Quasi-Distributions for PDFs and TMDs

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TMD Collaboration

Outline

- Motivation

Quasi-Parton Distribution Functions (PDFs)

- Physical picture & Factorization
- Continuum and Lattice Ingredients
- Results & Recent work

Quasi-Transverse Momentum Distributions (TMDs)

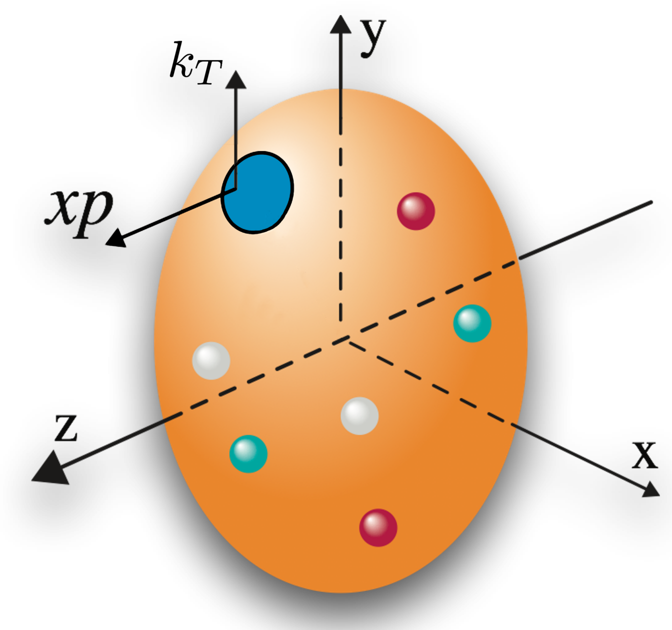
- Intro to TMD PDFs
- Non-perturbative anomalous dimension (CS Kernel)
- Quasi-TMDPDFs for Lattice QCD
- Results & Recent work

PDFs

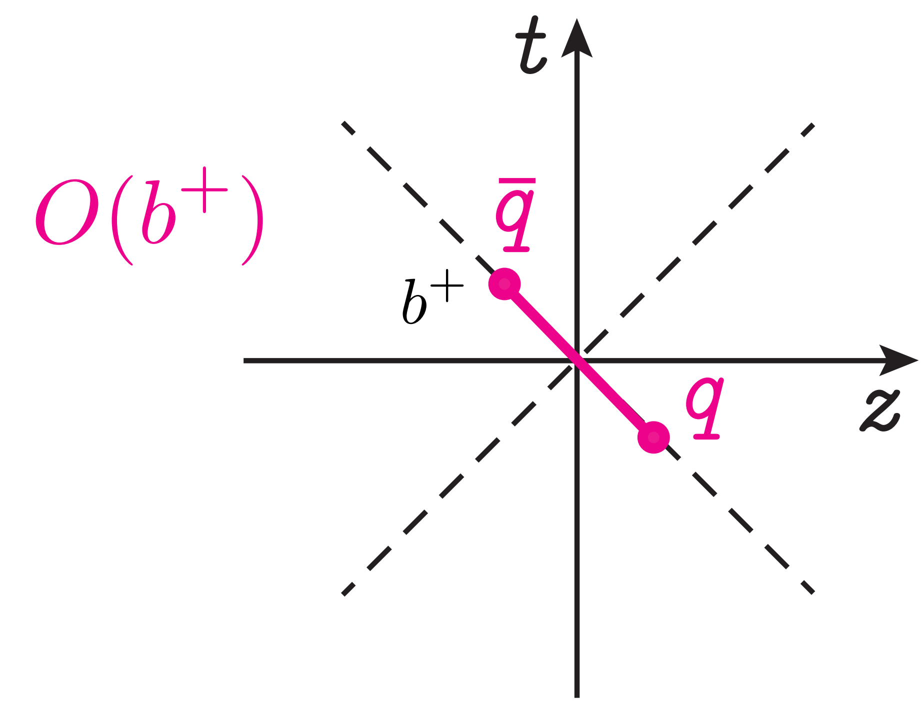
$f_{q/P}(x, \mu)$

longitudinal

$$f_{q/P}(x) = \int \frac{db^+}{4\pi}\, e^{-\frac{i}{2} b^+ x P^-} \langle P | O(b^+) | P \rangle$$



- **ubiquitous in description of collider physics processes**



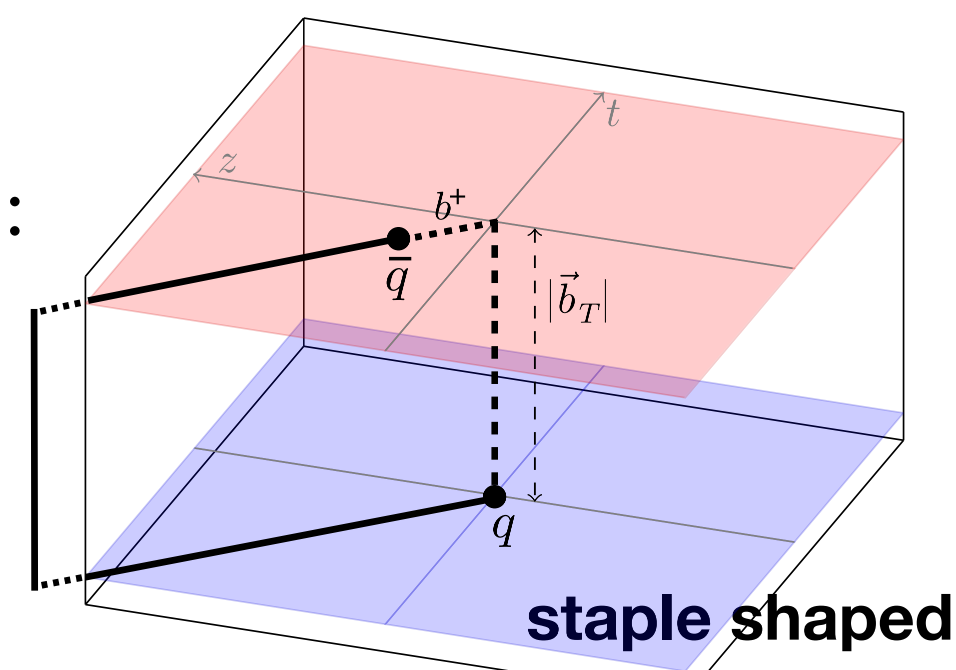
light-cone sensitive operators

TMDs

$f_{q/P}(x, k_T, \mu, \zeta)$

longitudinal & Transverse

$O(b^+, b_T)$:



staple shaped Wilson lines

- **key information about the structure of hadrons**

$$f_{q/P}(x, k_T) \sim \int \frac{db^+ d^2 b_T}{2(2\pi)^3}\, e^{-\frac{i}{2} b^+ x P^- + i \vec{b}_T \cdot \vec{k}_T} \langle P | O(b^+, b_T) | P \rangle$$

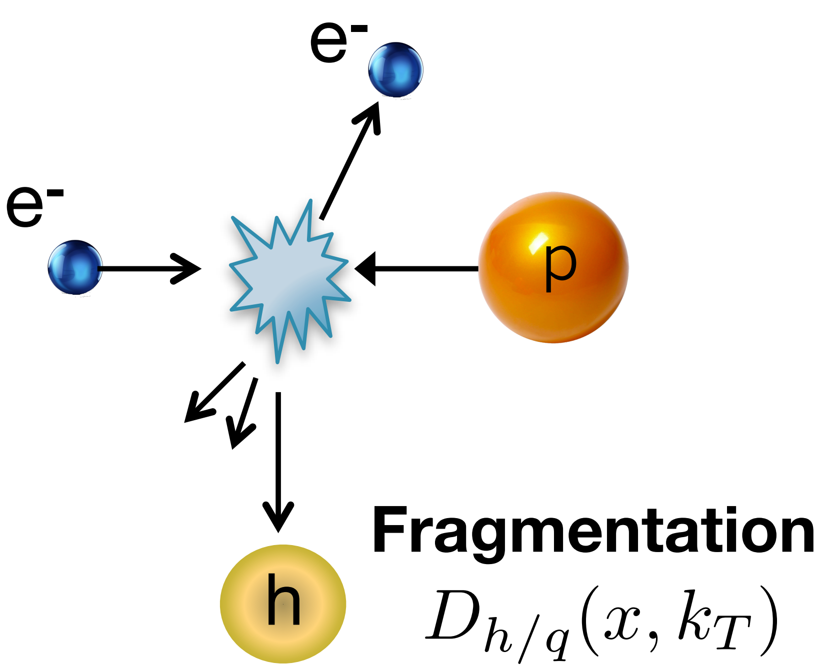
+ complications

TMD Factorization

(schematic)

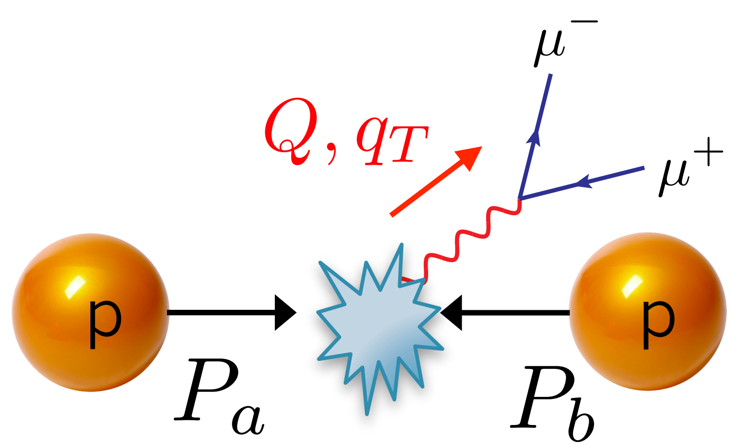
Semi-Inclusive DIS

$$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(x, k_T)$$



Drell-Yan

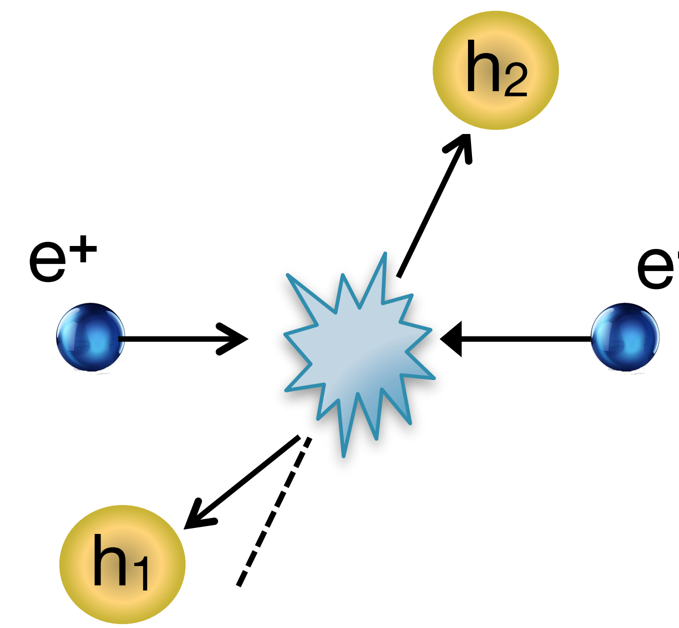
$$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$$



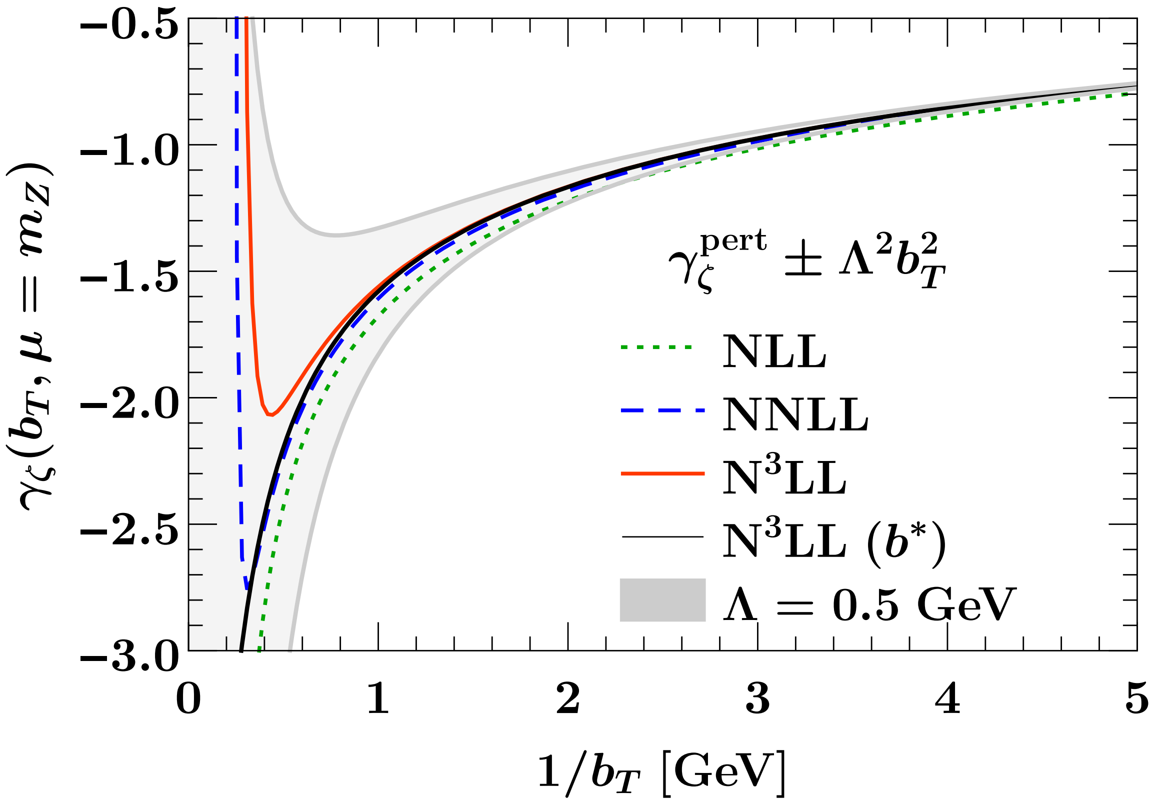
$$q_T \ll Q$$

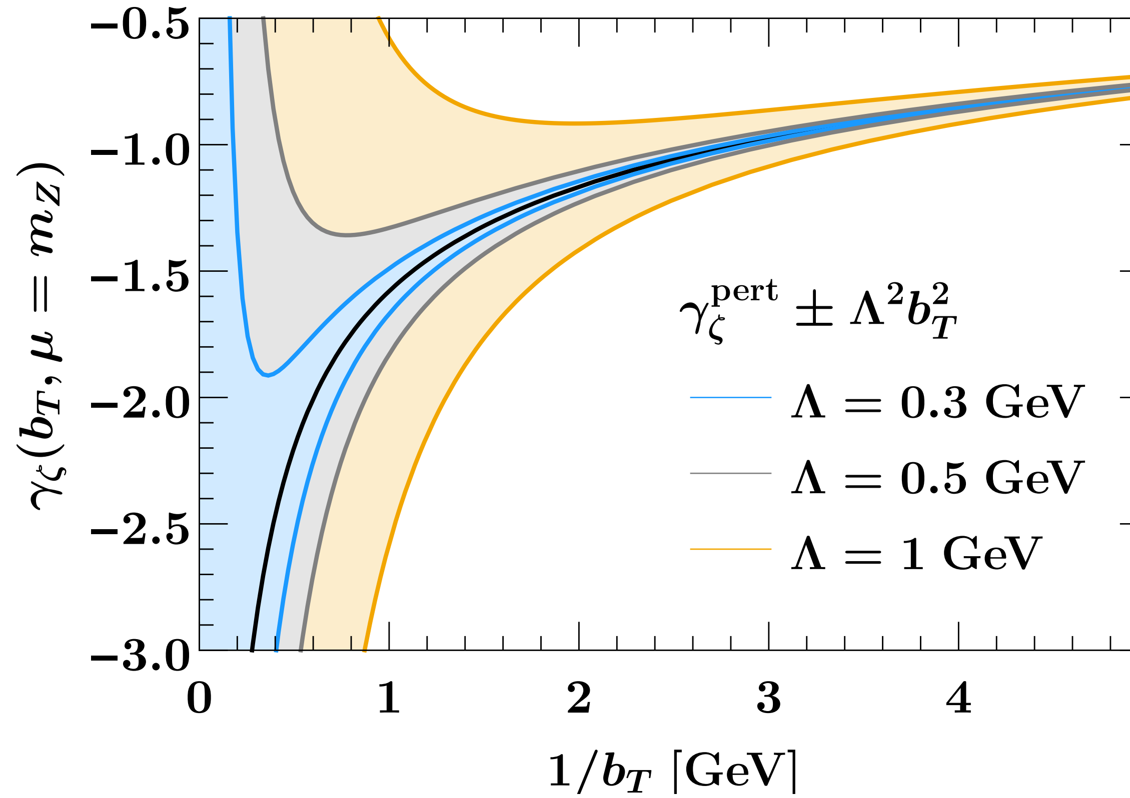
Dihadron in e+e-

$$\sigma \sim D_{h_1/q}(x, k_T) D_{h_2/q}(x, k_T)$$

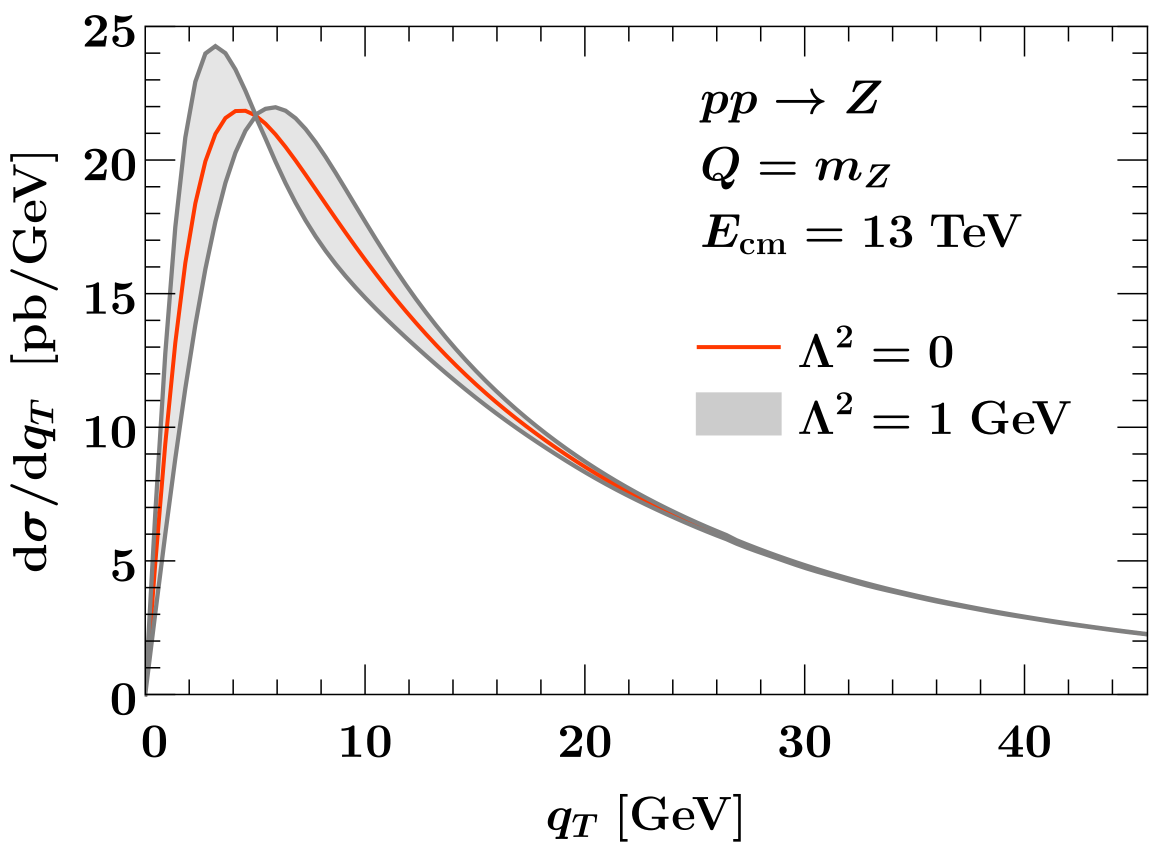


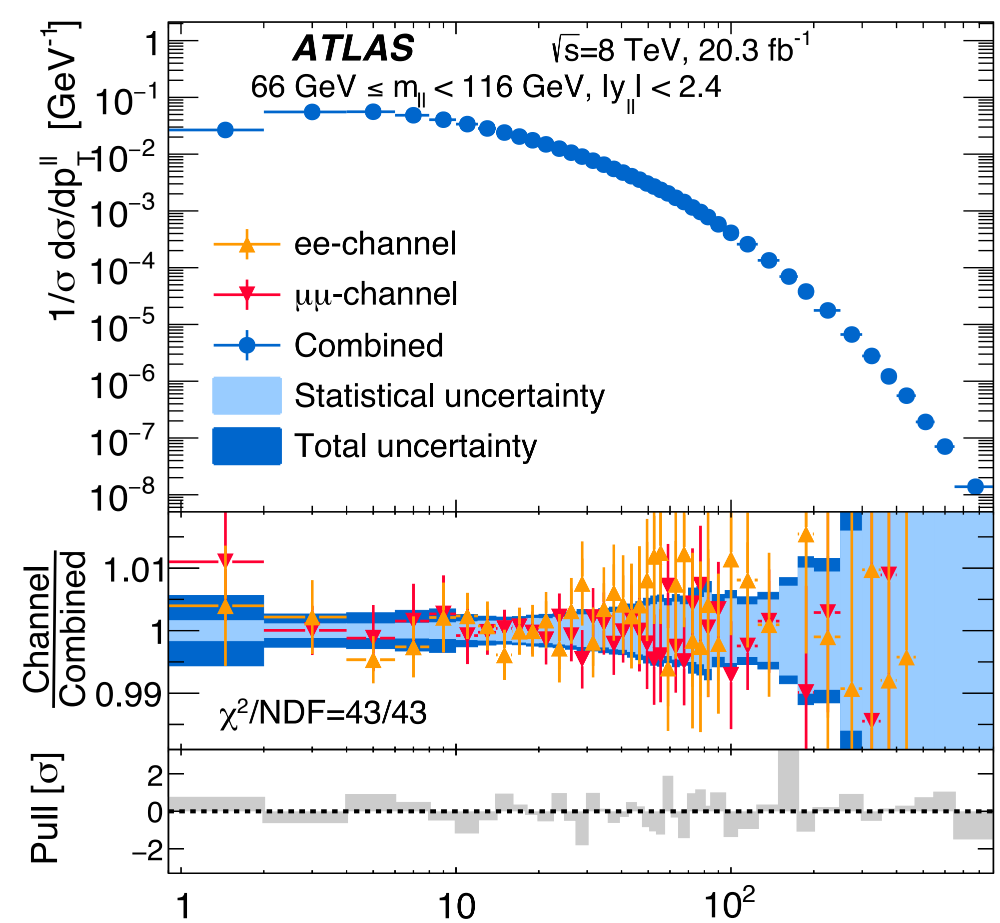
$\gamma_\zeta^q(\mu, b_T)$ Nonperturbative Contributions to Rapidity Anom. Dim.





Drell-Yan Cross Section:

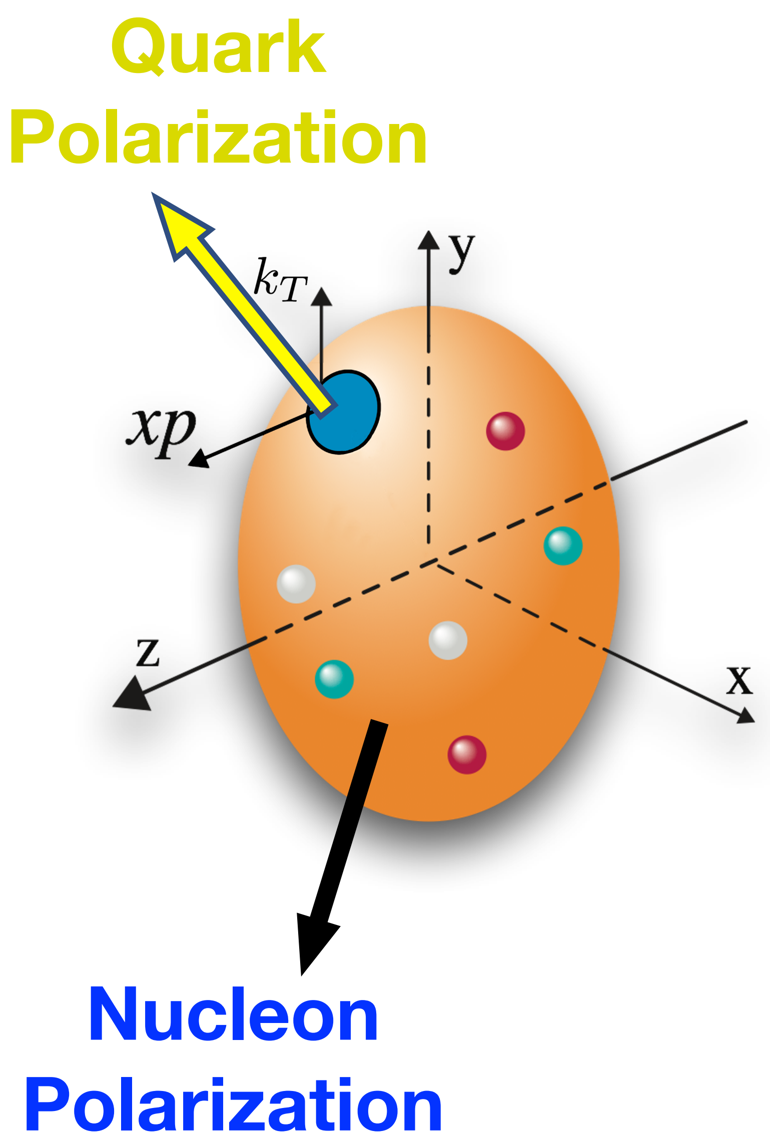




[ATLAS '15]

p$_T^{ll}$ [GeV]

TMDs with Polarization



Nucleon Polarization \ Quark Polarization	Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	$f_1(x,k_T^2)$		$h_1^\perp(x,k_T^2)$ - **Boer-Mulders**
L		$g_1(x,k_T^2)$ - **Helicity**	$h_{1L}^\perp(x,k_T^2)$ -
T	$f_{1T}^\perp(x,k_T^2)$ - **Sivers**	$g_{1T}(x,k_T^2)$ -	$h_1(x,k_T^2)$ - **Transversity** $h_{1T}^\perp(x,k_T^2)$ - **Pretzelosity**

Analogous tables for:

- **Gluons** $f_1 \to f_1^g$ etc
- **Fragmentation functions**
- **Nuclear targets** $S \neq \frac{1}{2}$

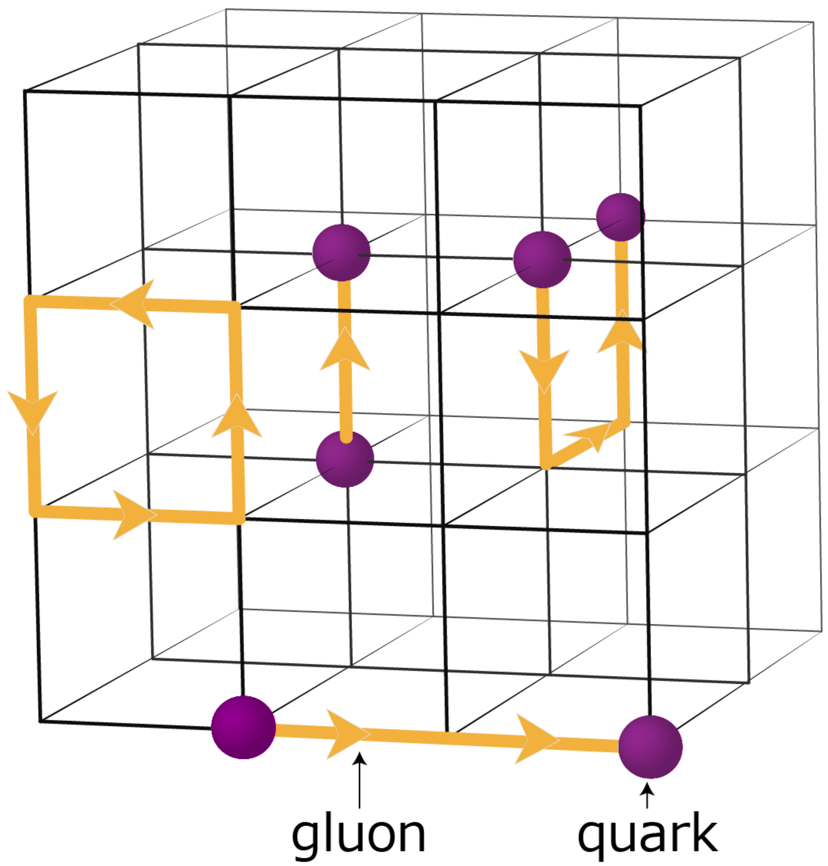
Lattice QCD

- **Nonperturbative calculations using discretized Euclidean path integral**

imaginary time $t = \mathrm{i}t_E$

$$\langle \mathcal{O} \rangle = \int D\psi D\bar{\psi} D\mathcal{A}\, \mathcal{O}\, e^{\mathrm{i}S} = \int D\psi D\bar{\psi} D\mathcal{A}\, \mathcal{O}\, e^{-S_E}$$

- **Need to be able to write calculations in euclidean form.**

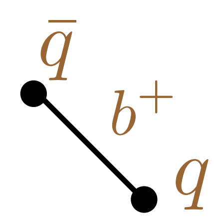


Obstacle for PDFs (and TMDPDFs):

- **PDF operators are intrinsically Minkowski, no direct Euclidean analog**

$$n_E^2 = 0 \;\Leftrightarrow\; n_E^\mu = 0$$

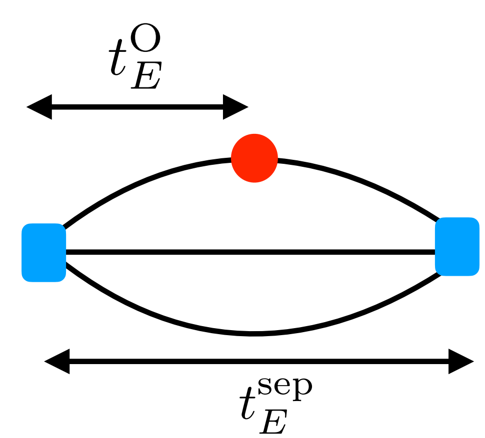
O_f :



- **Can compute moments**

$$a_n(\mu) = \int \mathrm{d}x\, x^{n-1} f_q(x,\mu)$$
$$= n^{\mu_1} \cdots n^{\mu_n} \langle P | \bar{q}(0) \gamma^{\mu_1} \mathrm{i}\overleftrightarrow{D}_{\mu_2} \cdots \mathrm{i}\overleftrightarrow{D}_{\mu_n} q(0) | P \rangle$$

but limited by power law operator mixing to $n \leq 3$



[Detmold et al ’02; Dolgov et al ’02]

Quasi-PDFs

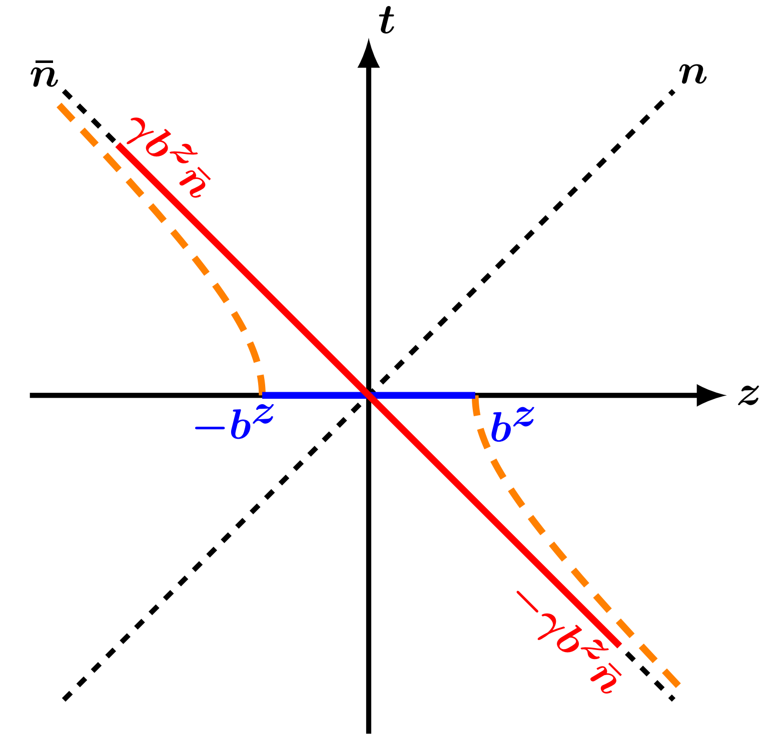
(Xiangdong Ji 2013)

- **Consider a purely spatial operator**

$$\tilde{f}_q(x, P^z, \epsilon) = \int \frac{db^z}{4\pi} e^{ib^z x P^z} \langle p(P) | \bar{q}(b^z) W_z(b^z, 0) \gamma^0 q(0) | p(P) \rangle$$

quasi-PDF

depends on P^z since not boost invariant



- **Relate to light-cone operator for PDF by a boost**

boost to $\mathcal{O}$ $\Leftrightarrow$ boost to proton state

? $\lim_{P^z \to \infty} \tilde{f}_q(x, P^z, \mu) = f_q(x, \mu)$ **No**

take $\Lambda_{\rm QCD} \ll P^z$ (finite large P^z) **"LaMET"**

- **quasi-PDF and PDF should have same IR physics**

- **Differences in UV accounted for by perturbative matching**

quasi-PDF Matching

$$\tilde{f}_i(x, P^z, \tilde{\mu}) = \int_{-1}^{1} \frac{dy}{|y|}\, C_{ij}\left(\frac{x}{y}, \frac{\tilde{\mu}}{P^z}, \frac{\mu}{yP^z}\right) f_j(y, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

quasi-PDF computable with Lattice QCD

Perturbative matching coefficient

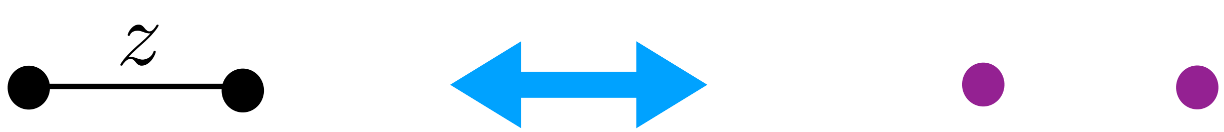
PDF

Power corrections

Proof with OPE:

Izubuchi, Ji, Jin, IS, Zhao '18 (see also Ma, Qiu '14, '17)

$\tilde{O}$:



$[\bar{\varphi}(0)\psi(0)]$ $[\bar{\psi}(z)\gamma_z\varphi(z)]$

$\mathcal{L}_\zeta = \bar{\varphi}\, iD_z \varphi$

product of local operators

$\zeta = zP^z$

$$\tilde{q}\left(x, \frac{\mu}{P_z}\right) \equiv \int \frac{d\zeta}{2\pi}\, e^{ix\zeta}\, \frac{1}{2P^z} \langle P|\tilde{O}_{\gamma^z}(z,\mu)|P\rangle = \int_{-1}^{1} dy \left[\int \frac{d\zeta}{2\pi} e^{ix\zeta} \sum_{n=0} C_n\left(\frac{\mu^2\zeta^2}{P_z^2}\right) \frac{(-i\zeta)^n}{n!} y^n\right] q\,(y,\mu)$$

$+$ higher-twist operators

$$= \int_{-1}^{1} \frac{dy}{|y|} \left[\int \frac{d\zeta}{2\pi} e^{i\frac{x}{y}\zeta} \sum_{n=0} C_n\left(\frac{\mu^2\zeta^2}{(yP^z)^2}\right) \frac{(-i\zeta)^n}{n!}\right] q\,(y,\mu)$$

$$= C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right)$$

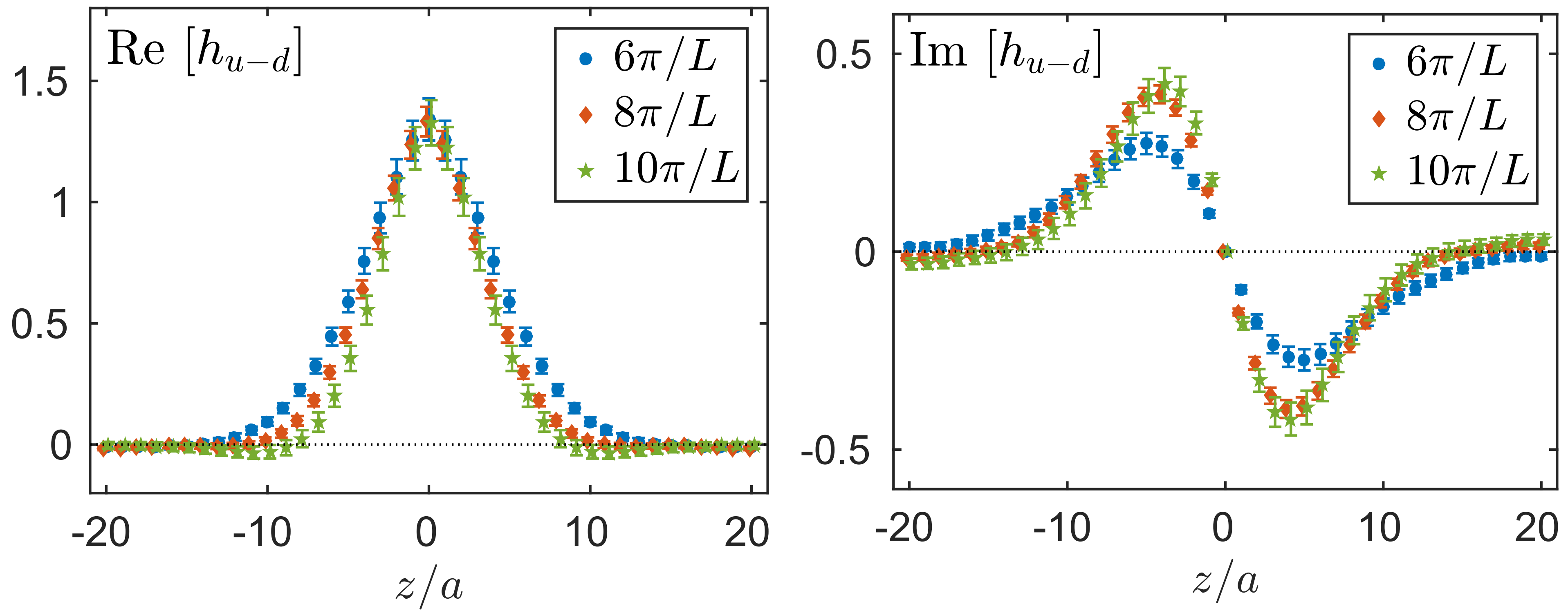
quasi-PDF Analysis

$$\tilde{f}(x, P^z, \tilde{\mu}) = \int_{-1}^{1} \frac{dy}{|y|}\, C\Big(\frac{x}{y}, \frac{\tilde{\mu}}{P^z}, \frac{\mu}{yP^z}\Big)\, f(y,\mu) \;+\; \mathcal{O}\Big(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\Big)$$

1. Lattice simulation of bare quasi PDF

eg. Alexandrou et al.(ETMC) '18

Position space:



quasi-PDF Analysis

$$\tilde{f}(x, P^z, \tilde{\mu}) = \int_{-1}^{1} \frac{dy}{|y|}\, C\left(\frac{x}{y}, \frac{\tilde{\mu}}{P^z}, \frac{\mu}{yP^z}\right) f(y,\mu) \;+\; \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

2. Renormalization & continuum limit

Proof of (continuum) renormalizability

$$\bar{\psi}_0(z)\frac{\Gamma}{2}W_0[z,0]\psi_0(0) = e^{\delta m|z|}\, Z_{j_1}Z_{j_2}\left[\bar{\psi}(z)\frac{\Gamma}{2}W[z,0]\psi(0)\right]_R$$

Ji, Zhang, Zhao ’18, Green et al. ’18, Ishikawa et al. ‘17

Operator mixing with broken chiral symmetry on lattice

Constantinou & Panagopoulos ’17, Chen et.al. ’17, Green et.al. ’18

eg. No $\mathcal{O}(a^0)$ mixing using γ^t

Perturbative renormalization (lattice perturbation theory)

Constantinou & Panagopoulos ’17, Ishikawa et.al. ’16, Xiong, Luu, Meissner ‘17

Nonperturbative renormalization (eg. RIMOM scheme)

Constantinou & Panagopoulos ’17, IS & Zhao ’18, Alexandrou et al.(ETMC) ’17,
Chen et al.(LP3) ’18, Liu et al. (LP3) ’18, Green et al. ’18, Monahan & Origins ’17, Monahan ’18
Ji, Liu, Schafer, Wang, Yang, Zhang, Zhao ‘20

Quasi-gluon PDF renormalization Zhang, Ji, Shaefer ’19, Li, Ma, Qiu ‘19

quasi-PDF Analysis

$$\tilde{f}(x, P^z, \tilde{\mu}) = \int_{-1}^{1} \frac{dy}{|y|}\, C\Big(\frac{x}{y}, \frac{\tilde{\mu}}{P^z}, \frac{\mu}{yP^z}\Big)\, f(y,\mu) \;+\; \mathcal{O}\Big(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\Big)$$

3. Subtraction of Power Corrections

Can determine proton mass dependent terms J.W. Chen et al.(LP3) '16

Renormalon analysis of power corrections Braun, Vladimiro, Zhang '19

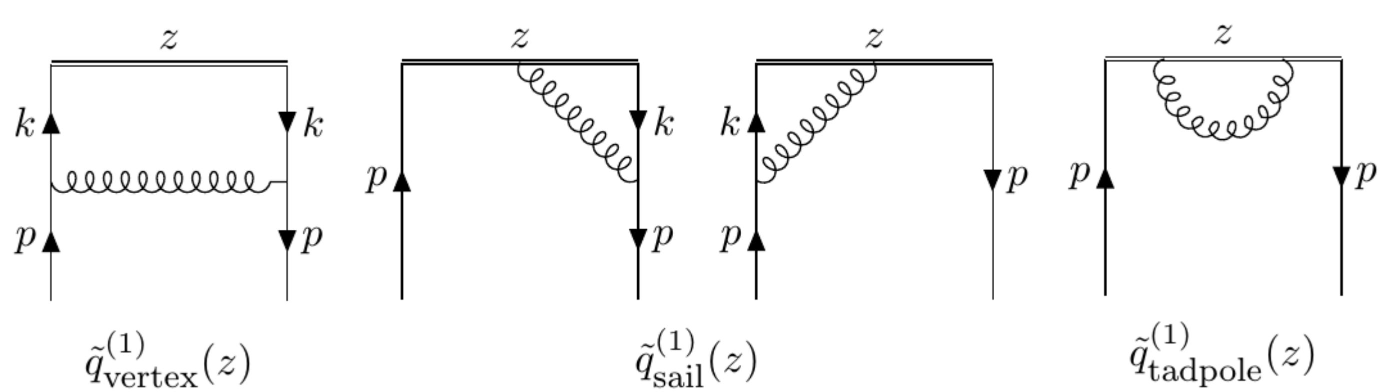
Liu, Chen '20

quasi-PDF Analysis

always $\overline{\mathrm{MS}}$

$$\tilde{f}(x, P^z, \tilde{\mu}) = \int_{-1}^{1} \frac{dy}{|y|}\, C\Big(\frac{x}{y}, \frac{\tilde{\mu}}{P^z}, \frac{\mu}{yP^z}\Big)\, f(y,\mu) \; + \; \mathcal{O}\Big(\frac{M^2}{P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{x^2 P_z^2}\Big)$$

4. Compute Matching Coefficients, including scheme conversion



quasi-PDF Analysis

always $\overline{\text{MS}}$

$$\tilde{f}(x,P^z,\tilde{\mu}) = \int_{-1}^{1}\frac{dy}{|y|}\, C\Big(\frac{x}{y},\frac{\tilde{\mu}}{P^z},\frac{\mu}{yP^z}\Big)\, f(y,\mu) \;+\; \mathcal{O}\Big(\frac{M^2}{P_z^2},\frac{\Lambda_{\text{QCD}}^2}{x^2P_z^2}\Big)$$

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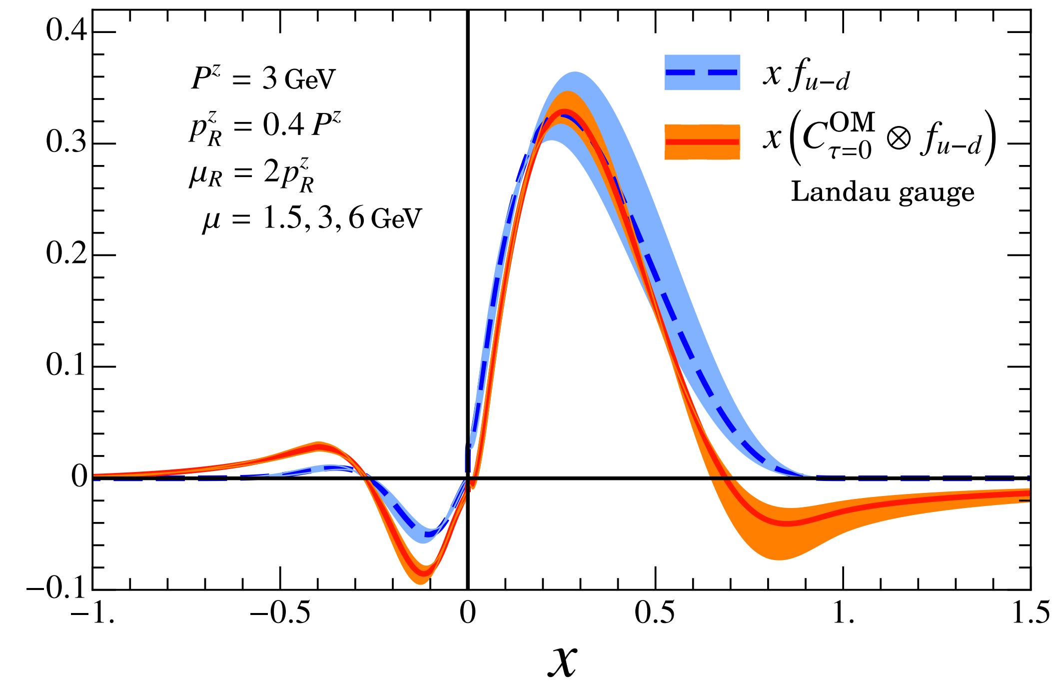


quasi-PDF scheme

Hard UV Cutoff — Xiong, Jie, Zhang, Zhao '14

RIMOM — IS, Zhao '18, Liu et al.(LP3) '18, Alexandrou et.al. '18

$\overline{\text{MS}}$ — Izubuchi, Ji, Jin, IS, Zhao '18 (1-loop non-singlet)

Wang et al.'18 & Zhang et al. '19 (singlet, gluon)

Chen, Wang, Zhu '20 (2-loop, also RIMOM), Li, Ma, Qiu '20 (2-loop)

Ratio Scheme & Modified $\overline{\text{MS}}$

Radyushkin '18, Izubuchi et al. '18, Alexandrou et al.(ETMC) '19

Note: various choices to be made, eg. scheme conversion prior/post Fourier transform

quasi-PDF Analysis

$$\tilde{f}(x, P^z, \tilde{\mu}) = \int_{-1}^{1} \frac{dy}{|y|} \, C\Big(\frac{x}{y}, \frac{\tilde{\mu}}{P^z}, \frac{\mu}{yP^z}\Big) \, f(y, \mu) \; + \; \mathcal{O}\Big(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\Big)$$

5. Invert C perturbatively (trivial), and extract f

Systematic Uncertainties:

- Excited state contamination
- Discretization effects
- Fourier transform with discrete data
- Extrapolation in Pz to remove Power Corrections
- Perturbative uncertainty from higher order matching

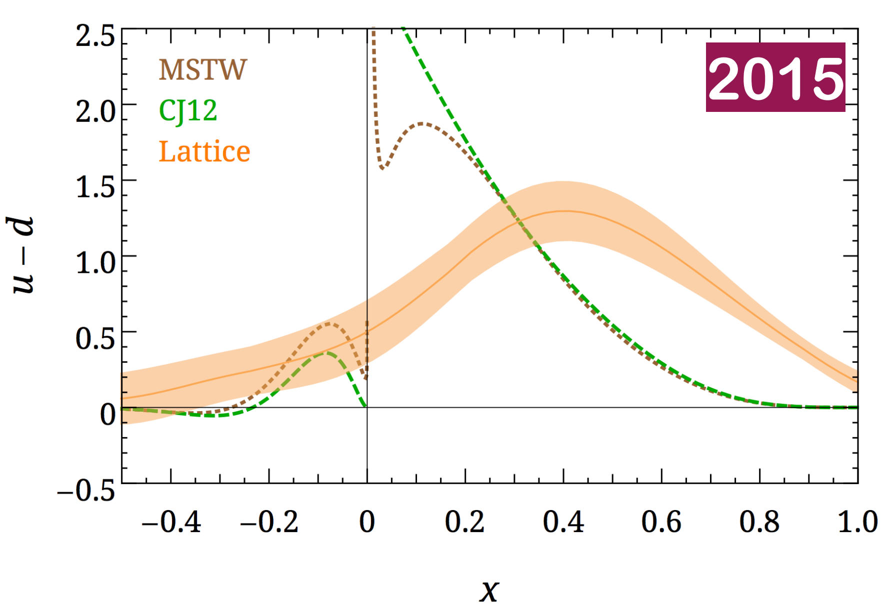
Lattice Results

Calculations at Physical Pion Mass

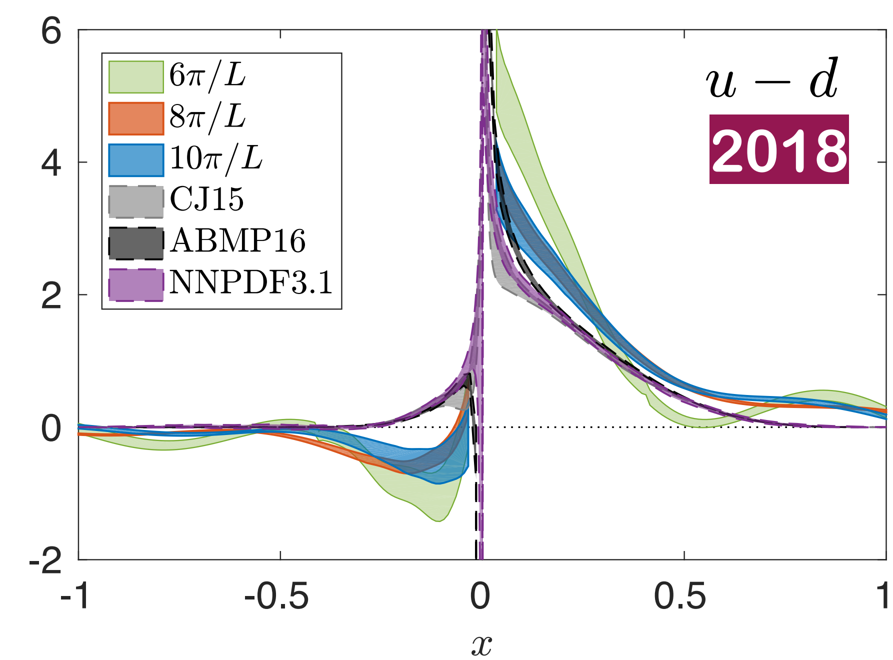
- [Lin et al (LP3)]: $P^z = \{2.2, \ 2.6, \ 3.0\} \ \text{GeV}$
- [Alexandrou et al (ETMC)]: $P^z = \{0.83, 1.11, 1.38\} \ \text{GeV}$

Results from ETMC (2018)

- Lattice size: $L = 4.5 \ \text{fm}$
- Match onto PDF at $\mu = 2 \ \text{GeV}$ (2-step matching)



- physical m_π
- renormalization
- matching



- Calculation at physical point is crucial
- Results become compatible with measurement for $P^z \gtrsim 1 \ \text{GeV}$

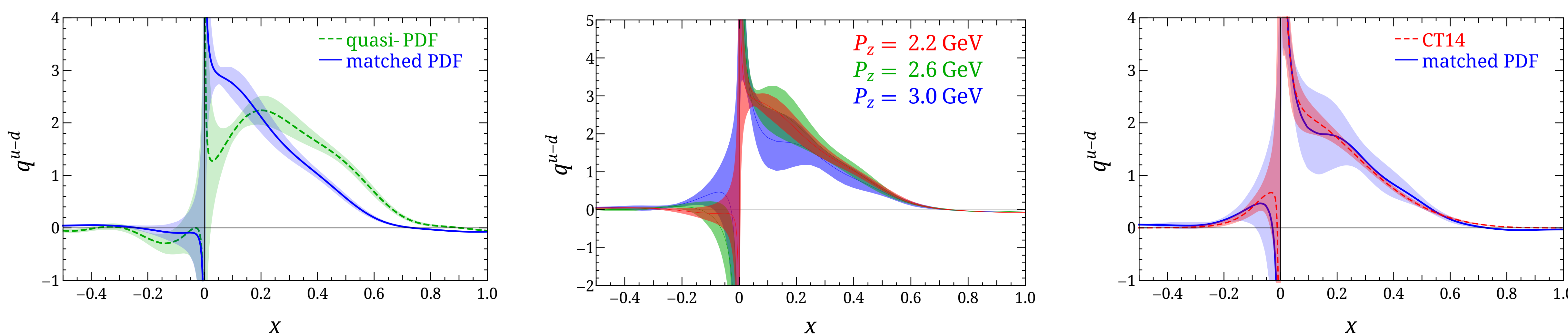
Lattice Results

Calculations at Physical Pion Mass

- [Lin et al (LP3)]: $P^z = \{2.2,\ 2.6,\ 3.0\}$ GeV
- [Alexandrou et al (ETMC)]: $P^z = \{0.83, 1.11, 1.38\}$ GeV

Results from LP3 (2018)

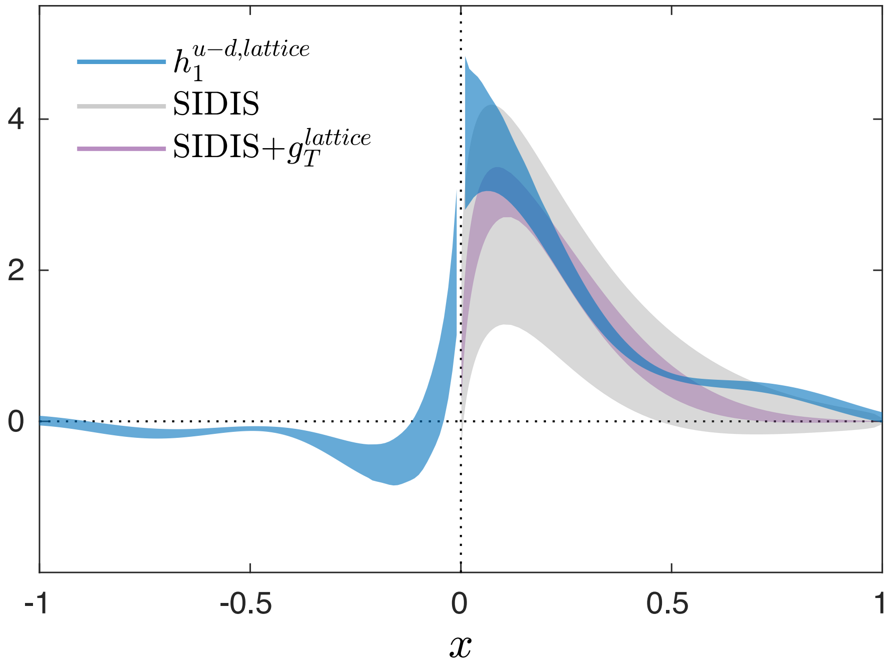
- Lattice size: $L = 5.8$ fm
- Match onto PDF at $\mu = 3.7$ GeV (1-step matching)



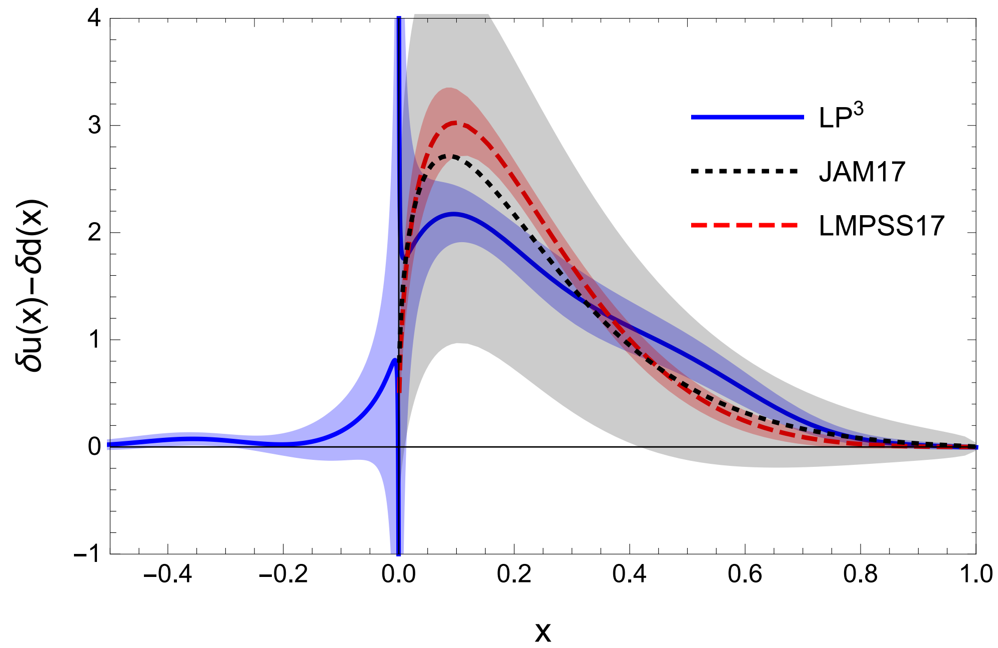
- Matching between quasi-PDF and PDF is crucial
- Results compatible with measurement

Lattice Results

PDFs with other spin-structures: quark transversity $\Gamma = \sigma^{ij}$



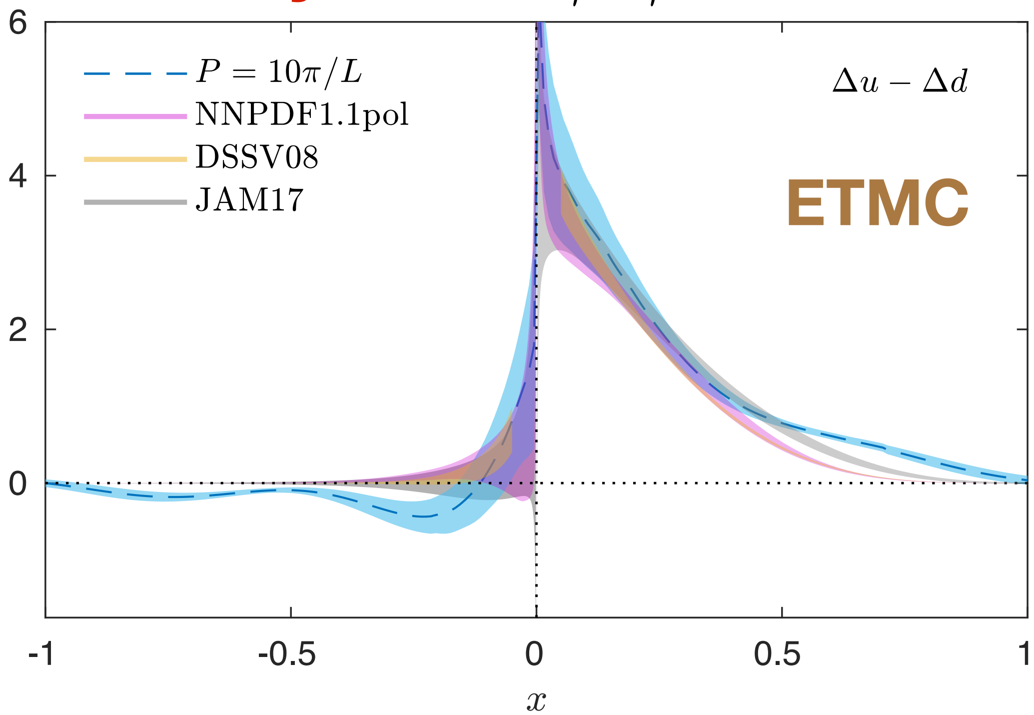
[Alexandrou et al (ETMC)]

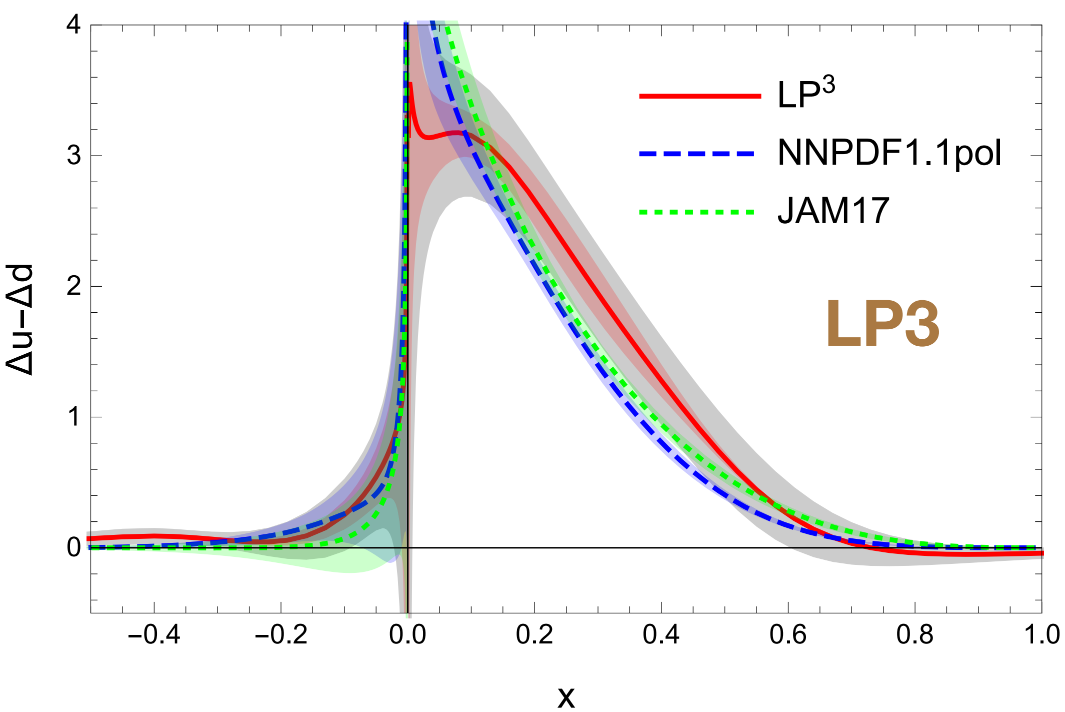


[Liu et al (LP3)]

Lattice results more precise than best extractions from SIDIS

quark helicity $\Gamma = \gamma^z \gamma^5$



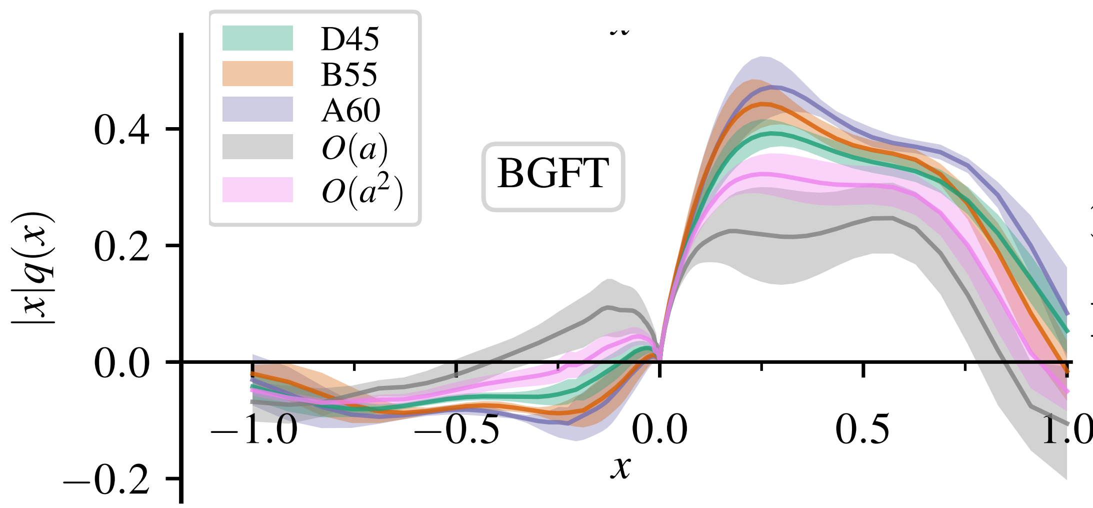


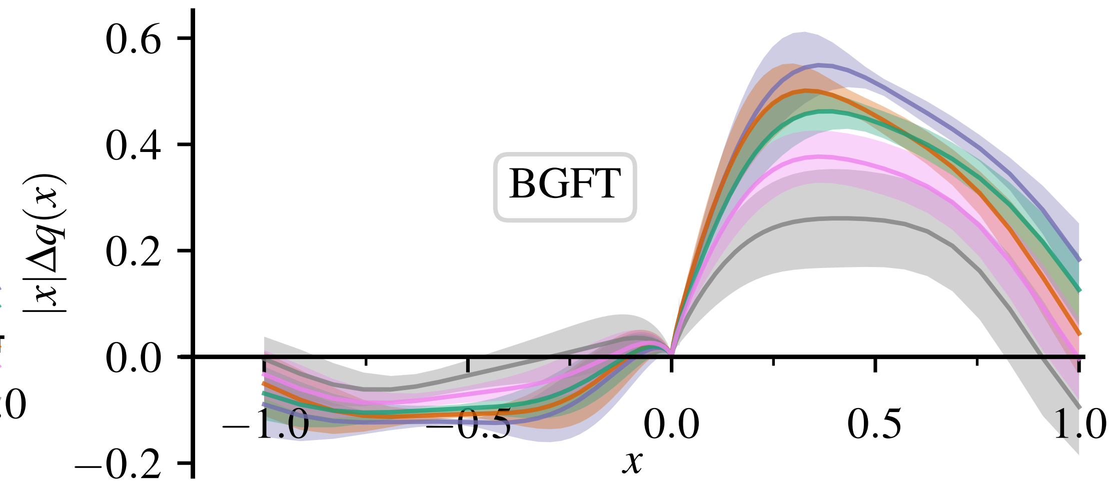
More Recent Results

Continuum Extrapolation $a \to 0$

Alexandrou et al. (ETMC)
2011.00864

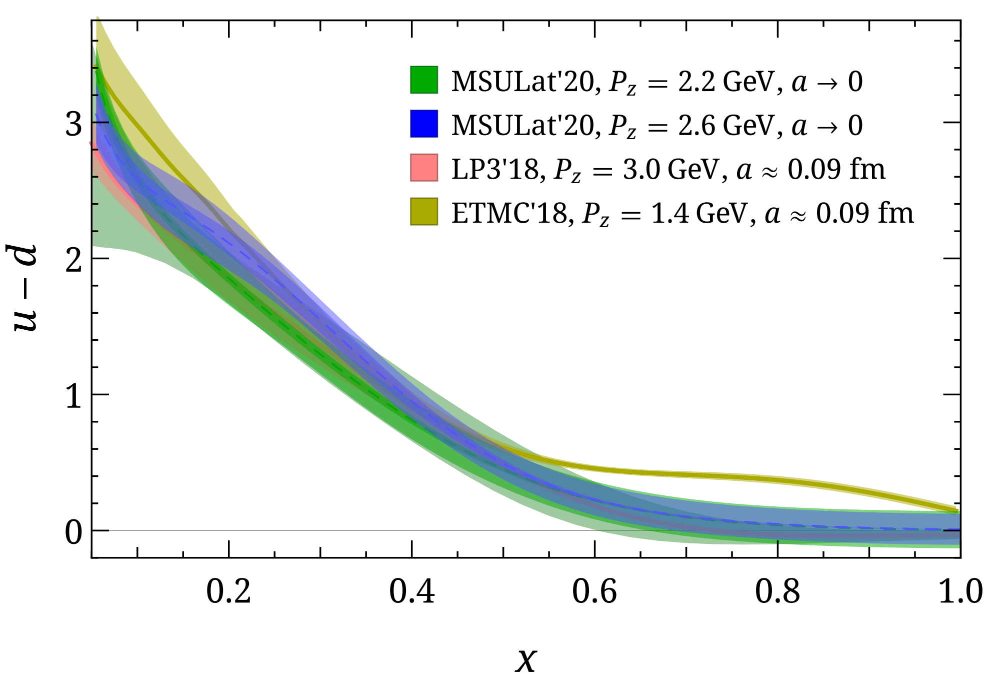
Name	a (fm)	m_π (MeV)
A60	0.0934(13)(35)	365
B55	0.0820(10)(36)	373
D45	0.0644(07)(25)	371

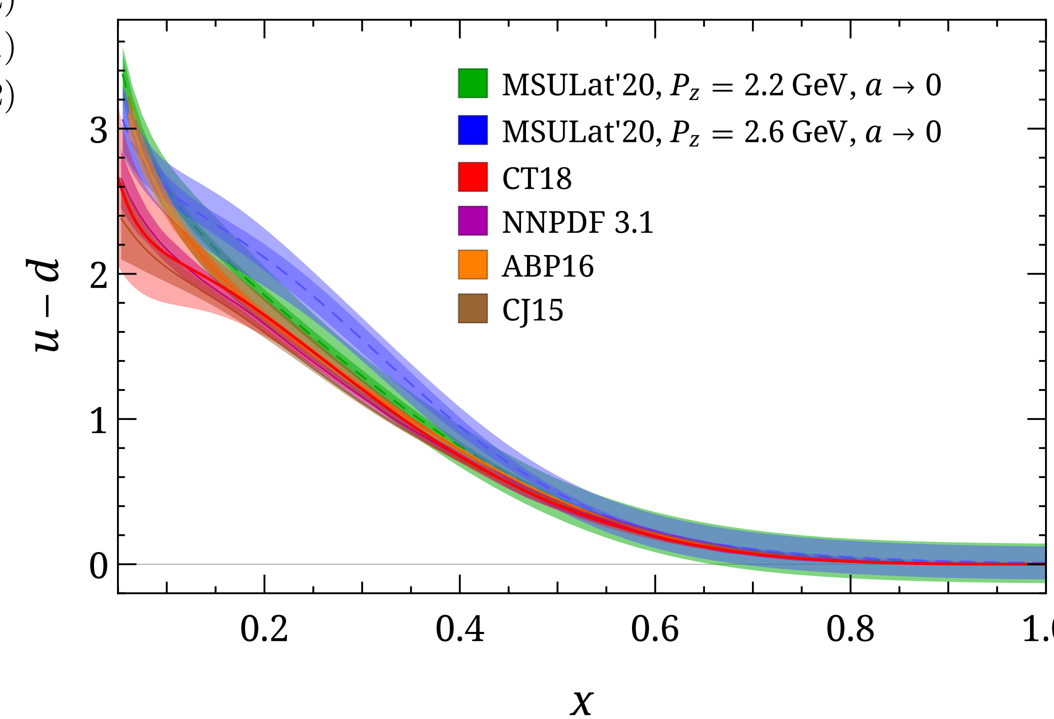




Lin, Chen, Zhang (MSULat)
2011.14971

Ensemble ID	a (fm)	$M_\pi^{\rm val}$ (MeV)
a12m310	0.1207(11)	310(3)
a12m220S	0.1202(12)	225(2)
a12m220	0.1184(10)	228(2)
a12m220L	0.1189(09)	228(2)
a09m130	0.0871(6)	138(1)
a06m310	0.0582(4)	320(2)





Other Recent Results

- **First lattice calculations of x-dependence of generalized parton distributions (off forward)** $E(x,\xi,t)\,,\ldots$

  Alexandrou et al.(ETMC) '20

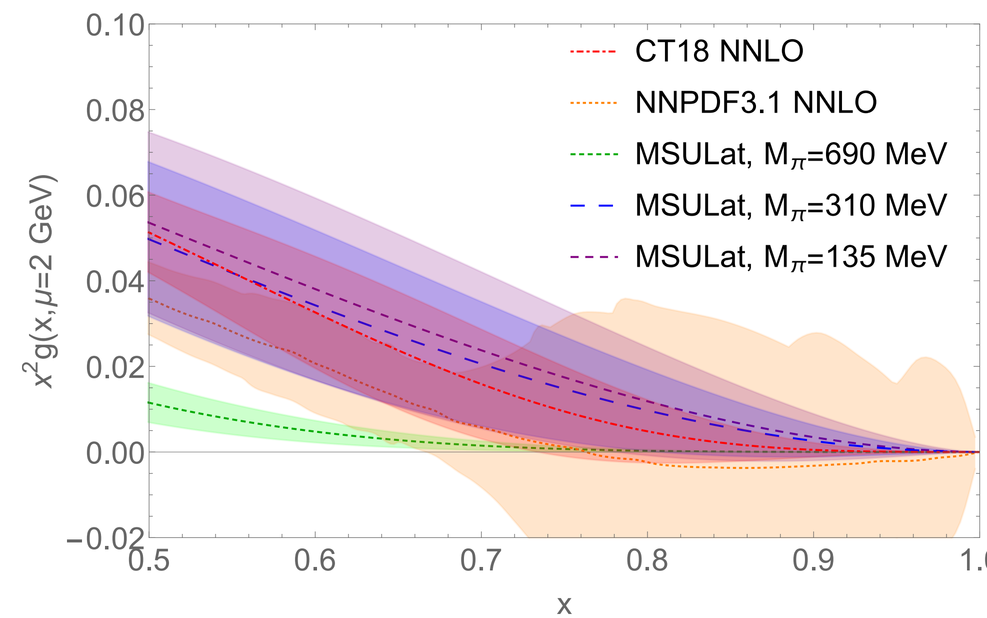
  Huey-Wen Lin '20

- **Parton distribution functions for the Δ^+ from quasi-PDFs** Chai et al. '20

- **Study of quasi-PDF matching for twist-3 PDFs** Bhattacharya et al. '20

- **First results for the gluon PDF at large x**

  Fan, Zhang, Lin '20



- **Results from the related Pseudo-PDF method**

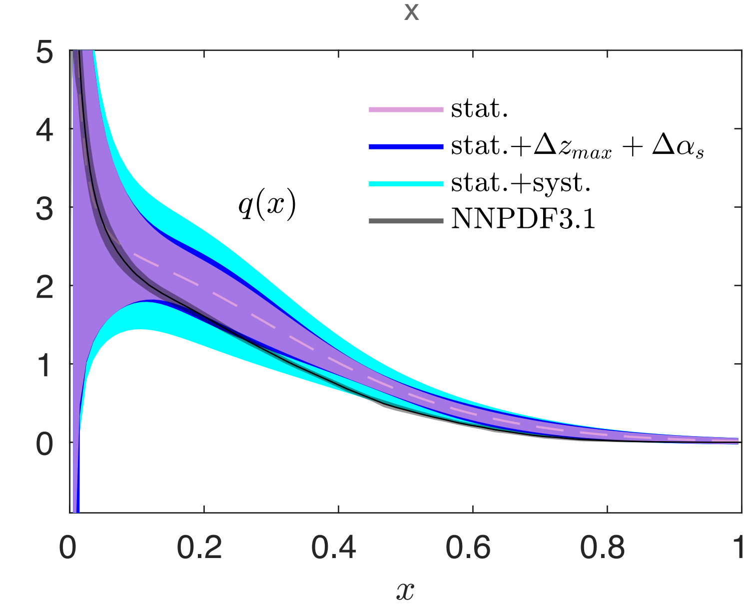
$$\mathcal{P}(x,z^2\mu^2)=\int_{|x|}^{1}\frac{dy}{|y|}\,\mathcal{C}\left(\frac{x}{y},\mu^2z^2\right)q(y,\mu)$$

Radyushkin '17, Orginos et al. '17

Joo et al. '20

Bhat, Cichy, Constantinou, Scapellato '20

Debbio, Giani, Karpie, Orginos, Radyushkin, Zafeiropoulos '20



TMD Factorization

nonperturbative when $k_T \sim b_T^{-1} \sim \Lambda_{\rm QCD}$

eg. Drell-Yan $q_T \ll Q$

$$\sigma(q_T, Q, Y) = H(Q,\mu) \int d^2\vec{b}_T\, e^{i\vec{q}_T\cdot\vec{b}_T}\, f_q(x_a, \vec{b}_T, \mu, \zeta_a)\, f_q(x_b, \vec{b}_T, \mu, \zeta_b) \left[1 + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)\right]$$

$\overline{\rm MS}$ **Hard function (virtual corrections)**

ζ **= Collins-Soper parameter**

$\zeta_a \zeta_b = Q^4$

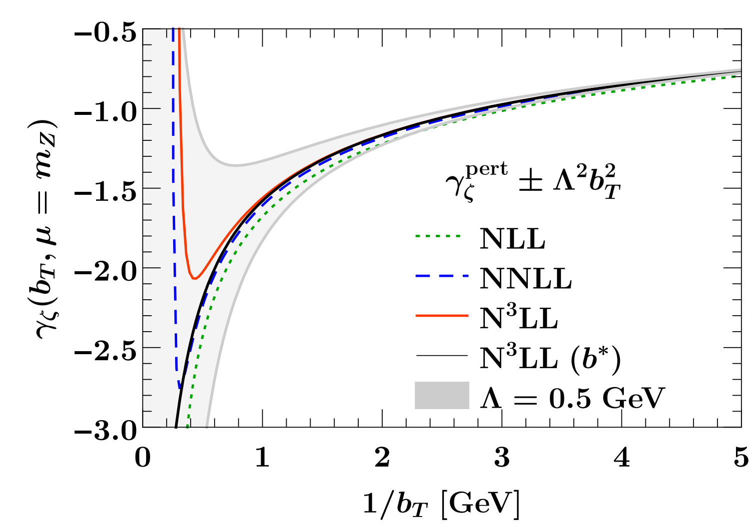
TMD Evolution: $\ln(Q/q_T)$

$$\mu \frac{d}{d\mu} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\mu^q(\mu, \zeta) = \Gamma^q_{\rm cusp}[\alpha_s(\mu)] \ln \frac{\mu^2}{\zeta} + \gamma_\mu^q[\alpha_s(\mu)]$$

$$\zeta \frac{d}{d\zeta} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\zeta^q(\mu, b_T) = -2\int_{1/b_T}^{\mu} \frac{d\mu'}{\mu'} \Gamma^q_{\rm cusp}[\alpha_s(\mu')] + \gamma_\zeta^q[\alpha_s(1/b_T)]$$

Collins-Soper Equation

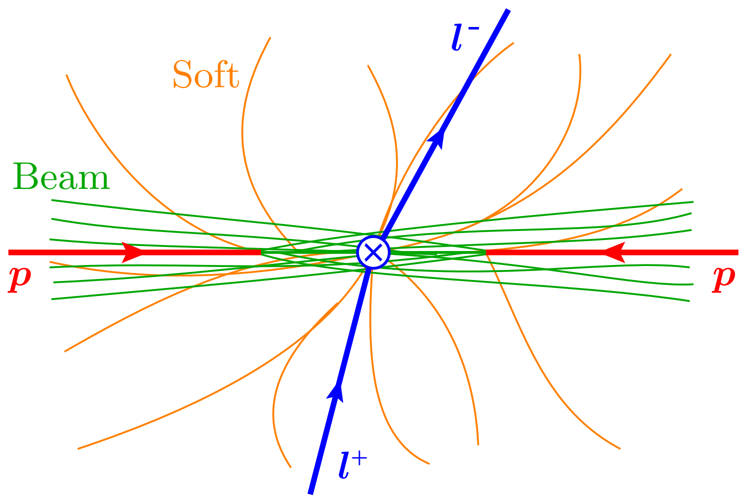
- $\gamma_\zeta^q(\mu, b_T)$ **is nonperturbative for** $b_T^{-1} \sim \Lambda_{\rm QCD}$
- **Log enhancement makes this the dominant nonperturbative effect**

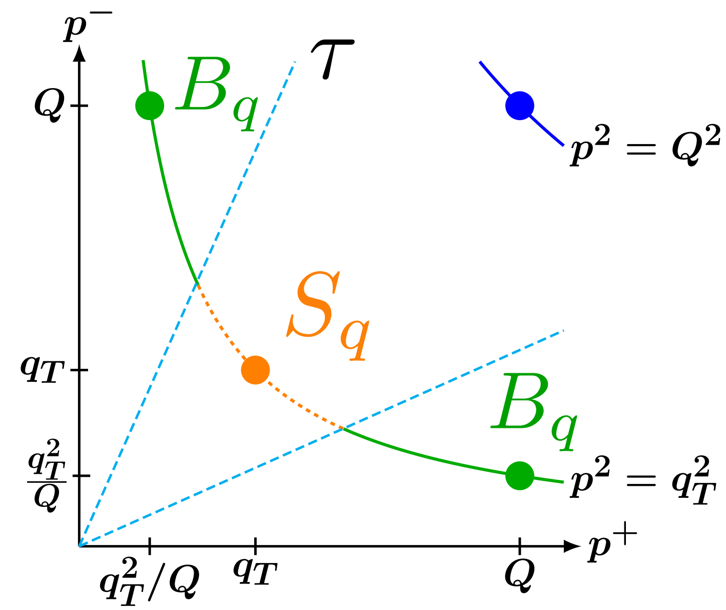


TMD Definitions

Beam Function **Soft factor**

$$f_q(x, \vec{b}_T, \mu, \zeta) = \lim_{\epsilon\to 0, \tau\to 0} Z_{\rm uv}(\epsilon, \mu, \zeta) B_q(x, \vec{b}_T, \epsilon, \tau, \zeta)\ \Delta_q(b_T, \epsilon, \tau)$$





contains

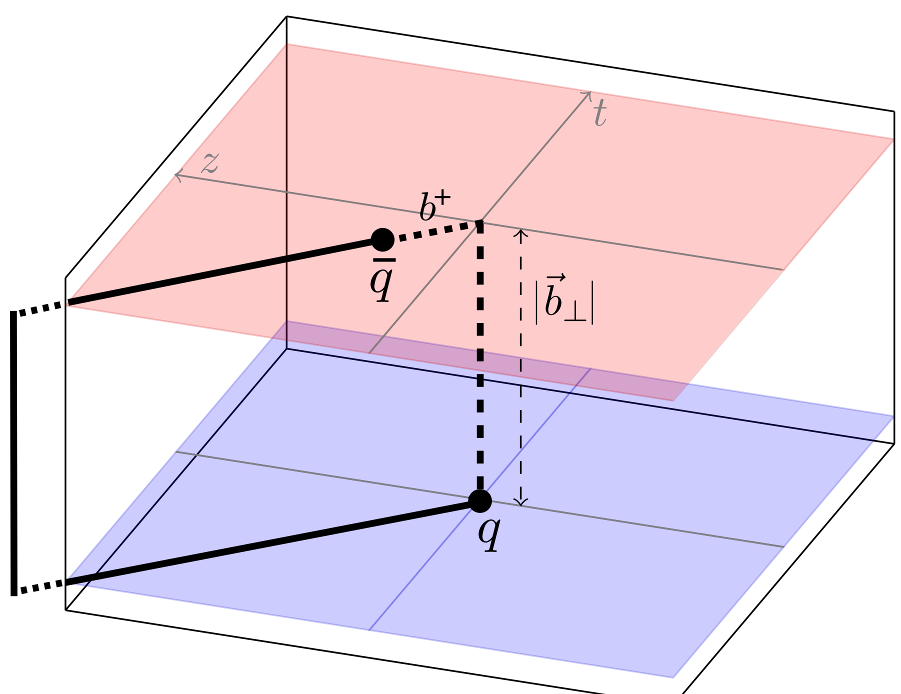
S_q & subtractions

$\Delta_q = 1/\sqrt{S_q}$

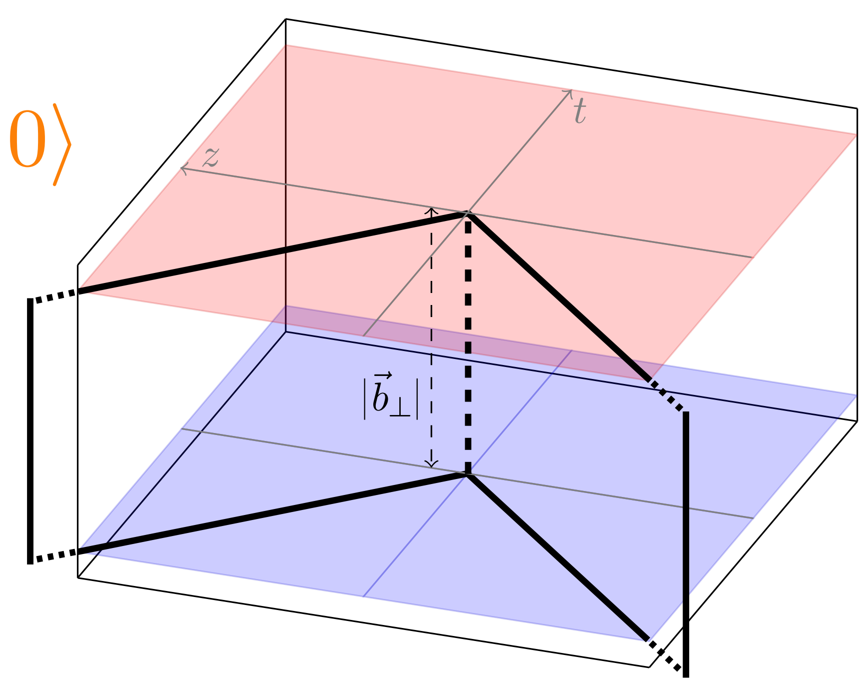
$$B_q = \mathrm{FT}_{b^+} \langle p|O_B|p\rangle$$

$$S_q = \langle 0|O_S|0\rangle$$

O_B :



O_S :



staple shaped Wilson lines

two light-cone directions
depends on color rep. (q or g)

Quasi-TMDPDFs

UV renormalization & scheme change

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = \int \frac{db^z}{2\pi} e^{ib^z(xP^z)} \lim_{\substack{a\to 0 \\ L\to\infty}} \tilde{Z}'_q(b^z, \mu, \tilde{\mu}) \tilde{Z}^q_{\rm uv}(b^z, \tilde{\mu}, a)$$

$$\times \tilde{B}_q(b^z, \vec{b}_T, a, L, P^z) \tilde{\Delta}^q_S(b_T, a, L)$$

quasi-Beam function

quasi-soft factor

a = lattice spacing (UV regulator)

- **needs to be computable with Lattice QCD**
- **must have same IR physics as TMDPDF**

(including $b_T \sim \Lambda_{\rm QCD}^{-1}$ dependence)

(isovector quark operators u-d, from here on)

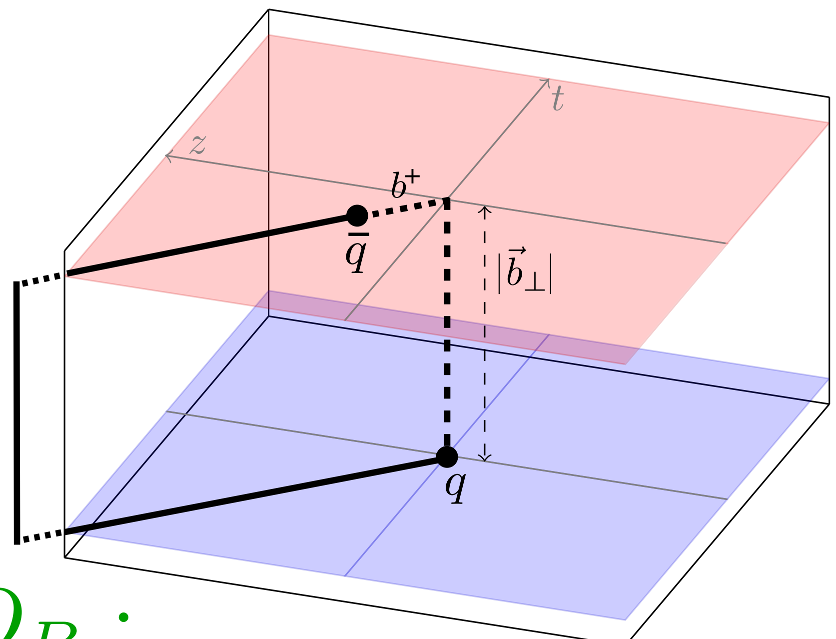
Quasi-Beam Functions

$$\tilde{B}_q(b^z, \vec{b}_T, a, L, P^z)$$

Beam Function

$$B_q = \langle p|O_B|p\rangle$$

O_B :

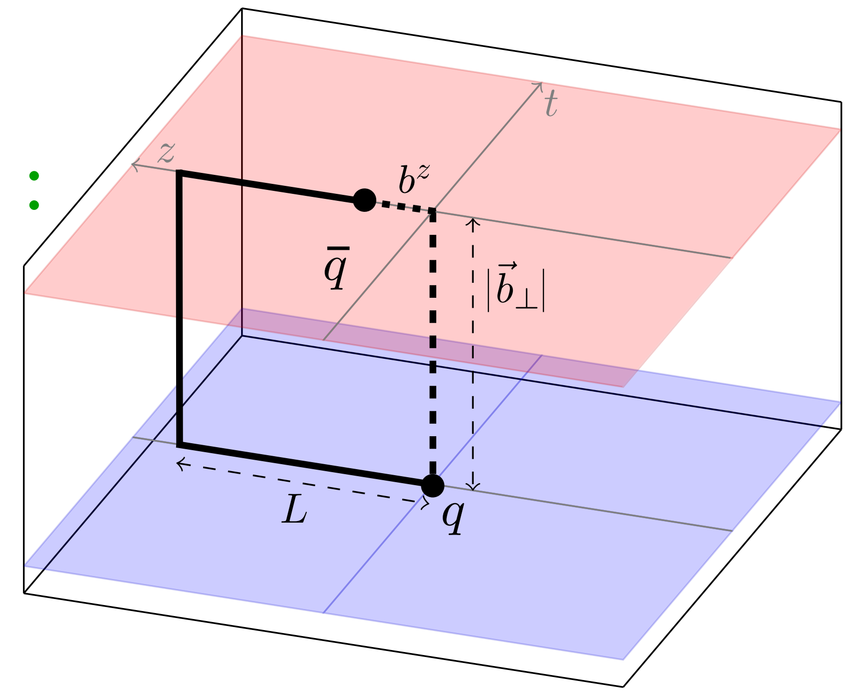


Connected by boost (for bare operators)

Natural Quasi-Beam Function

$$\tilde{B}_q = \langle p|\tilde{O}_B|p\rangle$$

$\tilde{O}_B$:



- **Finite length L for Wilson lines, regulates rapidity divergences** $\dfrac{1 - e^{-\mathrm{i}k^z L}}{k^z}$

$$\frac{1}{P^z} \ll b_T \ll L$$

- **Spatial lines, so have power law UV divergence** $\propto \text{length} = 2L + b_T - b^z$

Quasi-Soft Function

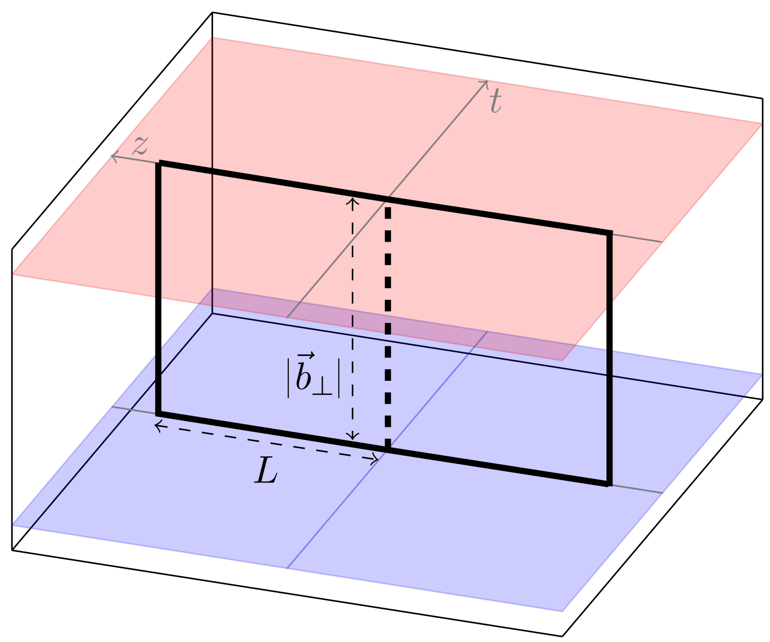
$\tilde{\Delta}_S^q = 1/\sqrt{\tilde{S}_q}$ $\tilde{S}_q = \langle 0|\tilde{O}_S|0\rangle$

- Cancel power law dependence on L, length = $2(2L+b_T)$
- Needed to reproduce infrared structure.
- Free to invent a $\tilde{O}_S$ to achieve this.

[Ebert, IS, Zhao '18]
[Ji, Sun, Xiong, Yuan '14]

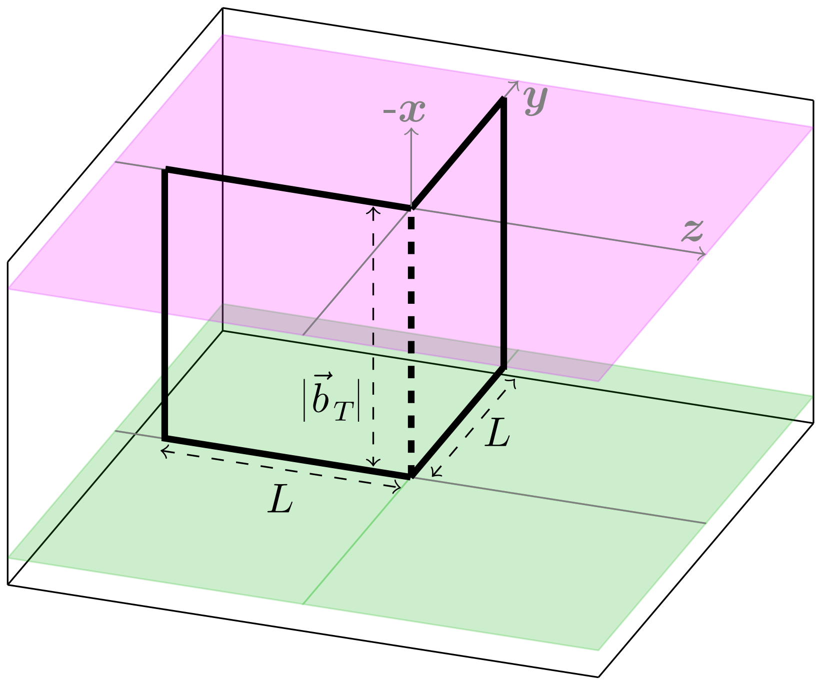
(1) "naive" quasi-soft

Invalid:
IR differs
@ 1-loop



(2) "bent" quasi-soft

IR agrees
@ 1-loop



(3) can be extracted from TMD factorization theorem for light meson form factor F
& quasi-TMD light meson wavefunction $\tilde{\phi}$ [Ji, Liu, Liu 1910.11415]

$$\tilde{S}_q = \frac{F(b_\perp, P\cdot P')}{\int dx dx' H(x,x',P,P')\tilde{\phi}(x',b_\perp,P')\tilde{\phi}^\dagger(x,b_\perp,P)}$$

IR then agrees
to all orders

Quasi-TMDPDF

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = \int \frac{db^z}{2\pi} e^{ib^z(xP^z)} \lim_{\substack{a\to 0 \\ L\to\infty}} \tilde{Z}'_q(b^z, \mu, \tilde{\mu}) \tilde{Z}^q_{\rm uv}(b^z, \tilde{\mu}, a)$$
$$\times \tilde{B}_q(b^z, \vec{b}_T, a, L, P^z) \tilde{\Delta}^q_S(b_T, a, L)$$

- **linear divergences in L cancel**
- $\tilde{Z}^q_{\rm uv}$ **multiplicative, and removes linear** b^z/a **divergence**
- $\tilde{Z}'_q$ **converts lattice friendly scheme (**$\tilde{\mu}$**) to** $\overline{\rm MS}$ **(**μ**)**

Relation between Quasi-TMDPDF & TMDPDF

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = C^{\rm TMD}(\mu, xP^z) \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln\frac{(2xP^z)^2}{\zeta}\right] f_q(x, \vec{b}_T, \mu, \zeta)$$

nonperturbative quasi-TMDPDF

perturbative kernel

nonperturbative CS kernel

nonperturbative TMDPDF

(Note: no convolution in x)

[Ebert, IS, Zhao '18]

[Ji, Liu, Liu '19]

Collins-Soper Kernel from Lattice

M. Ebert, IS, Y. Zhao, 1811.00026

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = C^{\rm TMD}(\mu, xP^z) \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln\frac{(2xP^z)^2}{\zeta}\right] f_q(x, \vec{b}_T, \mu, \zeta)$$

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\frac{C^{\rm TMD}(\mu, xP_2^z)\, \tilde{f}_q(x, \vec{b}_T, \mu, P_1^z)}{C^{\rm TMD}(\mu, xP_1^z)\, \tilde{f}_q(x, \vec{b}_T, \mu, P_2^z)}$$

quasi-Beam fns.

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\frac{C^{\rm TMD}(\mu, xP_2^z) \int db^z e^{ib^z xP_1^z}\, \tilde{Z}'_q\, \tilde{Z}^q_{\rm uv}\, \tilde{B}_q(b^z, \vec{b}_T, a, L, P_1^z)}{C^{\rm TMD}(\mu, xP_1^z) \int db^z e^{ib^z xP_2^z}\, \tilde{Z}'_q\, \tilde{Z}^q_{\rm uv}\, \tilde{B}_q(b^z, \vec{b}_T, a, L, P_2^z)}$$

- **needs $\tilde{B}_q$, $\tilde{Z}^q_{\rm uv}$, $\tilde{Z}'_q$, $C^{\rm TMD}$ (does not require $\tilde{\Delta}^q_S$)**
- **LHS independent of $P_1^z, P_2^z, x,$ hadron state, spin**
- **can setup $\tilde{Z}^q_{\rm uv}$ to remove power law divergences**

➡ **Important universal QCD function from Lattice QCD**

Ratios of proton $\tilde{B}_q$ s also studied by [Musch et al ’10’12; Engelhardt et al ’15; Yoon et al ’17]

$C^{\mathrm{TMD}}(\mu, xP^z)$ in $\overline{\mathrm{MS}}$ at 1-loop

[Ji, Jin, Yuan, Zhang, Zhao '18]
[Ebert, IS, Zhao '18]

$$C^{\mathrm{TMD}}(\mu, xP^z) = 1 + \frac{\alpha_s C_F}{4\pi}\left(-\ln^2\frac{(2xP^z)^2}{\mu^2} + 2\ln\frac{(2xP^z)^2}{\mu^2} - 4 + \frac{\pi^2}{6}\right) + \mathcal{O}(\alpha_s^2)$$

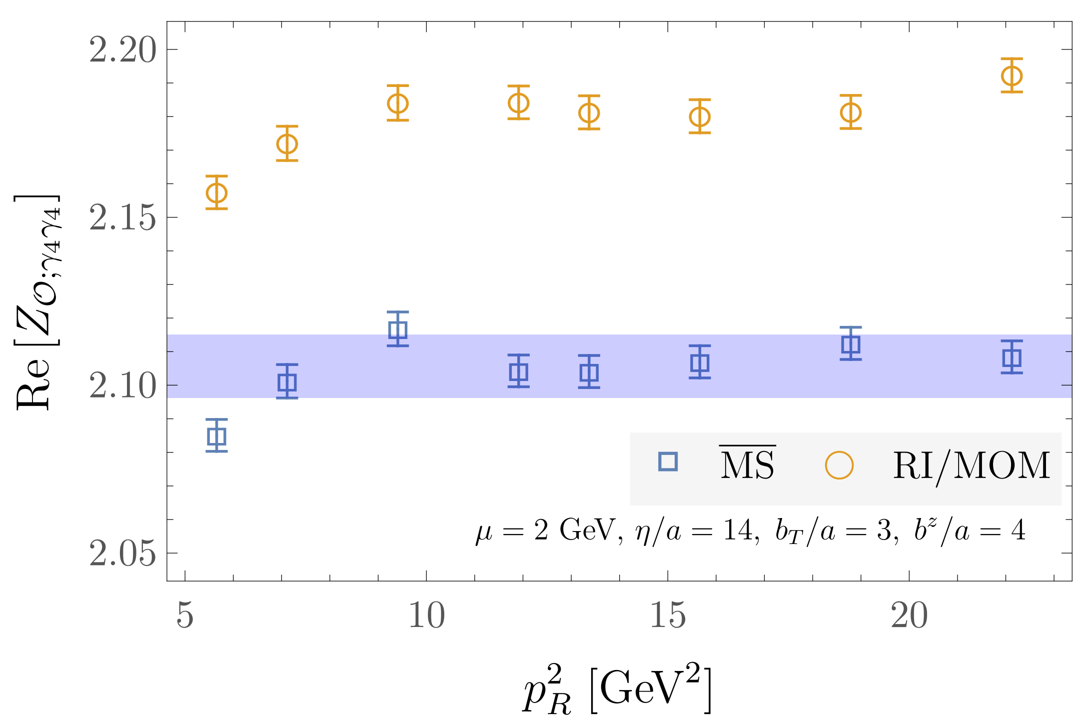
$\tilde{Z}'_q$ **conversion factor between RI/MOM scheme and $\overline{\mathrm{MS}}$ at 1-loop**

M. Ebert, IS, Y. Zhao, 1910.08569

$\tilde{Z}^q_{\mathrm{uv}}$ **Calculate Nonperturbatively on Lattice in an RI/MOM scheme**

P. Shanahan, M. Wagman, Y. Zhao, 1911.00800

$$Z_q^{-1}(p_R) Z^{\mathrm{RI'/MOM}}_{\mathcal{O}_{\Gamma\Gamma'}}(p_R) \Lambda^{\mathcal{O}_{\Gamma'}}_{\alpha\beta}(p)\Big|_{p^\mu = p^\mu_R} = \Lambda^{\mathcal{O}_{\Gamma};\mathrm{tree}}_{\alpha\beta}(p)$$



nf=0 (quenched)
improved Wilson fermions
smearing (Wilson flow) on gauge links
a=0.04, 0.06, 0.08 fm
volume ~ 2 fm
$m_\pi \sim 1.2\,\mathrm{GeV}, 340\,\mathrm{MeV}$
various L, b_T, b_z, p_R
full 16x16 mixing matrix

Lattice Results for Rapidity Anomalous Dimension

P. Shanahan, M. Wagman, Y. Zhao arXiv:2003.06063

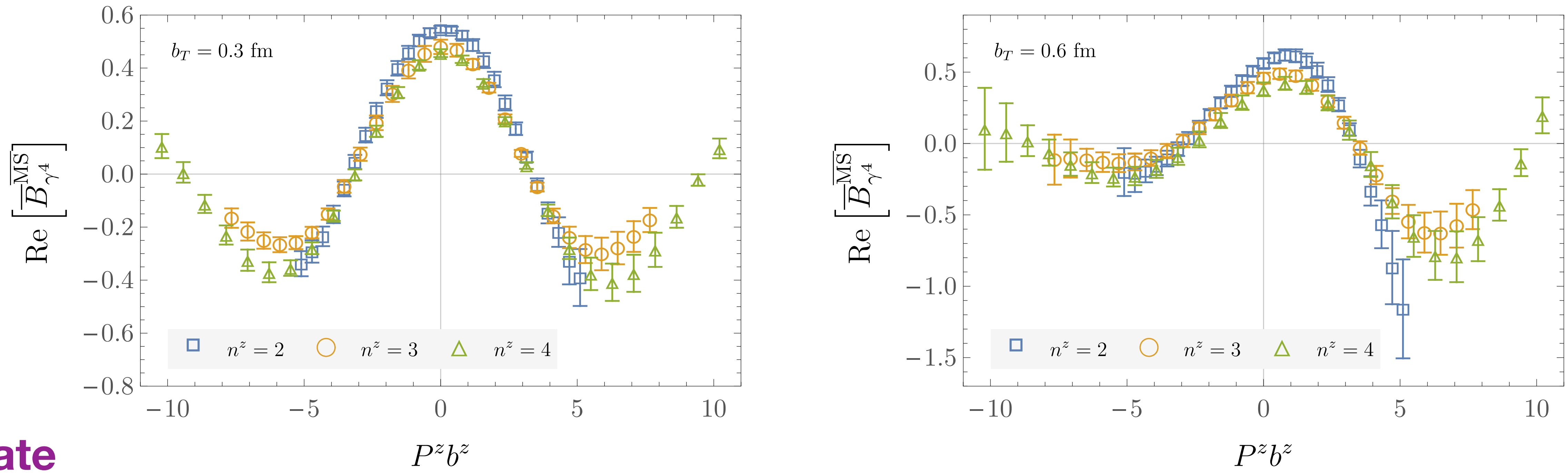
nf=0 (quenched) simulation

Exploits universality: uses 1.2 GeV pseudoscalar meson

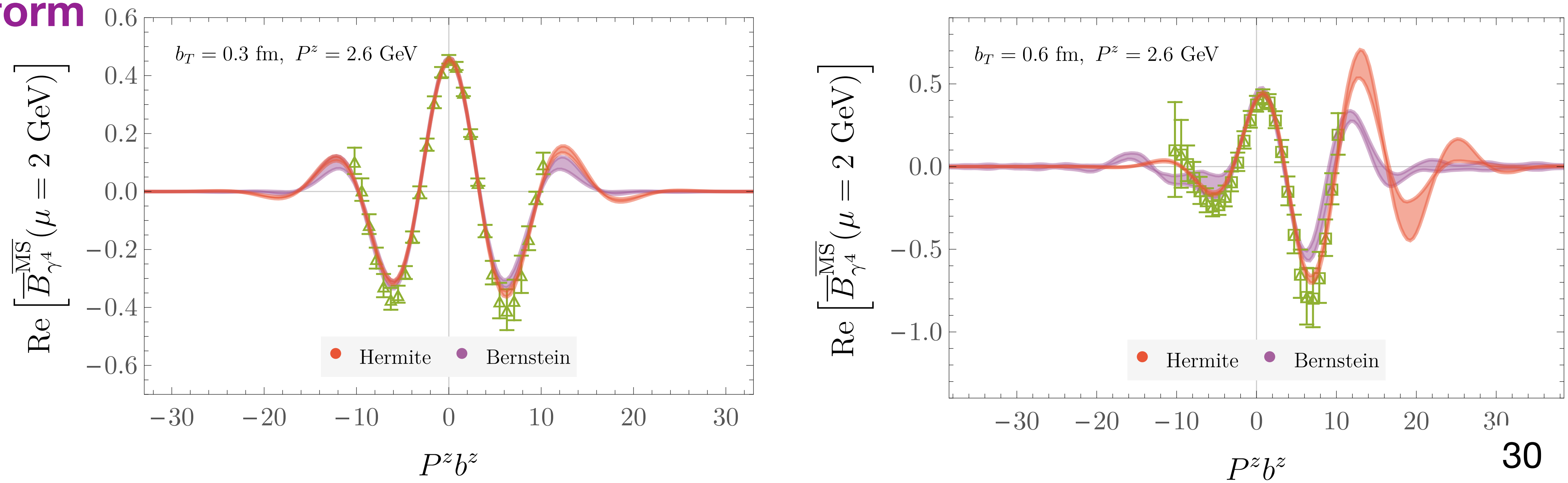
Includes nonperturbative renormalization

$P^z \in \{1.29, 1.94, 2.58\}$ GeV

Renormalized quasi-Beam Fn.



Fits to facilitate Fourier transform



Lattice Results for Rapidity Anomalous Dimension

P. Shanahan, M. Wagman, Y. Zhao arXiv:2003.06063

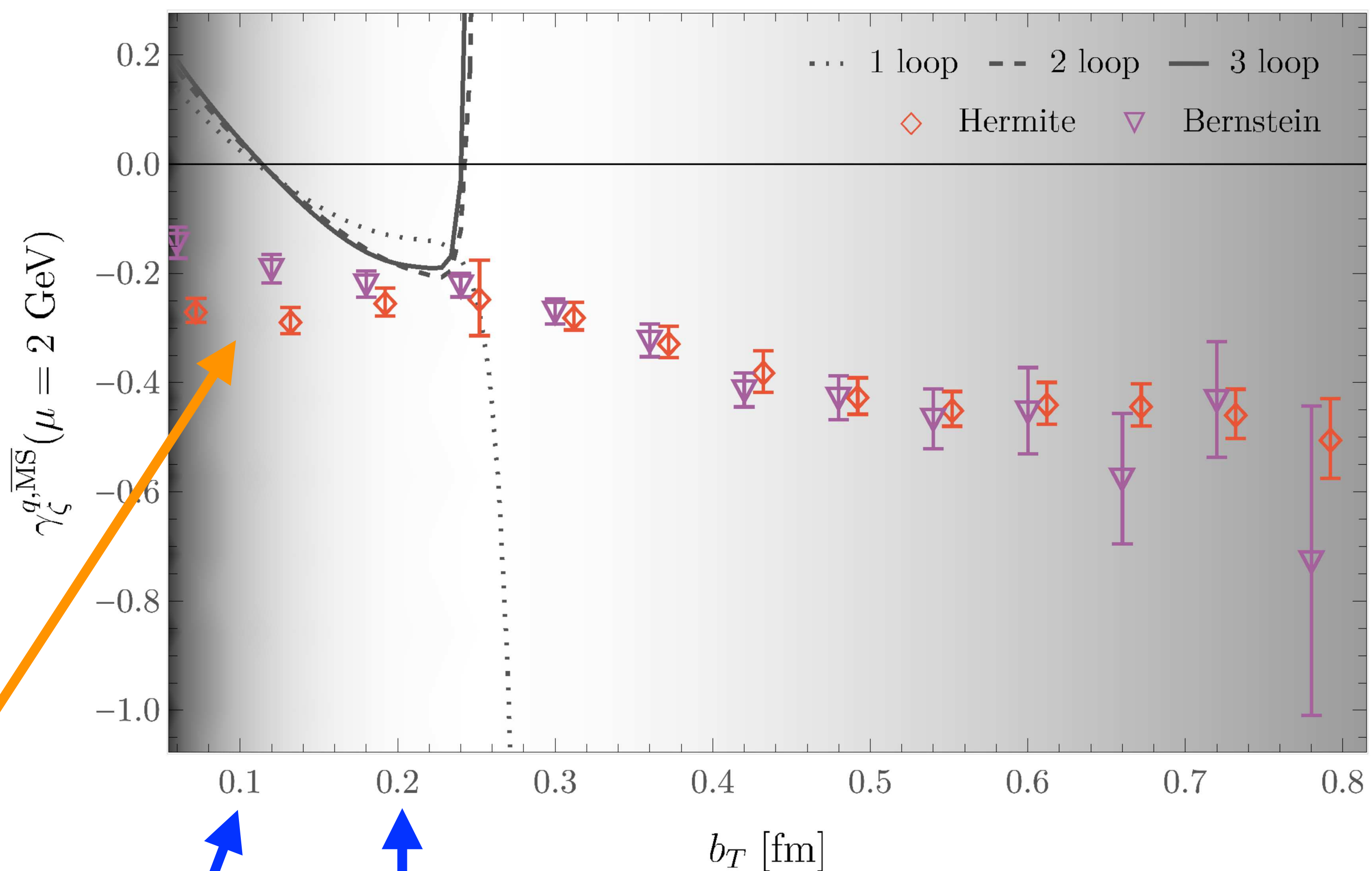
nf=0 (quenched) simulation

Exploits universality: uses 1.2 GeV pseudoscalar meson

Includes nonperturbative renormalization

$P^z \in \{1.29, 1.94, 2.58\}$ GeV

Result for Nonperturbative TMD Rapidity Anomalous Dimension (nf=0)



Larger $1/(b_T P^z)$ power corrections (not included in error bars)

(2 GeV)$^{-1}$ (1 GeV)$^{-1}$

Lattice Results for Rapidity Anomalous Dimension

P. Shanahan, M. Wagman, Y. Zhao arXiv:2003.06063

nf=0 (quenched) simulation

Exploits universality: uses 1.2 GeV pseudoscalar meson

Includes nonperturbative renormalization

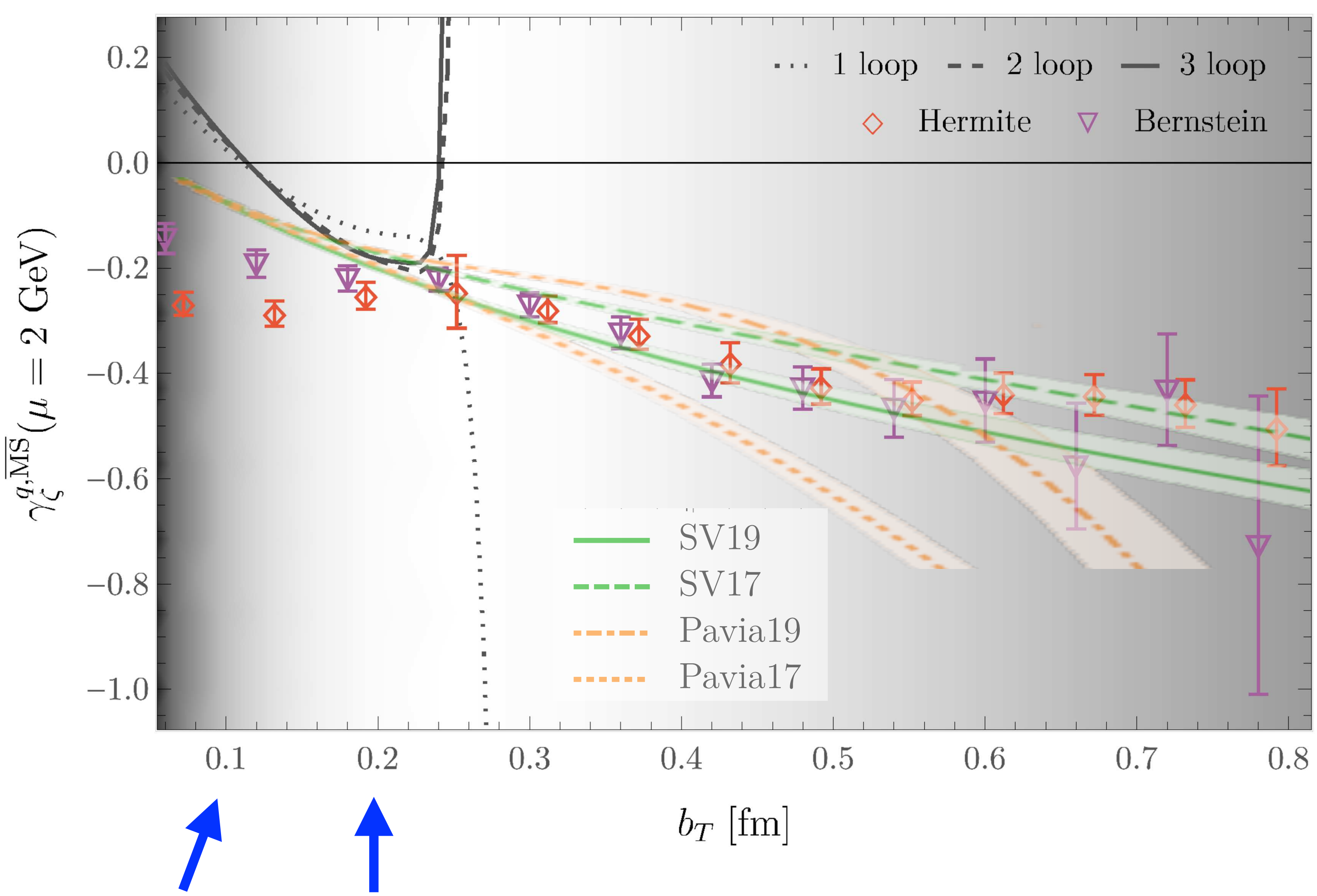
$$P^z \in \{1.29, 1.94, 2.58\} \ \text{GeV}$$

Result for Nonperturbative TMD Rapidity Anomalous Dimension (nf=0)

Can make a rough comparison to Pheno Models obtained in TMD fits
[overlay by Phiala Shanahan]

SV= Scimemi, Vladimirov

Pavia= Bacchetta et al.



$(2\ \text{GeV})^{-1}$ $(1\ \text{GeV})^{-1}$

TMD Soft Function Calculation

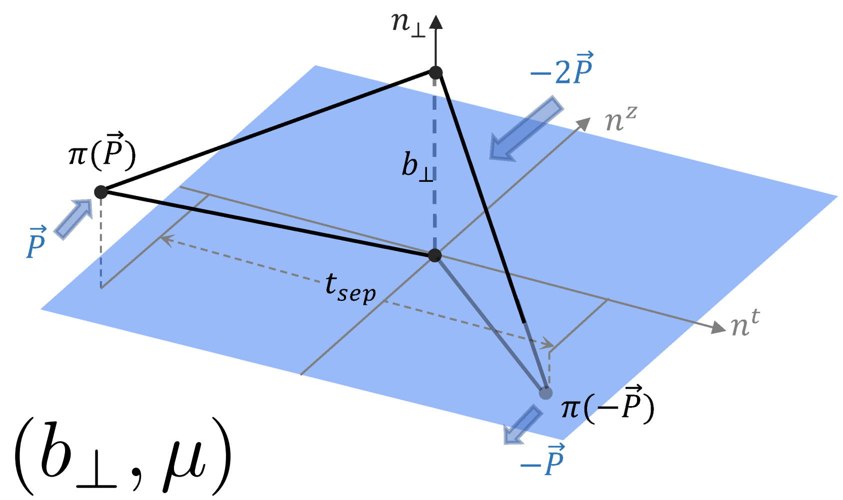
Zhang et al (LPC) 2005.14572

nf=2+1 simulation

Renormalization & Matching at tree level

$$F(b_\perp, P^z) = \langle \pi(-\vec{P}) | (\bar{q}_1 \Gamma q_1)(\vec{b})(\bar{q}_2 \Gamma q_2)(0) | \pi(\vec{P}) \rangle_c$$

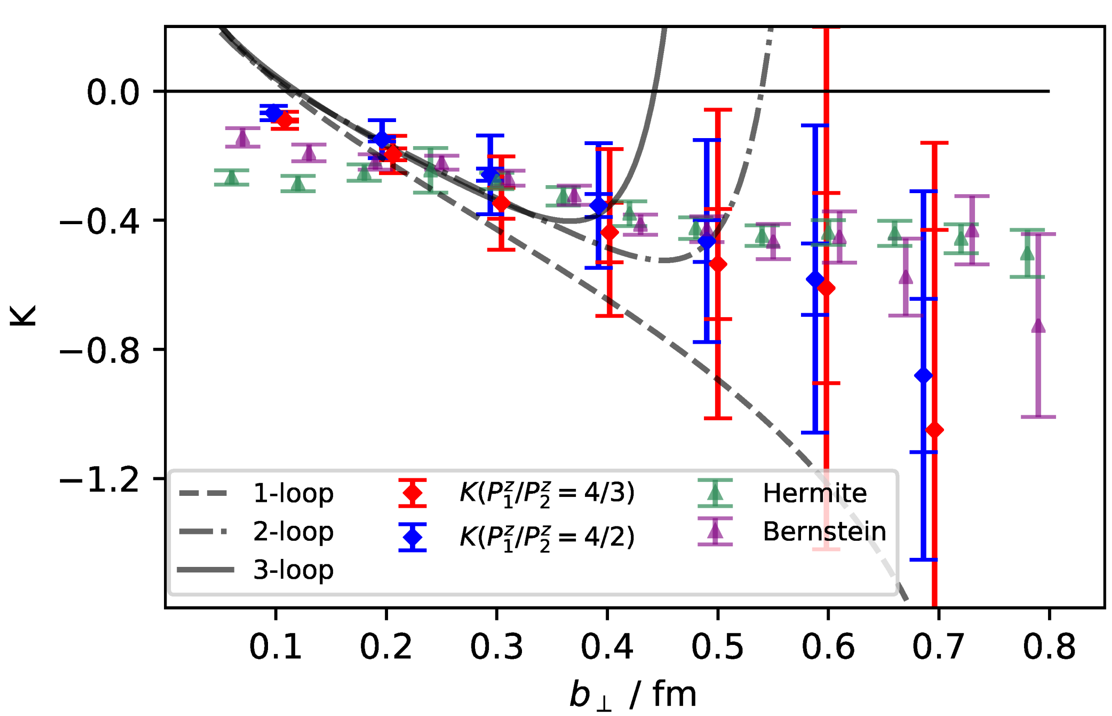
$$\tilde{S}_q = \frac{F(b_\perp, P\cdot P')}{\int dx dx' H(x,x',P,P') \widetilde{\phi}(x', b_\perp, P') \widetilde{\phi}^\dagger(x, b_\perp, P)}$$

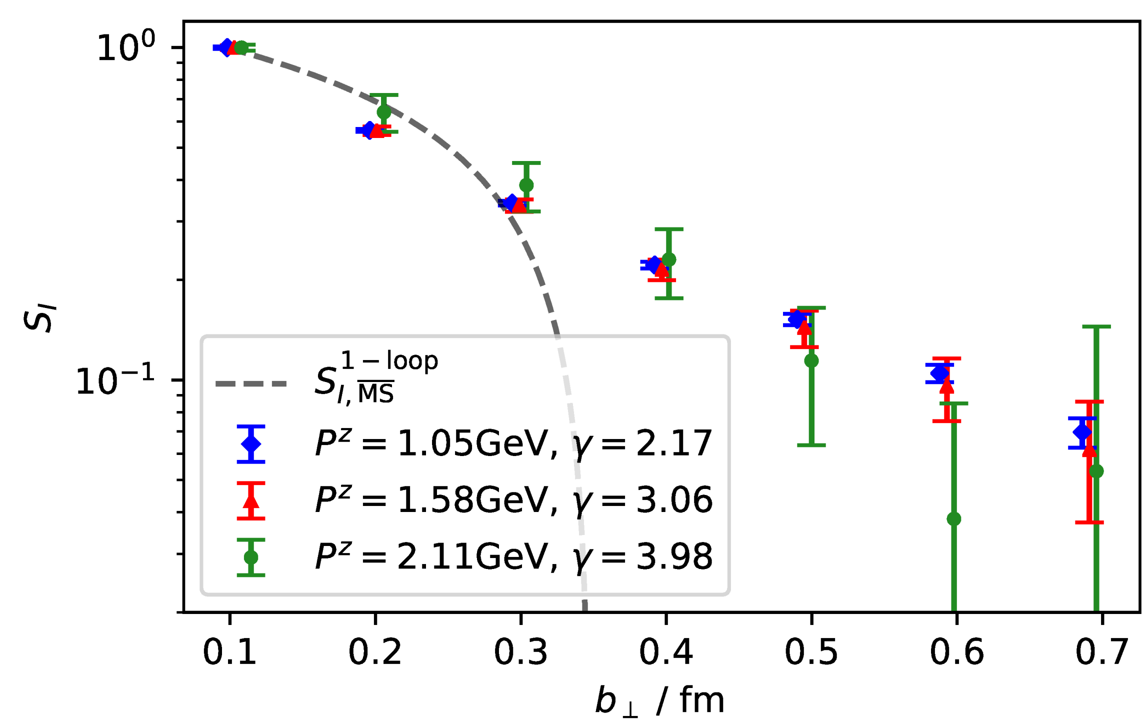


$$\tilde{S}_q(b_\perp, \mu, Y) = e^{Y\gamma_\zeta(\mu, b_\perp)} S_I^{-1}(b_\perp, \mu)$$

Rapidity Anom. Dim.

Intrinsic nonperturbative soft function





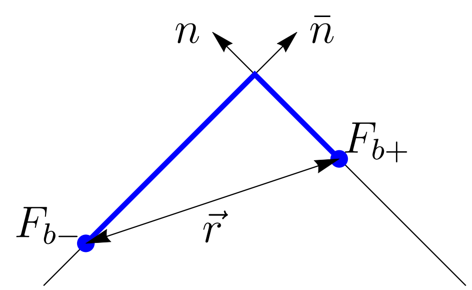
Other Recent Results

- **Rapidity Anom. Dimension: alternate methods, operators for power expansion, and models**

Alexey Vladimirov ‘20

$$\gamma_\zeta(\mu, b_\perp) = \gamma_\zeta^{[0]}(\mu, b_\perp) + b_\perp^2 \gamma_\zeta^{[2]}(\mu, b_\perp) + b_\perp^4 \gamma_\zeta^{[4]}(\mu, b_\perp) + \ldots$$

$$\gamma_\zeta^{[2]}(\mu, b_\perp) :$$



(see his talk on Wednesday)

- **Spin dependent quasi-TMD distributions**

$$\frac{g_{1L}(x, b_T, \mu, \zeta)}{f_1(x, b_T, \mu, \zeta)} = \frac{\tilde{g}_{1L}(x, b_T, \mu, P^z)}{\tilde{f}_1(x, b_T, \mu, P^z)}, \quad \frac{h_1(x, b_T, \mu, \zeta)}{f_1(x, b_T, \mu, \zeta)} = \frac{\tilde{h}_1(x, b_T, \mu, P^z)}{\tilde{f}_1(x, b_T, \mu, P^z)}, \quad \frac{h_{1T}^\perp(x, b_T, \mu, \zeta)}{f_1(x, b_T, \mu, \zeta)} = \frac{\tilde{h}_{1T}^\perp(x, b_T, \mu, P^z)}{\tilde{f}_1(x, b_T, \mu, P^z)}$$

Ebert, Schindler, IS, Zhao ’20,
(see also Vladimirov, Schaefer ’20)

$$\frac{f_1^\perp(x, b_T, \mu, \zeta)}{f_1(x, b_T, \mu, \zeta)} = \frac{\tilde{f}_1^\perp(x, b_T, \mu, P^z)}{\tilde{f}_1(x, b_T, \mu, P^z)}$$

Ji, Liu, Schaefer, Yuan ‘20

Nucleon Polarization	Quark Polarization: Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	$f_1(x, k_T^2)$		$h_1^\perp(x, k_T^2)$ - Boer-Mulders
L		$g_1(x, k_T^2)$ - Helicity	$h_{1L}^\perp(x, k_T^2)$ - Long-Transversity
T	$f_1^\perp(x, k_T^2)$ - Sivers	$g_{1T}(x, k_T^2)$ - Trans-Helicity	$h_1(x, k_T^2)$ - Transversity; $h_{1T}^\perp(x, k_T^2)$ - Pretzelosity

Summary

- quasi PDFs enable direct calculations of PDFs (and other light cone matrix elements) with Lattice QCD.
  Fairly mature field with lots of activity!

- quasi TMDs are a field in its early stages, but already show significant promise.   eg. rapidity anomalous dimension