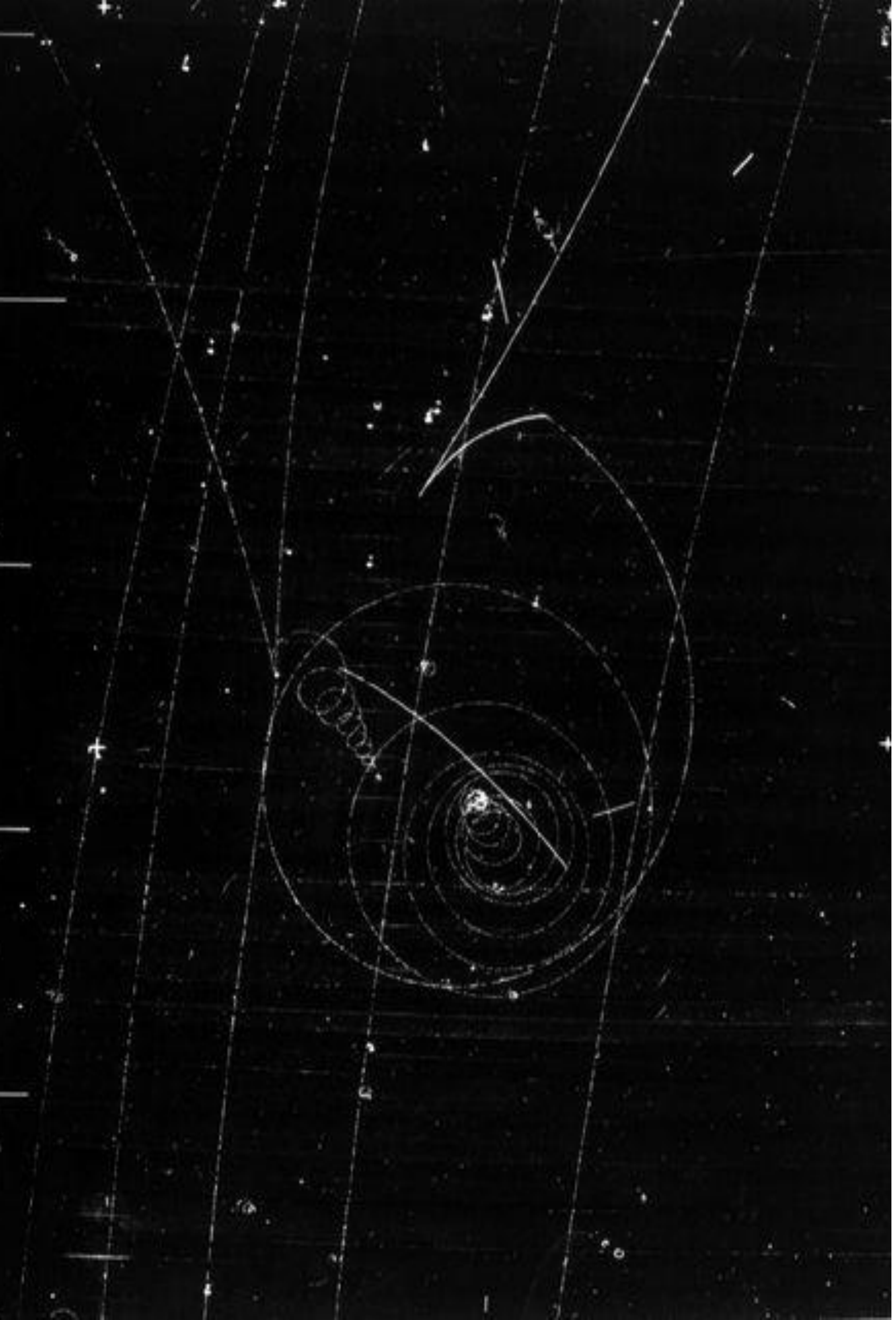

Lambda Polarizing Fragmentation Function from Belle e^+e^- data

Marco Zaccheddu - Università degli Studi di Cagliari & INFN

In collaboration with: Umberto D'Alesio, Francesco Murgia

Based on [Phys. Rev. D 102, 054001 (2020)]

Resummation, Evolution, Factorization 2020

Contents

- Introduction
- TMD FFs with Helicity Formalism
- $e^+e^- \to h^{\uparrow}{}_1 h_2 X$
- $e^+e^- \to \mathrm{h}_1(\mathrm{jet})X$
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- Conclusions

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Similar analysis:

[D. Callos, Z.B. Kang, and J. Terry, Phys. Rev. D 102, 096007 (2020)]

TMD Fragmentation Functions for quarks : Spin - ½ hadrons

Helicity density matrix

$$\rho^{i,s_i}_{\lambda_i,\lambda'_i} = \frac{1}{2}\begin{pmatrix} 1+P^i_z & P^i_x - iP^i_y \\ P^i_x + iP^i_y & 1-P^i_z \end{pmatrix}$$

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8 independent TMD Fragmentation Functions

		Hadron		
	Pol. States	U	L	T
Quark	U	$\hat{D}_{h/q}$		$\Delta\hat{D}^h_{S_Y/q}$
Quark	L		$\Delta\hat{D}^{h/q}_{S_Z/s_L}$	$\Delta\hat{D}^{h/q}_{S_X/s_L}$
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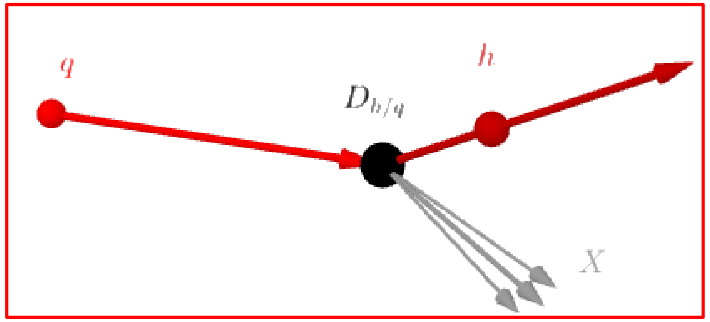
Unpolarized FF



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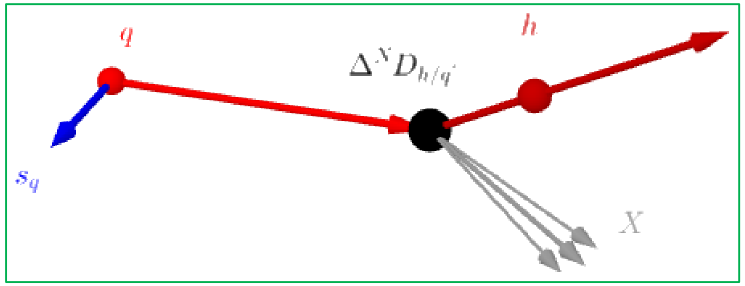
Collins FF



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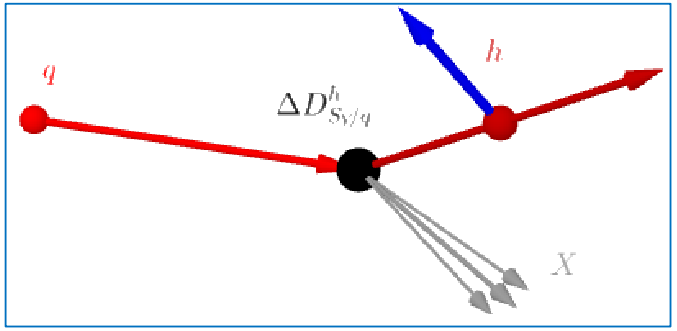

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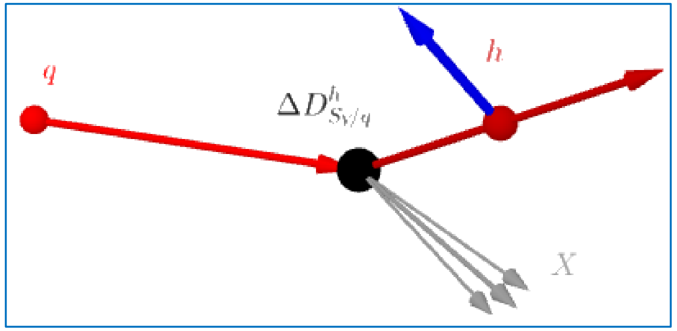

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Amsterdam notation:

Collins $\Delta^N D_{h/q^\uparrow} \longleftrightarrow H_1^\perp$

Polarizing $\Delta D^h_{S_Y/q} \longleftrightarrow D_{1T}^\perp$

$e^+e^- \to h_1 h_2 X$: Helicity Formalism

$$\rho^{h_1}_{\lambda_{h_1},\lambda'_{h_1}}\,\rho^{h_2}_{\lambda_{h_2},\lambda'_{h_2}}\,\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta dz_1 d^2\mathbf{p}_{\perp 1} dz_2 d^2\mathbf{p}_{\perp 2}}$$

$$=\sum_{q_c}\sum_{\{\lambda\}}\frac{1}{32\pi s}\frac{1}{4}\hat{M}_{\lambda_c\lambda_d,\lambda_a\lambda_b}\hat{M}^*_{\lambda'_c\lambda'_d,\lambda_a\lambda_b}\hat{D}^{\lambda_{h_1},\lambda'_{h_1}}_{\lambda_c,\lambda'_c}(z_1,\mathbf{p}_{\perp 1})\hat{D}^{\lambda_{h_2},\lambda'_{h_2}}_{\lambda_d,\lambda'_d}(z_2,\mathbf{p}_{\perp 2})$$

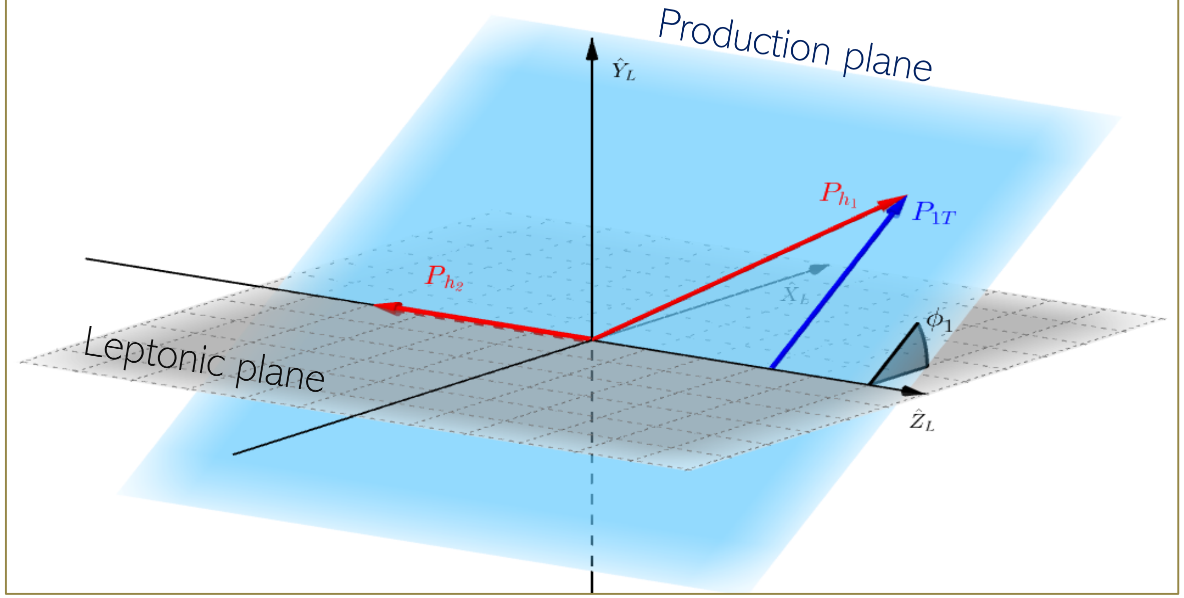
Scaling variables

- Light cone z
- Momentum fraction $z_p = 2|\mathbf{P}_h|/\sqrt{s}$
- Energy fraction $z_h = 2E_h/\sqrt{s}$

$$z_{h,p} \simeq z\left[1 \pm m_h^2/(z^2 s)\right]$$

Results consistent with [D. Boer, R. Jakob, and P.J. Mulders. Nucl. Phys. B 504 (1997)]

Hadron Frame



$e^+e^- \to h_1 h_2 X$: Helicity Formalism

Helicity matrices

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Scattering Amplitudes

TMD Fragmentation Functions

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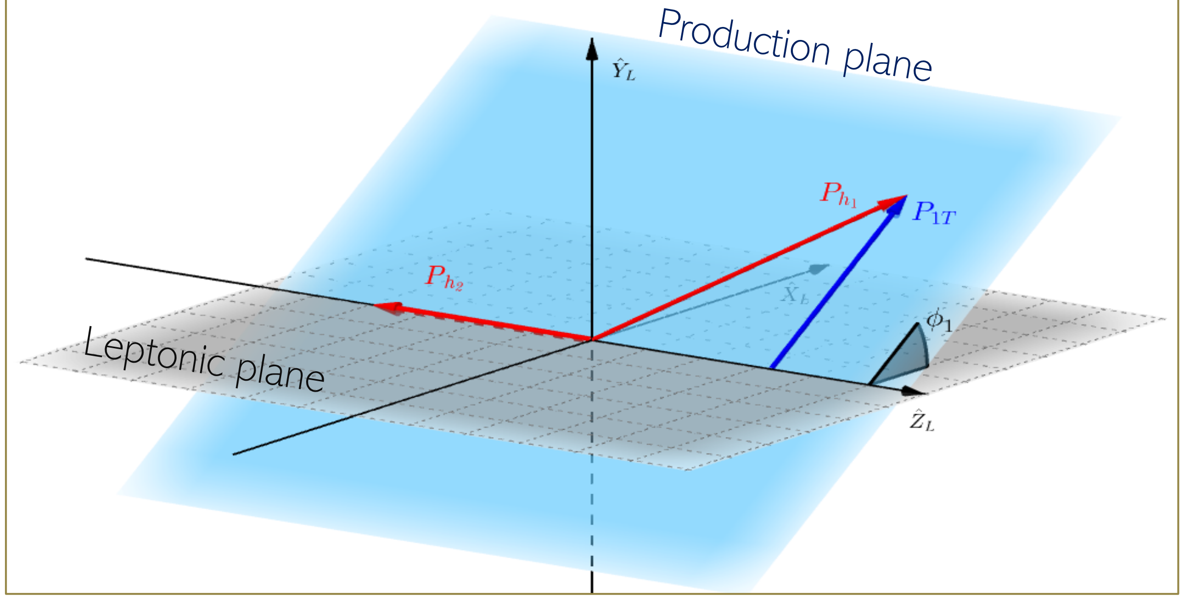

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Polarization vector

$$\mathcal{P}^{h_1} = P^{h_1}_x \hat{X}_1 + P^{h_1}_y \hat{Y}_1 + P^{h_1}_z \hat{Z}_1$$

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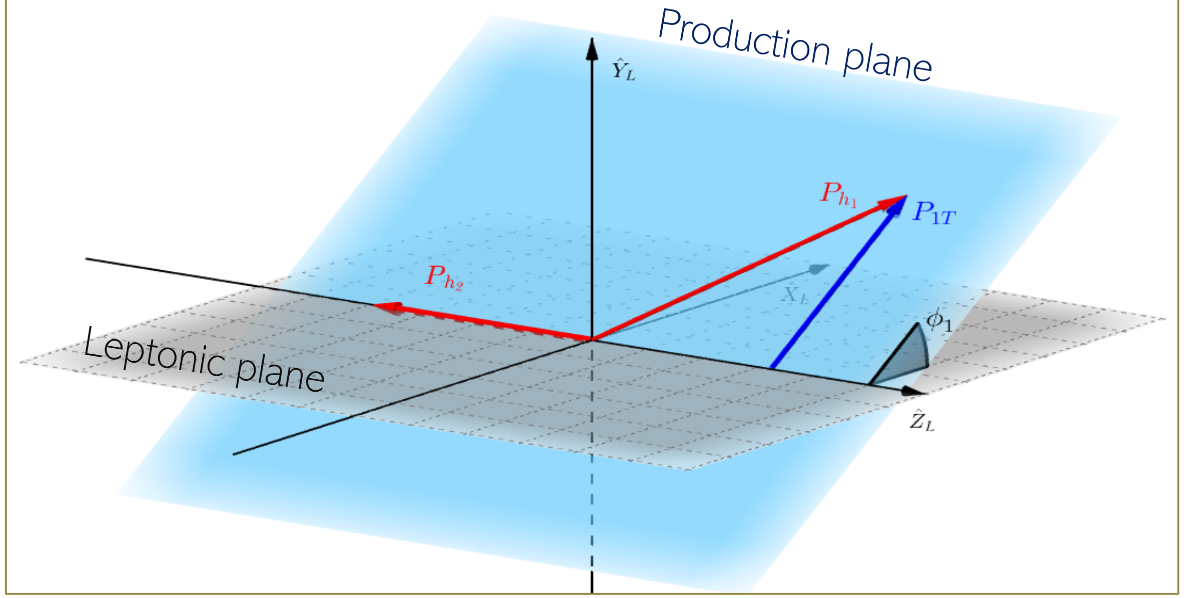

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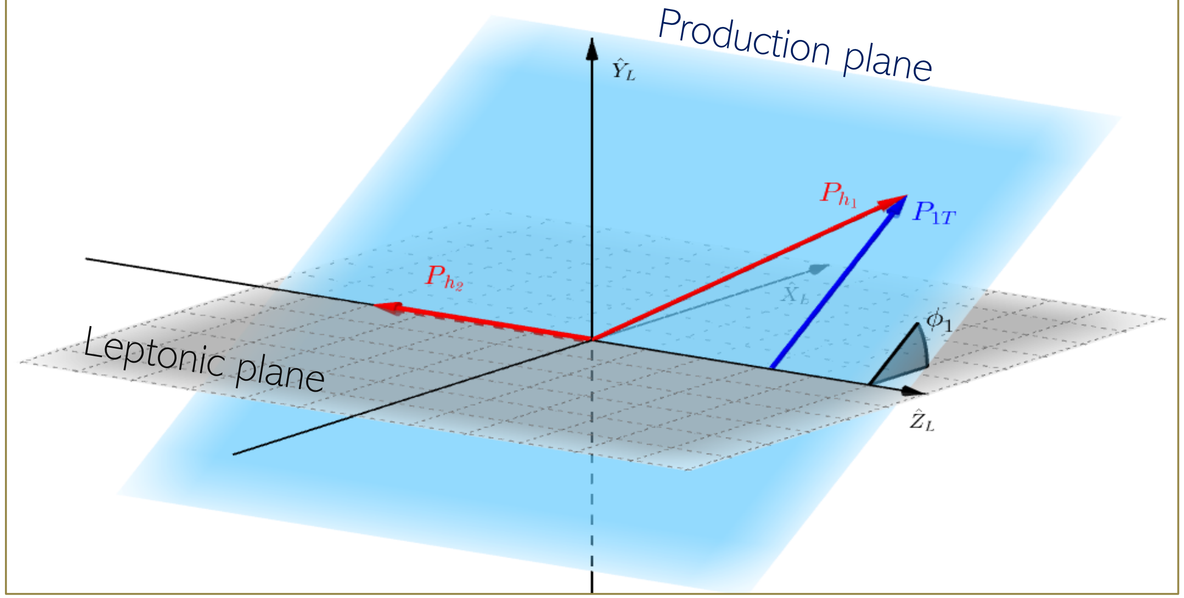

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$$\mathcal{P}^{h_1} = P_x^{h_1}\hat{X}_1 + P_y^{h_1}\hat{Y}_1 + P_z^{h_1}\hat{Z}_1$$

The polarization is measured along:

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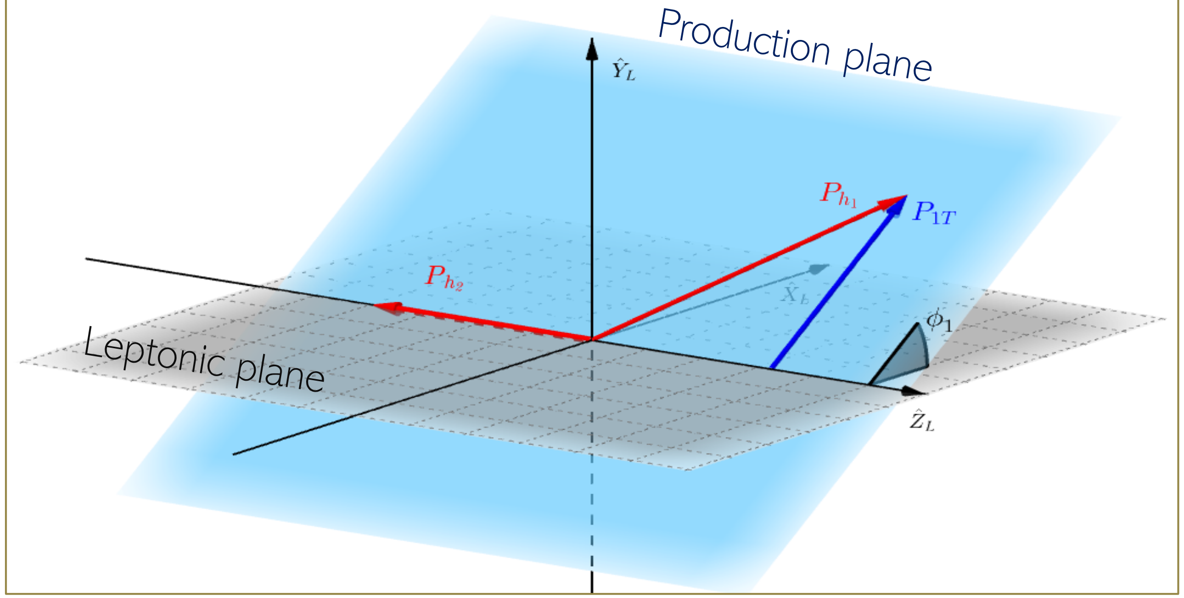

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The polarization projection along $\hat{n}$:

$$\mathcal{P}^{h_1}\cdot\hat{n} = P^{h_1}_x \cos\widetilde{\phi} + P^{h_1}_y \sin\widetilde{\phi}$$

$e^+e^- \to h^\uparrow{}_1 h_2 X$: Hadron Frame

Introduce $p_\perp$ - parameterization for FFs:

$$\Delta D^h_{S_Y/q}(z,p_\perp) = \Delta D^h_{S_Y/q}(z)\sqrt{2e}\frac{p_\perp}{M_{pol}}\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_{pol}}}{\pi\langle p_\perp^2\rangle_h}$$

$$D_{h/q}(z,p_\perp) = D_{h/q}(z)\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_h}}{\pi\langle p_\perp^2\rangle_h}$$

- Fixed energy scale $\sqrt{s} = 10.58$ GeV
- NO Evolution
- Data depend only on energy fraction z_Λ - $z_{\pi,K}$
- $\langle p_\perp^2\rangle_h = 0.2\ GeV^2$ width of unp. FF

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z dependence

Gaussian dependence on $p_\perp$

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$$d^2\mathbf{p}_{\perp 1} \to d^2\mathbf{P}_{T1}$$

$$\int d^2\mathbf{P}_{1T} d^2\mathbf{p}_{\perp 2}$$

$e^+e^- \to h^{\uparrow}{}_1 h_2 X$: Hadron Frame

Introduce $p_\perp$ - parameterization for FFs:

Linear term

$$\Delta D^h_{S_Y/q}(z, p_\perp) = \Delta D^h_{S_Y/q}(z)\sqrt{2e}\,\frac{p_\perp}{M_{pol}}\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_{pol}}}{\pi\langle p_\perp^2\rangle_h}$$

z dependence

Gaussian dependence on $p_\perp$

$$D_{h/q}(z, p_\perp) = D_{h/q}(z)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_h}}{\pi\langle p_\perp^2\rangle_h}$$

- Fixed energy scale $\sqrt{s} = 10.58$ GeV
- NO Evolution
- Data depend only on energy fraction z_Λ - $z_{\pi,K}$
- $\langle p_\perp^2\rangle_h = 0.2\ GeV^2$ width of unp. FF

$$d^2\mathbf{p}_{\perp 1} \to d^2\mathbf{P}_{T1}$$

$$\int d^2\mathbf{P}_{1T} d^2\mathbf{p}_{\perp 2}$$

$\longrightarrow$

$$\mathcal{P}_n(z_1, z_2) = \sqrt{\frac{e\pi}{2}}\,\frac{1}{M_{\rm pol}}\,\frac{\langle p_\perp^2\rangle_{\rm pol}^2}{\langle p_{\perp 1}^2\rangle}\,\frac{z_2}{\{[z_1(1 - m_{h_1}^2/(z_1^2 s))]^2\langle p_{\perp 2}^2\rangle + z_2^2\langle p_\perp^2\rangle_{\rm pol}\}^{1/2}} \times \frac{\sum_q e_q^2\,\Delta D_{h_1^\uparrow/q}(z_1) D_{h_2/\bar{q}}(z_2)}{\sum_q e_q^2\, D_{h_1/q}(z_1) D_{h_2/\bar{q}}(z_2)}$$

$e^+e^- \to h^\uparrow{}_1 h_2 X$: Hadron Frame

Introduce $p_\perp$ - parameterization for FFs:

Linear term

$$\Delta D^h_{S_Y/q}(z,p_\perp) = \underbrace{\Delta D^h_{S_Y/q}(z)}_{\text{z dependence}}\sqrt{2e}\,\frac{p_\perp}{M_{pol}}\,\underbrace{\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_{pol}}}{\pi\langle p_\perp^2\rangle_h}}_{\text{Gaussian dependence on } p_\perp}$$

z dependence

Gaussian dependence on $p_\perp$

$$D_{h/q}(z,p_\perp) = D_{h/q}(z)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_h}}{\pi\langle p_\perp^2\rangle_h}$$

- Fixed energy scale $\sqrt{s} = 10.58$ GeV
- NO Evolution
- Data depend only on energy fraction z_Λ - $z_{\pi,K}$
- $\langle p_\perp^2\rangle_h = 0.2\ GeV^2$ width of unp. FF

$$d^2\mathbf{p}_{\perp 1} \to d^2\mathbf{P}_{T1}$$

$$\int d^2\mathbf{P}_{1T}\,d^2\mathbf{p}_{\perp 2} \longrightarrow$$

$$\mathcal{P}_n(z_1,z_2) = \sqrt{\frac{e\pi}{2}}\,\frac{1}{M_{\rm pol}}\,\frac{\langle p_\perp^2\rangle_{\rm pol}^2}{\langle p_{\perp 1}^2\rangle}\,\frac{z_2}{\{[z_1(1-m_{h_1}^2/(z_1^2 s))]^2\langle p_{\perp 2}^2\rangle + z_2^2\langle p_\perp^2\rangle_{\rm pol}\}^{1/2}} \times \frac{\sum_q e_q^2\,\Delta D_{h_1^\uparrow/q}(z_1)D_{h_2/\bar q}(z_2)}{\sum_q e_q^2\,D_{h_1/q}(z_1)D_{h_2/\bar q}(z_2)}$$

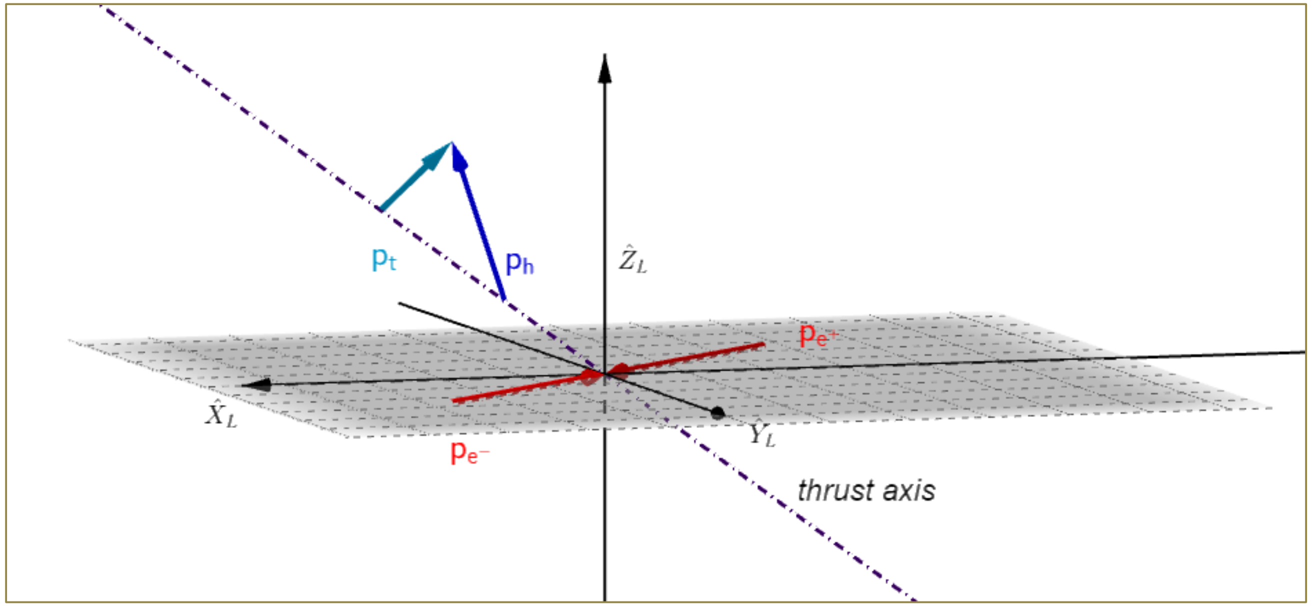
Consistent with [D. Callos, Z.B. Kang, and J. Terry, Phys. Rev. D 102, 096007 (2020)]

$e^+e^- \to h_1 + \text{(jet)}\ X$: Thrust Frame

The polarization is measured along:

$$\hat{n} = \hat{T} \times \hat{P}_{h_1}$$

- Data depend only on z_Λ - $p_\perp$
- Direct access to $p_\perp$ dependence



Within a phenomenological approach

$$\mathcal{P}_T(z_1, p_{\perp 1}) = \frac{\sum_q e_q^2\, \Delta D_{h_1^\uparrow/q}(z_1, p_{\perp 1})}{\sum_q e_q^2\, D_{h_1/q}(z_1, p_{\perp 1})}$$

Fit and Results (1)

Belle data: $\sqrt{s} = 10.58$ GeV

- 128 points $\Lambda + h$, in bins of the energy fractions z_Λ - $z_{\pi,K}$
- 32 points $\Lambda(jet)$, in bins of z_Λ - $p_\perp$

Polarizing FF parametrization:

Unpolarized FF

$$\Delta D^h_{S_Y/q}(z) = \mathcal{N}^p_q(z) \underbrace{D_{h/q}(z)}$$

$$\mathcal{N}^p_q(z) = N^p_q z^{\alpha_q}(1-z)^{\beta_q} \frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}$$

Unpolarized FF set adopted:

- DSS07 for π, K
- AKK08 for $\Lambda + \overline{\Lambda}$

$q\bar{q}$ FF separation for AKK08:

$$D_{\Lambda/\bar{q}}(z_p) = (1-z_p) D_{\Lambda/q}(z_p)$$

- Normalization factor: N^p_q, $|N^p_q| \leq 1$
- Shape for high and low z: α_q β_q

Fit and Results (2)

Data selection:

- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5 \ - 0.9]$ bin excluded → 96 data points
- $\Lambda(jet)$: $z_{\Lambda} = [0.5 \ - 0.9]$ bin excluded → 24 data points

Fit and Results (2)

Data selection:

- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5 \ - 0.9]$ bin excluded → 96 data points
- $\Lambda(jet)$: $z_{\Lambda} = [0.5 \ - 0.9]$ bin excluded → 24 data points

Fitted 8 parameters

Flav.	$\mathcal{N}_q^p$	α_q	β_q	$\langle p_\perp^2 \rangle_{pol}$
u	$\mathcal{N}_u^p$		β_u	$\langle p_\perp^2 \rangle_{pol}$
d	$\mathcal{N}_d^p$			
s	$\mathcal{N}_s^p$	α_s		
sea	$\mathcal{N}_{sea}^p$		β_{sea}	

Fit and Results (2)

Data selection:

- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5 \; - 0.9]$ bin excluded → 96 data points
- $\Lambda(jet)$: $z_{\Lambda} = [0.5 \; - 0.9]$ bin excluded → 24 data points

Fitted 8 parameters

Fitted Parameters Value	
Nu	$0.47^{+0.32}_{-0.20}$
Nd	$-0.32^{+0.13}_{-0.13}$
Ns	$-0.57^{+0.29}_{-0.43}$
Nsea	$-0.27^{+0.12}_{-0.20}$
α_s	$2.30^{+1.08}_{-0.91}$
β_{sea}	$2.60^{+2.60}_{-1.74}$
β_u	$3.50^{+2.33}_{-1.82}$
$< p_\perp^2 >_{pol}$	$0.10^{+0.02}_{-0.02}$

Full data Fit : $\chi^2_{dof} = 1.94$

Fit and Results (2)

Data selection:
- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5 \ - 0.9]$ bin excluded → 96 data points
- $\Lambda(jet)$: $z_\Lambda = [0.5 \ - 0.9]$ bin excluded → 24 data points

Fitted 8 parameters

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Ns	-0.57 $^{+0.29}_{-0.43}$
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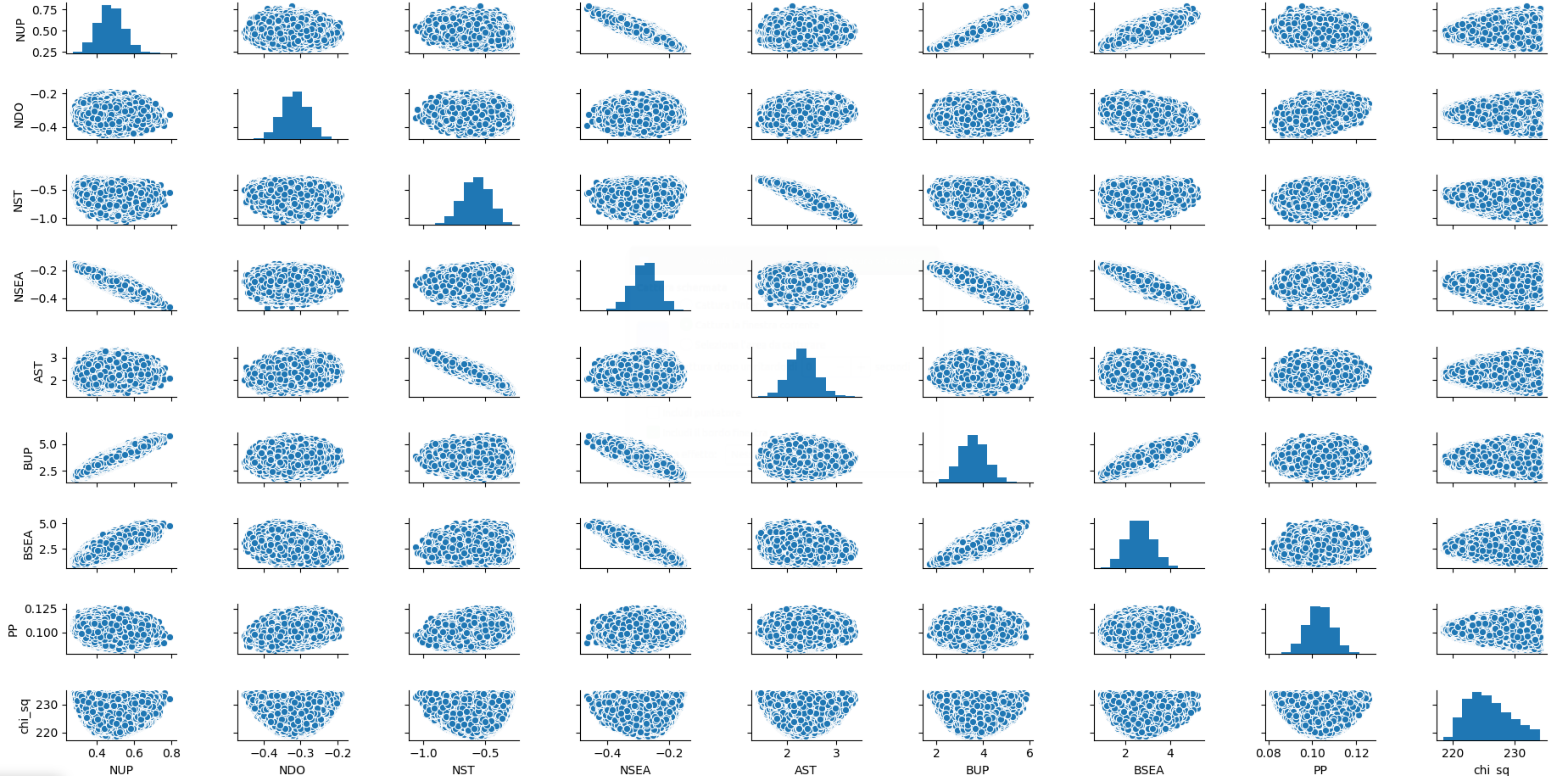
α_s	2.30 $^{+1.08}_{-0.91}$
β_{sea}	2.60 $^{+2.60}_{-1.74}$
β_u	3.50 $^{+2.33}_{-1.82}$
$< p_\perp^2 >_{pol}$	0.10 $^{+0.02}_{-0.02}$

Full data Fit : $\chi^2_{dof} = 1.94$

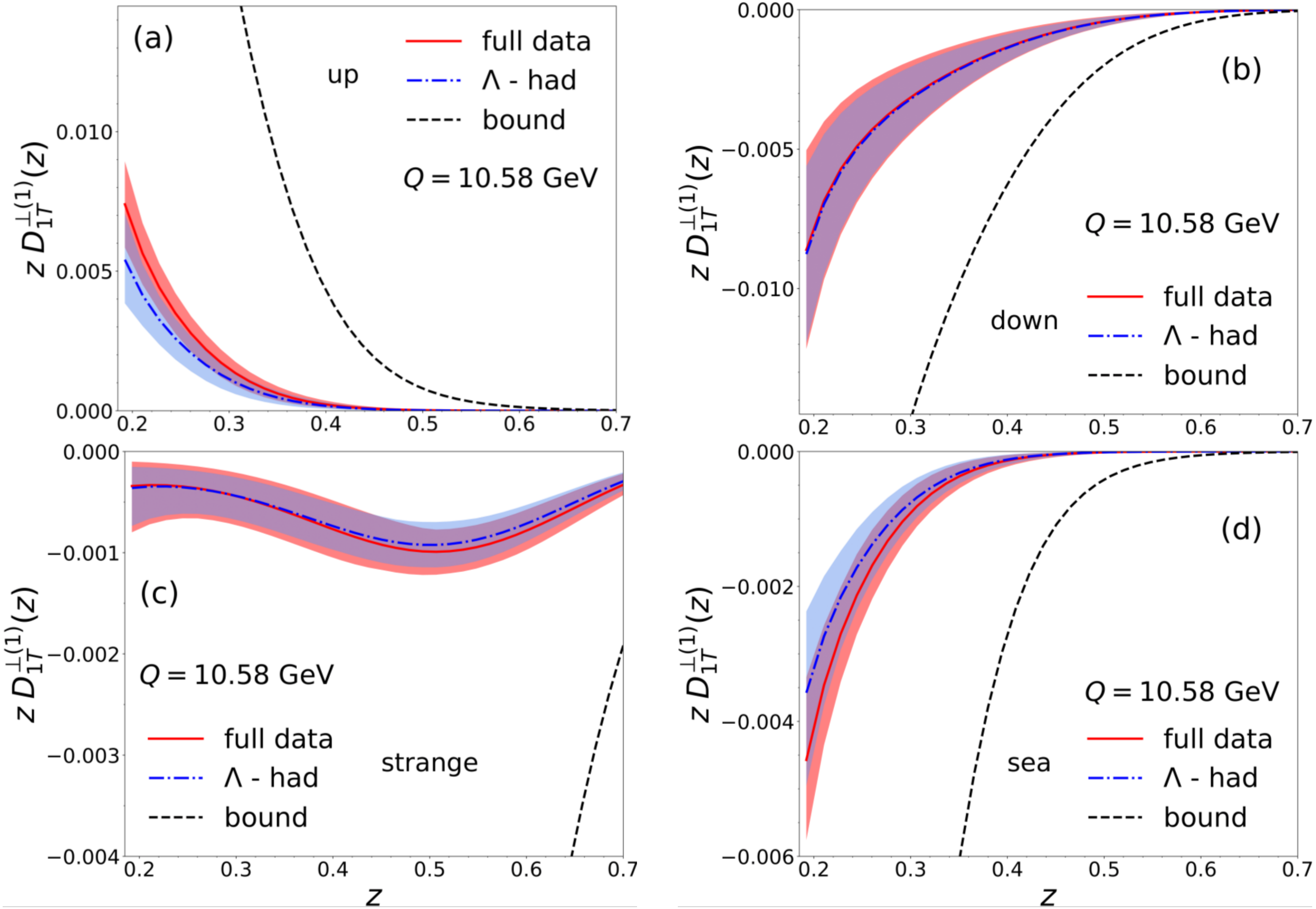
$\Lambda + \pi/K$ data Fit : $\chi^2_{dof} = 1.26$

The two extractions are consistent



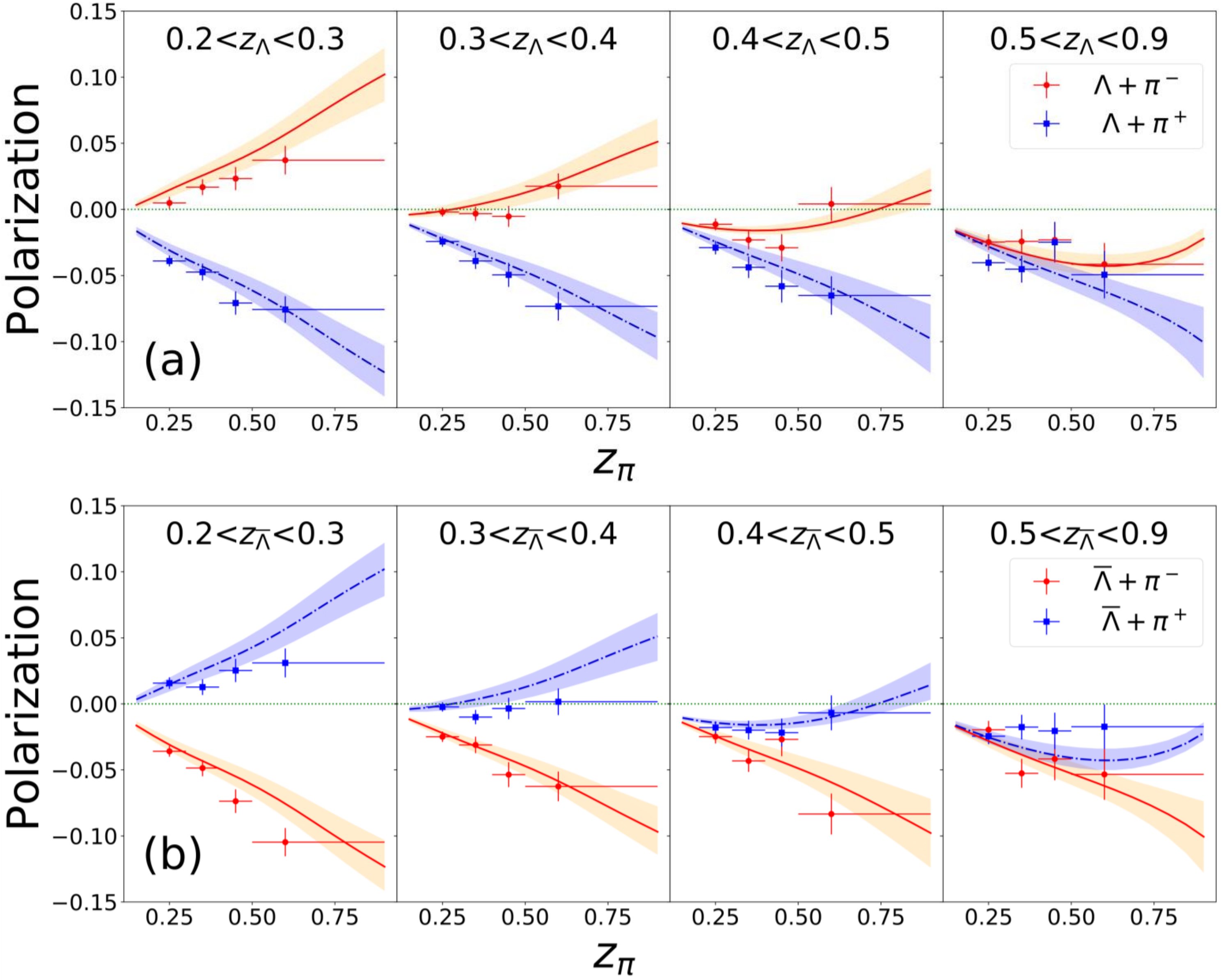
NUP
NDO
NST
NSEA
AST
BUP
BSEA
PP
chi_sq
chi sq

First moments



$$D_{1T}^{\perp(1)}(z) = \int d^2\mathbf{p}_\perp \frac{p_\perp}{2zm_h} \Delta D_{h^\uparrow/q}(z, p_\perp)$$

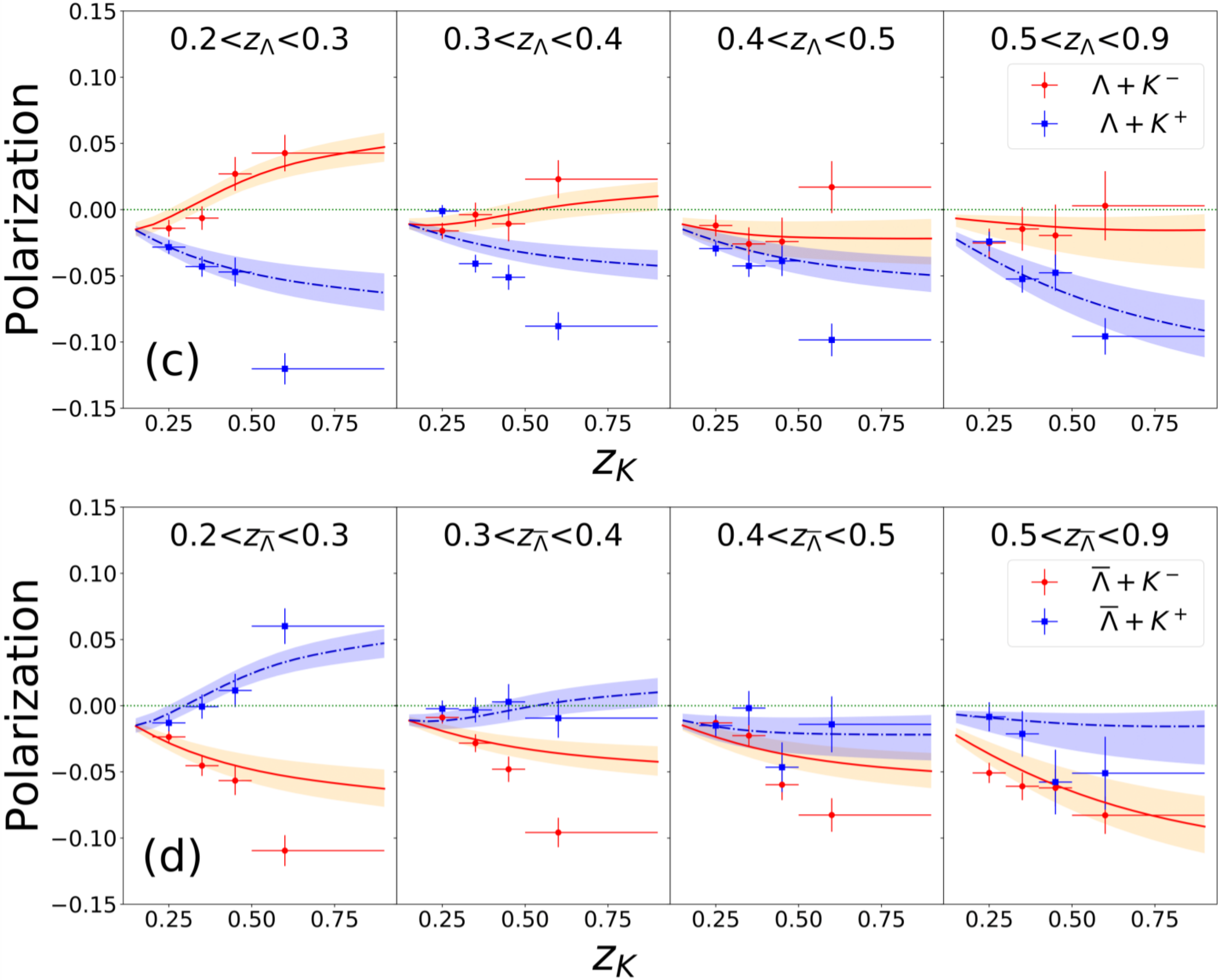
Lambda-pion



Bin excluded
$z_\pi = [0.5 - 0.9]$

$\chi^2_{dof} = 1.94$

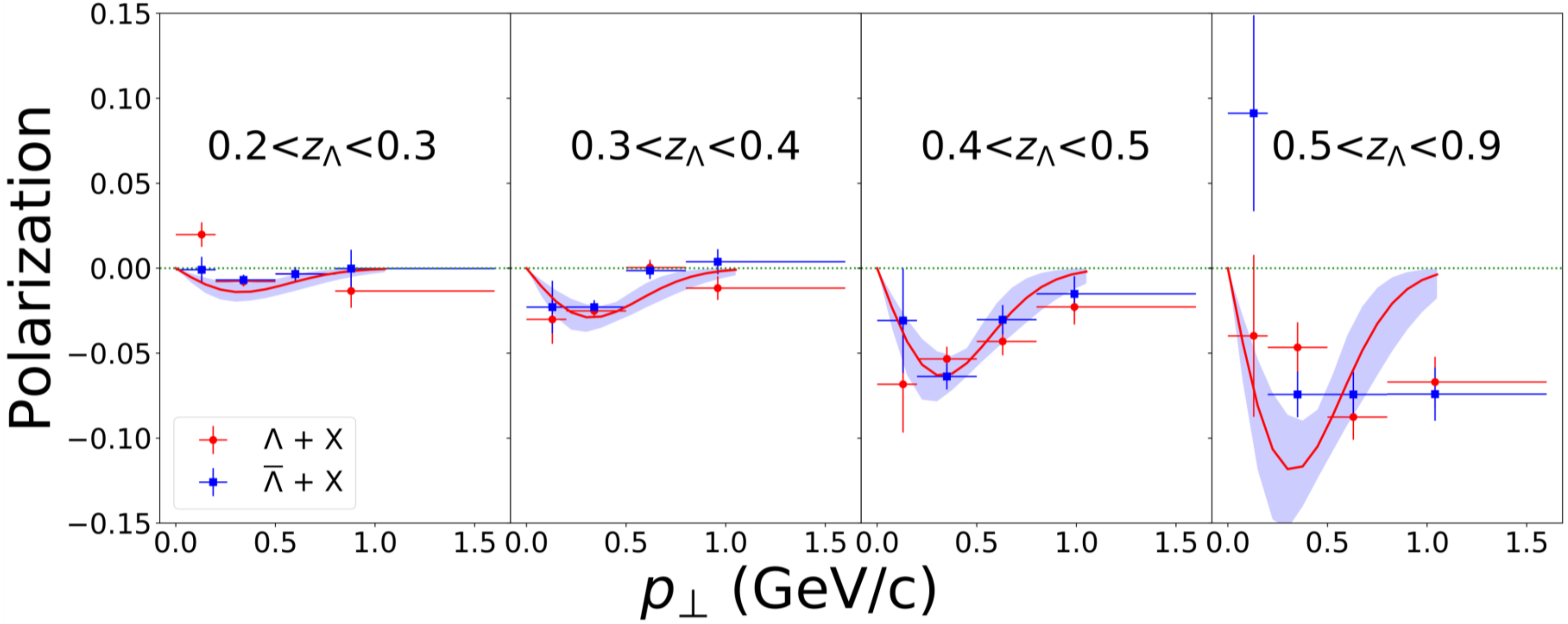
Lambda-kaon



Bin excluded
$z_K = [0.5 - 0.9]$

$\chi^2_{dof} = 1.94$

Lambda-jet



Bin excluded
$z_\Lambda = [0.5 \; - 0.9]$

$\chi^2_{dof} = 1.94$

Expected features:

- $P_T \; = 0$ when $p_\perp \; = 0$;
- $P_T\,(\Lambda) = P_T\,(\overline{\Lambda})$.

SiDIS – Polarized Lambda Production

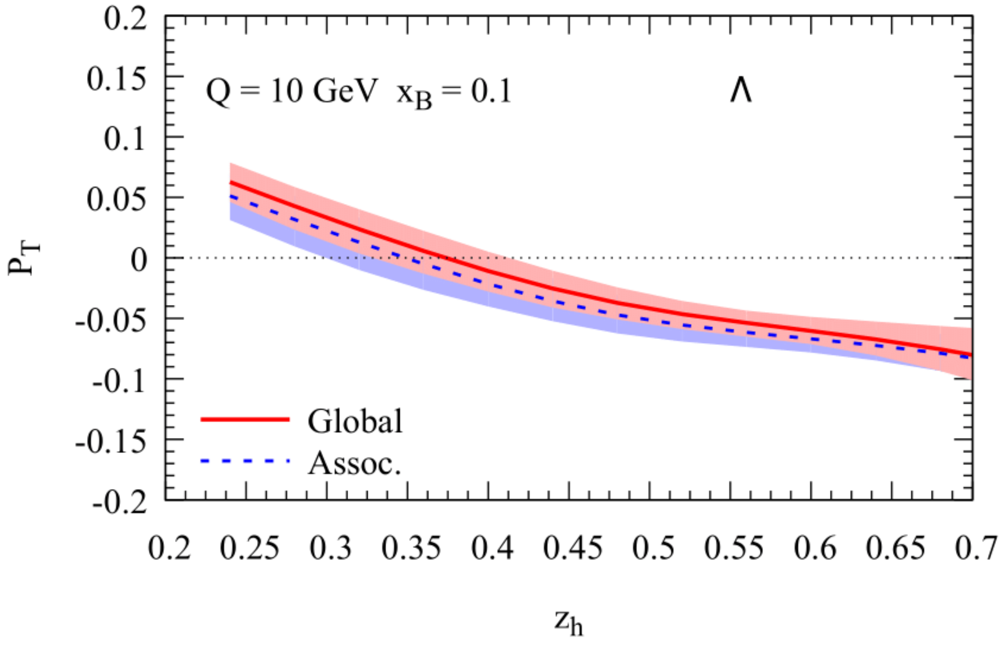
$$P_T(x_B, z_h) = \frac{\sqrt{2e\pi}}{2M_p} \frac{< p_\perp^2 >_p^2}{< p_\perp^2 >} \frac{1}{\sqrt{< p_\perp^2 >_p + \xi_p^2 < k_\perp^2 >}} \times \frac{\sum_q f_{q/P}(x_B) \Delta D_{h^\uparrow/q}(z_h)}{\sum_q f_{q/P}(x_B) D_{h/q}(z_h)}$$

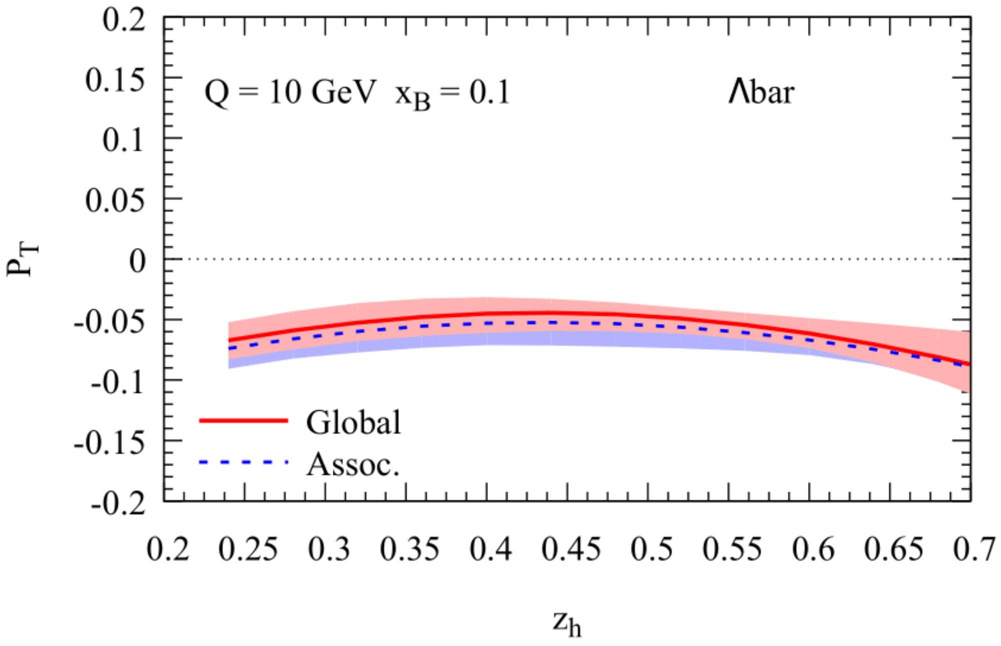
Kinematical configuration compatible with EIC

Prediction for the Λ polarization:

- $x_B = 0.1$

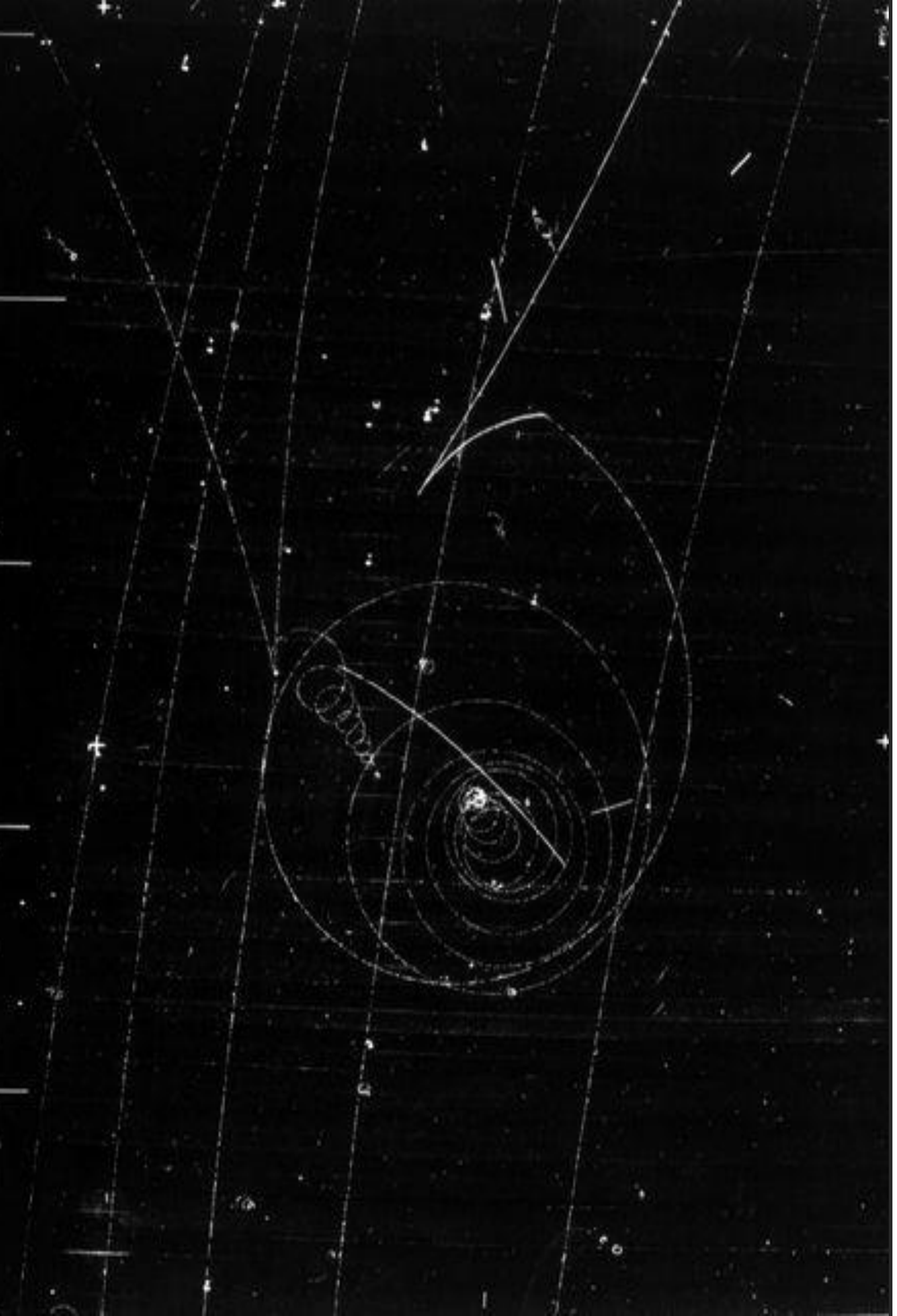
$$\xi_p = z_h \left(1 - \frac{m_h^2}{z_h^2 Q^2} \frac{x_B}{1 - x_B}\right)$$





Conclusions

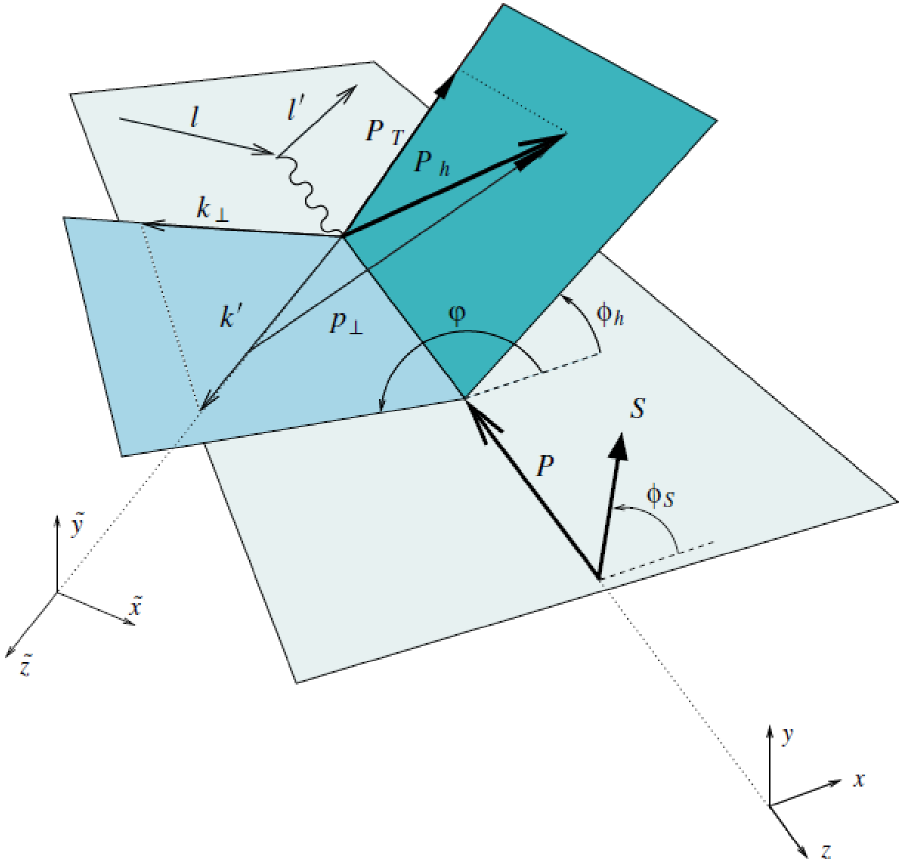
- First extraction of the Λ polarizing fragmentation function from Belle e^+e^- data;
- Clear separation in flavours: three different valence pFF needed;
- Sea-quark pFF also necessary;
- First indication of the $p_\perp$ dependence within a Gaussian Ansatz;
- Prediction for transverse Λ polarization in SiDIS at EIC.



Backup Slides

SiDIS – Polarized Lambda Production

$e^-P \rightarrow e^- \Lambda$



$$P_T(x_B, z_h) = \frac{\sqrt{2e\pi}}{2M_p}\frac{<p_\perp^2>_p^2}{<p_\perp^2>}\frac{1}{\sqrt{<p_\perp^2>_p + \xi_p^2 <k_\perp^2>}} \times \frac{\sum_q f_{q/P}(x_B)\Delta D_{h^\uparrow/q}(z_h)}{\sum_q f_{q/P}(x_B) D_{h/q}(z_h)}$$

Proton PDF

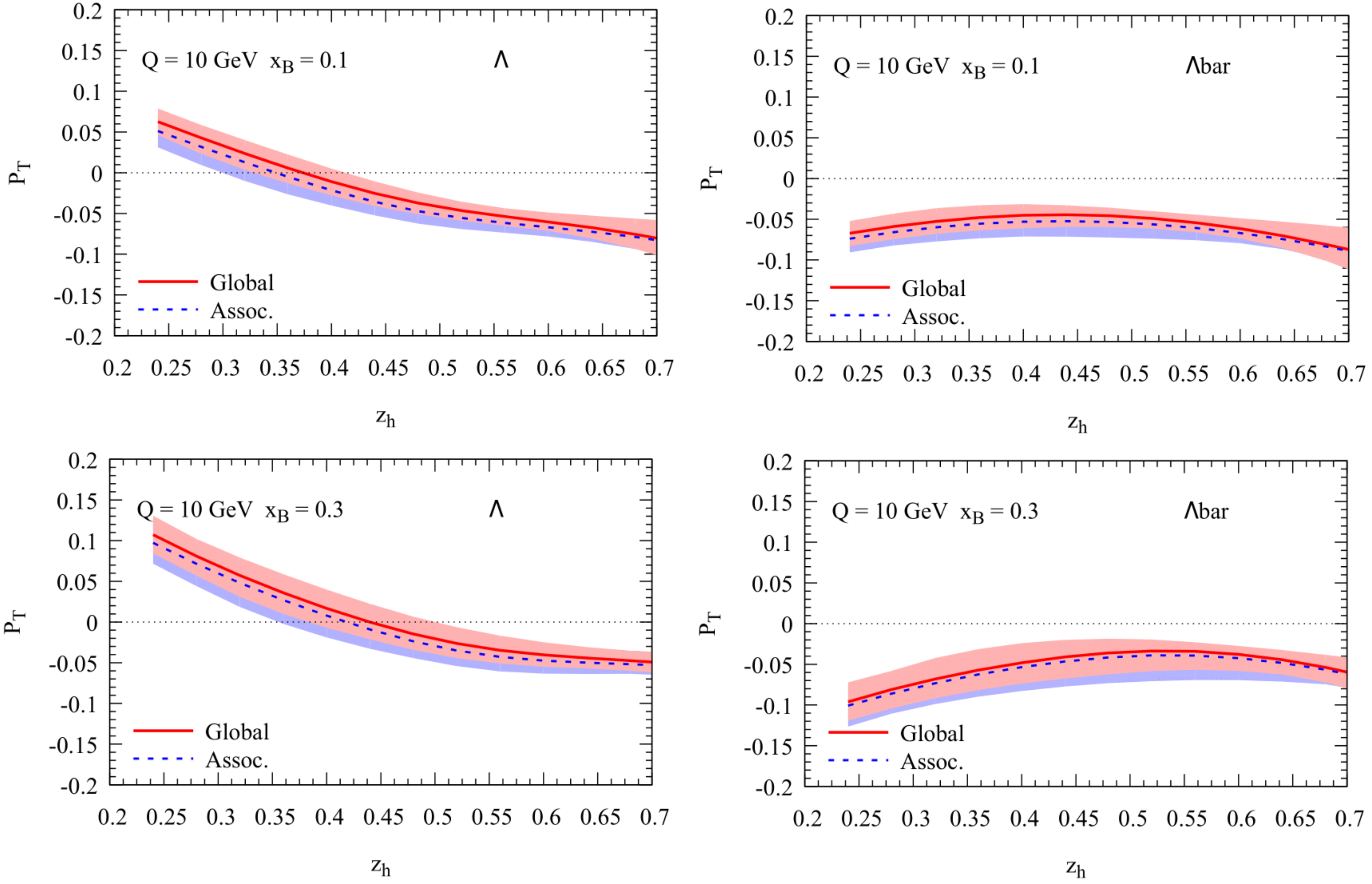
Lambda Polarizing FF

CTEQ6L1

x_B Bjorken-x
z_h energy fraction

$$\xi_p = z_h\left(1 - \frac{m_h^2}{z_h^2 Q^2}\frac{x_B}{1-x_B}\right)$$

SiDIS – Polarized Lambda Production



Prediction for the Λ polarization:

- x_B = 0.1
- x_B = 0.3

Statistical Uncertainty Band

Multivariate Normal Distribution

MINUIT:
- Best fit parameters $\mu : \mathcal{N}_q^p \, \alpha_q \; \beta_q \; \langle p_\perp^2 \rangle_p$
- Covariance matrix Σ
- Minimum Chi-square χ^2

$\mu \; , \; \Sigma \longrightarrow$

$$p(x;\mu,\Sigma) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right)$$

Generate a random set of parameter

$x: (\mathcal{N}_q^p \;\; \alpha_q \;\; \beta_q \;\; \langle p_\perp^2 \rangle_p)$

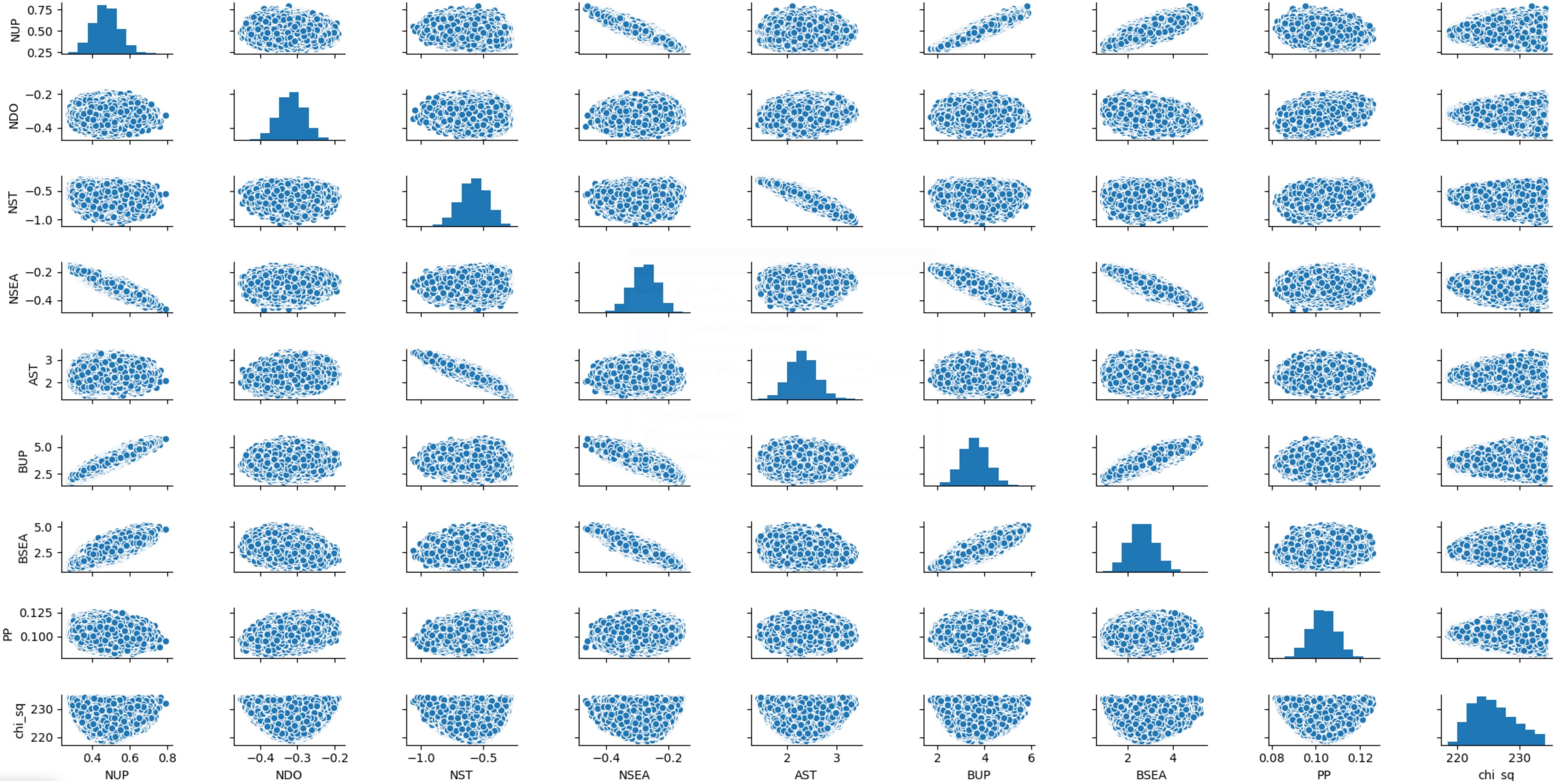
$\longrightarrow$ Calculate their own Chi-square

χ'^2

$\longrightarrow$ Keep set if

$$\chi^2 \leq \chi'^2 \leq \chi^2 + \Delta\chi^2$$

- Minimum Chi-square: χ^2 = 232,8
- Confidence interval $2\sigma \rightarrow$ 95,5% : $\Delta\chi^2$ = 15,79 for 8 parameters (χ^2 -distribution)


NUP
NDO
NST
NSEA
AST
BUP
BSEA
PP
chi_sq
chi_sq

$e^+e^- \to h^\uparrow{}_1 h_2 X$

h_1, h_2 unpolarized

$$\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta dz_1 d^2p_{\perp1}dz_2d^2p_{\perp2}}$$

Unpolarized FFs

$$=\frac{6e^4e_q^2}{64\pi\hat{s}}\Big\{D_{h_1/q}(z_1,p_{\perp1})D_{h_2/\bar{q}}(z_2,p_{\perp2})\left(1+\cos^2\theta\right)$$

Collins FFs

$$+\frac{1}{4}\sin^2\theta\Delta^N D_{h_1/q^\uparrow}(z_1,p_{\perp1})\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2,p_{\perp2})\cos(2\varphi_2+\phi_1^{h_1})\Big\}$$

h_1 polarized: X , h_2 unpolarized

$$P_X^{h_1}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta dz_1 d^2p_{\perp1}dz_2d^2p_{\perp2}}$$

$$=\frac{3e^4e_q^2}{64\pi\hat{s}}\Delta D^{h_1}_{S_X/s_T}(z_1,p_{\perp1})\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2,p_{\perp2})\sin^2\theta\sin(2\varphi_2+\phi_1^{h_1})$$

Collins FF

FF h_1 transv. Pol.

h_1 polarized: Y , h_2 unpolarized

$$P_Y^{h_1}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta dz_1 d^2p_{\perp1}dz_2d^2p_{\perp2}}$$

Polarizing FF

Unpolarized FF

$$=\frac{6e^4e_q^2}{64\pi\hat{s}}\Big\{\Delta D^{h_1}_{S_Y/q}(z_1,p_{\perp1})D_{h_2/\bar{q}}(z_2,p_{\perp2})\left(1+\cos^2\theta\right)$$

$$+\frac{1}{2}\sin^2\theta\Delta^- D^{h_1}_{S_Y/s_T}(z_1,p_{\perp1})\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2,p_{\perp2})\cos(2\varphi_2+\phi_1^{h_1})\Big\}$$

Collins FF

FF h_1 transv. Pol.

h_1 polarized: Z , h_2 unpolarized

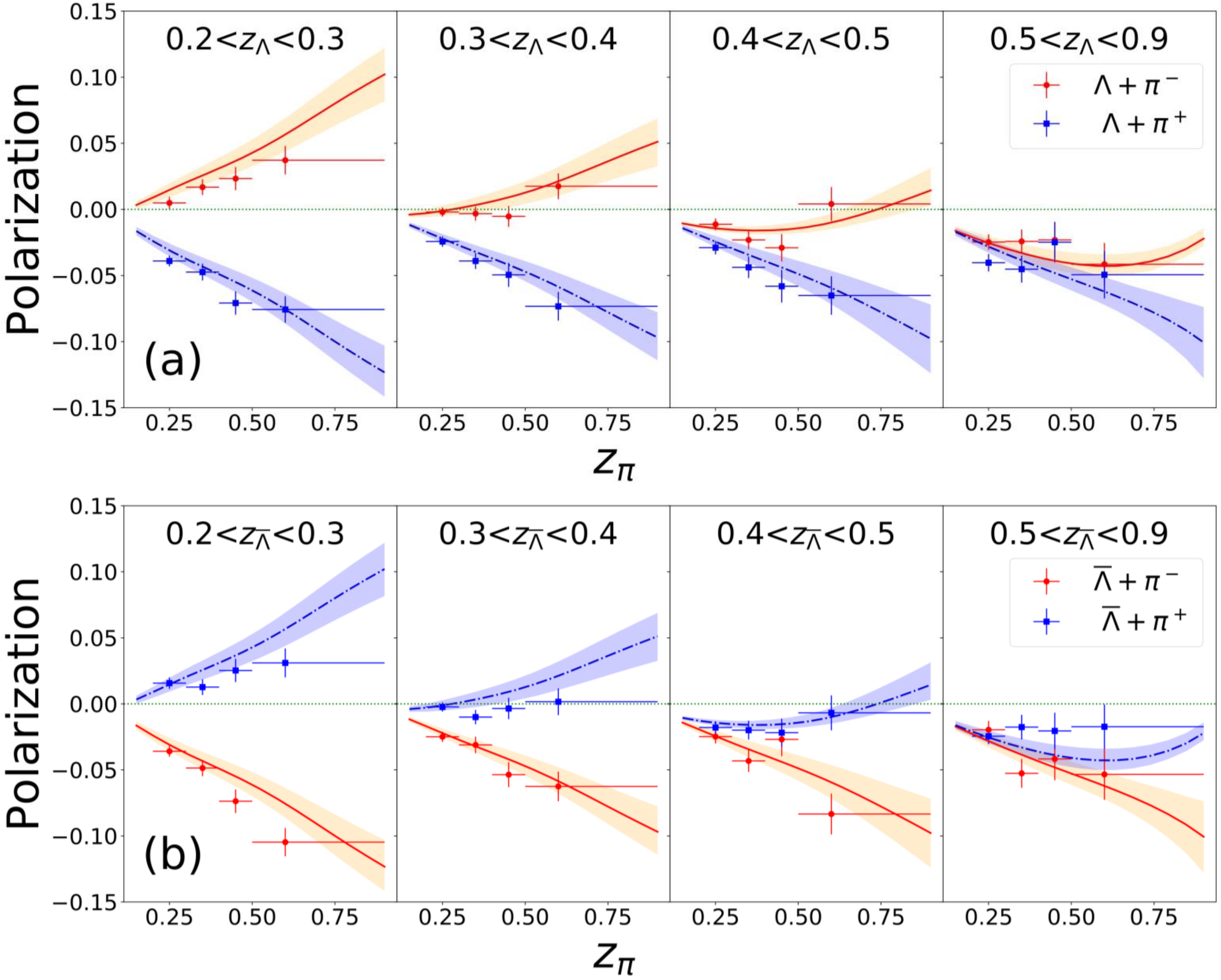
$$P_Z^{h_1}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{d\cos\theta dz_1 d^2p_{\perp1}dz_2d^2p_{\perp2}}$$

$$=\frac{3e^4e_q^2}{64\pi\hat{s}}\Delta D^{h_1}_{S_Z/s_T}(z_1,p_{\perp1})\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2,p_{\perp2})\sin^2\theta\sin(2\varphi_2+\phi_1^{h_1})$$

Collins FF

FF h_1 long. Pol.

Lambda-pion



Data fitted:

- $z_\pi < 0.50$

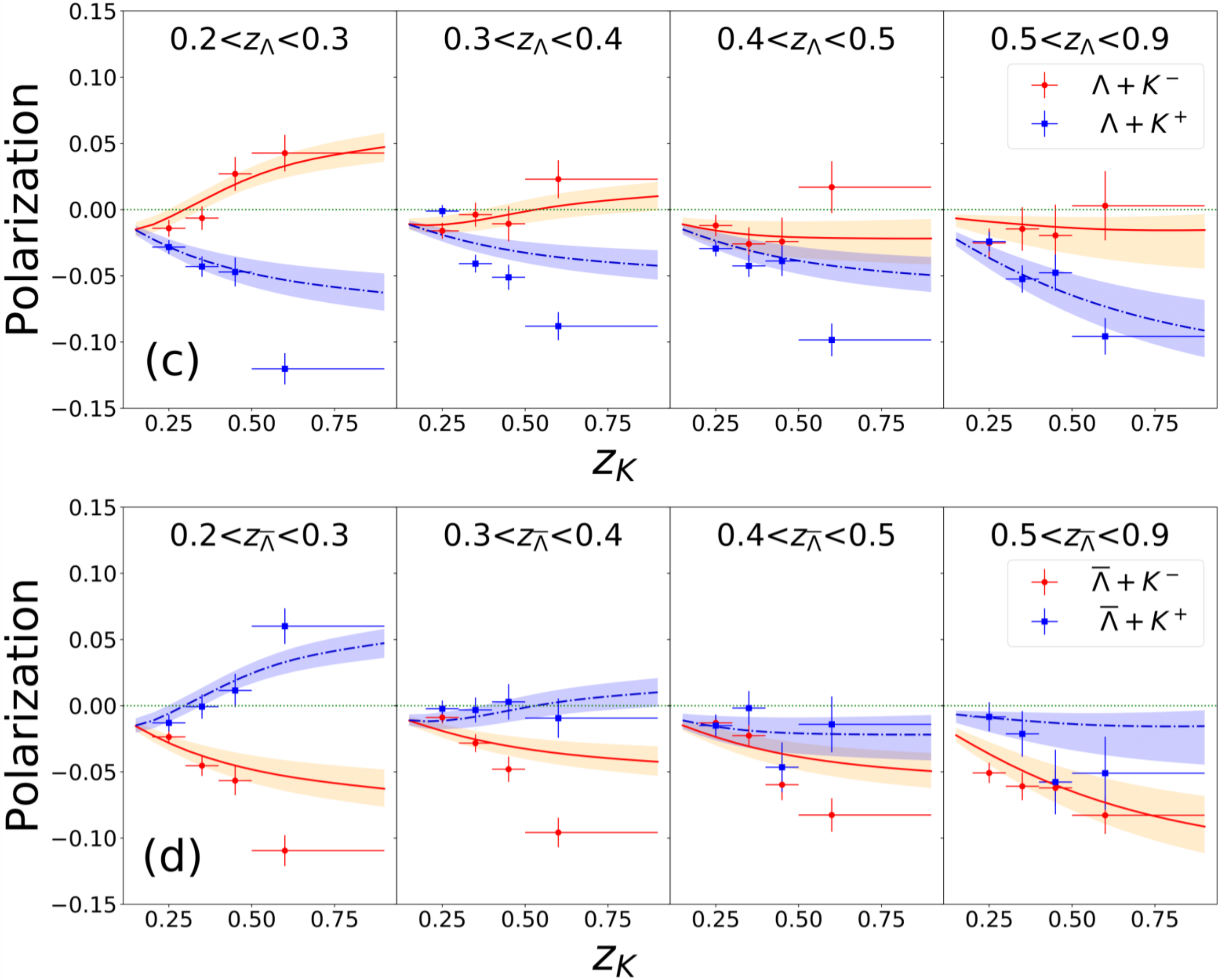
$\chi^2_{dof} = 1.94$

Lambda pion

- Charge-conjugation symmetry $P_n(\Lambda\pi^+) = P_n(\bar{\Lambda}\pi^-)$
- Data give information on the pFFs for u and d
- $P_n(\Lambda\pi^+)$ negative, dominated by down pFF
- $P_n(\Lambda\pi^-)$ positive, dominated by up pFF
- $P_n(\Lambda\pi^-)$ strong reduction due to the large suppression of the up pFF

- Small z_a sea pFFs become important and negative,
- $P_n(\Lambda\pi^+)$ up and down cancel each other, sea pFF leads to large, and negative, values of the transverse polarization
- $P_n(\Lambda\pi^-)$ sea pFF partial reduction of the up pFF

Lambda-kaon



Data fitted:

- $z_K < 0.50$

$\chi^2_{dof} = 1.94$

Lambda-kaon

- Charge-conjugation symmetry $P_n(\Lambda K^+) = P_n(\bar{\Lambda} K^-)$
- Similar pattern pion
- Data give information on the pFFs for u and s
- $P_n(\Lambda K^+)$ negative, dominated by strange pFF
- $P_n(\Lambda K^-)$ positive, dominated by up pFF
- $P_n(\Lambda K^-)$ strong reduction due to the large suppression of the up pFF

- Small z_a sea pFFs become important and negative,

$e^+e^- \to \mathrm{h}_1(\mathrm{jet})X$

For Spin-½ hadron production, two possibile cross sections :

Unpolarised hadron

$$\frac{d\sigma^{e^+e^-\to h_1(jet)X}}{d\cos\theta dz_1 d^2p_{\perp 1}} = \sum_{q_c} \frac{3e^4}{32\pi s} e_q^2 (1+\cos^2\theta)\hat{D}_{h/q}(z_1, p_{1\perp h_1})$$

Transversely polarised hadron

$$P_Y^{h_1}\frac{d\sigma^{e^+e^-\to h_1(jet)X}}{d\cos\theta dz_1 d^2p_{\perp 1}} = \sum_{q_c} \frac{3e^4}{32\pi s} e_q^2 (1+\cos^2\theta)\Delta D^{h_1}_{S_Y/q}(z_1, p_{1\perp h_1})$$

ratio →

Hadron polarisation

$$\mathcal{P}_T(z_1, p_{\perp 1}) = \frac{\sum_q e_q^2\, \Delta D_{h_1^\uparrow/q}(z_1, p_{\perp 1})}{\sum_q e_q^2\, D_{h_1/q}(z_1, p_{\perp 1})}$$

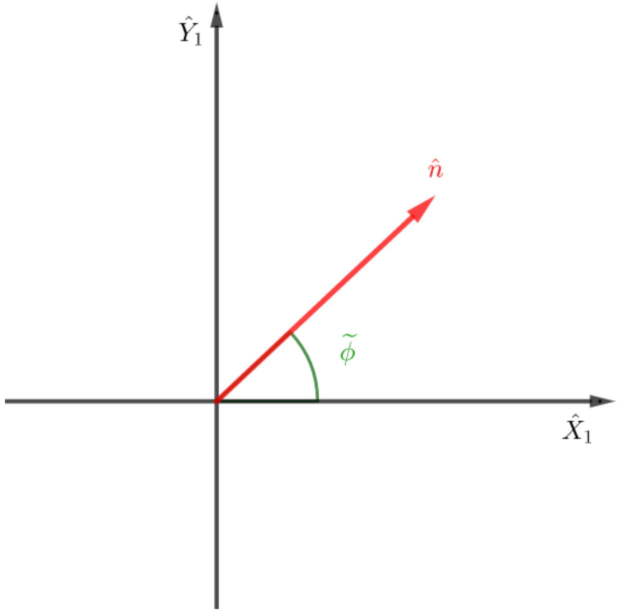
$$\hat{X}_{h_1} = \hat{Y}_{h_1} \times \hat{Z}_{h_1}$$

$$\hat{Y}_{h_1} = \frac{\hat{q}_1 \times P_{h1}}{|\hat{q}_1 \times P_{h1}|}$$

$$\hat{Z}_{h_1} = \frac{P_{h1}}{|P_{h1}|}$$

$$\mathcal{P}^{h_1} = P_x^{h_1} \hat{X}_1 + P_y^{h_1} \hat{Y}_1 + P_z^{h_1} \hat{Z}_1$$

$$\hat{n} = -\hat{P}_{h_2} \times \hat{P}_{h_1}$$



$$\mathcal{P}^{h_1} \cdot \hat{n} = P_x^{h_1} \cos \widetilde{\phi} + P_y^{h_1} \sin \widetilde{\phi}$$

$$\cos \widetilde{\phi} \simeq \frac{z_{h_1}}{z_{h_2}} \frac{p_{\perp 2}}{p_{\perp 1}} \sin(\phi_1 - \varphi_2)$$

$$\sin \widetilde{\phi} \simeq \frac{P_{1T}}{p_{\perp 1}} - \frac{z_{h_1}}{z_{h_2}} \frac{p_{\perp 2}}{p_{\perp 1}} \cos(\phi_1 - \varphi_2)$$