Resummation of fiducial power corrections in the Drell-Yan transverse momentum distribution

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Based on [ME, J. Michel, I. Stewart, F. Tackmann; 2006.11382]

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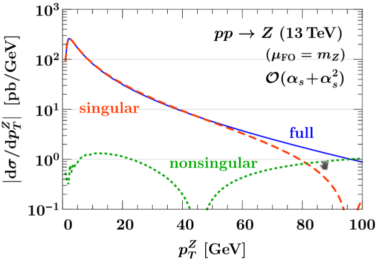
Motivation

q_T spectrum in inclusive Drell-Yan:



$$\frac{d\sigma}{dq_T^2} \sim \alpha_s \left[\frac{1}{q_T^2}(L + \cdots) + (L + \cdots) + \mathcal{O}(q_T^2) \right] + \alpha_s^2 \left[\frac{1}{q_T^2}(L^3 + \cdots) + (L^3 + \cdots) + \mathcal{O}(q_T^2) \right] + \cdots$$

where $L = \ln(Q^2/q_T^2)$

- Singular terms $\sim 1/q_T^2$ fully predicted by TMD factorization
- Subleading terms suppressed as $\mathcal{O}(q_T^2/Q^2)$
  - Calculated at NLO in [ME, Moult, Stewart, Tackmann, Vita, Zhu '18]
  - Observed numerically at NNLO [see e.g. MATRIX]

q_T spectrum in fiducial Drell-Yan:

- Subleading terms only suppressed as $\mathcal{O}(q_T/Q)$ [ME, Tackmann '19]
- Can be included through matching to fixed order
- Can they also be resummed?

q_T factorization with fiducial power corrections

Inclusive q_T factorization for Drell-Yan

Consider $p(P_a^\mu)p(P_b^\mu) \to Z/\gamma^*(q^\mu) \to \ell^-(p_1^\mu)\ell^+(p_2^\mu)$:

- Factorize matrix element: $\mathcal{M}_{pp\to V+X} = \mathcal{M}^\mu_{V\to L}\,\langle X|J_{V\mu}|pp\rangle$
  - See Georgios' talk for QCD+QED effects
- Factorize cross section accordingly: $\dfrac{\mathrm{d}\sigma}{\mathrm{d}^4q} = L_{\mu\nu}(q)W^{\mu\nu}(q, P_a, P_b)$

Inclusive Drell-Yan:

- $L_{\mu\nu}$ only depends on q: $L_{\mu\nu}(q) = \left(g_{\mu\nu} - \dfrac{q_\mu q_\nu}{q^2}\right) L(q^2)$
- Factorized cross section simplifies to

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q} = L(q^2)W(q, P_a, P_b)\,, \quad W = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W^{\mu\nu}$$

- Leading-power expansion of W recovers standard q_T factorization
- Inclusive W only depends on

$$q^2 \equiv Q^2\,, \quad P_{a,b}\cdot q = E_{\rm cm}\sqrt{Q^2 + q_T^2}\, e^{\pm Y}$$

  - Intuitively: azimuthal symmetry implies quadratic corrections in q_T^2 only

Fiducial q_T factorization for Drell-Yan

Consider $p(P_a^\mu)p(P_b^\mu) \to Z/\gamma^*(q^\mu) \to \ell^-(p_1^\mu)\ell^+(p_2^\mu)$:

- Allow for generic fiducial cuts Θ
- Factorize cross section accordingly:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q}(\Theta) = L_{\mu\nu}(q,\Theta)W^{\mu\nu}(q,P_a,P_b)$$

- $W^{\mu\nu}$ contains nine real independent structures:
  - Current conservation: $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$
  - Hermiticy: $W^{*\mu\nu} = W^{\nu\mu}$
- Decompose $W^{\mu\nu}$ accordingly:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q}(\Theta) = \sum_{i=-1}^{7}(L\cdot K_i)(W\cdot K_i) \equiv \sum_{i=-1}^{7} L_i(q,\Theta)W_i(q,P_a,P_b)$$

  - Decomposition independent of leptonic final state
- Strategy: construct K_i such that W_i can be conveniently expanded in $q_T \ll Q$
  - Projection onto polarization vectors in vector boson rest frame [Mirkes '92]

Power corrections to fiducial q_T spectrum

- Leading power:

$$\frac{d\sigma^{(0)}}{d^4q} = \sum_{i=-1,4} L_i^{(0)} W_i^{(0)}$$

- Next-to-leading power:

$$\frac{d\sigma^{(1)}}{d^4q} = \sum_{i=-1,2,4,5} L_i^{(1)} W_i^{(0)}$$

- Linear corrections arise *entirely* from leptonic tensor

W_i	Scaling	$L_i^{(0)}$	$g_i(\theta,\varphi)$
W_{-1}	$\sim \lambda^0$	✓	$1+\cos^2\theta$
W_4	$\sim \lambda^0$	✓	$\cos\theta$
W_2	$\sim \lambda^0$		$\sin^2\theta\cos(2\varphi)$
W_5	$\sim \lambda^0$		$\sin^2\theta\sin(2\varphi)$
W_0	$\sim \lambda^2$	✓	$1-\cos^2\theta$
W_1	$\sim \lambda^1$		$\sin(2\theta)\cos\varphi$
W_3	$\sim \lambda^{\geq 1}$		$\sin\theta\cos\varphi$
W_6	$\sim \lambda^{\geq 1}$		$\sin(2\theta)\sin\varphi$
W_7	$\sim \lambda^{\geq 1}$		$\sin\theta\sin\varphi$

- $W_{2,5}$ (Boer-Mulders effect) are suppressed as $\mathcal{O}(\lambda^2)$ in collinear factorization
- $W_{-1,4}$ is standard TMD factorization:

$$W_i^{(0)} = \sum_{a,b} H_{i\,ab} \int d^2\vec{b}_T\, e^{i\vec{b}_T\cdot\vec{q}_T} f_a(x_a,\vec{b}_T) f_b(x_b,\vec{b}_T)$$

- Can extend LP factorization / resummation to capture all linear corrections:

$$\frac{d\sigma^{(0+1)}}{d^4q} = \sum_{i=-1,2,4,5} (L_i^{(0)} + L_i^{(1)}) W_i^{(0)}$$

Power corrections to fiducial q_T spectrum

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W_7	$\sim \lambda^{\geq 1}$		$\sin\theta\sin\varphi$

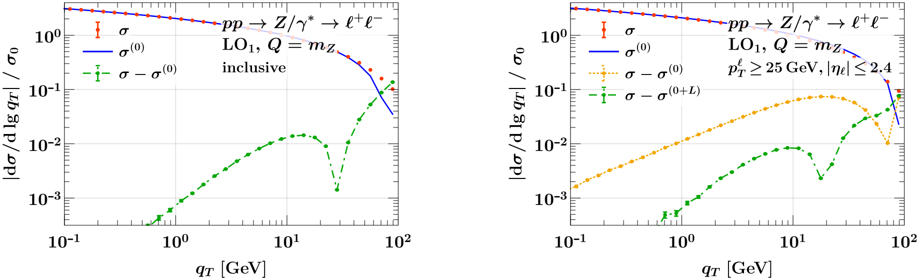
- Can extend LP factorization / resummation to capture all fiducial corrections:

$$\frac{\mathrm{d}\sigma^{(0+L)}}{\mathrm{d}^4 q} = \sum_{i=-1,2,4,5} L_i W_i^{(0)}$$

  - Exact L_i induce $\mathcal{O}(\lambda^2)$ correction depending on the tensor decomposition
  - In practice more convenient (and often more reasonable) than $\mathrm{d}\sigma^{(0+1)}$

Power corrections to fiducial q_T spectrum

Verify numerically:



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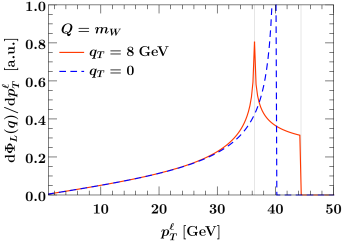
Application to leptonic observables

p_T^ℓ spectrum in $pp \to W^+ \to \ell^+ \nu_\ell$

- Naive LP factorization:

$$\frac{d\sigma^{(0)}}{d^4q\, dp_T^\ell} = \sum_{i=-1} L_i^{(0)}(q, p_T^\ell)\, W_i^{(0)}(q, P_a, P_b)$$

- LP kinematics bounds $p_T^\ell \leq Q/2$
  - $\sigma^{(0)}$ illdefined for $\left|p_T^\ell - \frac{Q}{2}\right| \lesssim \frac{q_T}{2}$



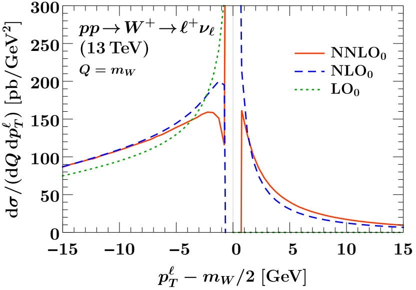
- To obtain correct singular results as $p_T^\ell \sim Q/2$: need to simultaneously expand L_i in $q_T \sim \left|p_T^\ell - Q/2\right| \ll Q$
- In practice: more convenient to keep leptonic tensor exact

$$\frac{d\sigma^{(0+L)}}{d^4q\, dp_T^\ell} = \sum_{i=-1,2,4,5} L_i W_i^{(0)}$$

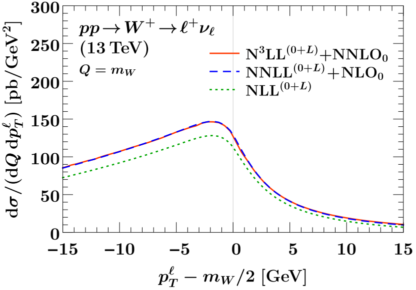
  - Avoids different expansions for $p_T^\ell \sim Q/2$ and $p_T^\ell \ll Q/2$
- Required in general when measurement induces additional small scale

p_T^ℓ spectrum with leptonic power corrections

Fixed order:



Resummed with leptonic corrections:



- Fixed order clearly breaks down at $p_T^\ell \approx m_W/2$
  - Perturbative convergence lost in peak region
- Cured by including leptonic corrections in $\dfrac{\mathrm{d}\sigma^{(0+L)}}{\mathrm{d}^4q\,\mathrm{d}p_T^\ell}$
  - Good perturbative convergence in peak region

Comparison to literature

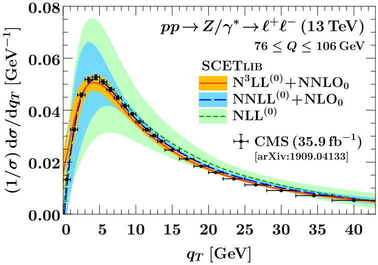
- [Balazs, Qiu, Yuan '95; Ellis, Ross, Veseli '97; Guzzi, Nadolsky, Wang '13] ($\longrightarrow$ `RESBOS`)
  [Scimemi, Vladimirov '18, Gutierrez-Reyes, Leal-Gomez, Scimemi '20] ($\longrightarrow$ `Artemide`)
  - obtain $\vec{q}_T$ and p_T^ℓ at NNLL$^{(0+L)}$ / N^{3}LL$^{(0+L)}$
  - implement exact lepton kinematics in Collins-Soper frame
  - no formal justification / discussion of ambiguities of this choice
- [Catani, de Florian, Ferrera, Grazzini '15; Camarda et al '19] ($\longrightarrow$ `DYRes,DYTurbo`)
  [Becher, Neumann' 20] (see Tobias' talk) ($\longrightarrow$ `CuTe-MCFM`)
  - obtain $\vec{q}_T$ and p_T^ℓ at NNLL$^{(0+L)}$ / N^{3}LL$^{(0+L)}$
  - boost event to split total $\vec{q}_T$ among incoming partons
  - [Catani et al '15]: ambiguity from this choice is considered a $\mathcal{O}(q_T/Q)$ effect
- [Monni, Re, Torrielli '17; Bizon et al '17 '18] ($\longrightarrow$ `RadISH`)
  - obtain $\vec{q}_T$ at N^{3}LL$^{(0)}$ without recoil
  - linear corrections from fixed-order matching (not applicable for p_T^ℓ)
- [This work] ($\longrightarrow$ `SCETlib`)
  - Formal justification of uniqueness of linear $\mathcal{O}(q_T/Q)$ corrections
  - First results of p_T^ℓ at N^{3}LL$^{(0+L)}$ including fiducial corrections

Comparison to data

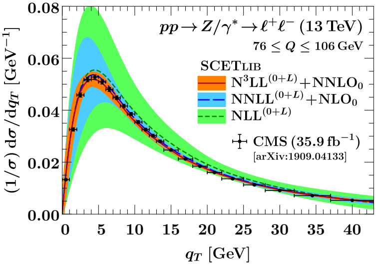
- Comparison to CMS 13 TeV measurement [1909.04133]
- See [2006.11382] for comparison to ATLAS 8 TeV measurement [1512.02192]
- See [2006.11382] for results for ϕ^*

Drell-Yan q_T spectrum

Resummed at leading power:



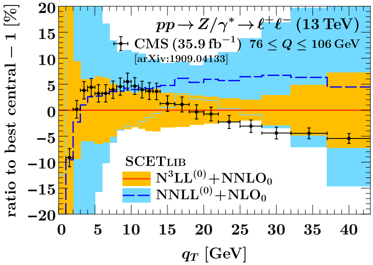
Resummed with fiducial corrections:



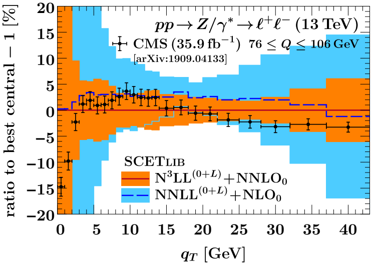
- Good agreement with data at N^{3}LL+NNLO$_0$, except in nonperturbative regime $q_T \lesssim 1$ GeV
- Perturbative uncertainties greatly reduced at higher orders
- Perturbative convergence further improves with inclusion of fiducial corrections

Drell-Yan q_T spectrum

Ratio to central $\mathrm{N^3LL}^{(0)}$:



Ratio to central $\mathrm{N^3LL}^{(0+L)}$:



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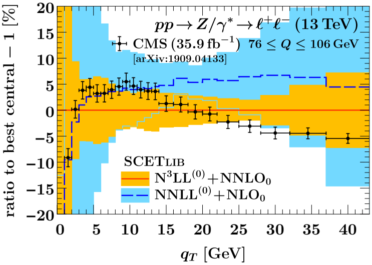

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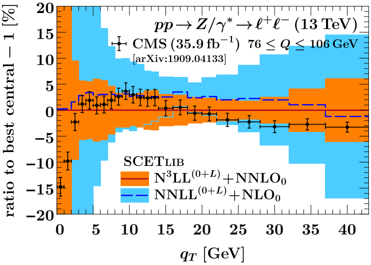
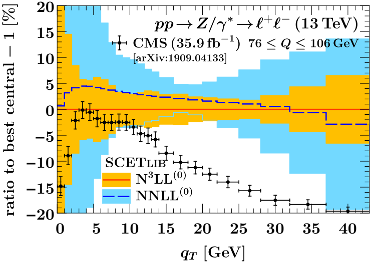


Without fixed-order matching:



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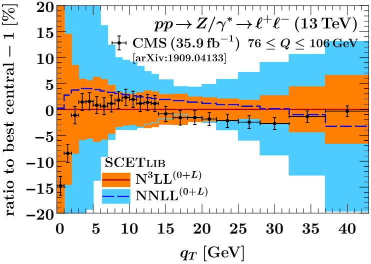

Conclusion

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- Showed that linear power corrections can be uniquely included in q_T factorization
  - Proof combines tensor decomposition in a vector boson rest frame with power counting in SCET
- Straightforward extension of LP factorization / resummation:

$$\frac{\mathrm{d}\sigma^{(0+L)}}{\mathrm{d}^4 q}(\Theta) = \sum_{i=-1,2,4,5} L_i(q,\Theta) W_i^{(0)}(q, P_a, P_b)$$

- Also holds for leptonic fiducial power corrections, such as p_T^ℓ near the peak:
  - Yields the *actual* leading power result in such singular regions
- Corollary: can be used to improve q_T subtractions ($\rightarrow$ see backup)
  - Necessary for observables such as p_T^ℓ near the peak
- Showed the numerical impact in data comparison for q_T and ϕ^*
  - Greatly improves convergence and agreement with data
  - Significantly reduces impact of fixed-order matching

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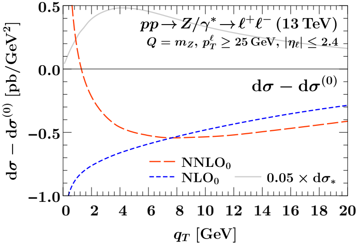
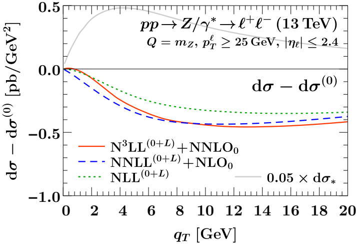
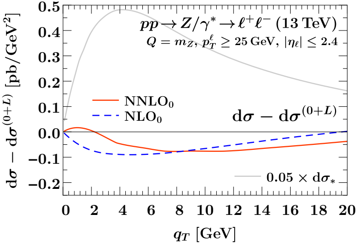
Thank you for your attention!

Backup slides

Effect of (un)resummed fiducial corrections







Hadronic tensor decomposition

- Key idea: decompose hadronic tensor using vector boson polarization [Mirkes '92]

$$W_{\lambda\lambda'} = \epsilon^{\mu}_{\lambda}\epsilon^{*\nu}_{\lambda'} W_{\mu\nu}$$

- Decomposition most natural in vector boson rest frame ($q^\mu = (Q, 0, 0, 0)$)

$$\epsilon^{\mu}_{\pm} = \frac{1}{\sqrt{2}}(x^\mu \mp i y^\mu)\,, \quad \epsilon^{\mu}_{0} = z^\mu$$

  - Our rest frame corresponds to the Collins-Soper (CS) frame

- Define hadronic tensor structures through suitable projections, e.g.

$$\begin{aligned} W_{-1} &= W_{++} + W_{--} &&= (x_\mu x_\nu + y_\mu y_\nu) W^{\mu\nu} \\ W_0 &= 2W_{00} &&= z_\mu z_\nu W^{\mu\nu} \\ &\dots \end{aligned}$$

- Can now systematically expand the W_i in $\lambda \sim q_T/Q$
  - Projectors admit straightforward expansion in q_T
  - Hadronic tensor $W_{\mu\nu}$ can be expanded in SCET

- Likewise: expand leptonic coefficients $L_i(q, \Theta) = P_i^{\mu\nu} L_{\mu\nu}(q, \Theta)$

Power expansion of the hadronic tensor

Expansion of $W_{\mu\nu}$:

- Match QCD current onto SCET: $J_V^\mu(x) \sim \gamma_\perp^\mu C_V^{(0)}(q^2)\mathcal{O}_{q\bar{q}}^{(0)}(x)\,[1 + \mathcal{O}(\lambda)]$
- Hadronic tensor at leading-power: $W^{\mu\nu} \propto g_\perp^{\mu\nu} W^{(0)}\,[1 + \mathcal{O}(\lambda)]$
- No hard operators contribute at $\mathcal{O}(\lambda)$ to inclusive spectrum
  [Feige, Kolodrubetz, Moult, Stewart '17; Moult, Stewart, Vita '19]

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Expansion of polarization vectors:

- Standard lightcone directions: $n_a^\mu = (1, 0, 0, 1) \,, \quad n_b^\mu = (1, 0, 0, -1)$
- Rest frame unit vectors:

$$x^\mu = n_\perp^\mu + \frac{q_T}{Q} \frac{n_a^\mu + n_b^\mu}{2} + \mathcal{O}(\lambda^2) \,, \quad y^\mu = \epsilon_\perp^{\mu\nu} n_{\perp\nu} \,, \quad z^\mu = \frac{n_a^\mu - n_b^\mu}{2}$$

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Expansion of structure functions:

$$W_{-1} = (x^\mu x^\nu + y^\mu y^\nu) W_{\mu\nu} = g_\perp^{\mu\nu} g_{\perp\,\mu\nu} W^{(0)} + \cdots \qquad = \mathcal{O}(\lambda^{-2})$$

$$W_0 = z^\mu z^\nu W_{\mu\nu} \qquad \sim n_{a,b}^\mu n_{a,b}^\nu g_{\perp\mu\nu} W^{(0)} + \cdots = \mathcal{O}(\lambda^0)$$

- Easy to obtain (bound on) scaling of all W_i in this fashion

Full list of hadronic structure functions

- Recall: $W_{\lambda\lambda'} = \epsilon^{\mu}_{\lambda}\epsilon^{*\nu}_{\lambda'} W_{\mu\nu}$
- Complete list of hadronic structure functions:

$$
\begin{aligned}
W_{-1} &= W_{++} + W_{--} &&= (x_\mu x_\nu + y_\mu y_\nu)\, W^{\mu\nu} \\
W_0 &= 2W_{00} &&= 2\, z_\mu z_\nu\, W^{\mu\nu} \\
W_1 &= -\frac{1}{\sqrt{2}}\big(W_{+0} + W_{0+} + W_{-0} + W_{0-}\big) &&= -(x_\mu z_\nu + x_\nu z_\mu)\, W^{\mu\nu} \\
W_2 &= -2\big(W_{+-} + W_{-+}\big) &&= 2\,(y_\mu y_\nu - x_\mu x_\nu)\, W^{\mu\nu} \\
W_3 &= -\sqrt{2}\big(W_{+0} + W_{0+} - W_{-0} - W_{0-}\big) &&= 2\mathrm{i}\,(y_\mu z_\nu - y_\nu z_\mu)\, W^{\mu\nu} \\
W_4 &= 2\big(W_{++} - W_{--}\big) &&= 2\mathrm{i}\,(x_\mu y_\nu - x_\nu y_\mu)\, W^{\mu\nu} \\
W_5 &= -\mathrm{i}\big(W_{+-} - W_{-+}\big) &&= -(x_\mu y_\nu + x_\nu y_\mu)\, W^{\mu\nu} \\
W_6 &= -\frac{\mathrm{i}}{\sqrt{2}}\big(W_{+0} - W_{0+} - W_{-0} + W_{0-}\big) &&= -(y_\mu z_\nu + y_\nu z_\mu)\, W^{\mu\nu} \\
W_7 &= -\mathrm{i}\sqrt{2}\big(W_{+0} - W_{0+} + W_{-0} - W_{0-}\big) &&= -2\mathrm{i}\,(x_\mu z_\nu - x_\nu z_\mu)\, W^{\mu\nu}
\end{aligned}
$$

Full list of hadronic structure functions

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- Complete list of hadronic structure functions:

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$$W_0 = 2\, z_\mu z_\nu\, W^{\mu\nu}$$

$$W_1 = -(x_\mu z_\nu + x_\nu z_\mu)\, W^{\mu\nu}$$

$$W_2 = 2\,(y_\mu y_\nu - x_\mu x_\nu)\, W^{\mu\nu}$$

$$W_3 = 2\mathrm{i}\,(y_\mu z_\nu - y_\nu z_\mu)\, W^{\mu\nu}$$

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W_5	$\sim \lambda^0$		$\sin^2\theta\sin(2\varphi)$
W_0	$\sim \lambda^2$	✓	$1-\cos^2\theta$
W_1	$\sim \lambda^1$		$\sin(2\theta)\cos\varphi$
W_3	$\sim \lambda^{\geq 1}$		$\sin\theta\cos\varphi$
W_6	$\sim \lambda^{\geq 1}$		$\sin(2\theta)\sin\varphi$
W_7	$\sim \lambda^{\geq 1}$		$\sin\theta\sin\varphi$

Power expansion of the leptonic tensor

- Consider $p(P_a^\mu)p(P_b^\mu) \to Z/\gamma^*(q^\mu) \to \ell^-(p_1^\mu)\ell^+(p_1^\mu)$
- Kinematic structure of leptonic tensor:

$$L_{\mu\nu}(p_1,p_2) \propto L_+(q^2)\big(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu} p_1\cdot p_2\big) + \mathrm{i}L_-(q^2)\,\epsilon^{\mu\nu\rho\sigma} p_{1\rho}p_{2\sigma}$$

  - Contains a parity-even (L_+) and parity-odd (L_-) component

- Parameterize $p^\mu_{1,2}$ in terms of CS angles θ, φ and project:

$$L_i(q,\Theta) = P_i^{\mu\nu} L_{\mu\nu} \propto L_{\pm(i)}(q^2) \int_{-1}^{1} \mathrm{d}\cos\theta \int_0^{2\pi} \mathrm{d}\varphi\, g_i(\theta,\varphi)\, \Theta(q,\theta,\varphi)$$

  - Projection onto $P_i^{\mu\nu}$ encoded in spherical harmonics $g_i(\theta,\varphi)$

- Expansion of $L_i(q,\Theta)$ in $q_T \ll Q$ depends on observable Θ
- *Often*: Θ is azimuthally symmetric at leading power

$$\Theta(q,\theta,\varphi) = \Theta^{(0)}(q,\theta)\big[1+\mathcal{O}(\lambda)\big]$$

  - All L_i except for $i = -1, 0, 4$ average out

q_T subtraction with fiducial corrections

- Inclusive cross section with cuts / observables X:

$$\sigma(X) = \sigma^{\rm sub}(X, q_T^{\rm cut}) + \int_{q_T^{\rm cut}}^{\infty} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma(X)}{\mathrm{d}q_T} + \Delta\sigma(X, q_T^{\rm cut})$$

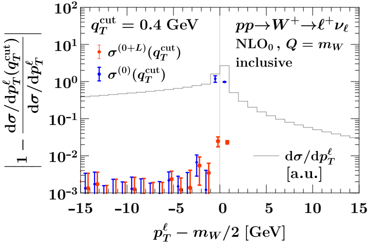
$$\Delta\sigma(X, q_T^{\rm cut}) = \int_0^{q_T^{\rm cut}} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma(X)}{\mathrm{d}q_T} - \sigma^{\rm sub}(X, q_T^{\rm cut})$$

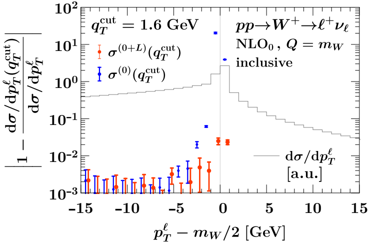
- $\sigma^{\rm sub}$ contains full singular limit as $q_T \to 0$
- Common choice: $\sigma^{\rm sub}(X, q_T^{\rm cut}) = \sigma^{(0)}(X, q_T^{\rm cut})$
  - For inclusive processes: $\Delta\sigma \sim \mathcal{O}(q_T^2/Q^2)$
  - With fiducial cuts: $\Delta\sigma \sim \mathcal{O}(q_T/Q)$
  - In singular regions of phase space: $\Delta\sigma \sim \mathcal{O}(1)$
- Enhanced corrections can be trivially avoided by choosing

$$\sigma^{\rm sub}(X, q_T^{\rm cut}) = \sigma^{(0+L)}(X, q_T^{\rm cut})$$

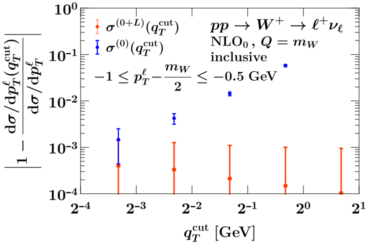
q_T subtraction for p_T^ℓ

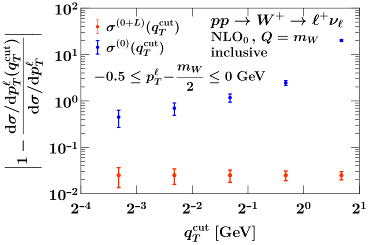
p_T^ℓ spectrum for fixed $q_T^{\rm cut}$:





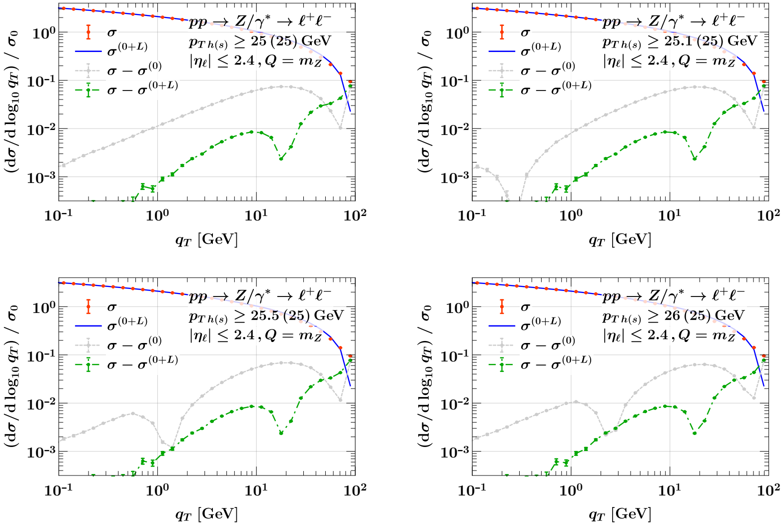
$q_T^{\rm cut}$ dependence of fixed p_T^ℓ bin:





q_T subtraction for Drell-Yan with (a)symmetric cuts

q_T spectrum for different cuts on harder (softer) lepton:



q_T subtraction for Drell-Yan with (a)symmetric cuts

$q_T^{\rm cut}$ dependence of total cross section for different cuts on harder (softer) lepton: