THE UNIVERSITY of EDINBURGH

DEC 7 2020 - DEC 11 2020 | Resummation, Evolution, Factorization 2020

Higgs Centre Workshop

HIGGS CENTRE FOR THEORETICAL PHYSICS

Factorization, evolution and resummation for heavy quarkonium production

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December 7th, 2020

Based on works done with Z.-B. Kang, K. Lee, Y.-Q. Ma, G. Nayak, G. Sterman, K. Watanabe, H. Zhang, …

Jefferson Lab

TMD Collaboration

U.S. DEPARTMENT OF ENERGY | Office of Science

JSA

Outline

- QCD, Factorization, Renormalization, and Resummation
- Factorization, renormalization and resummation for heavy quarkonium production
- Factorization, renormalization and resummation beyond the leading power
- Non-linear evolution, and quarkonium polarization
- Summary and outlook

QCD – Unprecedented intellectual challenge

- **Color confinement:**

  “Cross section” with identified hadron(s) is NOT perturbatively calculable

  $\sigma(S, M_Q, Q_s)$ **is NOT perturbative no matter how large** S **is!**

  With the hadronic scale: $Q_s \sim \Lambda_{\rm QCD}$

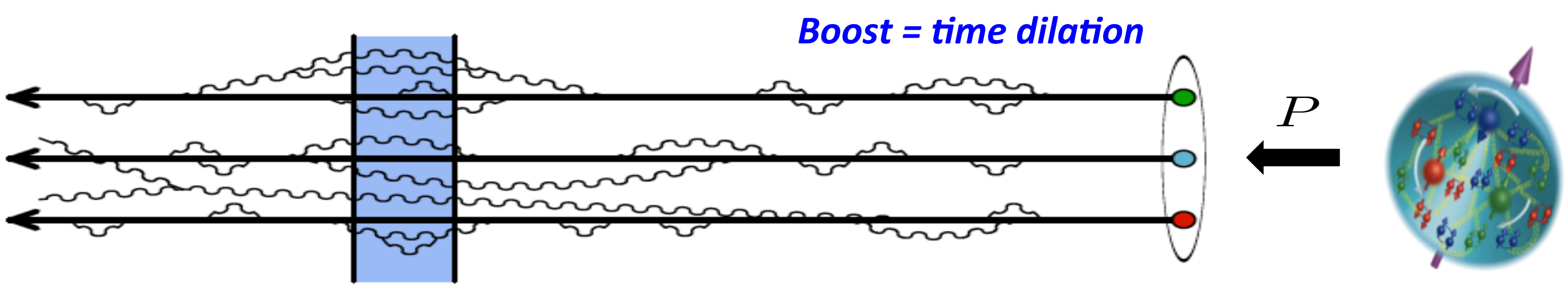
- **Asymptotic freedom:**

  Perturbative QCD could work for dynamics at short-distance: $1/Q$

  with a large momentum transfer: Q and $S \gtrsim Q \gg Q_s$

$$\sigma(Q, S, M_Q, Q_s) = \sigma^{\rm LP}(Q, S, M_Q, Q_s) \times \left[1 + \mathcal{O}\left(\frac{Q_s}{Q}\right)^n + \ldots\right]$$

- **Hard probe (t ~ 1/Q << fm):**



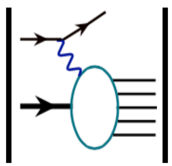
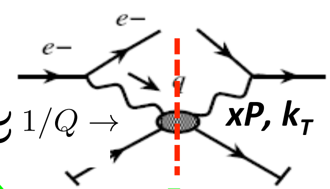
Probability to “catch” the parton!

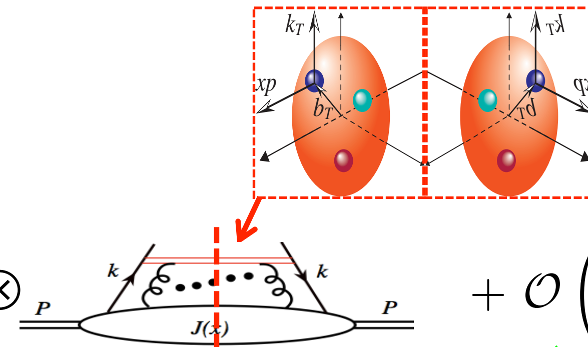
Factorization – Predictive Power

❑ Factorization – Approximation:

Leading non-perturbative hadronic information is factorized into universal functions

Ex: Single identified hadron – lepton-hadron DIS:

$\sigma_{\rm DIS}(x,Q^2) = \Big|$  $\Big|^2 \approx$ 

$\otimes$ 

$+ \mathcal{O}\left(\frac{Q_s}{Q}\right)^n$

The way out: **Factorization** | **Hard-part** *Probe* | **Parton-distribution** *Structure* | **Power corrections** *Approximation!*

$$\sigma_{\rm DIS}(x,Q^2,Q_s) = \underbrace{\sum_f c_f(x,Q^2/\mu^2)\otimes\phi_f(x,\mu^2)}_{\sigma^{\rm LP}_{\rm DIS}(x,Q^2,Q_s)}\left[1+\mathcal{O}\left(\frac{Q_s}{Q}\right)^n+\ldots\right]$$

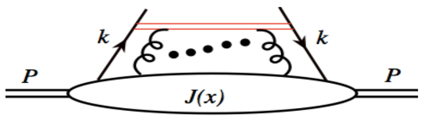
❑ Factorization – Predictive power:

Factorized non-perturbative information, e.g., $\phi_f(x,\mu^2)$ is universal,
Controllable power corrections, …

Factorization, Renormalization and Evolution

- **Factorization requires renormalization of nonlocal operators:**

$$\sigma_{\rm DIS}^{\rm LP}(x,Q^2,Q_s) = \sum_f c_f(x,Q^2/\mu^2)\otimes\phi_{f/h}(x,\mu^2,Q_s)$$



$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{4\pi} e^{ixp^+y^2}\langle h(p)|\overline{\psi}_q(0)\gamma^+\Phi(0,y^-)\psi_q(y^-)|h(p)\rangle + {\rm UVCT}(\mu^2)$$

- **Matching coefficients and factorization scheme:**

$$c_f^{(1)}(x,Q^2/\mu^2) = \sigma^{(1)}_{\rm DIS-q}(x,Q^2) - \sigma^{(0)}_{\rm DIS-q}(x,Q^2,Q_s)\otimes\phi^{(1)}_{q/q}(x,\mu^2,Q_s)$$

$$\propto \int_0^{Q^2}\frac{dk_T^2}{k_T^2} - \left[\int_0^\infty \frac{dk_T^2}{k_T^2} + \underbrace{{\rm UVCT}(\mu^2)}\right]$$

$$\longrightarrow\ \ln(Q^2/\mu^2)$$

Scheme-dependence of c_f
Leading to scheme dependence
of extracted PDFs, …

- **Factorization leads to evolution and resummation:**

$$\frac{d}{d\ln\mu^2}\,\sigma_{\rm DIS}^{\rm LP}(x,Q^2,Q_s) = \sum_f c_f(x,Q^2/\mu^2)\otimes\phi_{f/h}(x,\mu^2,Q_s) = 0$$

$$\Longrightarrow\ \frac{\partial}{\partial\ln\mu^2}\phi_{f/h}(x,\mu^2) = \sum_i \gamma_{f/i}(x,\alpha_s)\otimes\phi_{i/h}(x,\mu^2)$$

Solution of evolution
= Resummation

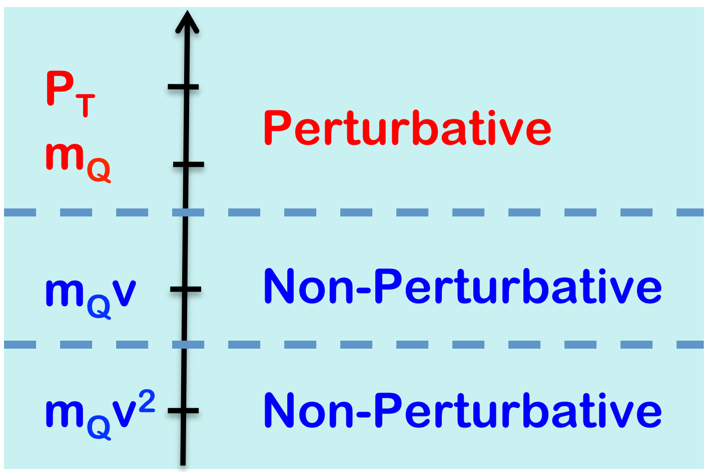
Heavy quarkonium production

- **One of the simplest QCD bound states:**

  Localized color charges (heavy mass), non-relativistic relative motion

  Charmonium: $v^2 \approx 0.3$ **Bottomonium:** $v^2 \approx 0.1$

- **Well-separated momentum scales – effective theory:**



Hard — Production of $Q\bar{Q}$ **[pQCD]**

Soft — Relative Momentum **[NRQCD]**

$\Lambda_{\rm QCD}$

Ultrasoft — Binding Energy **[pNRQCD]**

- **Cross sections and observed mass scales:**

$$\frac{d\sigma_{AB\to H(P)X}}{dy\,dP_T^2} \qquad \sqrt{S}, \qquad P_T, \qquad M_H,$$

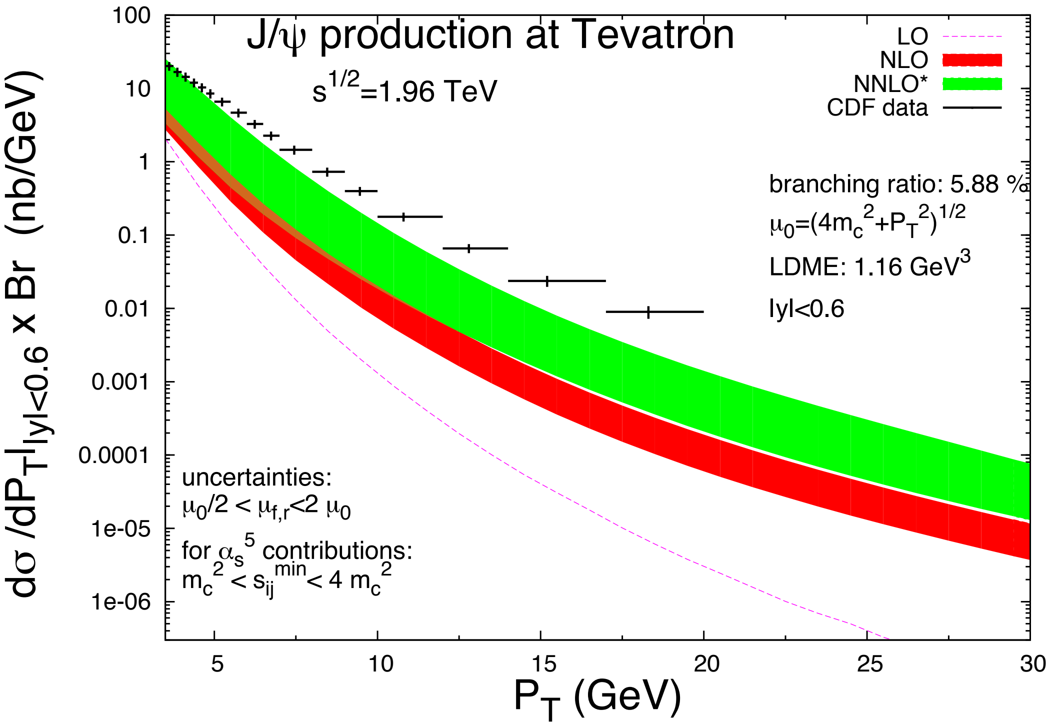
PQCD is "expected" to work for the production of heavy quarks

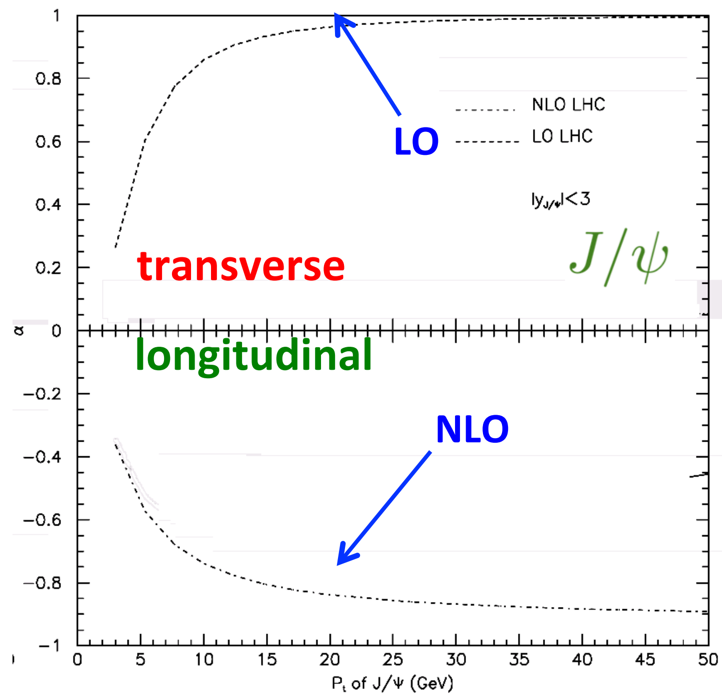
Difficulty = Emergence of a quarkonium from a heavy quark pair?

Color singlet model (CSM)

❑ **Effectively No parameter:**

Campbell, Maltoni, Tramontano (2007),
Artoisenet, Lansburg, Maltoni (2007),
Artoisenet, et al. (2008)





B. Gong et, al. PRL (2008)

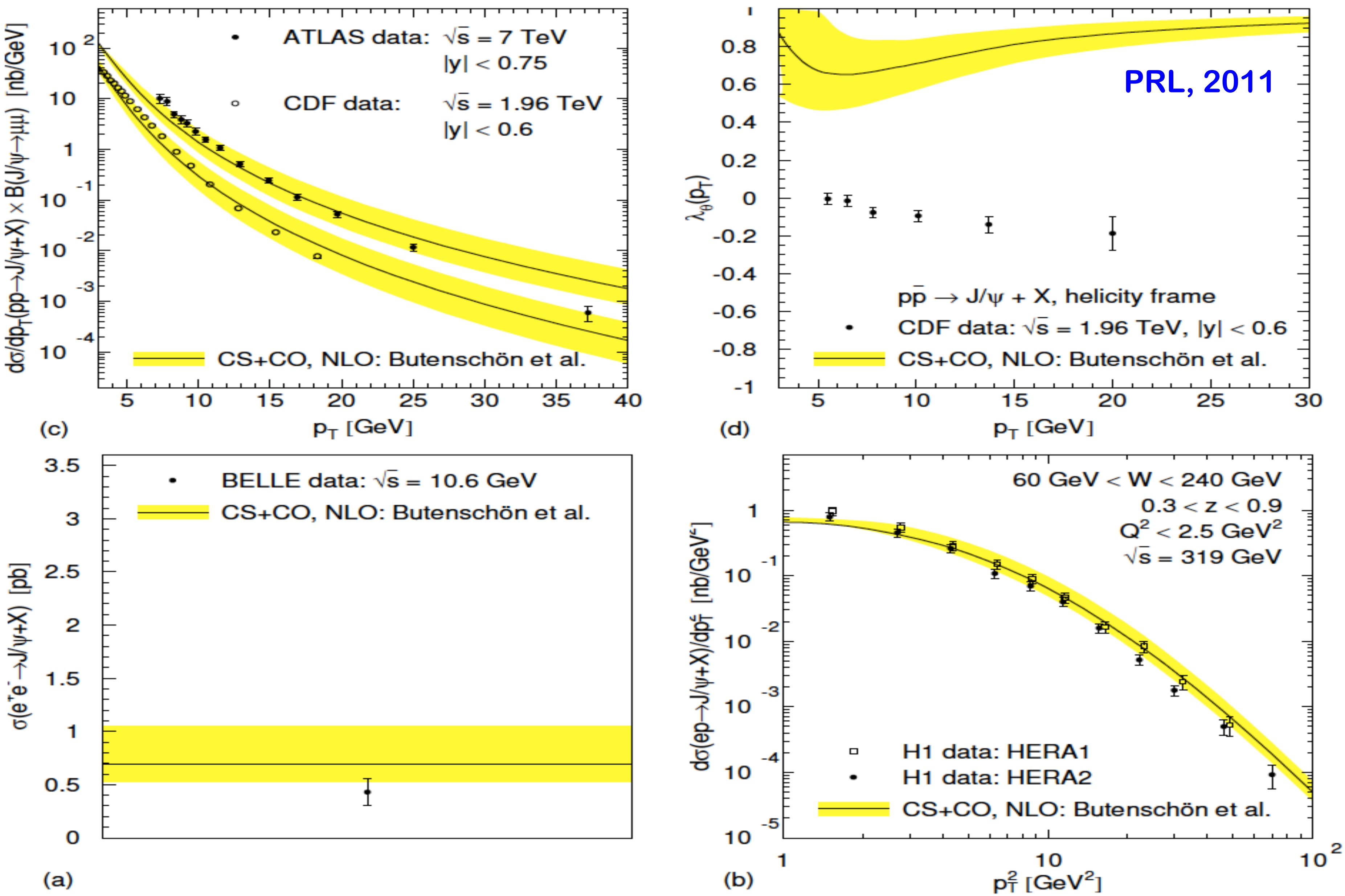
❑ **Issues:**

✧ How reliable is the perturbative expansion?

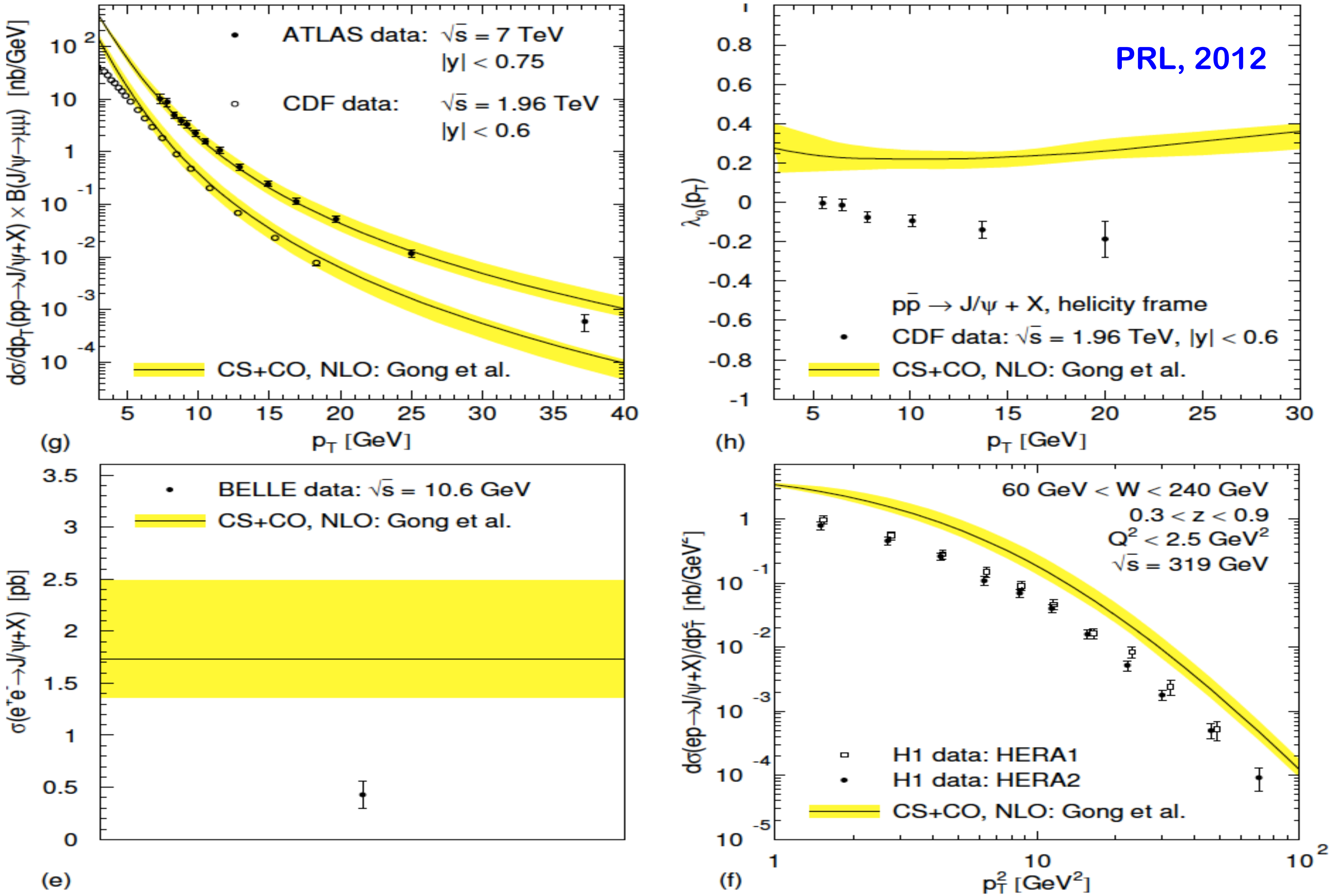
✧ S-wave: large corrections from high orders

✧ P-wave: Infrared divergent – CSM is not complete

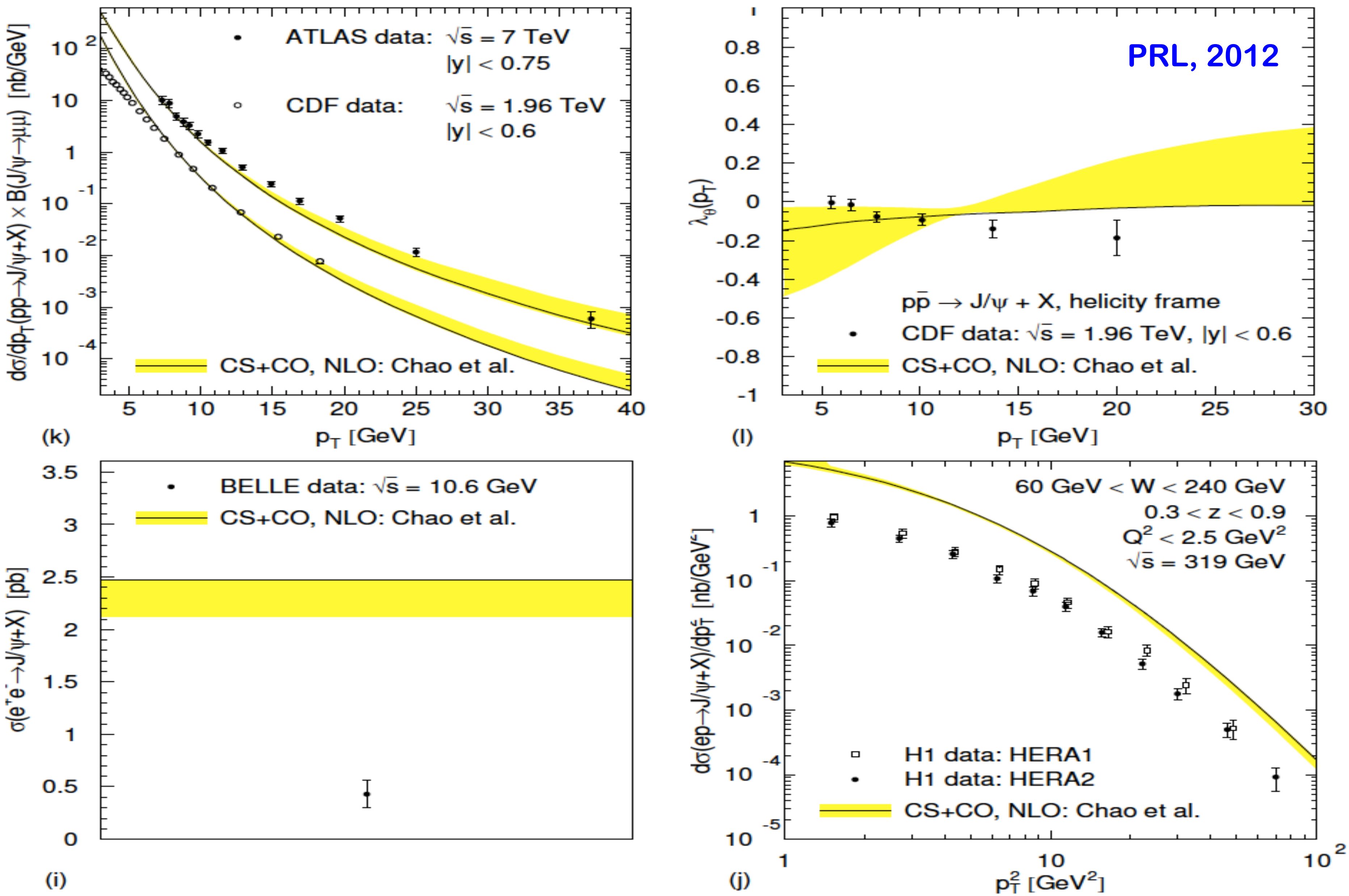
Challenges: NLO theory fits – Butenschoen et al.



Challenges: NLO theory fits – Gong et al.



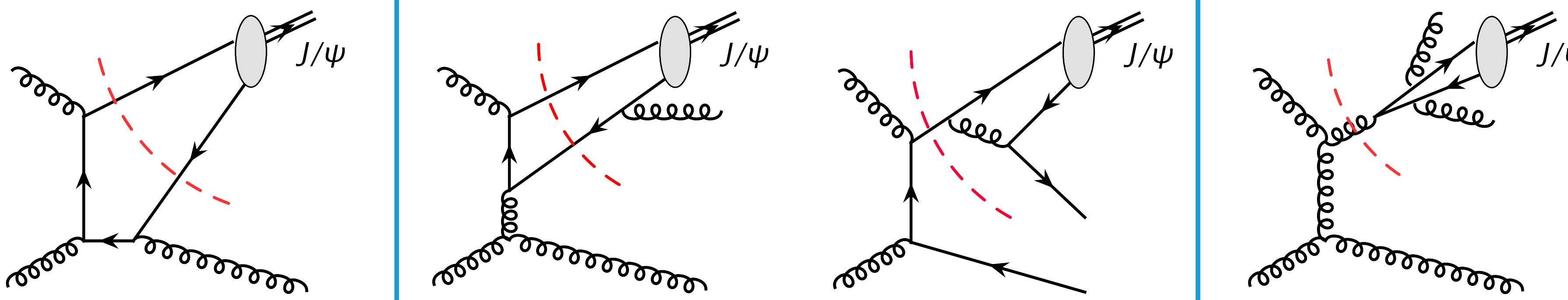
Challenges: NLO theory fits – Chao et al.



Why high orders in NRQCD are so large?

Kang, Qiu and Sterman, 2011

- **Consider J/ψ production in CSM:**



LO in α_s	NLO in α_s	NNLO in α_s
NNLP $\propto \alpha_s^3 \frac{m_Q^4}{p_T^8}$	**NLP in $1/P_T$** $\propto \alpha_s^4 \frac{m_Q^2}{p_T^6}$	**LP:** $\propto \alpha_s^5 \frac{1}{p_T^4}$

- **High-order correction receive power enhancement**
- **Expect no further power enhancement beyond NNLO**
- $[\alpha_s \ln(p_T^2/m_Q^2)]^n$ **ruins the perturbation series at sufficiently large p_T**

Leading order in α_s-expansion =\= leading power in $1/p_T$-expansion!

At high p_T fragmentation contribution dominant

Heavy quarkonium polarization

Ma et al. 2014

- **Polarization = input fragmentation functions:**
  - ✧ Partonic hard parts and evolution kernels are perturbative
  - ✧ Insensitive to the properties of produced heavy quarkonia
- **Projection operators – polarization tensors:**

$$\mathcal{P}^{\mu\nu}(p) \equiv \sum_{\lambda=0,\pm1} \epsilon_\lambda^{*\mu}(p)\epsilon_\lambda^{\nu}(p) = -g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2}$$

Unpolarized quarkonium

$$\mathcal{P}_T^{\mu\nu}(p) \equiv \frac{1}{2}\sum_{\lambda=\pm1} \epsilon_\lambda^{*\mu}(p)\epsilon_\lambda^{\nu}(p) = \frac{1}{2}\left[-g^{\mu\nu} + \frac{p^\mu n^\nu + p^\nu n^\mu}{p\cdot n}\right]$$

Transversely polarized quarkonium

$$\mathcal{P}_L^{\mu\nu}(p) \equiv \mathcal{P}^{\mu\nu}(p) - 2\mathcal{P}_T^{\mu\nu}(p) = \frac{1}{p^2}\left[p^\mu - \frac{p^2}{2p\cdot n}n^\mu\right]\left[p^\nu - \frac{p^2}{2p\cdot n}n^\nu\right]$$

Longitudinally polarized quarkonium

for produced the quarknium moving in +z direction with

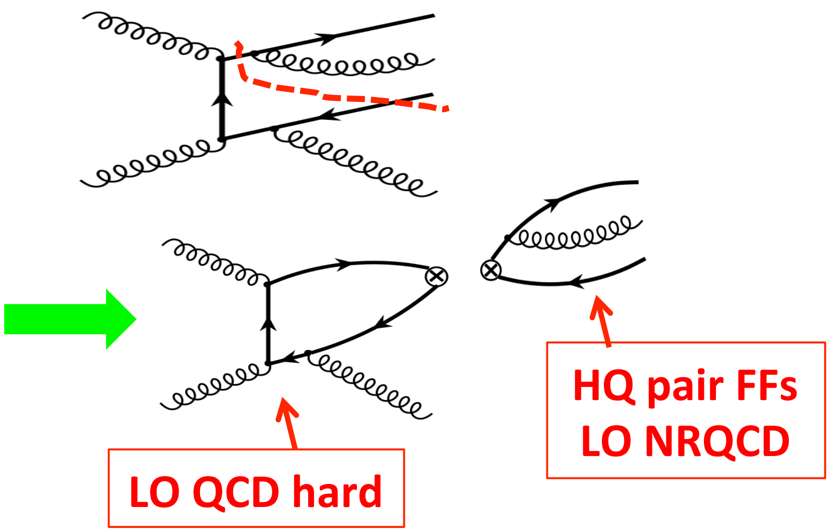
$$p^\mu = (p^+, p^-, p_\perp) = p^+(1, 0, \mathbf{0}_\perp) \qquad p^2 = n^2 = 0$$

$$n^\mu = (n^+, n^-, n_\perp) = (0, 1, \mathbf{0}_\perp) \qquad p\cdot n = p^+$$

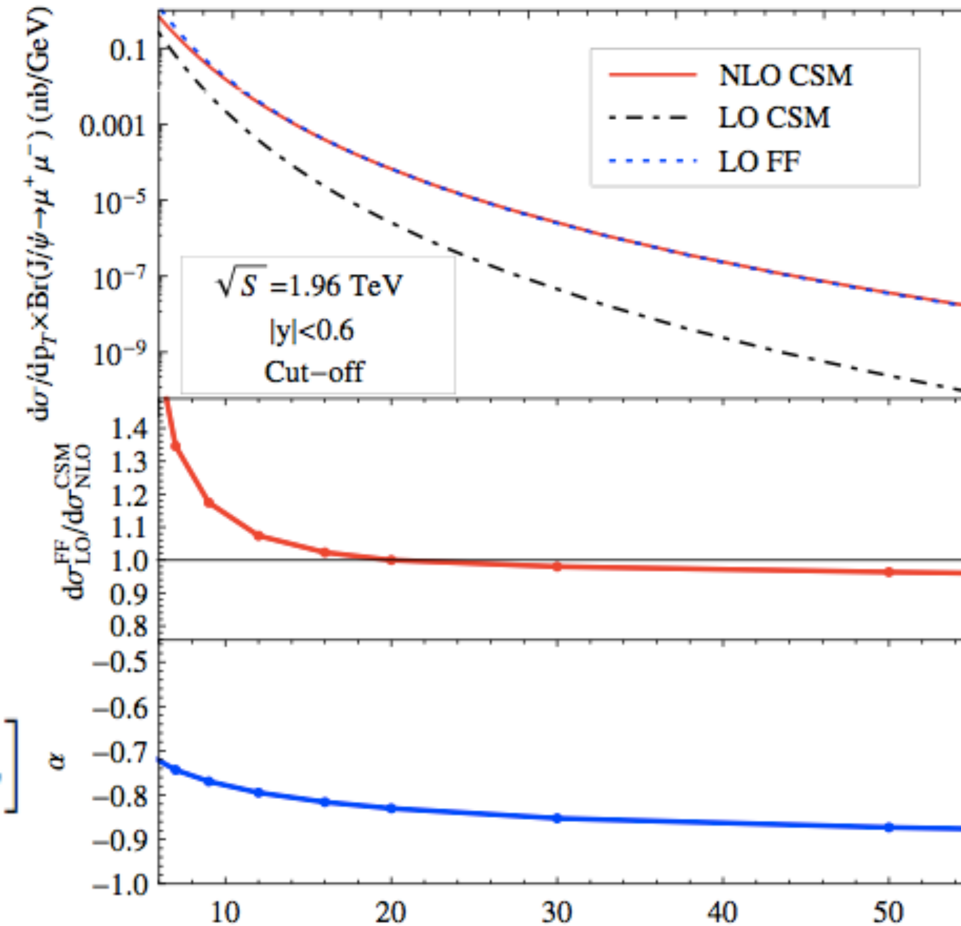
QCD factorization + NRQCD factorization

Kang, Qiu and Sterman, 2011

- Color singlet as an example:



$$\sigma_{\rm NRQCD}^{\rm (NLO)} \propto \left[d\hat{\sigma}^{A\,({\rm LO})}_{ab\to[Q\bar{Q}(v8)]} \otimes \mathcal{D}^{({\rm LO})}_{[Q\bar{Q}(v8)]\to J/\psi} + d\hat{\sigma}^{S\,({\rm LO})}_{ab\to[Q\bar{Q}(a8)]} \otimes \mathcal{D}^{({\rm LO})}_{[Q\bar{Q}(a8)]\to J/\psi} \right]$$



Reproduce NLO CSM for p_T > 10 GeV!

Cross section + polarization

Different kinematics, different approximation,
Dominance of different production channels!

QCD factorization approach when $P_T >> m_Q$

Nayak, Qiu, and Sterman, 2005
Kang, Ma, Qiu and Sterman, 2014 , …

❑ Factorization formalism:

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$

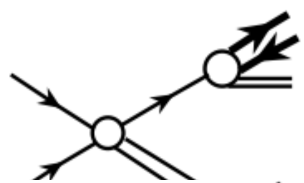
$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$

$$+ \mathcal{O}(m_Q^4/p_T^4)$$

❑ Production of the pairs:

$$\hat{p}_Q = \frac{1+\zeta}{2z}\hat{p}\,, \quad \hat{p}_{\bar{Q}} = \frac{1-\zeta}{2z}\hat{p}$$

✧ at $1/m_Q$:

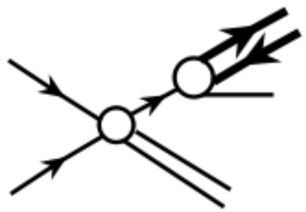


$$D_{i\to H}(z, m_Q, \mu_0)$$

✧ at $1/P_T$:

$$d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(P_{[Q\bar{Q}]}(\kappa), \mu)$$

✧ between: [$1/m_Q$, $1/P_T$]



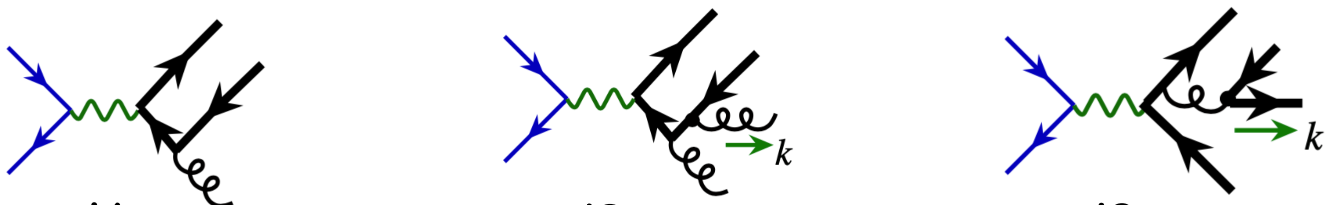
$$\frac{d}{d\ln(\mu)} D_{i\to H}(z, m_Q, \mu) = \ldots$$

$$+ \frac{m_Q^2}{\mu^2}\Gamma(z) \otimes D_{[Q\bar{Q}(\kappa)]\to H}(\{z_i\}, m_Q, \mu)$$

QCD factorization beyond leading power

- Beyond the leading power – e+e- as an example:

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$
$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z)$$
$$\otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$



d1 d2 d3

- Matching coefficients beyond the leading power: $H \to Q\bar{Q}$

$$d\hat{\sigma}^{(2)}_{e^+e^-\to Q\bar{Q}}(p_T) = d\sigma^{(2)}_{e^+e^-\to Q\bar{Q}}(p_T) - d\sigma^{(1)}_{e^+e^-\to Q'\bar{Q}'}(p_T) \otimes \mathcal{D}^{(1)}_{Q\bar{Q}/Q'\bar{Q}'} - d\sigma^{(0)}_{e^+e^-\to Q}(p_T) \otimes D^{(2)}_{Q\bar{Q}/Q}$$

d2: $\propto \int_0^{Q^2} \frac{dk_T^2}{k_T^2} - \left[\int_0^{\infty} \frac{dk_T^2}{k_T^2} + \mathrm{UVCT}(\mu^2)\right]$

d3: $\propto \int_0^{Q^2} \frac{dk_T^2}{(k_T^2)^{n-2}} - \left[\int_0^{\infty} \frac{dk_T^2}{(k_T^2)^{n-2}} + \mathrm{FACT}(\mu^2)\right]$

Cancel CO divergence, and UV finite, but should take care of oversubtraction

$n \geq 1$

Take are of the Pair produced between $1/m_Q$ and $1/pT$

See arXiv:2006.07375

Evolution of fragmentation functions

Kang, Ma, Qiu and Sterman, 2013

- **Independence of the factorization scale:**

$$\frac{d}{d\ln(\mu)}\sigma_{A+B\to HX}(P_T)=0$$

✧ **at Leading power in $1/P_T$:**

DGALP evolution

$$\frac{d}{d\ln\mu^2}D_{H/f}(z,m_Q,\mu)=\sum_j\frac{\alpha_s}{2\pi}\gamma_{f\to j}(z)\otimes D_{H/j}(z,m_Q,\mu)$$

✧ **next-to-leading power in $1/P_T$ – New non-linear evolution!**

$$\frac{d}{d\ln\mu^2}D_{H/f}(z,m_Q,\mu)=\sum_j\frac{\alpha_s}{2\pi}\gamma_{f\to j}(z)\otimes D_{H/j}(z,m_Q,\mu)$$
$$+\frac{1}{\mu^2}\sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s^2}{(2\pi)^2}\Gamma_{f\to[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta')\otimes\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q,\mu)$$

$$\frac{d}{d\ln\mu^2}\mathcal{D}_{H/[Q\bar{Q}(c)]}(z,\zeta,\zeta',m_Q,\mu)=\sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s}{2\pi}K_{[Q\bar{Q}(c)]\to[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta')$$
$$\otimes\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q,\mu)$$

- **Evolution kernels are perturbative:**

✧ **Set mass:** $m_Q\to 0$ **with a caution**

Non-perturbative input distributions

- **Sensitive to the properties of quarkonium produced:**

  Should, in principle, be extracted from experimental data

- **Large heavy quark mass and clear scale separation:**

  $\mu_0 \sim m_Q \gg m_Q\, v$ ➡ **Apply NRQCD to the FFs**

  ✧ **Single parton FFs – valid to two-loops:** Nayak, Qiu and Sterman, 2005

  $$D_{g\to J/\psi}(z,\mu_0,m_Q) \to \sum_{[Q\bar{Q}(c)]} \hat{d}_{g\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\langle\mathcal{O}_{[Q\bar{Q}(c)]}(0)\rangle|_{\rm NRQCD}$$

  Braaten, Yuan, 1994
  Ma, 1995, …
  Braaten, Chen, 1997
  Braaten, Lee, 2000,
  Ma, Qiu, Zhang, 2013
  …

  Complete LO+NLO for S, P states & NNLO for singlet S state

  ✧ **Heavy quark pair FFs – valid to one-loop:**

  $$\mathcal{D}_{[Q\bar{Q}(\kappa)]\to J/\psi}(z,\zeta,\zeta',\mu_0,m_Q) \to \sum_{[Q\bar{Q}(c)]} \hat{d}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(c)]}(z,\zeta,\zeta',\mu_0,m_Q)\langle\mathcal{O}_{[Q\bar{Q}(c)]}(0)\rangle_{\rm NRQCD}$$

  Kang, Ma, Qiu and Sterman, 2014

  Full LO+NLO for S, P states is now available Ma, Qiu, Zhang, 2013

- **No all-order proof of such factorization yet!**

  Reduce "many" unknown FFs to a few universal NRQCD matrix elements!

Next-to-leading power fragmentation – Ma et al.

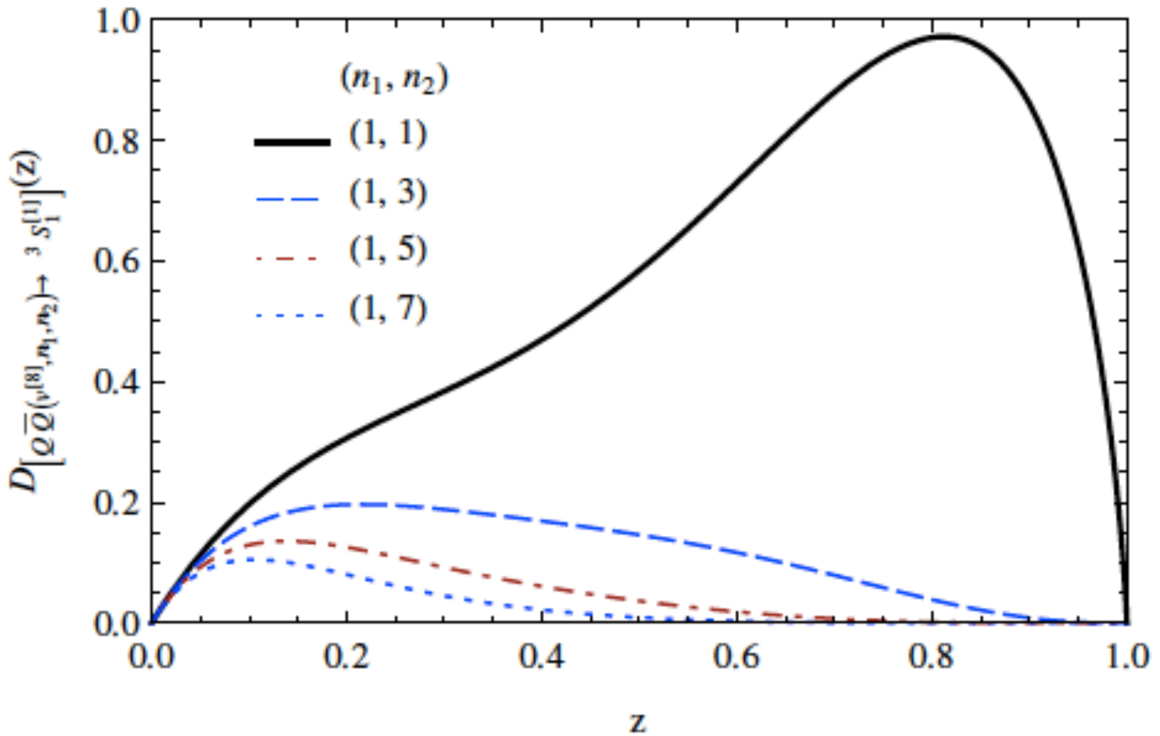
Ma, Qiu, Zhang, 2013

- **Heavy quark pair FFs:**

$$\mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}(z,\zeta_1,\zeta_2,\mu_0;m_Q) = \sum_{[Q\bar{Q}(n)]} \left\{ \hat{d}^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z,\zeta_1,\zeta_2,\mu_0;m_Q,\mu_\Lambda) + \left(\frac{\alpha_s}{\pi}\right) \hat{d}^{(1)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z,\zeta_1,\zeta_2,\mu_0;m_Q,\mu_\Lambda) + O(\alpha_s^2) \right\} \times \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m_Q^{2L+1}}$$

- **Moment of the FFs:**

$$\mathcal{D}^{[n_1,n_2]}(z) \equiv \int_{-1}^{1} \frac{d\zeta_1\, d\zeta_2}{4} \zeta_1^{n_1} \zeta_2^{n_2} \mathcal{D}(z,\zeta_1,\zeta_2)$$

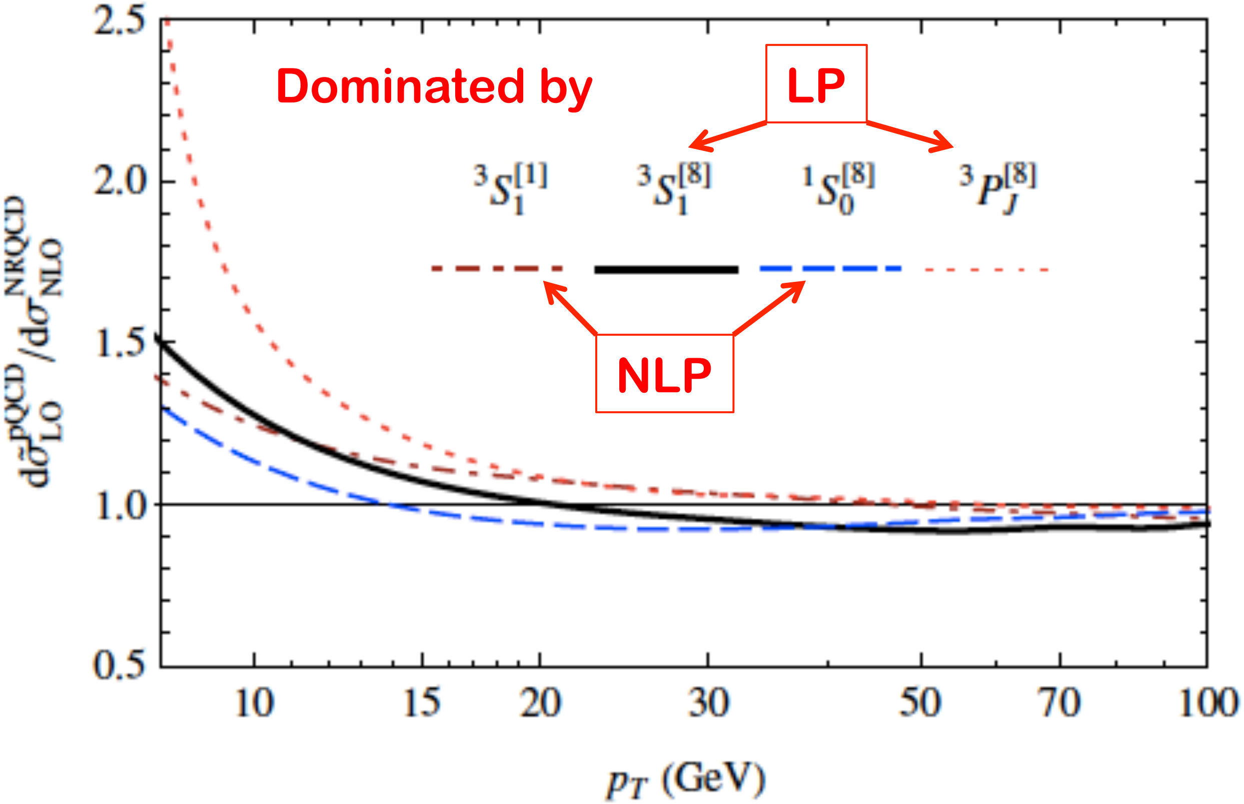


Next-to-leading power fragmentation – Ma et al.

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$

$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$

- Channel-by-channel comparison:



independent of NRQCD matrix elements

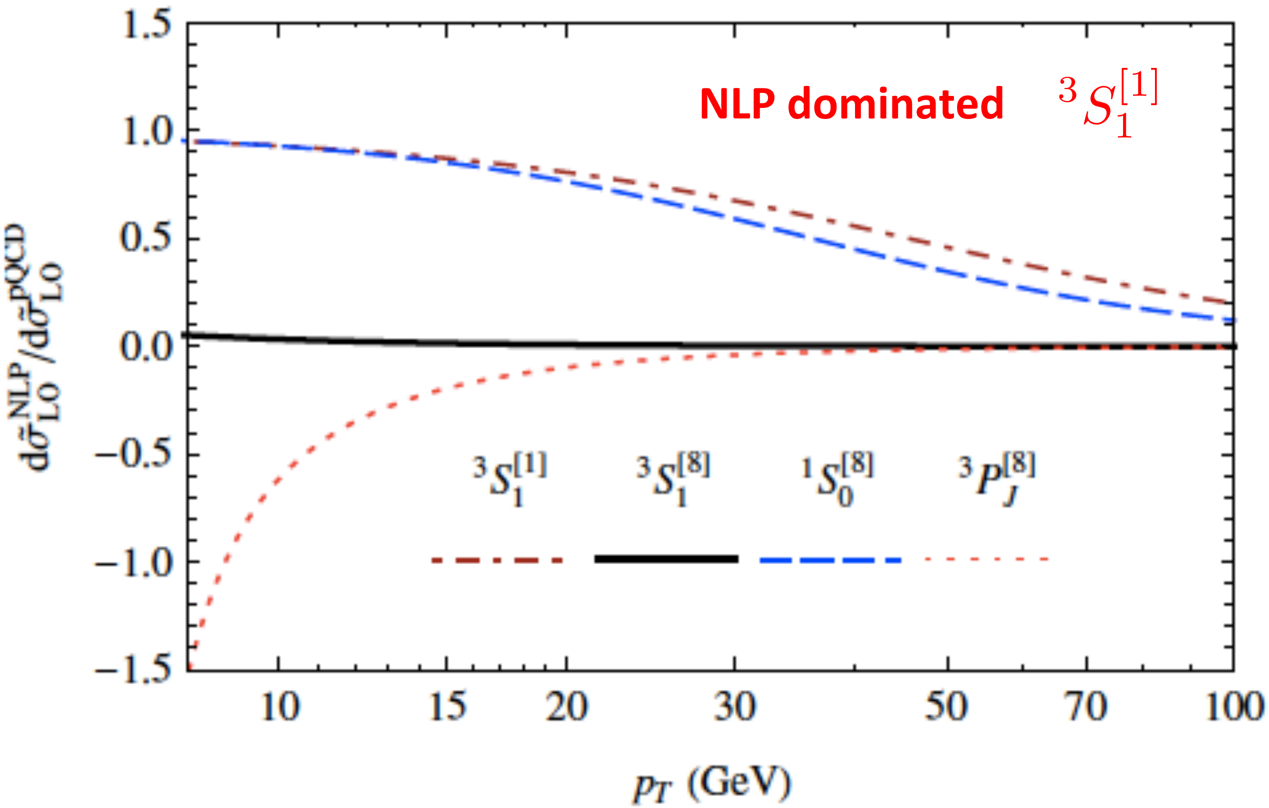
LO analytical results reproduce NLO NRQCD calculations (numerical)

Next-to-leading power fragmentation – Ma et al.

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$

$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$

- **LP vs. NLP (both LO):**



NLP dominated $^1S_0^{[8]}$ **for wide pT**

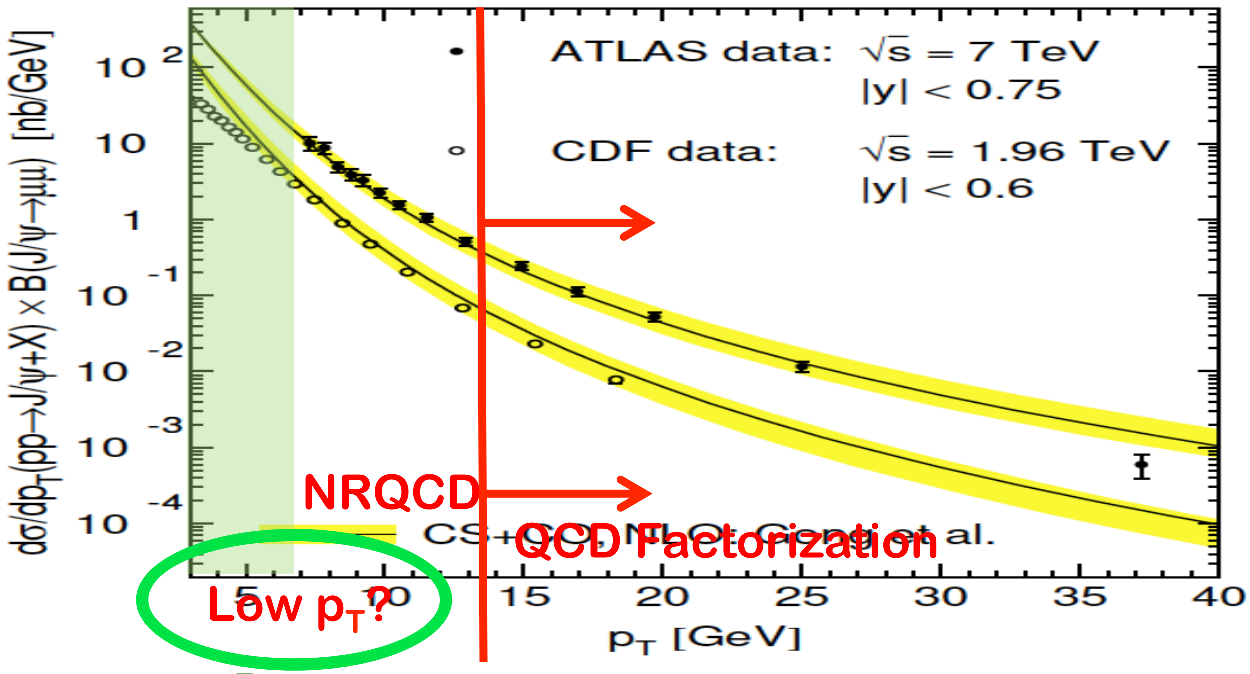
LP dominated $^3S_1^{[8]}$ and $^3P_J^{[8]}$

PT distribution is consistent with distribution of $^1S_0^{[8]}$

Matching between different approaches

- **Expectation:**

Kang, Ma, Qiu and Sterman, 2014



- **Matching for $p_T \to m_Q$:**

$$E_p \frac{d\sigma_{l+h\to H(p)+X}}{d^3p} \approx E_p \frac{d\sigma^{\rm PQCD}_{l+h\to H(p)+X}}{d^3p} - E_p \frac{d\sigma^{\rm PQCD-Asym}_{l+h\to H(p)+X}}{d^3p} + E_p \frac{d\sigma^{\rm NRQCD}_{l+h\to H(p)+X}}{d^3p}$$

Kang, Qiu, 2008

Lee, Qiu, Sterman, Watanabe, 2020

When $p_T >> m_Q$:

$$d\sigma_{l+h\to H(p)+X} \to d\sigma^{\rm PQCD}$$

No-logarithmic mass terms in $d\sigma^{\rm NRQCD}$ **vanish**

$d\sigma^{\rm PQCD-Asym}$ cancels $d\sigma^{\rm NRQCD}$

Summary

- It has been over 40 years since the discovery of J/Ψ, but, still not completely sure about its production mechanism

- NRQCD factorization is expected to work for $P_T \sim Q$, no all-order proof

- QCD factorization is shown to work for both LP and NLP at high P_T
  - Resummation of logarithms from $2m_Q$ to P_T
  - Non-linear evolution equation of single parton fragmentation function is needed for a consistent accuracy at Next-to-leading-power
  - Matching between high P_T to $P_T \sim m_Q$
  - Challenge for low P_T region or near the threshold

- Nuclear medium could be a good "filter" or a fermi-scale "detector" for studying the emergence of a quarkonium from a havey quark pair

  Not talked here

Thank you!