Factorisation in Colour Singlet Production

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Based on JHEP 1407 (2014) 110 (JG) and SciPost Phys. 3, 040 (2017) (Boer, van Daal, JG, Kasemets, Mulders)

FACTORISATION

Factorisation is a key tool to make predictions at the LHC.

For pp, several cases where factorisation has been shown to hold:

- **Collinear factorisation for DY**

$$\sigma = \int dx_A dx_B \hat{\sigma}_{ij\to X}(\hat{s} = x_A x_B s) f_i(x_A) f_j(x_B) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^4}\right)$$

- **TMD factorisation for DY**

$$W^{\mu\nu} = \frac{8\pi^2 s}{Q^2} \sum_f C_f^{\mu\nu}\left(\hat{k}_A, \hat{k}_B\right) \int d^2\mathbf{b}_T e^{i\mathbf{p}_T\cdot\mathbf{b}_T} \tilde{f}_f(x_A, \mathbf{b}_T; \zeta_A)\, \tilde{f}_{\bar{f}}(x_B, \mathbf{b}_T; \zeta_B) + \mathcal{O}\left(\frac{p_T^2}{Q^2}\right)$$

Bodwin Phys. Rev. 31 (1985) 2616, Collins, Soper, Sterman Nucl. Phys. B261 (1985) 104, Nucl. Phys. B308 (1988) 833, Collins, pQCD book

- **Collinear/TMD factorisation for 'double Drell-Yan'**

$$\sigma_{DPS}^{(A,B)} = \int dx_i dx'_i d^2\boldsymbol{y}\, \Phi^2(y\nu)\, F_{ik}(x_1, x_2, \boldsymbol{y}, \mu_A, \mu_B)\, F_{jl}(x'_1, x'_2, \boldsymbol{y}, \mu_A, \mu_B)\, \hat{\sigma}_{ij}^A(x_1 x'_1 s, \mu_A)\, \hat{\sigma}_{kl}^B(x_2 x'_2 s, \mu_B)$$

Diehl, JG, Ostermeier, Plößl, Schafer, JHEP 1601 (2016) 076, Diehl, JG, Schönwald, JHEP 1706 (2017) 083, Diehl, Ostermeier, Schafer, JHEP 1203 (2012) 089, Diehl, Nagar, JHEP 1904 (2019) 124, Vladimirov, JHEP 1804 (2018) 045 , Buffing, Diehl, Kasemets, JHEP 1801 (2018) 044

This talk: review **traditional QCD** ('CSS'-style) approach to factorisation, focussing on Glauber cancellation aspect.
Then discuss some model calculations with a few Glauber exchanges – illustrates why CSS Glauber cancellation works, when it fails, and where CSS proof could be improved.

LEADING REGIONS

Step 1: Consider arbitrary Feynman graph contributing to production of state of interest.

What **regions** of loop momenta can give **leading** contributions to cross section?

+ component - component T component

Hard: $k \sim Q(1,1,1)$

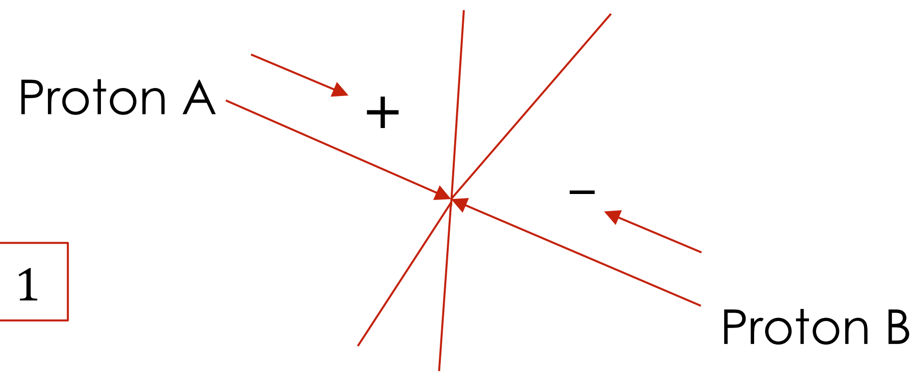
Collinear: $k \sim Q(1,\lambda^2,\lambda)$

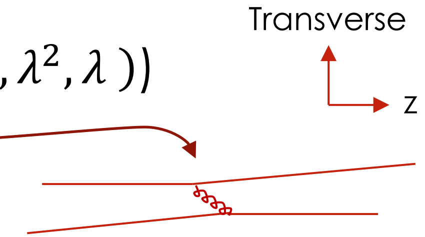
$\boxed{\lambda \ll 1}$

Soft: $k \sim Q(\lambda^n,\lambda^n,\lambda^n)$

Glauber: $|k^+k^-| \ll \boldsymbol{k}^2 \ll Q^2$

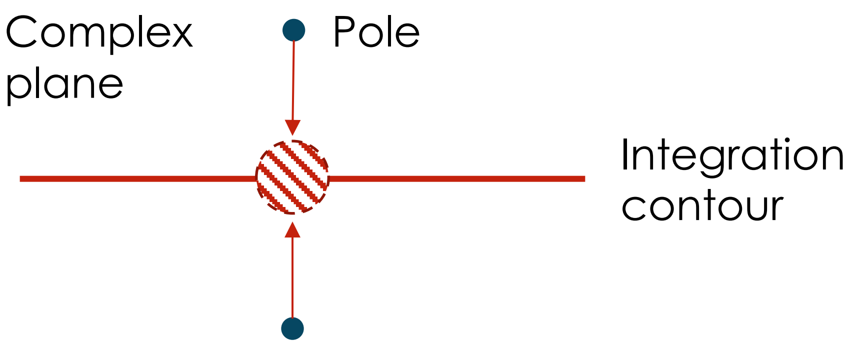
(e.g. $k \sim Q(\lambda^2,\lambda^2,\lambda\,)$)



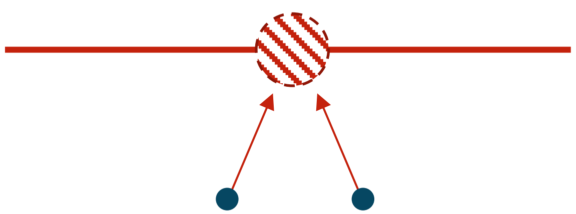


PINCH SINGULARITIES

More specifically, looking for **leading regions around pinch singularities**:



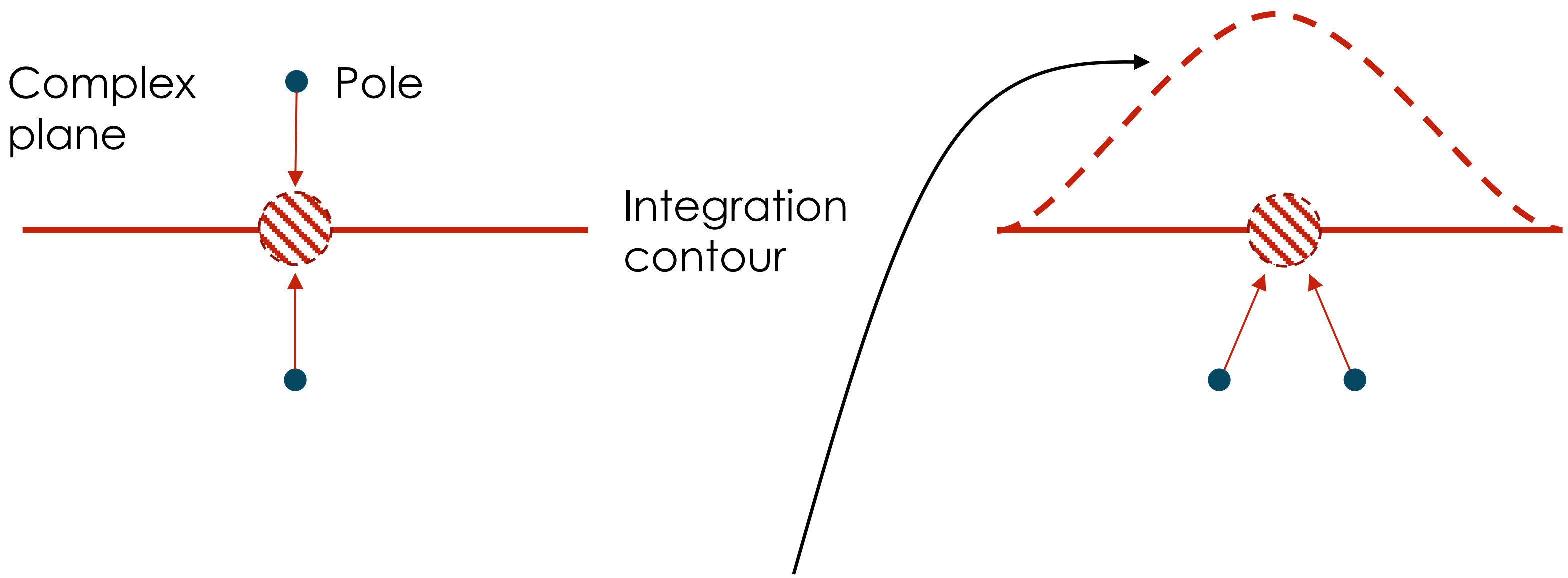
Pinch singularity



Non-pinched singularity

PINCH SINGULARITIES

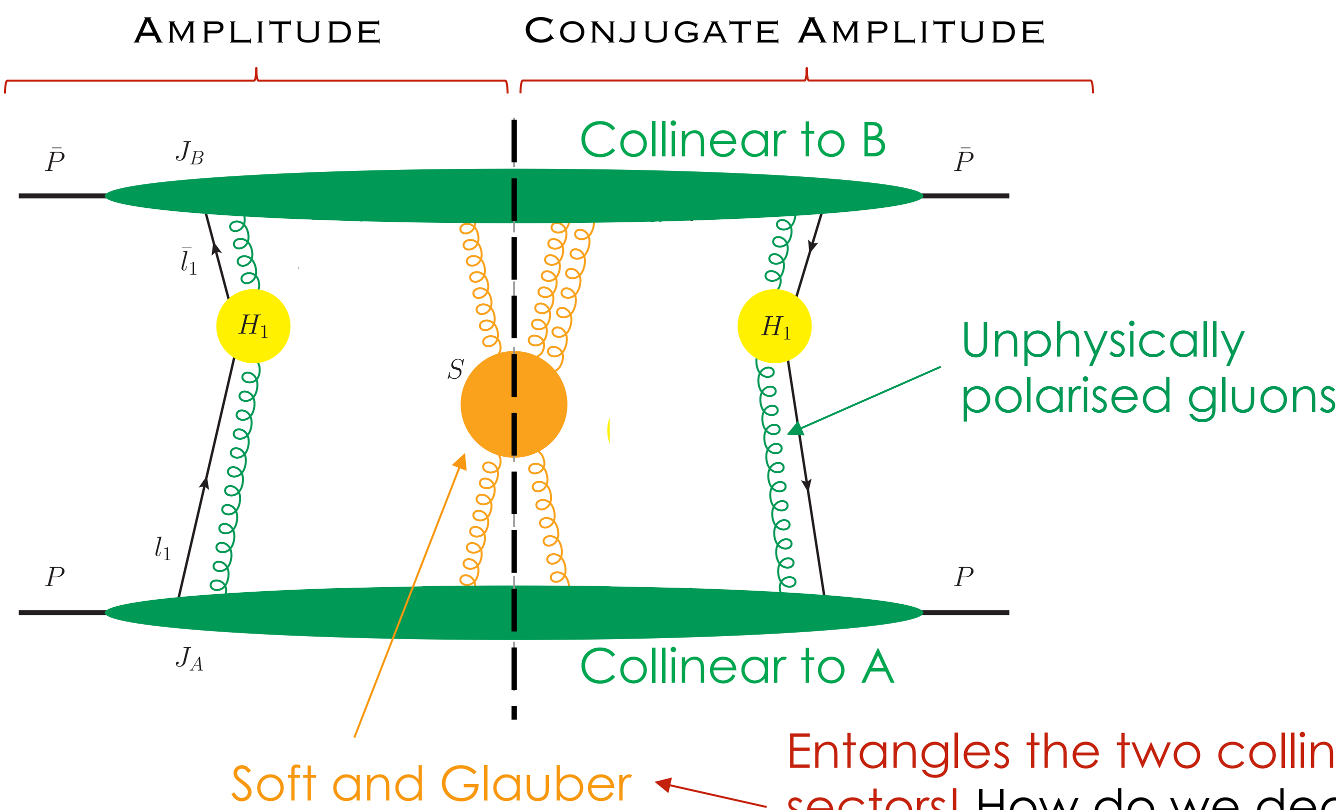
More specifically, looking for **leading regions around pinch singularities**:



If singularity is not pinched, can **deform contour in complex plane** away from poles into another momentum region
→ don't have to consider unpinched region explicitly.

LEADING REGIONS FOR DY

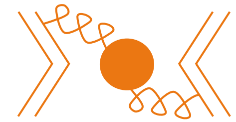
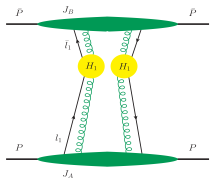
Leading region for TMD process:



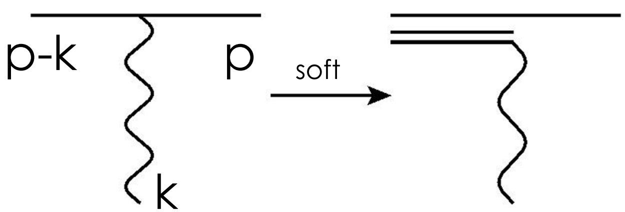
Entangles the two collinear sectors! How do we deal with these exchanges?

WARD IDENTITIES

If blob S only contained **central soft**, could strip attachments to collinear blobs using Ward identities:

 $\otimes$ 

Simple example:



Propagator denominator:

$$(p-k)^2 = -2p\cdot k + k^2 \overset{\text{soft}}{\rightarrow} -2p\cdot k$$

λ^n λ^{2n}

Eikonal piece

Same manipulation is **not possible** for Glauber gluons: → λ^2 λ^2

How do we get around this problem?

TREATING GLAUBERS

CSS strategy – try to show that pinched Glauber exchanges ‘**cancel**’. Works for DY, both TMD + collinear cross section.

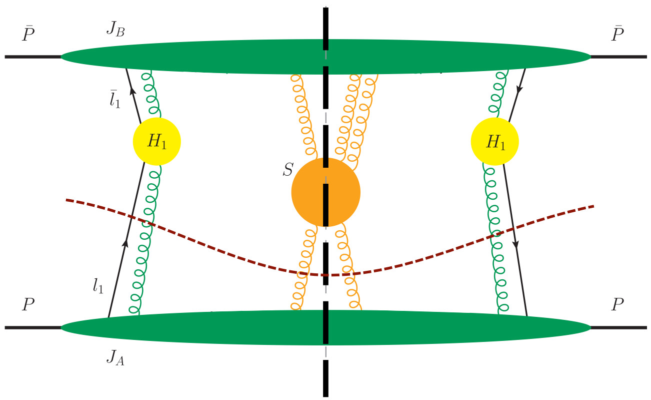
Bodwin Phys. Rev. 31 (1985) 2616, Collins, Soper, Sterman, Nucl. Phys. B308 (1988) 833, Collins, pQCD book

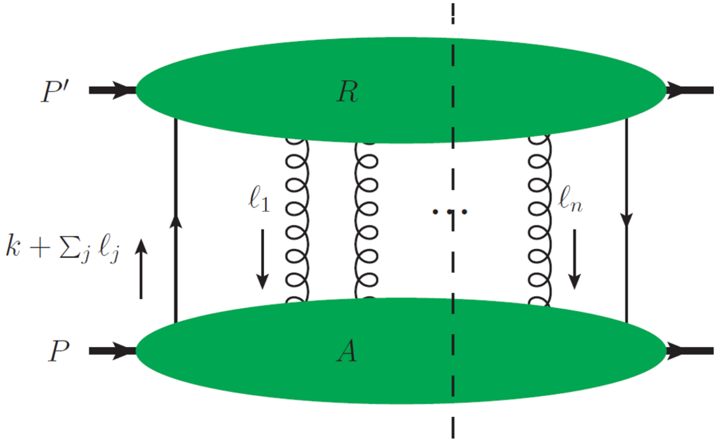
Then can **deform** integration contours away from Glauber region.

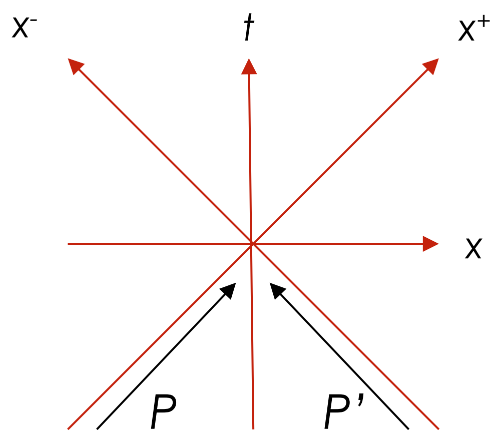
Let’s briefly review this.

CSS GLAUBER CANCELLATION PROOF

Partition graph into **collinear graph *A*** and **remainder *R***.







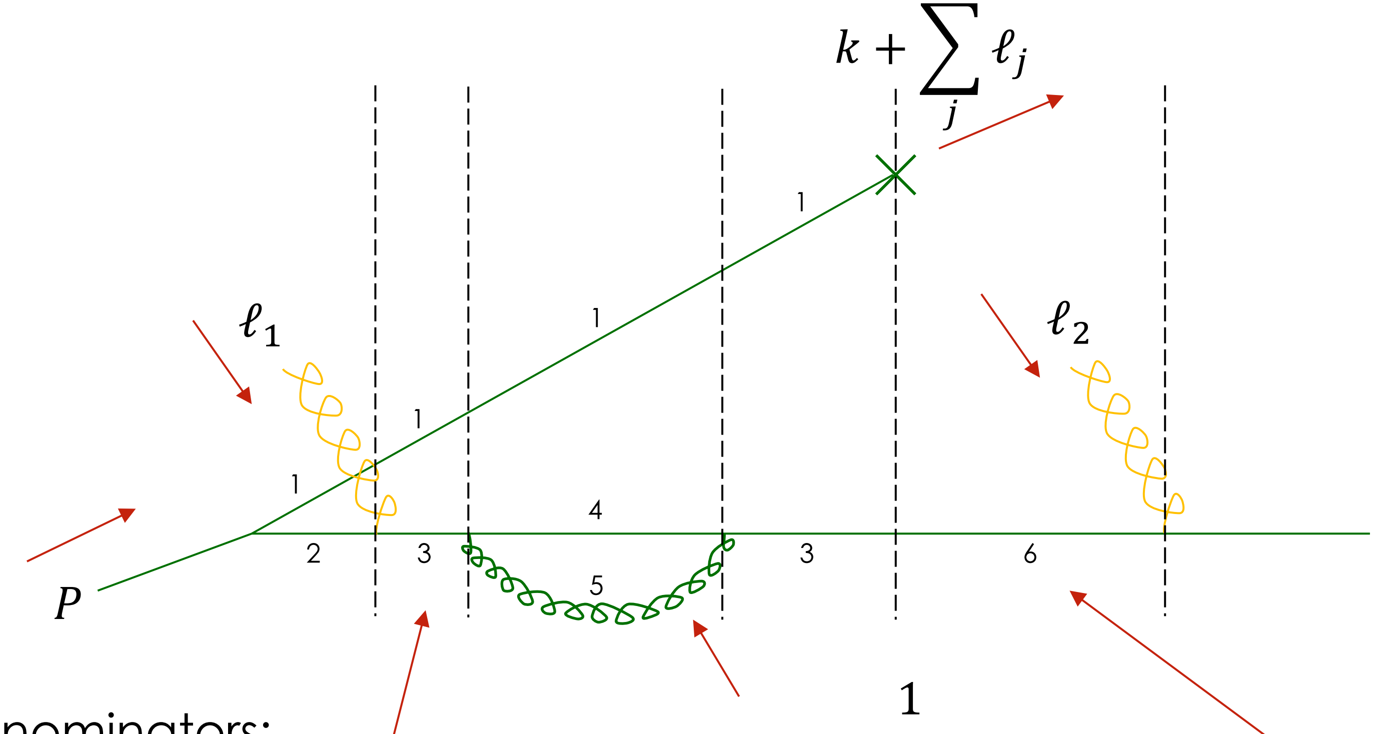
Consider A in x^+ ordered perturbation theory (**LCPT**)

LCPT

Sum over topologies + 'time' orderings. Have 'states' between vertices

At vertices, + and T conserved, - not, all lines on shell:

$$\kappa_L = \left(\kappa_L^+, \kappa_L^- = \frac{\boldsymbol{\kappa}_L^2 + m_L^2}{2\kappa_L^+}, \boldsymbol{\kappa}_L\right)$$



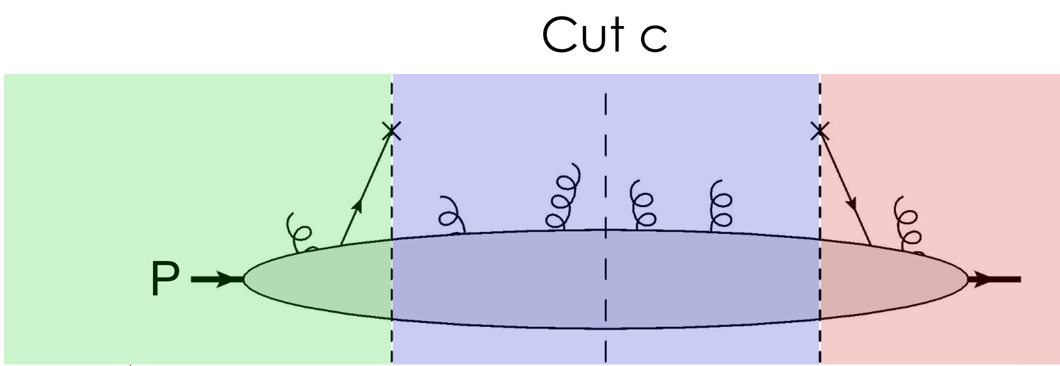
LC denominators:

$$\frac{1}{P^- + \ell_1^- - \kappa_1^- - \kappa_3^- + i\epsilon}$$

$$\frac{1}{P^- + \ell_1^- - \kappa_1^- - \kappa_4^- - \kappa_5^- + i\epsilon}$$

$$\frac{1}{P^- - k^- - \ell_2^- - \kappa_6^- + i\epsilon}$$

LCPT FOR COLLINEAR GRAPH



$$\prod_{\substack{\text{states } \xi\\ \xi<H}} \frac{1}{P^- + \sum\limits_{\substack{\text{vertices } j\\ j<\xi}} \ell_j^- - \sum\limits_{\substack{\text{lines } L\\ L\in\xi}} \kappa_L^- + i\epsilon}$$

$$\prod_{\substack{\text{states } \xi\\ H'<\xi}} \frac{1}{P^- - \sum\limits_{\substack{\text{vertices } j\\ j>\xi}} \ell_j^- - \sum\limits_{\substack{\text{lines } L\\ L\in\xi}} \kappa_L^- - i\epsilon}$$

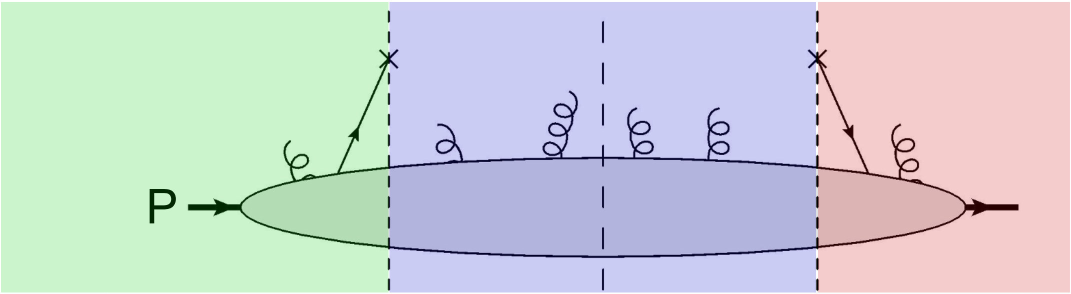
$$\sum_{F_A}\int_{-\infty}^{\infty}\frac{dk^-}{2\pi}F_T(k,\tilde{\ell}_j)$$

$$= \int_{-\infty}^{\infty}\frac{dk^-}{2\pi}\left\{\prod_{f=c+1}^{N}\frac{1}{P^- - k^- - \sum\limits_{j>f}\ell_j^- - D_f - i\epsilon}\right\}(2\pi)\delta\left(P^- - k^- - \sum_{j>c}\ell_j^- - D_c\right)\left\{\prod_{f=1}^{c-1}\frac{1}{P^- - k^- - \sum\limits_{j>f}\ell_j^- - D_f + i\epsilon}\right\}$$

ℓ_j^-s here are trapped at small values

CANCELLATION OF GLAUBER PINCHES

If remainder is independent of partioning of soft vertex attachments between $\mathcal{M}$ and $\mathcal{M}^*$, can sum over all cuts of A:



$$\sum_{F_A}\int_{-\infty}^{\infty}\frac{dk^-}{2\pi}F_T(k,\tilde{\ell}_j)$$

$$=\int_{-\infty}^{\infty}\frac{dk^-}{2\pi}\sum_{c=1}^{N}\left\{\prod_{f=c+1}^{N}\frac{1}{P^- - k^- - \sum_{j>f}\ell_j^- - D_f - i\epsilon}\right\}(2\pi)\delta\Big(P^- - k^- - \sum_{j>c}\ell_j^- - D_c\Big)\left\{\prod_{f=1}^{c-1}\frac{1}{P^- - k^- - \sum_{j>f}\ell_j^- - D_f + i\epsilon}\right\}$$

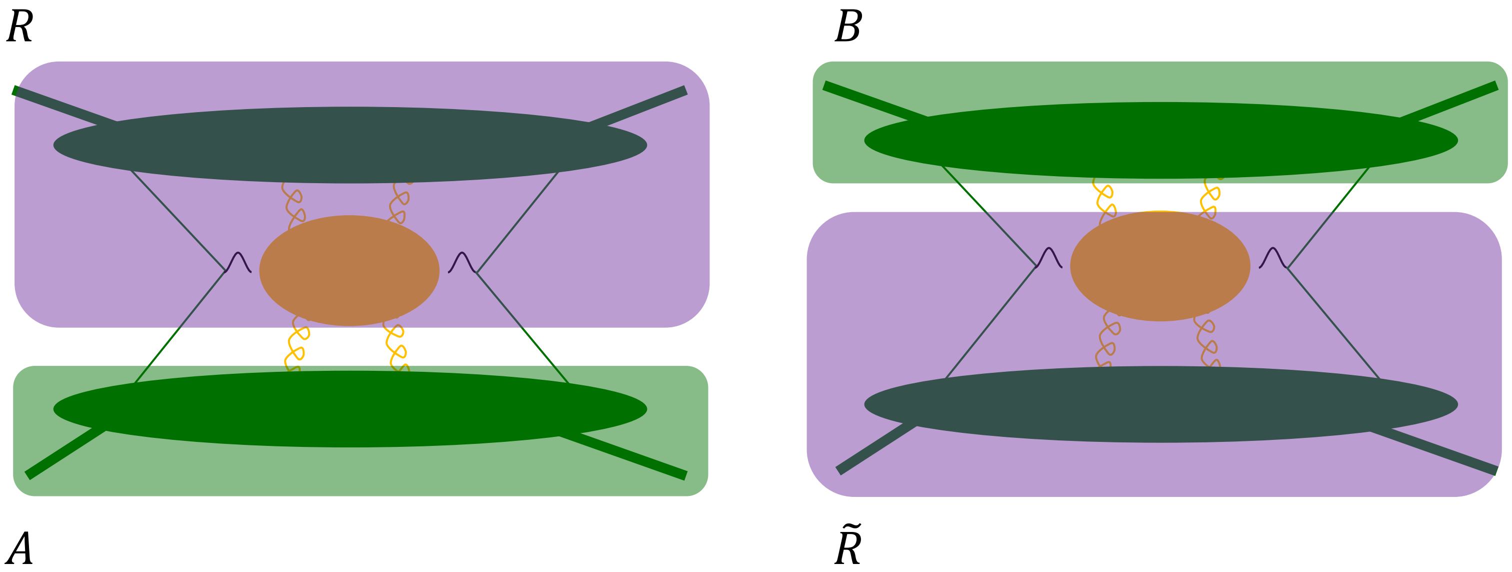
$$=\int_{-\infty}^{\infty}\frac{dk^-}{2\pi}\left\{i\prod_{f=1}^{N}\frac{1}{P^- - k^- - \sum_{j>f}\ell_j^- - D_f - i\epsilon} - i\prod_{f=1}^{N}\frac{1}{P^- - k^- - \sum_{j>f}\ell_j^- - D_f + i\epsilon}\right\}$$

$$=\begin{cases}1 & \text{if } N=1\\ 0 & \text{otherwise}\end{cases}$$

Cutkosky rules/ unitarity

Independence of R on partitioning is shown by studying R using x- ordered LCPT

CSS ARGUMENT: BOTH DIRECTIONS



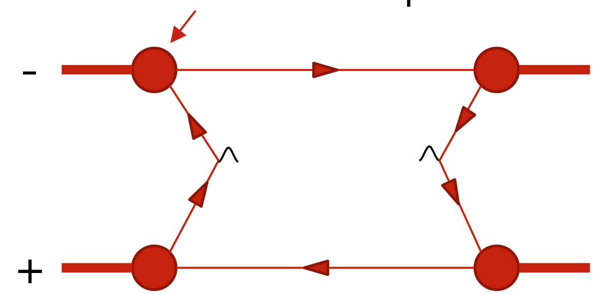
Could perform argument for both A and B: allows deformation of both ℓ_j^-s entering A and ℓ_j^+s entering B

Seemingly allows deformation of Glauber momenta into central soft region, or collinear to A/B.

UNITARITY CANCELLATION: ILLUSTRATION

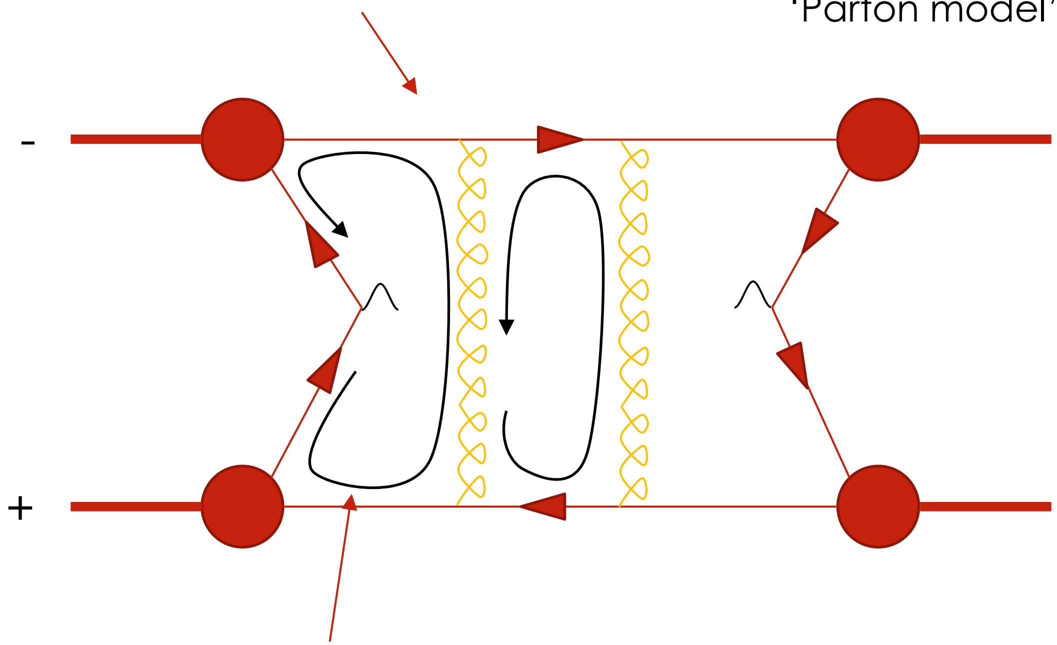
Can illustrate unitarity cancellation using a model calculation.

‘Meson’ composed from $q\bar{q}$



‘Parton model’ graph

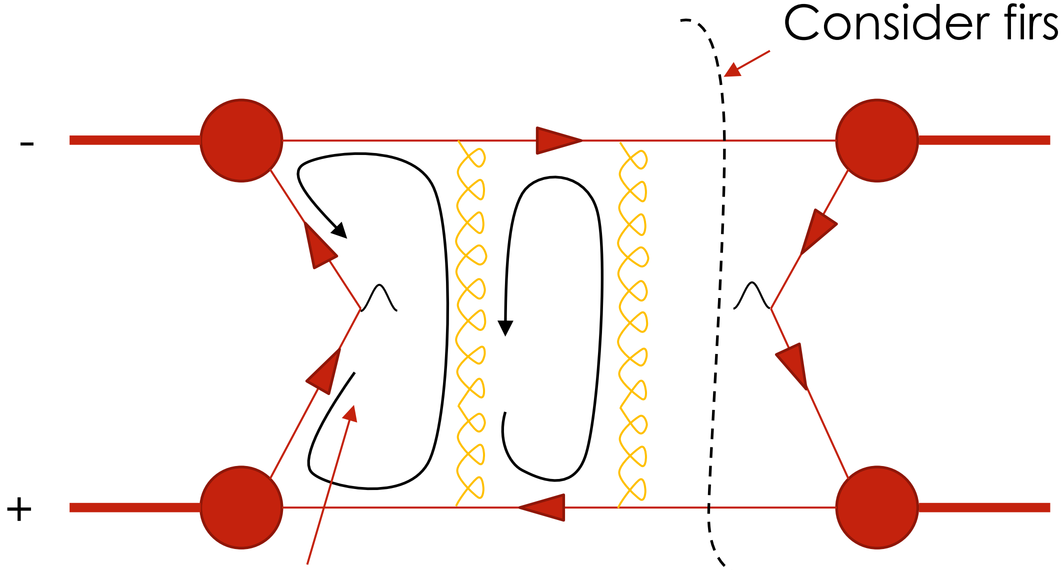
Consider two gluon exchanges between spectator particles



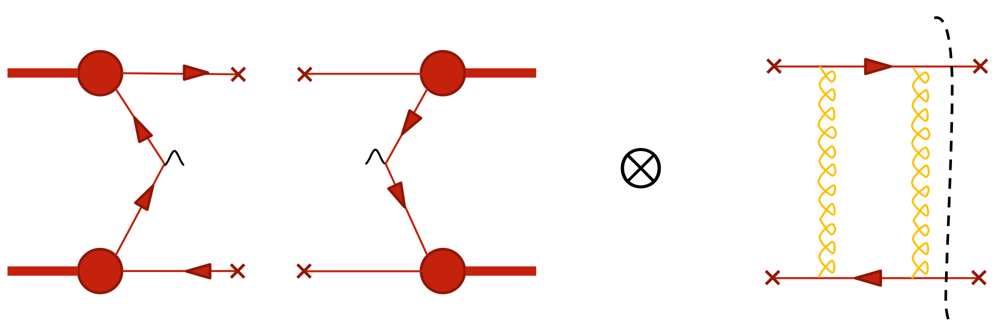
Plus and minus components of this loop are **trapped** in Glauber region

TWO GLAUBER EXCHANGES

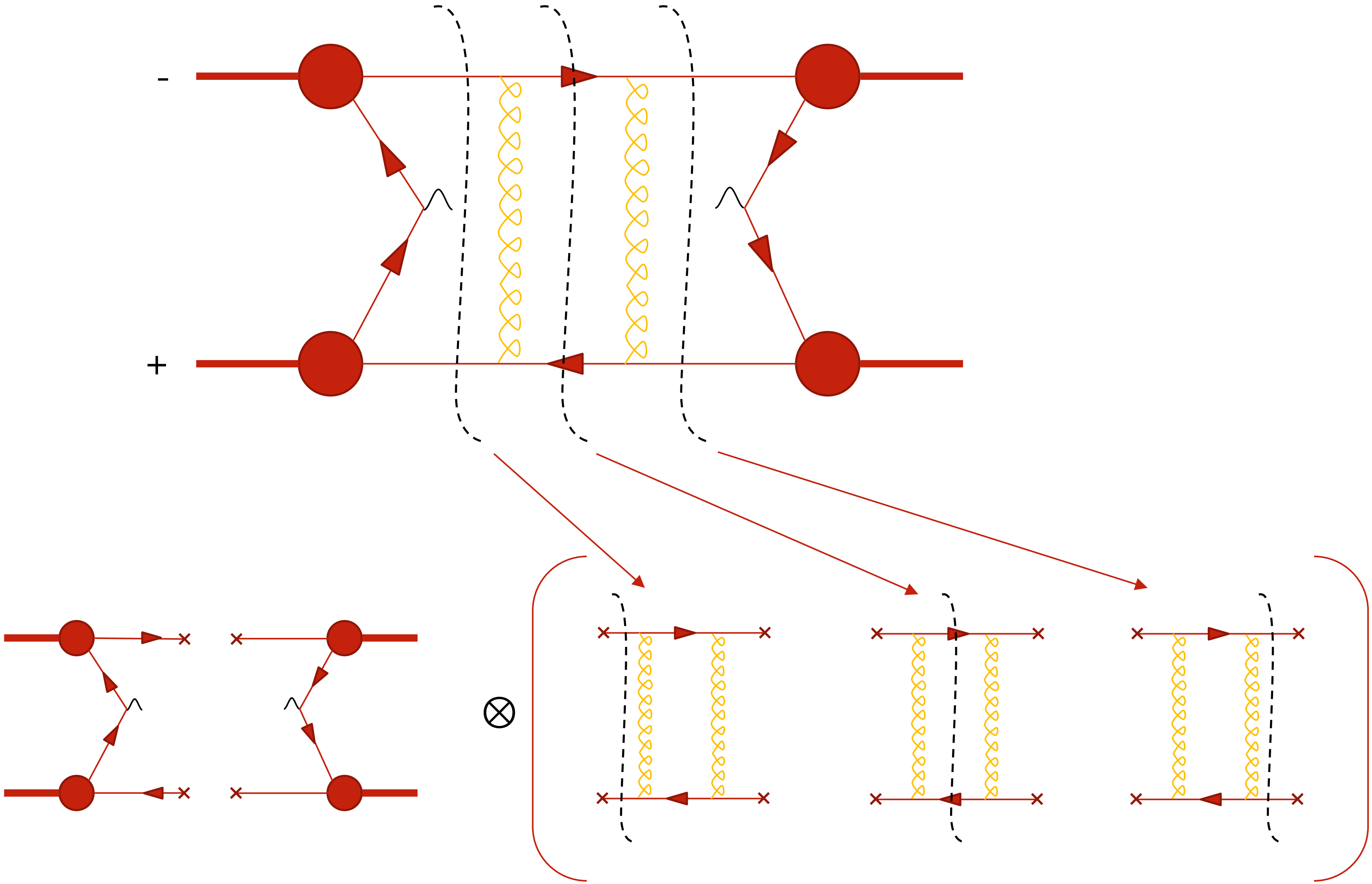
Consider first cut here



Perform + and – integrations of this loop momentum, close on poles of spectator legs



TWO GLAUBER EXCHANGES

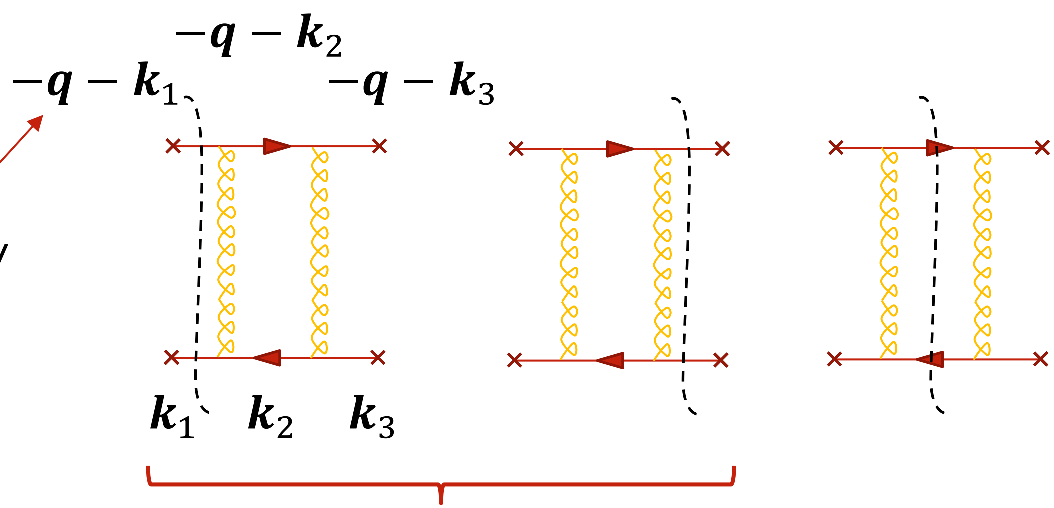


TWO GLAUBER EXCHANGES: UNITARITY CANCELLATION

Consider case where we measure p_T of V. For given momenta in the three decomposed graphs, the value of the measurement is **the same**.

➔ can **factor out the parton model graph and measurement** and add together the Glauber subgraphs:

$\boldsymbol{q}$ = p_T of V



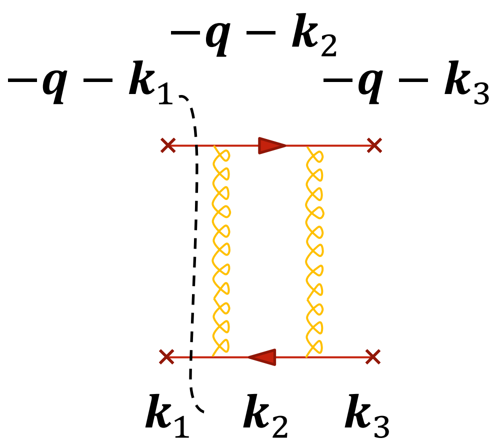
- Imaginary part + sum over internal cuts = 0

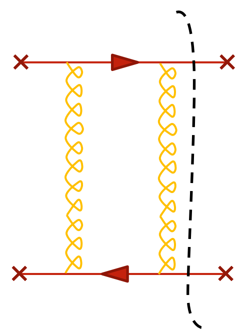
Cutkosky rules

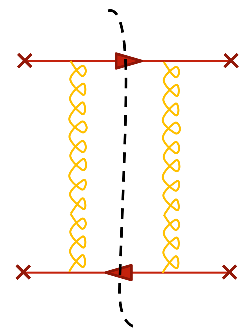
HADRONIC TRANSVERSE ENERGY

Consider instead measuring E_T:

$$E_T = \sum_{i \in X} \sqrt{\mathbf{p}_{Ti}^2 + m_i^2}$$







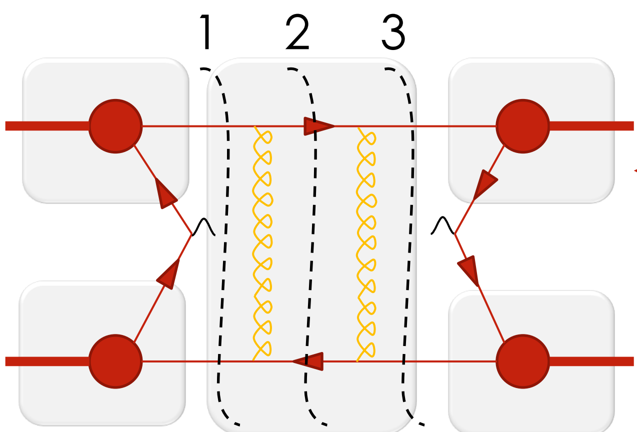
$E_T = |\boldsymbol{k}_1| + |\boldsymbol{q} + \boldsymbol{k}_1|$ $E_T = |\boldsymbol{k}_3| + |\boldsymbol{q} + \boldsymbol{k}_3|$ $E_T = |\boldsymbol{k}_2| + |\boldsymbol{q} + \boldsymbol{k}_2|$

CSS-style cancellation does not happen – factorisation breaking at two Glauber exchange level?

TWO GLAUBER EXCHANGES FOR E_T

For perturbative regime: i.e. 'meson' $\to q\bar{q}$ vertex is $g \to q\bar{q}$ vertex, neglect quark/hadron masses, **cancellation still occurs at this level**

Schwartz, Yan, Zhu, Phys. Rev. D 97, 096017 (2018)



In all of these pieces, a 'pure transverse' piece can be factored out

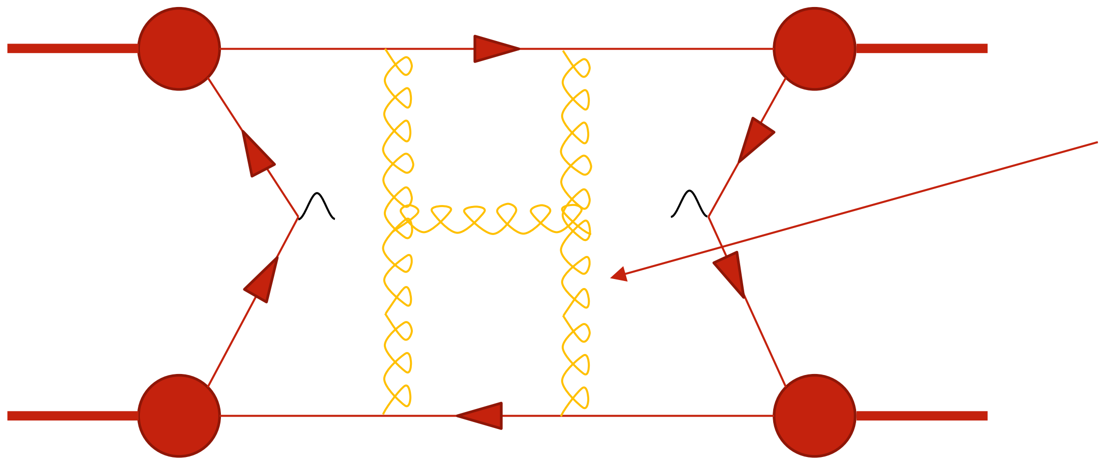
$$\text{Graph} = \int \frac{d^2q}{|\boldsymbol{q}|^2} \prod_i \frac{d^2\boldsymbol{k}_i}{|\boldsymbol{q}|^2} f\left(\frac{\boldsymbol{k}_i}{|\boldsymbol{q}|}, \frac{\boldsymbol{q}}{|\boldsymbol{q}|}\right)$$

$E_{T,1} = |\boldsymbol{k}_1| + |\boldsymbol{q} + \boldsymbol{k}_1|$ $\quad E_{T,2} = |\boldsymbol{k}_2| + |\boldsymbol{q} + \boldsymbol{k}_2|$ $\quad E_{T,3} = |\boldsymbol{k}_3| + |\boldsymbol{q} + \boldsymbol{k}_3|$

Rescale all momenta by $E_{T,1}/E_{T,i}$ - brings all measurements to same value, allows cancellation

FACTORISATION VIOLATION FOR E_T

This cancellation for E_T, unlike that for p_T, is rather **delicate** – relies on no scales appearing in factorized transverse piece

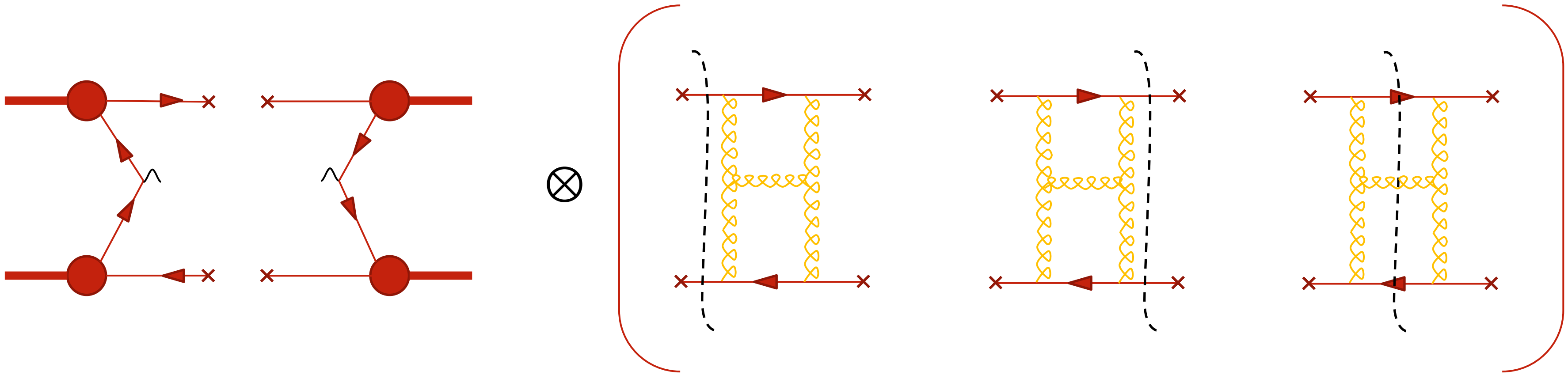


At next order this picture will be broken e.g. by this type of diagram (**ladder allows + and – sectors to 'communicate'**)

N.B. scaling argument for cancellation of 2-Glauber exchange **only works for single scale observables**. For **two-scale observable** (beam thrust differential in two hemispheres), explicit **factorisation violation** due to 2-Glauber exchange shown in Zeng, JHEP 10 (2015) 189

FACTORISATION VIOLATION AND MPI

Can view factorisation violating diagrams in the following way:



Primary process

Secondary low scale absorptive process

Secondary low scale scattering

These factorisation breaking effects are related to **additional low-scale scatters, or multiple parton interactions (MPI)**.

Glauber cancellation for the p_T observable works because this is **insensitive** to whether extra scatters occurred or not.

EXPLICIT FACTORIZATION STUDY

Explicit study of factorisation at work at the two gluon exchange level:
SciPost Phys. 3, 040 (2017) (Boer, van Daal, JG, Kasemets, Mulders)

Studied **azimuthal-angle-dependent part of TMD cross section** in Drell-Yan

$$\frac{d^6\sigma}{d\Omega\, dx_1 dx_2\, d^2\boldsymbol{q}} = \frac{\alpha^2}{N_c\, q^2} \sum_q e_q^2 \left\{ A(\theta)\, \mathcal{F}\left[f_1 \bar{f}_1\right] + B(\theta)\, \cos(2\phi)\, \mathcal{F}\left[w(\boldsymbol{k}_1, \boldsymbol{k_2})\, h_1^{\perp} \bar{h}_1^{\perp}\right] \right\},$$

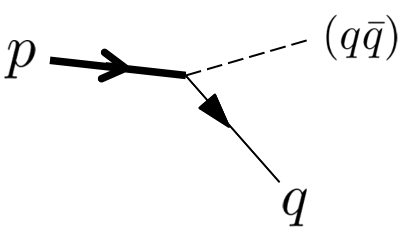
Unpolarised TMD

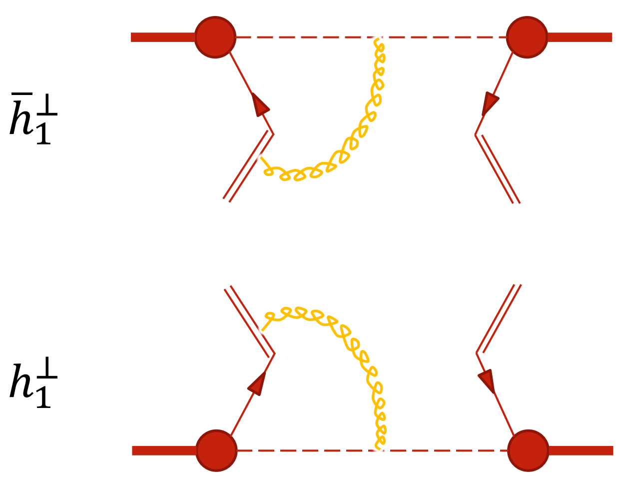
Collins-Soper angles

Boer-Mulders TMD
(measures correlation between quark transverse spin and transverse mtm)

MODEL

Model calculation: each hadron is a massive spin ½ particle than can split into a massless spin ½ quark and a massive scalar 'diquark' via a Yukawa-type interaction.





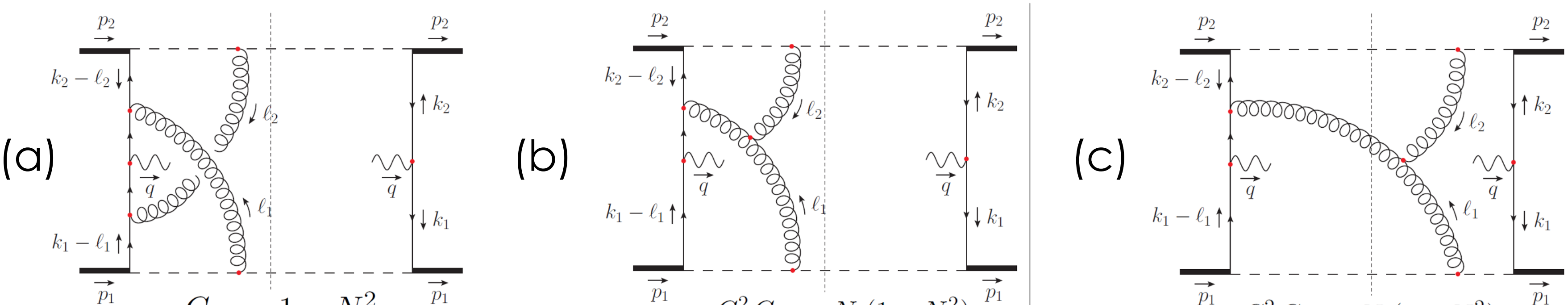
Leading contribution to factorised formula requires a **gluon exchange in each BM function**

Region of gluon momentum that contributes in $h_1^\perp$, $\bar{h}_1^\perp$ is **exactly Glauber region**

Colour factor of leading term in factorised formula = $(C_A C_F)^2/N_c$

CALCULATION AT 2G EXCHANGE LEVEL

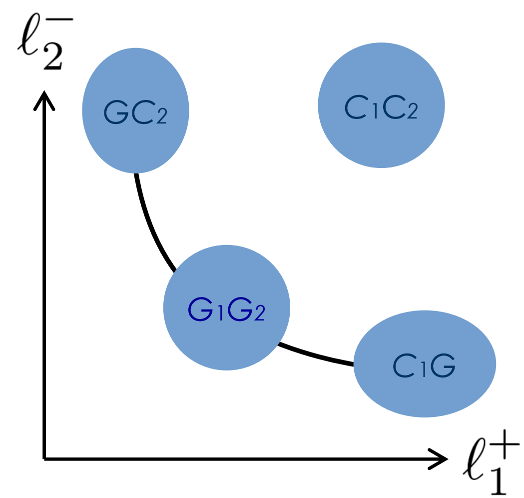
Compare to ab initio leading power cross section calculation. **Key diagrams**:



(a) $-\dfrac{C_F}{2} = \dfrac{1-N_c^2}{4N_c}$ (b) $-\dfrac{C_A^2 C_F}{2} = \dfrac{N_c(1-N_c^2)}{4}$ (c) $-\dfrac{C_A^2 C_F}{2} = \dfrac{N_c(1-N_c^2)}{4}$

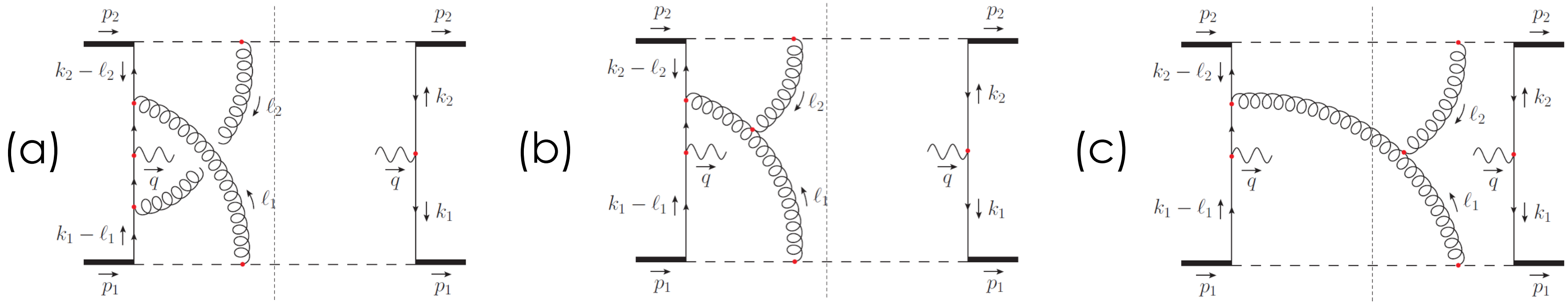
Full calculation difficult. Split into **leading power regions**

$G : (\lambda^2, \lambda^2, \lambda)Q$	Central Glauber
$G_1 : (\lambda, \lambda^2, \lambda)Q$	Left-moving Glauber
$G_2 : (\lambda^2, \lambda, \lambda)Q$	Right-moving Glauber
$C_1 : (1, \lambda^2, \lambda)Q$	Left-moving collinear
$C_2 : (\lambda^2, 1, \lambda)Q$	Right-moving collinear



CALCULATION: G1G2 REGION

Let's look in detail at **G_1G_2 region** calculation



$$d\sigma_{\rm BM} \propto C_{(a)}(I_{(a)} - N_C^2(I_{(b)} + I_{(c)}))$$

where $I_{(n)}$ are the integrals over larger (λ scaling) lightcone components :

$$I_{\rm (a)} \equiv \int \frac{d\ell_1^+}{2\pi}\,\frac{\nu^{\eta_1}|\ell_1^+|^{-\eta_1}}{\ell_1^+ + i\epsilon}\int \frac{d\ell_2^-}{2\pi}\,\frac{\nu^{\eta_2}|\ell_2^-|^{-\eta_2}}{\ell_2^- + i\epsilon}.$$

Just **initial-state poles** in ℓ_1^+, ℓ_2^-

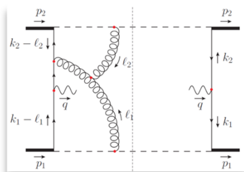
$$I_{\rm (b)} \equiv \int \frac{d\ell_1^+}{2\pi}\,\frac{\nu^{\eta_1}|\ell_1^+|^{-\eta_1}}{\ell_1^+ + i\epsilon}\int \frac{d\ell_2^-}{2\pi}\,\frac{2\ell_1^+\nu^{\eta_2}|\ell_2^-|^{-\eta_2}}{2\ell_1^+\ell_2^- - (\boldsymbol{\ell}_1 + \boldsymbol{\ell}_2)^2 + i\epsilon}.$$

Initial- and final-state poles

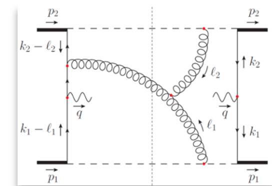
$$I_{\rm (c)} \equiv 4\pi i \int \frac{d\ell_1^+}{2\pi}\,\frac{\nu^{\eta_1}|\ell_1^+|^{-\eta_1}}{\ell_1^+ + i\epsilon}\int \frac{d\ell_2^-}{2\pi}\,\theta(-\ell_1^+)\,\ell_1^+\,\delta[2\ell_1^+\ell_2^- - (\boldsymbol{\ell}_1 + \boldsymbol{\ell}_2)^2]\,\nu^{\eta_2}|\ell_2^-|^{-\eta_2}.$$

CALCULATION: G1G2 REGION

Now:



$$I_{(\mathrm{b})} \equiv \int \frac{d\ell_1^+}{2\pi} \frac{\nu^{\eta_1}|\ell_1^+|^{-\eta_1}}{\ell_1^+ + i\epsilon} \int \frac{d\ell_2^-}{2\pi} \frac{2\ell_1^+ \nu^{\eta_2}|\ell_2^-|^{-\eta_2}}{2\ell_1^+\ell_2^- - (\boldsymbol{\ell}_1 + \boldsymbol{\ell}_2)^2 + i\epsilon}.$$



$$I_{(\mathrm{c})} \equiv 4\pi i \int \frac{d\ell_1^+}{2\pi} \frac{\nu^{\eta_1}|\ell_1^+|^{-\eta_1}}{\ell_1^+ + i\epsilon} \int \frac{d\ell_2^-}{2\pi}\, \theta(-\ell_1^+)\, \ell_1^+\, \delta[2\ell_1^+\ell_2^- - (\boldsymbol{\ell}_1 + \boldsymbol{\ell}_2)^2]\, \nu^{\eta_2}|\ell_2^-|^{-\eta_2}.$$

Final-state poles cancel **(ensured by unitarity!)**

$$\int \frac{d\ell_1^+}{2\pi} \frac{\nu^{\eta_1}|\ell_1^+|^{-\eta_1}}{\ell_1^+ + i\epsilon} \int \frac{d\ell_2^-}{2\pi} \frac{\nu^{\eta_2}|\ell_2^-|^{-\eta_2}}{\ell_2^- + i\epsilon} = I_{(a)}$$

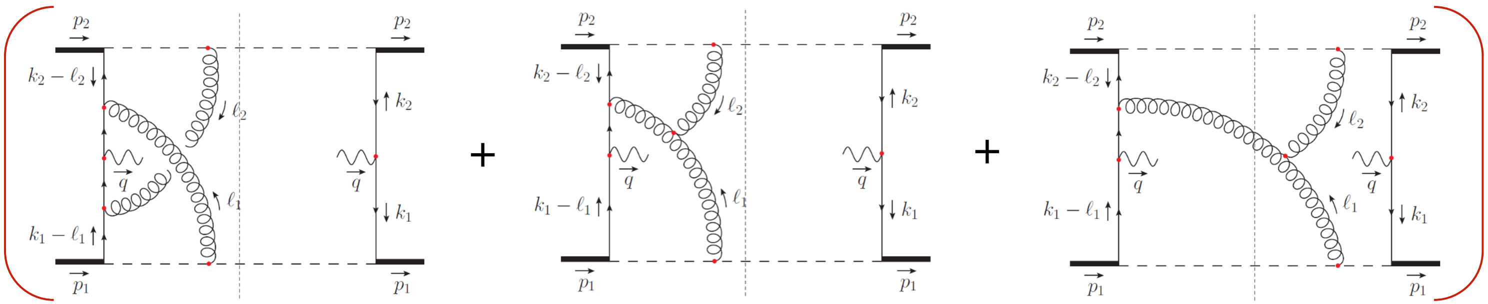
$$d\sigma_{\mathrm{BM}} \propto \overbrace{C_{(a)}(1 - N_C^2)}\, I_{(a)}$$

Restores colour factor predicted by factorisation! **(ensured by Ward identities!)**

CALCULATION: G1G2 REGION

All regions except for G_1G_2 turn out to give 0, &:

G_1G_2



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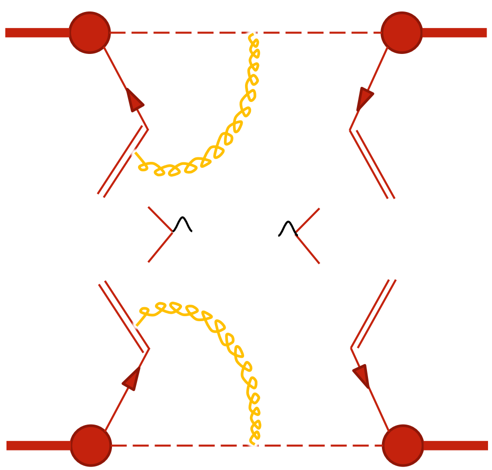
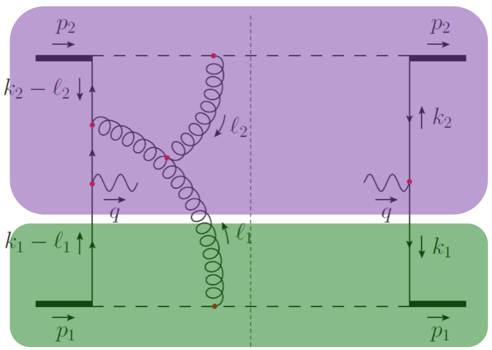


Double Glauber contribution absorbed into collinear functions!

CALCULATION IN CONTEXT OF CSS PROOF

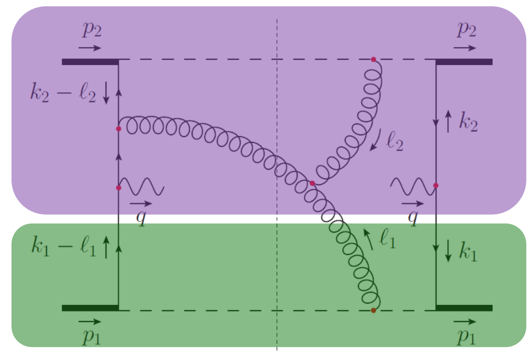
R

(b)



A

(c)



Can we use CSS argument with this partitioning, show that ℓ_1^- is not trapped? Would be needed to deform ℓ_1 into soft region

For A the argument goes through…

CALCULATION IN CONTEXT OF CSS PROOF

However, for R the argument **fails**, as R does depend on partitioning of soft vertices in A.

Culprit seems to be **numerator factor** coming from 3g vertex:

$$I_{(\mathrm{b})} \equiv \int \frac{d\ell_1^+}{2\pi} \frac{\nu^{\eta_1}|\ell_1^+|^{-\eta_1}}{\ell_1^+ + i\epsilon} \int \frac{d\ell_2^-}{2\pi} \frac{2\ell_1^+ \nu^{\eta_2}|\ell_2^-|^{-\eta_2}}{2\ell_1^+\ell_2^- - (\boldsymbol{\ell}_1 + \boldsymbol{\ell}_2)^2 + i\epsilon}.$$

$$I_{(\mathrm{c})} \equiv 4\pi i \int \frac{d\ell_1^+}{2\pi} \frac{\nu^{\eta_1}|\ell_1^+|^{-\eta_1}}{\ell_1^+ + i\epsilon} \int \frac{d\ell_2^-}{2\pi}\, \theta(-\ell_1^+)\ell_1^+ \delta[2\ell_1^+\ell_2^- - (\boldsymbol{\ell}_1 + \boldsymbol{\ell}_2)^2]\, \nu^{\eta_2}|\ell_2^-|^{-\eta_2}.$$

If these numerators were removed, could perform ℓ_1^+ using contours and R would be the same for the two cuts (& we'd have $I_{(b)} + I_{(c)} = 0$)

N.B. CSS cancellation proof does not consider numerator explicitly.

Highlights room for improvement in CSS proof – desirable to have a better proof that **describes which Glaubers can be deformed into soft, which into collinear.**

- Factorisation works for the total & TMD cross sections for colour singlet production. Reviewed CSS proof of cancellation of Glauber pinches. Argument is based on unitarity.

- CSS-style argument fails for hadronic transverse energy E_T. Breakdown of standard factorisation for E_T (and similar event shapes in pp) – requires at least two Glaubers exchanged.

- Showed explicitly that factorisation works for azimuthally-dependent part of TMD DY cross-section at two-loops in a model. Highlights a key point where CSS proof could be improved.