Quark and Gluon quasi-PDFs at low-x

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Outline

- Definition of quasi-pdfs;
- Light-ray operator with point-splitting: quasi-pdf frame;
- brief review of high-energy OPE;
- Results in the saddle point approximation;
- Results in the leading twist approximation;
- Conclusions.

Light-cone gluon and quark distributions are obtained from

light-cone gluon operator

$$\langle P|G^{a\,i-}(x^+)[nx^+,0]^{ab}G^{b\,i-}(0)|P\rangle$$

light-cone quark operator

$$\langle P|\bar{\psi}(x^+)\gamma^-[nx^+,0]\psi(0)|P\rangle$$

$$x^\mu = n^\mu x^+ + n'^\mu x^- + x_\perp^\mu\,, \qquad x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}}\,, \qquad x_\mu n^\mu = x^- \qquad x_\mu n'^\mu = x^+$$

$$P^- \gg 1$$

quasi-pdf

quasi-pdf as introduced by X. Ji in 2013; Motivation: study pdf on the lattice.

gluon quasi-distribution

$$g(x_B, \mu^2, P) = \int \frac{dx}{4\pi x_B P}\, e^{i\, x_B P\cdot x} \langle P| G^{3\mu}(x) \exp\left(-ig \int_0^x dx' A^3(x')\right) G_\mu{}^3(0)|P\rangle$$

quark quasi-distribution

$$q(x_B, \mu^2, P) = \int \frac{dx}{4\pi}\, e^{i\, x_B P\cdot x} \langle P| \bar{\psi}(x)\gamma^3 \exp\left(-ig \int_0^x dx' A^3(x')\right) \psi(0)|P\rangle$$

quasi-distribution studied by several groups

X. Xiong, X. Ji, J. H. Zhang and Y. Zhao (2014);

T. Izubuchi, X. Ji, L. Jin, I. W. Stewart and Y. Zhao (2018);

W. Wang, S. Zhao, R. Zhu (2018); W. Wang, J. H. Zhang, S. Zhao, R. Zhu (2019);

I. Balitsky, W. Morris, A. Radyuskin (2020);

. . .

quasi-pdf

gluon quasi-distribution

$$g(x_B, \mu^2, P) = \int \frac{dx}{4\pi x_B P} \, e^{i\, x_B P x} \langle P | G^{3\mu}(x) \exp\left(-ig \int_0^x dx' A^3(x') \right) G_\mu{}^3(0) | P \rangle$$

quark quasi-distribution

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We will calculate the low-x behavior of the quasi-pdfs in coordinate space in two ways

- saddle point approximation
- leading twist approximation

Tensor decomposition over invariant amplitudes of the gluon matrix element

$$\begin{aligned} M_{\mu\alpha;\lambda\beta} &\equiv \langle P|G_{\mu\alpha}(x)[x,0]G_{\lambda\beta}(0)|P\rangle \\ &= I_{1\mu\alpha;\lambda\beta}\mathcal{M}_{pp} + I_{2\mu\alpha;\lambda\beta}\mathcal{M}_{zz} + I_{3\mu\alpha;\lambda\beta}\mathcal{M}_{zp} \\ &\quad + I_{4\mu\alpha;\lambda\beta}\mathcal{M}_{pz} + I_{5\mu\alpha;\lambda\beta}\mathcal{M}_{ppzz} + I_{6\mu\alpha;\lambda\beta}\mathcal{M}_{gg} \end{aligned}$$

the amplitudes $\mathcal{M}$ are functions of the invariants x^2 and $x \cdot P$

light-cone Gluon distribution is obtained from

$$g_{\perp}^{\alpha\beta} M_{+\alpha;\beta_+}(x^+, P) = -2(P^-)^2 \mathcal{M}pp$$

$$M_{+i;+i} = M_{0i;0i} + M_{3i;3i} + M_{0i;3i} + M_{3i;0i}$$

$$M_{0i;i0} + M_{ji;ij} = 2p_0^2 \mathcal{M}_{pp}$$

At high energy (Regge limit) the transverse components are suppresed and we do not distinguish between the 0-component and the 3-component

From local operators to Light-ray operators

twist-two gluon operator

$$\mathcal{O}_F^j = F^a_{\mu_+} \nabla_+^{j-2} F_+^{\mu a}$$

anomalous dimension is singular at $j = 1$

$j \to 1 \quad \Leftrightarrow \quad x_B \to 0;$ at $\frac{\alpha_s}{j-1} \sim 1 \quad \Rightarrow$ resummation: BFKL eq.

So, near $j = 1$ we have the anomalous dimension to all orders via BFKL pomeron.

$j = 1$ is called *unphysical point*, the operator becomes non local and BFKL provides the analytic continuation of the anomalous dimension at this point.

From local operators to Light-ray operators

DIS at high-energy $\quad -q^2 = Q^2 \gg P^2 \qquad s = (P+q)^2 \gg Q^2$

$$\sigma^{\gamma^* p}(x_B, Q^2) = \int d\nu\, F(\nu)\, x_B^{-\aleph(\nu)-1} \left(\frac{Q^2}{P^2}\right)^{\frac{1}{2}+i\nu}$$

in DIS, the n-th moment of the structure function is

$$M_n = \int_0^1 dx_B\, x_B^{n-1} \sigma^{\gamma^* p}(x_B, Q^2) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} d\gamma\, \frac{F(\gamma)}{n-1-\aleph(\gamma)} \left(\frac{Q^2}{P^2}\right)^{\gamma}$$

$\aleph(\gamma)$ is the BFKL pomeron intercept.

Closing the contour on the poles we get the anomalous dimensions of the leading and higher twist operators at the *unphysical point*.

- For our task we could repeat the same steps as it is done in DIS.
- But we need the explicit form of the twist-two operator at the unphysical point: $F^a_{\mu+} \nabla_+^{-1} F_+^{\mu a}$

$\Rightarrow$ light-ray operator with point-splitting

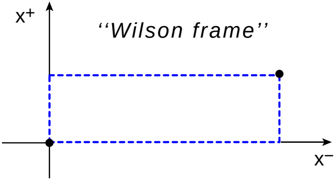
Balitsky(2013); Balitsky, Kazakov, Sobko (2013-2015)

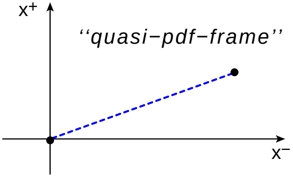
analytic continuation of local operator to light-ray operators
are singular in the BFKL approximation

$\Rightarrow$ analytic continuation of local operator to
light-ray operators with point-splitting

Wilson frame: gluon operator Balitsky(2013); Balitsky, Kazakov, Sobko (2013-2015)

quasi-pdf frame: this work (quark and gluon operators)





Light-ray operators with point-splitting quasi-pdf frame

gauge link: $[x,y] = \mathrm{Pexp}\left\{ig\int_0^1 du\,(x-y)_\mu A^\mu(xu+(1-u)y)\right\}$

Gluon $\mathcal{F}_n^j(x_\perp,y_\perp) \equiv \int_0^{+\infty} dx^+\,(x^+)^{1-j}\,\mathcal{F}_n(x^+;x_\perp,y_\perp)$

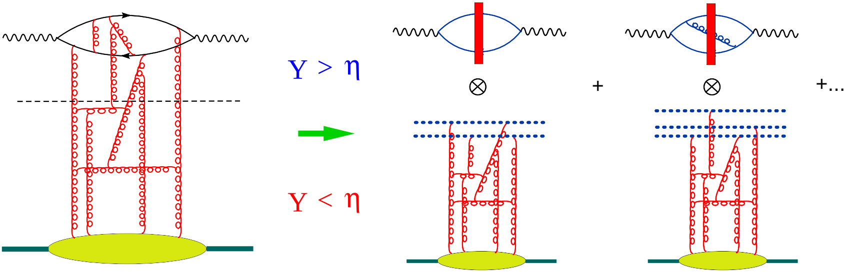
$$\mathcal{F}_n(x^+;x_\perp,y_\perp) \equiv \int dy^+ G^{a\,i-}(nx^+ + ny^+ + x_\perp)[nx^+ + ny^+ + x_\perp, ny^+ + y_\perp]^{ab} G^{b\,i-}(ny^+ + y_\perp)$$

Quark $\mathcal{Q}_n^j(x_\perp,y_\perp) \equiv \int_0^{+\infty} dx^+\,(x^+)^{-j}\,\mathcal{Q}_n(x^+;x_\perp,y_\perp)$

$$\mathcal{Q}_n(x^+;x_\perp,y_\perp) \equiv \int dx^+ \bar\psi(nx^+ + ny^+ + x_\perp)\gamma^-[nx^+ + ny^+ + x_\perp, ny^+ + y_\perp]\psi(ny^+ + y_\perp)$$

$$x^\mu = x^+ n^\mu + x^- n'^\nu + x_\perp^\mu$$

remind: High-energy OPE for DIS

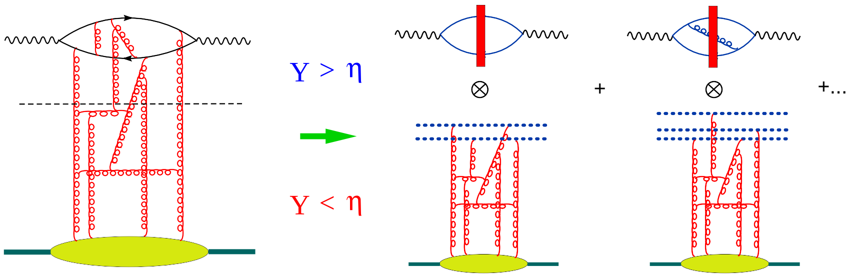


factorization scale: rapidity η

Rapidity Y > η - coefficient function ("impact factor")

Rapidity Y < η - matrix elements of (light-like) Wilson lines with rapidity divergence cut by η

$$U_x^\eta = \text{Pexp}\Big[ig\int_{-\infty}^{\infty} dx^+ A_+^\eta(x_+, x_\perp)\Big]$$



The high-energy operator product expansion is

$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2 \; I^{\mathrm{LO}}_{\mu\nu}(z_1,z_2,x,y)\mathrm{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$
$$+ \int d^2z_1 d^2z_2 d^2z_3 \; I^{\mathrm{NLO}}_{\mu\nu}(z_1,z_2,z_3,x,y)[\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}\mathrm{tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - N_c\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]$$

DIS at Leading Log Approximation at high-energy

$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2 \; I^{\rm LO}_{\mu\nu}(z_1, z_2, x, y){\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$

- Calculate LO Imapct factor: $I^{\rm LO}_{\mu\nu}(z_1, z_2, x, y)$
- Calculate evolution of matrix element ${\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$: BK/JIMWLK equation
  - we need only linear terms: BFKL
- Solve the evolution equation with initial condition: GBW/MV model
- Convolute the solution of the evolution equation with the impact factor

High-energy OPE for quasi-pdf-frame operators

$$\langle P|G^{a\,i-}(x)[x,0]G^{b,i-}(0)|P\rangle = \int d^2z_2 d^2z_z\, I_g(z_1,z_2;x)\langle P|\mathrm{Tr}\{U(z_1)U^\dagger(z_2)\}|P\rangle$$

$$\langle P|\bar\psi(x)\gamma^-[x,0]\psi(0)|P\rangle = \int d^2z_2 d^2z_z\, I_q(z_1,z_2;x)\langle P|\mathrm{tr}\{U(z_1)U^\dagger(z_2)\}|P\rangle$$

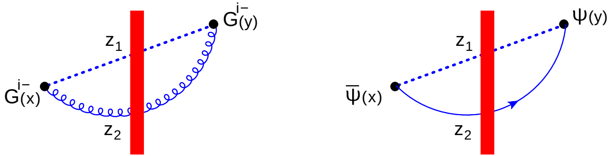
- Calculate coefficient functions (impact factors) I_g and I_q
- Convolute them with the solution of the evolution equation of relative matrix elements

High-energy OPE for quasi-pdf-frame operators

$$\langle P|G^{a\,i-}(x)[x,0]G^{b,i-}(0)|P\rangle = \int d^2z_2 d^2z_z\, I_g(z_1,z_2;x)\langle P|\mathrm{Tr}\{U(z_1)U^\dagger(z_2)\}|P\rangle$$

$$\langle P|\bar\psi(x)\gamma^-[x,0]\psi(0)|P\rangle = \int d^2z_2 d^2z_z\, I_q(z_1,z_2;x)\langle P|\mathrm{tr}\{U(z_1)U^\dagger(z_2)\}|P\rangle$$

Diagrams for the gluon impact factor I_g and quark impact factor I_q respectively



- Gluon: Tr trace in the adjoint representation;
- Quark: tr trace in the fundamental representation.

Linear evolution equation: BFKL equation

$\mathcal{U}(z_{12}) = 1 - \frac{1}{N_c}\mathrm{tr}\{U(z_{1\perp})U^\dagger(z_{2\perp})\}$ $\quad\frac{1}{z_{12}^2}\mathcal{U}^a(z_{12}) \equiv \mathcal{V}^a(z_{12})$ $\quad z_{12} = |z_{1\perp} - z_{2\perp}|$

$$2a\frac{d}{da}\,\mathcal{V}_a(z) = \frac{\alpha_s N_c}{\pi^2}\int d^2z'\Big[\frac{\mathcal{V}_a(z')}{(z-z')^2} - \frac{(z,z')\mathcal{V}_a(z)}{z'^2(z-z')^2}\Big]$$

a rapidity cut-off in coordinate space.

Linear evolution equation: BFKL equation

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a rapidity cut-off in coordinate space.

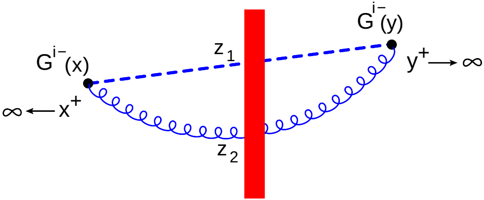
Solution

$$\mathcal{V}^a(z_{12}) = \int \frac{d\nu}{2\pi^2}\,(z_{12}^2)^{-\frac{1}{2}+i\nu}\left(\frac{a}{a_0}\right)^{\frac{\alpha_s N_c}{2\pi}\chi(\nu)}\int d^2\omega(\omega_\perp^2)^{-\frac{1}{2}-i\nu}\mathcal{V}^{a_0}(\omega_\perp)$$

$$\chi(\nu) = 2\psi(1) - \psi(\frac{1}{2}+i\nu) - \psi(\frac{1}{2}-i\nu)$$

initial condition $\mathcal{V}^{a_0}(\omega_\perp)$: GBW/MV model

Q_s saturation scale: initial condition for the high-energy evolution.



DIS at high-energy

$$Q \text{ fixed, } s \to \infty \quad \Rightarrow \quad x_B \sim \frac{Q^2}{s} \to 0$$

coordinate space

$$|x-y|_\perp \quad \text{fixed,} \quad x^+, y^+ \to \infty$$

coordinate space rapidity cut-off: $a = \frac{2x^+y^+}{(x-y)_\perp^2}$

I. Balitsky, G.A.C. (2009)

Results in the saddle point approximation

$$n^\mu = \frac{x^\mu}{|x|}\,, \qquad \bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$$

Gluon

$$n_\mu n_\nu \langle P|G^{a\alpha\mu}(x)[x,0]^{ab}G^{b\beta\nu}(0)|P\rangle$$
$$= g_\perp^{\alpha\beta}\frac{3N_c^2}{128}\frac{Q_s\sigma_0}{|x|}\frac{\exp\left\{-\frac{\ln^2 Q_s|x|}{14\zeta(3)\bar{\alpha}_s\ln(x\cdot P)}\right\}}{\sqrt{14\zeta(3)\bar{\alpha}_s\ln(x\cdot P)}}\,(x\cdot P)^{\bar{\alpha}_s\,4\ln 2}$$

Quark

$$n_\mu\langle P|\bar{\psi}(x)\gamma^\mu[x,0]\psi(0)|P\rangle$$
$$= \frac{i\,N_c}{32}Q_s\sigma_0\frac{\exp\left\{-\frac{\ln^2 Q_s|x|}{14\zeta(3)\bar{\alpha}_s\ln(x\cdot P)}\right\}}{\sqrt{14\zeta(3)\bar{\alpha}_s\ln(x\cdot P)}}\,(x\cdot P)^{\bar{\alpha}_s 4\ln 2}$$

Usual exponential growth of high-energy evolution

Best check: correlation functions in CFT

Supersymmetric light-ray operator $S^j(x_\perp)$ (of spin j) are analytic continuation of local operators

$$\langle S^j(z_{1\perp})S^{j'}(z_{2\perp})\rangle = \delta(j-j')\frac{C(j,\Delta)s^{j-1}}{[(z_{1\perp}-z_{2\perp})^2]^{\Delta-1}}\mu^{-2\gamma_{\rm an}}$$

Δ dimension of the operator; s is Mandelstam variable.
μ is the renormalization point; $\gamma_{\rm an}$ anomalous dimension.

correlation function for Gluon: in the limit $\alpha_s \ll \omega = j-1 \ll 1$ *quasi-pdf frame* agrees with *Wilson frame* which agrees with the general form of two-point correlator of light-ray operators.

$$\langle \mathcal{F}^j_n(x_\perp,y_\perp)\mathcal{F}^{j'}_{n'}(x'_\perp,y'_\perp)\rangle = -\frac{N_c^2}{2\pi}\frac{2^\omega\,\omega}{[(X-X')^2_\perp]^{2+\omega}}\left(\frac{(X-X')^2_\perp}{|\Delta_\perp||\Delta'_\perp|}\right)^{\omega+2\frac{\bar\alpha_s}{\omega}}\delta(\omega-\omega')$$

$X_\perp = \frac{x_\perp+y_\perp}{2}$, $X'_\perp = \frac{x'_\perp+y'_\perp}{2}$
$\Delta_\perp = (x-y)_\perp$, $\Delta'_\perp = (x'-y')_\perp$ $\Delta_\perp\Delta'_\perp$ is like the IR cut-off μ^{-2}

Best check: correlation functions in CFT

Supersymmetric light-ray operator $S^j(x_\perp)$ (of spin j) are analytic continuation of local operators

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Δ dimension of the operator; s is Mandelstam variable.
μ is the renormalization point; $\gamma_{\rm an}$ anomalous dimension.

correlation function for Quark (gluino): in the limit $\alpha_s \ll \omega = j-1 \ll 1$ agrees with the general form of two-point correlator of light-ray operators.

$$\langle \mathcal{Q}^j_n(x_\perp,y_\perp)\mathcal{Q}^{j'}_{n'}(x'_\perp,y'_\perp)\rangle = \frac{8}{\pi}\frac{2^\omega\,\omega}{[(X-X')^2_\perp]^{2+\omega}}\left(\frac{(X-X')^2_\perp}{|\Delta_\perp||\Delta'_\perp|}\right)^{\omega+2\frac{\bar\alpha_s}{\omega}}\delta(\omega-\omega')$$

$X_\perp = \frac{x_\perp+y_\perp}{2}$, $X'_\perp = \frac{x'_\perp+y'_\perp}{2}$
$\Delta_\perp = (x-y)_\perp$, $\Delta'_\perp = (x'-y')_\perp$ $\Delta_\perp\Delta'_\perp$ is like the IR cut-off μ^{-2}

leading twist approx. for quasi-pdf

$\bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$ $\qquad n^\mu = \frac{x^\mu}{|x|}$

Gluon

$$n_\mu n_\nu \langle P|G^{a\alpha\mu}(x)[x,0]^{ab}G^{b\beta\nu}(0)|P\rangle = \frac{g_\perp^{\alpha\beta}}{2}\frac{N_c\, Q_s^2\sigma_0}{4\pi\alpha_s}\left(\frac{\ln(x\cdot P)}{2\bar{\alpha}_s \ln Q_s|x|}\right)^{\frac{1}{2}} J_1(z)$$

Quark

$$n_\mu\langle P|\bar{\psi}(x)\gamma^\mu[x,0]\psi(0)|P\rangle = -\frac{iQ_s^2\sigma_0|x|}{12\pi\alpha_s}\left(\frac{\ln(x\cdot P)}{2\bar{\alpha}_s \ln Q_s|x|}\right)^{\frac{1}{2}} J_1(z)$$

$$z = \left[2\bar{\alpha}_s \ln\left(Q_s^2 x^2\right)\ln\left(2(x\cdot P)^2\right)\right]^{\frac{1}{2}}$$

Conclusions

- Saddle point approximation: quasi-pdfs grow with energy as expected
- Correlation function of quasi-pdfs quark and gluon operators agrees with the general result for two-point correlation function in CFT.
- Leading twist approximation in coordinate space for quasi-pdfs gives Bessel function with double log in the argument.