Multi-parton scattering amplitudes beyond 3 loops

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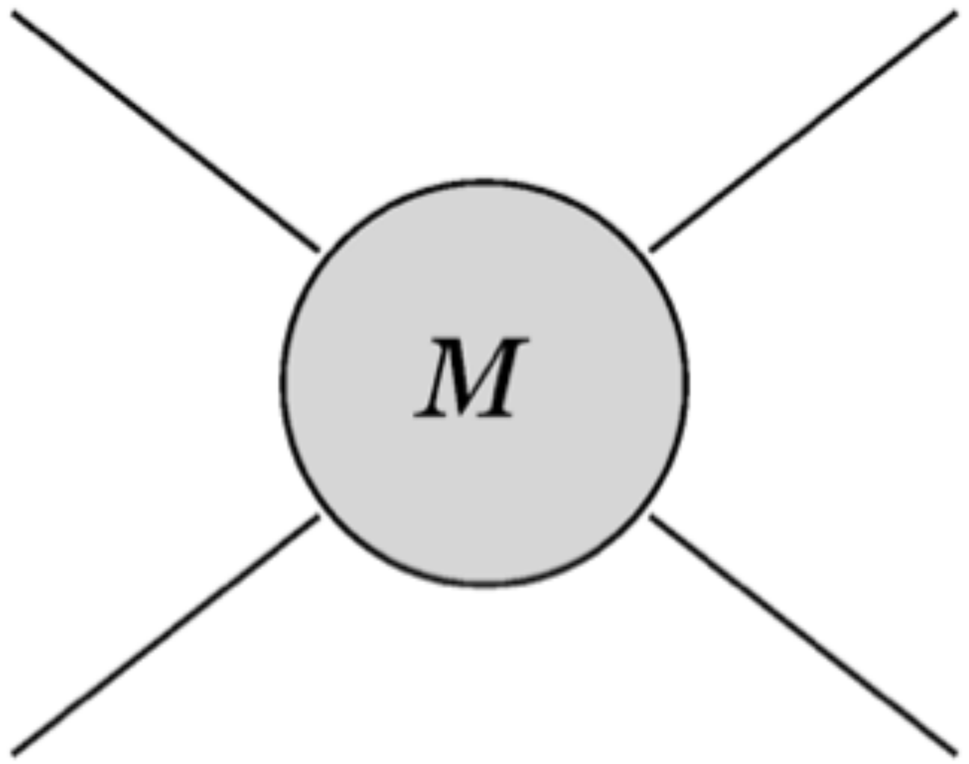
Plan of the talk

- Webs in multi-parton amplitudes
- Properties of web mixing matrices
- Results for web mixing matrices at 4 loops
- Summary

Multi-parton Scattering Amplitude
In IR limit

IR behaviour $\leftrightarrow$ Wilson line correlator



Soft matrix

$$\mathcal{S}_n\Big(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon\Big) \equiv \langle 0| \prod_{k=1}^{n} \Phi_{\beta_k}(\infty, 0)\, |0\rangle$$

Soft anomalous dimension

$$\mathcal{S}_n\Big(\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon\Big) = \mathcal{P} \exp\left[-\frac{1}{2}\int_0^{\mu^2} \frac{d\lambda^2}{\lambda^2}\, \mathbf{\Gamma}_n\Big(\beta_i \cdot \beta_j, \alpha_s(\lambda^2), \epsilon\Big)\right]$$

Diagrammatic Exponentiation
(A complementary approach)

Sum over Feynman diagrams
Kinematic factor $K(D)$
Color factor $C(D)$

$$S_n(\gamma_i) = \sum_D K(D)\, C(D)$$

Mitov, Sterman, Sung; 2010
Gardi, Laenen, Stavenga, White; 2010
Gardi, Smillie, White; 2011

$$S_n(\gamma_i) = \exp\left[\mathscr{W}_n(\gamma_i)\right]$$

Gardi, White; 2011
Dukes, Gardi, Steingrimsson, White; 2013
Gardi, Smillie, White; 2013

Same diagrams D,
modified colour factors $\widetilde{C}(D)$

$$\mathscr{W}(\gamma_i) = \sum_D K(D)\, \widetilde{C}(D)$$

Dukes, Gardi, McAslan, Scott, White; 2016

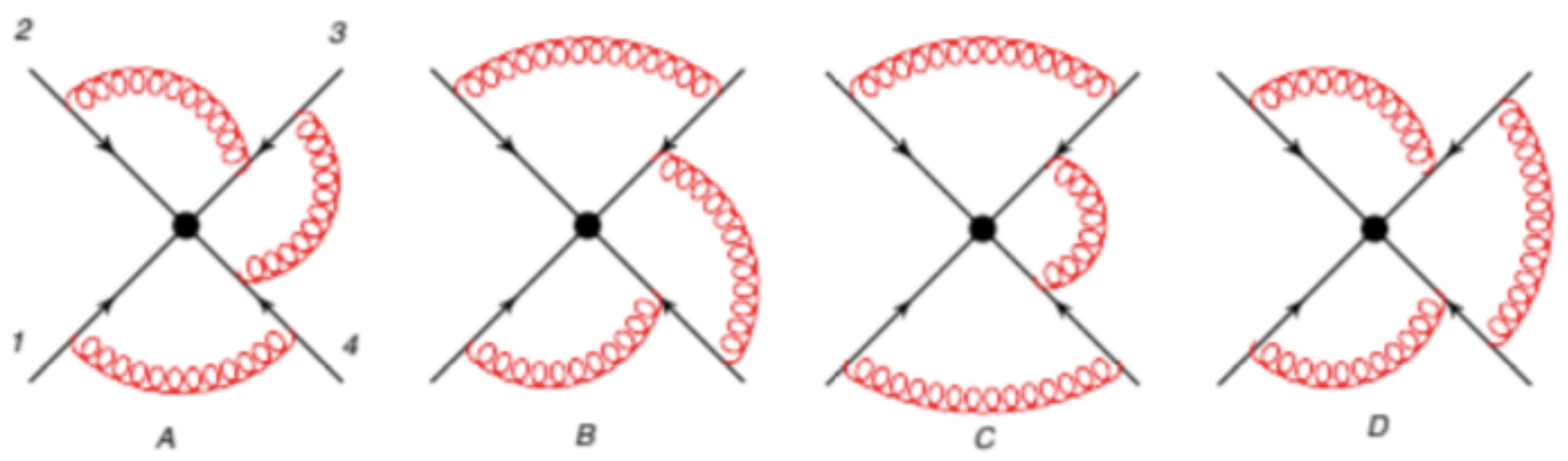
See also: Vladimirov, 2014-2017 for Alternative approach

For Eikonal Form factors these are called webs **Gatheral; Frenkel, Taylor; Sterman**

Multi-parton Webs

Web (w) : A set of diagrams closed under permutations of the gluon attachments on the Wilson lines.

(Gardi, Smillie, White, *et al*, *2010-2013*)



The exponent $W(\gamma_i)$
grouped into webs

$$S_n = \exp\left(\sum w\right)$$

$R_w(D, D')$
Web mixing matrix

$$S_n = \exp\left(\sum_{D,D'\in w} K(D)\, R_w(D,D')\, C(D)\right)$$

Properties of web mixing matrices

(Gardi, Smillie, White, et al 2010-2013)

Idempotent

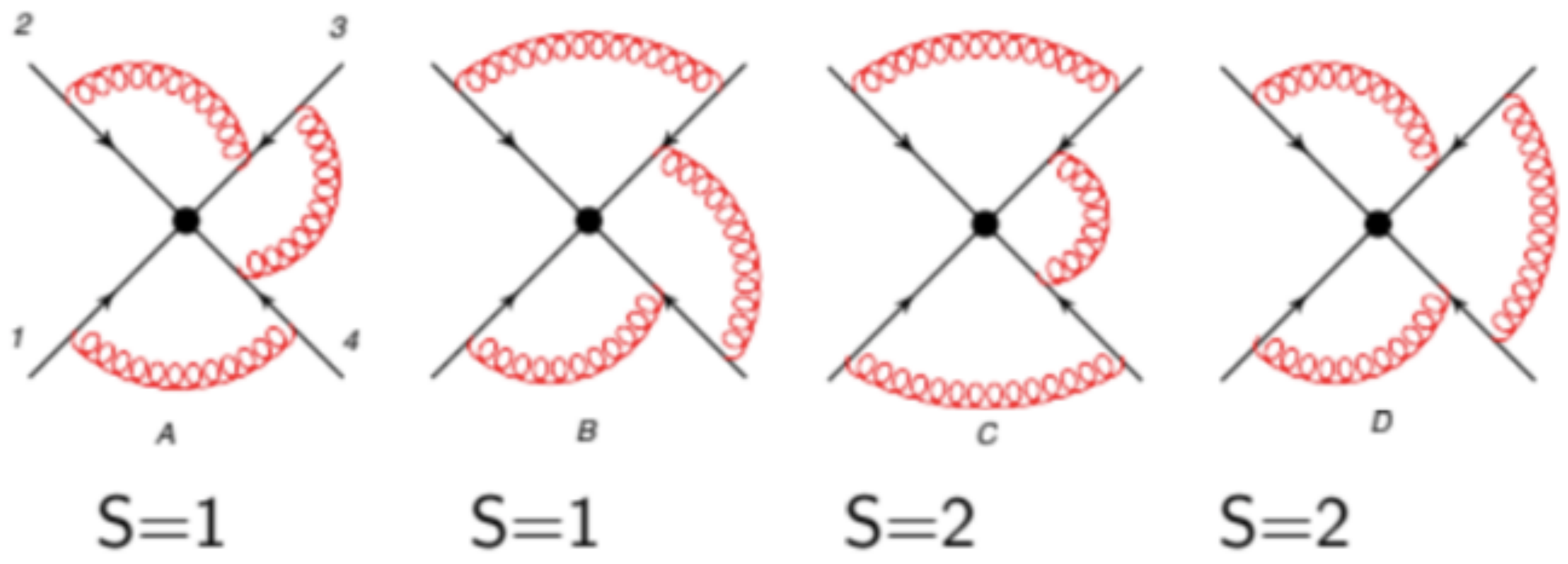
$R_w^2 = R_w$, Projection Operators

Projector

Non-abelian exponentiation theorem.

(Gardi, Smillie, White, 2013)

Projects onto colour factors that correspond to fully connected gluon diagrams



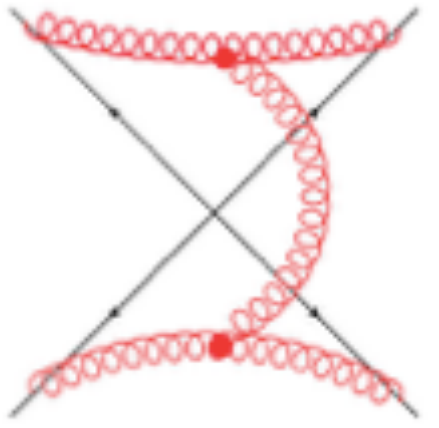
Row sum rule

$$\sum_{D'} R_w(D, D') = 0$$

Column sum rule (Conjecture)

$$\sum_{D} s(D)\, R_w(D, D') = 0$$

$\Longrightarrow$



Connection with Mathematical structures (Posets)

(Dukes, Gardi, McAslan, Scott, White)

Mixing matrices

Cwebs Set of diagrams built out of gluon correlators

Agarwal, Danish, Magnea, Pal, AT ; 2020

Replica Trick N_r identical copies of gauge fields are introduced, Wilson lines are replicated

Gardi, Laenen, Stavenga, White, 2010

See also: Vladimirov, 2014-2017

Replicated correlator

$$\mathcal{S}_n^{\text{repl.}}(\gamma_i) = \left[\mathcal{S}_n(\gamma_i)\right]^{N_r} = \exp\left[N_r\,\mathcal{W}_n(\gamma_i)\right]$$

Order N_r term

$$= \mathbf{1} + N_r\,\mathcal{W}_n(\gamma_i) + \mathcal{O}(N_r^2)$$

Combinatorics to extract ECF

- Assign replica number i to each connected gluon correlator
- Replica ordering operator to order colour generators $\mathbf{T}_k^i$ on each line
- # of hierarchies $h(m)$ between m replica numbers
- …
- Algorithm gives ECF

Inhouse Mathematica Code

The algorithm from generation of diagrams → computation ECF is implemented →Mixing matrices

Results at 4 loops

Agarwal, Danish, Magnea, Pal, AT ; 2020

Wilson line Correlators (**Cwebs**)	# of webs	Largest dimension of mixing matrix
5 legs	9	24
4 legs	21	24
3 legs	23	36
2 legs	8	36

Loop order (m)	Maximum number of hierarchies
1	**1**
2	3
3	13
4	75
5	541
6	**4683**

Fubini numbers
1,3,13,75,541,4683, …

Generating Function of Fubini numbers $h(m)$

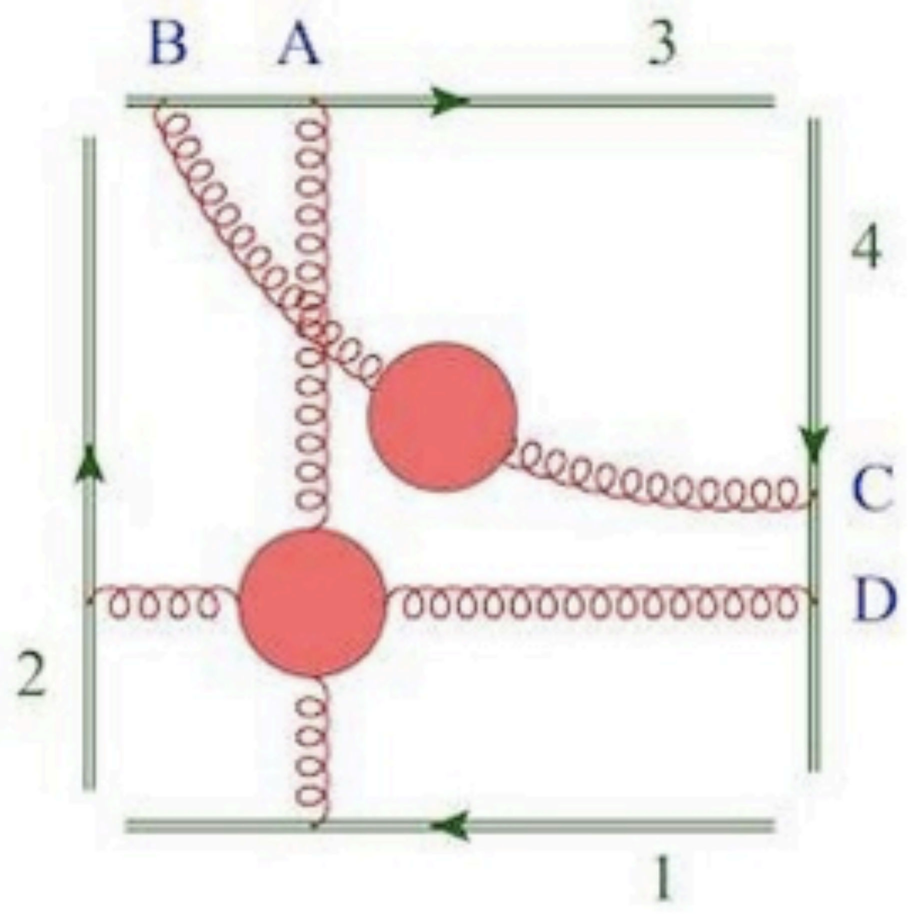
$$\frac{1}{2-\exp(x)} - 1 \equiv \sum_{m=1}^{\infty} h(m) \frac{x^m}{m!} .$$

Results at 4 loops

(One of the 4-leg webs)

$\mathbf{W}_{4,\mathrm{I}}^{(1,0,1)}(1,1,2,2)$

Agarwal, Danish, Magnea, Pal, AT ; 2020



Diagrams	Sequences	S-factors
C_1	$\{\{BA\},\{CD\}\}$	1
C_2	$\{\{BA\},\{DC\}\}$	0
C_3	$\{\{AB\},\{CD\}\}$	0
C_4	$\{\{AB\},\{DC\}\}$	1

$$R = \begin{pmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad D = (\mathbf{1}_3, 0)$$

Exponentiated Color factors

$$(YC)_1 = i f^{abg} f^{cdg} f^{edh} \mathbf{T}_1^a \mathbf{T}_2^b \mathbf{T}_3^e \mathbf{T}_3^c \mathbf{T}_4^h - i f^{abg} f^{cdg} f^{cej} \mathbf{T}_1^a \mathbf{T}_2^b \mathbf{T}_3^j \mathbf{T}_4^d \mathbf{T}_4^e ,$$

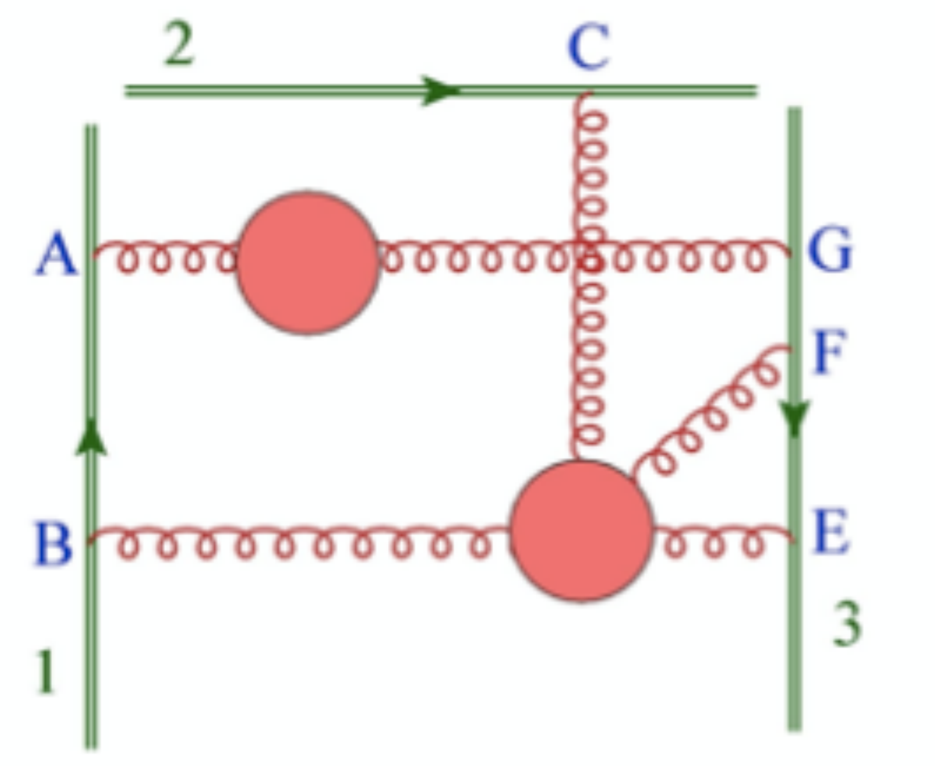
$$(YC)_2 = -i f^{abg} f^{cdg} f^{cej} \mathbf{T}_1^a \mathbf{T}_2^b \mathbf{T}_3^j \mathbf{T}_4^d \mathbf{T}_4^e ,$$

$$(YC)_3 = i f^{abg} f^{cdg} f^{edh} \mathbf{T}_1^a \mathbf{T}_2^b \mathbf{T}_3^e \mathbf{T}_3^c \mathbf{T}_4^h - f^{abg} f^{cdg} f^{cej} f^{edh} \mathbf{T}_1^a \mathbf{T}_2^b \mathbf{T}_3^j \mathbf{T}_4^h .$$

New Results at 4 loops

(3 and 2-leg webs)

AT et al (to appear)



Diagrams	Sequences	S-factors
C_1	$\{\{BA\},\{GFE\}\}$	0
C_2	$\{\{BA\},\{FGE\}\}$	0
C_3	$\{\{BA\},\{FEG\}\}$	1
C_4	$\{\{AB\},\{GFE\}\}$	1
C_5	$\{\{AB\},\{FGE\}\}$	0
C_6	$\{\{AB\},\{FEG\}\}$	0

$$R = \begin{pmatrix} 1 & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & 1 & 0 \\ 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 1 \end{pmatrix} \qquad D = (\mathbf{1}_5, 0)$$

Exponentiated Colour Factors

$$(YC)_1 = if^{afk}f^{bcg}f^{efg}\mathbf{T}_1^b\mathbf{T}_1^a\mathbf{T}_2^c\mathbf{T}_3^e\mathbf{T}_3^k + if^{aeh}f^{bcg}f^{efg}\mathbf{T}_1^b\mathbf{T}_1^a\mathbf{T}_2^c\mathbf{T}_3^h\mathbf{T}_3^f \\ -if^{abm}f^{bcg}f^{efg}\mathbf{T}_1^m\mathbf{T}_2^c\mathbf{T}_3^e\mathbf{T}_3^f\mathbf{T}_3^a$$

$$(YC)_2 = if^{afk}f^{bcg}f^{efg}\mathbf{T}_1^b\mathbf{T}_1^a\mathbf{T}_2^c\mathbf{T}_3^e\mathbf{T}_3^k - if^{abm}f^{bcg}f^{efg}\mathbf{T}_1^m\mathbf{T}_2^c\mathbf{T}_3^e\mathbf{T}_3^f\mathbf{T}_3^a$$

$$(YC)_3 = -if^{abm}f^{bcg}f^{efg}\mathbf{T}_1^m\mathbf{T}_2^c\mathbf{T}_3^e\mathbf{T}_3^f\mathbf{T}_3^a$$

$$(YC)_4 = if^{afk}f^{bcg}f^{efg}\mathbf{T}_1^a\mathbf{T}_1^b\mathbf{T}_2^c\mathbf{T}_3^e\mathbf{T}_3^k + if^{aeh}f^{bcg}f^{efg}\mathbf{T}_1^a\mathbf{T}_1^b\mathbf{T}_2^c\mathbf{T}_3^h\mathbf{T}_3^f$$

$$(YC)_5 = if^{afk}f^{bcg}f^{efg}\mathbf{T}_1^a\mathbf{T}_1^b\mathbf{T}_2^c\mathbf{T}_3^e\mathbf{T}_3^k$$

Summary

- Wilson line correlators obey diagrammatic exponentiation in terms of Cwebs.
- We have computed the colour structure of multi-parton Cwebs at 4 loops for 5,4,3 & 2 connected Wilson lines
- The mixing matrices obey the proven properties - Idempotency, Row sum rule, and Non-abelian exponentiation theorem.
- We observe that all the mixing matrices at 4 loops obey the conjectured Column sum rule.

Thank You!

Backup Slides

Soft Anomalous Dimension

IR behaviour of scattering amplitude $\leftrightarrow$ Wilson line correlator

Soft matrix

$$\mathcal{S}_n\Big(\beta_i\cdot\beta_j,\alpha_s(\mu^2),\epsilon\Big)\equiv\langle 0|\prod_{k=1}^{n}\Phi_{\beta_k}(\infty,0)\,|0\rangle$$

The Wilson line

$$\Phi_\beta(\infty,0)\equiv P\exp\left[ig\int_0^\infty d\lambda\,\beta\cdot\mathbf{A}(\lambda\beta)\right]$$

Soft anomalous dimension

$$\mathcal{S}_n\Big(\beta_i\cdot\beta_j,\alpha_s(\mu^2),\epsilon\Big)=\mathcal{P}\exp\left[-\frac{1}{2}\int_0^{\mu^2}\frac{d\lambda^2}{\lambda^2}\,\mathbf{\Gamma}_n\Big(\beta_i\cdot\beta_j,\alpha_s(\lambda^2),\epsilon\Big)\right]$$

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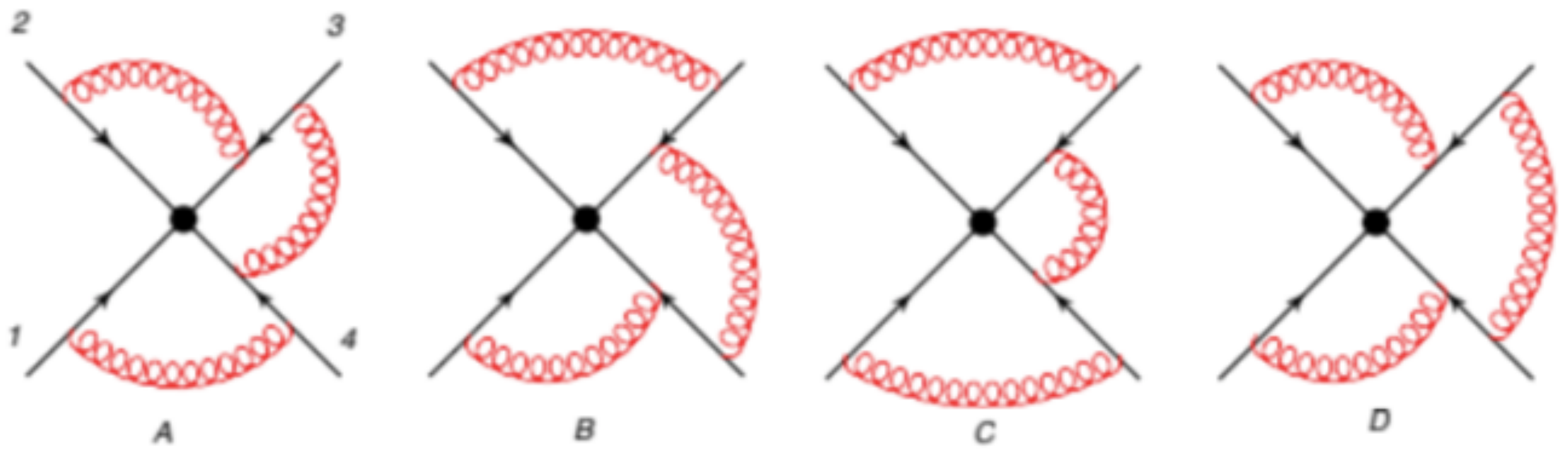
$$= \exp\left(\sum_{D,D'\in w} K(D)\, R_w(D,D')\, C(D)\right)$$

$R_w(D,D')$
Web mixing matrix

A 3 loop web
4×4 **mixing matrix**



S=1 S=1 S=2 S=2