yu. Ivanov, A.Papa. (2012)] (small-cone approximation)

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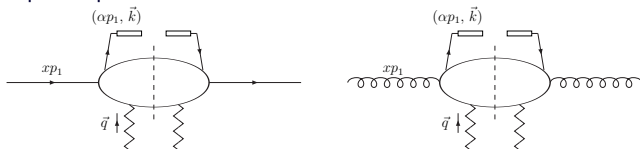
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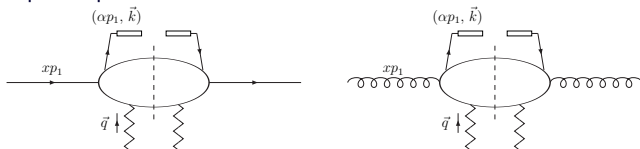
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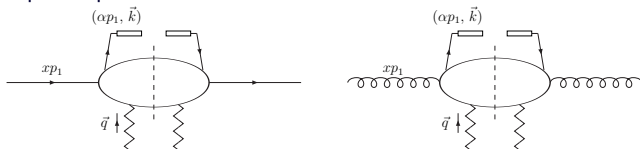
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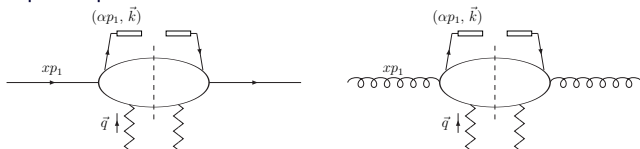
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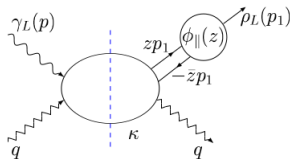
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$\gamma^* \rightarrow VLM$

$$\Phi_{1,2}(\vec{q}) = \alpha_s D_{1,2} \left[C_{1,2}^{(0)}(\vec{q}^2) + \bar{\alpha}_s C_{1,2}^{(1)}(\vec{q}^2) \right]$$

$$D_{1,2} = -\frac{4\pi e_q f_V}{N_c Q_{1,2}} \sqrt{N_c^2 - 1}$$



[D.Yu. Ivanov, M.I. Kotsky, A.P. (2004)]

- Leading order (photon virtuality Q^2):

$$C_{1,2}^{(0)}(\vec{q}^2) = \int_0^1 \frac{\vec{q}^2}{\vec{q}^2 + z\bar{z}Q^2} \phi_{||}(z, \mu_F) dz$$

- Next-to-leading order:

$$C_{1,2}^{(1)}(\vec{q}^2) = \frac{1}{4N_c} \int_0^1 \frac{\vec{q}^2}{\vec{q}^2 + z\bar{z}Q^2} \left[\tau(z) + \tau(1-z) \right] \phi_{||}(z, \mu_F) dz$$

- Collinear factorization reduces complexity.