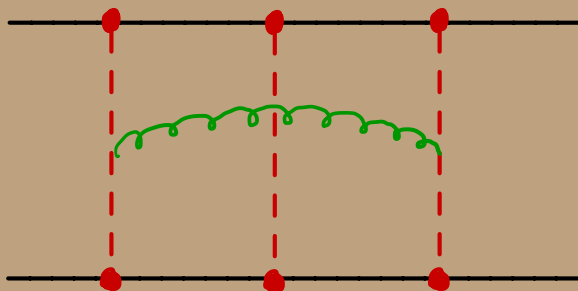


REGGE LIMITS FROM GLAUBER SCET



by GREGORY RIDGWAY

Based on work in progress

with IAN MOULT

and IAIN STEWART

for REF2020

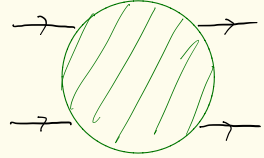


①

MOTIVATION

Goal: To understand the Regge limit at higher orders

→ Regge Limit: strong rapidity ordering.
At 4-point (this talk), $s \gg -t$.



→ Lots of work has been done

- Many Methods
- Effective Hamiltonians, [Balitsky, Fadin, Kuraev, Lipatov -1975; Review:0807.2605]
 - Integrable Spin Chains, [Lipatov 9311037; Fadeev + Karchemsky 9404173]
 - Wilson Line methods, [Balitsky 9509348; Caron-Huot 1309.6521; Falcioni, Gardi, Noguea, Verma]
 - ...

→ Interesting Structures

- Reggeization $\text{---} + \text{---} + \text{---} + \dots \ll \text{---}^{\text{"reggeon"}} \sim \frac{1}{-t} \left(\frac{s}{-t} \right)^{\alpha(t)}$ Regge Pole, Trajectory
- BFKL $\text{---} \text{---} \text{---} = \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \rightarrow \frac{\partial}{\partial Y} G_{\text{BFKL}} = K \otimes G_{\text{BFKL}}$

→ Many sophisticated methods rely on planar (large- N_c) limit and Conformal symmetries.

MOTIVATION

Some Open Questions:

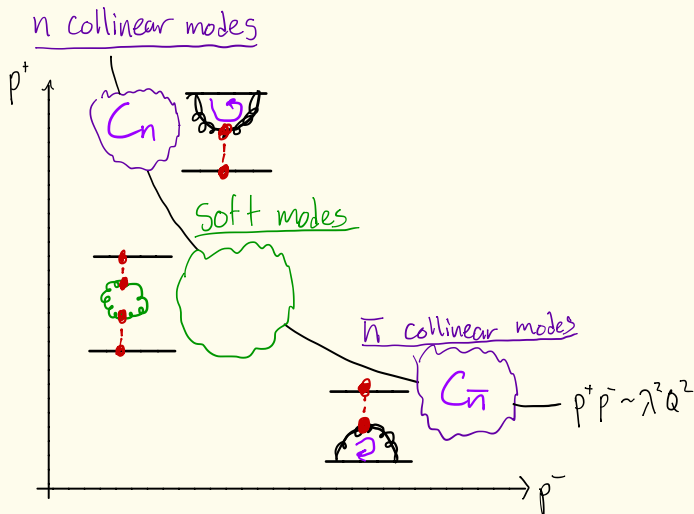
- What is the structure of non-pole solutions?
Non-planar corrections?
- How to extend BFKL to NNLL?
- How are the regge trajectory, Pomeron, etc. related to universal anomalous dimensions? What are their QFT definitions?
(e.g. Γ_{cusp} , $\gamma_{\text{collinear}}$ in $N=4\text{SYM}$ [0907.0243; 9607229])

We will delve into these problems with
SCET+Glauber [1601.04695]

3

How DOES SCET HELP?

An EFT based on expansions in momentum regions.



- Maintains gauge invariance within each sector independently
- Obeys unitarity and analyticity constraints as any QFT should.

SCET offers new perspectives on Regge phenomena

Collinear perspective

A field theory for the impact factors, $\{C_n, C_{\bar{n}}\}$. A very different, powerful, equivalent organization of the Regge limit.

Soft perspective

A field theory for the soft fluctuations. More familiar, analogous to traditional approaches (with important differences)

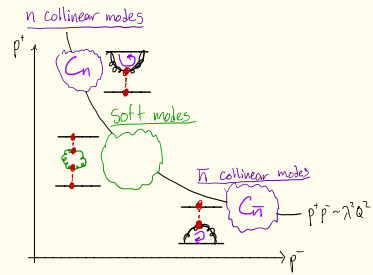
How DOES SCET HELP?

4

The EFT connects all sectors via

$$\mathcal{L}_{\text{SCET}}^{(0)} = \mathcal{L}_{\text{Collinear}} + \mathcal{L}_{\text{Soft}} + \mathcal{L}_{\text{Glauber}}$$

at leading power in $(\frac{-t}{s})$

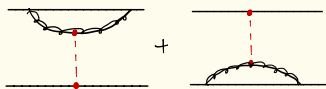


- **Glauber** operators mediate interactions between sectors
- Regge information is cast in a Field Theory language
 - ▶ Regge trajectory = anomalous dimension of 1-Glauber operator
 - ▶ Pomeron is a rapidity RGE of singlet projected multi-Glauber op.
 - ▶ BFKL is also a rapidity RGE (RRGE)
- **Soft-Collinear consistency**: anomalous dimensions can be calculated in either sector.

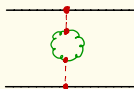
EXAMPLE: 1-Loop

5

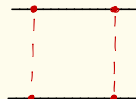
Collinear



Soft



Glauber



$$\begin{aligned}
 & \sim \gamma_1^{\delta_A} \left(-\frac{1}{t} + \log \left| \frac{s}{v^2} \right| \right) + \gamma_1^{\delta_A} \left(\frac{1}{t} + \log \left| \frac{v^2}{-t} \right| \right) - \gamma_1^{\delta_A} \left(\frac{i\pi}{2} \right) \\
 & = \gamma_v \left(\log \left| \frac{s}{-t} \right| - \frac{i\pi}{2} \right) \\
 & = \frac{\gamma_v}{2} \left(\log \left(\frac{s}{-t} \right) + \log \left(\frac{-s}{-t} \right) \right)
 \end{aligned}$$

- Resums logs using $v \frac{d}{dv} \sigma_i^{\delta_A} = \gamma_i^{\delta_A} \sigma_i^{\delta_A} \rightarrow \sigma_i^{\delta_A}(s) = \left(\frac{s}{-t} \right)^{\gamma_i^{\delta_A}} \sigma_i^{\delta_A}(-t)$ Reggeization ✓
- Rapidity anomalous dimension of 1-Glauber operator, $\gamma_1^{\delta_A}$, is minus the Regge trajectory, α_g .
- Soft-Collinear consistency: each sector produces the same γ_v
- Glauber loop necessary for
 - (1) Unitarity/QFT cutting rules
 - (2) Analytic properties

BEYOND LL

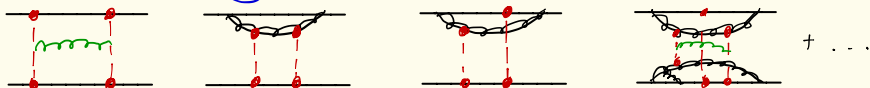
6

Higher order $\Rightarrow$ more Glauber's exchanged. Our task is to:

- identify the correct basis of operators

$$\sigma_n^{(R)} \equiv \{\sigma_1^{(B_1)}, \sigma_2^{(1)}, \sigma_2^{(B_1)}, \sigma_2^{(B_3)}, \sigma_2^{(2,7)}, \sigma_3^{(1)}, \sigma_3^{(B_1)}, \dots\}$$

- find/calculate diagrams that contribute to their anomalous dims.



- write their RRGEs

$$V \frac{d}{dV} \sigma_n^{(R)} = \gamma_n^{(R)} \otimes \sigma_n^{(R)}$$

- There is no conceptual barrier to obtaining non-trivial evolution equations, as are expected from non-planar contributions
- Will look very different between soft & Collinear sectors

Collinear: Trivializes relationship between planarity and Regge pole structure

Soft: More complicated: more diagrams, non-trivial cancellations, m-to-n Glauber transitions

Advantage: clarifies the universal t-channel color decomp

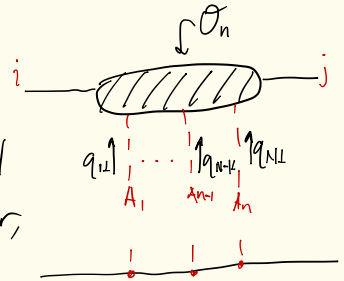
COLLINEAR PERSPECTIVE

7

Start on the collinear side

• Basis of operators: $\mathcal{O}_n^{(k)}(q_{1\perp}, \dots, q_{n\perp})$

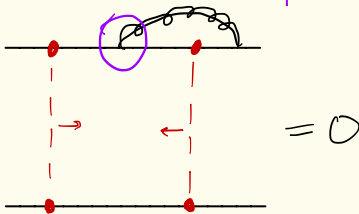
- ▶ n-Glauber exchange
- ▶ Color rep R flowing through the t-channel
- ▶ momentum $q_{i\perp}$ flowing through the i th Glauber,
 $(\sum \vec{q}_{i\perp})^2 = -t$ constraint



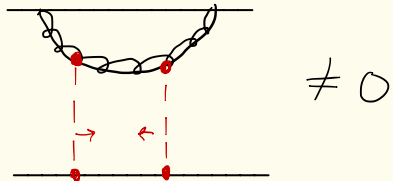
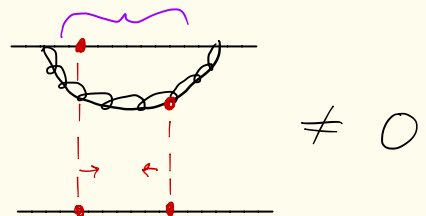
• Contributing diagrams:

- ▶ Greatly simplified by "collapse rule"

Obstruction to Glauber Collapse



no such obstruction



- ▶ Planar and non-planar contributions on equal footing

COLLINEAR PERSPECTIVE : PLANAR

Understanding the Planar limit.

Start with 1 collinear loop.

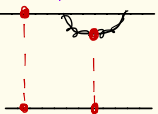
$n=1$: 1 diagram  $\sim \sigma_v \log\left(\frac{s}{v^2}\right)$

▶ one operator $\mathcal{O}_1^{\delta_A}$

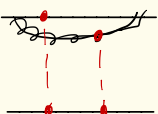
▶ $v \frac{1}{s} \mathcal{O}_1^{\delta_A} = \gamma_{10}^{\delta_A} \mathcal{O}_1^{\delta_A} \Rightarrow \mathcal{O}_1^{\delta_A}(v_2) = \left(\frac{v_2}{v_1}\right)^{\gamma_{10}^{\delta_A}} \mathcal{O}_1^{\delta_A}(v_1)$, pole solution

$n=2$:

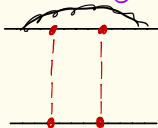
collapse rule



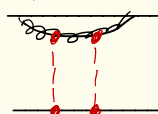
Non-Planar



(not divergent)

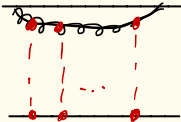


only contribution



▶ when $N_c \rightarrow \infty$, only δ_A , all else subleading $\Rightarrow \mathcal{O}_2^{\delta_A}(q_{1\perp}, q_{2\perp})$

▶ $\gamma_{20}^{\delta_A}(q_1, q_2) = \gamma_{20}^{\delta_A}(\vec{q}_1, \vec{q}_2)^2 = \gamma_{20}^{\delta_A}(-t) \Rightarrow$ pole solution again, $\left(\frac{v_2}{v_1}\right)^{\gamma_{20}^{\delta_A}} \mathcal{O}_2^{\delta_A}$

any n : only δ_A ; only ; $\gamma_{n0}^{\delta_A} = \gamma_{n0}^{\delta_A}(-t) \Rightarrow$ pole again, $\left(\frac{v_2}{v_1}\right)^{\gamma_{n0}^{\delta_A}} \mathcal{O}_n^{\delta_A}$

• Also, all $\gamma_{n0}^{\delta_A} = \gamma_{10}^{\delta_A} + \mathcal{O}\left(\frac{1}{N_c}\right)$

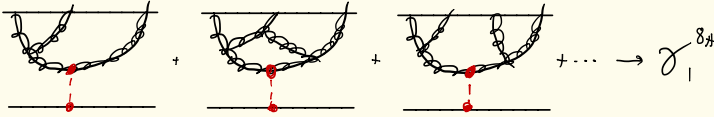
• Extend to more collinear loops...

COLLINEAR PERSPECTIVE : PLANAR

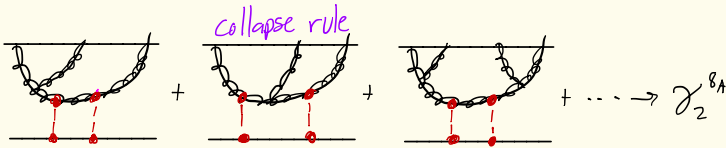
9

Multiple collinear loops

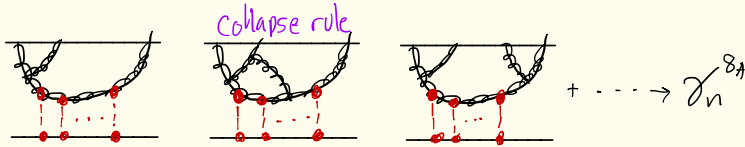
$n=1$:



$n=2$:

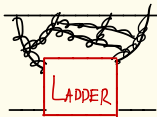


any n :



By the **collapse rule**, there is no interruption to the Glauber ladder. Two important consequences:

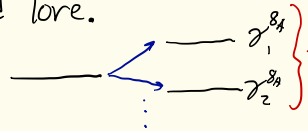
(1) only the sum $\sum q_i$ is exchanged $\Rightarrow \gamma_n^{8_A} = \gamma_n^{8_A}(-t)$,
leading to a pole solution



(2) Same contributing graphs for all $n \Rightarrow \gamma_n^{8_A} = \gamma_1^{8_A}$ still.

A simple diagrammatic derivation of standard lore.

The degeneracy of (2) will be broken at $1/t_c$

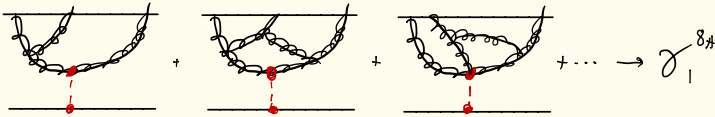


DEFINITION OF REGGE TRAJECTORY

10

Given these gauge invariant operators with separate RG's, it is natural to define the regge trajectory to all orders as follows

Definition: the regge trajectory is (minus) the anomalous dimension of the 1-Glauber exchange operator, $\mathcal{O}_1^{\text{gl}}$


$$\text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \dots \rightarrow \gamma_1^{\text{gl}}$$

Has desirable properties

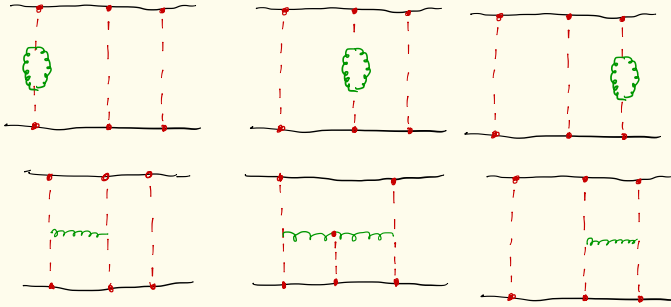
(e.g. correct IR behavior [Del Duca, Duhr, Gardi, Magnea, White, 1109.3581])

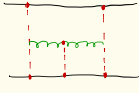
SOFT PERSPECTIVE : PLANAR

11

Much harder to see, many more planar diagrams

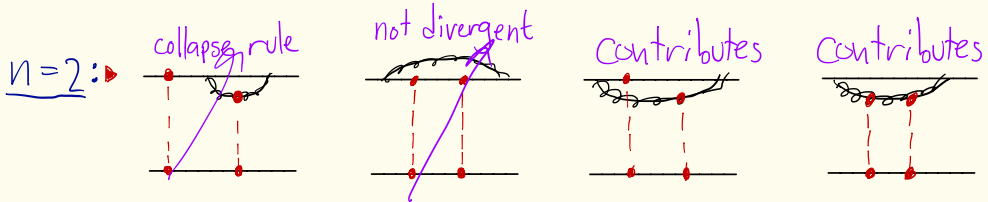
$n=3$:



- cancellations occur so that $\gamma_3^{8_A} = \gamma_3^{8_A}(-t) \Rightarrow \text{pole}$.
- Non-vanishing transition graphs (e.g. ).
- This complexity also seen in other methods
[See e.g. Caron-Huot, Gardi, White, 1701.05241, which also features transitions]

COLLINEAR PERSPECTIVE: NON-PLANAR 12

Conceptually straightforward in EFT—just include non-planar graphs and sub-leading color channels



▶ $\mathcal{O}_2^{(R)}(q_{1\perp}, q_{2\perp})$ for other $R \neq 8_A$

▶ $\chi_2^{8_A}(-t)$ doesn't change (well known—consequence of unitarity)

▶ $\chi_2^{(1)}(q_1, q_2)$ depends on $q_{1\perp}, q_{2\perp}$ separately $\Rightarrow$ Regge cut solution

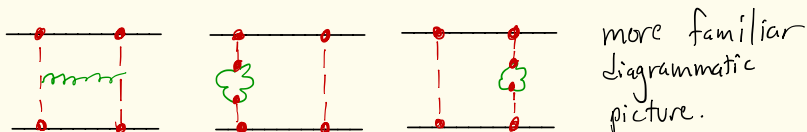
$$\nu \frac{d}{d\nu} \mathcal{O}_2^{(1)}(\vec{q}_1, \vec{q}_2) = \int d\vec{q}'_1 d\vec{q}'_2 \delta(\vec{q}'_1 + \vec{q}'_2 - \vec{q}) \gamma(\vec{q}_1, \vec{q}'_1) \mathcal{O}_2^{(1)}(\vec{q}'_1)$$

reproduces Pomeron Equation

▶ Other color channels work out as they should

$$\nu \frac{d}{d\nu} \mathcal{O}_2^{(R)} = \chi_2^{(R)} \otimes \mathcal{O}_2^{(R)} \text{ for } R \in (1, 8_A, 8_S, 10, \bar{10}, 27)$$

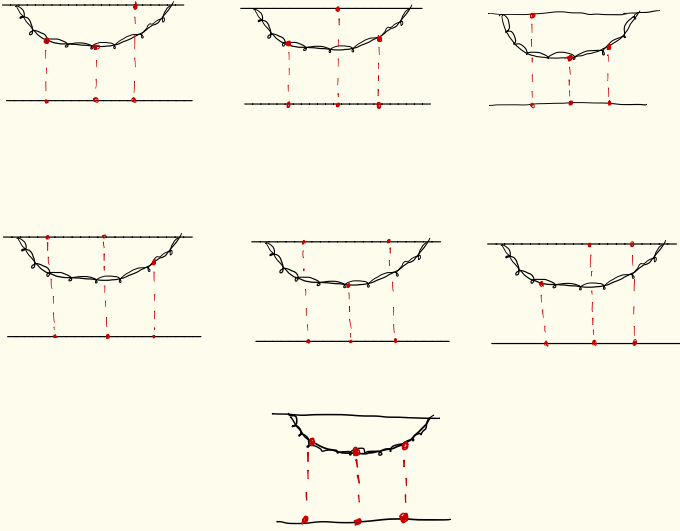
• From the Soft perspective, we recover the same results



COLLINEAR PERSPECTIVE: NON-PLANAR

(13)

$n=3$:



► $\mathcal{O}_3^{(R)}(q_{1\perp}, q_{2\perp}, q_{3\perp})$

► $\mathcal{Z}_3^{8_A}(-t)$ has $\frac{1}{N_c}$ corrections

► These graphs yield the evolution equations

$$\nu \frac{d}{d\nu} \mathcal{O}_3^{(R)}(q_{1\perp}, q_{2\perp}, q_{3\perp}, \nu) = \int d\vec{q}_{1\perp}' d\vec{q}_{2\perp}' d\vec{q}_{3\perp}' \mathcal{Z}_3^{(R)}(q_{i\perp}, q_{i\perp}') \mathcal{O}_3^{(R)}(q_{i\perp})$$

with $q_{1\perp} + q_{2\perp} + q_{3\perp} = q_{i\perp}$

- Will give the all-orders (in α_s) structure at NNLL including non-planar contributions
- Currently performing cross-checks (IR & Soft consistency)

FLANK

You