Large-momentum effective theory for partons and light-cone physics

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Outline

- Three representations for partons
- Large momentum expansion as an EFT
- Going beyond collinear partons
- Conclusions

- Refs: X. Ji, Sci.China Phys.Mech.Astron. 57 (2014) 1407-1412
  X. Ji, Liu, Liu, Zhang, Zhao, 2004.03543
  X. Ji, 2007.06613

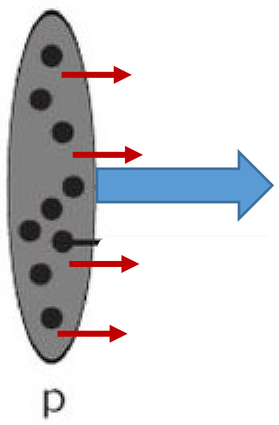
Three representations for partons

Feynman's parton model

- When the proton travels at $v \sim c$, one can assume the proton travels exactly at v=c, or the proton momentum is

$$p=E=\infty$$

(Infinite momentum frame, IMF)



- Proton may be considered as a collection of interaction-free particles: partons

Momentum distribution

- High-energy scattering in impulse approximation in NR systems probes momentum distribution

$$n\left(\vec{k}\right) = \left|\psi\left(\vec{k}\right)\right|^2$$

$$\sim \int \psi^*(\vec{r})\psi(0)e^{i\vec{k}\vec{r}}d^3r$$

$$\sim \int \langle\Omega\left|\hat{\psi}^+(\vec{r})\hat{\psi}(0)\right|\Omega\rangle e^{i\vec{k}\vec{r}}d^3r$$

- Mom.dis. are related to pure-space (Euclidean) correlations, generally amenable for Monte Carlo simulations.

PDF a la Feynman

- Consider the mom.dis. of constituents in a hadron

$$f(k^z, P^z) = \int d^2k_\perp \, f(k^z, k_\perp, P^z)$$

which depends on P^z because of relativity.

(H is not invariant under boost K)

- PDF is a result of the P^z → ∞ limit,

$$f(k^z, P^z) \rightarrow_{P^z \rightarrow \infty} f(x) \quad with\, x = \frac{k^z}{P^z},$$

Euclidean formulation of partons

- Calculate the Euclidean correlation

$$C(\lambda) = \langle P^z = \infty|\overline{\psi}(z)\Gamma\psi(0)|P^z = \infty\rangle$$

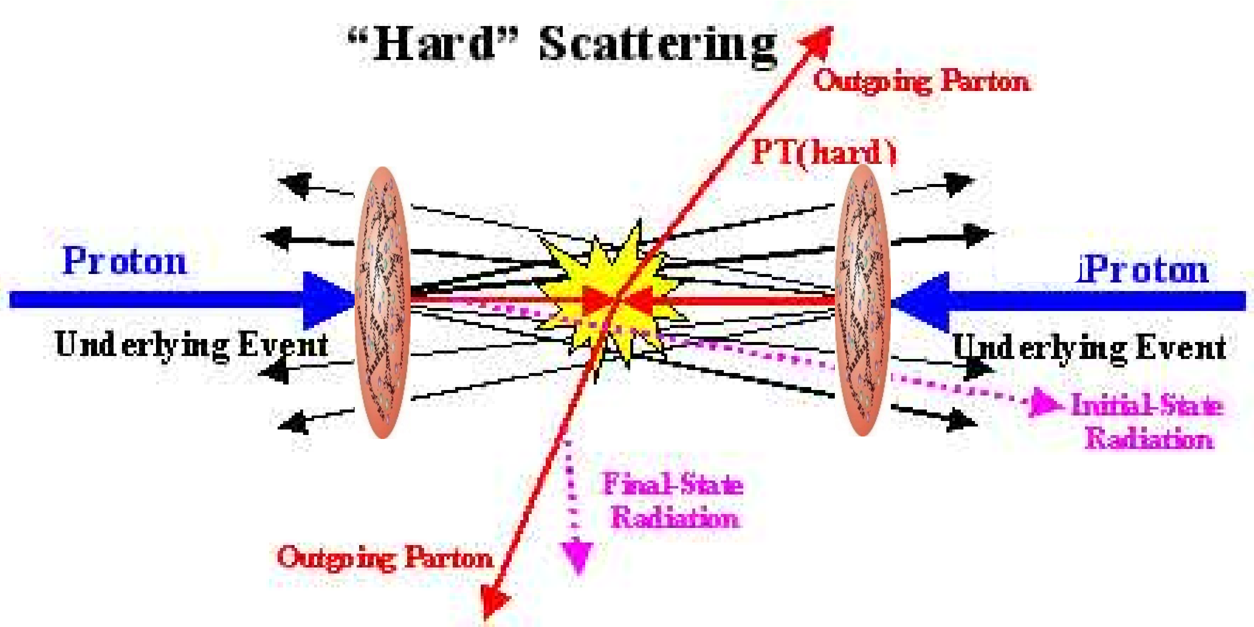
$$\lambda = \lim_{P^z\to\infty, z\to 0}(zP^z).$$

- Parton distribution

$$f(x) = \frac{1}{2P^+}\int \frac{d\lambda}{2\pi} e^{ix\lambda} C(\lambda) \ .$$

- Nobody has any use of this

Hard scattering theory



- Factorization theorems: The scattering cross sections are factorized in terms of PDFs and parton x-section.

$$\sigma = \int dx_a dx_b f_{a/A}(x_a) f_{b/B}(x_b) \hat{\sigma}$$

Light-front collinear modes

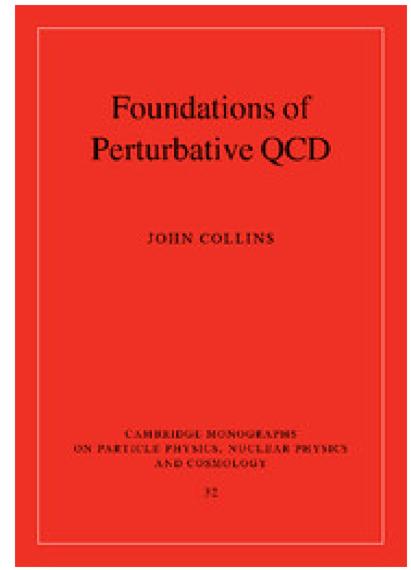
- In lagrangian formulation of parton physics, the partons are represented by collinear modes in QCD

$$\psi(\lambda n)\,,\; n^2 = 0$$

λ is the distance along the LF

- Parton physics is related to correlations of these fields along n with distance λ.

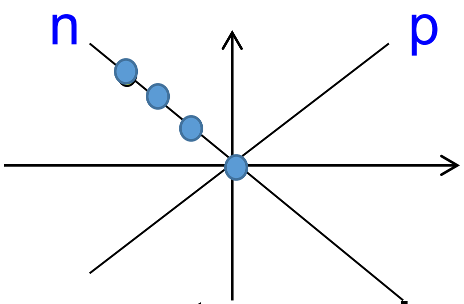
e.g. Soft-collinear Effective Theory (SCET), Bauer



Partons as LF correlations

- Probes operators) are light-cone correlations

$$\hat{O} = \phi_1(\lambda_1 \mathrm{n})\phi_2(\lambda_2 n)\ldots\phi_k(\lambda_k n)$$



- The matrix elements are independent of hadron momentum, and they can be calculated in the states in the rest frame.

Field Theory in IMF

- What does an object look like when travelling at infinite momentum or speed of light?
- S. Weinberg (scalar QFT)

  Dynamics at infinite momentum

  Phys. Rev. 150 (1966) 1313-1318

- All kinematic infinities can be removed from the calculations, resulting a set of rules for Hamiltonian perturbation theory ("old-fashioned p.t.")
- The result is similar to a "non-relativistic" theory.

Dirac's form of dynamics

- The Weinberg's rules exactly correspond to what Dirac proposed in 1949.
- Paul A.M. Dirac,

Forms of Relativistic Dynamics,

Rev. Mod. Phys. 21 (1949) 392-399.

"Front form"

or Light-front quantization (LFQ)

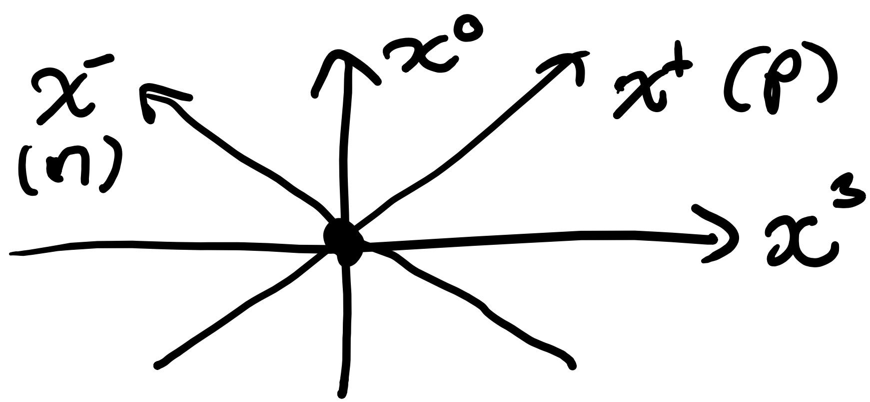
Light-front quantization (Brodsky et al)

- Quantizing the theory with “new coordinates”

$$x^+ = \frac{x^0 + x^3}{\sqrt{2}}, \quad x^- = \frac{x^0 - x^3}{\sqrt{2}}$$

by treating x^+ as the new “time”

x^- as the new “space” dimension.



Summary

Origin	States	operators	Rep
Feynman	P=∞	Euclidean	“Schrodinger”
QCD Factorization	P finite	Light-front	“Heisenberg”
Light-front quatization	P=∞	Light-front	Hamiltonian

Large-momentum expansion as an EFT

Large momentum expansion

- Approximate p=∞ by a large P
- This is what we frequently do in QCD

Lattice QCD: approximate a continuum theory by a discrete one.

cut-off $\Lambda \to \infty$, on lattice $\Lambda = \pi/a$

0.1 fm ~ 2 GeV

HQET: $\epsilon = \Lambda_{QCD}/m_Q$

using $m_Q = \infty$ to approximate

m_c =1.5 GeV, m_b = 4.2 GeV

How good is the finite-P approximation?

- Assuming the P-> ∞limit exists, one can naïve make a large-P expansion,

$$f(k^z, P^z) = f(x) + f_2(x)(M/P^z)^2 + ...$$

where M is a bound-state scale,

P^z is a large-momentum scale.

- $\epsilon = \left(\frac{M}{P^z}\right)^2$ is the expansion parameter

M =1 GeV, P_z=2 GeV, ϵ=1/4

the expansion may already work.

QFT subtleties

- There is a UV cut-off Λ_{UV}, $f(k^z, P^z)$ is not analytic at $P^z = \infty$! (ln Pz)
- There are two possible $P^z \to \infty$ limits:
  1. $P^z \ll \Lambda_{UV} \to \infty$, IMF limit (lattice QCD)
  2. $P^z \gg \Lambda_{UV} \to \infty$ LFQ limit (HEP PDF)
- Momentum distribution is calculated with the limit 1), but PDF is defined in limit 2).
- Solution: matching in EFT

The IR physics is the same, and the difference is perturbative!

Large-momentum expansion in field theory

- For finite momentum, one can have a factorization formula for large γ□ $(2-5)$:

$$\tilde{f}(y, P^z) = \int Z(y/x, xP^z/\mu) f(x,\mu) dx$$
$$+ \mathcal{O}\Big(\frac{\Lambda^2_{\rm QCD}}{y^2(P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(1-y)^2(P^z)^2}\Big),$$

- **Power counting:**

Large scales: parton and hadron remnants momenta

The expansion works only for x away from 0, 1

Where an EFT?

- An EFT integrates out the d.o.f. Q, outside the model space P.

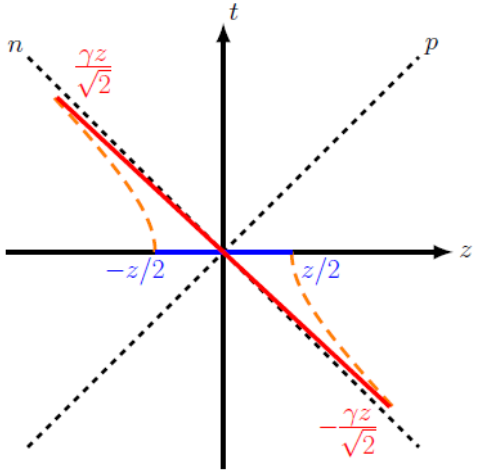
  P+Q=1

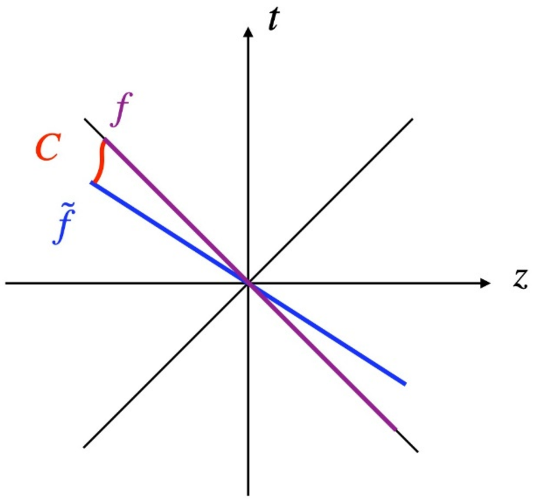
- LaMET P-space contains all modes with momentum between 0 and P^z , and cutoff $\Lambda_{UV} \gg P^z$.
- LF correlations contain all momentum range, and Q-Space contains all modes between P^z and ∞
- Similar to lattice QCD approximating the continuum theory, where finite lattice spacing effects are accounted by high-dim operators.

An EFT expansion

- Partons in EFT expansion (all P dependence, a sort of regulator, cancels)

$$f(x,\mu) = \tilde{f}(x,P) \circ C\left(x,\frac{\mu}{P}\right) + \widetilde{f_2}(x,P) \circ \frac{C_2\left(x,\frac{\mu}{P}\right)}{P^2} + \ldots$$





Calculating the x-dependence!

- Phenomenologically, the x-dependence of PDFs has been modelled or parametrized in all types of fitting.
- In EFT expansion, x-dependence is obtained point by point through calculation.

Relationship between LaMET and quasi distributions

- LaMET is a systematic expansion for calculating ALL parton properties of hadrons, including collinear PDFs, TMD-PDFs, GPDs, DA, as well as Light-Front Wave Functions.
- Quasi distributions are natural set up for carrying out LaMET, however, there are infinite many choices for making expansions due to universality.

Partons and critical phenomena

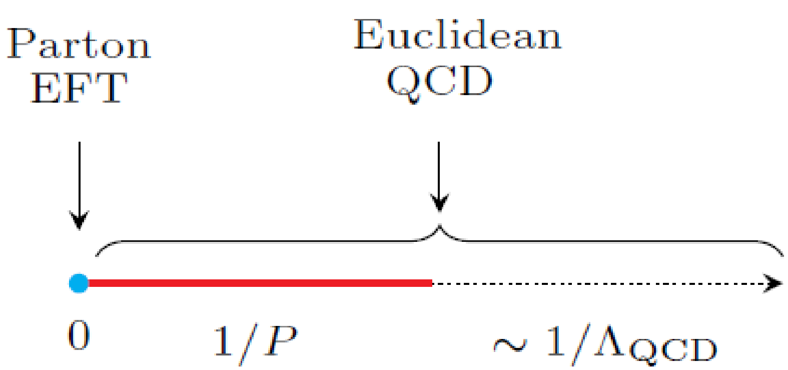
- Fourier trans. of PDFs gives a small-x behavior,

$$f(x) \rightarrow x^{-\alpha}$$

- When FT back to position space, one has

$$C(\lambda) \sim \lambda^{\alpha-1}$$

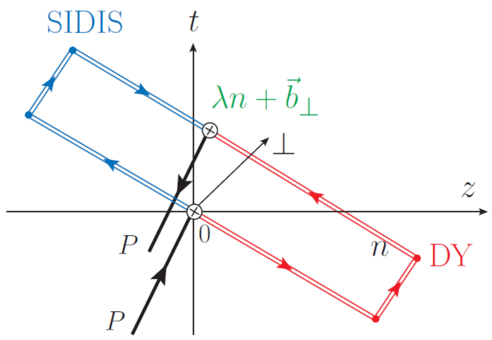
This corresponds to "infinite correlation" length

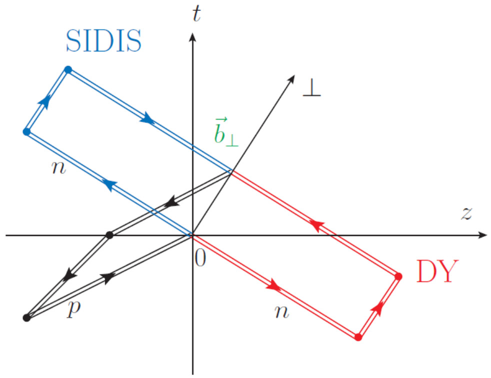


Going beyond collinear partons

TMD-PDFs

- A very important nucleon observable, many phenomenology related to spin physics (Sivers effect etc).
- It took sometime to figure out the correct definition

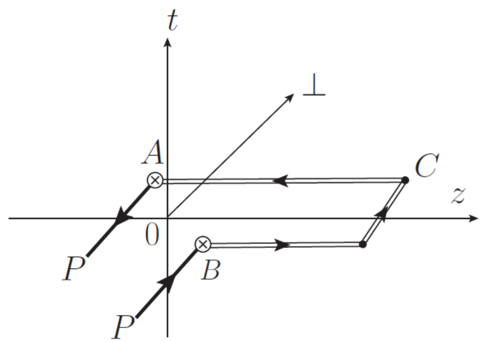


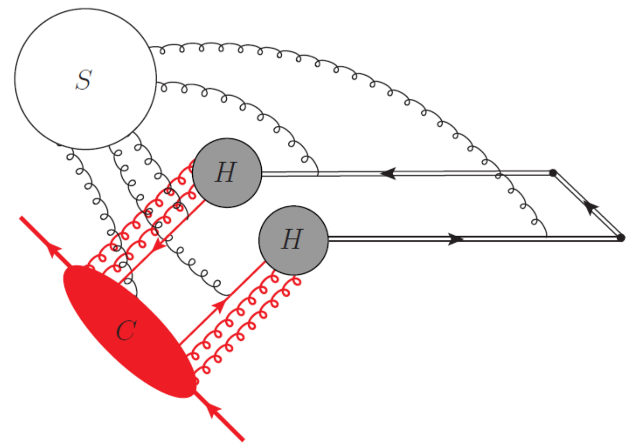


Echevarria, Idilbi, Scimemi (2013), Collins & Rogers (2013)

Quasi-TMDPDF and factorization

Ji et al., PRD91,074009 (2015); PRD99,114006(2019) Ji, Liu, Liu, PLB, 1911.03840
Ebert, Stewart, Zhao, PRD99,034505 (2019), JHEP09,037(2019); arXiv:1910.08569



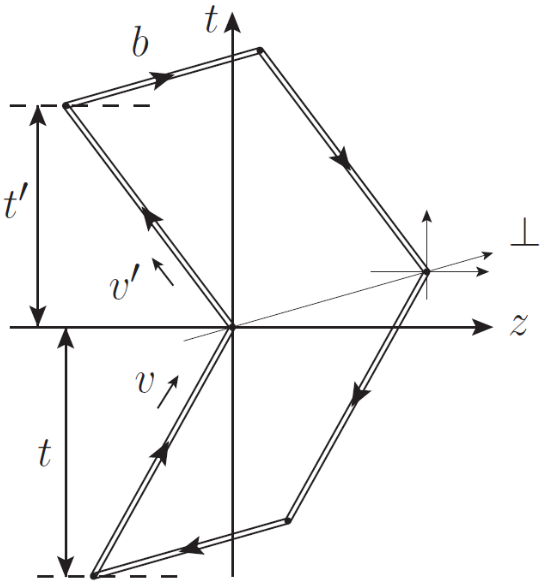


$$\tilde{f}(x, b_\perp, \mu, \zeta_z)\sqrt{S_r(b_\perp, \mu)}$$
$$= H\left(\frac{\zeta_z}{\mu^2}\right) e^{K(b_\perp,\mu)\ln(\frac{\zeta_z}{\zeta})} f^{\rm TMD}(x, b_\perp, \mu, \zeta) + ...$$

$$\mu\frac{d}{d\mu}\ln H\left(\frac{\zeta_z}{\mu^2}\right) = \Gamma_{\rm cusp}\ln\frac{\zeta_z}{\mu^2} + \gamma_C$$

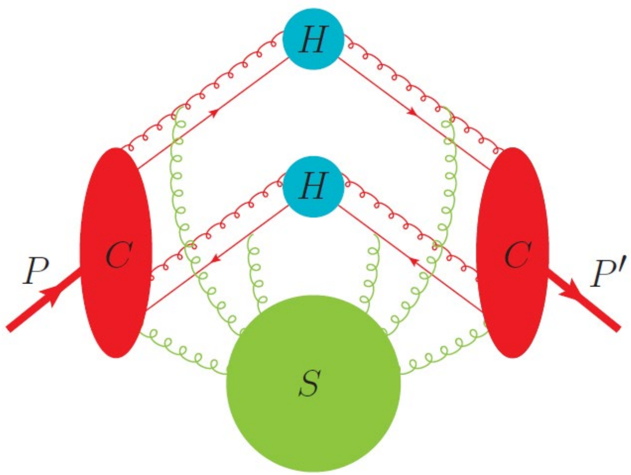
Soft factor

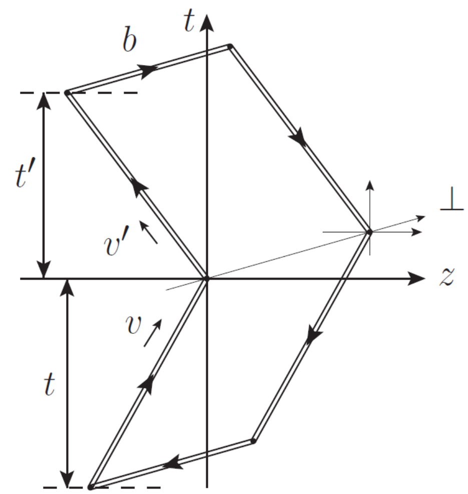
- Soft factor is not a parton distribution.
- It represents soft-radiation cross section for two opposite moving color charges.
- It involves two light-cone directions.
- It has been calculated to 3-loops in pert. theory (H.X.Zhu et al)



Soft factor in LaMET:

- Approximating light-cone Wilson-lines by high-momentum states

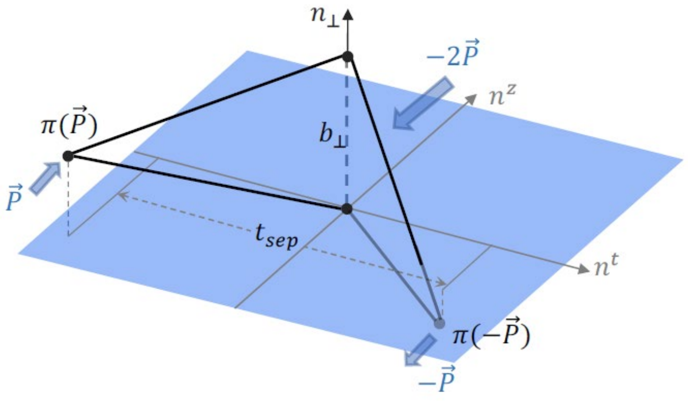


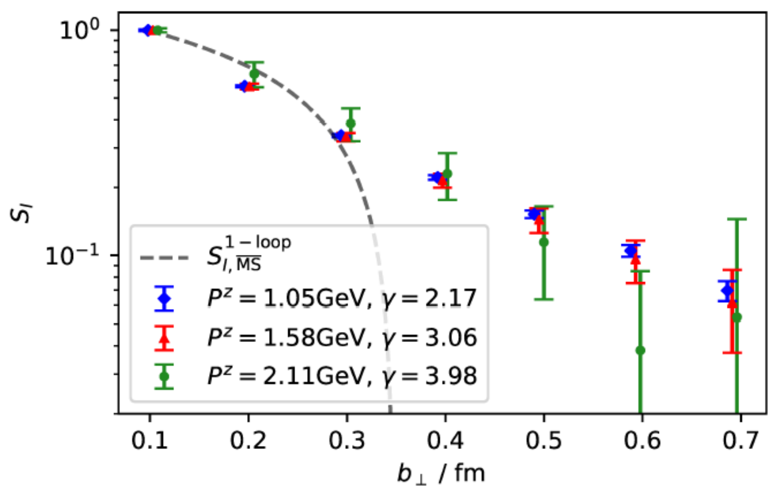


Factorization of form-factor of light-meson

Form-factor of heavy-quark pair

soft factor on lattice





Q. A. Zhang et al, Lattice Parton Collaboration (LPC)
Phys.Rev.Lett. 125 (2020) 19, 192001

Light-Front Wave-Functions

- LF quantization focuses on the WFs, from which everything can be calculated: a very ambitious goal! Brodsky et al. Phys. Rept. 301 (1998)
- However, there are a number of reasons this approach has not been very successful.
- LaMET provides the practical way to calculate non-perturbative WF, at least for lowest few components. Ji & Liu to be published.
- All WF can be computed as gauge-invariant matrix elements

$$\left\langle 0 \left| \hat{O}\left(z_1, \vec{b}_1, z_2, \vec{b}_2 \ldots, z_k, \vec{b}_k\right) \right| P \right\rangle$$

Conclusions

- Partons can be approximately systematically on lattice using a large-momentum hadron matrix elements.

$$\frac{\Lambda_{QCD}}{P} \ll 1$$

- LaMET3.0 (~ 5% error for x ~(0.1-0.9))

  two-loop matching

  P=3 GeV

  Improved non-pert renormalization
- 1% accuracy in 10-20 years