Factorized approach to radiative corrections for lepton-hadron semi-inclusive deep inelastic scatterings

Jianwei Qiu

Theory Center, Jefferson Lab

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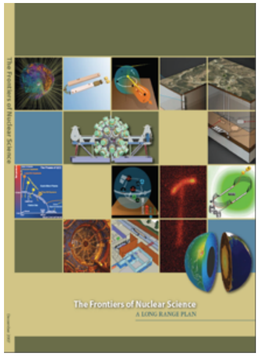
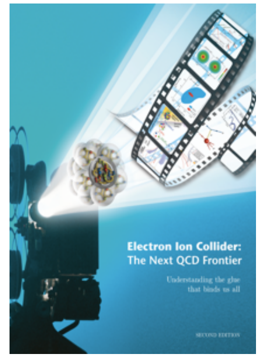
Based on works done with T. Liu, W. Melnitchouk, N. Sato

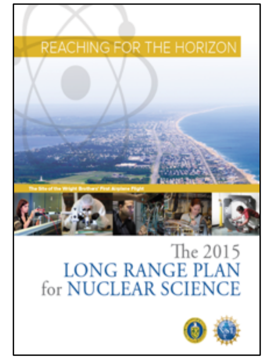
U.S. - based Electron-Ion Collider

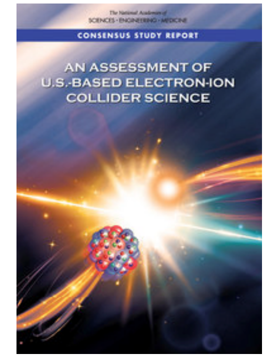
- A long journey, a joint effort of the full community:

…









…

"… answer science questions that are compelling, fundamental, and timely, and help maintain U.S. scientific leadership in nuclear physics."

… three profound questions:

How does the mass of the nucleon arise?
How does the spin of the nucleon arise?
What are the emergent properties of dense systems of gluons?

- On January 9, 2020:

The U.S. DOE announced the selection of BNL as the site for the Electron-Ion Collider

A new era to explore the emergent phenomena of QCD!

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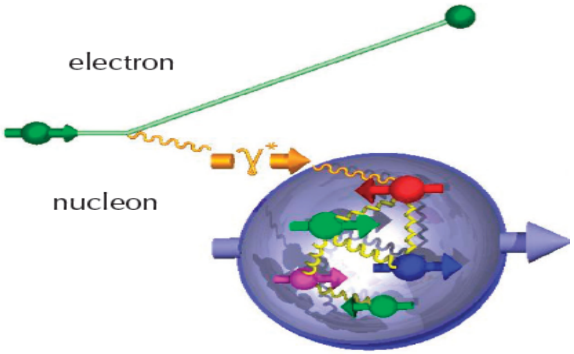
High energy lepton-hadron scattering

- ❑ **A new generation of the "Rutherford" experiment:**



✧ **A controlled "probe" – virtual photon**

✧ **Can either break or not break the hadron**

One facility covers all!

✧ **Inclusive events:** e+p/A → e'+X

Detect only the scattered lepton in the detector

(Modern Rutherford experiment!)

✧ **Semi-Inclusive events:** e+p/A → e'+h(p,K,p,jet)+X

Detect the scattered lepton in coincidence with identified hadrons/jets

(Initial hadron is broken – confined motion! – cleaner than h-h collisions)

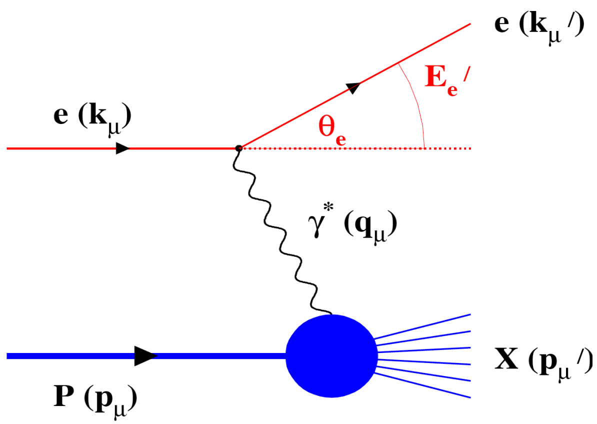
✧ **Exclusive events:** e+p/A → e'+ p'/A'+ h(p,K,p,jet)

Detect every things including scattered proton/nucleus (or its fragments)

(Initial hadron is NOT broken – tomography! – almost impossible for h-h collisions)

Inclusive inelastic lepton-hadron scattering

- **Approximation of one-photon exchange:**



$Q^2 = -(k-k')^2$ → **Measure of the resolution**

$y = P.(k-k')/P.k$ → **Measure of inelasticity**

$x_B = Q^2/2P.(k-k')$

→ **Measure of momentum fraction of the struck quark in a proton**

$Q^2 = S\, x_B\, y$

$$E'\frac{d\sigma}{d^3l'} = \frac{\alpha_{\rm EM}^2}{2\pi s}\int d^4q \sum_X \left| \langle k'|j_\mu|k\rangle \frac{1}{q^2}\langle X|J^\mu|P\rangle\right|^2 (2\pi)^4\delta^4(P+q-X)\,\delta^4(q-k+k')$$

$$= \frac{2\alpha_{\rm EM}^2}{Q^4 s}\, L^\mu(k,k';q)\, W_{\mu\nu}(q,P)$$

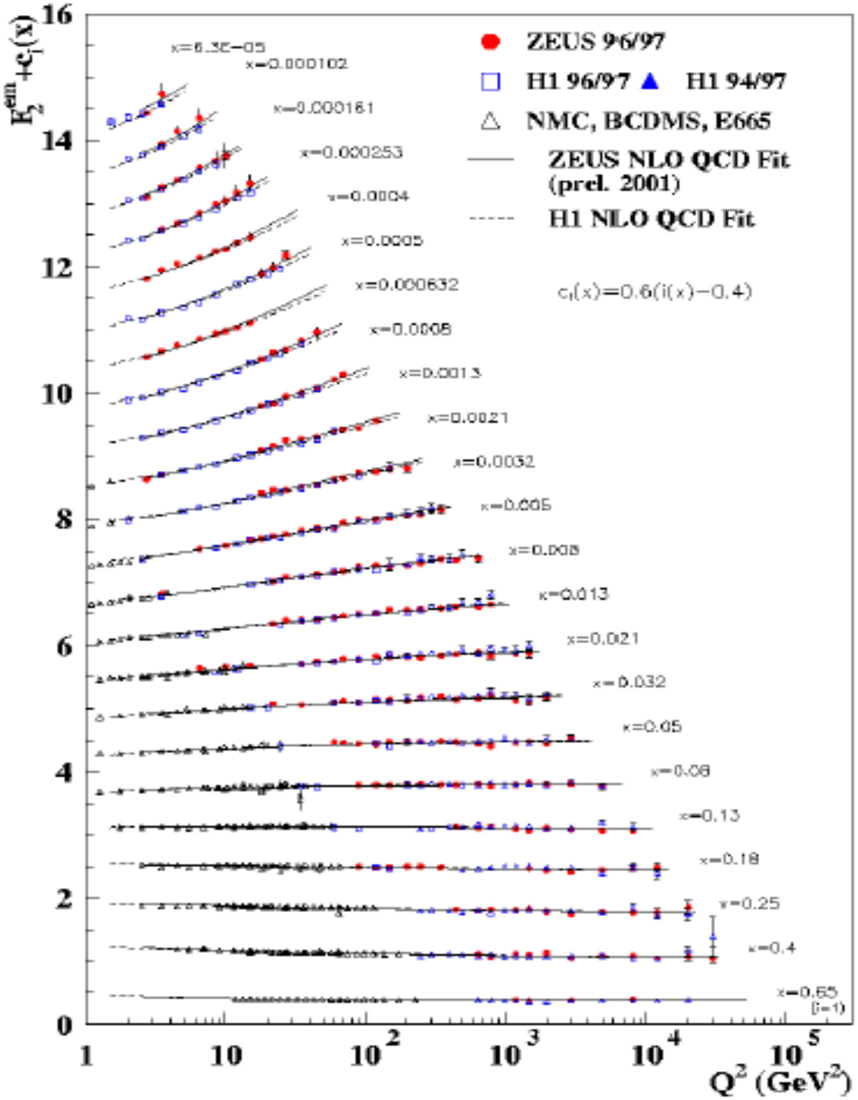
- **Deep inelastic scattering (DIS) structure functions:**

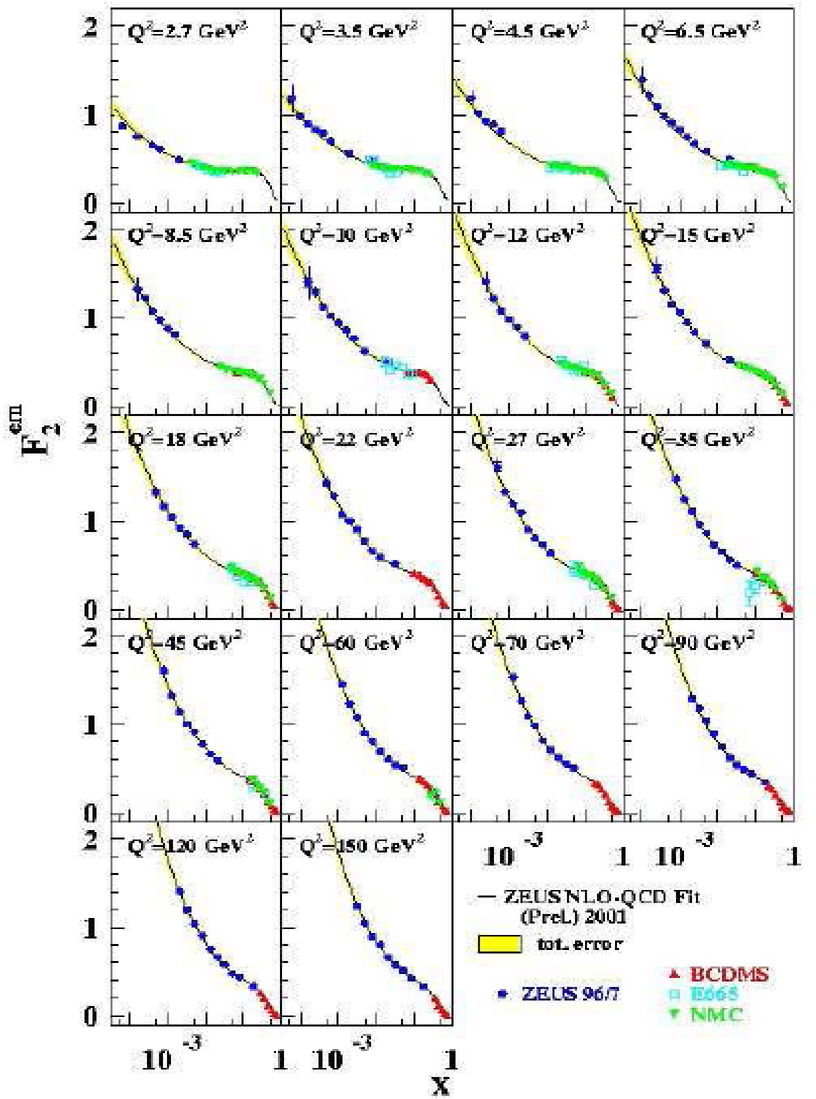
$$W_{\mu\nu}(q,P) = \frac{1}{4\pi}\sum_X (2\pi)^4\delta^4(P+q-X)\,\langle P|J_\mu(0)|X\rangle\,\langle X|J_\nu(0)|P\rangle + \text{spin}\ldots$$

$$= -\widetilde{g}_{\mu\nu}F_1(x_B,Q^2) + \frac{\widetilde{P}_\mu\widetilde{P}_\nu}{P\cdot q}F_2(x_B,Q^2) + \text{spin}\ldots$$

$$\widetilde{g}_{\mu\nu} = -g_{\mu\nu} + q_\mu q_\nu/q^2 \qquad \widetilde{P}_\mu = \widetilde{g}_{\mu\nu}P^\nu$$

Inclusive inelastic lepton-hadron scattering





A very successful story of QCD, QCD Factorization, and QCD evolution!

Extraction of Parton Distribution Functions (PDFs) – hadron structure

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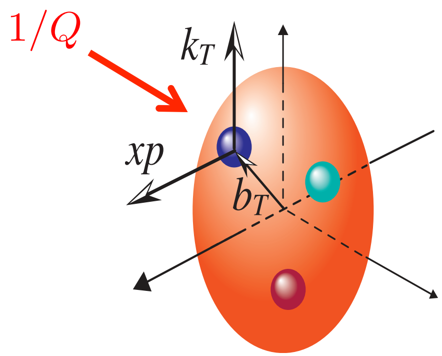
New-type probes for 3D hadron structure

- Single scale hard probes is too "localized":



- It pins down the particle nature of quarks and gluons
- But, not very sensitive to the detailed structure of hadron ~ fm
- Transverse confined motion: k_T ~ 1/fm << Q
- Transverse spatial position: b_T ~ fm >> $1/Q$

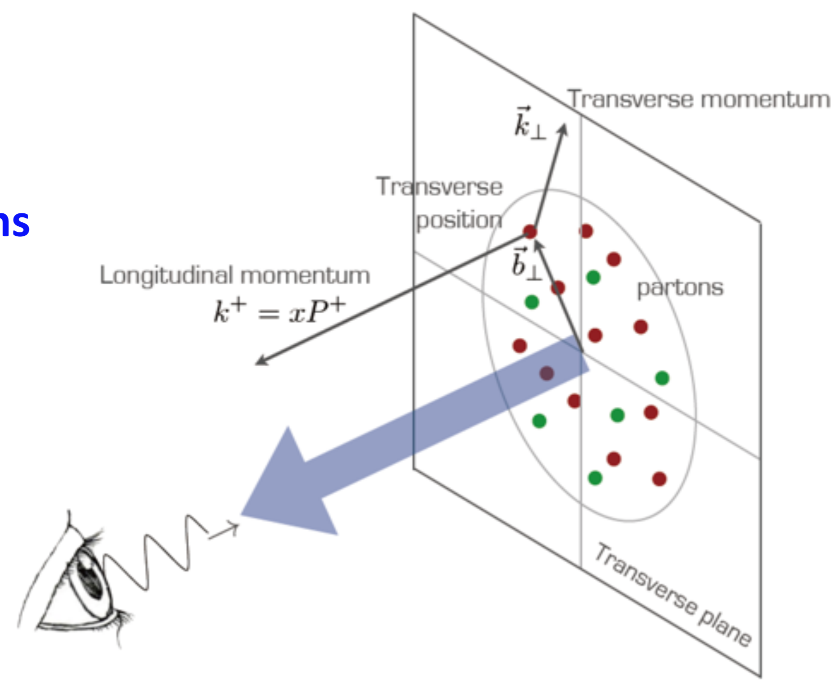
- Need new type of "Hard Probes" – Physical observables with TWO Scales:

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

Hard scale: Q_1 To localize the probe
particle nature of quarks/gluons

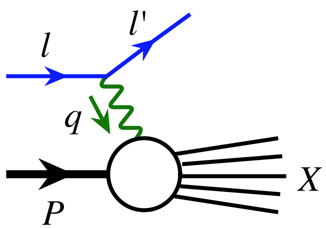
"Soft" scale: Q_2 could be more sensitive to the
hadron structure ~ 1/fm

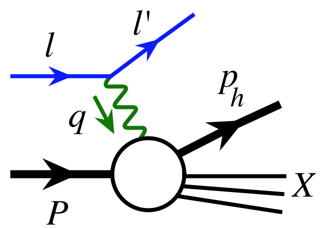
Hit the hadron "very hard" without breaking it,
clean information on the structure!

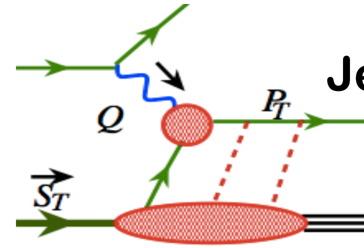


Semi-inelastic lepton-hadron scattering

❑ Semi-inclusive DIS:





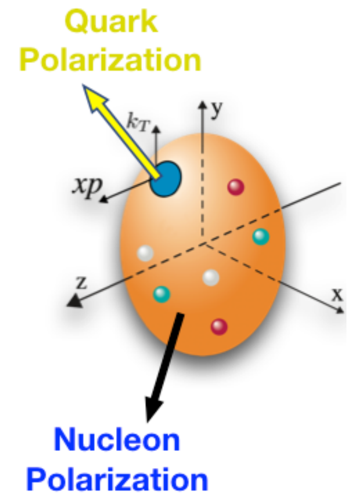


Jet, π, J/ψ, …

Scale: Q^2 $\quad Q^2 \gg P_{h_T}^2$ $\quad f(x, k_T, Q)$ **- TMDs**

In photon-hadron frame! **Parton's confined motion, …**

		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1(x,k_T^2)$		$h_1^\perp(x,k_T^2)$ - Boer-Mulders
	L		$g_1(x,k_T^2)$ - Helicity	$h_{1L}^\perp(x,k_T^2)$ - Long-Transversity
	T	$f_1^\perp(x,k_T^2)$ - Sivers	$g_{1T}(x,k_T^2)$ - Trans-Helicity	$h_1(x,k_T^2)$ - Transversity; $h_{1T}^\perp(x,k_T^2)$ - Pretzelosity



- Gluon: $f_q \rightarrow f_g$
- FFs
- Nuclei: $s \neq \frac{1}{2}$

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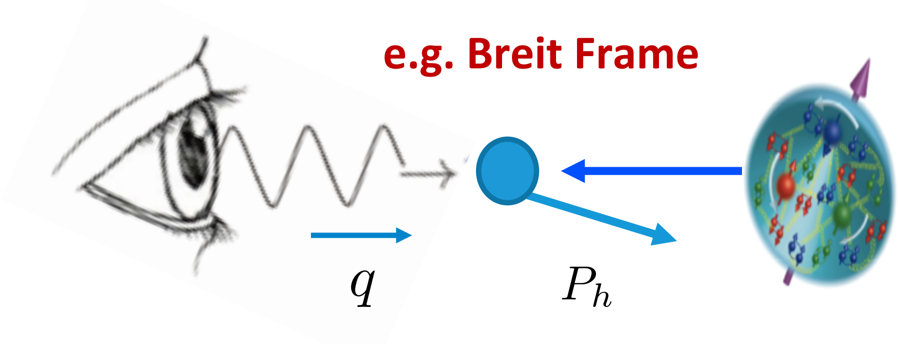
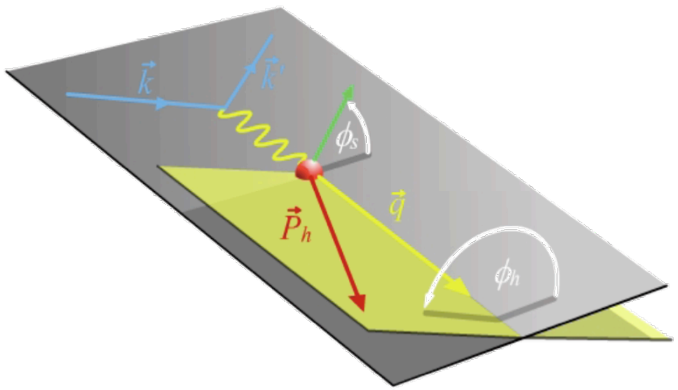
Semi-inelastic lepton-hadron scattering

- Photon-hadron frame:



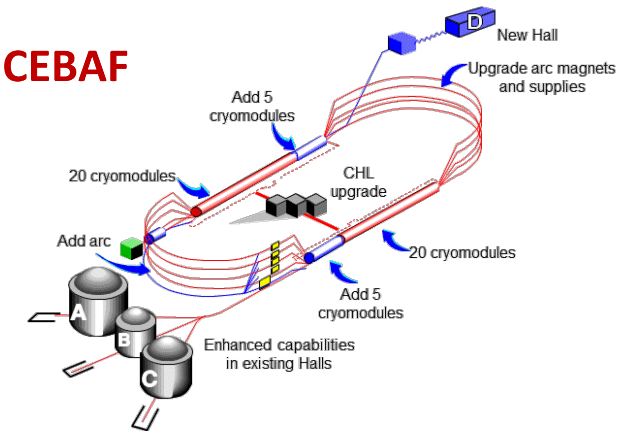
- Leptonic + hadronic planes:

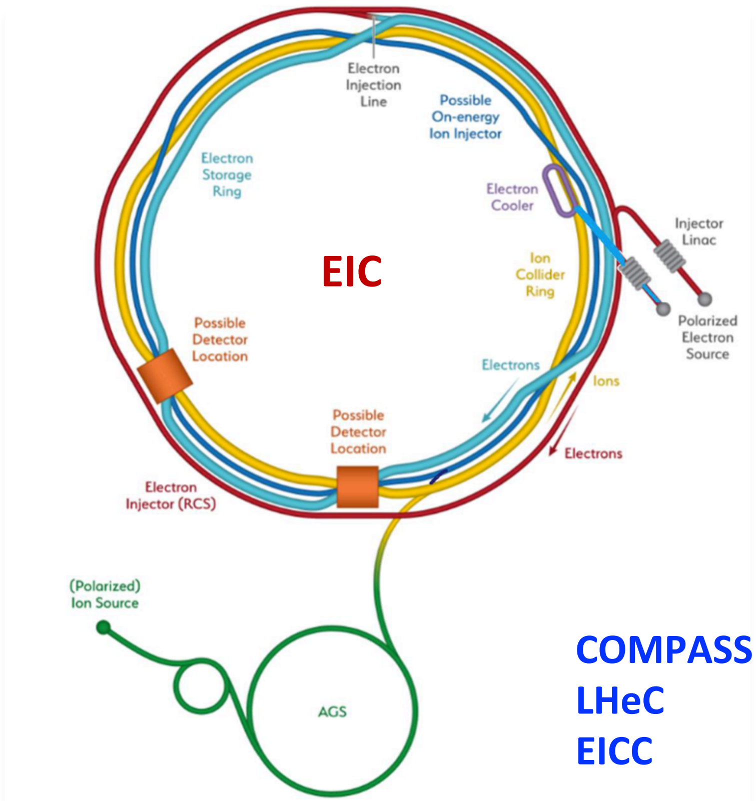


$$A_{UT}^{Collins} \propto \langle \sin(\phi_h + \phi_S) \rangle_{UT} \propto h_1 \otimes H_1^{\perp}$$

$$A_{UT}^{Sivers} \propto \langle \sin(\phi_h - \phi_S) \rangle_{UT} \propto f_{1T}^{\perp} \otimes D_1$$

$$A_{UT}^{Pretzelosity} \propto \langle \sin(3\phi_h - \phi_S) \rangle_{UT} \propto h_{1T}^{\perp} \otimes H_1^{\perp}$$





COMPASS
LHeC
EICC

QED radiative corrections

- **Collision of charged particles triggers radiation!**

  - **Inelastic collision breaks the proton:**

    QCD radiation ⟶ QCD evolution, high order corrections, resummation, …

  - **QED radiation – emission of photon from lepton and quark, …**

    Well-studied topic – too many references to list here

    L.W. Mo and Y.S. Tsai, Rev. Mod. Phys. 41 (1969) 205

    D.Y. Bardin, et al. Z. Phys. C 42 (1989) 679

    If not precisely observed, emission of real photon will

    - **change the inelastic cross section, …**
    - **change the kinematics – the meaning of x_B, Q^2, …**
    - **make the photon-hadron frame ill-defined – crucial for SIDIS, …**
    - **make the angular modulation between leptonic and hadronic planes inaccurate – critical for separating various TMDs, …**

How big the effect is?

How precisely we can account for this effect?

What could be the impact for the future EIC?, or CEBAF in a near term, …

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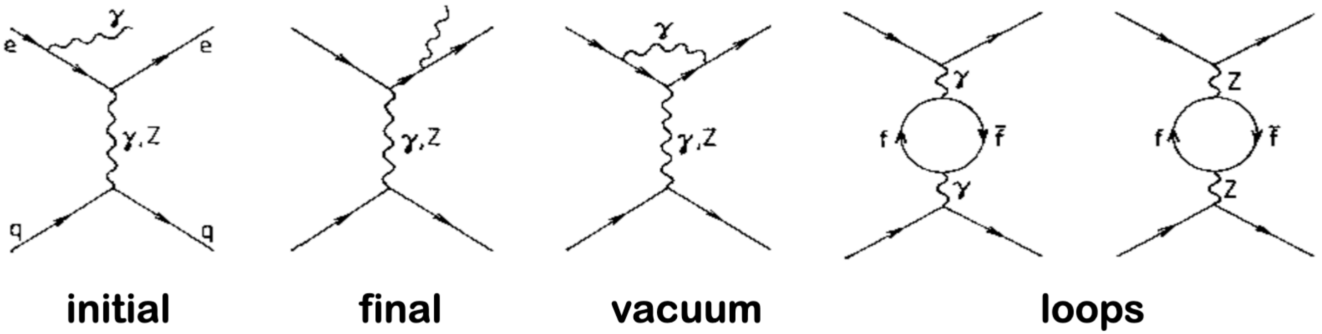
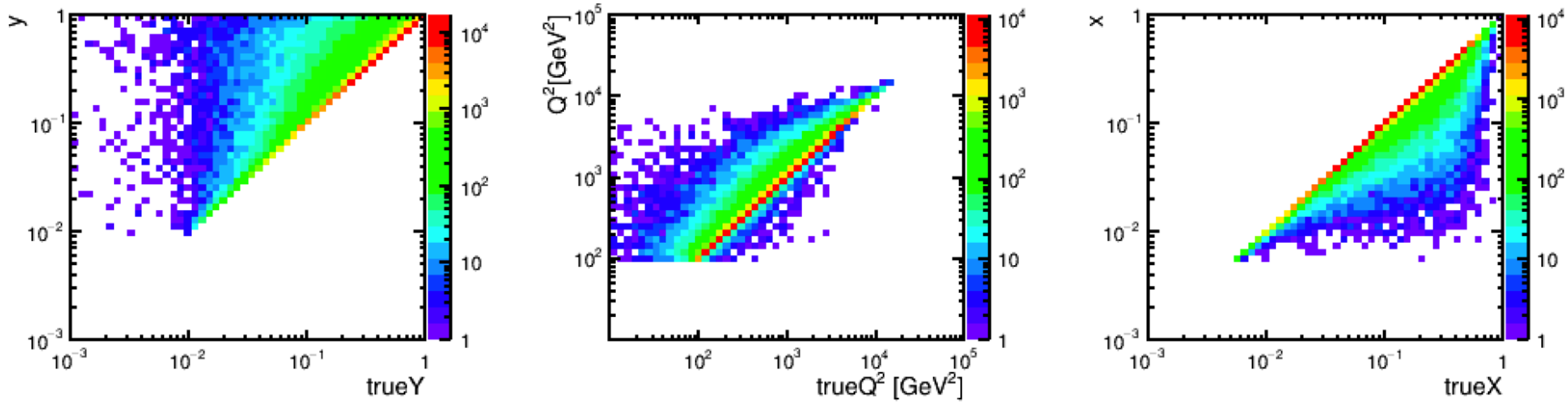
QED radiative corrections

See Xiaoxuan Chu
@2nd EIC YR workshop

❑ Kinematics is smeared by radiative corrections:

Data sample : Int L = 10 fb^{-1}, Kinematics settings: $0.01<y<0.95$, 10^2 $GeV^2 < Q^2 < 10^5$ GeV^2





Instead of a straight line – linear correlation,

the kinematic variables, y, Q^2 , x_B, from the leptons are smeared so much to make them different from what the scattered "quark" experienced!

Trouble with the "photon-hadron" frame?!

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QED radiative corrections

- Radiative correction factor is too big to be comfort:

See B. Badelek et al.
Z Phys C 66 (1995) 591

$$\eta(x,y) = \frac{\sigma_{1\gamma}}{\sigma_{\text{meas}}}$$

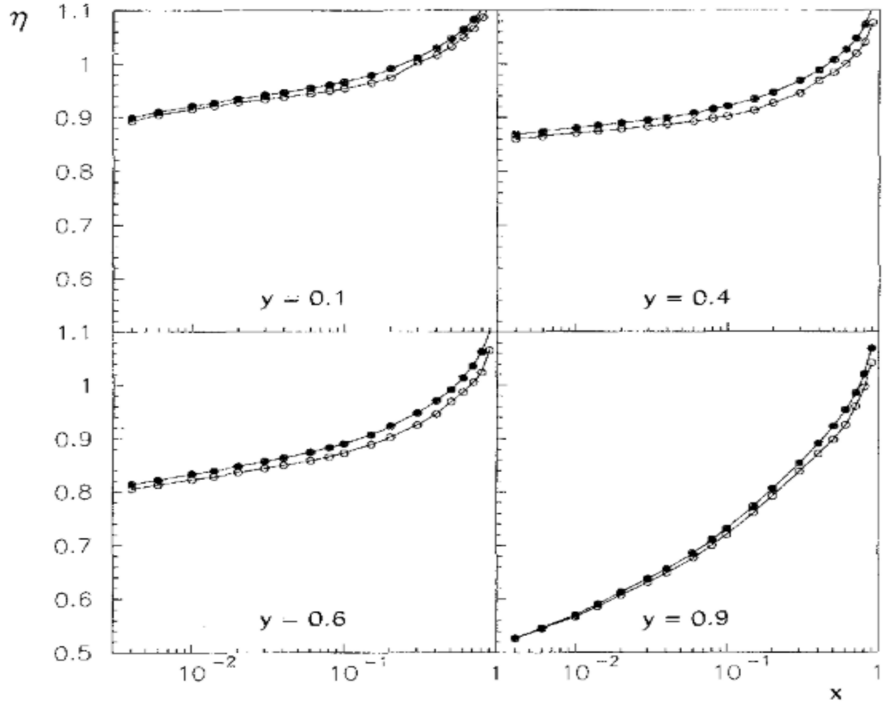

Fig. 5. Radiative correction factor η calculated in FERRAD35 (open symbols) and TERAD86 (closed symbols) for the muon – proton scattering at 280 GeV

Radiative corrections are very large, exceeding 50% at low x and high y region!

Recall: $y = \frac{2P \cdot q}{2P \cdot l}$ $\quad x_B = \frac{Q^2}{2P \cdot q}$ $\quad Q^2 = x_B\, y\, S$

larger phase-space for shower (smaller x_B), Larger momentum transfer (larger y)
larger radiative corrections!

Fits to EIC kinematics?!

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QED radiative corrections

❑ Radiative corrections can cause trouble, …

Can mimic new/unexpected physics, if not handled correctly, *e.g.*

- "HERMES effect", with apparent enhancement of nuclear $R=\sigma_L/\sigma_T$ ratio at $x_B < 0.03$ and $Q^2 < 2$ GeV^2
  - Original paper: Ackerstaff et al., PLB 475, 386 (2000)
  - Erratum: Airapetian et al., PLB 567, 339 (2003)
  - Interesting physics interpretations of original data [*e.g.*, Miller, Brodsky, Karliner, PLB 481, 245 (2000)]
- Nuclear EMC effect, with discrepancy between early EMC and BCDMS data at low x_B (enhancement rather than "shadowing")
  - Coulomb corrections have not always been consistently applied [*e.g*. Solvignon, Gaskell, Arrington, AIP Conf. Proc. 1160, 155 (2009)]
- …

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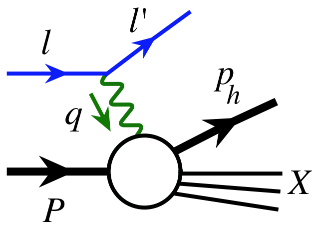
Questions

❑ **Radiative corrections – Born kinematics:**



$$\sigma_{\text{Measured}} = RC \otimes \sigma_{\text{No QED Radiation}}$$

- **Uniquely determined "q" – a clean and controllable EM probe**
- **A well-defined hadronic tensor – DIS Structure Functions**

$$W_{\mu\nu}(q,P) = \frac{1}{4\pi}\sum_X (2\pi)^4\delta^4(P+q-X)\,\langle P|J_\mu(0)|X\rangle\,\langle X|J_\nu(0)|P\rangle + \text{spin}\ldots$$
$$= -\widetilde{g}_{\mu\nu}F_1(x_B,Q^2) + \frac{\widetilde{P}_\mu\widetilde{P}_\nu}{P\cdot q}F_2(x_B,Q^2) + \text{spin}\ldots$$

- **OPE works – QCD factorization to all powers (or twists)**

$$F_i(x_B,Q^2) = \sum_f C_i(x_B,x;Q^2,\mu^2)\otimes f(x,\mu^2) + \mathcal{O}(1/Q^2)$$

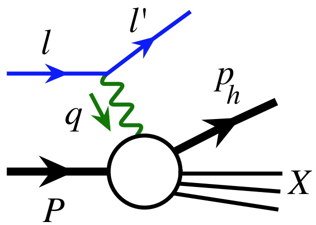
But, ○ **Traditional approach of Mo and Tsai:**

- **depends on unphysical parameter separating soft and hard regions of the phase space of radiated photon in order to cancel infrared divergences**
- **is not easily transferrable to application to other processes, *e.g*. SIDIS**

○ **Differences between different schemes (*e.g*., Mo-Tsai and Bardin-Shumeiko) can be as large or larger than some systematic errors in the data analysis [Badelek et al., Z. Phys. C66 (1995) 591]**

Is the Born kinematics necessary for extracting PDFs or TMDs?

Questions



- **Radiative corrections – Born kinematics:**

$$\sigma_{\text{Measured}} = RC \otimes \sigma_{\text{No QED Radiation}}$$

- **The "photon-hadron" frame is critical for SIDIS**
- **QCD factorization is proved to be valid for all P_T, as long as Q^2 is sufficiently large**
  - **Low P_{hT} ($P_{hT} << Q$) – TMD factorization:**

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f(x, k_\perp) \otimes \mathcal{D}_{f\to h}(z, p_\perp) \otimes \mathcal{S}(k_{s\perp}) + \mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

  - **High P_{hT} ($P_{hT} \sim Q$) – Collinear factorization:**

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q, P_{h\perp}, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{P_{h\perp}}, \frac{1}{Q}\right)$$

  - **P_{hT} Integrated - Collinear factorization:**

$$\sigma_{\text{SIDIS}}(Q, x_B, z_h) = \tilde{H}(Q, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{Q}\right)$$

  - **Very high $P_{hT} >> Q$ – Collinear factorization:**

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \sum_{abc} \hat{H}_{ab\to c} \otimes \phi_{\gamma\to a} \otimes \phi_b \otimes D_{c\to h} + \mathcal{O}\left(\frac{1}{Q}, \frac{Q}{P_{h\perp}}\right)$$

But, **Radiative corrections can change the "direction" and "value" of the "q" and make the "real" Q^2 to be small (and very small !)**

Is the Breit frame experimentally attainable?
Can we achieve the factorization for extracting the same distributions without requiring the Born kinematics?

Basic ideas for our new approach

- **Do not try to invent any new scheme to treat QED radiation to match to the Born kinematics – NO Radiative Correction !**

- **Develop a reliable formalism that can extract the PDFs, TMDs and parton correlation functions, systematically with controllable and consistent approximations, without requiring the "one-photon" approximation and the "photon-hadron" frame !**

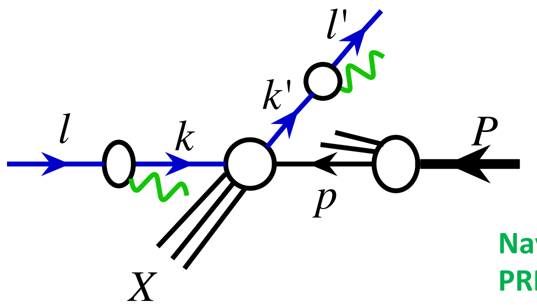
- **Generalize the QCD factorization to include Electroweak theory**
  - **QED radiation is a part of the production cross sections**
  - **QED radiation is treated in the same way as QCD radiation is treated**

Note: our new approach is more relevant for high-energy process where a large GeV scale exists and collinear logarithms caused by radiation are important

Inclusive inelastic deep inelastic scattering

- **Inclusive DIS**
  = Inclusive production of a high transverse momentum lepton
  in lepton-hadron collision frame

$$e(l) + h(P) \to e'(l') + X$$



Nayak, Qiu, Sterman
PRD72 (2005) 114012

- **Factorization proof:**
  = Factorization proof of single hadron production hadronic collisions

$$E' \frac{d\sigma_{eh\to e'X}}{d^3l'} \approx \frac{1}{2s} \sum_{i,j,a} \int_{z_L}^{1} \frac{d\zeta}{\zeta^2} \int_{x_L}^{1} \frac{d\xi}{\xi} D_{e'/j}(\zeta) f_{i/e}(\xi) \int_{x_h}^{1} \frac{dx}{x} f_{a/h}(x)\, \hat{H}^{(m,n)}_{ia\to j}(\xi,\zeta,x;l')$$

$m:$ QED power
$n:$ QCD power

$$s = (P+l)^2 \approx 2P\cdot l$$

i, j, a — **include all QED and light flavor QCD particles**

In the following discussion, we take valence approximation: $i = j = e$

$f_{i/e}(\xi)$ — **Lepton distribution functions (LDFs)**
$D_{e/j}(\zeta)$ — **Lepton fragmentation functions (LFFs)**
} **include all collinear sensitivities as** $m_e \to 0$

$\hat{H}_{ia\to j}$ — **Infrared safe, insensitive to** $m_e \to 0 \quad m_q \to 0$

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Inclusive inelastic deep inelastic scattering

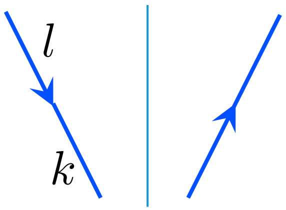
❑ Lepton distribution:

QED gauge link

$$f_{e/l}(\xi) = \int \frac{dy^-}{4\pi} e^{i\xi\, l^+ y^-} \langle l | \overline{\psi}(0) \gamma^+ \Phi(0, y^-) \psi(y^-) | l \rangle$$

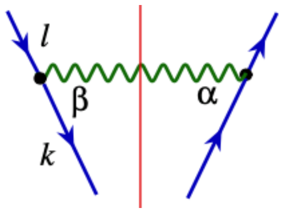
Similar to the definition of quark PDFs

- **Leading order:**



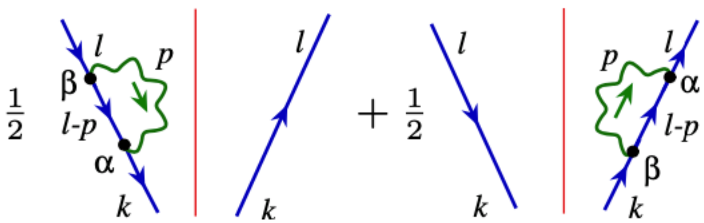
$$f_{e/l}^{(0)}(\xi) = \frac{1}{4l \cdot n} \mathrm{Tr}\left[\gamma \cdot n \gamma \cdot l\right] \delta(\xi - \frac{k \cdot n}{l \cdot n}) d^4k \delta^4(k - l) \delta_{el}$$
$$= \delta(\xi - 1)\delta_{el}$$

- **Next-to-Leading order (MSbar scheme):**



$$f_{e/e}^{\mathrm{Real}(1)}(\xi, \mu^2) = \frac{\alpha_{\mathrm{em}}}{2\pi} \left[\frac{1+\xi^2}{1-\xi} \ln\left(\frac{\mu^2}{(1-\xi)m_e}\right)\right]$$

$$f_{e/e}^{(1)}(\xi, \mu^2) = \frac{\alpha_{\mathrm{em}}}{2\pi} \left[\frac{1+\xi^2}{1-\xi} \ln\left(\frac{\mu^2}{(1-\xi)m_e}\right)\right]_+$$



❑ Resummation:

$$\mu^2 \frac{d}{d\mu^2} f_{e/e}(\xi, \mu^2) = \int_\xi^1 \frac{d\xi'}{\xi'} P_{ee}(\xi/\xi', \alpha) f_{e/e}(\xi', \mu^2) \qquad P_{e/e}^{(1)}(z, \alpha) = \frac{\alpha_{\mathrm{em}}}{2\pi}\left[\frac{1+z^2}{1-z}\right]_+$$

QED DGLAP evolution with QED kernels

Inclusive inelastic deep inelastic scattering

- **Lepton fragmentation function:**

QED gauge link

$$D_{l/e}(\zeta) = \int \frac{dy^-}{4\pi} e^{il^+y^-/\zeta} \frac{\zeta}{2} \mathrm{Tr}\left[\gamma^+ \langle 0|\overline{\psi}(0)\Phi(0,\infty)|l,X\rangle\langle\psi(y^-)\Phi(y^-,\infty)|0\rangle\right]$$

Similar to definition of quark FFs

- **Leading order:**

$$D^{(0)}_{l/e}(\zeta) = \delta(\zeta - 1)\delta_{el}$$

- **Next-to-Leading order (MSbar scheme):**

$$D^{(1)}_{l/e}(\zeta) = \frac{\alpha_{\rm em}}{2\pi}\left[\frac{1+\zeta^2}{1-\zeta}\ln\left(\frac{(1-\zeta)\mu^2}{\Delta E^2}\right)\right]_+$$

ΔE^2

invariant mass resolution of the radiated photon

$\Delta E^2 = 0.01\ \mathrm{GeV}^2$

Used in numerical calculations

- **Resummation:**

LFFs obey the same QED DGLAP evolution of the LPFs

- **Input distributions at the input scale** $\mu_0^2 \sim m_e^2$

Unlike input distributions for PDFs, which are non-perturbative, input distribution for LDFs and LFFs are perturbatively calculable, …

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Inclusive inelastic deep inelastic scattering

❑ LO Factorized inclusive DIS cross section:

$$E' \frac{d\sigma^{(0)}_{eh\to e'X}}{d^3l'} \approx \frac{2\alpha^2_{\rm EM}}{s} \sum_q \int_{z_L}^1 \frac{d\zeta}{\zeta^2} \int_{x_L}^1 \frac{d\xi}{\xi} D_{e/e}(\zeta) f_{e/e}(\xi) \int_{x_h}^1 \frac{dx}{x} e_q^2 \, f_{q/h}(x)\, \delta(x - \frac{-\xi t}{\xi\zeta s + u})$$

$$\times \left[\frac{(x\xi\zeta s)^2 + (xu)^2}{(\xi t)^2}\right] \left[\frac{\zeta}{(\xi\zeta s) + u}\right]$$

LO in hard part, but include all orders resummation into PDFs, FFs

$$\Rightarrow \frac{4\alpha^2_{\rm em}}{Q^2 s} \left[F_1(x_B, Q^2) + \frac{1-y}{x_B y^2} F_2(x_B, Q^2)\right]$$

With LO relation: $F_2(x_B) = 2x_B F_1(x_B) = \sum_q e_q^2 \, x_B f_{q/h}(x_B)$

$$f_{e/e}(\xi) \approx f^{(0)}_{e/e}(\xi) \qquad \xi = \zeta = 1$$

$$D_{e/e}(\zeta) \approx D^{(0)}_{e/e}(\zeta)$$

❑ NLO fixed order QED correction:

By taking: $f_{e/e}(\xi) \approx f^{(0)}_{e/e}(\xi) + f^{(1)}_{e/e}(\xi)$ **or**: $\hat{H}^{(m,n)} \approx \hat{H}^{(2,0)} + \hat{H}^{(3,0)}$

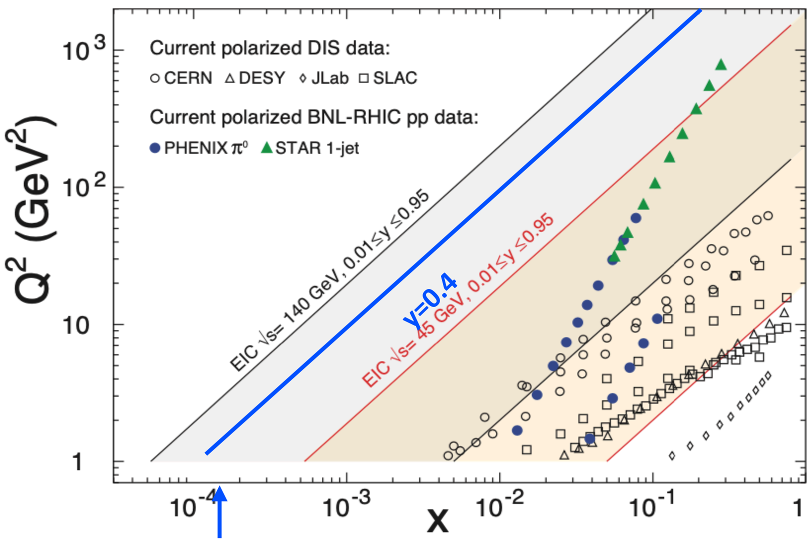
or: $D_{e/e}(\zeta) \approx D^{(0)}_{e/e}(\zeta) + D^{(1)}_{e/e}(\zeta)$

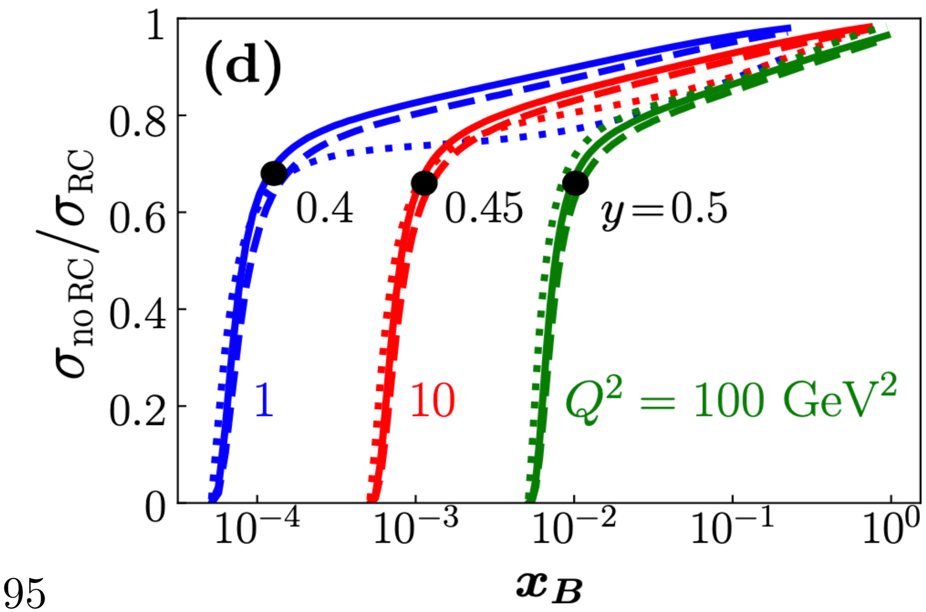
Inclusive inelastic deep inelastic scattering

- **Numerical impact of QED contribution at EIC ($\sqrt{S}=140\ \mathrm{GeV}$):**

$$\frac{\sigma_{\rm noRC}}{\sigma_{\rm RC}} \leftrightarrow \frac{\sigma_{1\gamma}}{\sigma_{\rm measured}} = \eta(x_B, y)$$

B. Badelek et al.
Z Phys C 66 (1995) 591





Almost the same as $\sqrt{S} = 90\ \mathrm{GeV}$ at $y = 0.95$

- **Renormalization and factorization scale choice:**

$\sigma_{\rm noRC} = E' \dfrac{d\sigma}{d^3 l'}$ With $f_{e/e}(\xi) \approx f^{(0)}_{e/e} = \delta(\xi - 1)$ and $D_{e/e}(\zeta) \approx D^{(0)}_{e/e} = \delta(\zeta - 1)$

$\mathrm{NLO}:$ With $f_{e/e}(\xi,\mu) \approx f^{(0)}_{e/e}(\xi) + f^{(1)}_{e/e}(\xi,\mu^2)$ and $D_{e/e}(\zeta,\mu) \approx D^{(0)}_{e/e}(\zeta) + D^{(1)}_{e/e}(\zeta,\mu^2)$

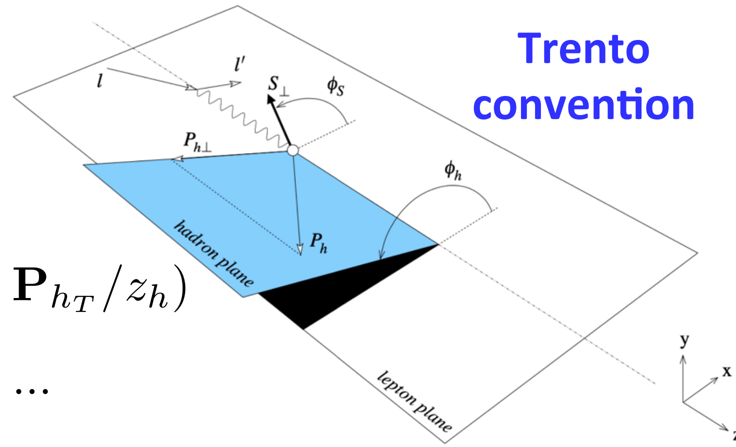
$\mathrm{RES}_{\rm I}$: DGLAP evolved with 0th input $f_{e/e} \approx f^{(0)}_{e/e}$ and $D_{e/e} \approx D^{(0)}_{e/e}$ at $\mu_0^2 = m_e^2$

$\mathrm{RES}_{\rm II}$: DGLAP evolved with input $f_{e/e} \approx f^{(0)}_{e/e} + f^{(1)}_{e/e}$, $D_{e/e} \approx D^{(0)}_{e/e} + D^{(1)}_{e/e}$

Semi-inclusive DIS

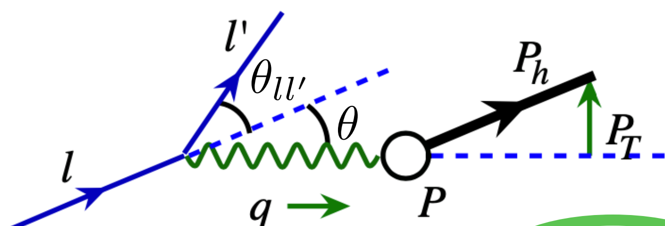
- **Photon-Hadron frame - "Born" kinematic:**

$$E_h \frac{d\sigma}{dx_B dQ^2 d^3P_h} \approx \frac{\alpha^2}{S} \frac{1+(1-y)^2}{y} \frac{z_h}{Q^2}$$
$$\times \int d^2\mathbf{p}_T d^2\mathbf{p}_{h_T} \delta^{(2)}\left(\mathbf{p}_T - \mathbf{p}_{h_T} - \mathbf{P}_{h_T}/z_h\right)$$
$$\times D_{h/j}(z_h, \mathbf{p}_{h_T}) f_{i/h}(x, \mathbf{p}_T) + \ldots$$

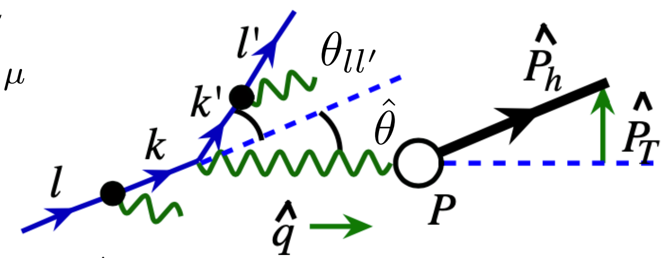


Trento convention

- **QED radiation – NO "Born" kinematic:**



$$q^\mu = (l - l')^\mu$$
$$\hat{q}^\mu = (k - k')^\mu$$



$q^\mu \neq \hat{q}^\mu$ $\quad \theta \neq \hat{\theta} \quad P_T \neq \hat{P}_T$

$$E_h \frac{d\sigma}{dx_B dQ^2 d^3P_h} \approx \int \frac{d\zeta}{\zeta^2} D_{e/e}(\zeta) \int d\xi \; \phi_{e/e}(\xi) E_h \frac{d\hat{\sigma}}{d\hat{x}_B d\hat{Q}^2 d^3P_h} + \ldots$$

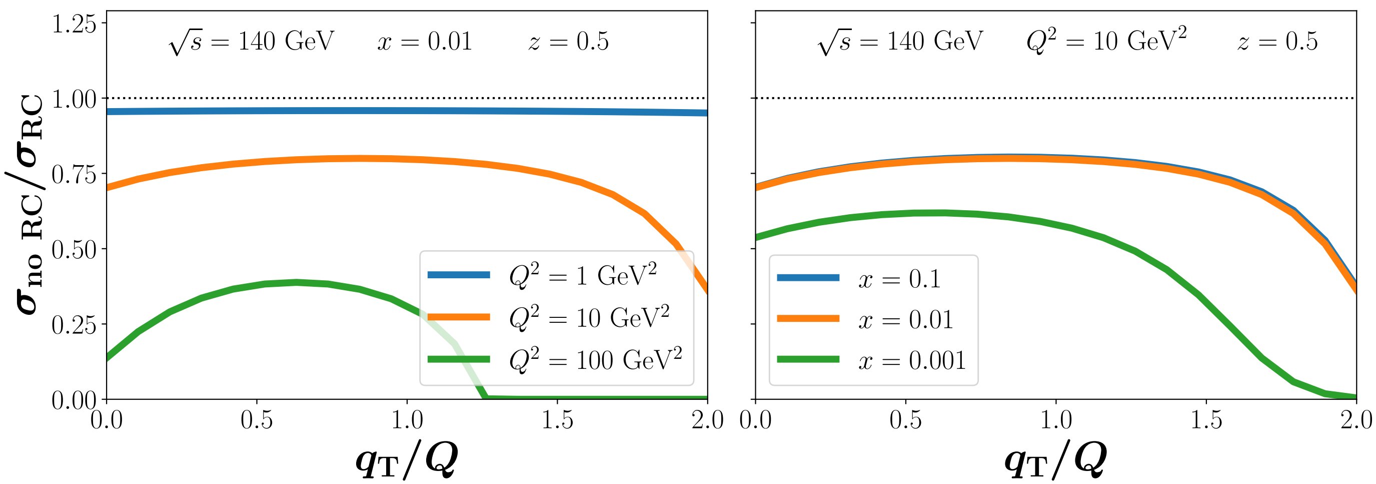
Collinear factorization for QED:

$$\hat{Q}^2 = -(k-k')^2 = \frac{\xi}{\zeta} Q^2 \qquad \hat{x}_B = \frac{\hat{Q}^2}{2p \cdot \hat{q}} = x_B \frac{\xi y}{\xi\zeta - (1-y)}$$

Jefferson Lab

Semi-inclusive DIS

- QED radiation changes P_T , magnitude of P_T spectrum:



$$q_T = P_T$$

In photon-hadron Frame without QED radiation

QED radiation also changes the angular modulation and how to extract different TMDs

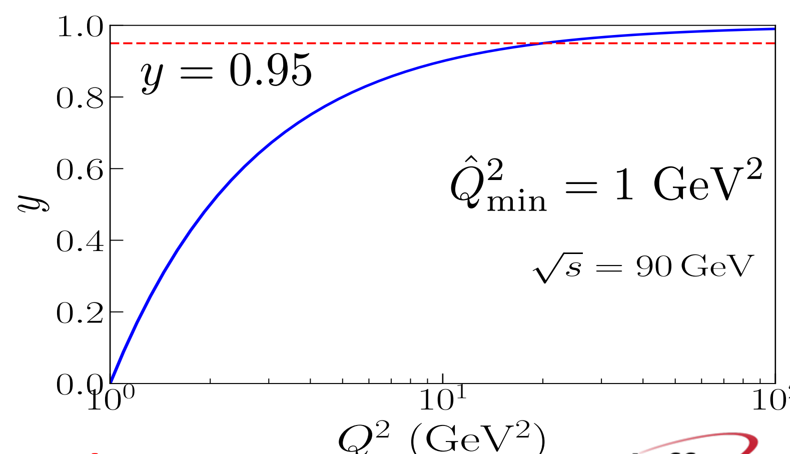
- QED radiation makes the hard scale to be softer!

$$\hat{Q}^2 = Q^2 \left(\frac{\xi}{\zeta}\right)$$

$$\hat{Q}^2_{\text{min}} = Q^2 \left(\frac{1-y}{1-x_B y}\right) \to 0$$

as $y \to 1$

Reduce the kinematic reach



With QED radiation, No photon-hadron frame!

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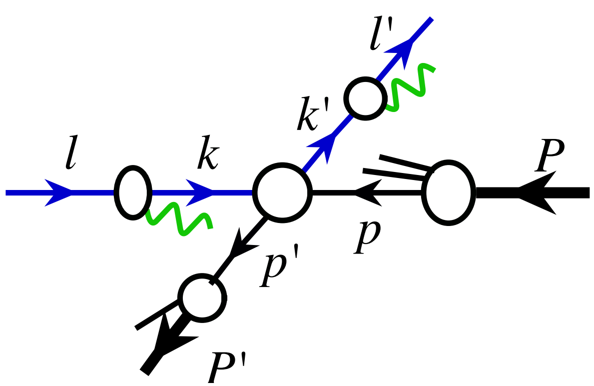
Semi-inclusive DIS

- **Our proposal:**

  Semi-inclusive DIS cross section
  = inclusive electron + hadron (or jet) cross section in lepton-hadron collisions

(lepton or lepton jet)



(hadron or jet)

- **Transverse plane to the colliding axis:**

$$\overline{\mathbf{P}}_T \equiv \frac{1}{2}\left(\boldsymbol{\ell}'_T - \boldsymbol{P}_{hT}\right)$$

$$\overline{\mathbf{p}}_T \equiv \boldsymbol{\ell}'_T + \boldsymbol{P}_{hT}$$

- **Collinear factorization:**

$$|\overline{\mathbf{P}}_T| \sim |\overline{\mathbf{p}}_T|$$

- **TMD factorization:**

$$|\overline{\mathbf{P}}_T| \gg |\overline{\mathbf{p}}_T|$$

"Hard" scale = $\overline{\mathbf{P}}_T$

"Soft" scale = $\overline{\mathbf{p}}_T$

Momentum imbalance between two particles (or jet(s))

- **The "Parton frame":**

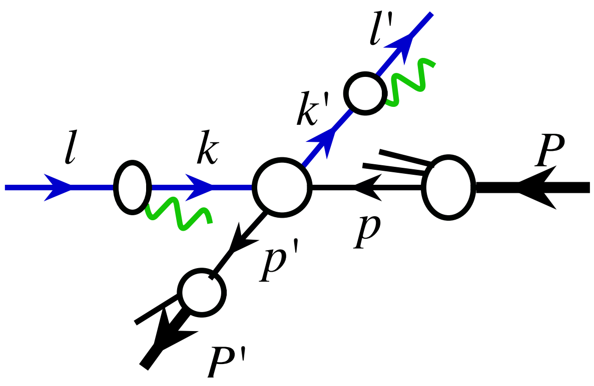
  Boost along the collision axis, such that $y_{\ell'} + y_h = 0$

  $y_{\ell'}$ Rapidity of the observed lepton (or jet) in the lab frame

  y_h Rapidity of the observed hadron (or jet) in the lab frame

Semi-inclusive DIS

❑ Collinear factorization:



Effectively the same as two-jet (or hadron) cross section at a hadron collider

CO-Functions

$$\frac{d\sigma_{\text{SIDIS}}}{dy_{\ell'}\, dy_h\, d^2\overline{\mathbf{P}}_T\, d^2\overline{\mathbf{p}}_T} = \frac{1}{2s}\sum_{i,j,a,b} f_{i/e}\otimes D_{e/j}\otimes f_{a/h}\otimes D_{h'/b}\otimes H^{(m,n)}_{i+a\to j+b+X} + \mathcal{O}(1/|\overline{\mathbf{P}}|, 1/|\overline{\mathbf{p}}|)$$

❑ TMD factorization:

For final-state jet(s): $D \to J$
or fully calculated H_J without D's

TMDs

$$\frac{d\sigma_{\text{SIDIS}}}{dy_{\ell'}\, dy_h\, d^2\overline{\mathbf{P}}_T\, d^2\overline{\mathbf{p}}_T} = \frac{1}{2s}\sum_{i,j,a,b} \tilde{f}_{i/e}\otimes \tilde{D}_{e/j}\otimes \tilde{f}_{a/h}\otimes \tilde{D}_{h'/b}\otimes \tilde{H}^{(m,n)}_{i+a\to j+b+X} + \mathcal{O}(|\overline{\mathbf{p}}|/|\overline{\mathbf{P}}|)$$

The "+"-direction of the final-state TMDs are defined in the "parton frame" where the observed lepton (or jet) and hadron (or jet) are about back-to-back

Only two hadrons (or jet) are observed. Factorization breaking effect, identified by Collins & Qiu (2007) and Mulders & Rogers (2010), is not relevant here

Summary and outlook

- **Radiative corrections are very important for lepton-hadron scattering**
  - Especially difficult for a consistent treatment beyond the inclusive DIS
  - No well-defined photon-hadron frame, if we cannot recover all QED radiation
  - Radiative corrections are more important for events with high momentum transfers and large phase space to shower – such as those at the EIC

- **We proposed a factorization based treatment of QED radiation**
  - QED radiation is a part of production cross sections, treated in the same way as radiation from quarks and gluons
  - No artificial and/or process dependent scale(s) introduced for treating QED radiation, other than the standard factorization scale
  - All perturbatively calculable hard parts are IR safe for both QCD and QED
  - All lepton mass or resolution sensitivity are included into "Universal" lepton distribution and fragmentation functions (or jet functions)

Thank you!

Special thanks to JLab experimental colleagues for helpful discussions!