Forward trijet production at LHC

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in collaboration with

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Explicit k_T-employing factorization

TMD factorization

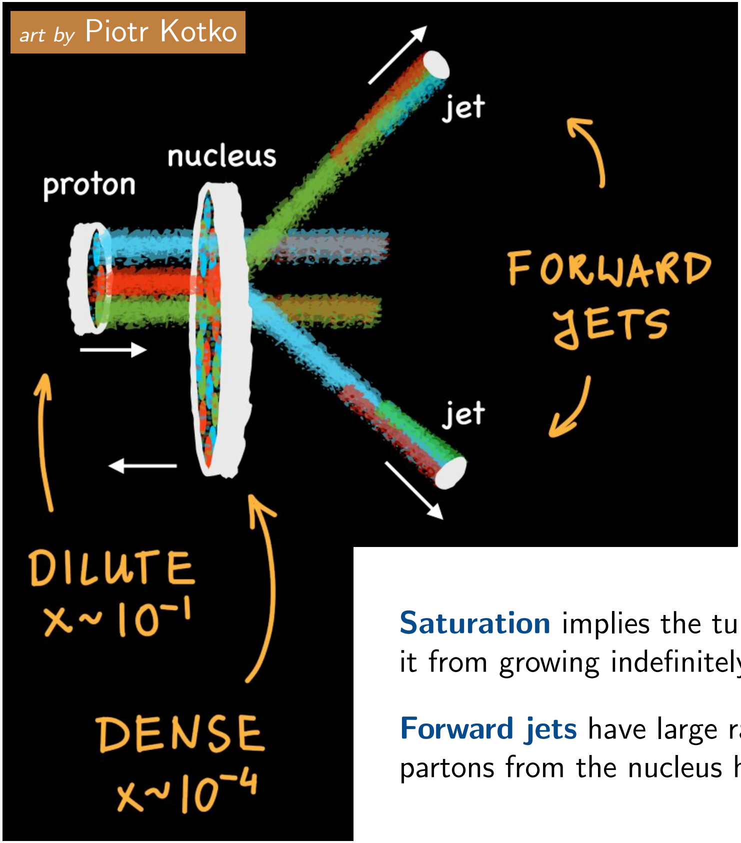
- holds at leading power in k_T/μ
- on-shell parton-level matrix elements
- Transverse Momentum Dependent PDFs,
  evolve via the Collins-Soper-Sterman equations,
  re-sum large logs of k_T/μ

High energy factorization

$$d\sigma_{hh} = \sum_{a,b} \int dx_1 \frac{d^2k_{T1}}{\pi} \int dx_2 \frac{d^2k_{T2}}{\pi}\, \mathcal{F}_a(x_1, k_{T1})\, \mathcal{F}_b(x_2, k_{T2})\, d\sigma_{ab}(x_1, k_{T1}, x_2, k_{T2})$$

- focus on small-x, not neglecting powers of k_T/μ
- off-shell parton-level matrix elements
- Transvers Momentum Dependent, or un-integrated, PDFs,
  evolve to resum logs of $1/x$,
  e.g. with BFKL or CCFM equations, or their non-linear extensions,

QCD evolution, dilute vs. dense, forward jets



A dilute system carries a few high-x partons contributing to the hard scattering.

A dense system carries many low-x partons.

At high density, gluons are imagined to undergo recombination, and to saturate.

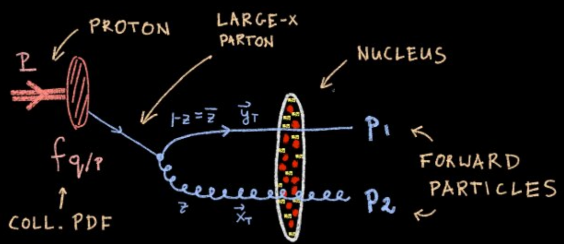
This is modeled with non-linear evolution equations, involving explicit non-vanishing k_T.

Saturation implies the turnover of the gluon density, stopping it from growing indefinitely for small x.

Forward jets have large rapidities, and trigger events in which partons from the nucleus have small x.

art by Piotr Kotko

pA (dilute-dense) collisions within CGC



$$\frac{d\sigma_{qA\to 2j}}{d^3p_1\,d^3p_2} \sim \int \frac{d^2x}{(2\pi)^2}\frac{d^2x'}{(2\pi)^2}\frac{d^2y}{(2\pi)^2}\frac{d^2y'}{(2\pi)^2}\, e^{-i\vec{p}_{T1}\cdot(\vec{x}_T-\vec{x}'_T)}e^{-i\vec{p}_{T2}\cdot(\vec{y}_T-\vec{y}'_T)}$$

$$\times\,\psi_z^*\left(\vec{x}'_T-\vec{y}'_T\right)\psi_z\left(\vec{x}_T-\vec{y}_T\right)$$ ← QUARK WAVE FUNCTION

$$\times\Big\{S_x^{(6)}\left(\vec{y}_T,\vec{x}_T,\vec{y}'_T,\vec{x}'_T\right)-S_x^{(4)}\left(\vec{y}_T,\vec{x}_T,\bar{z}\vec{y}'_T+z\vec{x}'_T\right)$$
$$-S_x^{(4)}\left(\bar{z}\vec{y}_T+z\vec{x}_T,\vec{y}'_T,\vec{x}'_T\right)-S_x^{(2)}\left(\bar{z}\vec{y}_T+z\vec{x}_T,\bar{z}\vec{y}'_T+z\vec{x}'_T\right)\Big\}$$

CORRELATORS OF WILSON LINES

$$S_x^{(2)}\left(\vec{y}_T,\vec{x}_T\right)=\frac{1}{N_c}\left\langle \mathrm{Tr}\,U(\vec{y}_T)U^\dagger(\vec{x}_T)\right\rangle_x$$

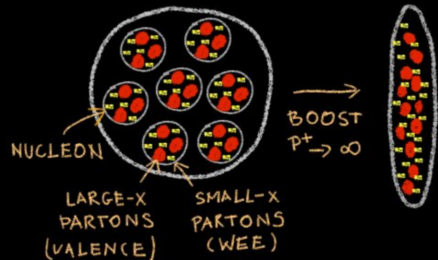
$$S_x^{(4)}\left(\vec{z}_T,\vec{y}_T,\vec{x}_T\right)=\frac{1}{2C_FN_c}\left\langle \mathrm{Tr}\left[U(\vec{z}_T)U^\dagger(\vec{y}_T)\right]\mathrm{Tr}\left[U(\vec{y}_T)U^\dagger(\vec{x}_T)\right]\right\rangle_x - S_x^{(2)}\left(\vec{z}_T,\vec{x}_T\right)$$

etc...

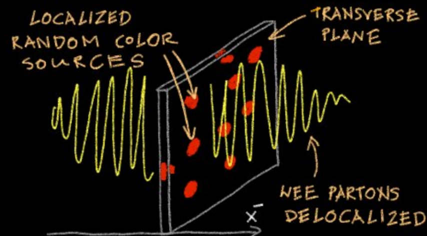
$$U(\vec{x}_T)=\mathscr{P}\exp\left\{ig\int_{-\infty}^{+\infty}dx^+\,A_a^-\left(x^+,\vec{x}_T\right)t^a\right\}$$

[C. Marquet, 2007]

COLOR FIELD OF THE NUCLEUS



[L. McLerran, R. Venugopalan, 1993]



Large-x partons — the color source for wee partons:

$$\left(D_\mu F^{\mu\nu}\right)_a\left(x^-,\vec{x}_T\right)=\delta^{\nu+}\rho_a\left(\vec{x}_T\right)\delta\left(x^-\right)$$

RANDOM DISTRIBUTION OF COLOR SOURCES

AVERAGE OVER COLOR SOURCES
GAUSSIAN FUNCTIONAL
B-JIMWLK EVOLUTION IN X → $\mathscr{W}_x[\rho]$

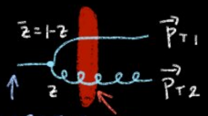
[Balitsky-Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner, 1996-2002]

CGC & TMD — Leading twist study

art by Piotr Kotko

FORWARD DIJET PRODUCTION IN CGC

[F. Dominguez, C. Marquet, B. Xiao, F. Yuan, 2011]



$$\vec{k}_T = \vec{P}_{T1} + \vec{P}_{T2}$$

$$\vec{P}_T = \bar{z}\vec{P}_{T1} - z\vec{P}_{T2}$$

$$x_2 \ll x_1$$

LEADING POWER LIMIT

$$Q_s \sim k_T \ll P_T \sim \mu$$

Q_s: SATURATION SCALE; μ: HARD SCALE

$$\frac{d\sigma_{pA\to 2j+X}}{dy_1 dy_2 d^2p_{T1} d^2p_{T2}} \sim \sum_{a,c,d} f_{a/p}(x_1,\mu) \sum_i H^{(i)}_{ag\to cd}(k_T=0)\, \mathcal{F}^{(i)}_{ag}(x_2,k_T)$$

$H^{(i)}_{ag\to cd}$: ON-SHELL HARD FACTORS

$\mathcal{F}^{(i)}_{ag}$: TMD GLUON DISTRIBUTIONS (SMALL-X LIMIT)

Equivalence of leading power CGC and TMD ‘factorization’ was recently shown for dijet+photon process.

[T. Altinoluk, R. Boussarie, C. Marquet, P. Taels, 2018]

SMALL-X LIMIT OF TMD GLUON DISTRIBUTIONS

$$\mathcal{F}^{(i)}_{ag}(x,k_T) \sim \int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3} e^{ixP^-\xi^+ - i\vec{k}_T\cdot\vec{\xi}_T} \langle P|\,\mathrm{Tr}\left[\hat{F}^{i-}\left(\xi^+,\vec{\xi}_T,\xi^-=0\right)\mathcal{U}_{C_1}\hat{F}^{i-}(0)\,\mathcal{U}_{C_2}\right]|P\rangle$$

$x \to 0$

DEPENDENCE ON X IS ONLY VIA THE SMALL-X EVOLUTION

For example:

$$\mathcal{F}^{(1)}_{qg} \sim \int \frac{d^2x_T d^2y_T}{(2\pi)^4} k_T^2 e^{-i\vec{k}_T\cdot(\vec{x}_T-\vec{y}_T)} \langle \mathrm{Tr}\left[U(\vec{x}_T)U^\dagger(\vec{y}_T)\right]\rangle_x$$

Intensively studied:

[D. Kharzeev, Y. Kovchegov, K. Tuchin, 2003]
[B. Xiao, F. Yuan, 2010]
[F. Dominguez, C. Marquet, B. Xiao, F. Yuan, 2011]
[A. Metz, J. Zhou, 2011]
[E. Akcakaya, A. Schafer, J. Zhou, 2012]
[C. Marquet, E. Petreska, C. Roiesnel, 2016]
[I. Balitsky, A. Tarasov, 2015, 2016]
[D. Boer, P. Mulders, J. Zhou, Y. Zhou, 2017]
[C. Marquet, C. Roiesnel, P. Taels, 2018]
[Y. Kovchegov, D. Pitonyak, M. Sievert, 2017,2018]
[T. Altinoluk, R. Boussarie, 2019]

art by Piotr Kotko

Small-x Improved TMD Factorization (ITMD)

Factorization formula for forward dijets in p-p and p-A collisions

[PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, 2015]

$$\frac{d\sigma_{pA\to 2j+X}}{dy_1 dy_2 d^2p_{T1} d^2p_{T2}} \sim \sum_{a,c,d} f_{a/p}(x_1,\mu) \sum_{i=1,2} K^{(i)}_{ag\to cd}(k_T)\, \Phi^{(i)}_{ag\to cd}(x_2,k_T)$$

RAPIDITY

TRANSVERSE MOMENTA

COLLINEAR PROTON PDF

GAUGE INVARIANT OFF-SHELL HARD FACTORS

TMD GLUON DISTRIBUTIONS AT SMALL-X

$x_2 \ll x_1$ $\quad |\vec{p}_{T1}+\vec{p}_{T2}| = k_T$

TWO PER CHANNEL

$(g^*q\to gq,\ g^*g\to gg,\ g^*g\to q\bar{q})$

ITMD factorization formula has been proven from the Color Glass Condensate (CGC) theory.

⇒ RESUMMATION OF KINEMATIC TWISTS AND NEGLECTING GENUINE TWSTS.

[T. Altinoluk, R. Boussarie, PK, 2019]

$$\Lambda_{QCD} \ll Q_s \ll P_T$$

SATURATION SCALE

ITMD* factorization for more than 2 jets

We want to establish a similar factorization for more than 2 jets.

However, the ITMD formalism does not account for linearly polarized gluons in unpolarized target.

Such a contribution is absent for massless 2-particle production in CGC theory, but does appear in heavy quark production (Marquet, Roiesnes, Taels 2018), in the correlation limit for 3-parton final-states (Altinoluk, Boussarie, Marquet, Taels 2020), and can be concluded to be present from 3-jet formulae in CGC (Iancu, Mulian 2019).

This contribution cannot staightforwardly be formulated in terms of gauge-invariant off-shell hard scattering amplitudes

$$\sum_{i,j} \mathcal{M}_i^* \left(\frac{k_T^{(i)} k_T^{(j)}}{2|\vec{k}_T|^2} (\mathcal{F} + \mathcal{H}) + \frac{q_T^{(i)} q_T^{(j)}}{2|\vec{q}_T|^2} (\mathcal{F} - \mathcal{H}) \right) \mathcal{M}_j \quad , \quad \vec{q}_T \cdot \vec{k}_T = 0$$

$\sum_i \mathcal{M}_i k_T^{(i)}$ is gauge invariant while $\sum_i \mathcal{M}_i q_T^{(i)}$ is not. For dijets, it happens that $\mathcal{F} = \mathcal{H}$.

In the following only the manifestly gauge-invariant contribution is included, hence the designation ITMD*.

ITMD* factorization for more than 2 jets

Schematic hybrid (non-ITMD) factorization fomula

$$d\sigma = \sum_a \int dx_1 d^2k_T \int dx_2 \, d\Phi_{g^* a \to n} \, \frac{1}{\mathrm{flux}_{ga}} \, \mathcal{F}_g(x_1, k_T, \mu) \, f_a(x_2, \mu) \sum_{\mathrm{color}} \left| \mathcal{M}^{(\mathrm{color})}_{g^* a \to n} \right|^2$$

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Color connection representation: turn adjoint gluon indices a into fundamental indices i, j

$$\tilde{\mathcal{M}}^{\cdots\, {}^{i}_{j} \,\cdots} \equiv \mathcal{M}^{\cdots\, a \,\cdots} \, (\sqrt{2} T^a)^i_j$$

$$\sum_{\mathsf{color}} \left| \mathcal{M}^{(\mathsf{color})} \right|^2 = \sum_{i_1, i_2, \ldots, i_{n+2}} \; \sum_{j_1, j_2, \ldots, j_{n+2}} \left(\tilde{\mathcal{M}}^{i_1 i_2 \ldots i_{n+2}}_{j_1 j_2 \ldots j_{n+2}} \right)^* \left(\tilde{\mathcal{M}}^{i_1 i_2 \ldots i_{n+2}}_{j_1 j_2 \ldots j_{n+2}} \right)$$

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Decomposition into partial amplitudes (Kanaki, Papadopoulos 2000; Maltoni, Paul, Stelzer, Willenbrock 2003)

$$\tilde{\mathcal{M}}^{i_1 i_2 \ldots i_{n+2}}_{j_1 j_2 \ldots j_{n+2}} = \sum_{\sigma \in S_{n+2}} \delta^{i_1}_{j_{\sigma(1)}} \delta^{i_2}_{j_{\sigma(2)}} \cdots \delta^{i_{n+2}}_{j_{\sigma(n+2)}} \mathcal{A}_\sigma$$

ITMD* factorization for more than 2 jets

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$$\tilde{\mathcal{M}}^{i_1 i_2 \ldots i_{n+2}}_{j_1 j_2 \ldots j_{n+2}} = \sum_{\sigma \in S_{n+2}} \delta^{i_1}_{j_{\sigma(1)}} \delta^{i_2}_{j_{\sigma(2)}} \cdots \delta^{i_{n+2}}_{j_{\sigma(n+2)}} \, \mathcal{A}_\sigma$$

Color sum in terms of a color matrix

$$\sum_{\text{color}} \left| \mathcal{M}^{(\text{color})} \right|^2 = \sum_{\sigma \in S_{n+2}} \sum_{\tau \in S_{n+2}} \mathcal{A}^*_\sigma \, \mathcal{C}_{\sigma\tau} \, \mathcal{A}_\tau$$

$$\mathcal{C}_{\sigma\tau} = \sum_{i_1, i_2, \ldots, i_{n+2}} \sum_{j_1, j_2, \ldots, j_{n+2}} \delta^{i_1}_{j_{\sigma(1)}} \delta^{i_2}_{j_{\sigma(2)}} \cdots \delta^{i_{n+2}}_{j_{\sigma(n+2)}} \delta^{i_1}_{j_{\tau(1)}} \delta^{i_2}_{j_{\tau(2)}} \cdots \delta^{i_{n+2}}_{j_{\tau(n+2)}} = N_c^{\lambda(\sigma,\tau)}$$

ITMD* factorization for more than 2 jets

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$$\mathcal{F}_g \sum_{\text{color}} \left| \mathcal{M}^{(\text{color})} \right|^2 = \mathcal{F}_g \sum_{i_1, i_2, \ldots, i_{n+2}} \; \sum_{j_1, j_2, \ldots, j_{n+2}} \left(\tilde{\mathcal{M}}^{i_1 i_2 \ldots i_{n+2}}_{j_1 j_2 \ldots j_{n+2}} \right)^* \left(\tilde{\mathcal{M}}^{i_1 i_2 \ldots i_{n+2}}_{j_1 j_2 \ldots j_{n+2}} \right)$$

ITMD* factorization for more than 2 jets

Schematic hybrid (non-ITMD) factorization fomula

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ITMD* formula: replace

$$\mathcal{F}_g \sum_{\text{color}} \left| \mathcal{M}^{(\text{color})} \right|^2 = \mathcal{F}_g \sum_{i_1, i_2, \ldots, i_{n+2}} \; \sum_{j_1, j_2, \ldots, j_{n+2}} \left(\tilde{\mathcal{M}}^{i_1 i_2 \ldots i_{n+2}}_{j_1 j_2 \ldots j_{n+2}} \right)^* \left(\tilde{\mathcal{M}}^{i_1 i_2 \ldots i_{n+2}}_{j_1 j_2 \ldots j_{n+2}} \right)$$

with (Bomhof, Mulders, Pijlman 2006; Bury, Kotko, Kutak 2018)

$$(N_c^2 - 1) \sum_{i_1, \ldots, i_n} \; \sum_{j_1, \ldots, j_{n+2}} \; \sum_{\bar{\imath}_1, \ldots, \bar{\imath}_{n+2}} \; \sum_{\bar{\jmath}_1, \ldots, \bar{\jmath}_{n+2}} \left(\tilde{\mathcal{M}}^{i_1 i_2 \cdots i_{n+2}}_{j_1 j_2 \cdots j_{n+2}} \right)^* \left(\tilde{\mathcal{M}}^{\bar{\imath}_1 \bar{\imath}_2 \cdots \bar{\imath}_{n+2}}_{\bar{\jmath}_1 \bar{\jmath}_2 \cdots \bar{\jmath}_{n+2}} \right)$$

$$\times \, 2 \int \frac{d^4\xi}{(2\pi)^3 P^+} \delta(\xi_+) \, e^{ik\cdot\xi} \left\langle P \middle| \left(\hat{F}^+(\xi) \right)^{j_1}_{i_1} \left(\hat{F}^+(0) \right)^{\bar{\jmath}_1}_{\bar{\imath}_1} \left(\mathcal{U}^{[\lambda_2]} \right)_{i_2 \bar{\imath}_2} \left(\mathcal{U}^{[\lambda_2]\dagger} \right)^{j_2 \bar{\jmath}_2} \cdots \right.$$

$$\left. \cdots \left(\mathcal{U}^{[\lambda_{n+2}]} \right)_{i_{n+2} \bar{\imath}_{n+2}} \left(\mathcal{U}^{[\lambda_{n+2}]\dagger} \right)^{j_{n+2} \bar{\jmath}_{n+2}} \middle| P \right\rangle$$

where P is the light-like momentum of the hadron (with $P^- = 0$), and $k^\mu = xP^\mu + k_T^\mu$,
where $\hat{F}$ is the field strenght,
and $\mathcal{U}^\pm$ is a Wilson line from 0 to ξ via a “staple-like detour” to $\pm\infty$ depending on the type and state (initial/final) of parton.

ITMD* factorization for more than 2 jets

Schematic hybrid (non-ITMD) factorization fomula

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$$\mathcal{F}_g \sum_{\mathrm{color}} \left|\mathcal{M}^{(\mathrm{color})}\right|^2 = \mathcal{F}_g \sum_{i_1,i_2,\dots,i_{n+2}} \sum_{j_1,j_2,\dots,j_{n+2}} \left(\tilde{\mathcal{M}}^{i_1 i_2 \dots i_{n+2}}_{j_1 j_2 \dots j_{n+2}}\right)^* \left(\tilde{\mathcal{M}}^{i_1 i_2 \dots i_{n+2}}_{j_1 j_2 \dots j_{n+2}}\right)$$

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where P is … $P^\mu + k_T^\mu$,

where $\hat{F}$ is …

and $\mathcal{U}^\pm$ is … ling on the

type and st…

$$\tilde{\mathcal{M}}^{i_1 i_2 \dots i_{n+2}}_{j_1 j_2 \dots j_{n+2}} = \sum_{\sigma \in S_{n+2}} \delta^{i_1}_{j_{\sigma(1)}} \delta^{i_2}_{j_{\sigma(2)}} \cdots \delta^{i_{n+2}}_{j_{\sigma(n+2)}} \mathcal{A}_\sigma$$

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ITMD* formula: replace

$$\mathcal{F}_g \sum_{\text{color}} \left|\mathcal{M}^{(\text{color})}\right|^2 = \mathcal{F}_g \sum_{\sigma\in S_{n+2}} \sum_{\tau\in S_{n+2}} \mathcal{A}^*_\sigma\, \mathcal{C}_{\sigma\tau}\, \mathcal{A}_\tau \quad , \quad \mathcal{C}_{\sigma\tau} = N_c^{\lambda(\sigma,\tau)}$$

with "TMD-valued color matrix"

$$(N_c^2 - 1) \sum_{\sigma\in S_{n+2}} \sum_{\tau\in S_{n+2}} \mathcal{A}^*_\sigma\, \tilde{\mathcal{C}}_{\sigma\tau}(x, |k_T|)\, \mathcal{A}_\tau \quad , \quad \tilde{\mathcal{C}}_{\sigma\tau}(x, |k_T|) = N_c^{\bar{\lambda}(\sigma,\tau)} \tilde{\mathcal{F}}_{\sigma\tau}(x, |k_T|)$$

where each function $\tilde{\mathcal{F}}_{\sigma\tau}$ is one of 10 functions

$$\mathcal{F}^{(1)}_{qg} \quad , \quad \mathcal{F}^{(2)}_{qg} \quad , \quad \mathcal{F}^{(3)}_{qg}$$

$$\mathcal{F}^{(1)}_{gg} \quad , \quad \mathcal{F}^{(2)}_{gg} \quad , \quad \mathcal{F}^{(3)}_{gg} \quad , \quad \mathcal{F}^{(4)}_{gg} \quad , \quad \mathcal{F}^{(5)}_{gg} \quad , \quad \mathcal{F}^{(6)}_{gg} \quad , \quad \mathcal{F}^{(7)}_{gg}$$

ITMD* factorization for more than 2 jets

$$\mathcal{F}_{qg}^{(1)}(x,k_T)=\left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle \quad , \quad \left\langle \cdots \right\rangle = 2\int \frac{d^4\xi}{(2\pi)^3 P^+}\delta(\xi_+)\,e^{ik\cdot\xi}\left\langle P\middle|\cdots\middle|P\right\rangle$$

$$\mathcal{F}_{qg}^{(2)}(x,k_T)=\left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{qg}^{(3)}(x,k_T)=\left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(1)}(x,k_T)=\left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(2)}(x,k_T)=\frac{1}{N_c}\left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\right]\mathrm{Tr}\left[\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(3)}(x,k_T)=\left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(4)}(x,k_T)=\left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[-]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(5)}(x,k_T)=\left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(6)}(x,k_T)=\left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(7)}(x,k_T)=\left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

ITMD gluons

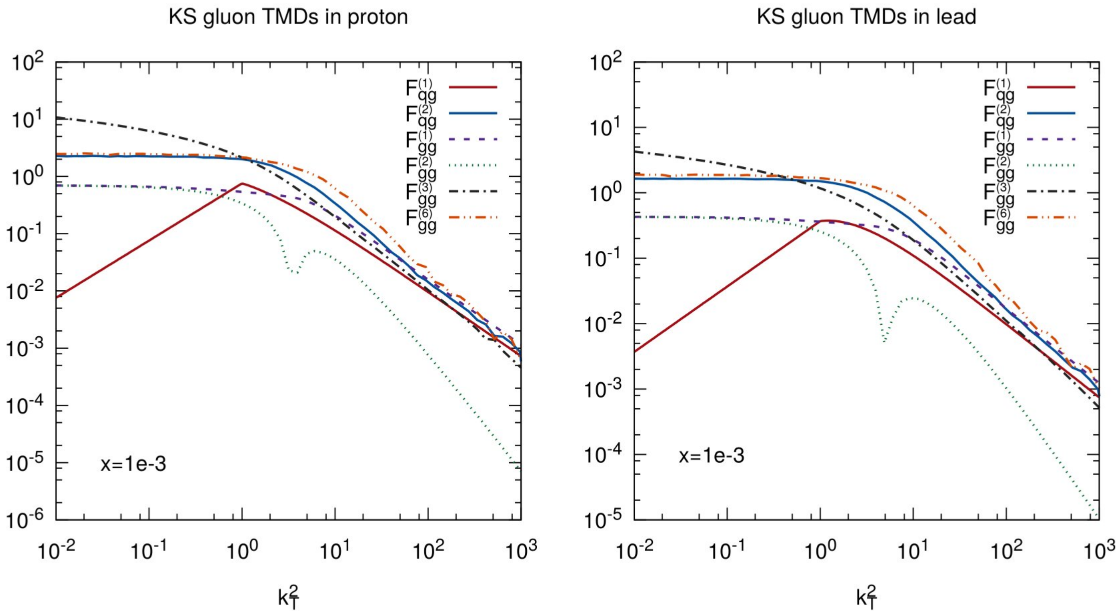
Start with dipole distribution $\mathcal{F}_{qg}^{(1)}(x, k_T) = \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$ evolved via the BK equation formulated in momentum space supplemented with subleading corrections and fitted to F_2 data (Kutak, Sapeta 2012)

All other distribution appearing in dijet production, $\mathcal{F}_{qg}^{(2)}, \mathcal{F}_{gg}^{(1)}, \mathcal{F}_{gg}^{(2)}, \mathcal{F}_{gg}^{(6)}$, in the mean-field approximation (AvH, Marquet, Kotko, Kutak, Sapeta, Petreska 2016).

This is, at leading order in $1/N_c$. In this approximation, the same distributions suffice for trijets.

ITMD gluons

Bury, AvH, Kotko, Kutak 2020

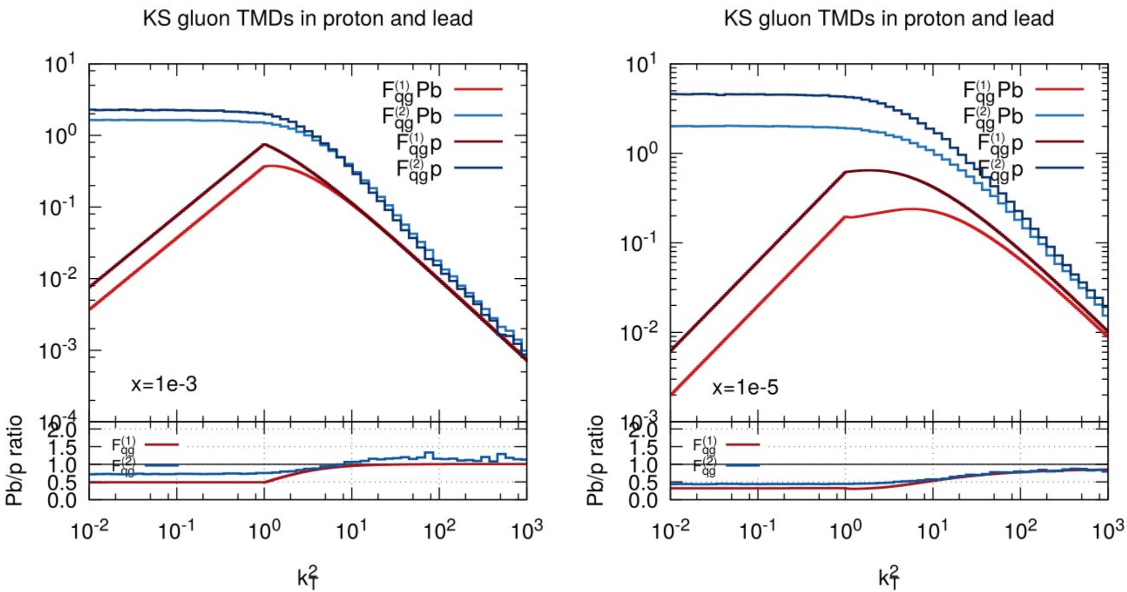


Dependence of $\mathcal{F}^{(1)}_{qg}$ on k_T below 1GeV approximated by power-like fall-off. For higher values of $|k_T|$ it is a solution to the BK equation.

TMDs decrease as $1/|k_T|$ for increasing $|k_T|$, except $\mathcal{F}^{(2)}_{gg}$, which decreases faster (even becomes negative, absolute value shown here).

ITMD gluons

Bury, AvH, Kotko, Kutak 2020



Ratio Pb/p is smaller than 1 for small x,
but can become larger than 1 for moderate x and large $|k_T|$.

Set up

We consider p-p and p-Pb collisions at 5.02TeV producing at least 3 jets with forward rapidities $3.2 < |y_1^*, y_2^*, y_3^*| < 4.9$ in the CM frame.

Jet definition: $\Delta R > 0.5$, $p_T > 20\text{GeV}$

renormalization/factorization scale: $(p_{T\,1} + p_{T\,2} + p_{T\,3})/3$

Collinear PDFs: CTEQ10NLO from LHAPDF6

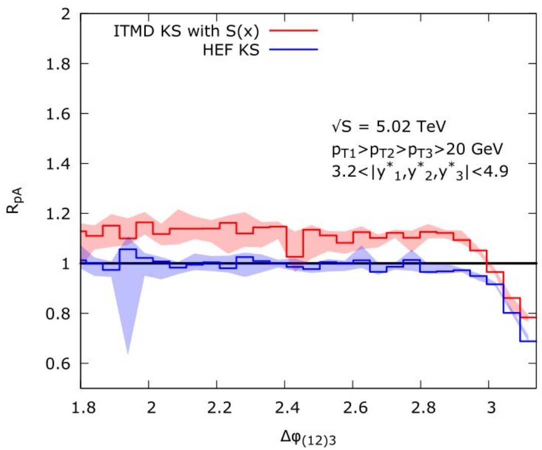
Include all partonic processes with 5 light flavors with an (off-shell) gluon and a quark or gluon in the initial state.

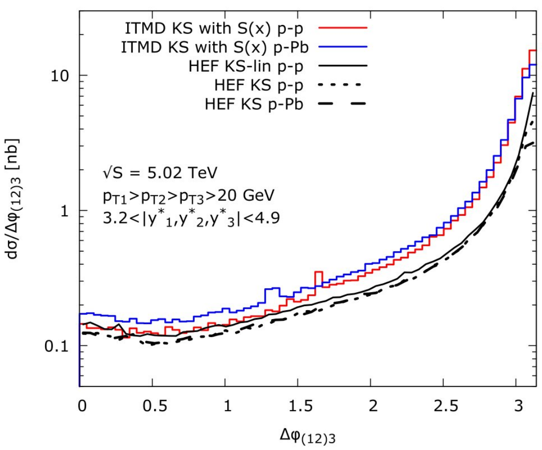
observables:
$\Delta\phi_{12}$ (angle between 2 hardest jets),
$\Delta\phi_{13}$ (angle between hardest jet and 3$^{\text{rd}}$ hardest jet),
$\Delta\phi_{(12)3}$ (angle between the sum of the two hardest and the 3$^{\text{rd}}$ hardest jet. Is sensitive to momentum inbalance)

Nuclear modification ratio $R_{pA} = \dfrac{1}{A}\dfrac{d\sigma^{pPb}/d\mathcal{O}}{d\sigma^{pp}/d\mathcal{O}}$ where A is the number of nucleons

Calculations performed independently with LxJet (Kotko) and KaTie (AvH 2018)

Results





$S(x)$ refers to the x-dependent treatment of the nuclear target area, guaranteeing unitarity.

Saturation effect for $\Delta\phi_{(12)3} \approx \pi$, enhancement of pPb result for $\Delta\phi_{(12)3} < \pi$ due to broadening of the TMD distributions.

ITMD* normalization significantly larger than HEF, due to different shape and normalization of the extra TMDs present in ITMD* but not in HEF.

Summary

- small-x Improved TMD factorization allows to consistently include saturation effects in calculations for forward dijets
- we extended ITMD factorization to ITMD* for more than 2 jets, and performed explicit calculations for 3 jets
- we observe significant saturation effects in the nuclear modification factor for momentum inbalance-sensitive observable
- we observe significant differences between results from ITMD* and k_T/high-energy factorization, implying strong discriminating potential
- multi-(say more than 2)-jet observables are interesting for small-x physics (see also Van Haevermaet, AvH, Kotko, Kutak, Van Mechelen 2020)

Thank you for your attention.