Perturbative uncertainties in the Drell-Yan spectrum at low q_T

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RGEs in QCD

- Generic RGE in QCD:

R = (α_s, PDF, TMD)

$$\frac{d\ln R}{d\ln\mu} = \gamma(\alpha_s(\mu))$$

$$\gamma(\alpha_s(\mu)) = \frac{\alpha_s(\mu)}{4\pi}\sum_{n=0}^{k}\left(\frac{\alpha_s(\mu)}{4\pi}\right)^n \gamma^{(n)}$$

- Start with α_s

k = highest-order known in β-expansion

$$\frac{d\ln a_s}{d\ln\mu} = \overline{\beta}(a_s(\mu)) = a_s(\mu)\sum_{n=0}^{k} a_s^n(\mu)\beta^{(n)}$$

- Knowledge of $\beta^{(k)}$ allows to resum N^{k}LL tower of logs
- Question: is it possible to find an analytical expression for each tower?

α_s

- Answer at LL: YES

$$a_s^{\mathrm{LL}}(\mu) = \frac{a_s(\mu_0)}{1 - \beta^{(0)} a_s(\mu_0) \ln\left(\frac{\mu}{\mu_0}\right)}$$

- *exact solution of RGE for k=0*
- *sum of the full LL series*

- Answer beyond LL: NO

No exact solution of RGE for k>0: transcendental equations

NLL (k=1)

$$-\frac{1}{\alpha_s(\mu)} + \frac{1}{\alpha_s(\mu_0)} + b_1 \ln\left(\frac{\alpha_s(\mu_0)(1 + b_1\alpha_s(\mu))}{\alpha_s(\mu)(1 + b_1\alpha_s(\mu_0))}\right) = \beta_0 \ln\left(\frac{\mu}{\mu_0}\right)$$

NNLL (k=2)

$$-\frac{1}{\alpha_s(\mu)} + \frac{1}{\alpha_s(\mu_0)} + b_1 \ln\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right) + \left(b_1^2 - b_2\right)\left(\alpha_s(\mu) - \alpha_s(\mu_0)\right) = \beta_0 \ln\left(\frac{\mu}{\mu_0}\right)$$

Numerical implementations based on these equations satisfy RGE exactly

α_s

- But what if we insist on having an analytical expression?

Expand α_s in previous equations and solve to obtain

$$a_s^{\mathrm{NLL}}(\mu) = a_s^{\mathrm{LL}}(\mu)\left[1 - b_1 a_s^{\mathrm{LL}}(\mu)\ln\left(\frac{a_s(\mu_0)}{a_s^{\mathrm{LL}}(\mu)}\right)\right]$$

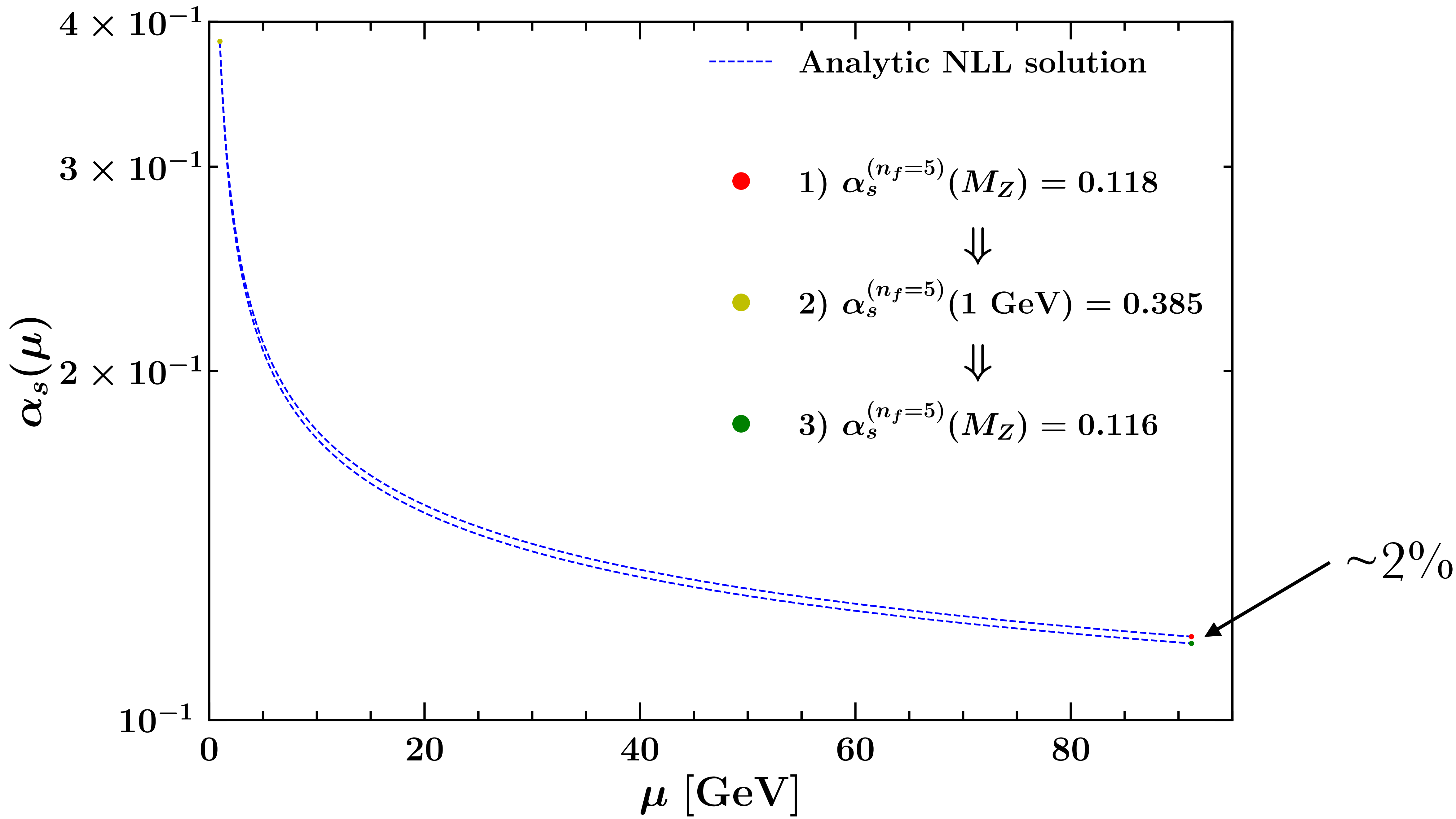
$$a_s^{\mathrm{NNLL}}(\mu) \;=\; a_s^{\mathrm{LL}}(\mu)\left[1 + b_1\left[a_s^{\mathrm{NLL}}(\mu) + b_1\left(a_s^{\mathrm{LL}}(\mu)\right)^2\right]\ln\left(\frac{a_s^{\mathrm{LL}}(\mu)}{a_s(\mu_0)}\right) + (b_2 - b_1^2)a_s^{\mathrm{LL}}(\mu)\left[a_s^{\mathrm{LL}}(\mu) - a_s(\mu_0)\right]\right]$$

⋮

These equations satisfy RGE for α_s
UP TO subleading terms

The perturbative hysteresis: α_s

$$a_s^{\mathrm{NLL}}(\mu) = a_s^{\mathrm{LL}}(\mu)\left[1 - b_1 a_s^{\mathrm{LL}}(\mu)\ln\left(\frac{a_s(\mu_0)}{a_s^{\mathrm{LL}}(\mu)}\right)\right]$$



PDFs

- RGE for PDFs
- solution of RGE
- γ expansion
- link to β

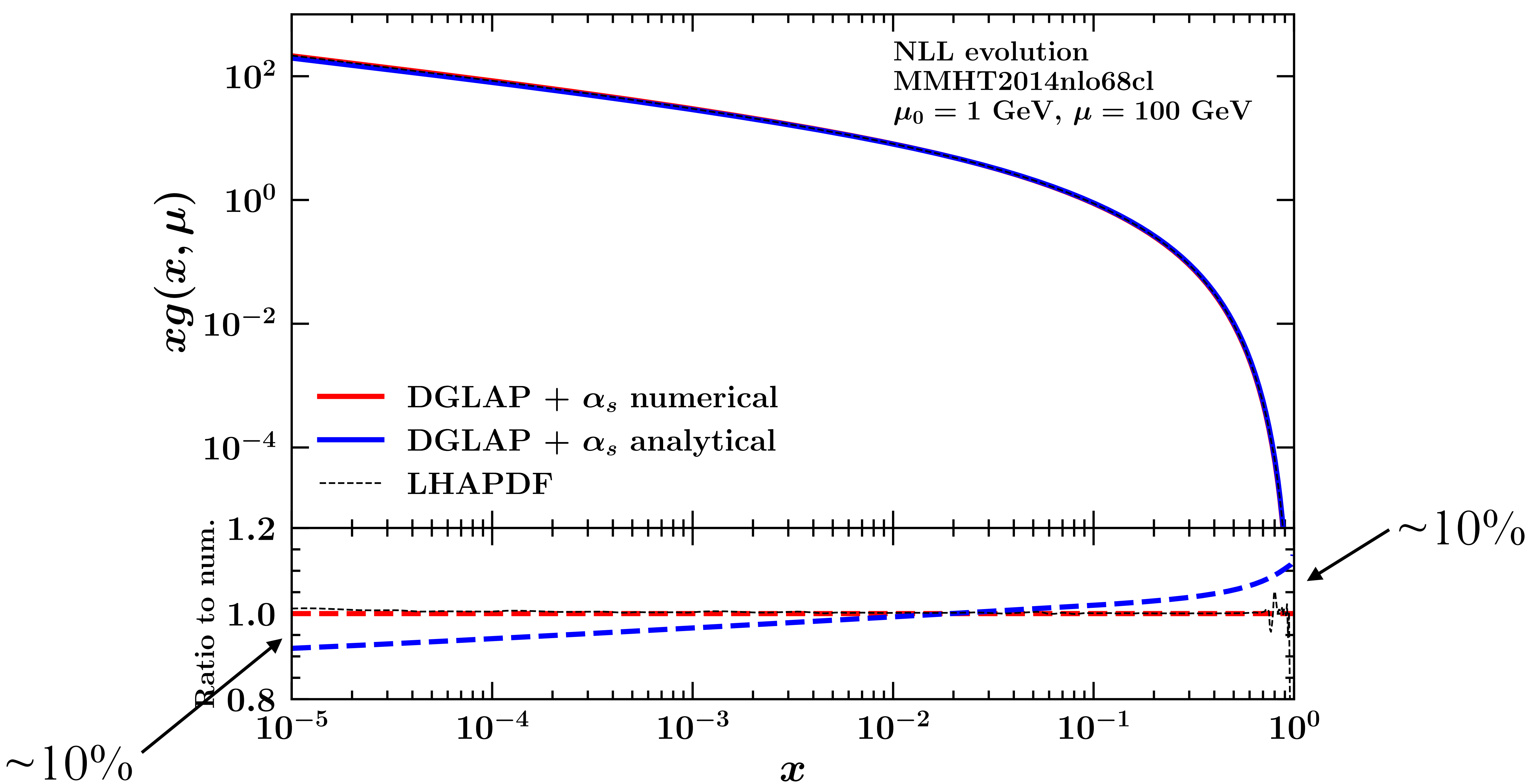
$$
\begin{aligned}
f(\mu) &= \exp\left[\int_{\mu_0}^{\mu} d\ln\mu'\,\gamma(a_s(\mu'))\right] f(\mu_0) \\
&= \exp\left[\sum_{n=0}^{k}\gamma^{(n)}\int_{\mu_0}^{\mu} d\ln\mu'\,a_s^{n+1}(\mu')\right] f(\mu_0) \\
&= \prod_{n=0}^{k}\exp\left[\gamma^{(n)} I_n\right]\, f(\mu_0),
\end{aligned}
$$

$$
I_n = \int_{a_s(\mu_0)}^{a_s(\mu)} da_s \left(\frac{a_s^n}{\overline{\beta}(a_s)}\right).
$$

Again: numerical vs. fully analytical solution

The perturbative hysteresis: PDF

$$f^{\mathrm{NLL}}(\mu) = \left[1 + \frac{1}{\beta^{(0)}}\left(\gamma^{(1)} - b_1\gamma^{(0)}\right)\left(a_s^{\mathrm{LL}}(\mu) - a_s(\mu_0)\right)\right]\left(\frac{a_s^{\mathrm{NLL}}(\mu)}{a_s(\mu_0)}\right)^{\frac{\gamma^{(0)}}{\beta^{(0)}}} f(\mu_0)\,.$$



Sudakov

$$S = -\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2}\left[A\left(\alpha_s\left(q^2\right)\right)\ln\frac{M^2}{q^2} + \tilde{B}\left(\alpha_s\left(q^2\right)\right)\right]$$

RGE evolution of TMDs

numerical α_s
numerical Sudakov

example: NLL integrand

$$\ln(\frac{M^2}{q^2})\left(A_1\alpha_s^{NLL} + A_2(\alpha_s^{NLL})^2\right) + B_1\alpha_s^{NLL}$$

soft-gluon resummation

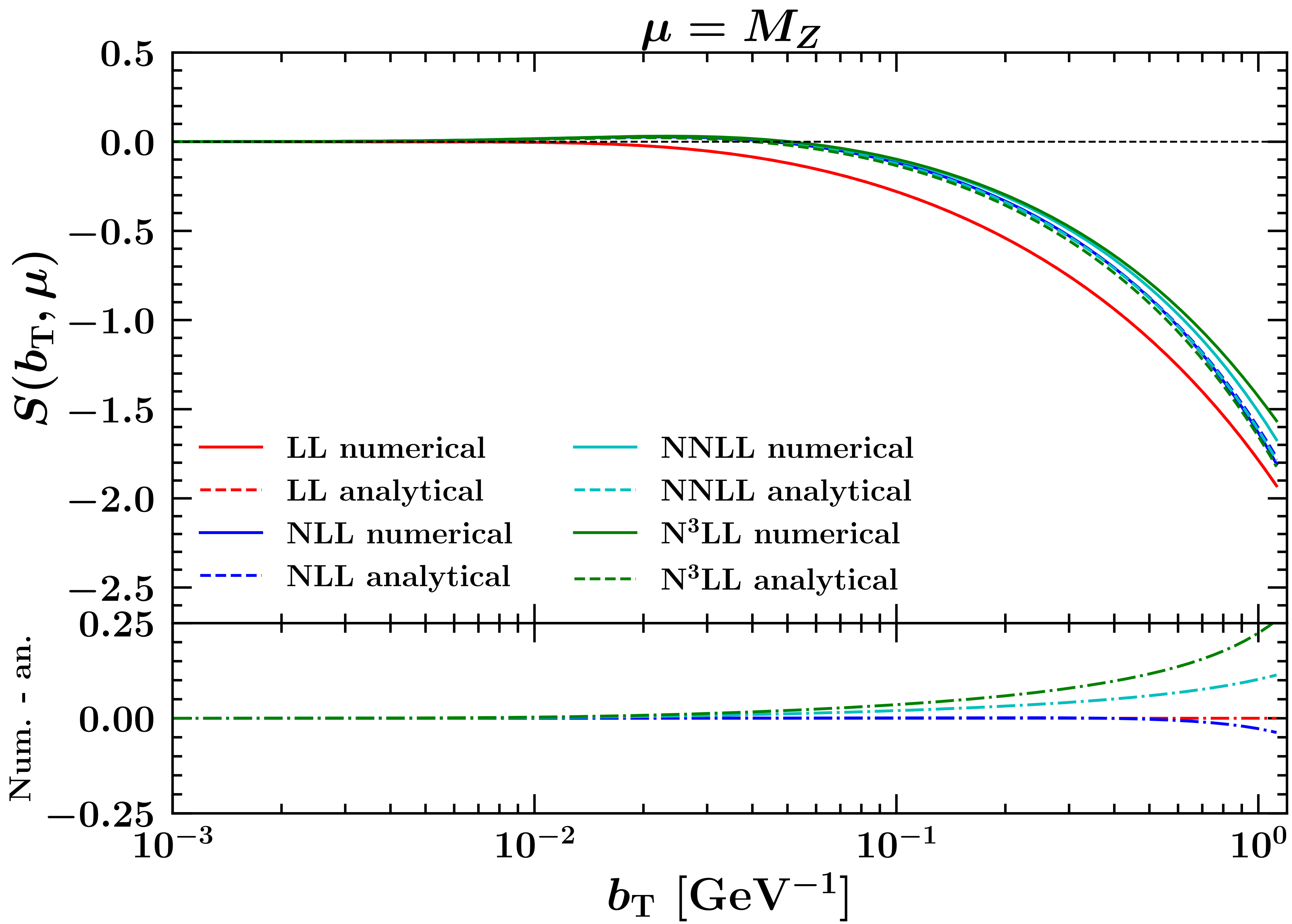
analytical α_s
analytical Sudakov

$$S = Lg_1(\alpha_s\beta_0 L) + g_2(\alpha_s\beta_0 L) + \alpha_s g_3(\alpha_s\beta_0 L) + \dots$$

example: NLL integrand

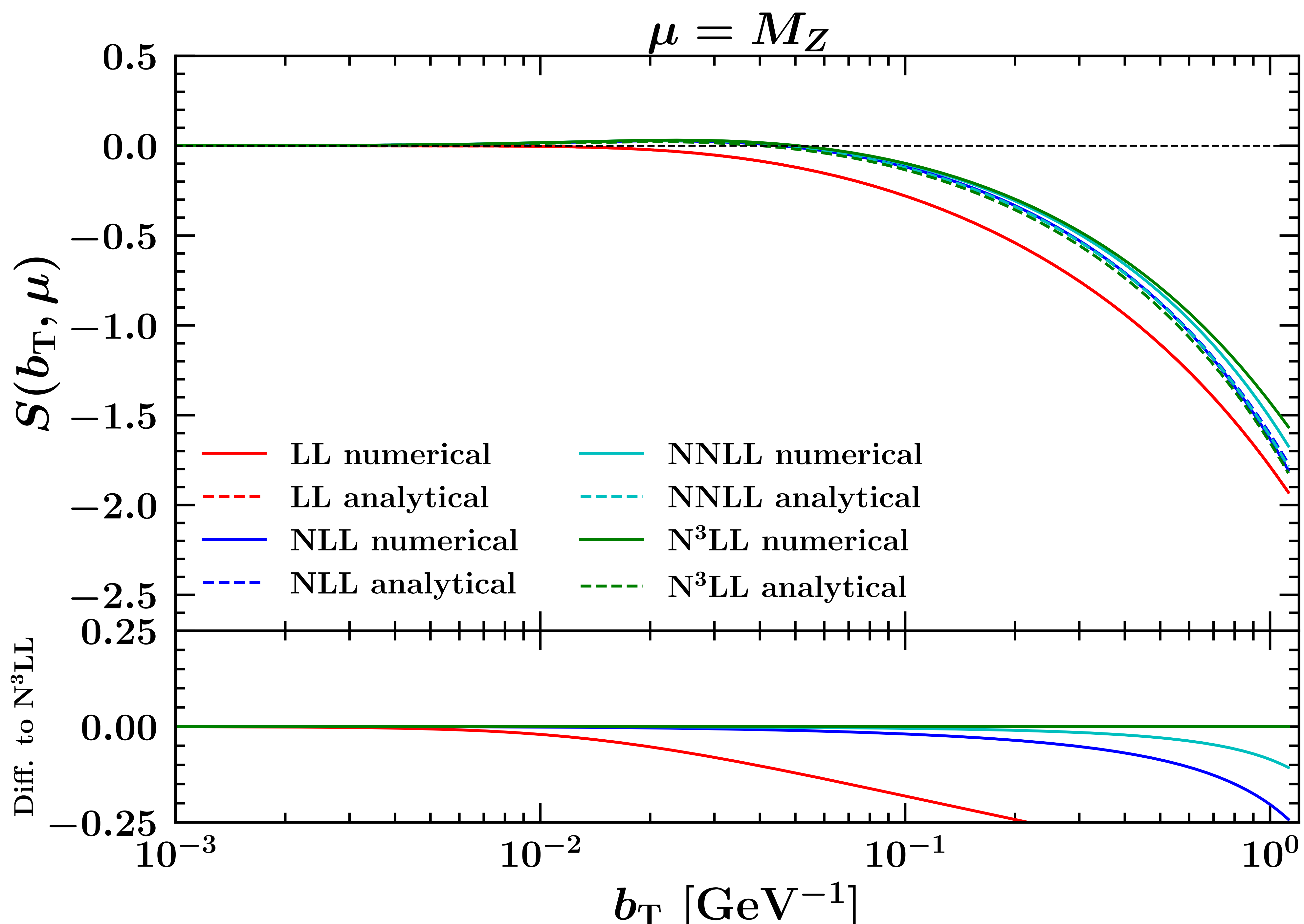
$$\ln(\frac{M^2}{q^2})\left(A_1\alpha_s^{NLL} + A_2(\alpha_s^{LL})^2\right) + B_1\alpha_s^{LL}$$

The perturbative hysteresis: Sudakov



- No difference at LL (as expected)
- No differences at low b_T (high q_T) because of → Modified logs: $\ln(\mu^2 b^2/b_0^2 + 1)$
- **Increasing** difference at high b_T (low q_T) (?)

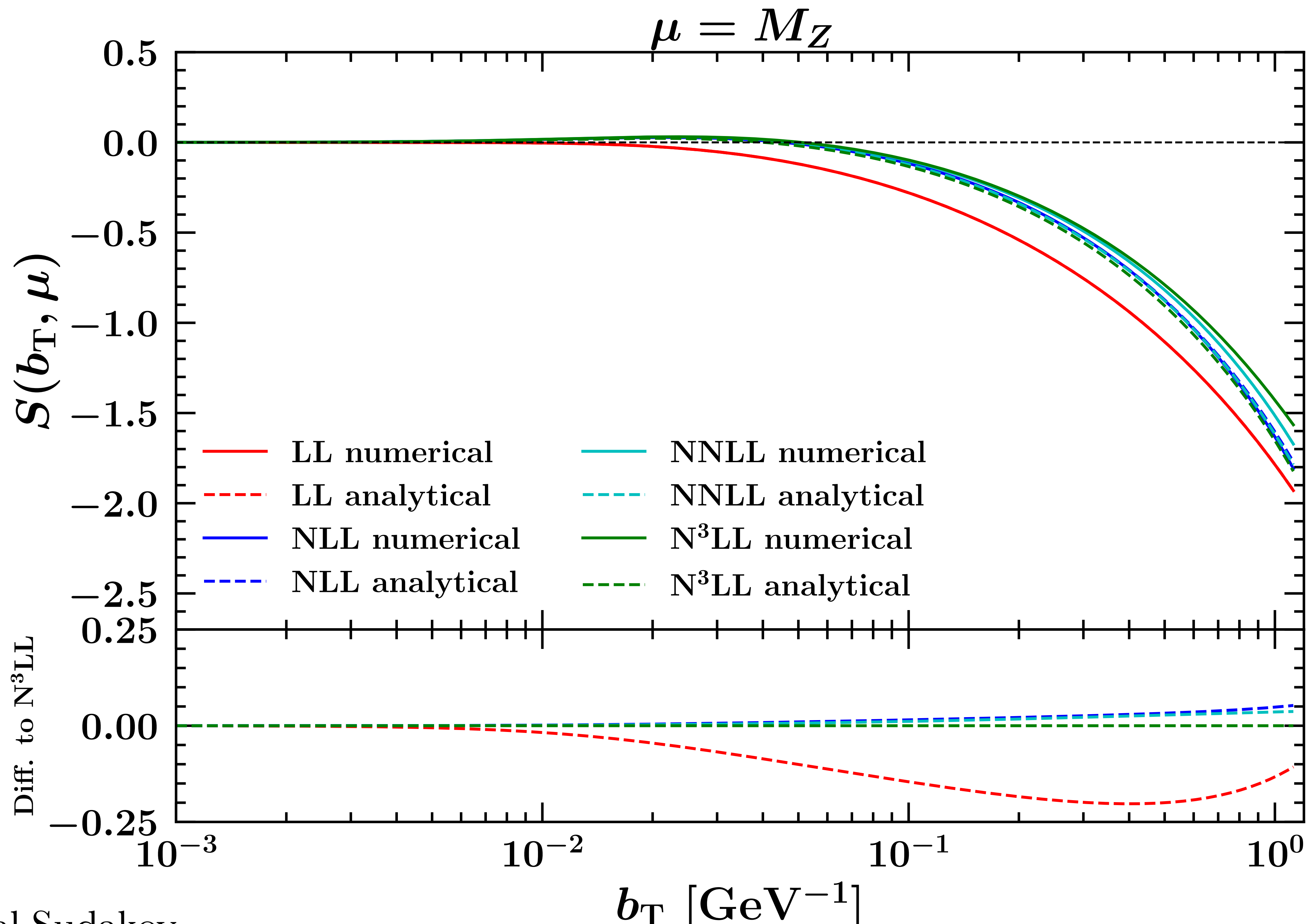
The perturbative hysteresis: Sudakov



- Numerical Sudakov
- Good convergence towards N3LL

Modified logs: $\ln(\mu^2 b^2/b_0^2 + 1)$

The perturbative hysteresis: Sudakov



- Analytical Sudakov
- Faster convergence towards N3LL
- (N3LL - NNLL) > (NNLL - NLL) (?)
- What next?

Modified logs: $\ln(\mu^2 b^2 / b_0^2 + 1)$

Outlook

- Drell-Yan spectrum at low q_T is a perfect playground to exploit the full potentiality of the TMD and the soft-gluon resummation frameworks

- We are not suggesting the adoption of one particular framework: we simply want to better understand the differences and to give a fair estimate of theoretical uncertainties

Open questions

- Scale variation is usually employed to estimate theoretical uncertainties from subleading contributions: would it induce a kind of "double counting" due to the perturbative mismatch in analytical vs. numerical codes?

- Is this perturbative mismatch (at least partially) responsible for discrepancies we observe between TMD codes and soft-gluon resummation codes?

- Can we identify a sensible recipe to compare predictions based on the TMD framework and the soft-gluon resummation framework?