QCD+QED q_T factorization for W/Z production and decay.

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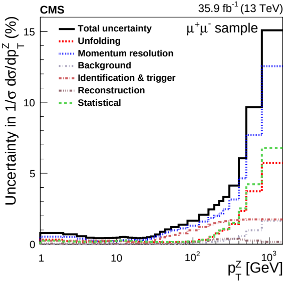
Work in collaboration with J. Michel, F. Tackmann
[in preparation]

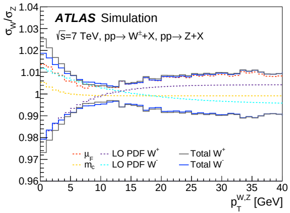
Resummation, Evolution, Factorization 2020

Motivation for $pp \to Z/W \to L+X$.

- Experimental measurement of $\mathbf{d}\boldsymbol{\sigma}/\mathbf{d}\boldsymbol{p}_T^Z$ yield $\lesssim \mathbf{0.5\%}$ uncertainty
- Thorough understanding of Z is necessary for analyses related to M_W measurement [ATLAS, 1701.07240]
- Also, small $p_T^W < 40\,\mathrm{GeV}$ region is relevant for M_W determination
- QCD resummation for p_T known to high order $\Rightarrow$ See talks by Markus and Tobias.
- *It is time* to start thinking about often neglected contributions like $\mathbf{QED}$



[CMS 1909.04133]



[ATL-PHYS-PUB-2017-021]

Overview of QED effects.

How much **QCD** + **QED** complicates things?

- Pertubative quantities $\mathcal{F} = \sum \mathcal{F}^{(n,m)} \alpha_s^n \alpha_e^m$
- Coupled β functions
- α_s, α_e & RGE running is affected [GB, F.J. Tackmann, J. Talbert; '19]

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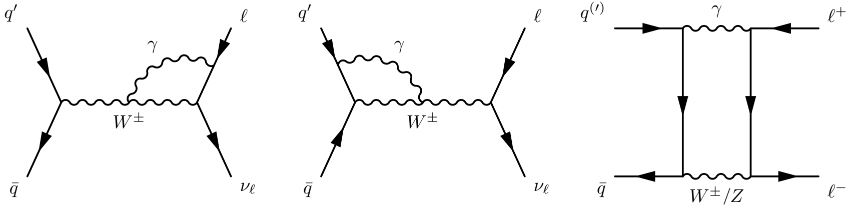
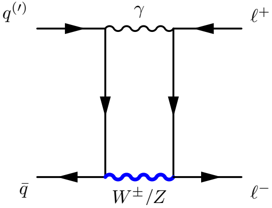
- α_s, α_e & RGE running is affected [GB, F.J. Tackmann, J. Talbert; '19]
- Quarks and leptons interact with each other via γ



Intermediate – Final (FI) **Initial – Intermediate (InI)** **Initial – Final (IFI)**

- Cannot separately investigate radiative corrections in $W_{\mu\nu}$ and $L^{\mu\nu}$
- q_T factorization is broken $\rightarrow$ is this the end of it?

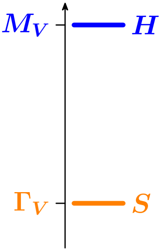
Heavy Vector Effective Theory (HVET).



- Work in the vicinity of the $V = Z, W$ resonance

$$\frac{P^2 - M_V^2}{M_V} \sim \Gamma_V \ll Q \sim M_V$$

- **Unstable particle EFT** [Beneke, Chapovsky, Signer, Zanderighi; '04]



- Hard modes $\sim \mathcal{O}(M_V)$
- Soft modes $\sim (\Gamma_V, \Gamma_V, \Gamma_V)$
- Collinear modes $\sim (M_V, \Gamma_V, \sqrt{M_V \Gamma_V})$

- Like HQET but with a heavy vector boson V_v^μ instead of Q_v

Heavy Vector Effective Theory (HVET).

Decompose vector boson momenta in label and residual

$$P^\mu = M_V v^\mu + k^\mu$$

$$\frac{1}{P^2 - M_V^2 + \mathrm{i} M_V \Gamma_V} \sim \frac{1}{2M_V} \frac{1}{v \cdot k + \mathrm{i}\Gamma_V/2}$$

- Interactions with only $k \sim \mathcal{O}(\Gamma_V)$ keep V near mass shell

$$\mathcal{L}_{\text{HVET}} = 2M_V V_v^{\mu\dagger} \left(v \cdot D_s - \frac{\Delta}{2} \right) V_\mu^v$$

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{HVET}} + \mathcal{L}_s^{\text{QED}} + \mathcal{L}_{\text{SCET}}$$

$$V_\mu^v(x) = e^{\mathrm{i} M_V v \cdot x} (-g_{\mu\nu} + v_\mu v_\nu) V^\nu(x)$$

- $\mathrm{Im}\Delta = -\Gamma_V$ but otherwise equivalent to the residual mass in HQET

Protofactorization.

- Full theory considerations for a generic measurement $\mathcal{O}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{O}} \sim \sum_X \int \mathrm{d}\Phi_{L,X} \int \mathrm{d}^4x\,\mathrm{d}^4y\;\delta^{(4)}(p_a + p_b - p_1 - p_2 - p_X)$$

$$\times \langle pp|\bar{\mathrm{T}}[J_H^\dagger(0)J_L(y)]\hat{\mathcal{O}}|LX\rangle\langle LX|\mathrm{T}[J_L^\dagger(x)J_H(0)]|pp\rangle$$

$$\sim \int \mathrm{d}^4x\,\mathrm{d}^4y\,\mathrm{d}^4z\,\langle pp|\bar{\mathrm{T}}[J_H^\dagger(z)J_L(y+z)]\hat{\mathcal{O}}\mathrm{T}[J_L^\dagger(x)J_H(0)]|pp\rangle$$

! Cannot disentangle J_H and J_L

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! Cannot disentangle J_H and J_L

- Match $J_{H,L}$ to collinear quarks & leptons

$$J_{H,L}(x) \to \sum_{\{n_i\}} \int \{\mathrm{d}\omega_i\}\, C_{H,L}(\{\omega_i\})\, e^{-\mathrm{i}\mathcal{P}\cdot x}\left[V_\mu^{v\dagger} \bar{\chi}_{n_i,-\omega_i} \Gamma^\mu \chi_{n_j,\omega_j}\right](x)$$

Follow the standard procedure

→ Recombine label & residual momenta

→ Get explicit phases for residual momenta that couldn't be recombined

→ $\int \mathrm{d}^4z \to \delta^{(4)}(\tilde{p}_a + \tilde{p}_b - \tilde{p}_1 - \tilde{p}_2)$ label momentum conservation

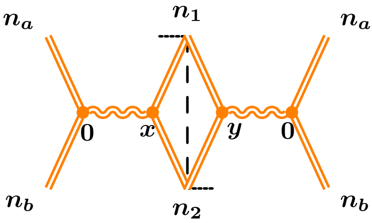
Protofactorization.

- EFT cross-section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{O}} \sim \int \mathrm{d}\omega_{a,b} \int \mathrm{d}^4 p_{1,2}\, \delta^{(4)}(\tilde{p}_a + \tilde{p}_b - \tilde{p}_1 - \tilde{p}_2)$$
$$\times H^{\mu\nu}(\omega_{a,b}) L_{\mu\nu}(\omega_{1,2}) B_{n_a}(x_a) B_{n_b}(x_b)$$
$$\times \int \mathrm{d}^4x\, \mathrm{d}^4y\, J_{n_1}(p_1) J_{n_2}(p_2) e^{\mathrm{i}(p_1+p_2-Mv)\cdot(x-y)}$$
$$\times \left\langle [\mathcal{S}^\dagger_{ab} V_v](0) [V^\dagger_v \mathcal{S}^\dagger_{12}](y) \hat{\mathcal{O}}_s [\mathcal{S}_{12} V_v](x) [V^\dagger_v \mathcal{S}_{ab}](0) \right\rangle$$

With $\mathcal{S}_{ij} = Y^\dagger_{n_j} Y_{n_i}$

- Hadronic $H(\omega_{a,b}) = |C_H|^2$
- Leptonic $L(\omega_{1,2}) = |C_L|^2$
- No hard matching coeff. that 'sees' all 4 legs
- Inclusive lepton jet functions
- Non-local soft function
  - (n_a, n_b, n_1, n_2) Wilson lines
  - Connected with HVET Wilson lines



Protofactorization.

- EFT cross-section

$$\frac{d\sigma}{d\mathcal{O}} \sim \int d\omega_{a,b}\, \delta^{(4)}(\tilde{p}_a + \tilde{p}_b - \tilde{p}_1 - \tilde{p}_2)$$
$$\times H^{\mu\nu}(\omega_{a,b}) L_{\mu\nu}(\omega_{1,2}) B_{n_a}(x_a) B_{n_b}(x_b)$$
$$\times \int d^4r\, d^4p_{1,2}\;\; S(r) J_{n_1}(p_1) J_{n_2}(p_2) \delta^{(4)}(\underbrace{p_1 + p_2 - Mv}_{\mathcal{O}(\Gamma_V)} - r)$$

- Convolution between soft & jet function

  Anticipated by [Beneke, Chapovsky, Signer, Zanderighi; '04]

$\rightarrow$ Final state radiation deforms the line-shape

$$S(r) = \begin{cases} \text{Contains the line shape} \\ \text{Encodes soft radiation off the resonance} \\ \text{Captures IFI \& FSR radiation} \end{cases}$$

$$S^{(0)}(r) = \left| \frac{1}{v \cdot r - \Delta/2} \right|^2 \rightarrow \text{Not a } \delta(r) \text{ at LO!}$$

- Consistency on the decay side suggests that $Z_s = Z_s^{\text{thrust}}$!
- Checked at $\mathcal{O}(\alpha_e)$ ✔

Lepton & Measurement definitions.

Typical lepton definition used by expreriments

- **Bare:** after QED radiation
- **Born:** before (inclusive over) QED radiation
- **Dressed:** clustering of QED radiation with a cone/jet algorithm

$\rightarrow$ Define suitable Born leptons

$$p_i^\mu = \bar{n}_i \cdot \Big[\omega_i \tfrac{n_i}{2} + k_i^+ \tfrac{\bar{n}_i}{2} + \underbrace{\sum_j \Theta_{ij} l_{s,j}}_{l_1^-, l_2^-}\Big] \frac{n_i^\mu}{2}$$

Measure $\mathcal{O} = \{Q^2, Y, \Delta y\} \equiv \Phi_L$

- Close to resonance Q^2 is constrained $\rightarrow$ sensitivity to residual momenta
- Otherwise $\{Y, \Delta y\}$ are unconstrained $\rightarrow$ defined by Born level kinematics

$$\delta\big(Q^2 - (p_1 + p_2)^2\big)\, \delta\Big(Y - \tfrac{1}{2}\ln\big[\tfrac{n_b\cdot(p_1+p_2)}{n_a\cdot(p_1+p_2)}\big]\Big)$$
$$= \delta\big(\omega_{a,b} - e^{\pm Y}(Q - l)\big)\Big[1 + \mathcal{O}\big(\tfrac{\Gamma_V}{Q}\big)\Big]$$

with $\quad l = l_1^- \frac{v\cdot n_1}{2} + l_2^- \frac{v\cdot n_2}{2} \sim \mathcal{O}(\Gamma_V)$

EFT cross section.

- The EFT cross section now looks like this

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_L} \sim H^{\mu\nu}(Q) L_{\mu\nu}(Q) B_{n_a}\Big(\frac{Qe^{Y}}{E_{\rm cm}}\Big) B_{n_b}\Big(\frac{Qe^{-Y}}{E_{\rm cm}}\Big)$$
$$\times \int \mathrm{d}(v\cdot r)\, \mathrm{d}l_{1,2}^{-} \int \mathrm{d}k_{1,2}^{+}\; S\big(v\cdot r, l_1^-, l_2^-\big) J_{n_1}(\omega_1 k_1^+) J_{n_2}(\omega_2 k_2^+)$$
$$\times\, \delta\Big(\underbrace{Q - M_V}_{\mathcal{O}(\Gamma_V)} - k_1^+ - k_2^+ - \frac{l_1^-}{2} - \frac{l_2^-}{2} - v\cdot r\Big)$$

  - The fact that Q is *restricted* to be near M_V induces an $\mathcal{O}(\Gamma_V)$ sensitivity which modifies the soft function
  - Anywhere else (B, J, H, L) expand l away
  - Physical picture is that the parts of the radiation recovered by clustering with the leptons *should* be 'known' to the propagator
  - Define a new soft function

$$S(v\cdot\bar{r}) \equiv \int \mathrm{d}(v\cdot r)\, \mathrm{d}l_{1,2}^{-}\; \delta\Big((v\cdot\bar{r}) - (v\cdot r) - \frac{l_1^-}{2} - \frac{l_2^-}{2}\Big)\, S(v\cdot r, l_1^-, l_2^-)$$

EFT cross section.

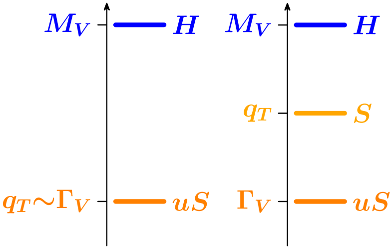
- Putting everything together and setting $\Delta Q \equiv Q - M_V$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_L} \sim H^{\mu\nu}(Q) L_{\mu\nu}(Q)\, B_{n_a}\Big(\frac{Qe^{Y}}{E_{\mathrm{cm}}}\Big) B_{n_b}\Big(\frac{Qe^{-Y}}{E_{\mathrm{cm}}}\Big) \times \int \mathrm{d}k_1^+\, \mathrm{d}k_2^+\; S(\Delta Q - k_1^+ - k_2^+) J_{n_1}(\omega_1 k_1^+) J_{n_2}(\omega_2 k_2^+)$$

  - Since J_{n_1}, J_{n_2} and $L_{\mu\nu}$ didn't change, then $S(v \cdot \bar{r})$ must renormalize same way as $S(v \cdot r)$
  - Therefore, also $S(v \cdot \bar{r})$ renormalizes like thrust!
  - Explicit check at $\mathcal{O}(\alpha_e)$ ✔
  - Note that $S(v \cdot \bar{r})$, $S(v \cdot r)$ differ in their finite pieces
  - IFI/FI/InI/II/ISR: $R + V = \mathcal{O}(\epsilon^0) \Rightarrow \mathrm{NLL}'$ effects

Measuring q_T: Regimes, Modes & Measurement.

- Let's measure in addition q_T



- **Regime I**: $q_T \sim \Gamma_V \ll M_V \sim Q$
  - uSofts $(\Gamma_V, \Gamma_V, \Gamma_V) \sim (q_T, q_T, q_T)$

- **Regime II**: $\Gamma_V \ll q_T \ll M_V \sim Q$
  - Softs $\sim (q_T, q_T, q_T)$
  - uSofts $\sim (\Gamma_V, \Gamma_V, \Gamma_V)$

! In Regime II only uSoft interactions keep the V_v^μ near its mass shell

Measuring q_T: Regimes, Modes & Measurement.

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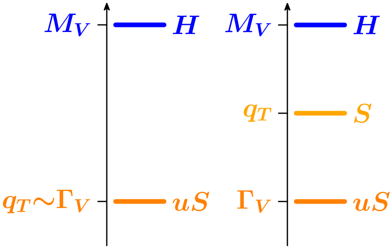

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  - uSofts $\sim (\Gamma_V, \Gamma_V, \Gamma_V)$

! In Regime II only uSoft interactions keep the V_v^μ near its mass shell

- Complete measurement

$$\begin{aligned}\mathcal{O} &= \Theta_1\big[\delta(l_1^- - k_1^-)\theta(k_2^- - k_1^-)\delta(l_2^-)\delta^{(2)}(q_T)\big] + (1 \leftrightarrow 2)\\ &\quad + \Theta_{ab}\big[\delta(l_1^-)\delta(l_2^-)\delta^{(2)}(q_T - k_T)\big]\\ &\text{with} \quad \Theta_{ab} + \Theta_1 + \Theta_2 = 1\end{aligned}$$

$\rightarrow$ Added Θ_{ab} region to cover the whole phase space

Factorization in **Regime 1**: $\Gamma_V \sim q_T \ll M_V$.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_L \mathrm{d}q_T} \sim H_{\mu\nu}(Q) L^{\mu\nu}(Q) \int \mathrm{d}^2 k_{T_a}\, \mathrm{d}^2 k_{T_b}\, \mathrm{d}k_1^+ \mathrm{d}k_2^+$$
$$\times B(k_{T_a}, \omega_a) B(k_{T_b}, \omega_b)\, J_{n_1}\left(\omega_1 k_1^+\right) J_{n_2}\left(\omega_2 k_2^+\right)$$
$$\times S_I\left(q_T - k_{T_a} - k_{T_b}, \Delta Q - k_1^+ - k_2^+\right)$$

- Production: q_T convolution between S_I and q_T beam func'
- Decay: ΔQ convolution between S_I and inclusive jet func'

For the Z:

- Consistency on the decay side still holds as previously (thrust-like)
- Consistency on the production side is like $2 \to 0$ QCD Drell-Yan
- IFI don't contribute to the pole structure

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_L \mathrm{d}q_T} \sim H_{\mu\nu}(Q) L^{\mu\nu}(Q) \int \mathrm{d}^2 k_{T_a}\, \mathrm{d}^2 k_{T_b}\, \mathrm{d}k_1^+ \mathrm{d}k_2^+$$
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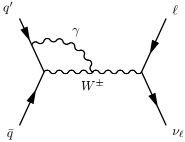
For the W:

- IFI/FI don't contribute to the pole structure
- Naively we would expect (abelianization) that $\Gamma_0 \sim Q_q Q_{q'}$
- But InI modify it with an extra term $\rightarrow \Gamma_0 \sim Q_q^2 + Q_{q'}^2$
- Also rapidity anomalous dim. get modified $\gamma_\nu^0, \Gamma_0 \sim Q_q^2 + Q_{q'}^2$
- Dictated from collinear gauge invariance, nontrivial!

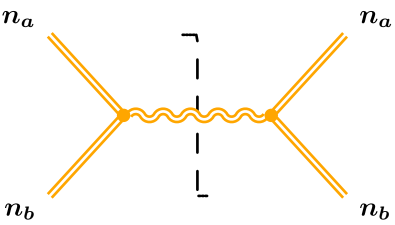


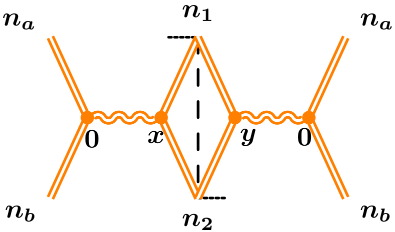
Factorization in **Regime 2**: $\Gamma_V \ll q_T \ll M_V$.

- Assume the stronger limit $\Gamma_V \ll q_T$ and use that hierarchy to re-factorize
- The requirement to be around the peak $Q \sim M_V$ still holds

This implies that at the scale q_T

$$S_I(q_T, \bar{r}) = C_{II}(q_T) \times S_{II}(\bar{r}) \Big[1 + \mathcal{O}(\tfrac{\Gamma_V}{q_T}) \Big]$$





- **Physical picture?**
  Since softs $\sim \mathcal{O}(q_T)$ take V_v^μ far off shell, it can only interact with usofts $\sim \mathcal{O}(\Gamma_V)$ therefore any radiation on the decay side is power suppressed

Factorization in **Regime 2**: $\Gamma_V \ll q_T \ll M_V$.

The EFT cross-section in Regime 2 $\sim \boldsymbol{\Gamma_V} \ll \boldsymbol{q_T} \ll \boldsymbol{M_V} \sim \boldsymbol{Q}$

$$\frac{\mathrm{d}\boldsymbol{\sigma}}{\mathrm{d}\boldsymbol{\Phi_L}\mathrm{d}\boldsymbol{q_T}} = \boldsymbol{W(Q, Y, q_T)L(Y, \Delta y, \Delta Q)}$$

with

$$\boldsymbol{W} = \boldsymbol{H(Q)} \int \mathrm{d}^2\boldsymbol{k_{T_a}}\mathrm{d}^2\boldsymbol{k_{T_b}}\, \boldsymbol{C_{II}(q_T - k_{T_a} - k_{T_b})B_{n_a}(k_{T_a}, \omega_a)B_{n_b}(k_{T_b}, \omega_b)}$$

$$\boldsymbol{L} = \boldsymbol{H_L(Q)} \int \mathrm{d}\boldsymbol{k_1^+}\mathrm{d}\boldsymbol{k_2^+}\, \boldsymbol{S_{II}(\Delta Q - k_1^+ - k_2^+)J_{n_1}(\omega_1 k_1^+)J_{n_2}(\omega_2 k_2^+)}$$

- $\boldsymbol{C_{II}}$ is a Wilson coeff. at the scale $\boldsymbol{q_T}$
  - describes soft ISR and InI/II that contributes to $\boldsymbol{q_T}$
  - For $\boldsymbol{W}$: Not the usual $S_{\mathrm{DY}} \Rightarrow$ three-prong soft function
  - IFI/FI/FSR are power suppressed
- $\boldsymbol{S_{II}}$ is the same as in ‘protofactorization’ ($\boldsymbol{S(v \cdot \bar{r})}$)
  - contains the line shape and describes usoft interactions with $\boldsymbol{V_v}$
  - convoluted with lepton jet functions $\Rightarrow$ FSR modifies line shape

Factorization in **Regime 2**: $\Gamma_V \ll q_T \ll M_V$.

The EFT cross-section in Regime 2 $\sim \Gamma_V \ll q_T \ll M_V \sim Q$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_L \mathrm{d}q_T} = W(Q, Y, q_T) L(Y, \Delta y, \Delta Q)$$

with

$$W = H(Q) \int \mathrm{d}^2 k_{T_a} \mathrm{d}^2 k_{T_b}\, C_{II}(q_T - k_{T_a} - k_{T_b}) B_{n_a}(k_{T_a}, \omega_a) B_{n_b}(k_{T_b}, \omega_b)$$

$$L = H_L(Q) \int \mathrm{d}k_1^+ \mathrm{d}k_2^+\, S_{II}(\Delta Q - k_1^+ - k_2^+) J_{n_1}\left(\omega_1 k_1^+\right) J_{n_2}\left(\omega_2 k_2^+\right)$$

What about consistency?

- From Regime I to Regime II, only $S_I = C_{II} \times S_{II}$ changed
- That implies that the poles of C_{II} and S_{II} are those of S_I
- Consistency works separately for W and L

Recap.

We saw...

- A QCD-QED factorization for $\dfrac{d\sigma}{d\Phi_L dq_T}$ for explicit production and decay
  - **Regime 1:** $\Gamma_V \sim q_T \ll M_V$
    - Non local soft function $S(k_T, v \cdot \bar{r})$ that contains the line shape
    - convoluted on the production side with q_T beam functions
    - and on the decay side with inclusive lepton jet functions (Born leptons) $\Rightarrow$ line shape modification!
  - **Regime 2:** $\Gamma_V \ll q_T \ll M_V$
    - We recover q_T factorization!
    - W captures all the q_T dependence mainly from ISR
    - L contains V^μ line-shape and decay captures FSR
    - $W^\pm$ RGE running is modified by $\gamma_\nu^0, \Gamma_0 \sim Q_q^2 + Q_{q'}^2$

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Thank you!