All-plus helicity off-shell gauge invariant multigluon amplitudes at one loop

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Motivations / Context

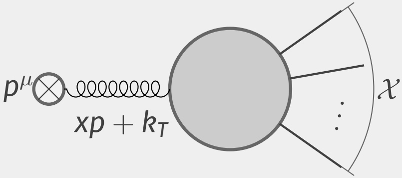
Formulation

$$d\sigma(ab \to \mathcal{X}) = \sum_{i,j} \int \frac{d^2\mathbf{k}_{T,i}}{\pi} dx_i dx_j\, f_{i,a}(x_i, \mathbf{k}_{T,i}, \mu) f_{j,b}(x_j, \mu) \times$$
$$\times d\hat{\sigma}(i^* j \to \mathcal{X})(x_i, \mathbf{k}_{T,i}, x_j, \mu)$$

- with $k_i = xp_i + k_{T,i}$

Adrian Dumitru, Arata Hayashigaki, and Jamal Jalilian-Marian. The Color glass condensate and hadron production in the forward region. ***Nucl. Phys. A*, 765:464–482, 2006** Cyrille Marquet. Forward inclusive dijet production and azimuthal correlations in p(A) collisions. ***Nucl. Phys. A*, 796:41–60, 2007** M. Deak, F. Hautmann, H. Jung, and K. Kutak. Forward Jet Production at the Large Hadron Collider. ***JHEP*, 09:121, 2009**

S. Catani, M. Ciafaloni, and F. Hautmann. High-energy factorization and small x heavy flavor production. ***Nucl. Phys. B*, 366:135–188, 1991**

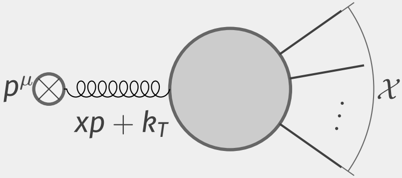


$+$ terms required for gauge invariance

Calculation

Two approaches :

- Lipatov's high energy effective action
- Explicit construction of the terms required by gauge invariance



$+$ terms required for gauge invariance

2nd approach

One way : embedding the off-shell process into an on-shell one

- effective at tree-level

  A. van Hameren, P. Kotko, and K. Kutak. Helicity amplitudes for high-energy scattering. ***JHEP*, 01:078, 2013**

- implemented in KaTie (also for 2 off-shell gluons)

  A. van Hameren. KaTie : For parton-level event generation with k_T-dependent initial states. ***Comput. Phys. Commun.*, 224:371–380, 2018**

Processes considered

$$g^* g^+ \cdots g^+$$

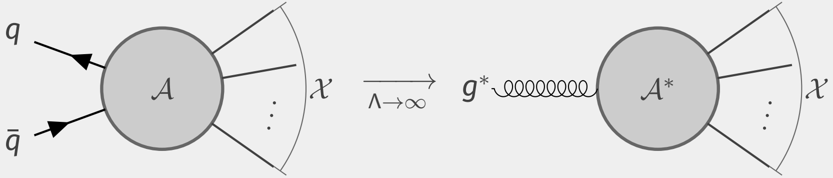
- 1st step to generalize our method at one-loop
- Important in forward particle production processes
- (Feasibility)

Color ordering

$$\mathcal{M}^{a_1,\ldots,a_n}_{\lambda_1,\ldots,\lambda_n}(k_1,\ldots,k_n) = \sum_{\text{perm.}(2\cdots n)} \mathrm{Tr}\left(t^{a_1}t^{a_2}\ldots t^{a_n}\right) \times \times\mathcal{A}\left(1^{(\lambda_1)}, 2^{(\lambda_2)}, \ldots, n^{(\lambda_n)}\right)$$

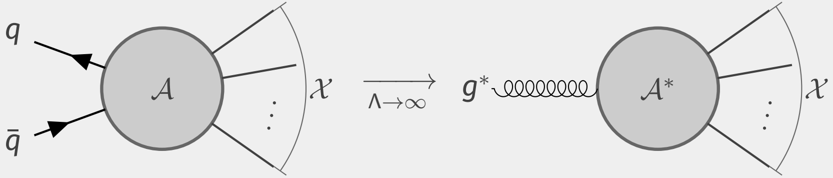
- Use of spinor-helicity methods

Method



On-shell process

- use of auxiliary quark line



Limit

- Λ parametrize the quark-antiquark pair momenta
- $k^\mu = xp^\mu + k_T^\mu$
- Limit taken such as the coupling to the quark line is eikonal

$$\lim_{\Lambda\to\infty}\left(\frac{x|k_T|}{g_s\Lambda}\mathcal{A}\left(\bar{q}(k_1)q(k_2)\mathcal{X}\right)\right) = \mathcal{A}^*\left(g^*(k)\mathcal{X}\right)$$

Auxiliary quark line kinematics

$$k_1^\mu = \Lambda p^\mu + \alpha q^\mu + \beta k_T^\mu \,,$$
$$k_2^\mu = (x - \Lambda) p^\mu - \alpha q^\mu + (1 - \beta) k_T^\mu \,,$$
$$\text{where} \quad \alpha = \frac{-\beta^2 k_T^2}{2\Lambda p \cdot q} \quad , \quad \beta = \frac{1}{1 + \sqrt{1 - x/\Lambda}}$$

High energy kinematics

$$k^\mu = k_1^\mu + k_2^\mu = x p^\mu + k_T^\mu$$

k_T decomposition

$$k_T^\mu = -\bar{\kappa} e^\mu - \bar{\kappa}^* e_*^\mu \,,$$

with $\quad e^\mu = \frac{1}{2}\langle p|\gamma^\mu|q] \quad , \quad e_*^\mu = \frac{1}{2}\langle q|\gamma^\mu|p]$

and $\quad \bar{\kappa} = \frac{\kappa}{[pq]} = \frac{\langle q|\not{k}|p]}{2p\cdot q} \quad , \quad \bar{\kappa}^* = \frac{\kappa^*}{\langle qp\rangle} = \frac{\langle p|\not{k}|q]}{2p\cdot q}$

we notice that $\quad k_T^2 = -\kappa\kappa^*$

Expansion in Λ

$$k_1^\mu = \Lambda p^\mu + \left(\frac{1}{2} + \frac{x}{8\Lambda}\right) k_T^\mu - \frac{k_T^2}{8\Lambda p \cdot q} q^\mu + \mathcal{O}(\Lambda^{-2}) \,,$$
$$k_2^\mu = (x - \Lambda) p^\mu + \left(\frac{1}{2} - \frac{x}{8\Lambda}\right) k_T^\mu + \frac{k_T^2}{8\Lambda p \cdot q} q^\mu + \mathcal{O}(\Lambda^{-2})$$

Spinors equivalence

$$|1\rangle \to \sqrt{\Lambda}\,|p\rangle \;\;,\;\; |1] \to \sqrt{\Lambda}\,|p] \;\;,\;\; |2\rangle \to \sqrt{\Lambda}\,|p\rangle \;\;,\;\; |2] \to -\sqrt{\Lambda}\,|p] \;\;.$$

$$\langle 12\rangle = -\kappa^* \quad , \quad [12] = -\kappa$$

On-shell limit

$$\lim_{|k_T|\to 0} \mathcal{A}_n^{*(0)}(g^*\mathcal{X}) = \frac{|k_T|}{\kappa^*}\mathcal{A}_n^{(0)}(g^-\mathcal{X}) + \frac{|k_T|}{\kappa}\mathcal{A}_n^{(0)}(g^+\mathcal{X})$$

A. van Hameren. BCFW recursion for off-shell gluons. ***JHEP*****, 07:138, 2014**

Equivalence between auxiliary partons

Using auxiliary quarks or auxiliary gluons has been shown to be equivalent a tree level.

A. van Hameren. Calculating off-shell one-loop amplitudes for k_T-dependent factorization: a proof of concept. **2017**

Results

Amplitudes calculated

1. $g^* g^+ g^+$
2. $g^* g^+ g^+ g^+$
3. $g^* g^+ g^+ g^+ g^+$
4. $g^* g^+ \cdots g^+$

Amplitudes calculated

1. $g^* g^+ g^+$
2. $g^* g^+ g^+ g^+$
3. ~~$g^* g^+ g^+ g^+ g^+$~~
4. $g^* g^+ \cdots g^+$

$\bar{q}qgg$ amplitude

$$\mathcal{A}_4^{(1)}(1_{\bar{q}}^-, 2_q^+, 3^+, 4^+) = -\frac{ig_s^4}{16\pi^2}\left[\frac{1}{2}\left(1+\frac{1}{N_c^2}\right)+\frac{1}{3}\left(1+\frac{n_s-n_f}{N_c}\right)\frac{s_{23}}{s_{12}}\right]\frac{\langle 12\rangle[24]}{\langle 23\rangle\langle 34\rangle}$$

Zvi Bern, Lance Dixon, and David A Kosower. One-loop corrections to two-quark three-gluon amplitudes. ***Nuclear Physics B*, 437(2):259–304, 1995**

$\bar{q}qgg$ amplitude

$$\mathcal{A}_4^{(1)}(1_{\bar{q}}^-, 2_q^+, 3^+, 4^+) = -\frac{ig_s^4}{16\pi^2}\left[\frac{1}{2}\left(1+\frac{1}{N_c^2}\right) + \frac{1}{3}\left(1+\frac{n_s-n_f}{N_c}\right)\frac{s_{23}}{s_{12}}\right]\frac{\langle 12\rangle[24]}{\langle 23\rangle\langle 34\rangle}$$

Λ prescription

$$\mathcal{A}_3^{*(1)}(g^*, 3^+, 4^+) = -\frac{ig_s^3}{24\pi^2}\left(1+\frac{n_s-n_f}{N_c}\right)\frac{x|k_T|}{\kappa^2} p\cdot k_3 \frac{[p3][p4]}{\langle p3\rangle\langle p4\rangle}$$

gggg amplitude

$$\mathcal{A}_4^{(1)}(1^-,2^+,3^+,4^+) = \frac{ig_s^4}{48\pi^2}\left(1+\frac{n_s-n_f}{N_c}\right)\frac{\langle 24\rangle[24]^3}{[12]\langle 23\rangle\langle 34\rangle[41]}$$

Zvi Bern, Lance Dixon, and David A Kosower. One-loop corrections to two-quark three-gluon amplitudes. ***Nuclear Physics B*, 437(2):259–304, 1995**

gggg amplitude

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Lipatov's effective action result

$$\gamma_{+,+,+}^{aa_3a_4} = \frac{ig_s^3}{3\pi^2} f^{aa_3a_4}(n_f - N_c)\frac{|k_T|}{\kappa^2} p \cdot k_3 \frac{[p3][p4]}{\langle p3\rangle\langle p4\rangle}$$

Maxim Nefedov. ONE-LOOP CORRECTIONS TO MULTISCALE EFFECTIVE VERTICES IN THE EFT FOR MULTI-REGGE PROCESSES IN QCD. **2019**

$\bar{q}qggg$ amplitude

$$\mathcal{A}_5^{(1)}(1_{\bar{q}}^-, 2_q^+, 3^+, 4^+, 5^+) = -\frac{ig_s^5}{32\pi^2}\left(1+\frac{1}{N_c^2}\right)\frac{\langle 12\rangle[23]\langle 31\rangle + \langle 14\rangle[45]\langle 51\rangle}{\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$
$$-\frac{ig_s^5}{48\pi^2}\left(1+\frac{n_s-n_f}{N_c}\right)\left(\frac{\langle 13\rangle[34]\langle 41\rangle^2}{\langle 12\rangle\langle 34\rangle^2\langle 45\rangle\langle 51\rangle} + \frac{\langle 14\rangle\langle 24\rangle[45]\langle 51\rangle}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle^2} + \frac{[23][25]}{[12]\langle 34\rangle\langle 45\rangle}\right).$$

Zvi Bern, Lance Dixon, and David A Kosower. One-loop corrections to two-quark three-gluon amplitudes. ***Nuclear Physics B*, 437(2):259–304, 1995**

$\bar{q}qggg$ amplitude

$$\mathcal{A}_5^{(1)}(1_{\bar{q}}^-, 2_q^+, 3^+, 4^+, 5^+) = -\frac{ig_s^5}{32\pi^2}\left(1+\frac{1}{N_c^2}\right)\frac{\langle 12\rangle[23]\langle 31\rangle + \langle 14\rangle[45]\langle 51\rangle}{\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$
$$-\frac{ig_s^5}{48\pi^2}\left(1+\frac{n_s-n_f}{N_c}\right)\left(\frac{\langle 13\rangle[34]\langle 41\rangle^2}{\langle 12\rangle\langle 34\rangle^2\langle 45\rangle\langle 51\rangle} + \frac{\langle 14\rangle\langle 24\rangle[45]\langle 51\rangle}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle^2} + \frac{[23][25]}{[12]\langle 34\rangle\langle 45\rangle}\right).$$

Λ prescription

$$\mathcal{A}_4^{*(1)}(g^*, 3^+, 4^+, 5^+) = -\frac{ig_s^4}{48\pi^2}\frac{x|k_T|\left(1+\frac{n_s-n_f}{N_c}\right)}{\kappa^*\langle p3\rangle\langle 34\rangle\langle 45\rangle\langle 5p\rangle}$$
$$\times\left[\langle p3\rangle^2\langle p4\rangle^2\frac{[34]}{\langle 34\rangle} + \langle p4\rangle^2\langle p5\rangle^2\frac{[45]}{\langle 45\rangle} - \frac{\kappa^*}{\kappa}s_{p3}s_{p5}\right]$$

ggggg amplitude

$$\mathcal{A}_5^{(1)}(1^-,2^+,3^+,4^+,5^+) = \frac{ig_s^5}{48\pi^2}\frac{\left(1+\frac{n_s-n_f}{N_c}\right)}{[12]\langle 23\rangle\langle 34\rangle\langle 45\rangle[51]} \times \Bigg[(s_{23}+s_{34}+s_{45})[25]^2$$
$$-[24]\langle 43\rangle[35][25] - \frac{[12][15]}{\langle 12\rangle\langle 15\rangle}\left(\langle 12\rangle^2\langle 13\rangle^2\frac{[23]}{\langle 23\rangle}+\langle 13\rangle^2\langle 14\rangle^2\frac{[34]}{\langle 34\rangle}+\langle 14\rangle^2\langle 15\rangle^2\frac{[45]}{\langle 45\rangle}\right)\Bigg]$$

Zvi Bern, Lance Dixon, and David A Kosower. ONE-LOOP CORRECTIONS TO TWO-QUARK THREE-GLUON AMPLITUDES. ***NUCLEAR PHYSICS B*, 437(2):259–304, 1995**

Results $\backslash$ g^*ggg

$ggggg$ amplitude

$$\mathcal{A}_5^{(1)}(1^-,2^+,3^+,4^+,5^+) = \frac{ig_s^5}{48\pi^2}\frac{\left(1+\frac{n_s-n_f}{N_c}\right)}{[12]\langle 23\rangle\langle 34\rangle\langle 45\rangle[51]} \times \Bigg[(s_{23}+s_{34}+s_{45})[25]^2$$

$$-[24]\langle 43\rangle[35][25] - \frac{[12][15]}{\langle 12\rangle\langle 15\rangle}\left(\langle 12\rangle^2\langle 13\rangle^2\frac{[23]}{\langle 23\rangle} + \langle 13\rangle^2\langle 14\rangle^2\frac{[34]}{\langle 34\rangle} + \langle 14\rangle^2\langle 15\rangle^2\frac{[45]}{\langle 45\rangle}\right)\Bigg]$$

Λ prescription

$$\mathcal{A}_4^{*(1)}(g^*,3^+,4^+,5^+) = \frac{ig_s^4}{48\pi^2}\frac{x|k_T|\left(1+\frac{n_s-n_f}{N_c}\right)}{\kappa^*\langle p3\rangle\langle 34\rangle\langle 45\rangle[5p]} \times \Bigg[s_{p3}[p5]^2$$

$$-\frac{\kappa[p5]}{\kappa^*\langle p5\rangle}\left(\langle p3\rangle^2\langle p4\rangle^2\frac{[34]}{\langle 34\rangle} + \langle p4\rangle^2\langle p5\rangle^2\frac{[45]}{\langle 45\rangle}\right)\Bigg]$$

Λ prescription

$$\mathcal{A}_4^{*(1)}(g^*,3^+,4^+,5^+) = -\frac{ig_s^4}{48\pi^2}\frac{x|k_T|\left(1+\frac{n_s-n_f}{N_c}\right)}{\kappa^*\langle p3\rangle\langle 34\rangle\langle 45\rangle\langle 5p\rangle} \times\left[\langle p3\rangle^2\langle p4\rangle^2\frac{[34]}{\langle 34\rangle}+\langle p4\rangle^2\langle p5\rangle^2\frac{[45]}{\langle 45\rangle}-\frac{\kappa^*}{\kappa}s_{p3}s_{p5}\right]$$

On-shell limit

$$A_4^{*(1)}(R,3^+,4^+,5^+) = \frac{|k_T|}{\kappa^*}A_4^{*(1)}(g^-,3^+,4^+,5^+)+\frac{|k_T|}{\kappa}A_4^{*(1)}(g^+,3^+,4^+,5^+)$$

Λ prescription

$$\mathcal{A}_4^{*(1)}(g^*,3^+,4^+,5^+) = -\frac{ig_s^4}{48\pi^2}\frac{x|k_T|\left(1+\frac{n_s-n_f}{N_c}\right)}{\kappa^*\langle p3\rangle\langle 34\rangle\langle 45\rangle\langle 5p\rangle} \times \Bigg[\underbrace{\langle p3\rangle^2\langle p4\rangle^2\frac{[34]}{\langle 34\rangle}+\langle p4\rangle^2\langle p5\rangle^2\frac{[45]}{\langle 45\rangle}}_{g^*\to g^-} - \underbrace{\frac{\kappa^*}{\kappa}s_{p3}s_{p5}}_{g^*\to g^+}\Bigg]$$

On-shell limit

$$A_4^{*(1)}(R,3^+,4^+,5^+) = \frac{|k_T|}{\kappa^*}A_4^{*(1)}(g^-,3^+,4^+,5^+) + \frac{|k_T|}{\kappa}A_4^{*(1)}(g^+,3^+,4^+,5^+)$$

$\bar{q}qg\cdots g$ amplitude

$$\mathcal{A}^{(1)}_{n+1}(1^-_{\bar{q}}, 2^+_q, 3^+, \cdots, (n+1)^+) = \frac{ig_s^{n+1}}{32\pi^2}\left(1+\frac{1}{N_c^2}\right)\frac{\sum_{l=3}^{n}\langle 1|\not{K}_{2\cdots l}\not{k}_l|1\rangle}{\langle 23\rangle\cdots\langle (n+1)1\rangle} + \frac{ig_s^{n+1}}{48\pi^2}\left(1+\frac{n_s-n_f}{N_c}\right)\frac{S_1+S_2}{\langle 12\rangle\langle 23\rangle\cdots\langle (n+1)1\rangle},$$

with
$$S_1 = \sum_{j=3}^{n}\frac{\langle 2j\rangle\langle 1(j+1)\rangle\langle 1|\not{K}_{j,j+1}\not{K}_{(j+1)\cdots(n+1)}|1\rangle}{\langle j(j+1)\rangle},$$

$$S_2 = \sum_{j=3}^{n-1}\sum_{l=j+1}^{n}\frac{\langle 1|\not{K}_{j\cdots l}\not{K}_{(l+1)\cdots(n+1)}|1\rangle^2\langle 2|\not{K}_{j\cdots l}\not{K}_{(l+1)\cdots(n+1)}|1\rangle}{\langle 1|\not{K}_{(l+1)\cdots(n+1)}\not{K}_{j\cdots l}|(j-1)\rangle\langle 1|\not{K}_{(l+1)\cdots(n+1)}\not{K}_{j\cdots l}|j\rangle} \times \frac{\langle (j-1)j\rangle\langle l(l+1)\rangle\langle 1|\not{K}_{2\cdots(j-1)}[\mathcal{F}(j,l)]^2\not{K}_{(l+1)\cdots(n+1)}|1\rangle}{\langle 1|\not{K}_{2\cdots(j-1)}\not{K}_{j\cdots l}|l\rangle\langle 1|\not{K}_{2\cdots(j-1)}\not{K}_{j\cdots l}|(l+1)\rangle s_{j\cdots l}},$$

where $\mathcal{F}(j,l) = \sum_{i=j}^{l-1}\sum_{m=i+1}^{l}\not{k}_i\not{k}_m$.

Zvi Bern, Lance J Dixon, and David A Kosower. LAST OF THE FINITE LOOP AMPLITUDES IN QCD. *PHYSICAL REVIEW D*, 72(12):125003, 2005

Λ prescription

$$\mathcal{A}_n^{*(1)}(g^*,3^+,\cdots,(n+1)^+) = \frac{ig_s^n x|k_T|}{48\pi^2}\left(1+\frac{n_s-n_f}{N_c}\right)\frac{U_1^*+U_2^*+U_3^*}{\kappa^*\langle p3\rangle\langle 34\rangle\cdots\langle np\rangle}\,,$$

$$\text{with } U_1^* = \sum_{j=3}^{n}\frac{\langle pj\rangle\langle p(j+1)\rangle\langle p|\not{K}_{j,j+1}\not{K}_{(j+1)\cdots(n+1)}|p\rangle}{\langle j(j+1)\rangle}\,,$$

$$U_2^* = \sum_{j=4}^{n-1}\sum_{l=j+1}^{n}\frac{\langle p|\not{K}_{j\cdots l}\not{K}_{(l+1)\cdots(n+1)}|p\rangle^3}{\langle p|\not{K}_{(l+1)\cdots(n+1)}\not{K}_{j\cdots l}|(j-1)\rangle\langle p|\not{K}_{(l+1)\cdots(n+1)}\not{K}_{j\cdots l}|j\rangle}$$

$$\times\frac{\langle (j-1)j\rangle\langle l(l+1)\rangle\langle p|\not{K}'_{3\cdots(j-1)}[\mathcal{F}(j,l)]^2\not{K}_{(l+1)\cdots(n+1)}|p\rangle}{\langle p|\not{K}_{3\cdots(j-1)}\not{K}_{j\cdots l}|l\rangle\langle p|\not{K}_{3\cdots(j-1)}\not{K}_{j\cdots l}|(l+1)\rangle s_{j\cdots l}}\,,$$

$$U_3^* = \sum_{l=4}^{n}\frac{\langle p|\not{K}_{3\cdots l}\not{K}_{(l+1)\cdots(n+1)}|p\rangle^3}{\langle p|\not{K}_{(l+1)\cdots(n+1)}\not{K}_{3\cdots l}|p\rangle\langle p|\not{K}_{(l+1)\cdots(n+1)}\not{K}_{3\cdots l}|3\rangle}$$

$$\times\frac{\langle p3\rangle\langle l(l+1)\rangle[p|[\mathcal{F}(3,l)]^2\not{K}_{(l+1)\cdots(n+1)}|p\rangle}{\kappa^*[p|\not{K}_{3\cdots l}|l\rangle[p|\not{K}_{3\cdots l}|(l+1)\rangle s_{3\cdots l}}\,.$$

$g \cdots g$ amplitude

$$\mathcal{A}^{(1)}_{n+1}(1^-, 2^+, 3^+, \cdots, (n+1)^+) = \frac{i g_s^{n+1}}{48\pi^2} \left(1 + \frac{n_s - n_f}{N_c}\right) \frac{T_1 + T_2}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle},$$

$$\text{with } T_1 = \sum_{j=2}^{n} \frac{\langle 1j\rangle\langle 1(j+1)\rangle\langle 1|\not{K}_{j,j+1}\not{K}_{(j+1)\cdots(n+1)}|1\rangle}{\langle j(j+1)\rangle},$$

$$T_2 = \sum_{j=3}^{n-1}\sum_{l=j+1}^{n} \frac{\langle 1|\not{K}_{j\cdots l}\not{K}_{(l+1)\cdots(n+1)}|1\rangle^3}{\langle 1|\not{K}_{(l+1)\cdots(n+1)}\not{K}_{j\cdots l}|(j-1)\rangle\langle 1|\not{K}_{(l+1)\cdots(n+1)}\not{K}_{j\cdots l}|j\rangle} \times \frac{\langle (j-1)j\rangle\langle l(l+1)\rangle\langle 1|\not{K}_{2\cdots(j-1)}[\mathcal{F}(j,l)]^2\not{K}_{(l+1)\cdots(n+1)}|1\rangle}{\langle 1|\not{K}_{2\cdots(j-1)}\not{K}_{j\cdots l}|l\rangle\langle 1|\not{K}_{2\cdots(j-1)}\not{K}_{j\cdots l}|(l+1)\rangle s_{j\cdots l}}.$$

Zvi Bern, Lance J Dixon, and David A Kosower. Last of the finite loop amplitudes in QCD. ***Physical Review D*, 72(12):125003, 2005**

CONCLUSION

- We calculated expression for the following amplitudes at loop-level in high energy factorization :
    - $g^*g^+g^+$ (in agreement with Lipatov's effective action result)
    - $g^*g^+g^+g^+$
    - $g^*g^+g^+g^+g^+$
    - $g^*g^+\cdots g^+$
- The obtained amplitudes have the expected properties (same as tree-level ones) :
    - On-shell limit
    - Equivalence in the use of auxiliary quarks or auxiliary gluons

Outlook

- Expanding this embedding method to non-finite amplitudes
- Addressing real correction
- Automatize NLO calculations in k_T-factorization / ITMD factorization

Thanks for your attention!

Find more at arXiv:2008.07916

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