“Bottomonia production and polarization in the NRQCD with $k_T$-factorization”

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Abdulov N.A., Lipatov A.V., Eur. Phys. J. C 79, 830 (2019)
Abdulov N.A., Lipatov A.V., Eur. Phys. J. C 80, 486 (2020)
https://arxiv.org/abs/2011.13401

- Last decades, the $J/\psi$ and $\Upsilon(nS)$ productions have been actively studied after the discovery of a strong discrepancy between theoretical predictions within the framework of the color singlet model (CS) and the data obtained at the Tevatron.
  M. Krämer, Prog. Part. Nucl. Phys.47, 141 (2001)
  N. Brambillaet al., Eur. Phys. J. C71, 1534 (2011)
  J.P. Lansberg, arXiv:1903.09185
- There are two approaches to describe the formation of heavy quarkonia, that are discussed in the literature: the color singlet and the color octet (CO) models based on the NRQCD.
  G. Bodwin, E. Braaten, G. Lepage, Phys. Rev. D51, 1125 (1995).
  P. Cho, A.K. Leibovich, Phys. Rev. D53, 150 (1996); Phys. Rev. D53, 6203 (1996)
- The CS model in LO fails to describe the experimental data at the Tevatron and LHC. A somewhat better description is achieved when NLO and NNLO* corrections are taken into account.

- NRQCD is an effective field theory, which is used to describe the production of heavy quarkonia, based on the expansion in $v$ and $\alpha_s$:

$$|\psi_Q\rangle = O(1)|Q\bar{Q}[^3S_1^{(1)}]\rangle + O(v)|Q\bar{Q}[^3P_J^{(8)}]g\rangle + O(v^2)|Q\bar{Q}[^3S_1^{(1,8)}]gg\rangle + \\ + O(v^2)|Q\bar{Q}[^1S_0^{(8)}]g\rangle + O(v^2)|Q\bar{Q}[^3D_J^{(1,8)}]gg\rangle + \ldots$$

  where ${}^{2S+1}L_J^{(a)}$ is an intermediate Fock state with spin $S$, orbital angular momentum $L$, total angular momentum $J$ and color representation $a$ from long-distance non-perturbative matrix elements (NMEs).

- In NLO NRQCD a good description of the data on $\boldsymbol{p}_t$ is achieved by fitting the NME values, which play the role of free parameters and determine the probability of quarkonium production. But the values of these NMEs obtained from different sets are very different from each other.

  *K.-T. Chao, Y.-Q. Ma, H.-S. Shao, K. Wang, Y.-J. Zhang, Phys. Rev. Lett. 108, 242004 (2012).*
  *B. Gong, L.-P. Wan, J.-X. Wang, H.-F. Zhang, Phys. Rev. Lett. 110, 042002 (2013)*

- However, until now, a simultaneous description of the $p_t$ spectra and polarization observables has not been achieved: theoretical predictions show that the octet contribution with a transverse polarization dominates in the production of $S$-wave quarkonia at large $p_t$. But this contradicts experimental data from CDF, CMS and LHCb.
- Also, the results of NME calculations obtained in the NRQCD for $J/\psi$ contradict the experimental data for $\eta_c$.

  *H.-F. Zhang, Z. Sun, W.-L. Sang, R. Li, Phys. Rev. Lett. 114, 092006 (2015).*

  *M. Butenschön, Z. G. He, B.A. Kniehl, Phys. Rev. Lett.114, 092004 (2015)*
- The solution to this problem was proposed in the framework of the depolarization model, which was proposed by S. Baranov and based on classical multipole expansion. In this approach, it was possible to solve the problem above for the entire charmonia family.
  S.P. Baranov, Phys. Rev. D93, 054037 (2016).
  S.P. Baranov, A.V. Lipatov, Eur. Phys. J. C 79, 621 (2019).
  S.P. Baranov, A.V. Lipatov, Phys. Rev. D 100, 114021 (2019)

- The main goal of our research is to study the processes of $S$- and $P$-wave botomonia [$\Upsilon(nS), \chi_{bJ}(mP)$] production and their polarization at high energies within the framework of the depolarization model.
- When considering the processes of $\Upsilon(3S)$ production, the contribution from the decay of $\chi_{bJ}(3P)$ was taken into account,
for $\Upsilon(2S)$ — $\chi_{bJ}(3P)$, $\chi_{bJ}(2P)$ and $\Upsilon(3S)$,
and for $\Upsilon(1S)$ — $\chi_{bJ}(3P)$, $\chi_{bJ}(2P)$, $\chi_{bJ}(1P)$, $\Upsilon(3S)$, $\Upsilon(2S)$ and cascade decays.
- To describe the perturbative production of the $b\bar{b}$ pair, the $k_T$-factorization approach is used with both CS and CO contributions.
- The NME for $\Upsilon(nS)$ and $\chi_{bJ}(mP)$ was determined using the data obtained by the CMS, ATLAS and LHCb at energies of 7, 8 and 13 TeV in different kinematic regions.

# The $k_T$-factorization approach

- In the $k_T$-factorization approach LO includes high-order QCD corrections (NLO + NNLO + ...) in the form of TMD parton distribution functions $f(x, \boldsymbol{k}_t^2, \mu^2)$.
- These functions are computed as solutions to the BFKL and CCFM equations. The CCFM equation also takes into account terms proportional to $\sim \alpha_S^n \ln^n(1/x)$ and $\sim \alpha_S^n \ln^n(1/(1-x))$. For large x the CCFM equation is equivalent to the DGLAP equations.
- Here we apply several TMD gluon distribution functions to describe the cross sections for the inclusive production of $\Upsilon(nS)$: A0, JH'2013set1, JH'2013set2, PB'2018set2 and MD'2018.

H. Jung, arXiv:hep-ph/0411287.
F. Hautmann, H. Jung, Nucl. Phys. B 883, 1 (2014).
A.B. Martinez et al., Phys. Rev. D 99, 074008 (2019).
N.A. Abdulov, H. Jung, A.V. Lipatov, G.I. Lykasov, and M.A. Malyshev, Phys. Rev. D 98, 054010 (2018)

## Basic formulas

We consider the subprocesses of fusion of two off-shell gluons:

$$g^*(k_1) + g^*(k_2) \to \Upsilon[{}^3S_1^{(1)}](p) + g(k),$$

$$g^*(k_1) + g^*(k_2) \to \Upsilon[{}^1S_0^{(8)}, {}^3S_1^{(8)}, {}^3P_J^{(8)}](p).$$

$$g^*(k_1) + g^*(k_2) \to b\bar{b} \to \chi_{bJ}[{}^3P_J^{(1)}, {}^3S_1^{(8)}](p) \to \Upsilon(p_1) + \gamma(p_2),$$

Projectors onto states $n = {}^{2s+1}L_J^{(a)}$ with the corresponding quantum numbers:

$$\prod[{}^1S_0] = \gamma_5(\hat{p}_b + m_b)/M^{1/2},$$

$$\prod[{}^3S_1] = \hat{\epsilon}(S_z)(\hat{p}_b + m_b)/M^{1/2},$$

$$\prod[{}^3P_J] = (\hat{p}_{\bar{b}} - m_b)\hat{\epsilon}(S_z)(\hat{p}_b + m_b)/M^{3/2},$$

## Basic formulas

The probability of a quarkonium production from two quarks depends on $\Psi^{(a)}(q)$, where $q$ is related to the momenta of quarks and quarkonium, as

$$p_b = p/2 + q, p_{\bar{b}} = p/2 - q.$$

Next, we integrate the product of the amplitude $A$, expanded in a series around $q = 0$, and $\Psi^{(a)}(q)$:

$$A(q)\Psi^{(a)}(q) = A|_{q=0}\Psi^{(a)}(q) + q^\alpha(\partial A/\partial q^\alpha)|_{q=0}\Psi^{(a)}(q) + \dots,$$

since A is no longer dependent on $q$, we get:

$$\int \frac{d^3q}{(2\pi)^3}\Psi^{(a)}(q) = \frac{1}{\sqrt{4\pi}}\mathcal{R}^{(a)}(0),$$

$$\int \frac{d^3q}{(2\pi)^3}q^\alpha\Psi^{(a)}(q) = -i\epsilon^\alpha(L_z)\frac{\sqrt{3}}{\sqrt{4\pi}}\mathcal{R}'^{(a)}(0),$$

where $\mathcal{R}^{(a)}(x)$ — radial wave function.

## Basic formulas

- NMEs are related to wave function as

$$\langle\mathcal{O}[^{2S+1}L_J^{(a)}]\rangle = 2N_c(2J+1)|\mathcal{R}^{(a)}(0)|^2/4\pi,$$

$$\langle\mathcal{O}[^{2S+1}L_J^{(a)}]\rangle = 6N_c(2J+1)|\mathcal{R}'^{(a)}(0)|^2/4\pi$$

  for $S$- and $P$-wave states, where $N_c = 3$.
- For CS $\Upsilon(nS)$ and $\chi_{bJ}(mP)$ NME values are well known. These values can be obtained from the experimentally measured width of the decay of quarkonium into lepton pair, or they can be calculated within the framework of potential models. For CO, the corresponding NMEs are derived from experimental data.
- Also for P-wave states, the following relation is taken into account:

$$\langle\mathcal{O}[^3P_J^{(8)}]\rangle = (2J+1)\langle\mathcal{O}[^3P_0^{(8)}]\rangle$$

## Depolarization model

- In the works of S. Baranov, N. Zotov and A. Lipatov a new mechanism was proposed for the transition of an octet quark pair to a singlet state, which leads to depolarization of quarkonia at medium and large $\boldsymbol{p}_t$.
  This mechanism was used to solve the problem of the simultaneous description of both the spectrum in terms of $\boldsymbol{p}_t$ and polarization for the charmonia family.

  *S. P. Baranov, A. V. Lipatov and N. P. Zotov, Eur. Phys. J. C 75, 455 (2015).*
  *S. P. Baranov, A. V. Lipatov and N. P. Zotov, Phys. Rev. D 93, 094012 (2016).*
  *S. P. Baranov and A. V. Lipatov, Phys. Rev. D 96, 034019 (2017).*

- After the production of the octet state, a soft gluon with energy $E \sim \Lambda_{QCD}$ is emitted. This step is described by the multipole expansion dominated by an electric dipole (E1) transition:

$$A_{tr}({}^3P_1^{(8)} \to \Upsilon + g) = g_1 e^{\mu\nu\alpha\beta} k_\mu^{(g)} \epsilon_\nu^{(CO)} \epsilon_\alpha^{(\Upsilon)} \epsilon_\beta^{(g)},$$

$$A_{tr}({}^3P_2^{(8)} \to \Upsilon + g) = g_2 p_\mu^{(CO)} \epsilon_{\alpha\beta}^{(CO)} \epsilon_\alpha^{(\Upsilon)} [k_\mu^{(g)} \epsilon_\beta^{(g)} - k_\beta^{(g)} \epsilon_\mu^{(g)}].$$

  In the case of ${}^3P_J^{(8)}$ one E1 transition occurs, and in the case of ${}^3S_1^{(8)}$ — two transitions.

- The depolarization model describes the transitions to CS and does not depend on the formation of CO states.

Summation over the polarizations of incoming gluons is performed according to the rules of the $k_T$-factorization approach:

$$\overline{\epsilon^{\mu}\epsilon^{*\nu}} = \boldsymbol{k}_t^{\mu}\boldsymbol{k}_t^{\nu}/\boldsymbol{k}_t^2,$$

and for quarkonium:

$$\Sigma\epsilon^{\mu}\epsilon^{*\nu} = 3(l_1^{\mu}l_2^{\nu} + l_1^{\nu}l_2^{\mu} - \frac{M^2}{2}g^{\mu\nu})/M^2$$

This expression is equivalent to the standard expression $\Sigma\epsilon^{\mu}\epsilon^{*\nu} = -g^{\mu\nu} + p^{\mu}p^{\nu}/M^2$, but more suitable for fitting polarized observables.

The cross section for CS looks like:

$$\sigma = \int \frac{1}{16\pi(x_1 x_2 S)^2} f_g(x_1, \boldsymbol{k}_{1t}^2, \mu^2) f_g(x_2, \boldsymbol{k}_{2t}^2, \mu^2) \overline{|A|^2} \times$$

$$\times\, d\boldsymbol{p}_t^2 d\boldsymbol{k}_{1t}^2 d\boldsymbol{k}_{2t}^2 dy dy_g \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi},$$

for CO:

$$\sigma = \int \frac{2\pi}{x_1 x_2 S F} f_g(x_1, \boldsymbol{k}_{1t}^2, \mu^2) f_g(x_2, \boldsymbol{k}_{2t}^2, \mu^2) \overline{|A|^2} d\boldsymbol{k}_{1t}^2 d\boldsymbol{k}_{2t}^2 dy \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi},$$

where $f_g(x, \boldsymbol{k}_t^2, \mu^2)$ — TMD gluon density function in a proton, $F = 2\lambda^{1/2}(\hat{s}, k_1^2, k_2^2)$, $\lambda(x, y, z)$ — kinematic function, $\hat{s} = (k_1 + k_2)^2$, $\overline{|A|^2}$ — amplitudes of the corresponding subprocesses, $\phi$ — azimuthal angles of initial gluons with a fraction of momentum $x$ and with momentum $\boldsymbol{k}_t$.

Calculations show that the shape of the cross sections in terms of $p_t$ is practically the same for the states $\Upsilon[^3S_1^{(8)}](nS)$ and $\chi_{bJ}[^3S_1^{(8)}](nP)$ in all kinematic regions and at different energies. For example, r values for the A0 set:

$$r^{\Upsilon(3S)} = \frac{\sum\limits_{J=0}^{2}(2J+1)\,Br_J d\sigma[\chi_{bJ},{}^3S_1^{(8)}]/dp_T}{d\sigma[\Upsilon,{}^3S_1^{(8)}]/dp_T} = 0.674 \pm 0.003,$$

$$r^{\Upsilon(2S)} = \frac{\sum\limits_{J=0}^{2}(2J+1)\,Br_J d\sigma[\chi_{bJ},{}^3S_1^{(8)}]/dp_T}{d\sigma[\Upsilon,{}^3S_1^{(8)}]/dp_T} = 1.008 \pm 0.005,$$

$$r^{\Upsilon(1S)} = \frac{\sum\limits_{J=0}^{2}(2J+1)\,Br_J d\sigma[\chi_{bJ},{}^3S_1^{(8)}]/dp_T}{d\sigma[\Upsilon,{}^3S_1^{(8)}]/dp_T} = 1.743 \pm 0.010$$

## NMEs determination

Considering the previous relationship, we use experimental data to fit NMEs and a linear combination of NME octets:

$$M_r = \langle \mathcal{O}^{\Upsilon}[^3S_1^{(8)}]\rangle + r\langle \mathcal{O}^{\chi_{b0}}[^3S_1^{(8)}]\rangle,$$

and then using the ratio obtained by the LHCb Collaboration,

$$R^{\chi_b(nP)}_{\Upsilon(nS)} = \sum_{J=0}^{2} \frac{\sigma(pp \to \chi_{bJ}(nP) + X)}{\sigma(pp \to \Upsilon(nS) + X)} \times Br(\chi_{bJ} \to \Upsilon(nS) + \gamma),$$

we find the required NME values.

Next, we found that the distribution ratios in terms of $p_T$ for $\Upsilon[^3P_J^{(8)}]$, $\chi_b[^3P_1^{(1)}]$ and $\chi_b[^3P_2^{(1)}]$ contributions to the production of $\Upsilon(1S)$ as well can be approximated by a constant.

We introduce:

$$r_1 = \frac{B(\chi_{b2}(1P) \to \Upsilon(1S) + \gamma) d\sigma[\chi_{b2}(1P), {}^3P_2^{(1)}]/dp_T}{B(\chi_{b1}(1P) \to \Upsilon(1S) + \gamma) d\sigma[\chi_{b1}(1P), {}^3P_1^{(1)}]/dp_T},$$

$$r_2 = \frac{\sum_{J=0}^{2} (2J+1)\, d\sigma[\Upsilon(1S), {}^3P_J^{(8)}]/dp_T}{B(\chi_{b1}(1P) \to \Upsilon(1S) + \gamma) d\sigma[\chi_{b1}(1P), {}^3P_1^{(1)}]/dp_T}.$$

For example, $r_1 = 0.91 \pm 0.02$ and $r_2 = 104 \pm 2$ for the A0 set.

Then we introduce the linear combination

$$M_{r_1 r_2} = \langle \mathcal{O}^{\chi_{b1}(1P)}[^3P_1^{(1)}]\rangle + r_1 \langle \mathcal{O}^{\chi_{b2}(1P)}[^3P_2^{(1)}]\rangle + r_2 \langle \mathcal{O}^{\Upsilon(1S)}[^3P_0^{(8)}]\rangle,$$

which, as in the case of $M_r$, we extract from the experimental data on the distribution of $\Upsilon(1S)$ in terms of $p_T$.

Then, we use the LHCb data at $\sqrt{s} = 7$ and 8 TeV for the ratio

$$R^{\chi_b(1P)}_{\Upsilon(1S)} = \sum_{J=1}^{2} \frac{\sigma(pp \to \chi_{bJ}(1P) + X)}{\sigma(pp \to \Upsilon(1S) + X)} \times B(\chi_{bJ} \to \Upsilon(1S) + \gamma),$$

from which we extract $\langle \mathcal{O}^{\Upsilon(1S)}[^3S_1^{(8)}]\rangle$, $\langle \mathcal{O}^{\chi_{b0}(1P)}[^3S_1^{(8)}]\rangle$, $\langle \mathcal{O}^{\Upsilon(1S)}[^3P_0^{(8)}]\rangle$ and linear combination $M_{CS} = \langle \mathcal{O}^{\chi_{b1}(1P)}[^3P_1^{(1)}]\rangle + r_1 \langle \mathcal{O}^{\chi_{b2}(1P)}[^3P_2^{(1)}]\rangle$.

Finally, we use the CMS and LHCb data at $\sqrt{s} = 8$ TeV for the ratio

$$R^{\chi_{b2}(1P)}_{\chi_{b1}(1P)} = \frac{\sigma(\chi_{b2}(1P))}{\sigma(\chi_{b1}(1P))}.$$

- The corresponding uncertainties were obtained using the Student's t-distribution with a confidence $P = 80\%$

| $\Upsilon(3S)$ | A0 | JH'2013 set 1 | KMR | NLO NRQCD |
|---|---|---|---|---|
| $\langle \mathcal{O}^{\Upsilon(3S)}[^3S_1^{(1)}]\rangle/\mathrm{GeV}^3$ | 3.22 | 3.22 | 3.22 | 3.54 |
| $\langle \mathcal{O}^{\Upsilon(3S)}[^1S_0^{(8)}]\rangle/\mathrm{GeV}^3$ | 0.0 | 0.0 | 0.0 | $0.0145 \pm 0.0116$ |
| $\langle \mathcal{O}^{\Upsilon(3S)}[^3S_1^{(8)}]\rangle/\mathrm{GeV}^3$ | $0.018 \pm 0.001$ | $0.007 \pm 0.002$ | $0.006 \pm 0.001$ | $0.0132 \pm 0.0020$ |
| $\langle \mathcal{O}^{\Upsilon(3S)}[^3P_0^{(8)}]\rangle/\mathrm{GeV}^5$ | 0.0 | $0.09 \pm 0.03$ | $0.073 \pm 0.006$ | $-0.0027 \pm 0.0025$ |
| $\langle \mathcal{O}^{\chi_{b0}(3P)}[^3P_0^{(1)}]\rangle/\mathrm{GeV}^5$ | 2.84 | 2.84 | 2.84 | 2.57 |
| $\langle \mathcal{O}^{\chi_{b0}(3P)}[^3S_1^{(8)}]\rangle/\mathrm{GeV}^3$ | $0.016 \pm 0.003$ | $0.009 \pm 0.001$ | $0.005 \pm 0.001$ | $0.0069 \pm 0.0014$ |

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- The corresponding uncertainties were obtained using the Student's t-distribution with a confidence $P = 80\%$

| $\Upsilon(2S)$ | A0 | JH'2013 set 1 | KMR | NLO NRQCD |
|---|---|---|---|---|
| $\langle \mathcal{O}^{\Upsilon(2S)}[^3S_1^{(1)}]\rangle/\mathrm{GeV}^3$ | 4.15 | 4.15 | 4.15 | 4.63 |
| $\langle \mathcal{O}^{\Upsilon(2S)}[^1S_0^{(8)}]\rangle/\mathrm{GeV}^3$ | 0.0 | 0.0 | 0.0 | $0.0062 \pm 0.0198$ |
| $\langle \mathcal{O}^{\Upsilon(2S)}[^3S_1^{(8)}]\rangle/\mathrm{GeV}^3$ | $0.016 \pm 0.004$ | $0.002 \pm 0.006$ | $0.0019 \pm 0.0006$ | $0.0222 \pm 0.0024$ |
| $\langle \mathcal{O}^{\Upsilon(2S)}[^3P_0^{(8)}]\rangle/\mathrm{GeV}^5$ | $0.014 \pm 0.009$ | $0.19 \pm 0.05$ | $0.14 \pm 0.02$ | $-0.0013 \pm 0.0043$ |
| $\langle \mathcal{O}^{\chi_{b0}(2P)}[^3P_0^{(1)}]\rangle/\mathrm{GeV}^5$ | 2.61 | 2.61 | 2.61 | 0.39 |
| $\langle \mathcal{O}^{\chi_{b0}(2P)}[^3S_1^{(8)}]\rangle/\mathrm{GeV}^3$ | $0.0181 \pm 0.0007$ | $0.0117 \pm 0.0007$ | $0.0074 \pm 0.0004$ | $0.0109 \pm 0.0014$ |

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- The corresponding uncertainties were obtained using the Student's t-distribution with a confidence $P = 80\%$

| $\Upsilon(1S)$ | A0 | JH'2013 set 1 | KMR | NLO NRQCD |
|---|---|---|---|---|
| $\langle \mathcal{O}^{\Upsilon(1S)}[^3S_1^{(1)}] \rangle/\mathrm{GeV}^3$ | 8.39 | 8.39 | 8.39 | 9.28 |
| $\langle \mathcal{O}^{\Upsilon(1S)}[^1S_0^{(8)}] \rangle/\mathrm{GeV}^3$ | 0.0 | 0.0 | 0.0 | $0.136 \pm 0.0243$ |
| $\langle \mathcal{O}^{\Upsilon(1S)}[^3S_1^{(8)}] \rangle/\mathrm{GeV}^3$ | $0.016 \pm 0.006$ | $0.0038 \pm 0.0019$ | $0.0029 \pm 0.0019$ | $0.0061 \pm 0.0024$ |
| $\langle \mathcal{O}^{\Upsilon(1S)}[^3P_0^{(8)}] \rangle/\mathrm{GeV}^5$ | $0.07 \pm 0.03$ | $0.20 \pm 0.10$ | $0.18 \pm 0.06$ | $-0.0093 \pm 0.005$ |
| $\langle \mathcal{O}^{\chi_{b0}(1P)}[^3P_0^{(1)}] \rangle/\mathrm{GeV}^5$ | 2.30 | 2.30 | 2.30 | 2.03 |
| $\langle \mathcal{O}^{\chi_{b1}(1P)}[^3P_1^{(1)}] \rangle/\mathrm{GeV}^5$ | $7 \pm 3$ | $11 \pm 5$ | $9 \pm 2$ | 6.09 |
| $\langle \mathcal{O}^{\chi_{b2}(1P)}[^3P_2^{(1)}] \rangle/\mathrm{GeV}^5$ | $2.4 \pm 1.9$ | $6 \pm 4$ | $6 \pm 2$ | 10.15 |
| $\langle \mathcal{O}^{\chi_{b0}(1P)}[^3S_1^{(8)}] \rangle/\mathrm{GeV}^3$ | $0.008 \pm 0.002$ | $0.0020 \pm 0.0011$ | $0.0015 \pm 0.0012$ | $0.0094 \pm 0.0006$ |

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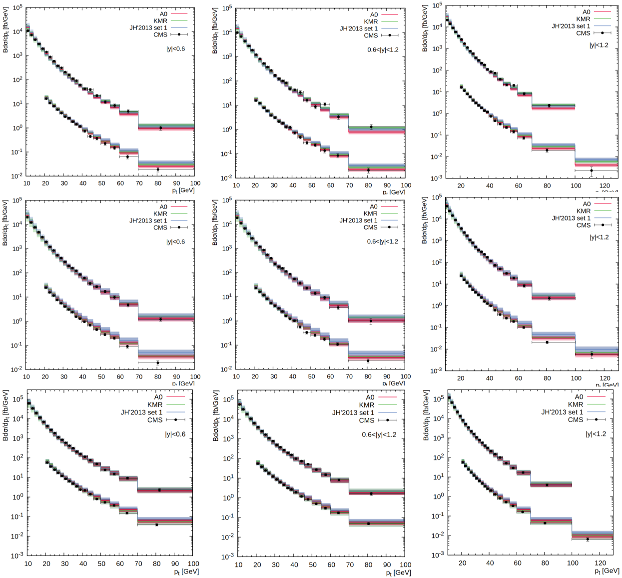
- Transverse momentum distribution $\Upsilon(nS)$ at $\sqrt{s} = 7$ TeV (upper histograms) and $\sqrt{s} = 13$ TeV (lower, divided by 100).

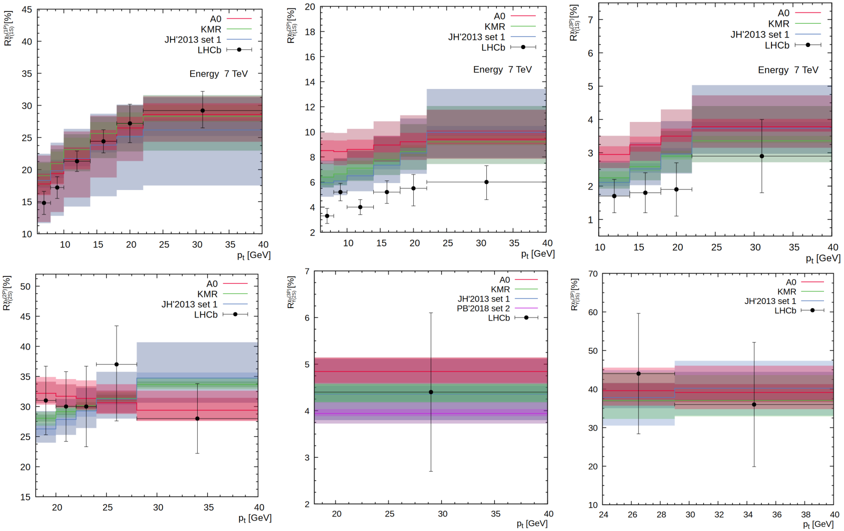
$\Upsilon(3S)$

$\Upsilon(2S)$

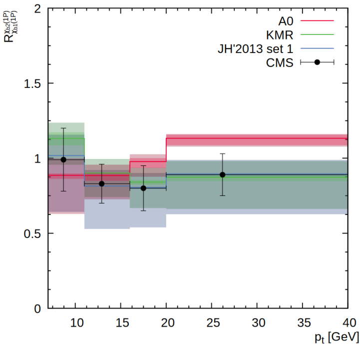
$\Upsilon(1S)$

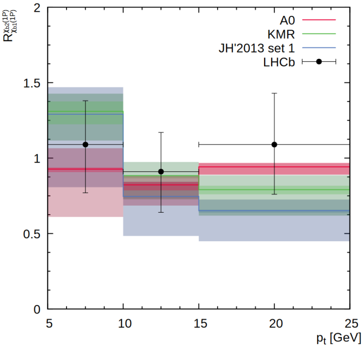


- Relations $R^{\chi_b(mP)}_{\Upsilon(nS)}$ as a function of the transverse momentum $\Upsilon(nS)$ at $\sqrt{s} = 7$ and 8 TeV.



- Relations $R^{\chi_{b\mathbf{2}}(\mathbf{1}P)}_{\chi_{b\mathbf{1}}(\mathbf{1}P)}$ as a function of the transverse momentum $\Upsilon(1S)$ at $\sqrt{s} = 8$ TeV.



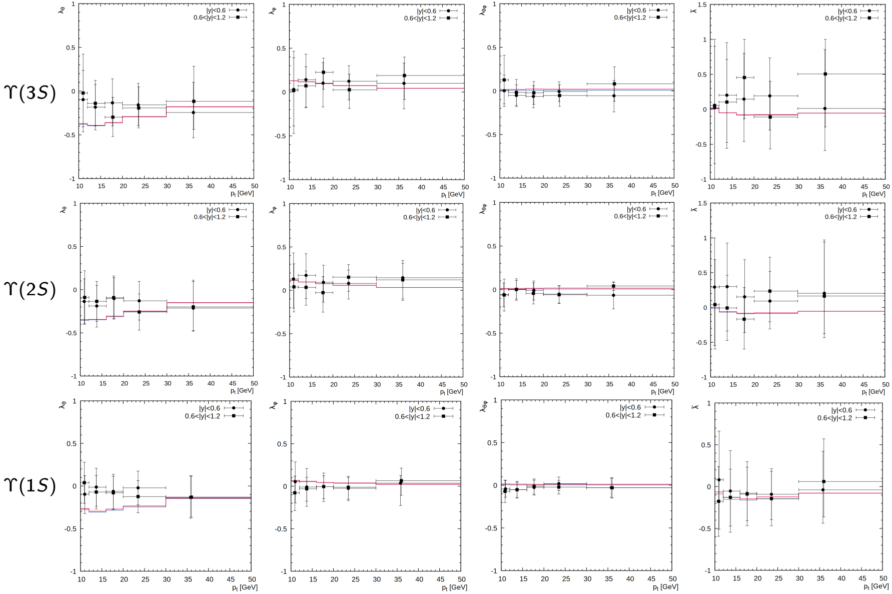



- After fitting the NME values, the angular parameters $\lambda_\theta$, $\lambda_\phi$ and $\lambda_{\theta\phi}$ can be calculated, which were measured experimentally by the CMS and CDF Collaborations.
  The angular distribution of the final leptons is as follows:

$$\frac{d\sigma}{d\cos\theta^* d\phi^*} \sim \frac{1}{3+\lambda_\theta}(1+\lambda_\theta\cos^2\theta^* + \lambda_\phi\sin^2\theta^*\cos 2\phi^* + \lambda_{\theta\phi}\sin 2\theta^*\cos\phi^*),$$

  where $\theta^*$ и $\phi^*$ — polar and azimuthal angles of leptons in the quarkonia rest frame.
- All parameters are calculated in three frames: Collins-Soper, helicity and perpendicular helicity frames.
- Additionally, the parameter $\tilde{\lambda} = (\lambda_\theta + 3\lambda_\phi)/(1-\lambda_\phi)$ was calculated.

- Polarization parameters $\Upsilon(nS)$ in the CS system at $\sqrt{s} = 7$ TeV.



# Conclusion

- We have considered the $\Upsilon(nS)$ and $\chi_b(mP)$ productions at the Tevatron and LHC in the framework of $k_T$-factorization approach. Our consideration is based on the off-shell production amplitudes for hard partonic subprocesses (including both color singlet and color octet contributions), NRQCD formalism for the formation of bound states and TMD gluon densities in a proton.
- Treating the nonperturbative color octet transitions in terms of multipole radiation theory and taking into account feed-down contributions from the radiative $\chi_b(mP)$ decays, we extracted long-distance non-perturbative NRQCD matrix elements for $\Upsilon(nS)$ and $\chi_b(mP)$ mesons.
- We achieved good agreement with experimental data on the transverse momentum distribution and on the relations $R^{\chi_b(mP)}_{\Upsilon(nS)}$ and $R^{\chi_{b\mathbf{2}}(1P)}_{\chi_{b\mathbf{1}}(1P)}$
- We find only weak or zero polarization in the all kinematic regions, that agrees with the CMS and CDF measurements.
- This proposed approach allowed us to solve the known problem of the simultaneous description of the $p_T$ spectra and polarization observables in the framework of QCD.

# Backup slides