

EXTRACTION OF SIVERS FUNCTIONS FROM SIDIS AND DRELL-YAN DATA WITH TMD EVOLUTION

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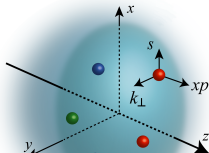
Resummation, Evolution, Factorization 2020

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Based on work in collaboration with Alexei Prokudin and Alexey Vladimirov

[arXiv:2012.05135](https://arxiv.org/abs/2012.05135)

Transverse Momentum Dependent distributions



Dudek *et al.* Eur. Phys. J. A 48 (2012)

Leading Twist TMDs



Nucleon Spin

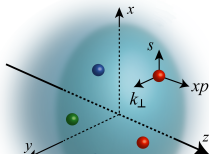


Quark Spin

| | | Quark Polarization | | |
|----------------------|---|-------------------------------|------------------------------|---|
| | | Un-Polarized (U) | Longitudinally Polarized (L) | Transversely Polarized (T) |
| Nucleon Polarization | U | $f_1 = $ | | $h_1^\perp = $ - Boer-Mulders |
| | L | | $g_{1L} = $ - Helicity | $h_{1L}^\perp = $ - |
| | T | $f_{1T}^\perp = $ - Sivers | $g_{1T}^\perp = $ - | $h_1 = $ - Transversity $h_{1T}^\perp = $ - |

Accardi *et al.* Eur. Phys. J. A 52 (2016)

Transverse Momentum Dependent distributions



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Leading Twist TMDs



Nucleon Spin

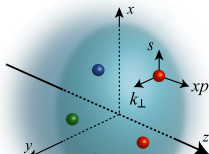


Quark Spin

| | | Quark Polarization | | |
|----------------------|---|--|--|---|
| | | Un-Polarized (U) | Longitudinally Polarized (L) | Transversely Polarized (T) |
| Nucleon Polarization | U | $f_1 = \text{Nucleon Spin} \cdot \text{Quark Spin}$ | | $h_1^\perp = \text{Nucleon Spin} \cdot \text{Quark Spin} - \text{Nucleon Spin} \cdot \text{Quark Spin}$ Boer-Mulders |
| | L | | $g_{1L} = \text{Nucleon Spin} \cdot \text{Quark Spin} - \text{Nucleon Spin} \cdot \text{Quark Spin}$ Helicity | $h_{1L}^\perp = \text{Nucleon Spin} \cdot \text{Quark Spin} - \text{Nucleon Spin} \cdot \text{Quark Spin}$ |
| | T | $f_{1T}^\perp = \text{Nucleon Spin} \cdot \text{Quark Spin} - \text{Nucleon Spin} \cdot \text{Quark Spin}$ Sivers | $g_{1T}^\perp = \text{Nucleon Spin} \cdot \text{Quark Spin} - \text{Nucleon Spin} \cdot \text{Quark Spin}$ | $h_1 = \text{Nucleon Spin} \cdot \text{Quark Spin} - \text{Nucleon Spin} \cdot \text{Quark Spin}$ Transversity $h_{1T}^\perp = \text{Nucleon Spin} \cdot \text{Quark Spin} - \text{Nucleon Spin} \cdot \text{Quark Spin}$ |

Accardi *et al.* Eur. Phys. J. A 52 (2016)

Transverse Momentum Dependent distributions



Dudek *et al.* Eur. Phys. J. A 48 (2012)

$$f_{1T}^{\perp \text{SIDIS}} = -f_{1T}^{\perp \text{DY}}$$

Leading Twist TMDs



Nucleon Spin



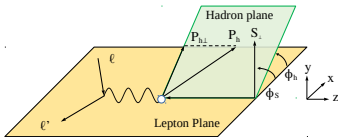
Quark Spin

| | | Quark Polarization | | |
|----------------------|---|--|--|---|
| | | Un-Polarized (U) | Longitudinally Polarized (L) | Transversely Polarized (T) |
| Nucleon Polarization | U | $f_1 = \odot$ | | $h_1^{\perp} = \uparrow \ominus - \downarrow \ominus$ Boer-Mulders |
| | L | | $g_{1L} = \odot \rightarrow - \ominus \rightarrow$ Helicity | $h_{1L}^{\perp} = \uparrow \rightarrow - \downarrow \rightarrow$ |
| | T | $f_{1T}^{\perp} = \odot \uparrow - \ominus \downarrow$ Sivers | $g_{1T}^{\perp} = \odot \rightarrow - \ominus \rightarrow$ | $h_1 = \uparrow \ominus - \downarrow \ominus$ Transversity $h_{1T}^{\perp} = \uparrow \rightarrow - \downarrow \rightarrow$ |

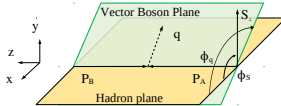
Accardi *et al.* Eur. Phys. J. A 52 (2016)

Single Spin Asymmetries

SIDIS



Drell-Yan, W^\pm/Z production



$$\frac{d\sigma}{d\mathcal{PS}} = \sigma_0 \left\{ F_{UU,T} + |S_\perp| \sin(\phi_h - \phi_S) F_{UT,T}^{\sin(\phi_h - \phi_S)} \right\}$$

$$\frac{d\sigma}{d\mathcal{PS}} = \sigma_0 \left\{ F_{UU}^1 + |S_T| \sin(\varphi - \phi_S) F_{TU}^1 \right\}$$

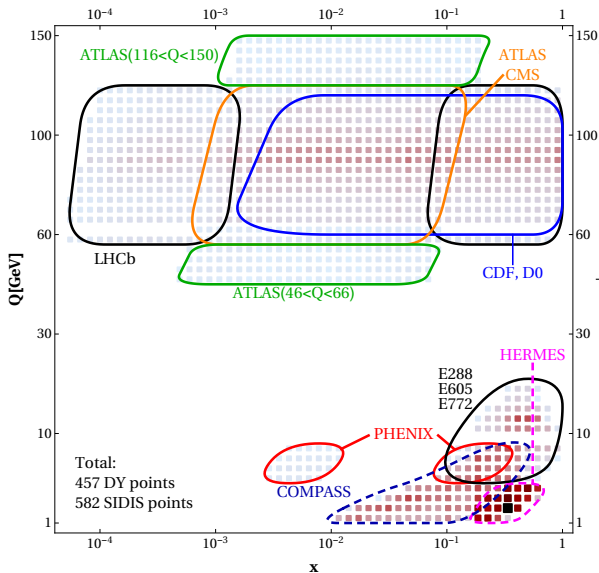
$$A_{UT}^{\sin(\phi_h - \phi_S)} \equiv \frac{F_{UT,T}^{\sin(\phi_h - \phi_S)}}{F_{UU,T}} = -M \frac{\mathcal{B}_1^{\text{SIDIS}} [f_{1T}^\perp D_1]}{\mathcal{B}_0^{\text{SIDIS}} [f_1 D_1]}$$

$$A_{TU} \equiv \frac{F_{TU}^1}{F_{UU}^1} = -M \frac{\mathcal{B}_1^{\text{DY}} [f_{1T}^\perp f_2]}{\mathcal{B}_0^{\text{DY}} [f_1 f_2]}$$

$$\mathcal{B}_n^{\text{SIDIS}} [fD] \equiv \sum_q e_q^2 \int_0^\infty \frac{bdb}{2\pi} b^n J_n \left(\frac{b|P_{hT}|}{z} \right) \\ \times f_{q \leftarrow h_1}(x, b; \mu, \zeta_1) D_{q \rightarrow h_2}(z, b; \mu, \zeta_2)$$

$$\mathcal{B}_n^{\text{DY}} [f_1 f_2] \equiv \sum_q e_q^2 \int_0^\infty \frac{bdb}{2\pi} b^n J_n (b|q_T|) \\ \times f_{1; q \leftarrow h_1}(x_1, b; \mu, \zeta_1) f_{2; \bar{q} \leftarrow h_2}(x_2, b; \mu, \zeta_2)$$

Unpolarized TMD



SV19: Scimemi, Vladimirov
arXiv:1912.06532

| | |
|---------------|--------------------|
| DY: | 457 points |
| SIDIS: | 582 points |
| TOTAL: | 1039 points |

NNLO: $\chi_{global}^2/N_{pt} = 0.95$

N³LO: $\chi_{global}^2/N_{pt} = 1.05$

Scimemi, Vladimirov arXiv:1912.06532



- TMD distributions depend on ultra-violet μ and rapidity ζ renormalization scales and the evolution is dictated by a pair of differential equations
- We use the ζ -prescription (the reference scale $(\mu, \zeta) = (\mu, \zeta_\mu(b))$ is selected from equipotential line of the field anomalous dimension that passes through the saddle point)
- The reference TMD distribution is independent on μ and the solution of evol. equations can be written in simpler form

$$f_{1T,q\leftarrow h}^\perp(x, b; \mu, \zeta) = \left(\frac{\zeta}{\zeta_\mu(b)} \right)^{-\mathcal{D}(b, \mu)} f_{1T,q\leftarrow h}^\perp(x, b)$$

- The function $f_{1T,q\leftarrow h}^\perp(x, b) = f_{1T,q\leftarrow h}^\perp(x, b; \mu, \zeta_\mu(b))$ on rhs is the **optimal Sivers** function
- $\zeta_\mu(b)$ is a calculable function of the universal non-perturbative Collins-Soper kernel $\mathcal{D}(b, \mu)$, parametrized as

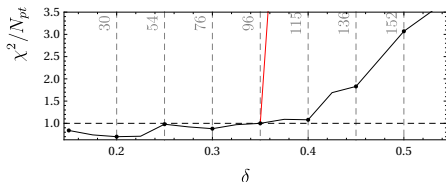
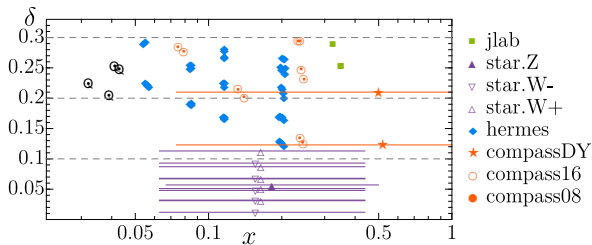
$$\mathcal{D}(b, \mu) = \mathcal{D}_{\text{resum}}(b^*, \mu) + c_0 b b^*, \quad b^* = b / \sqrt{1 + (b / (2 \text{ GeV}^{-1}))^2}$$

- N³LO expressions are used for $\zeta_\mu(b)$ and $\mathcal{D}_{\text{resum}}$

Data selection

- TMD factorization applies in the limit of large Q and small relative transverse momentum δ . Selection criteria:

$$\langle Q \rangle > 2 \text{ GeV} \quad \text{and} \quad \delta^{\text{SIDIS}} = \frac{|P_{h\perp}|}{zQ}, \quad \delta^{\text{DY}} = \frac{|q_T|}{Q} < 0.3$$



DY: 13/14 pt

SIDIS: 63/388 pt

TOTAL: 76/402 pt

Parametrization of Siverson function

- We do not use small- b matching for Siverson functions (presence of unknown twist-3 distributions, besides Qiu-Sterman function)
- Optimal Siverson function is a generic NP function, extracted from the data, parametrized as

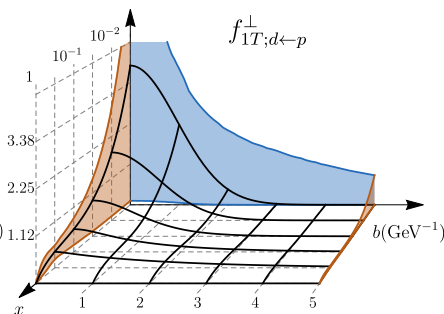
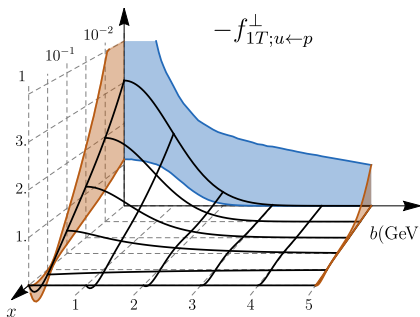
$$f_{1T;q\leftarrow h}^\perp(x, b) = N_q \frac{(1-x)x^{\beta_q}(1+\epsilon_q x)}{n(\beta_q, \epsilon_q)} \exp\left(-\frac{r_0 + xr_1}{\sqrt{1+r_2x^2b^2}}b^2\right)$$

$$n(\beta, \epsilon) = (3+\beta+\epsilon+\epsilon\beta)\beta! / (\beta+3)! \Rightarrow \int_0^1 dx f_{1T;q\leftarrow h}^\perp(x, 0) = N_q$$

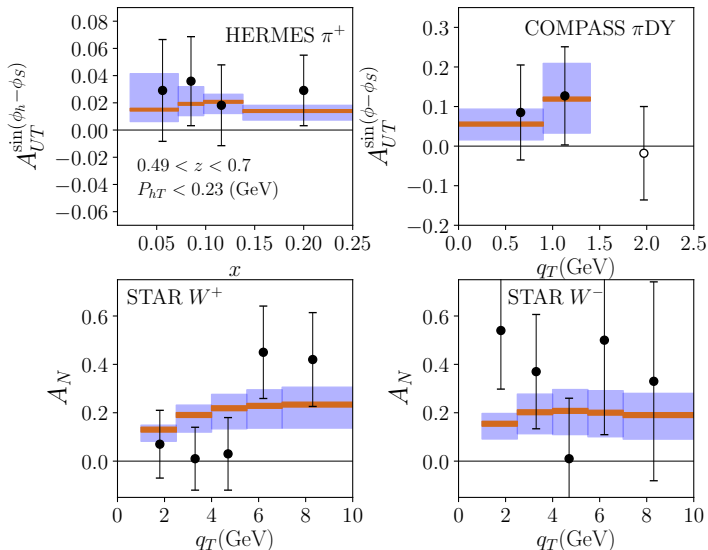
- Separate functions for u , d , s , single *sea* Siverson function for \bar{u} , \bar{d} and \bar{s} ; $\beta_s = \beta_{sea}$ and $\epsilon_s = \epsilon_{sea} = 0$, since small- x behavior is not restricted by data
- 12 free parameters in total
- QS function is then obtained from the small- b limit of the extracted Siverson function

Fit result

| Name | χ^2/N_{pt} [SIDIS] | χ^2/N_{pt} [DY] | χ^2/N_{pt} [total] |
|-------------------------------|-------------------------|----------------------|-------------------------|
| SIDIS at NNLO | 0.88 | 1.29 no fit | 0.95 |
| SIDIS+DY at NNLO | 0.90 | 0.94 | 0.91 |
| SIDIS at N ³ LO | 0.87 | 1.23 no fit | 0.93 |
| SIDIS+DY at N ³ LO | 0.88 | 0.90 | 0.88 |



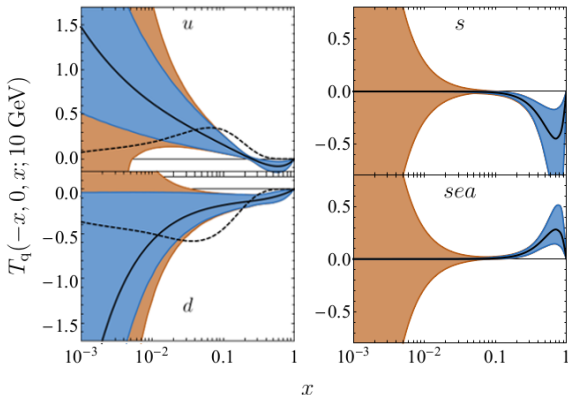
Description of the data



Qiu-Sterman function

$$T_q(-x, 0, x; \mu_b) = -\frac{1}{\pi} f_{1T; q \leftarrow h}^\perp(x, b) \quad G^{(+)} = \pm(|T_u| + |T_d|) - \text{gluon QS function}$$

$$-\frac{\alpha_s(\mu_b)}{4\pi^2} \int_x^1 \frac{dy}{y} \left[\frac{\bar{y}}{N_c} f_{1T; q \leftarrow h}^\perp\left(\frac{x}{y}, b\right) + \frac{3y^2 \bar{y}}{2x} G^{(+)}\left(-\frac{x}{y}, 0, \frac{x}{y}; \mu_b\right) \right] + \mathcal{O}(a_s^2, b^2)$$



Dashed line - JAM20 results (Cammarota *et al.*)

Conclusions

- We performed the extraction of the Sivers function that consistently utilizes previously extracted unpolarised proton and pion TMDs, and uses SIDIS, pion-induced Drell-Yan, and W^\pm/Z -boson production experimental data
- The extraction is performed at N³LO perturbative precision within the ζ -prescription that allows us to unambiguously relate the Sivers function and QS function
- Our results compare well in magnitude with the existing extractions
- We confirm the signs of Sivers functions for u and d quarks while we also obtain non negligible Sivers functions for anti-quarks
- The analysis was done with **artemide** package
- The fitting codes and the results of the extraction (in the form of replica-distributions for model parameters) are publicly available