

# Discussion session: fragmentation

Yiannis Makris - Andrea Signori

INFN Pavia - University of Pavia & Jefferson Lab

REF 2020 workshop

December 9, 2020



Istituto Nazionale di Fisica Nucleare



# Outline

## Topics

The Belle TMD-thrust data

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We begin with **short contributions** (5 min each) to **kick-start the discussion** (intro + 3 **Theoretical** + 2 **Experimental** talks):

- ▶ The Belle TMD-thrust data

1902.01552 - PRD

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- T Factorization of  $e^+e^- \rightarrow H X$  cross section, differential in  $z_h$ ,  $P_T$  and thrust in the 2-jet limit

M. Boglione, A. Simonelli - 2011.07366

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**E** What's next in Belle (II), CLAS12, ...

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**E** Jets and jet-substructure observables

J. Osborn

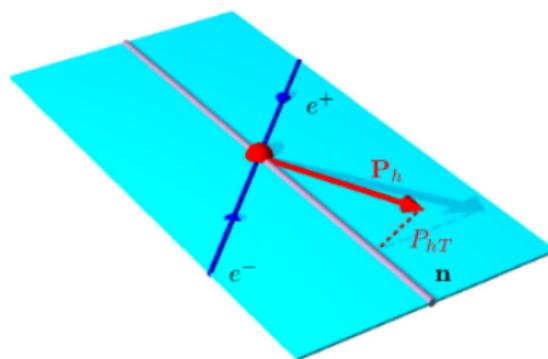
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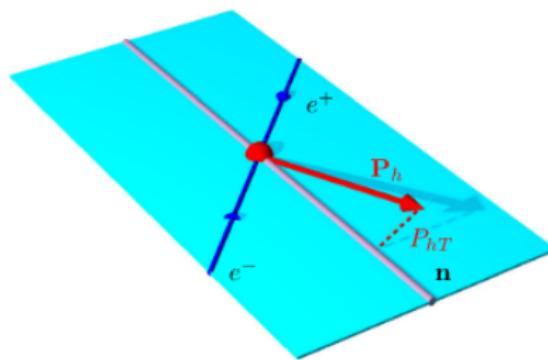
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R. Seidl et al. [1902.01552 - PRD](#)  
and supplemental material



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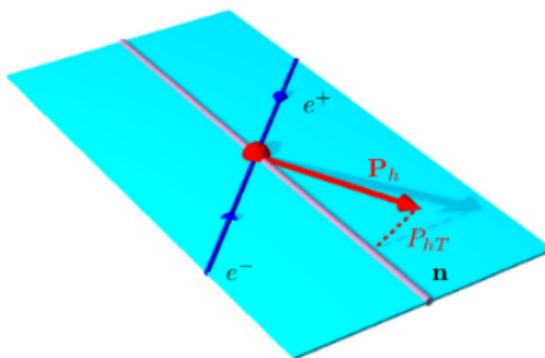
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Transverse-momentum-dependent (TMD) single-hadron fragmentation:

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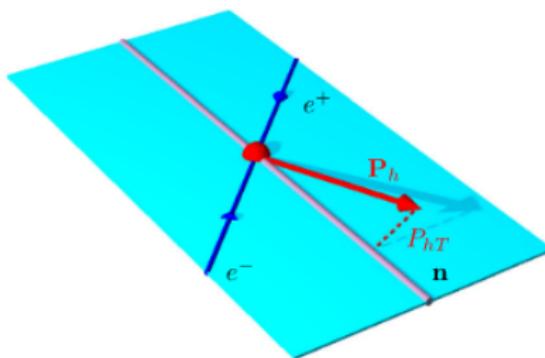


Transverse-momentum-dependent (TMD) single-hadron fragmentation:

- ▶ final state hadron:  $h$

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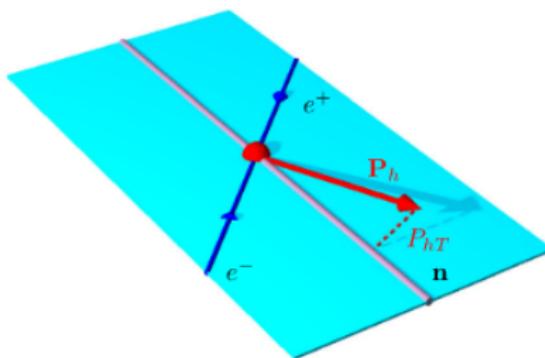


Transverse-momentum-dependent (TMD) single-hadron fragmentation:

- ▶ final state hadron:  $h$
- ▶ incoming leptons:  $e^+e^-$

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R. Seidl et al. [1902.01552 - PRD](#)  
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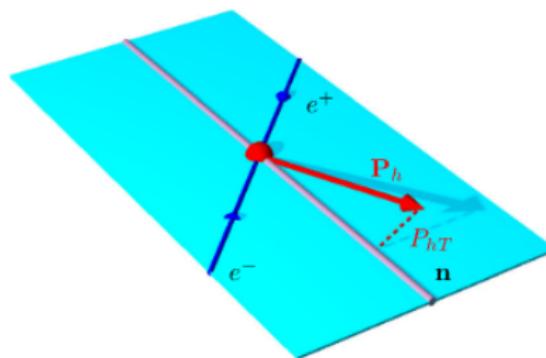


Transverse-momentum-dependent (TMD) single-hadron fragmentation:

- ▶ final state hadron:  $h$
- ▶ incoming leptons:  $e^+e^-$
- ▶ thrust axis:  $\vec{n}$

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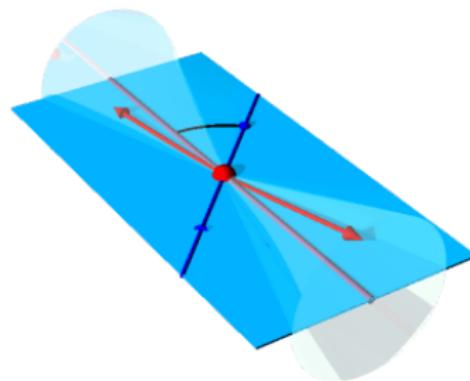


Transverse-momentum-dependent (TMD) single-hadron fragmentation:

- ▶ final state hadron:  $h$
- ▶ incoming leptons:  $e^+e^-$
- ▶ thrust axis:  $\vec{n}$
- ▶ the hadronic transverse momentum  $\vec{P}_{hT}$  is measured relative to the thrust axis  $\vec{n}$

# Thrust

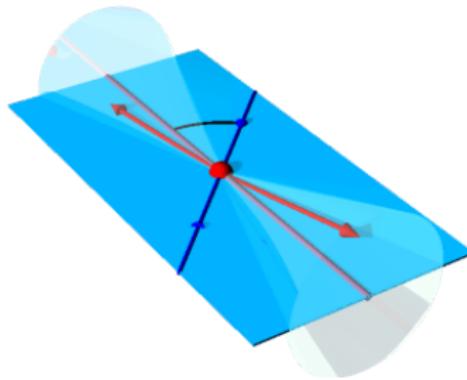
R. Seidl et al. [1902.01552 - PRD](#)  
[and supplemental material](#)



The axis  $\vec{n}$  such that  $T = \max_{\vec{n}} \frac{\sum_i \vec{p}_i \cdot \vec{n}}{\sum_i \vec{p}_i}$

# Thrust

R. Seidl et al. [1902.01552 - PRD](#)  
[and supplemental material](#)

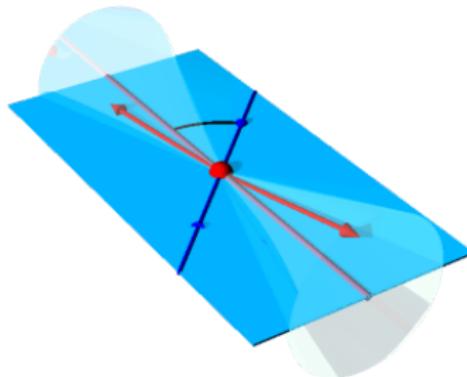


$$\text{The axis } \vec{n} \text{ such that } T = \max_{\vec{n}} \frac{\sum_i \vec{p}_i \cdot \vec{n}}{\sum_i \vec{p}_i}$$

- ▶ the axis that maximizes the sum of the longitudinal momentum components

# Thrust

R. Seidl et al. [1902.01552 - PRD](#)  
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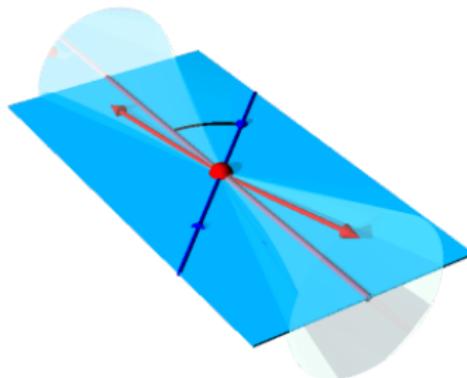


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- ▶ the axis that maximizes the sum of the longitudinal momentum components
- ▶  $T \rightarrow 1/2$  isotropic event

# Thrust

R. Seidl et al. [1902.01552 - PRD](#)  
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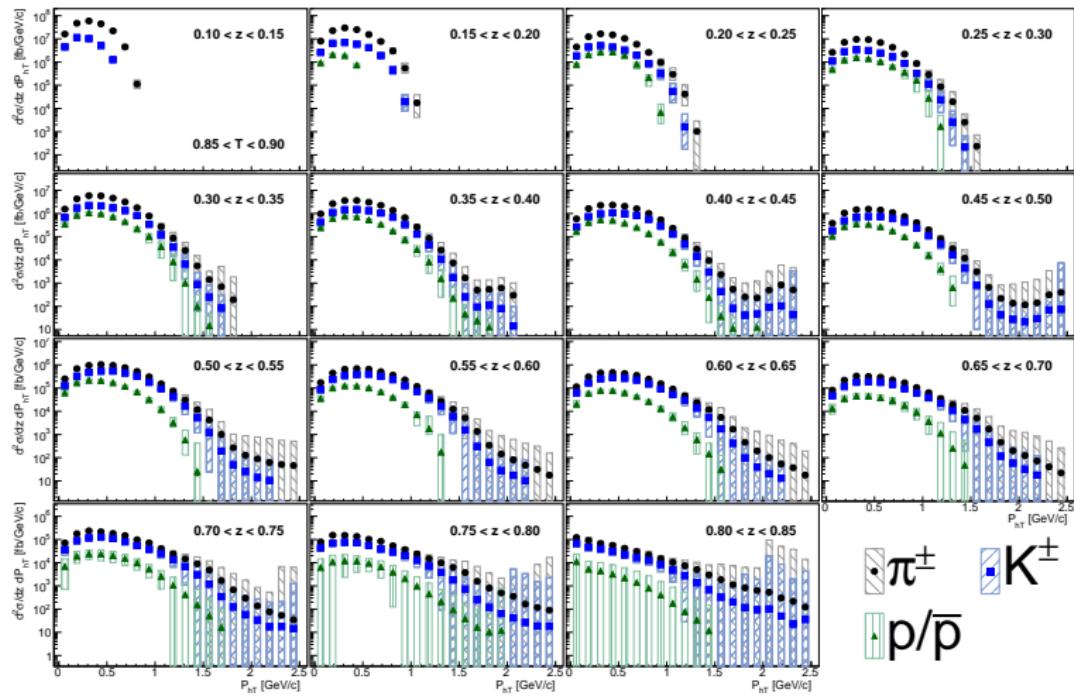


$$\text{The axis } \vec{n} \text{ such that } T = \max_{\vec{n}} \frac{\sum_i \vec{p}_i \cdot \vec{n}}{\sum_i \vec{p}_i}$$

- ▶ the axis that maximizes the sum of the longitudinal momentum components
- ▶  $T \rightarrow 1/2$  isotropic event
- ▶  $T \rightarrow 1$  back-to-back event

# $e^+e^- \rightarrow hX$ at Belle

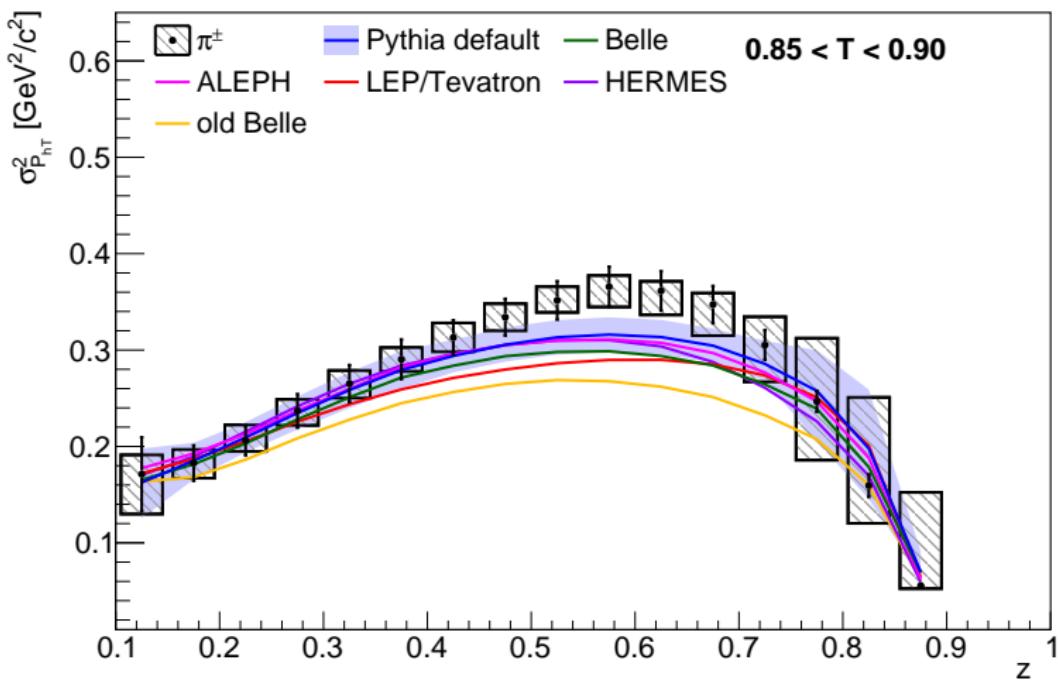
R. Seidl et al. 1902.01552 - PRD  
and supplemental material



Differential cross section  $d^2\sigma/dz dP_{hT}$  in bins of  $z$  and thrust  $0.85 < T < 0.90$

# $e^+e^- \rightarrow hX$ at Belle

R. Seidl et al. 1902.01552 - PRD  
and supplemental material



Gaussian widths in  $P_{hT}$  for the cross section as a function of  $z$   
compared to various PYTHIA tunes (pions)



College | Physical Sciences  
Mani L. Bhaumik Institute  
for Theoretical Physics



# Factorization and resummation on single hadron TMD with the thrust axis

DingYu Shao  
UCLA

REF2020, online  
Dec 7-11 , 2020

Based on 2007.14425 with Zhong-bo Kang and Fanyi Zhao

**Case-I:**  $e^- e^+ \rightarrow h_1 h_2 + X$

**Global observable, standard TMD factorization**

**Case-II:**  $e^- e^+ \rightarrow h + X$

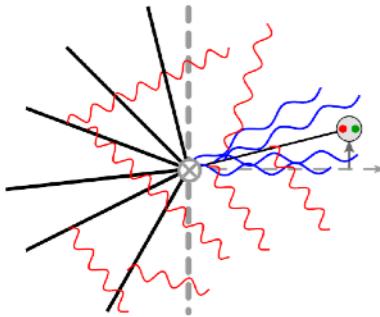
**Non-global observable; new TMD factorization**

$$\frac{d\sigma}{d^2 q_T} \sim H \otimes D_{h_1} \otimes D_{h_2} \otimes S$$

Collins, "Foundations of perturbative QCD"

$$\frac{d\sigma}{d^2 q_T} \sim D_h \otimes \mathcal{H} \otimes \mathcal{S}$$

Kang, DYS, Zhao '20



**hard:**  $p_h \sim Q(1, 1, 1)$

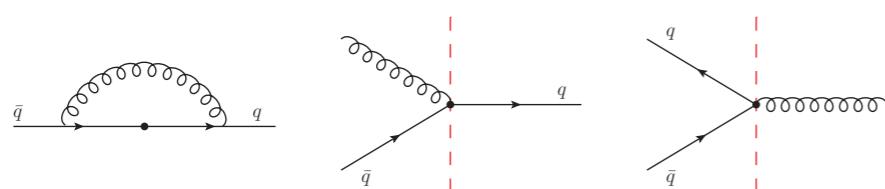
**collinear:**  $p_c \sim Q(\lambda^2, 1, \lambda)$        $\lambda = j_T/Q \ll 1$

**soft:**  $p_s \sim Q(\lambda, \lambda, \lambda)$

**Factorization formula:  
(neglecting NGLs)**

$$\frac{d\sigma}{dz_h d^2 \vec{j}_T} = \sigma_0 \sum_{i=q, \bar{q}, g} e_q^2 \int d^2 \vec{k}_T d^2 \vec{\lambda}_T \delta^{(2)} \left( \vec{j}_T - \vec{k}_T - z_h \vec{\lambda}_T \right) \mathcal{H}^i(Q, \mu) D_{h/i}(z_h, k_T, \mu, \zeta^2/\nu) \mathcal{S}_i(\lambda_T, \mu, \nu)$$

**NLO hard function:**



Divergences are half of the hard function in case-I

**NLO soft function:**

$$\begin{aligned} & \frac{\alpha_s C_F}{2\pi^2} \frac{e^{\epsilon \gamma_E}}{\Gamma(1-\epsilon)} \int \frac{dk^+ dk^-}{2} \left( \frac{\mu^2}{\vec{\lambda}_T^2} \right)^\epsilon \frac{2n \cdot \bar{n}}{k^+ k^-} \delta^+(k^+ k^- - \vec{\lambda}_T^2) \left| \frac{\nu}{2k_z} \right|^\eta \theta \left( 1 - \frac{k^+}{k^-} \right) \\ &= \frac{\alpha_s}{2\pi} C_F \left[ \frac{2}{\eta} \left( -\frac{1}{\epsilon} - \ln \left( \frac{\mu^2}{\mu_b^2} \right) \right) + \frac{1}{\epsilon^2} - \frac{1}{\epsilon} \ln \left( \frac{\nu^2}{\mu^2} \right) \right] \end{aligned}$$

Divergences are half of the soft function in case-I

**Factorization formula (full story)**

$$\frac{d\sigma}{dz_h d^2 \vec{j}_T} = \sum_{i=q, \bar{q}, g} \int \frac{d^2 \vec{b}}{(2\pi)^2} e^{i \vec{b} \cdot \vec{j}_T / z_h} \sum_{m=2}^{\infty} \frac{1}{N_c} \text{Tr}_c \left[ \mathcal{H}_m^i(\{\underline{n}\}, Q, \mu) \otimes \mathcal{S}_m(\{\underline{n}\}, b, \mu, \nu) \right] D_{h/i}(z_h, b, \mu, \zeta/\nu^2)$$

"Multi-Wilson-line structure" Becher, Neubert, Rothen, DYS '15,... (see Becher's talk)

A similar structure is also mentioned in Boglione & Simonelli '20 (see Simonelli's talk)

## NLL resummation formula:

$$\frac{d\sigma}{dz_h d^2 \vec{j}_T} = \sigma_0 \sum_{i=q,\bar{q}} e_i^2 \int_0^\infty \frac{b db}{2\pi} J_0(b j_T / z_h) e^{-S_{\text{pert}}(\mu_{b*}, \mu_h) - S_{\text{NP}}(b, Q_0, Q)} \frac{1}{z_h^2} D_{h/i}(z_h, \mu_{b*}) U_{\text{NG}}(\mu_{b*}, \mu_h)$$

**b\*-prescription to avoid Landau pole**       $b_* = b/\sqrt{1 + b^2/b_{\max}^2}$        $\mu_{b*} = 2e^{-\gamma_E}/b_*$

**QCD evolution between  $Q$  and  $j_T$**

**Linear part:**       $S_{\text{pert}}(\mu_b, \mu_h) = \int_{\mu_b}^{\mu_h} \frac{d\mu}{\mu} \left[ \Gamma_{\text{cusp}}(\alpha_s) \ln \left( \frac{Q^2}{\mu^2} \right) - 2\gamma^{D_q}(\alpha_s) - \gamma^S(\alpha_s) \right]$

**Non-linear part:**       $U_{\text{NG}}(\mu_{b*}, \mu_h) = \exp \left[ -C_A C_F \frac{\pi^2}{3} u^2 \frac{1 + (au)^2}{1 + (bu)^c} \right]$

Dasgupta, Salam '01

**Non-perturbative corrections:**       $j_T \sim \Lambda_{\text{QCD}}$

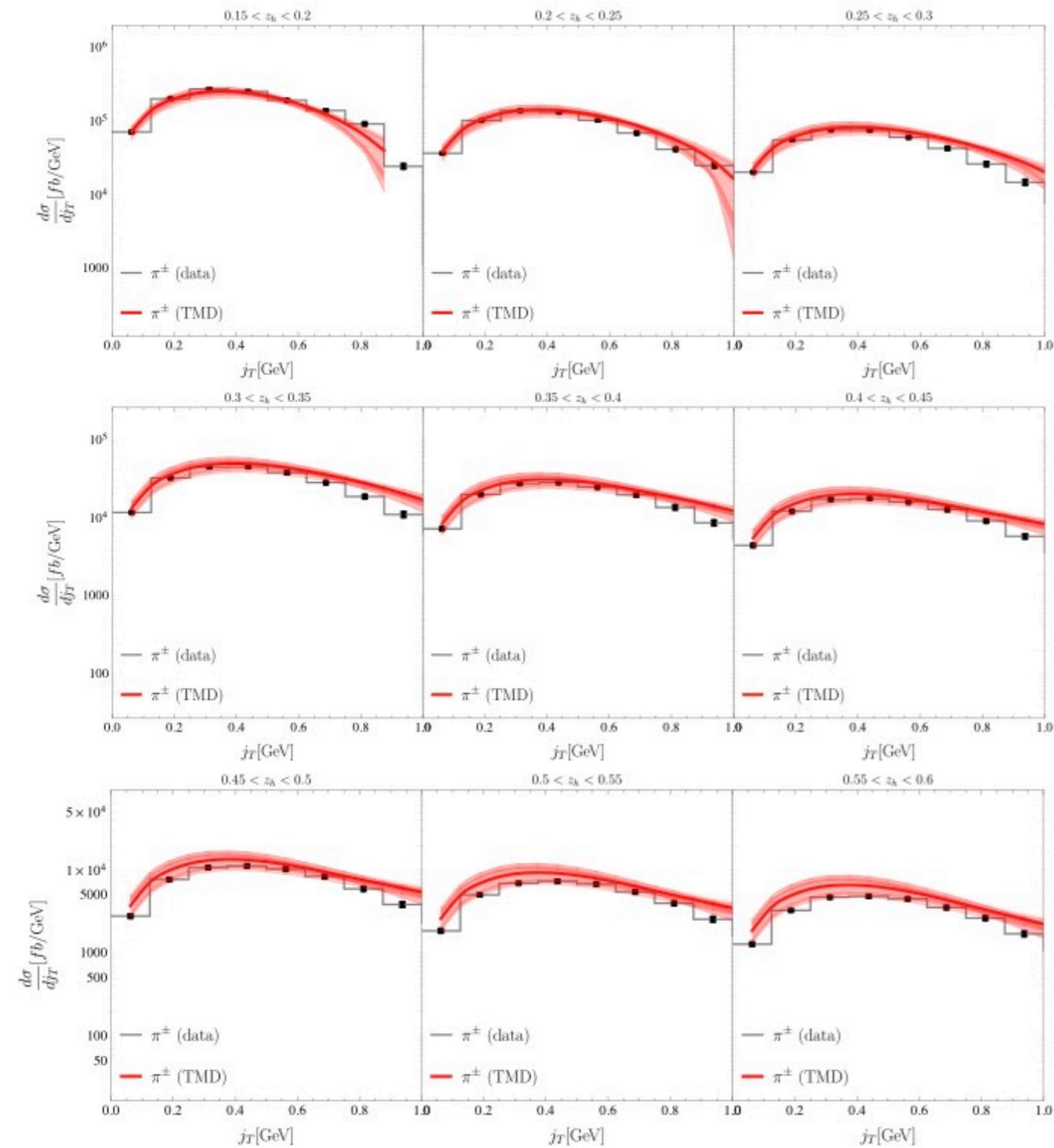
$$S_{\text{NP}}(b, Q_0, Q) = \frac{g_2}{2} \ln \left( \frac{b}{b_*} \right) \ln \left( \frac{Q}{Q_0} \right) + \frac{g_h}{z_h^2} b^2 \quad \text{fitted in standard (global) TMD processes}$$

Sun, Isaacson, Yuan, Yuan '14

**Non-perturbative collinear FFs**       $D_{h/i}(z_h, \mu_{b*})$       **(DSS2014)**

de Florian, *et.al.* '15

# Numerical results



- Belle data ([1902.01552](#)) was originally presented in different thrust bins
- Since the theoretical formalism we have developed is inclusive in thrust, we thus combine the experimental data to obtain the entire region  $0.5 < T < 1.0$
- Our TMD resummation formula gives a good description of the shape of  $j_T$  distribution as  $z_h < 0.65$
- As  $z_h > 0.65$ , one needs to also include threshold resummation effects

# Joint resummation formula: (refactorization of TMD FF)

Li, Neill, Zhu '16 & Lustermans, Waalewijn, Zeune '16

$$\frac{d\sigma}{dz_h d^2 \vec{j}_T} = \sigma_0 \sum_{i=q,\bar{q}} \int_0^\infty \frac{b db}{2\pi} J_0(b j_T/z_h) \frac{1}{z_h^2} \int_{z_h}^1 \frac{dz}{z} e^{-\hat{S}_{\text{pert}}(\mu_{b*}, \mu_h) - \hat{S}_{\text{NP}}(b, Q_0, Q)} \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \frac{1}{1-z} D_{h/i}(z_h/z, \mu_h) U_{\text{NG}}(\mu_{b*}, \mu_h)$$

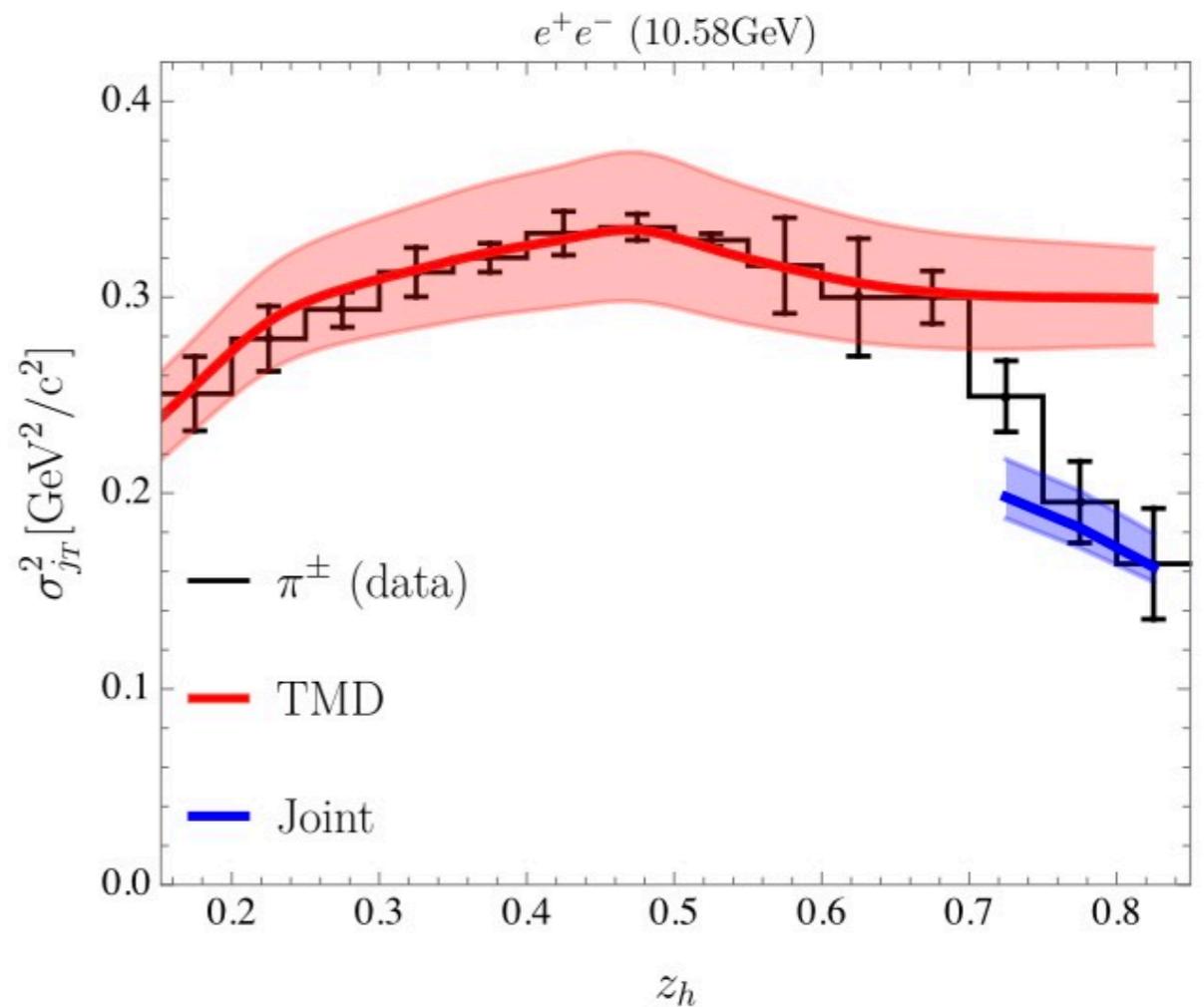
**with:**

$$\hat{S}_{\text{pert}}(\mu_b, \mu_h) = \int_{\mu_b}^{\mu_h} \frac{d\mu}{\mu} \left[ \Gamma_{\text{cusp}}(\alpha_s) \ln \left( \frac{(1-z)^2 Q^2}{\mu^2} \right) \right]$$

$$\hat{S}_{\text{NP}}(b, Q_0, Q) = \frac{g_2}{2} \ln \left( \frac{b}{b_*} \right) \ln \left[ \frac{Q(1-z_h)}{Q_0} \right] + \frac{g_h}{z_h^2} b^2$$

$$\frac{d\sigma}{dz_h d^2 \vec{j}_T} \propto \frac{1}{\pi \sigma_{j_T}^2} \exp \left( -j_T^2 / \sigma_{j_T}^2 \right)$$

- **The Gaussian width of the  $j_T$  distribution given by the TMD formalism freeze to a certain value.**
- **After including joint threshold and TMD resummation effects, the theoretical predictions are consistent with the data**



# Overview

arXiv: 2009.11871 (Y. Makris, F. Ringer, W.J. Waalewijn)

In progress (Y. Makris, J.K.L. Michel, F. Ringer, W.J. Waalewijn)

## Joint TMD and Thrust resummation:

- Factorization within SCET
- All elements for up to NNLL provided

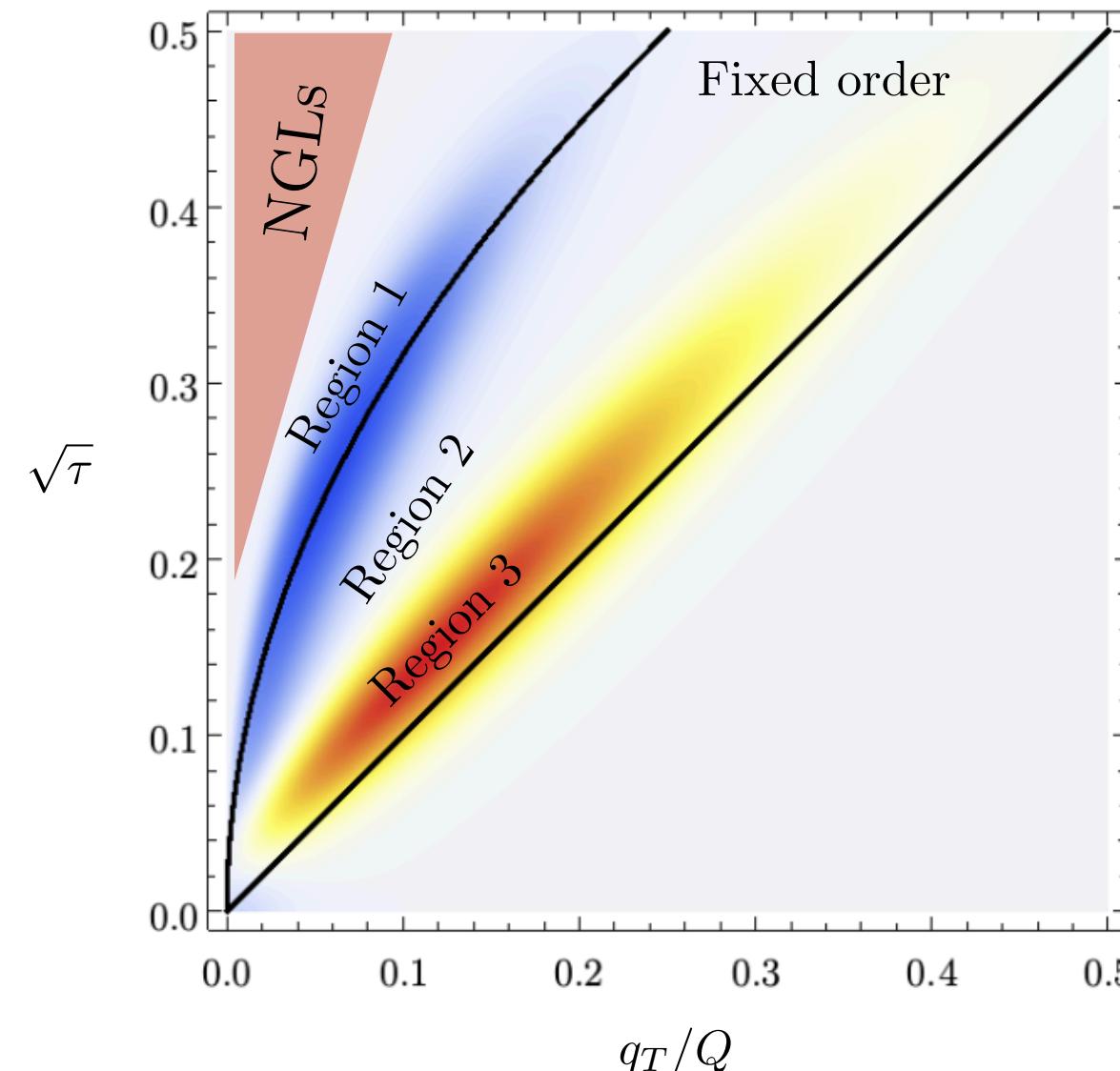
## Regions

- Region I  $1 \gg \sqrt{\tau} \gg q_T/Q \sim \tau$
- Region 2  $1 \gg \sqrt{\tau} \gg q_T/Q \gg \tau$
- Region 3  $1 \gg \sqrt{\tau} \sim q_T/Q \gg \tau$
- Fixed order QCD

## NGLs: $q_T \ll Q\tau$

- NGLs not relevant for the three regions
- We give the LO contributions:  $\mathcal{O}(\alpha_s^2)$

$$S^{\text{NG}}\left(\frac{q_T}{\tau Q}\right) = 1 - \frac{\alpha_s^2 C_F C_A}{12} \ln^2\left(\frac{q_T}{\tau Q}\right) + \mathcal{O}(\alpha_s^3)$$



# Factorization-Evolution-Resummation

---

**Region 1**  $\sqrt{\tau} \gg q_T/Q \sim \tau$

$$d\sigma \sim H(Q, \mu) J(\tau, \mu) \otimes_{\tau} S(\tau, b, \mu, \nu) \times D_{i \rightarrow h}(b, z_h, \mu, \nu)$$

**Double differential soft function**    **Universal (unsubtracted) TMDFF**

**Region 2**  $\sqrt{\tau} \gg q_T/Q \gg \tau$

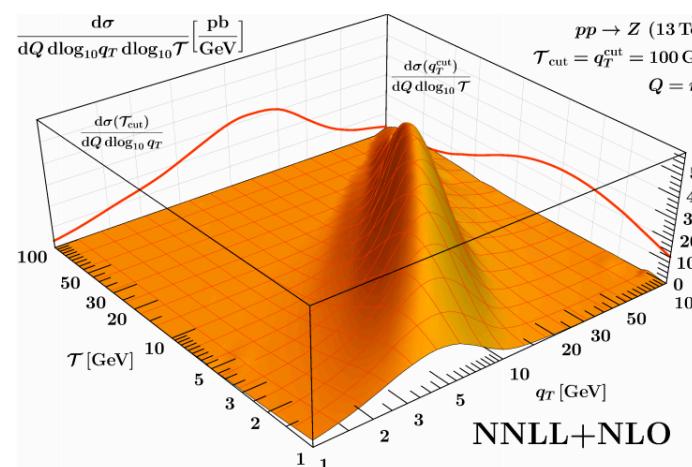
$$d\sigma \sim H(Q, \mu) J(\tau, \mu) \otimes_{\tau} S(\tau, \mu) \otimes_{\tau} C(\tau, b, \mu, \nu) \times D_{i \rightarrow h}(b, z_h, \mu, \nu)$$

**Region 3**  $\sqrt{\tau} \sim q_T/Q \gg \tau$

$$d\sigma \sim H(Q, \mu) J(\tau, \mu) \otimes_{\tau} S(\tau, \mu) \otimes_{\tau} G_{i \rightarrow h}(\tau, b, z_h, \mu)$$

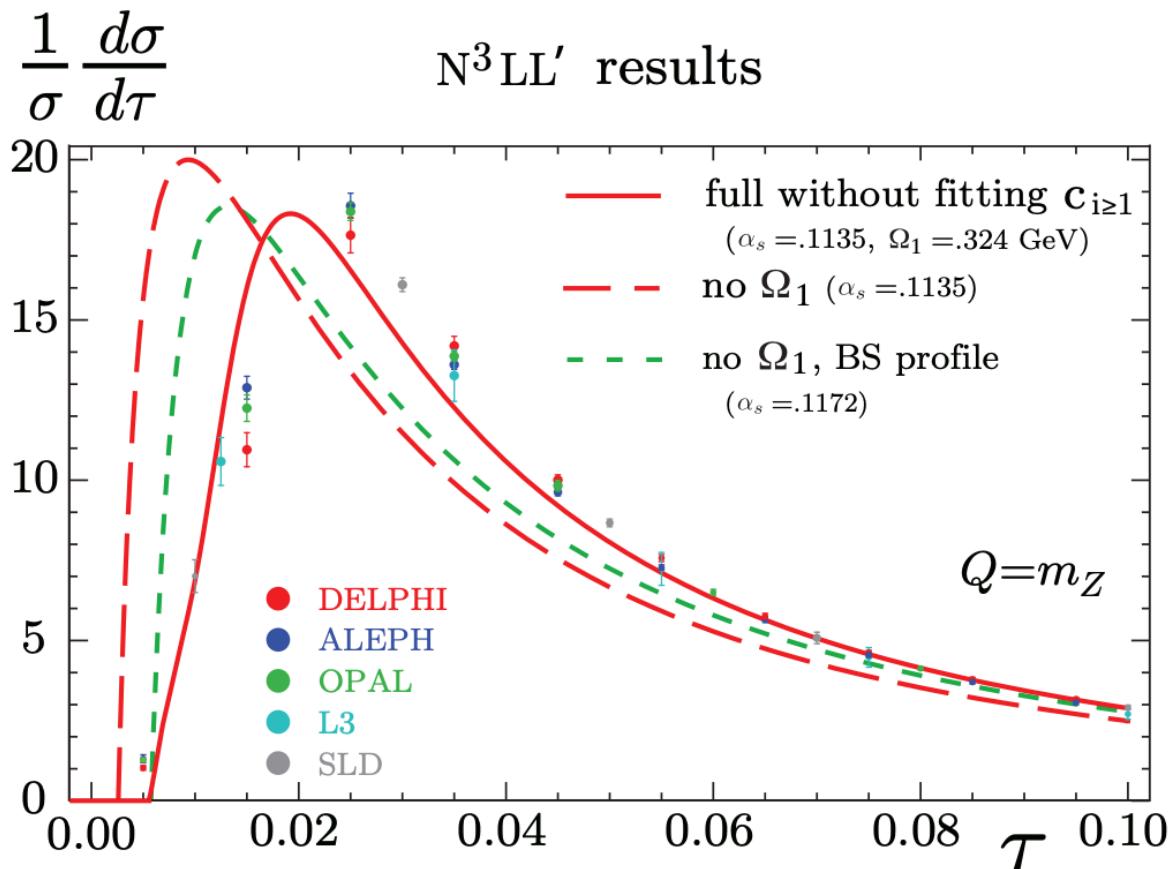
**Double differential  
Fragmenting jet function  
No rapidity divergences**

**Drell-Yan:**



arXiv: 1901.03331 (G. Lustermans,  
J. K. L. Michel, F. J. Tackmann, and W. J. Waalewijn)

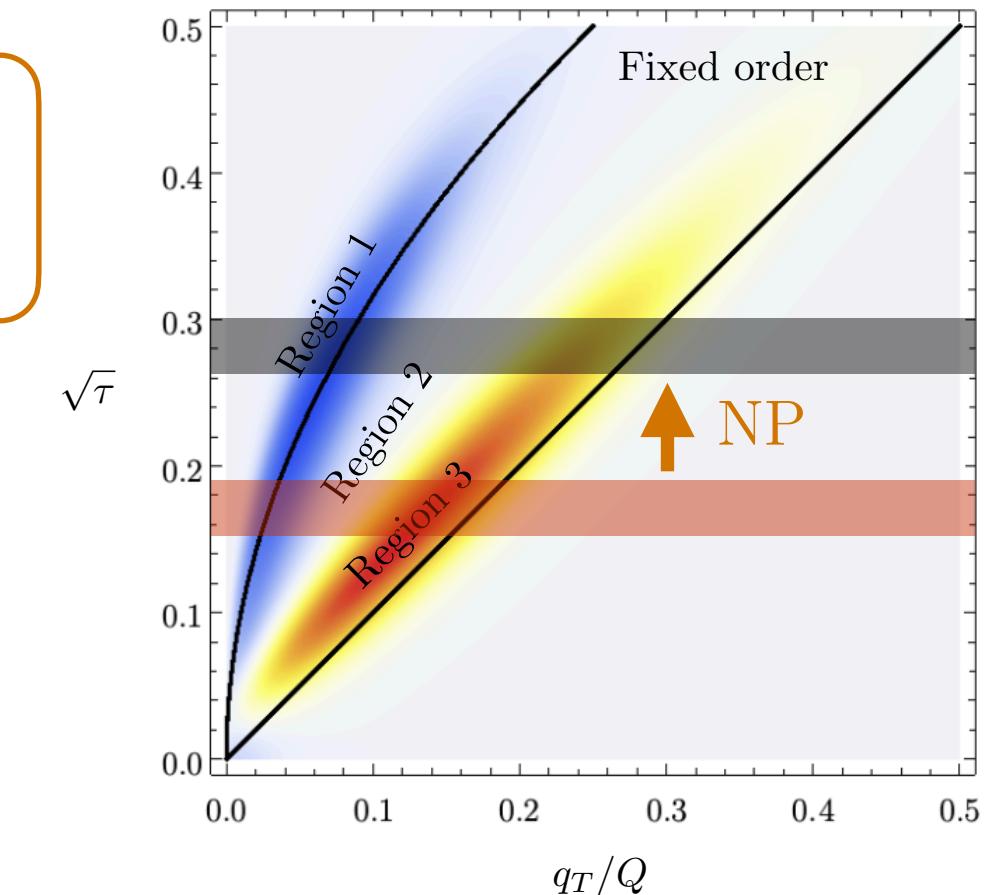
# Non-perturbative effects (naive first expectations)



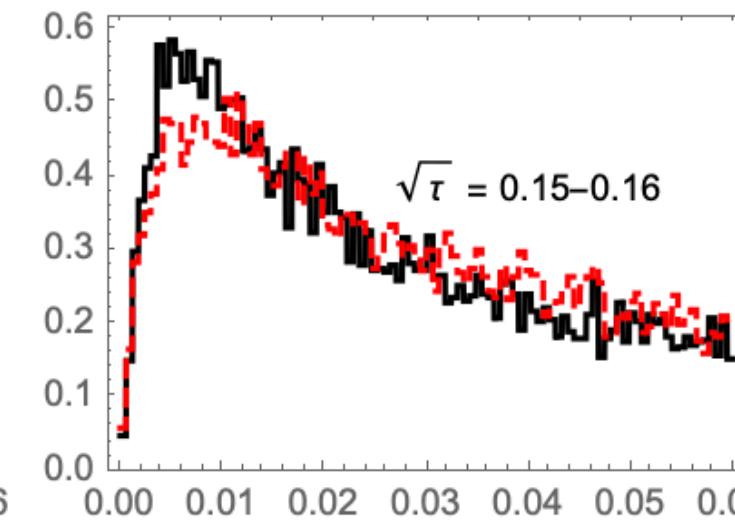
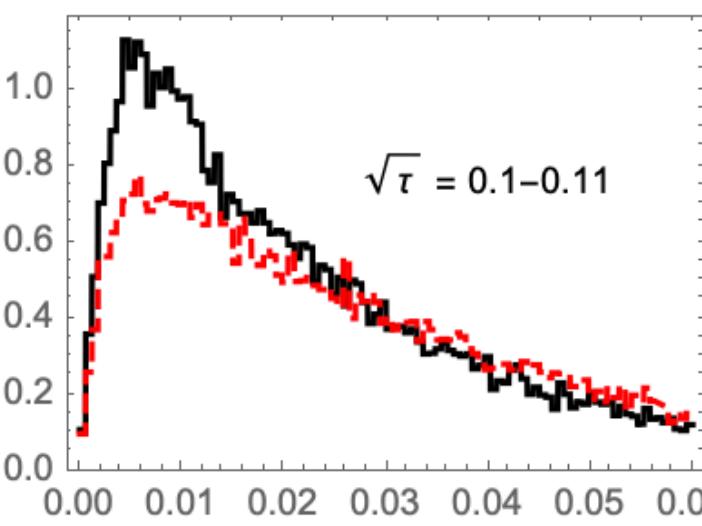
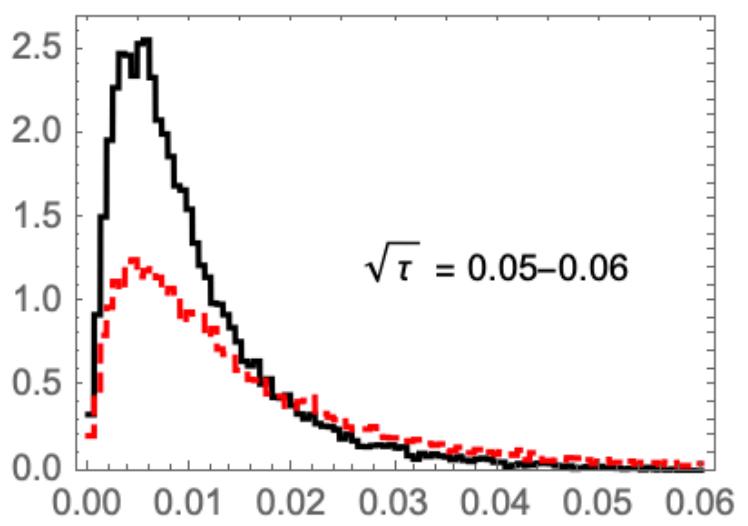
arXiv: 0611061 (C. Lee, G. Sterman)

$$\frac{d\sigma}{d\tau}(\tau) \xrightarrow{\text{NP}} \frac{d\sigma}{d\tau}\left(\tau - \frac{\Lambda}{Q}\right)$$

Figure from: arXiv:1006.3080  
(R. Abbate, M. Fickinger, A.H. Hoang,  
V. Mateu, and I.W. Stewart)



PYTHIA 8



Hadrons

Partons

# Boglione, Simonelli, 2007.13674, 2011.07366

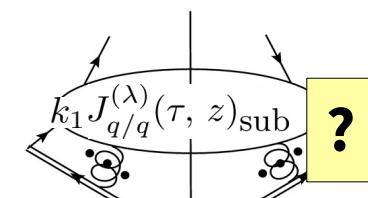
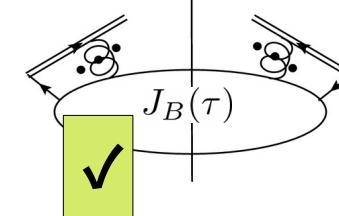
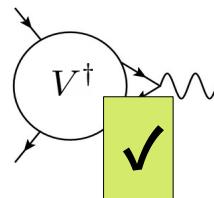
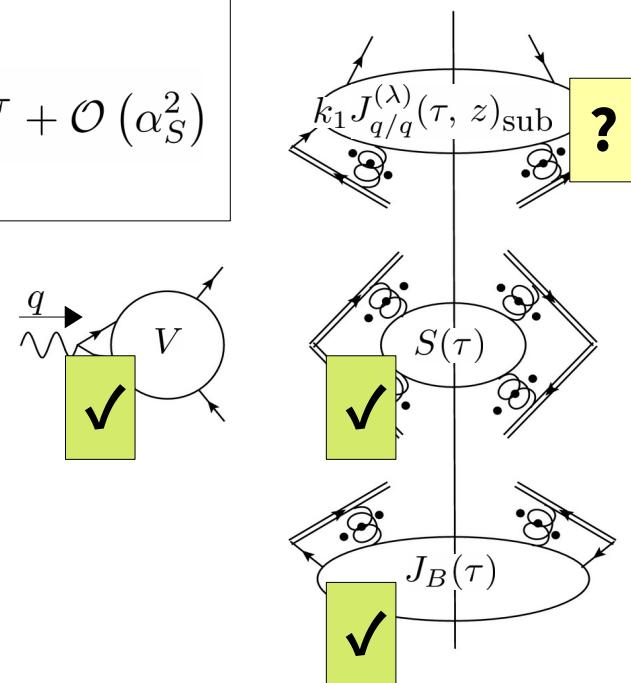
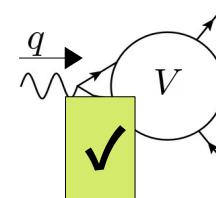
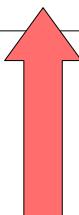
- $e^+e^- \rightarrow HX$  cross section, differential in  $z_h, P_T$  and  $T$ , factorized (CSS) as a convolution of a partonic cross section (computable in pQCD) and a TMD FF.
- Result valid to all orders and computed at NLO, NLL.
- Predictions in exceptional agreement with BELLE experimental measurements.

$$\frac{d\sigma}{dz_h dT dP_T^2} = \pi \sum_f \int_{z_h}^1 \frac{dz}{z} \left( \frac{d\hat{\sigma}_f}{dz_h/z dT} \right) D_{1,\pi^\pm/f}(z, P_T, Q, (1-T)Q^2)$$

Partonic Cross Section (pQCD)

$$\frac{d\hat{\sigma}_f}{dz dT} \stackrel{\text{NLO}}{=} -\sigma_B e_f^2 N_C \frac{\alpha_S}{4\pi} C_F \delta(1-z) \frac{3 + 8 \log \tau}{\tau} e^{-\frac{\alpha_S}{4\pi} 3C_F \log^2 \tau} + \mathcal{O}(\alpha_S^2)$$

NO THRUST RESUMMED



# Boglione, Simonelli, 2007.13674, 2011.07366

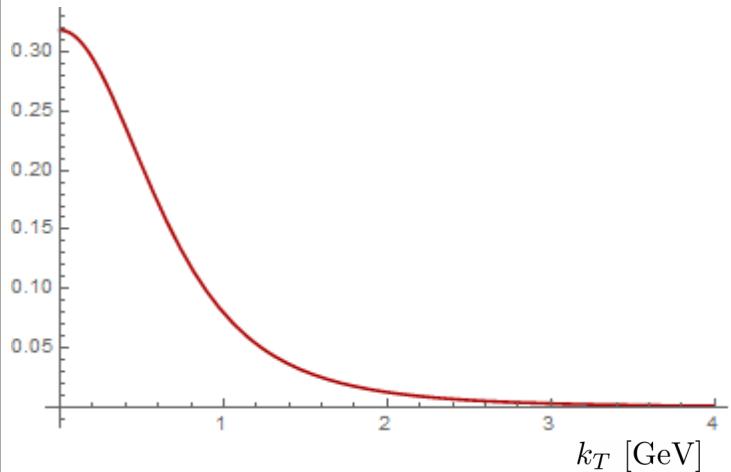
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Power-law

$$\mathcal{FT}\{M_D\} = \frac{\Gamma(p)}{\pi \Gamma(p-1)} \frac{m^{2(p-1)}}{(k_T^2 + m^2)^p}$$

$m = 1 \text{ GeV}, p = 2$

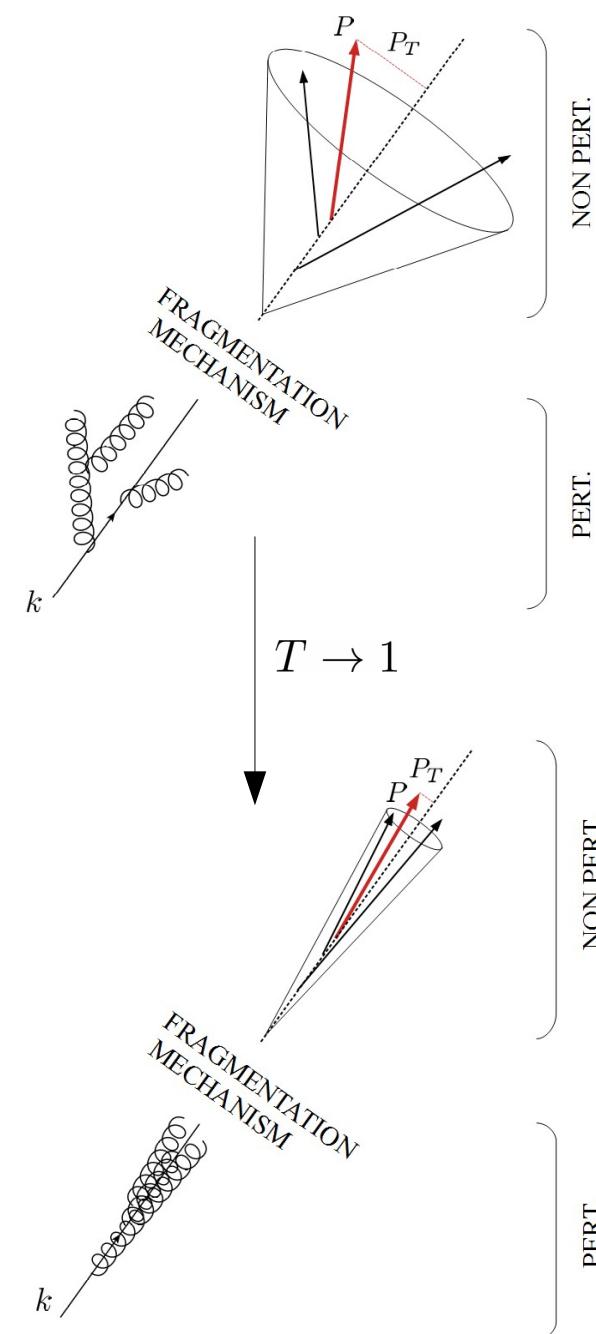


Unpolarized TMD FF

NNFF10NLO

$$\begin{aligned} \widetilde{D}_{1, \pi^\pm/f}(z, b_T; Q, \tau Q^2) &= \frac{1}{z^2} \sum_k [d_{\pi^\pm/k} \otimes \mathcal{C}_{k/f}] (\mu_b) \times \\ &\exp \left\{ \frac{1}{4} \tilde{K} \log \frac{\tau Q^2}{\mu_b^2} + \int_{\mu_b}^Q \frac{d\mu'}{\mu'} \left[ \gamma_D - \frac{1}{4} \gamma_K \log \frac{\tau Q^2}{\mu'^2} \right] \right\} \times \\ &(M_D)_{f, \pi^\pm}(z, b_T) \exp \left\{ -\frac{1}{4} g_K(b_T) \log \left( \tau \frac{z_h^2 Q^2}{M_H^2} \right) \right\} \end{aligned}$$

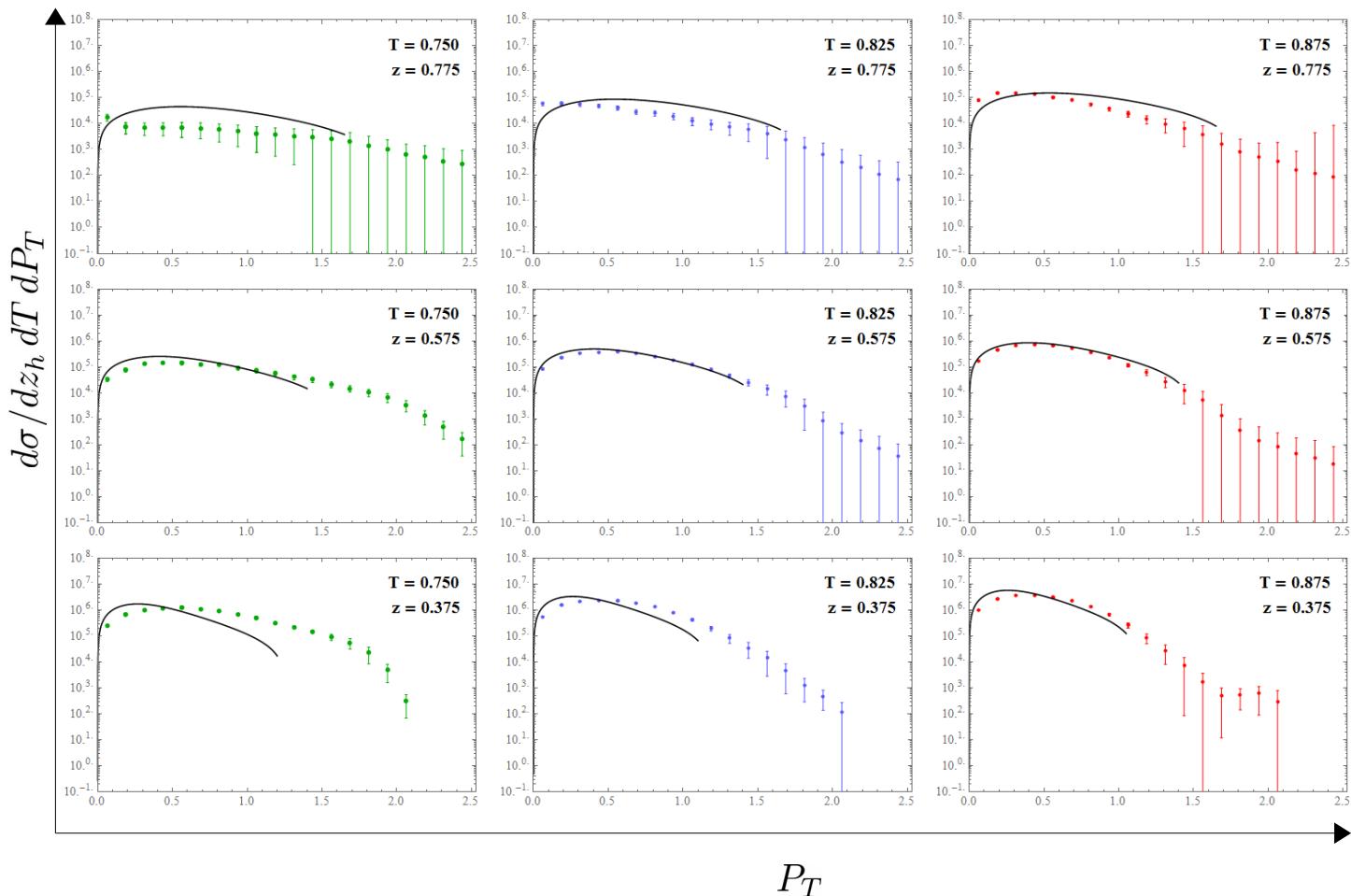
Quadratic behavior  $g_K(b_T) = a b_T^2, a = 0.05 \text{ GeV}^2$



$$\frac{d\sigma}{dz_h dT dP_T^2} = \pi \sum_f \int_{z_h}^1 \frac{dz}{z} \frac{d\hat{\sigma}_f}{dz_h/z dT} D_{1, \pi^\pm/f}(z, P_T, Q, (1-T)Q^2)$$

Rapidity cut-off

## Thrust dependence (First Phenomenological analysis – prototype)



# Discussion

# Discussion contribution REF2020 – Fragmentation/Hadronization

Anselm Vossen

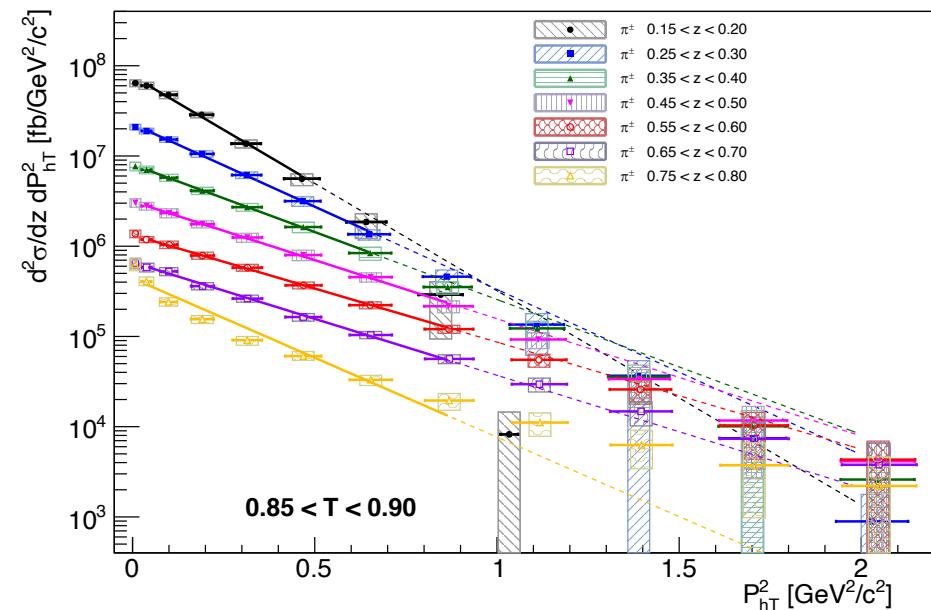


Research supported by the  
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**ENERGY** | Office of  
Science

**Duke**  
UNIVERSITY

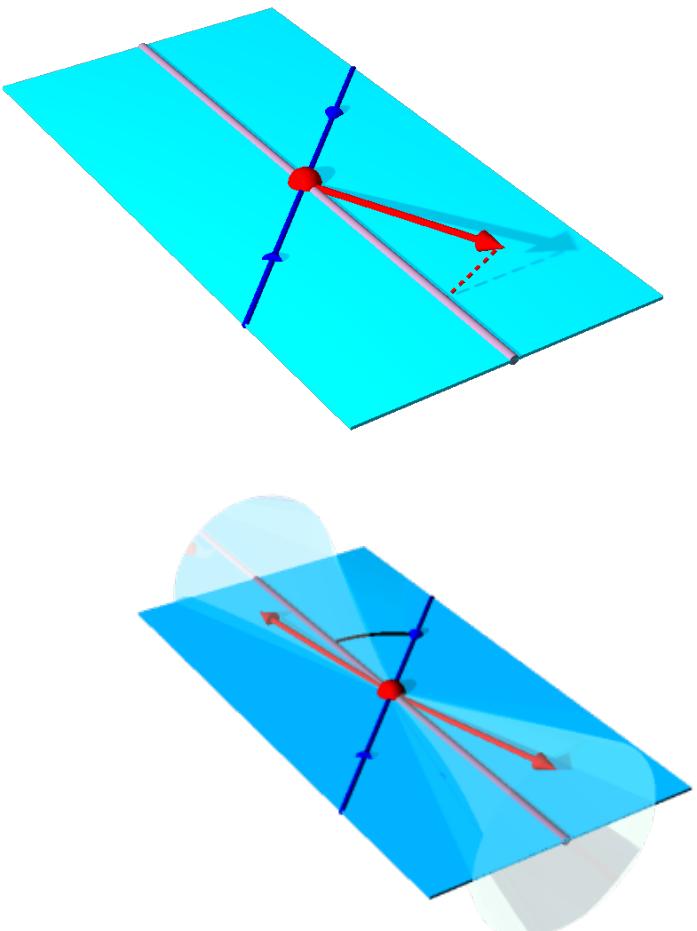
**Jefferson Lab**

# Current and near-term future at Belle



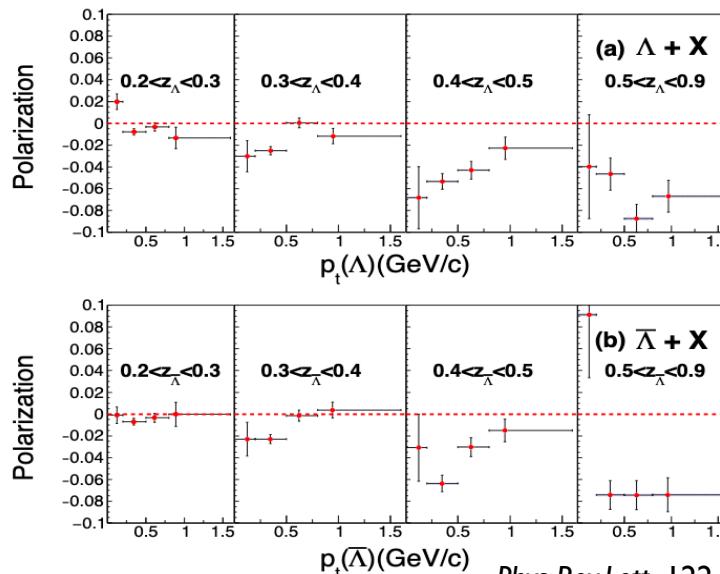
- Observation of  $k_t$  dependence wrt thrust axis
- Next:  $k_T$  dependence in back-to-back measurements
- In Jets?

*Thrust axis*

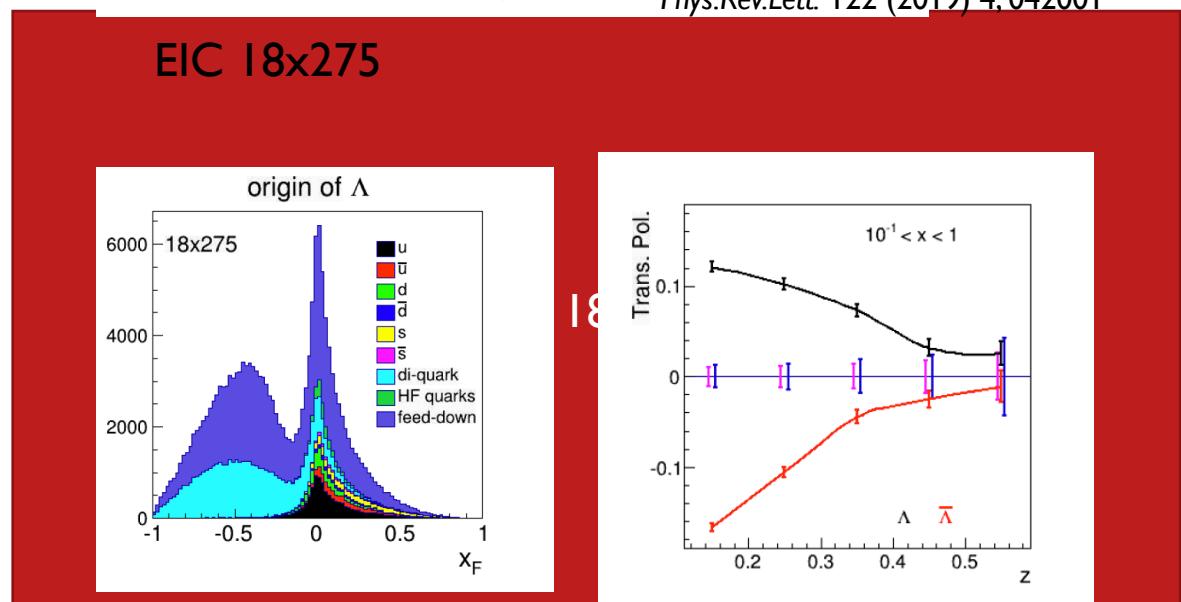


# Polarized Fragmentation at Belle/Belle II and JLab/EIC

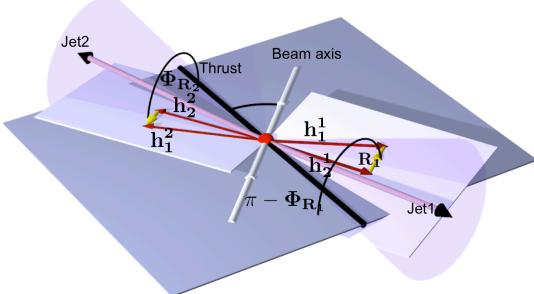
- Observation of polarized FFs at Belle
- What do we learn from fragmentation in jets?
- Next: Measure twist3  $D_T$
- Program at Jlab and EIC
- Issues in interpretation
  - Feed-down
  - Gluons (EIC)



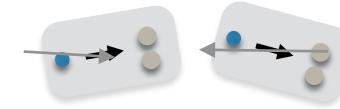
Phys.Rev.Lett. 122 (2019) 4, 042001



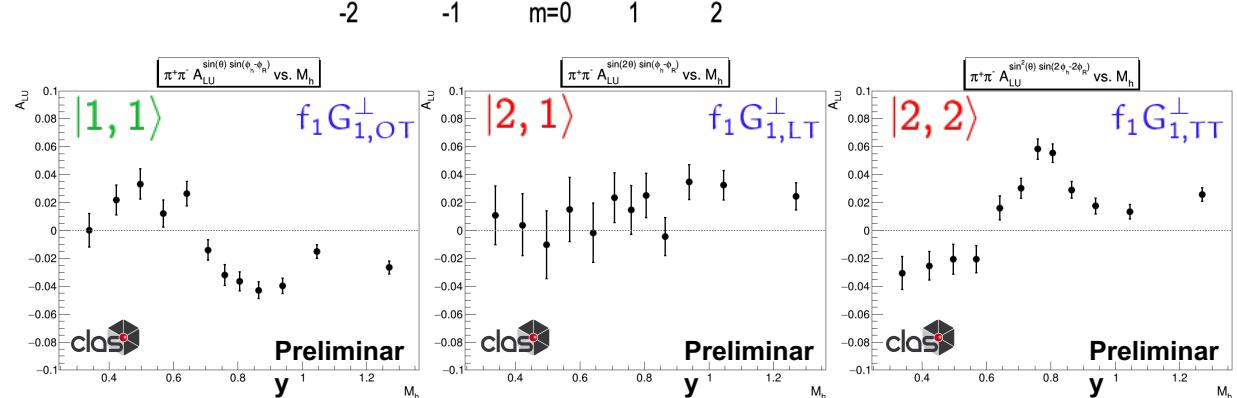
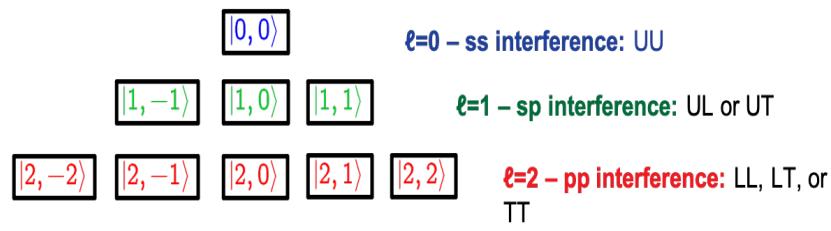
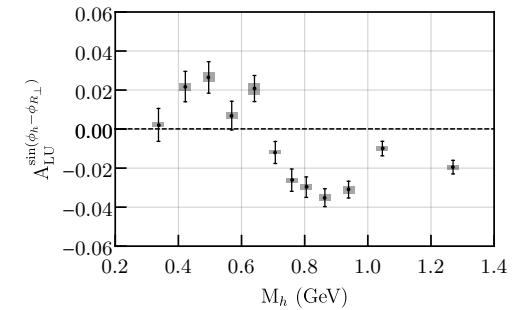
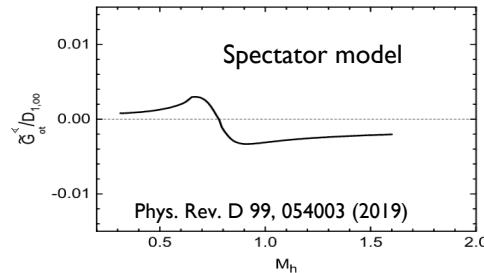
# Di-hadrons in SIDIS, $e^+ e^-$



- Additional degrees of freedom
- Carry rel OAM
- Interplay with single hadrons, e.g. transverse momentum
- Started comprehensive program at Jlab, Belle, Belle II, EIC
- Idea to measure BM w/o Cahn, less higher order corrections
- More applications?
- (also single hadron program e.g. Collins ongoing)
- Belle pioneering, Belle II precision



First di-hadron TMD measurement at CLAS12!



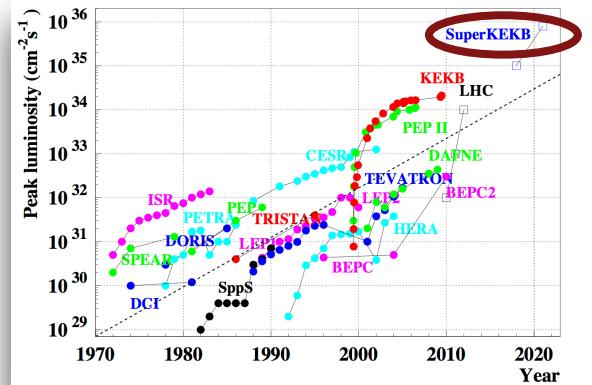
# Jets and more at Belle II

*Snowmass 2021 Letter of Interest:*  
QCD and Hadronization Studies at Belle II

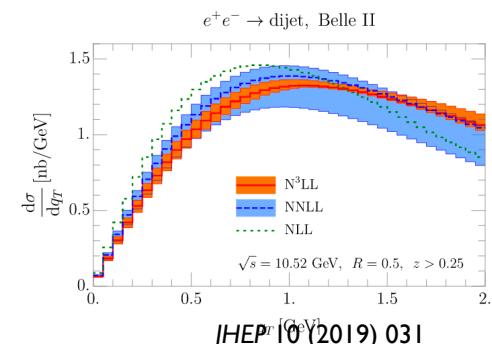
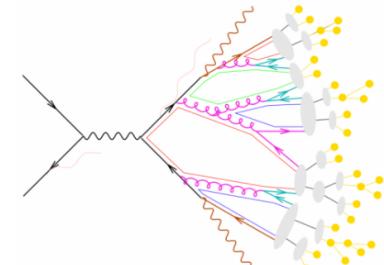
on behalf of the U.S. Belle II Collaboration

**Corresponding Author:**

Anselm Vossen (Duke University), anselm.vossen@duke.edu



- Comprehensive Hadronization Program at Belle II
- Precision FFs for EIC
- Study of jets (e.g.  $q_T$  spectrum, jet-hadron, )
- Energy correlation
- MC comparison
  - Implementation in Rivet → Is this interesting for phenomenology too?
  - Correlation of strangeness and baryon numbers
  - ...?
- Plan to have polarized beams at Belle II



# Experimental perspectives on hadronization

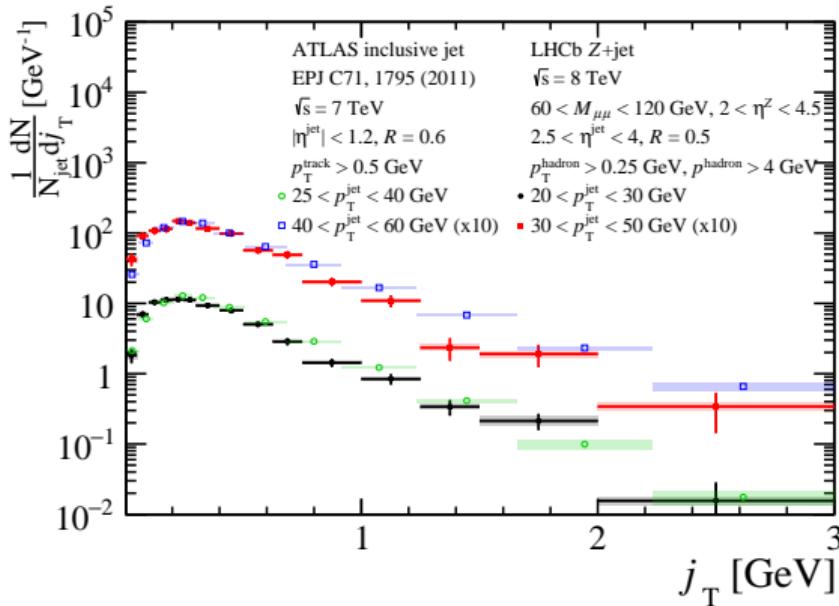
Resummation, Evolution, Factorization Workshop

Joe Osborn, ORNL  
December 10, 2020

ORNL is managed by UT-Battelle, LLC for the US Department of Energy



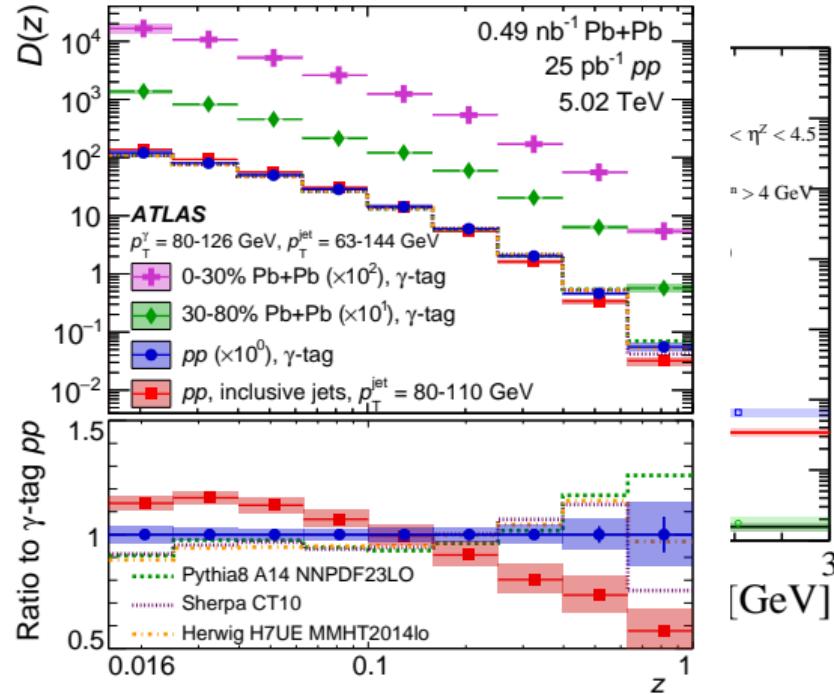
# Recent experimental advances



Phys. Rev. Lett. 123, 232001 (2019)

- Experimentalists starting to use processes to better identify parton $\rightarrow$ hadron relations
  - $\gamma/Z^0 + \text{jet}$  (predominantly  $u/d \rightarrow \text{hadrons}$ )

# Recent experimental advances

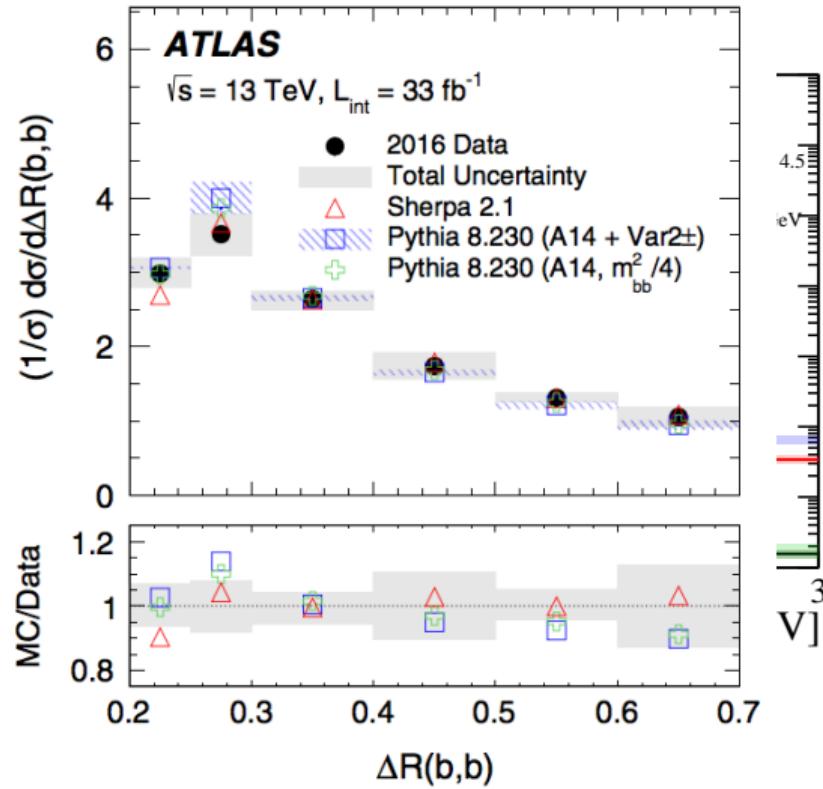


Phys. Rev. Lett. 123, 042001 (2019)

- Experimentalists starting to use processes to better identify parton $\rightarrow$ hadron relations
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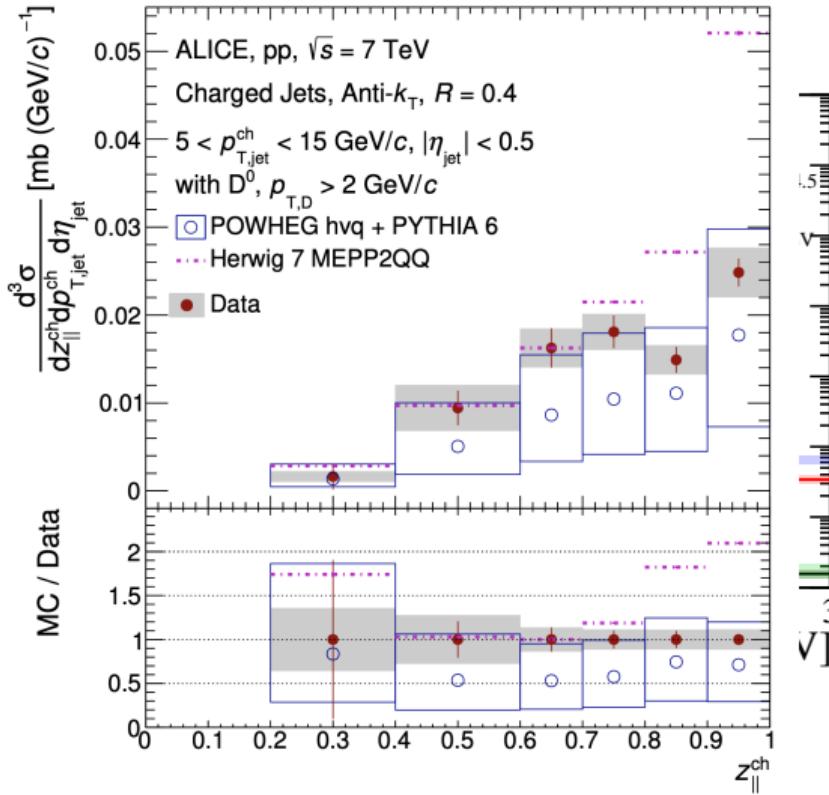
# Recent experimental advances

Phys. Rev. D 99, 052004 (2019)



- Experimentalists starting to use processes to better identify parton $\rightarrow$ hadron relations
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  - $g \rightarrow b\bar{b}$

# Recent experimental advances

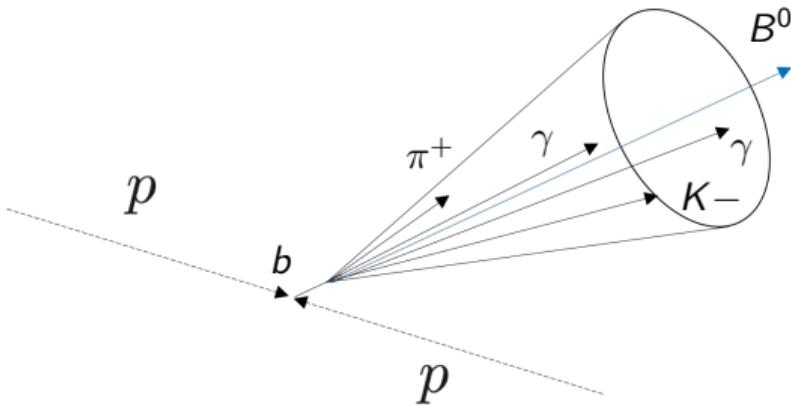


- Experimentalists starting to use processes to better identify parton  $\rightarrow$  hadron relations

- $\gamma/Z^0 + \text{jet}$  (predominantly  $u/d \rightarrow \text{hadrons}$ )
- $g \rightarrow b\bar{b}$
- $c \rightarrow D^0$
- ...

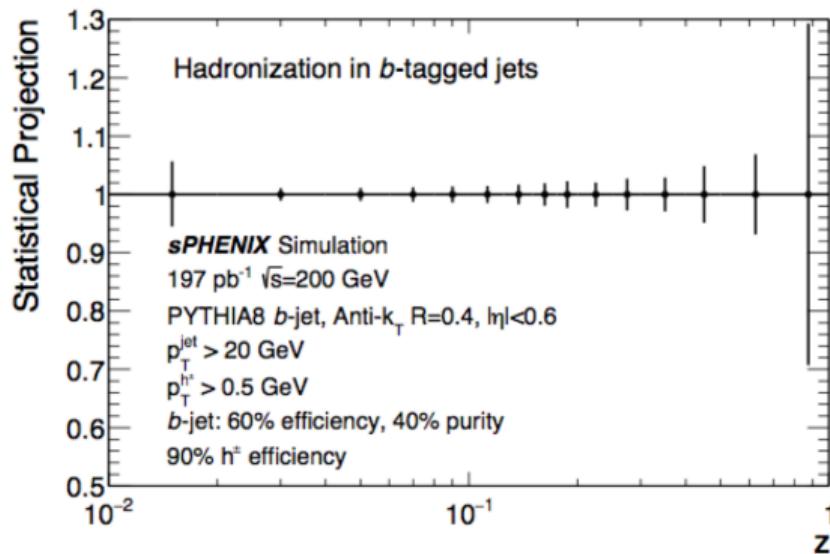
# Plans for the future

- Continue along the direction of connecting parton-to-hadron → what can be done with PID?
  - PID at STAR, ALICE, and LHCb give possibility of PIDed hadron-in-jet
    - More at EIC...



# Plans for the future

- Continue along the direction of connecting parton-to-hadron → what can be done with PID?
  - PID at STAR, ALICE, and LHCb give possibility of PIDed hadron-in-jet
    - More at EIC...
- sPHENIX and STAR at RHIC in the 2020's - Collins asymmetries and heavy flavor jets



# Discussion points

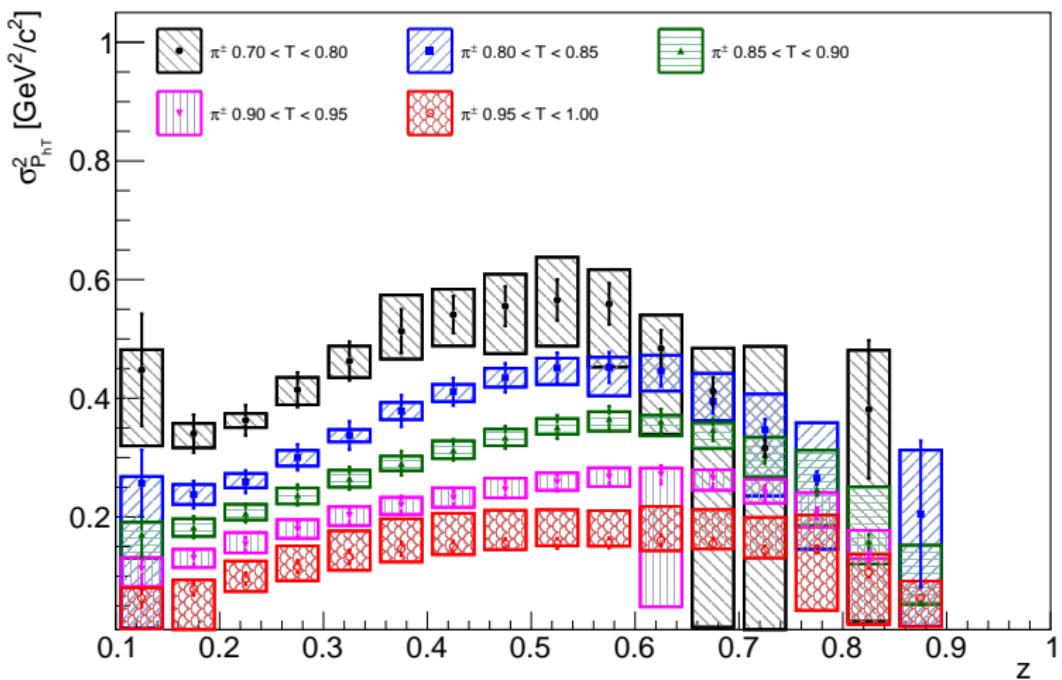
- What should experimentalists be thinking about in their measurements?
  - Are multidifferential measurements of  $(z, j_T)$  possible in hadronic collisions? See e.g. PRD 99, 112006 (2019), PRC 100, 064901 (2019)
  - Currently low  $z$  is difficult to handle theoretically - what can we learn from soft contributions to jets?
- Can we push towards measuring correlations rather than single hadrons?
  - e.g.  $K^+K^-$  correlations to test string breaking type picture?
- What about baryons?
  - Typically (at hadron colliders) measure “charged hadrons.” What can we learn about the physical process of hadronization from e.g. baryon vs. meson comparisons? Or more exotic baryons like  $\Lambda_s$  etc.? See e.g. PRL 124, 172301 (2020) etc.

# Discussion

# Backup

# $e^+e^- \rightarrow hX$ at Belle

R. Seidl et al. 1902.01552 - PRD  
and supplemental material



Gaussian widths in  $P_{hT}$  for the cross section as a function of  $z$  and thrust  
(pions)

# Relation to other formalisms in 2009.11871 & 2011.07366

The authors consider joint thrust and TMD resummation

$$1 - \tau = T = \max_{\hat{t}} \frac{\sum_i |\hat{t} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|}$$

**Our EFT**  
 $\tau \sim \mathcal{O}(1)$

Regime:	1: $\sqrt{\tau} \gg q_T/Q \sim \tau$	2: $\sqrt{\tau} \gg q_T/Q \gg \tau$
$n$ -collinear	$(q_T^2/Q, Q, q_T)$	$(q_T^2/Q, Q, q_T)$
$\bar{n}$ -collinear	$Q(1, \tau, \sqrt{\tau})$	$Q(1, \tau, \sqrt{\tau})$
$n$ -collinear-soft		$(\tau Q, q_T^2/(\tau Q), q_T)$
soft	$q_T(1, 1, 1)$	$\tau Q(1, 1, 1)$

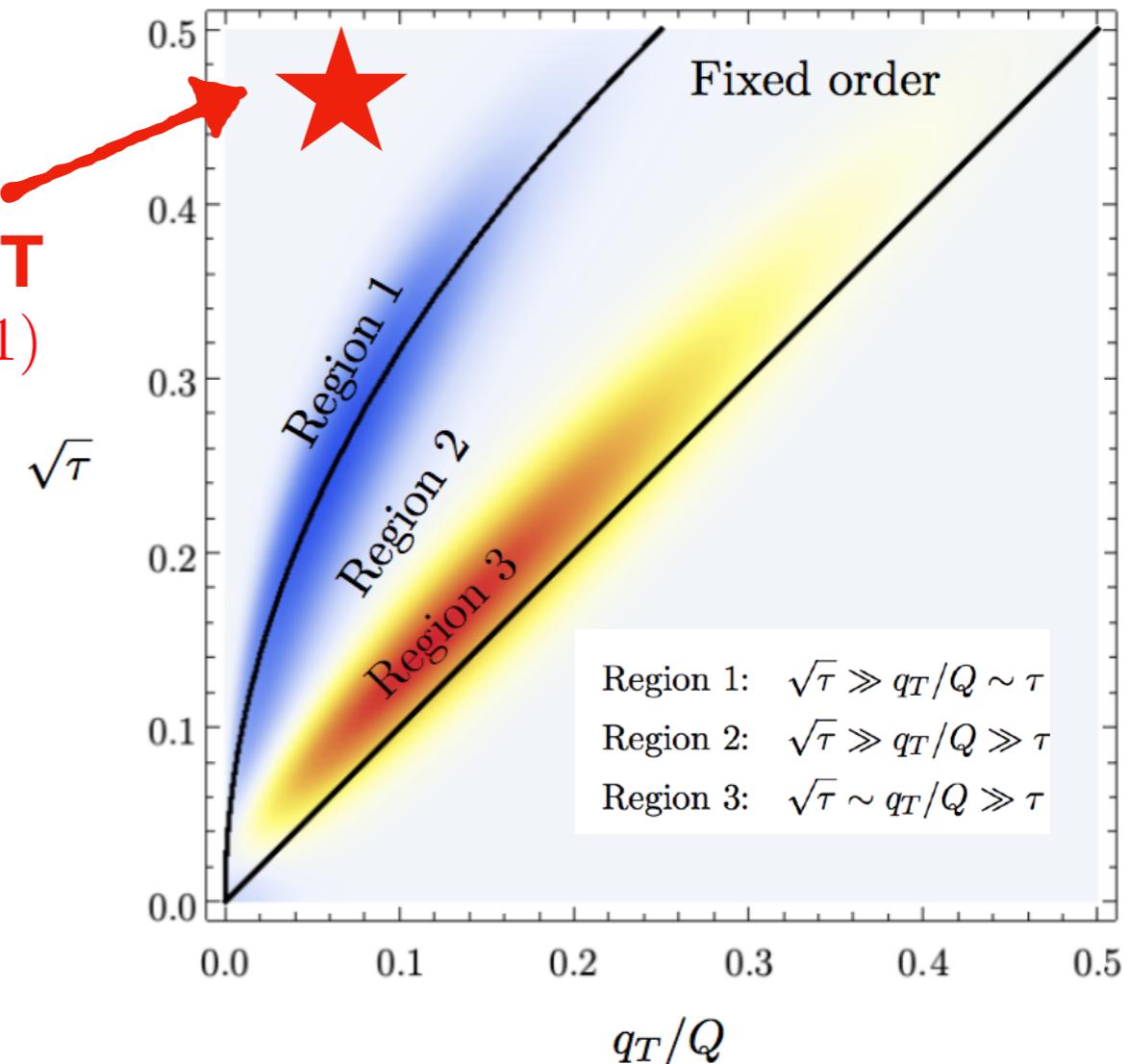
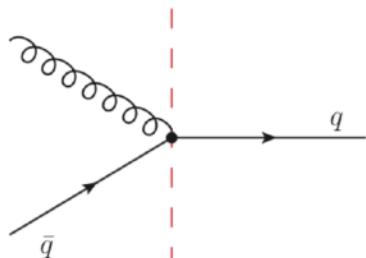


Fig from 2009.11871

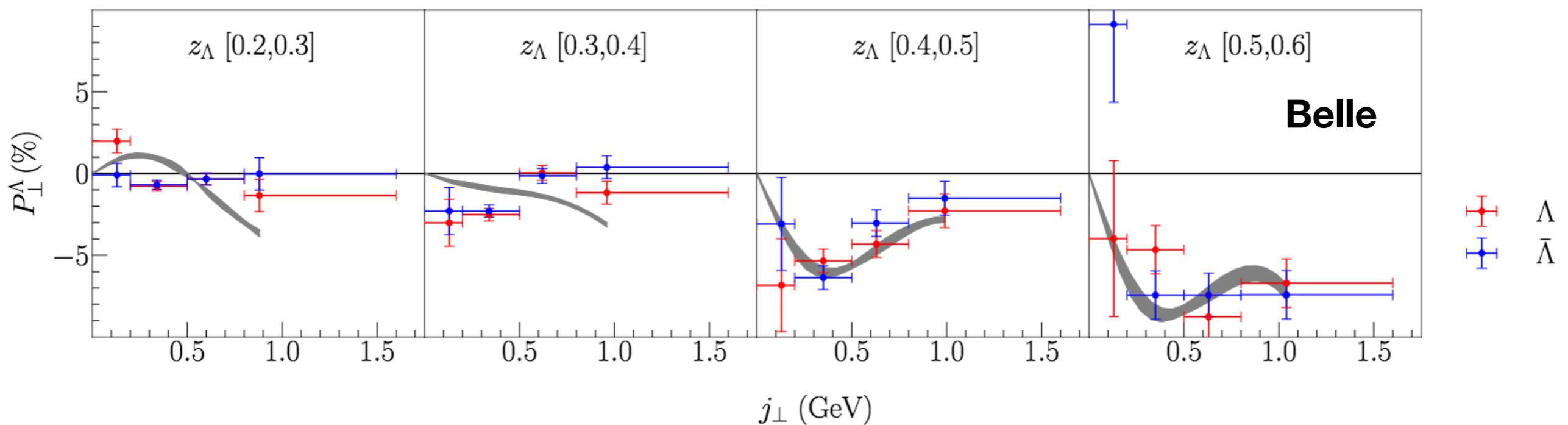
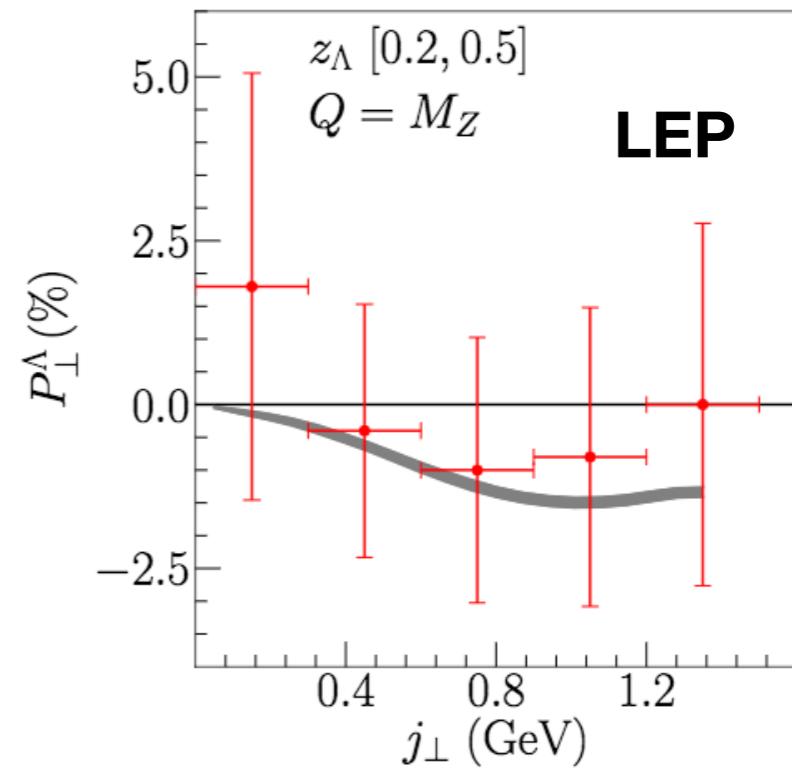
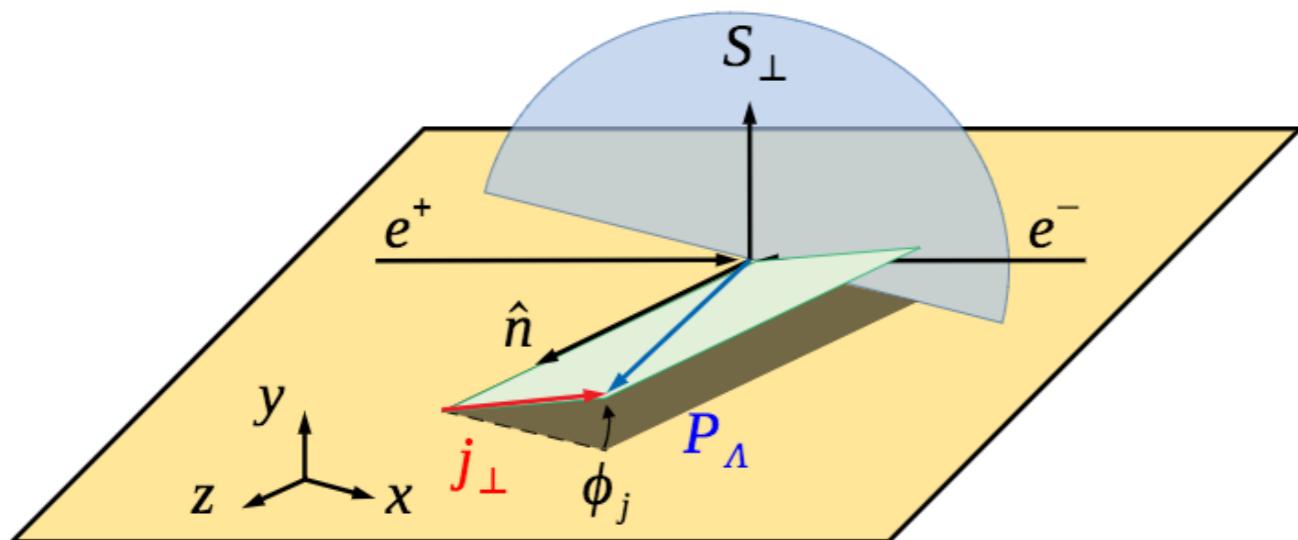
NLO hard function



NLO soft function is same in the hemisphere containing the identified hadron while scaleless in the other hemisphere

# Transverse polarized $\Lambda$ hyperon production

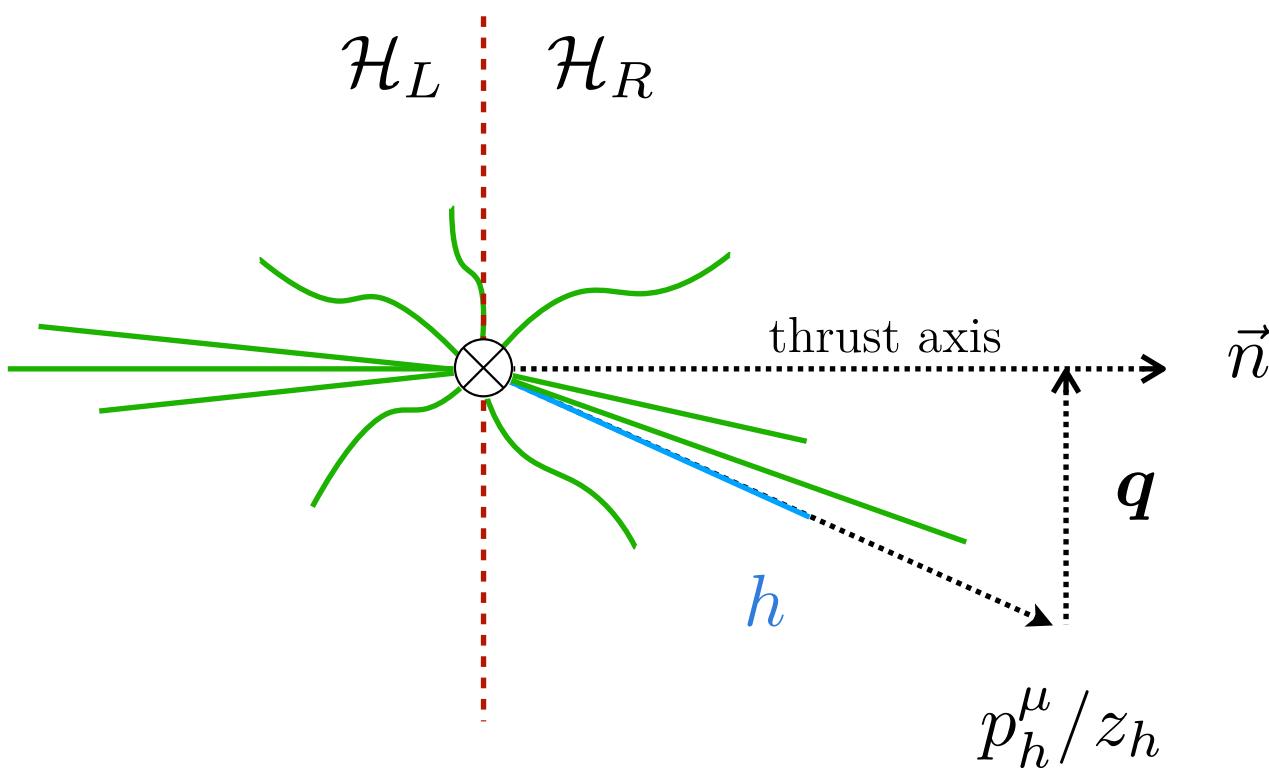
Gamberg, Kang, DYS, Terry, Zhao 2012.XXXXX



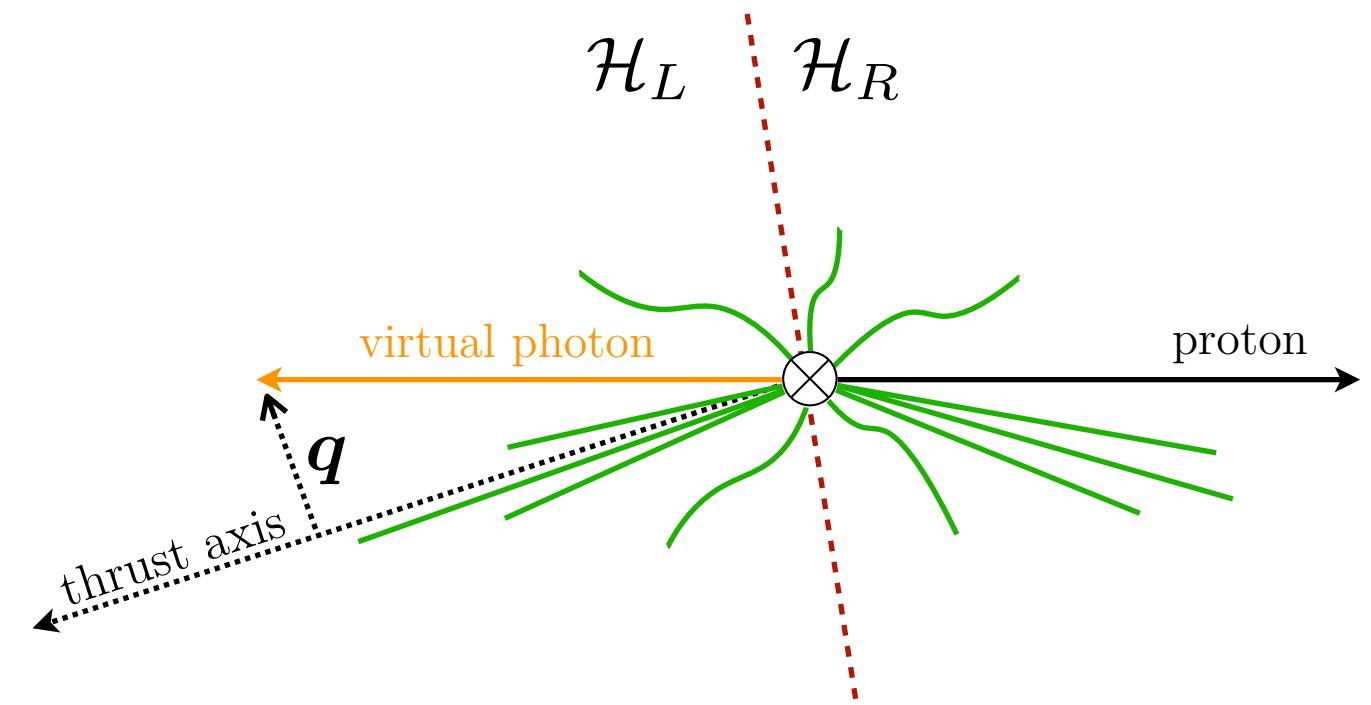
# Thrust - TMDs

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electron-positron annihilation



DIS



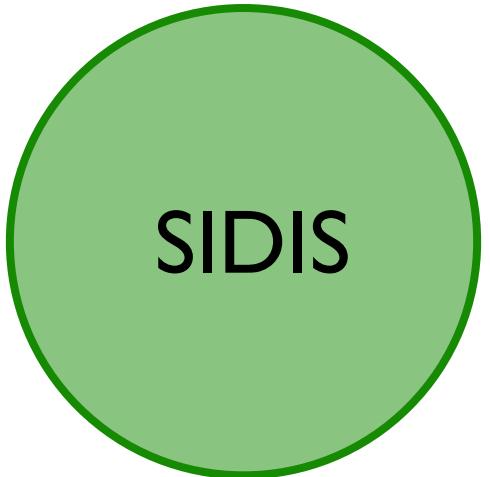
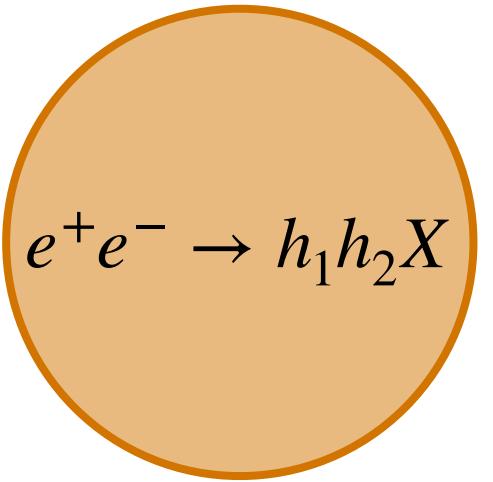
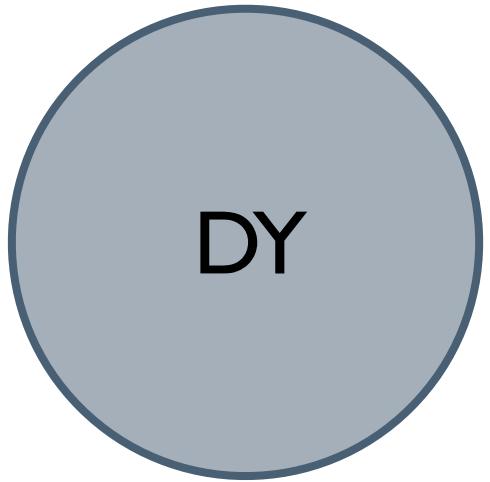
Transverse momentum sensitive about R-hemisphere (contrast with SIDIS)

Thrust is a global observable

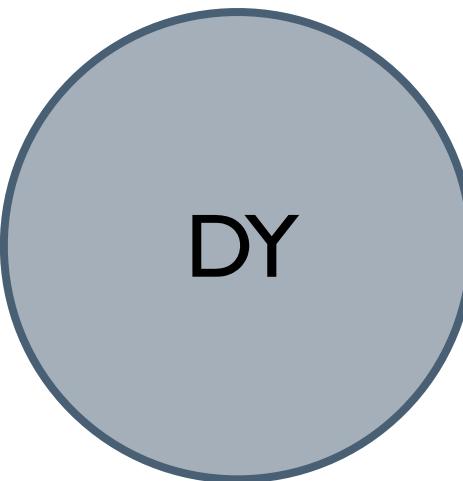
# Thrust - TMDs

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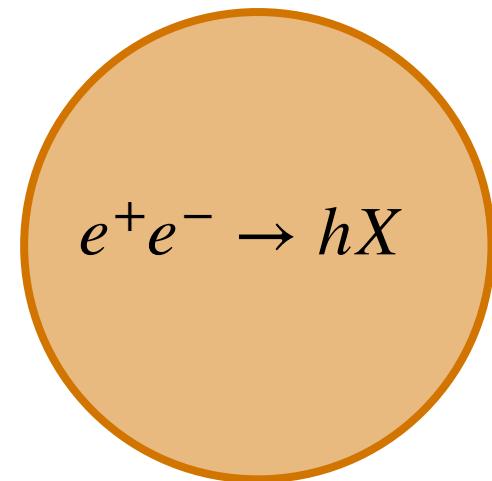
Conventional TMDs



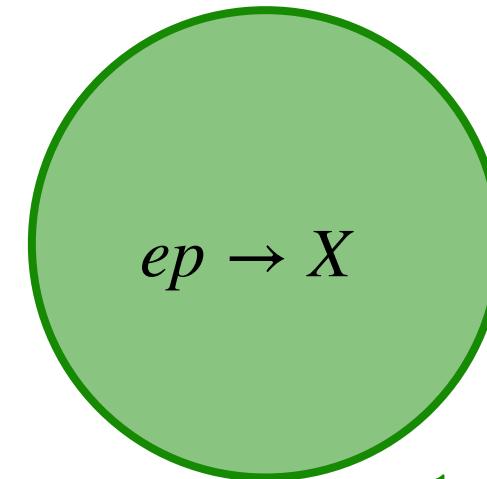
“Thrust” TMDs



0-jettiness



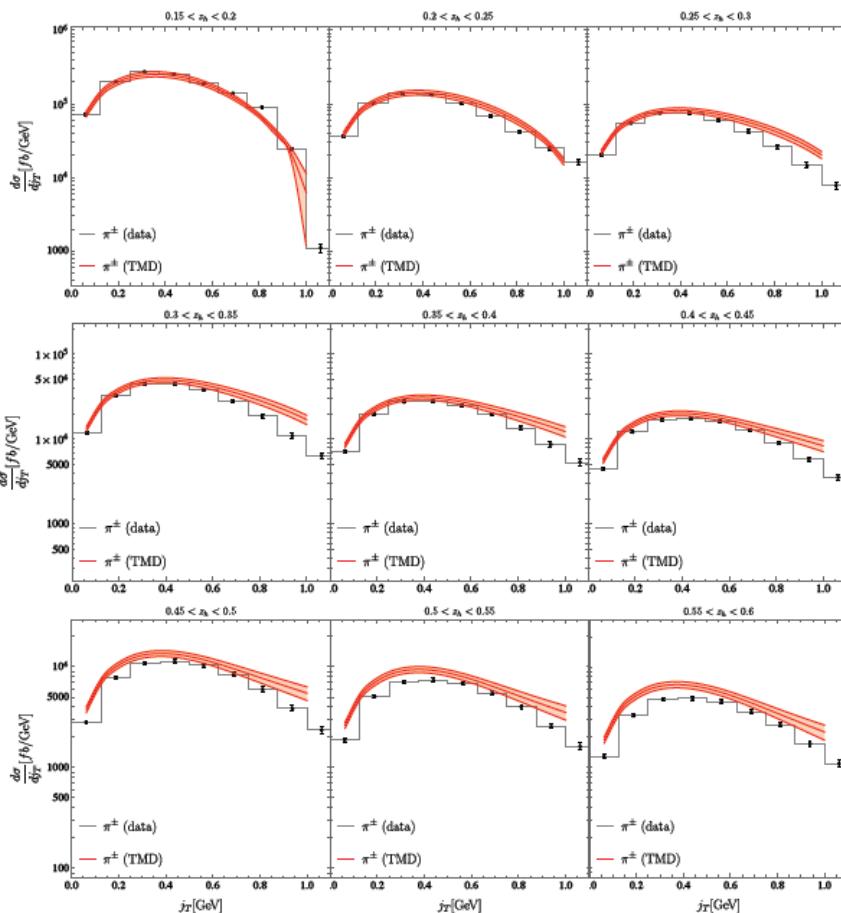
thrust



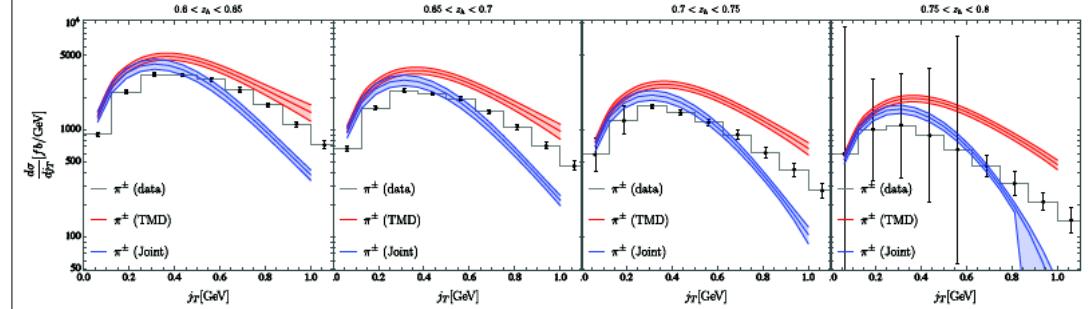
1-jettiness

- Two different kinematic regions are considered,  $j_T \ll Q$  and  $j_T \ll Q(1 - z_h) \ll Q$ .
- Using effective theory methods,  $\log Q/j_T$  and  $\log(1 - z_h)$  are resummed to all orders.
- Thrust dependence integrated out.

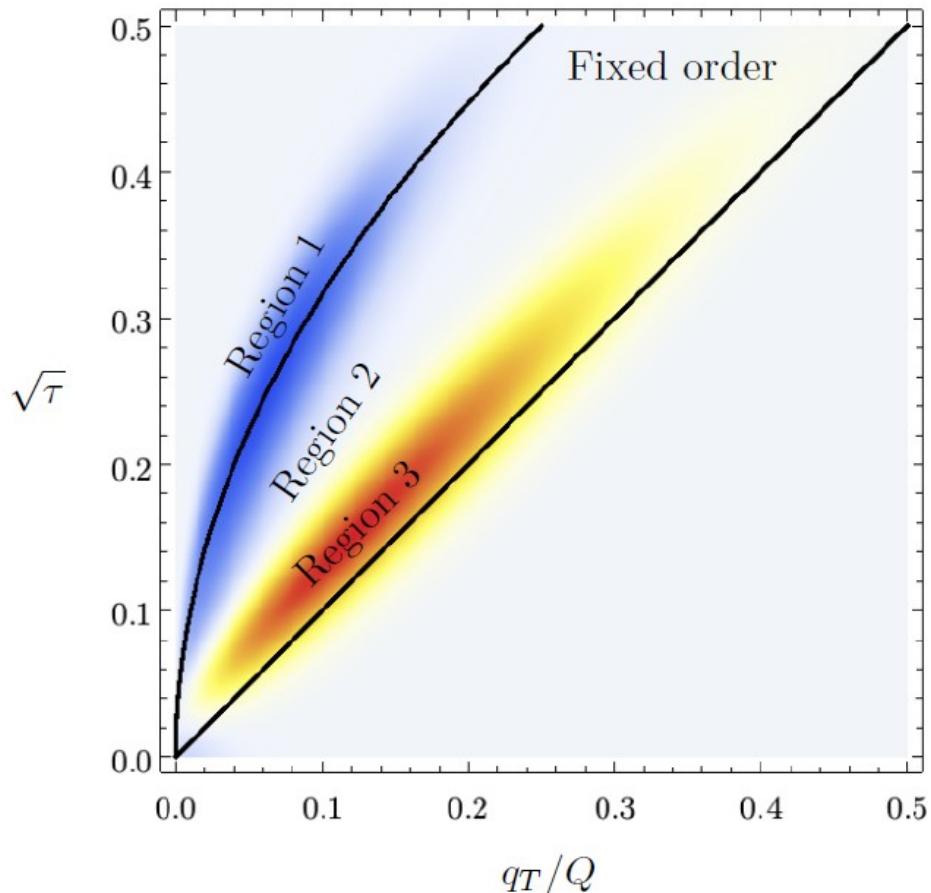
Differential Cross Section ( $\pi^\pm$ ) as function of  $j_T$  TMD resummation applied.



$j_T$ -distribution ( $\pi^\pm$ ) given by TMD resummation (red band) and joint TMD and threshold resummation (blue band)



- Three different kinematic regions are identified.
- Factorization theorems within SCET for each individual region provide the cross section of  $e^+e^- \rightarrow HX$ , differential in  $z_h$ ,  $P_T$  and  $T$ .
- Joint resummation of the transverse momentum and thrust spectrum at NNLL accuracy.
- Final result obtained by matching the cross sections in the three kinematic regions.



Region 1:  $\sqrt{\tau} \gg q_T/Q \sim \tau$

Region 2:  $\sqrt{\tau} \gg q_T/Q \gg \tau$

Region 3:  $\sqrt{\tau} \sim q_T/Q \gg \tau$

Matching:

$$d\sigma = d\sigma_2|_{\mu_2} + [d\sigma_1 - d\sigma_2]_{\mu_1} + [d\sigma_3 - d\sigma_2]_{\mu_3} + \\ + [d\sigma_{FO} - d\sigma_1 - d\sigma_3 - d\sigma_2]_{\mu_{FO}}$$