BERNHARD MISTLBERGER SLAC

COLLINEAR EXPANSIONS OF CROSS SECTIONS

With **Markus A. Ebert and Gherardo Vita**

arXiv:2006.03055 arXiv:2006.03056 arXiv:2006.05329

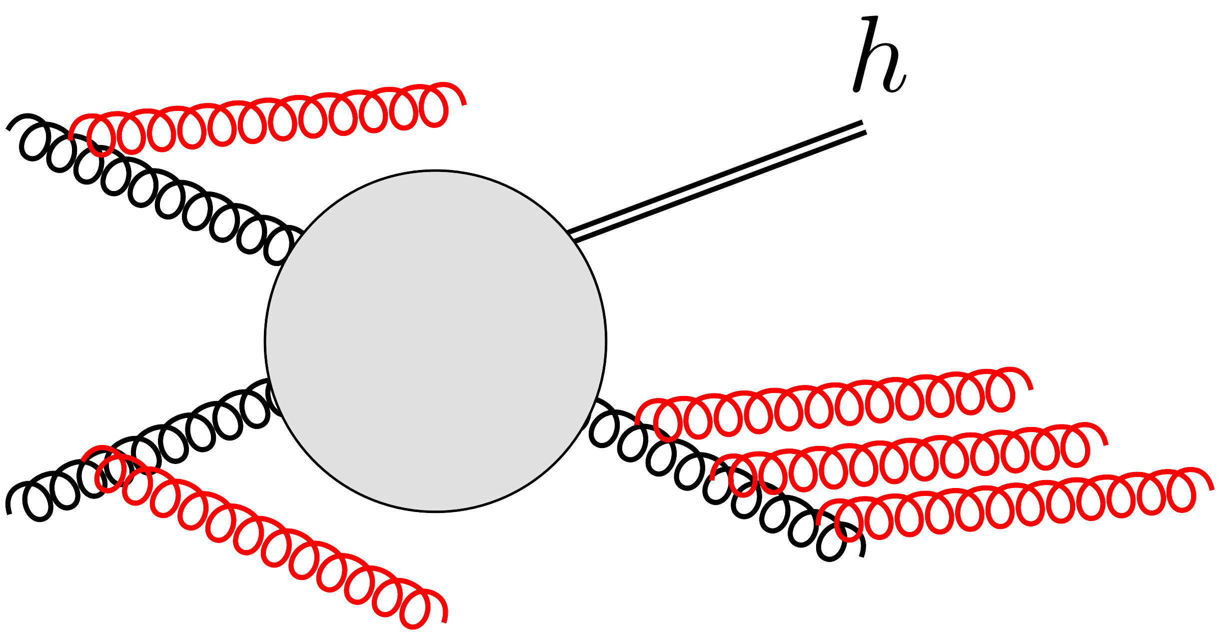
OUTLINE

1. **Motivation to look at Kinematic Limits**
2. **Expanding around Collinear Limits**
3. **Applications**

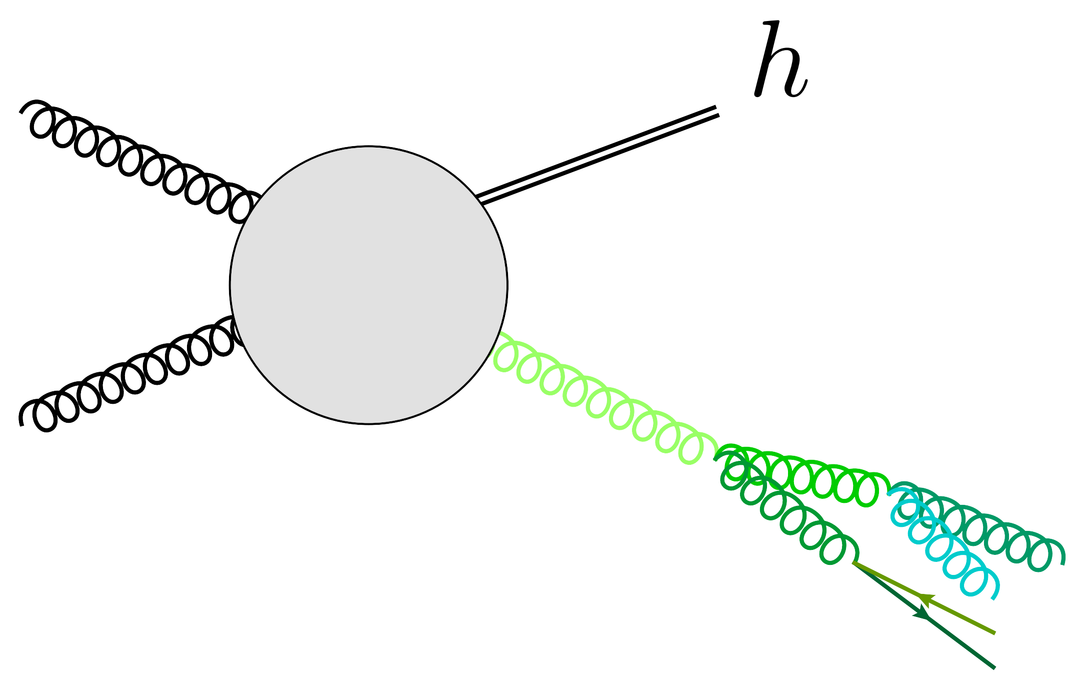
Introduction: Interesting Kinematic Limits

- Hard scattering processes are described in terms of the scattering of particles.

Soft Particles



Collinear Particles



- Breakdown of perturbation theory in limit of small particle energies or angles.

KINEMATIC LIMITS OF CROSS SECTIONS

Motivation:

Improve our Understanding and our Ability to make Predictions

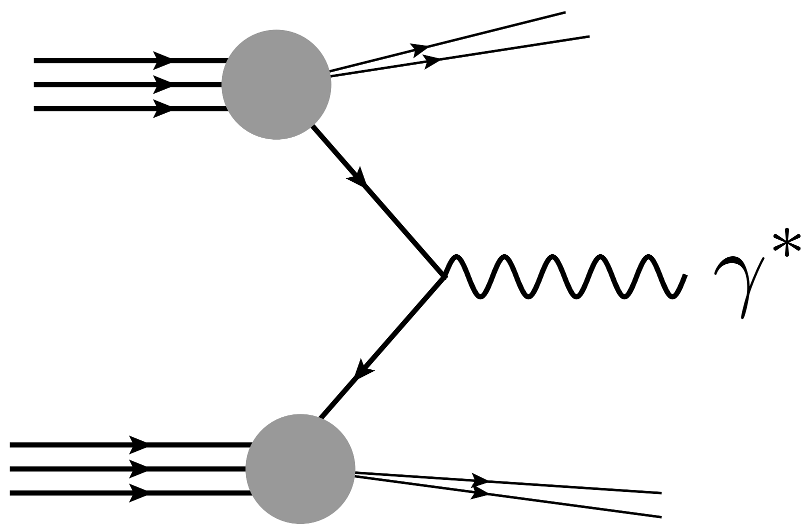
- Learn about structure of kinematic limits
- Improve fixed order precision predictions by obtaining ingredients and understanding for subtraction schemes.
- Obtain universal quantities for resummed cross sections.
- Find new ways to approximate perturbative cross sections.
- Study the universal structure of QCD beyond the leading term in kinematic expansions (beyond leading power)
- …

OUTLINE

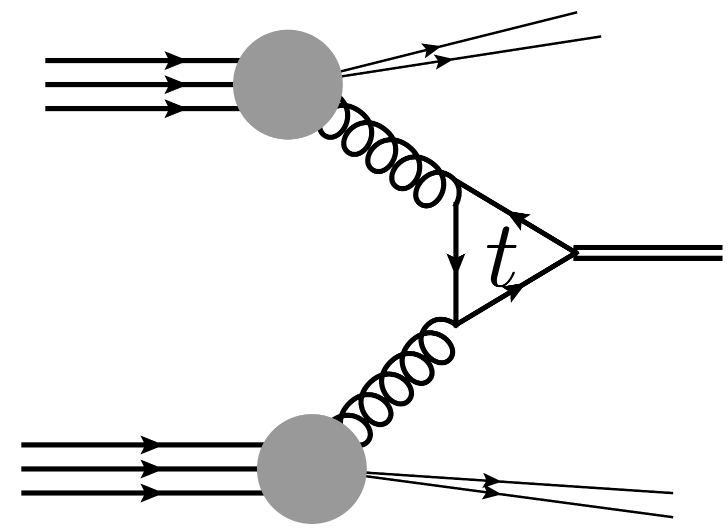
1. Motivation to look at Kinematic Limits
2. Expanding around Collinear Limits
3. Applications

FOCUS ON PRODUCTION OF A COLOUR SINGLET

Drell - Yan



Higgs

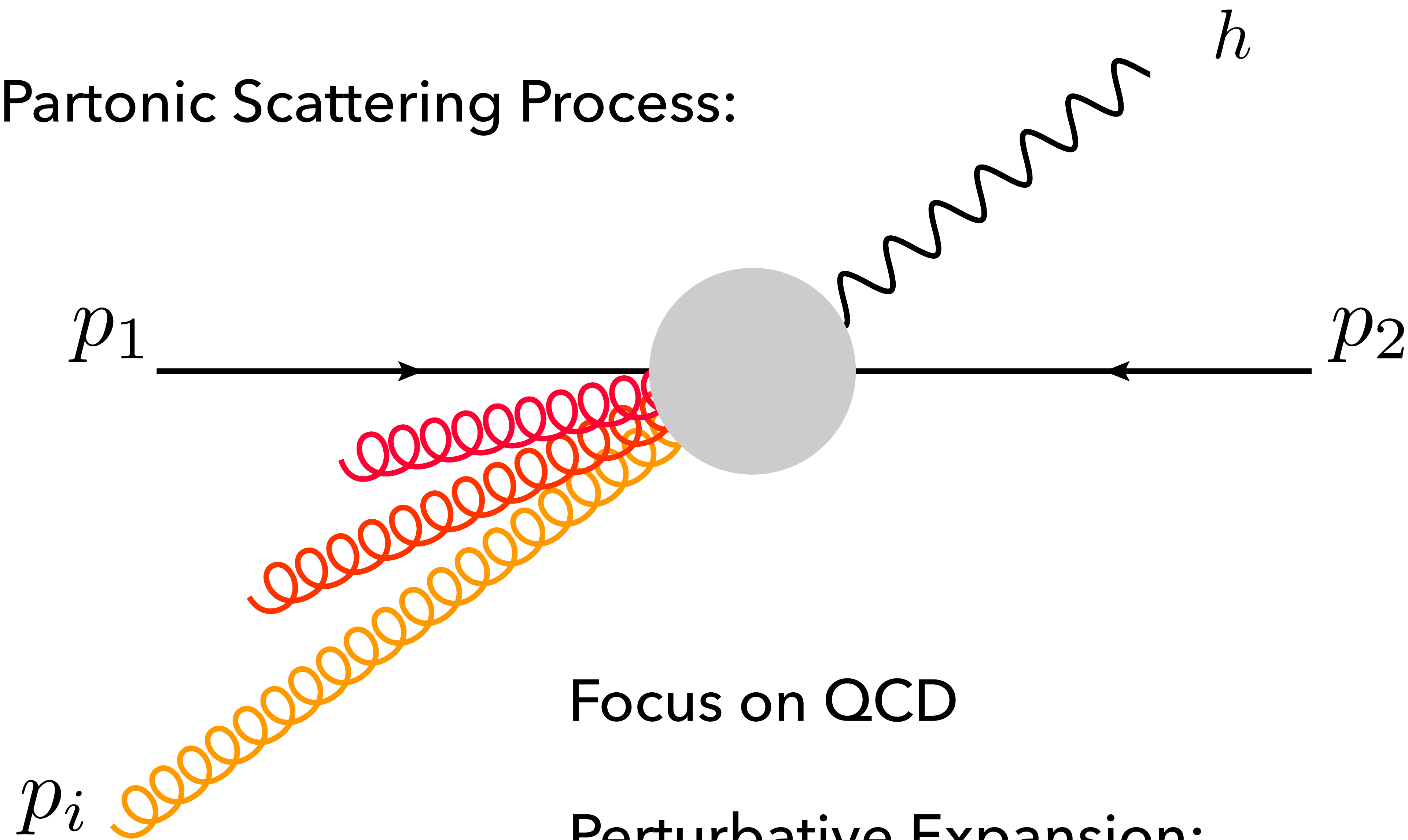


- Having these specific processes in mind.
- Applicable to much larger class of processes.

$$\gamma^* \; W \; Z \;\; H \quad bbH \quad H^* \; G \qquad WH \;\; ZH$$

PERTURBATIVE EXPANSIONS

Partonic Scattering Process:



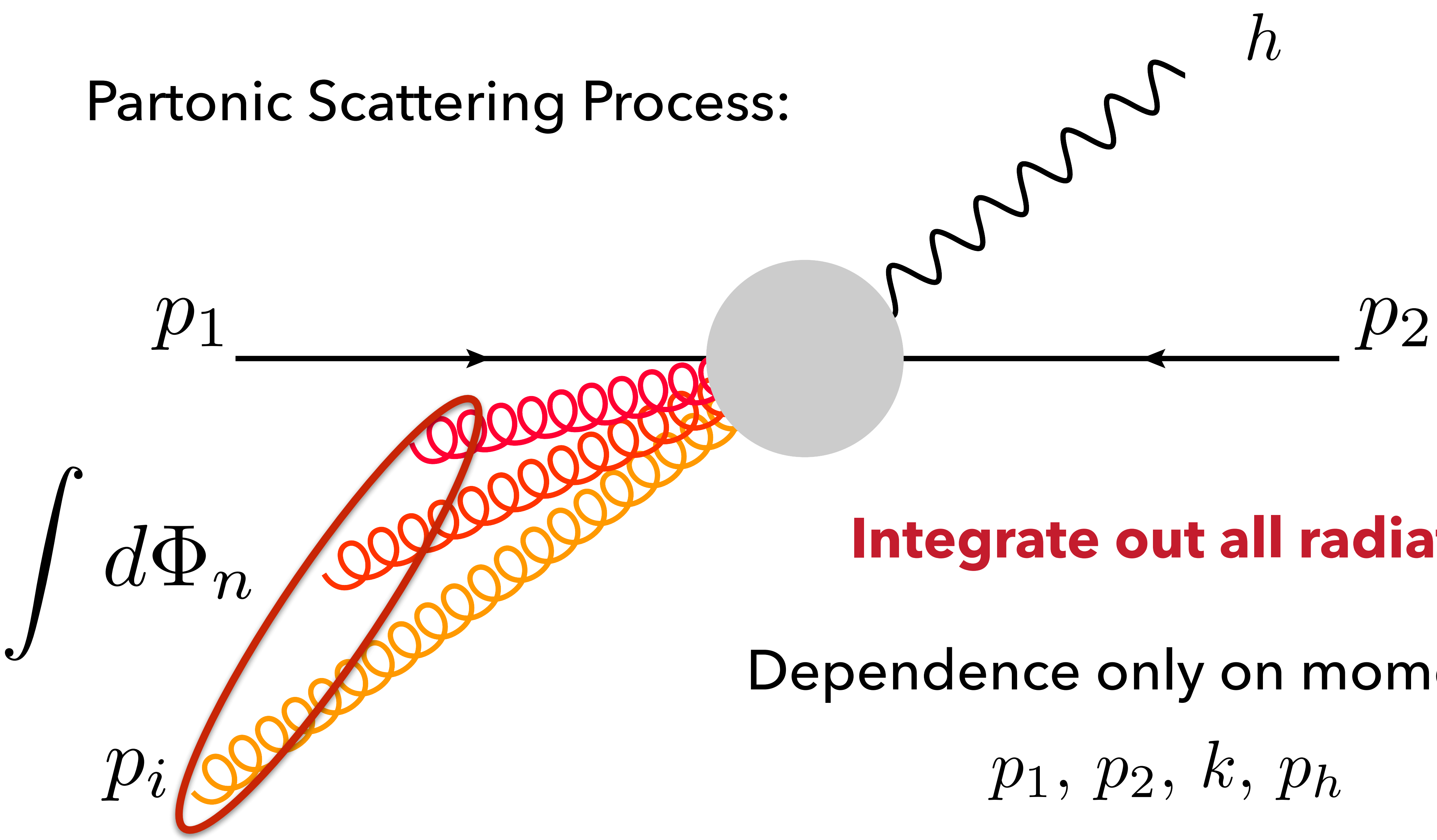
Focus on QCD

Perturbative Expansion:

$$\hat{\sigma} = \hat{\sigma}^{(0)} + \alpha_S^1 \hat{\sigma}^{(1)} + \alpha_S^2 \hat{\sigma}^{(2)} + \alpha_S^3 \hat{\sigma}^{(3)} \ldots$$

INTEGRATE OUT RADIATION

Partonic Scattering Process:



Integrate out all radiation

Dependence only on momenta

$$p_1,\ p_2,\ k,\ p_h$$

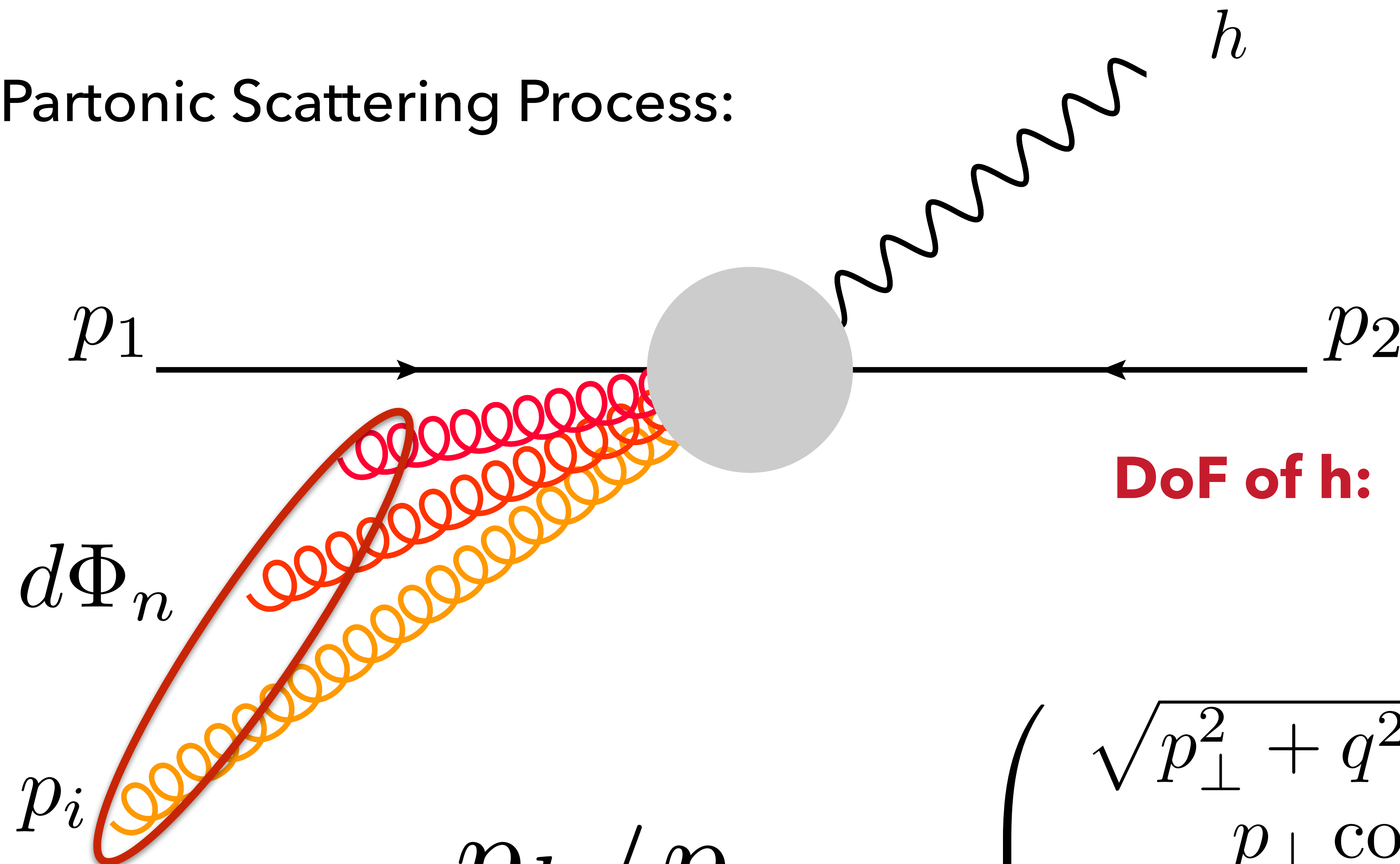
$$k = \sum_i p_i$$

Momentum conservation:

$$p_1 + p_2 + p_h + k = 0$$

INTEGRATE OUT RADIATION

Partonic Scattering Process:



$$\int d\Phi_n$$

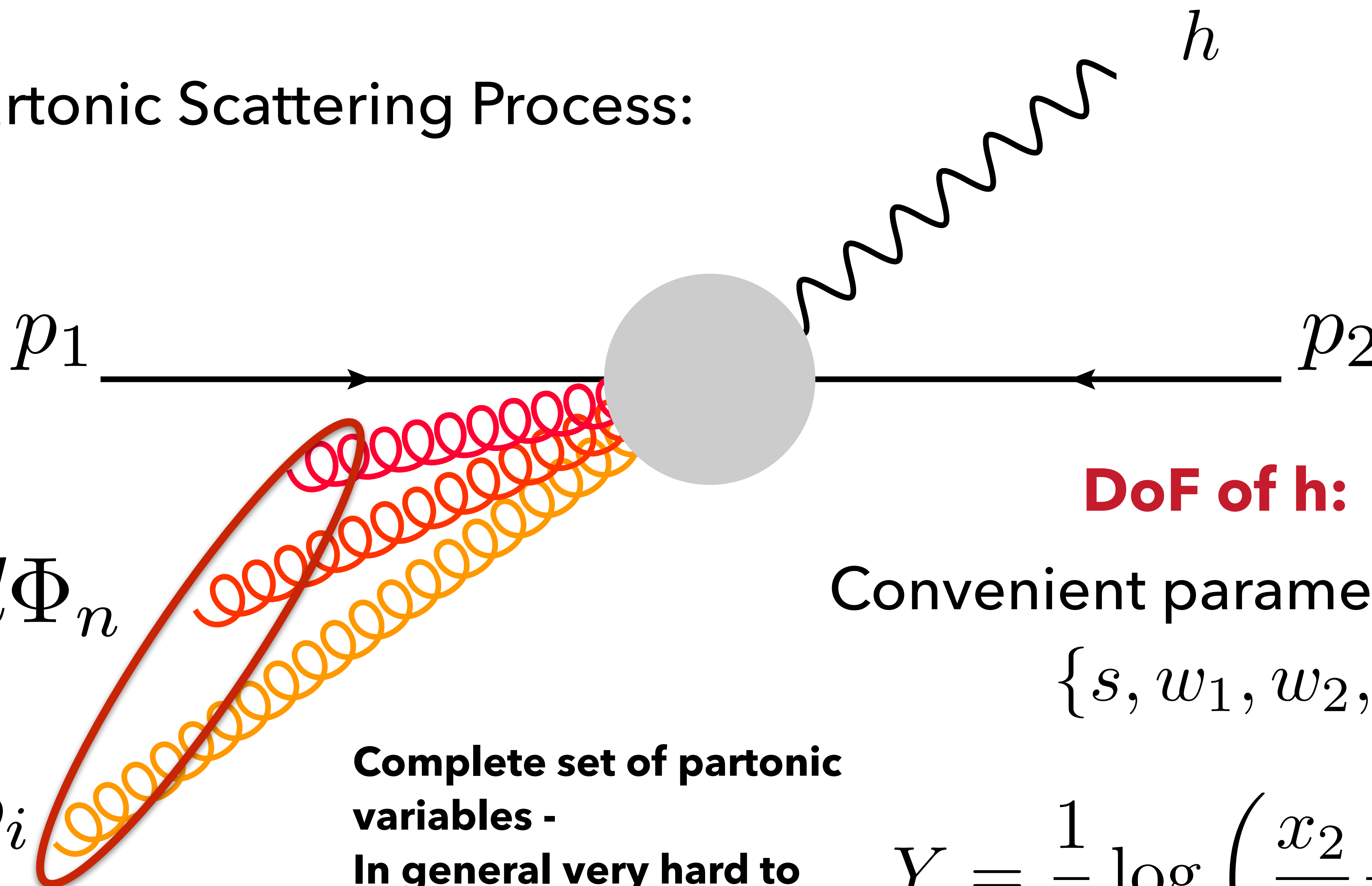
$$k = \sum_i p_i$$

DoF of h:

$$p_h / p_{\gamma^*} = \begin{pmatrix} \sqrt{p_\perp^2 + q^2} \cosh Y \\ p_\perp \cos\phi \\ p_\perp \sin\phi \\ \sqrt{p_\perp^2 + q^2} \sinh Y \end{pmatrix}$$

INTEGRATE OUT RADIATION

Partonic Scattering Process:



$$\int d\Phi_n$$

DoF of h:

Convenient parametrisation

$$\{s, w_1, w_2, x\}$$

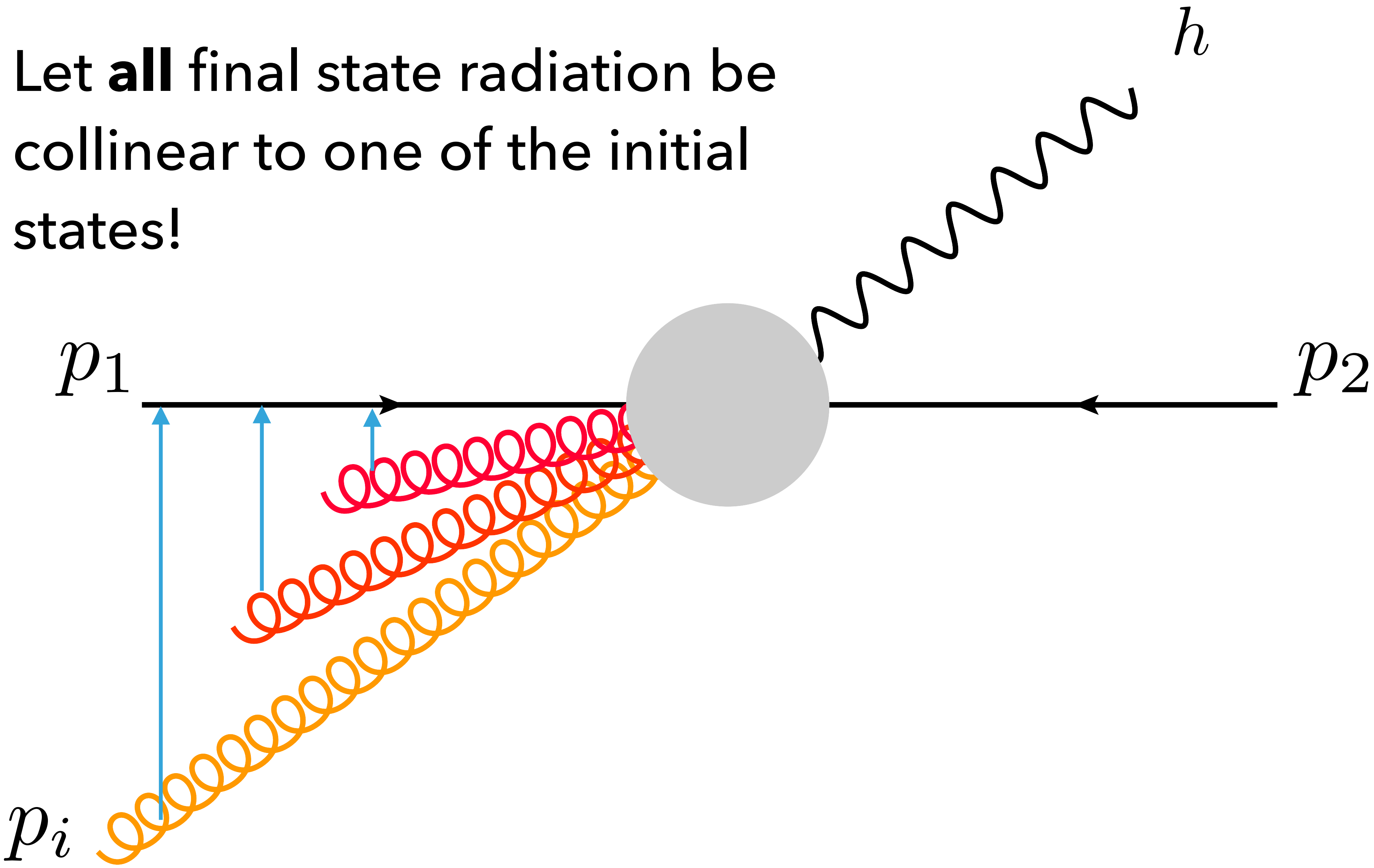
Complete set of partonic variables - In general very hard to compute!

$$Y = \frac{1}{2}\log\left(\frac{x_2}{x_1}\frac{1-w_1}{1-w_2}\right)$$

$$k = \sum_i p_i \qquad s = (p_1 + p_2)^2 \qquad p_\perp^2 = s w_1 w_2 (1-x)$$

EXPANDING AROUND COLLINEAR LIMITS

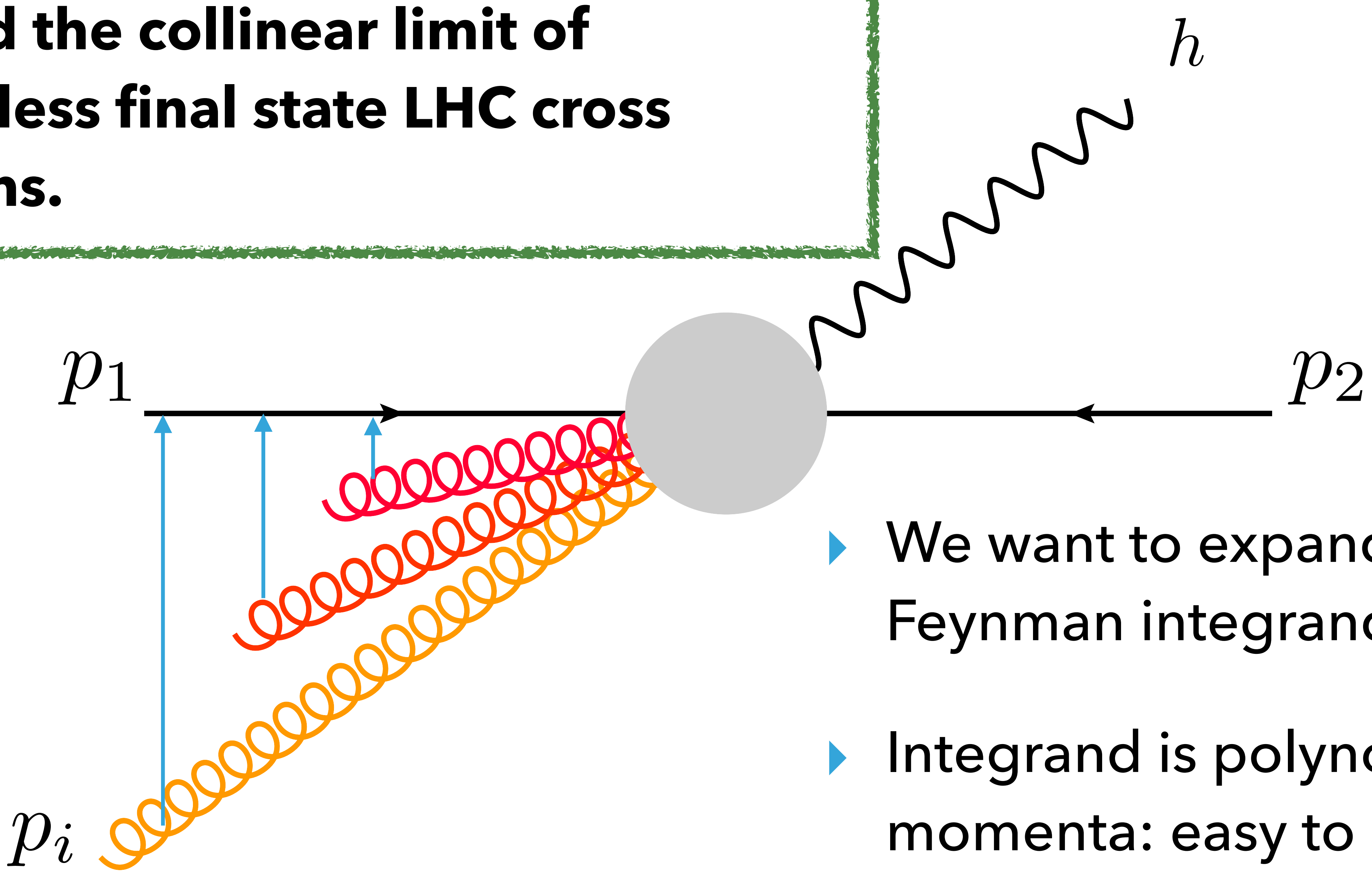
Let **all** final state radiation be collinear to one of the initial states!



EXPANDING AROUND COLLINEAR LIMITS

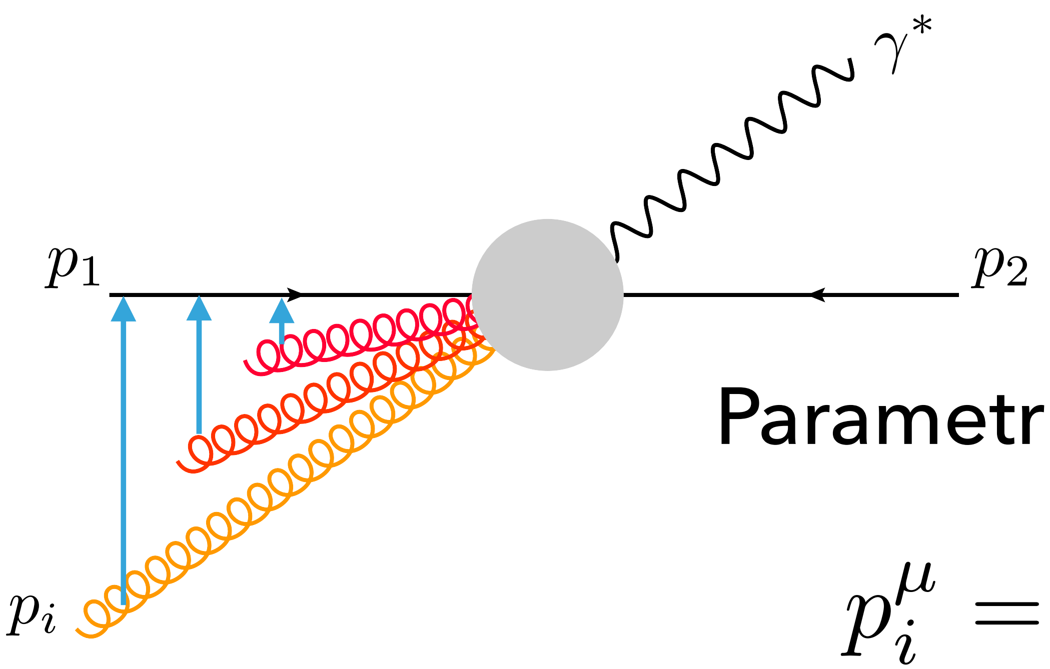
Goal:

Formulate a systematic expansion around the collinear limit of colourless final state LHC cross sections.



- We want to expand the Feynman integrand!
- Integrand is polynomial in momenta: easy to expand!
- Work in DimReg with $d = 4 - 2\epsilon$

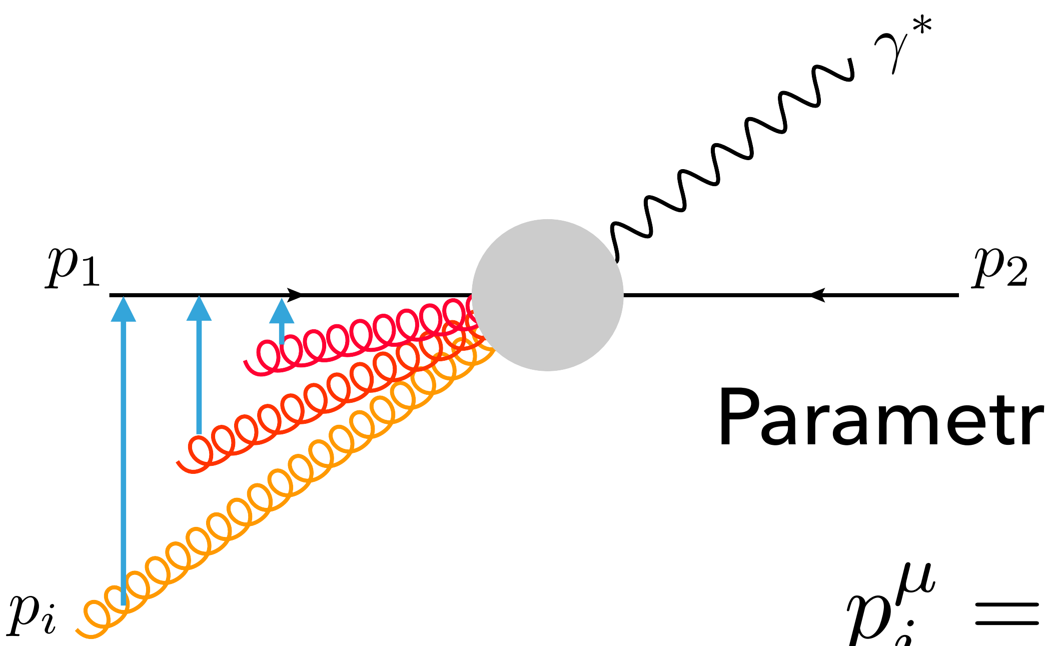
RESCALING TRANSFORMATION



Parametrise all final state momenta.

$$p_i^\mu = \frac{(2p_2p_i)}{s}p_1^\mu + \frac{(2p_1p_i)}{s}p_2^\mu + p_{i,\perp}^\mu.$$

RESCALING TRANSFORMATION



Parametrise all final state momenta.

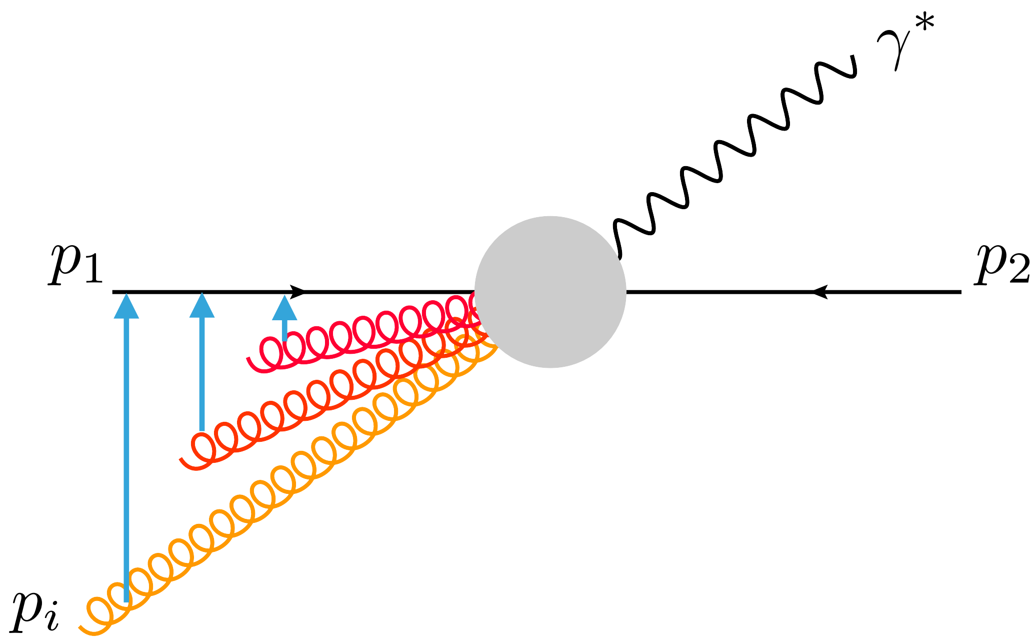
$$p_i^\mu = \frac{(2p_2p_i)}{s}p_1^\mu + \frac{(2p_1p_i)}{s}p_2^\mu + p_{i,\perp}^\mu.$$

Define a rescaling transformation.

$$p_i \to \lambda\frac{(2p_2p_i)}{s}p_1^\mu + \frac{(2p_1p_i)}{s}p_2^\mu + \sqrt{\lambda}p_{i,\perp}^\mu.$$

Artificial, small expansion parameter: λ

Same as for SCET / CSS / Splitting functions.



Transformation on variables:

$$s \to s$$
$$x \to x$$
$$w_1 \to w_1$$

$$w_2 \to \lambda w_2$$

Rescaling transformation.

$$p_i \to \lambda \frac{(2p_2 p_i)}{s} p_1^\mu + \frac{(2p_1 p_i)}{s} p_2^\mu + \sqrt{\lambda} p_{i,\perp}^\mu .$$

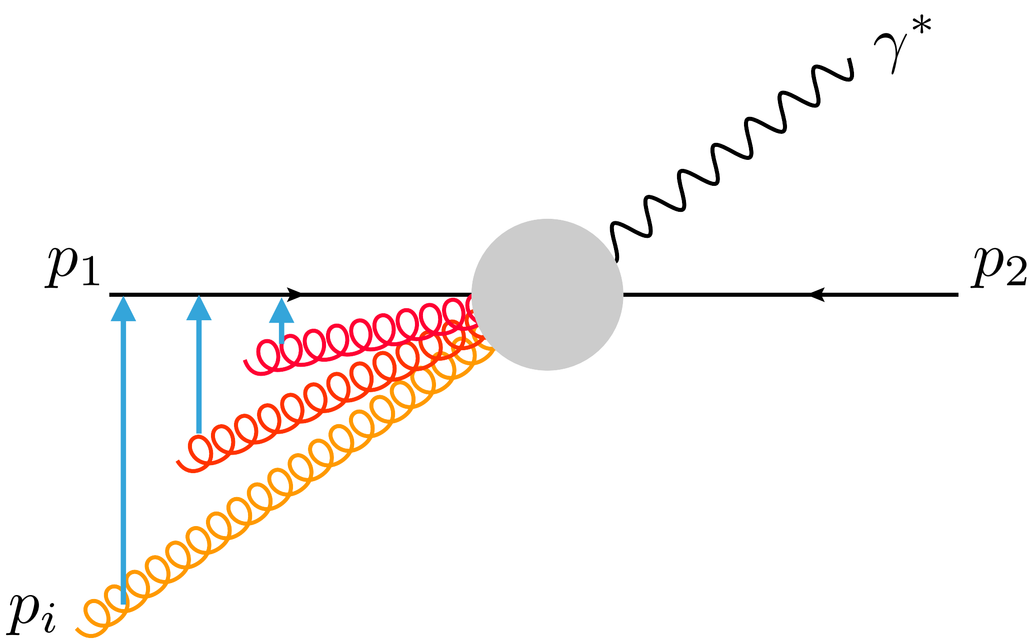
Variables:

$$w_i = -\frac{2p_i k}{2p_1 p_2} \qquad x = \frac{k^2}{s w_1 w_2}$$

At the integrated level the expansion will automatically yield an expansion in

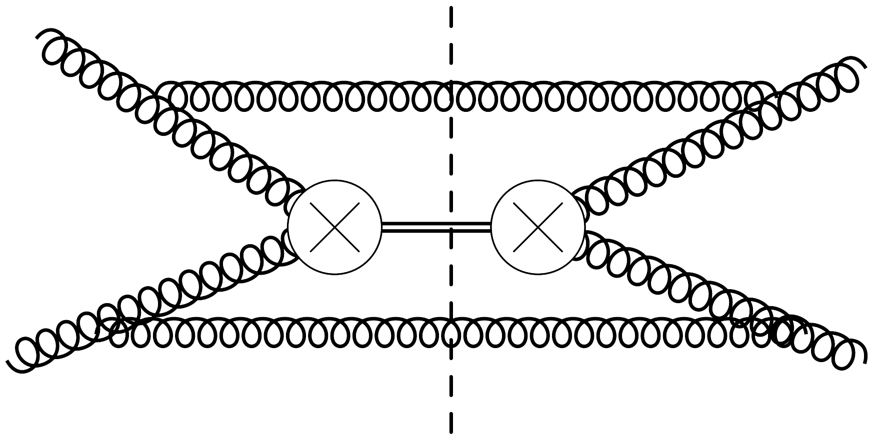
$$w_2$$

COLLINEAR EXPANSION



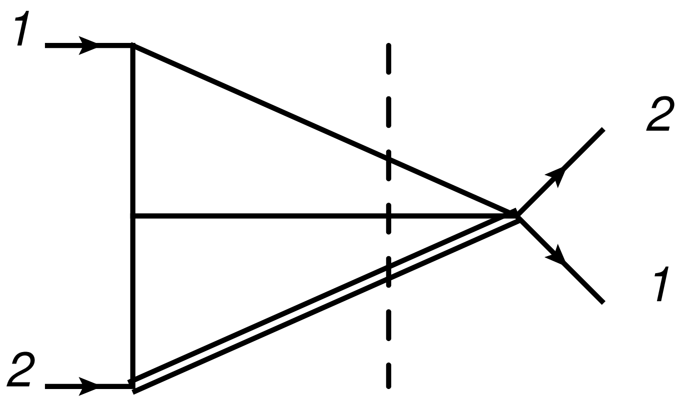
- We want to expand the Feynman integrand!
- Integrand is polynomial in momenta: easy to expand!

- Example: Phase Space Integrals:



PHASE SPACE INTEGRALS

- A simple double-real phase space integral:



$$= \int d\Phi_2^{\text{diff}} \frac{1}{(p_1+p_3)^2(p_1+p_3+p_4)^2}$$

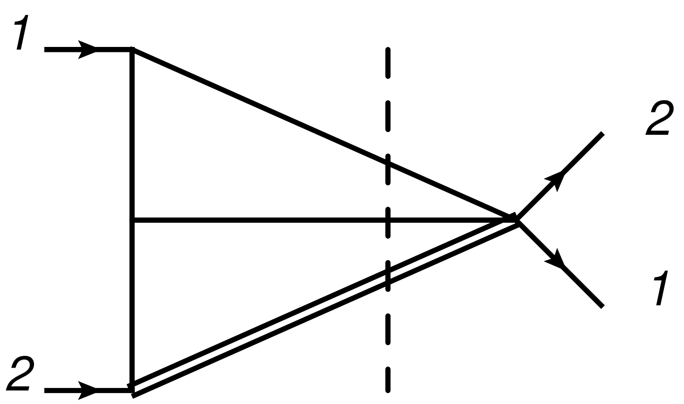
- One propagator rescales non-trivially

$$\frac{1}{(p_1+p_3+p_4)^2} = \frac{1}{2p_1p_3+2p_1p_4+2p_3p_4} \to \frac{1}{2p_1p_3+2p_1p_4+2\lambda p_3p_4}$$

$$= \sum_{i=0}^{\infty}(-\lambda)^i \frac{(2p_3p_4)^i}{(2p_1p_3+2p_1p_4)^{1+i}}.$$

PHASE SPACE INTEGRALS

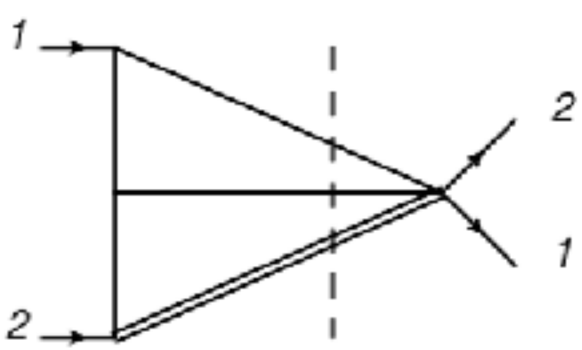
- Expanding our integral



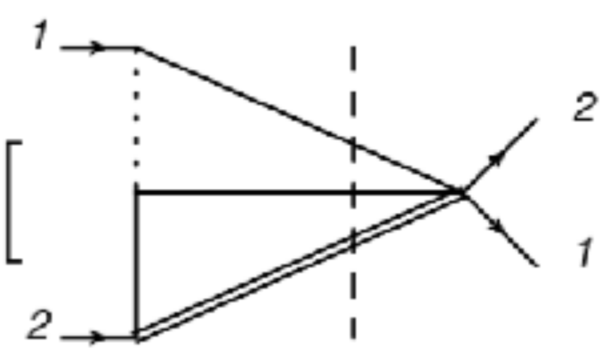
$$\rightarrow \sum_{i=0}^{\infty} (-\lambda)^{1+i-2\epsilon} \int d\Phi_2^{\text{diff}} \frac{(2p_3p_4)^i}{(p_1+p_3)^2(2p_1p_3+2p_1p_4)^{1+i}}.$$

~Collinear Wilson line!

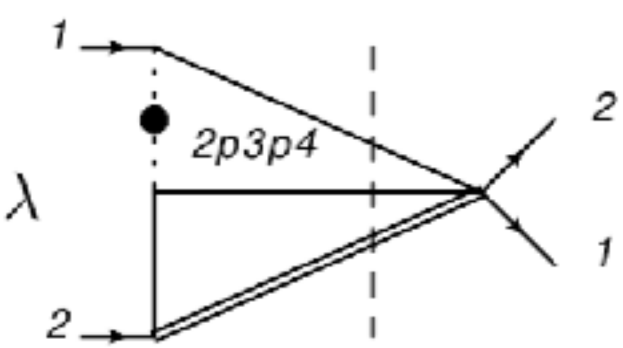
- Diagrammatic representation for terms in our expansion



$\rightarrow \lambda^{1-2\epsilon} \Big[$



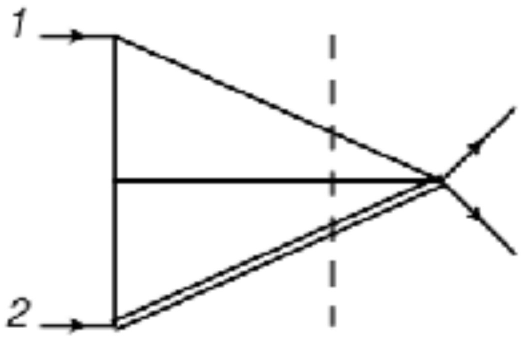
$- \lambda$



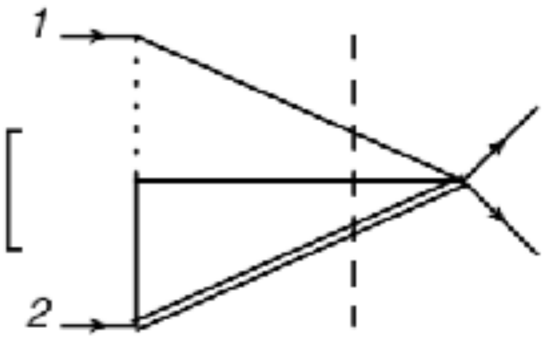
$+ \mathcal{O}(\lambda^2) \Big]$

PHASE SPACE INTEGRALS

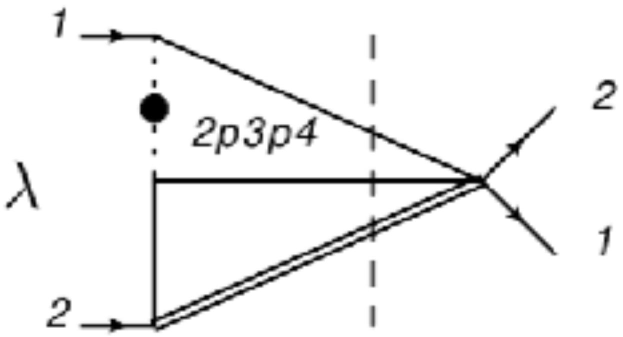
- Diagrammatic representation for terms in our expansion



$$\to \lambda^{1-2\epsilon}\Big[$$

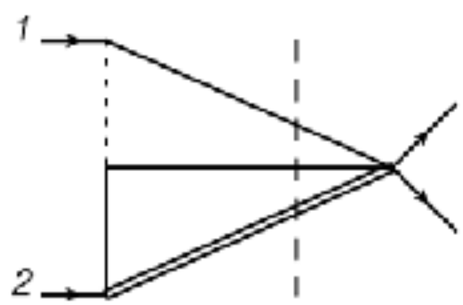


$$-\,\lambda$$

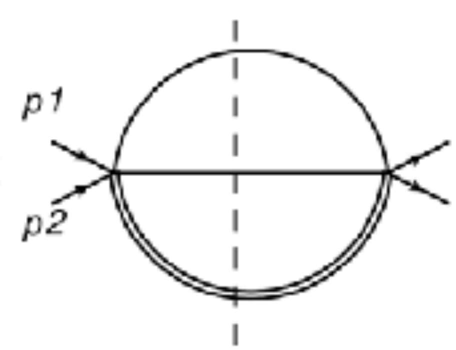


$$+\,\mathcal{O}(\lambda^2)\Big]$$

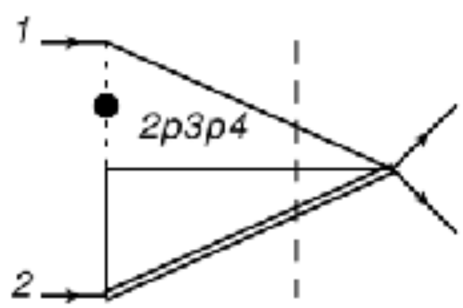
- IBP identities!



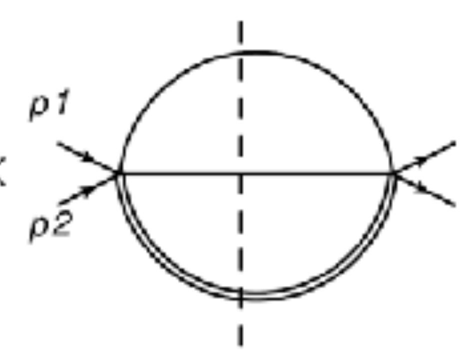
$$= -\frac{1-2\epsilon}{w_1^2\epsilon}\times$$



.

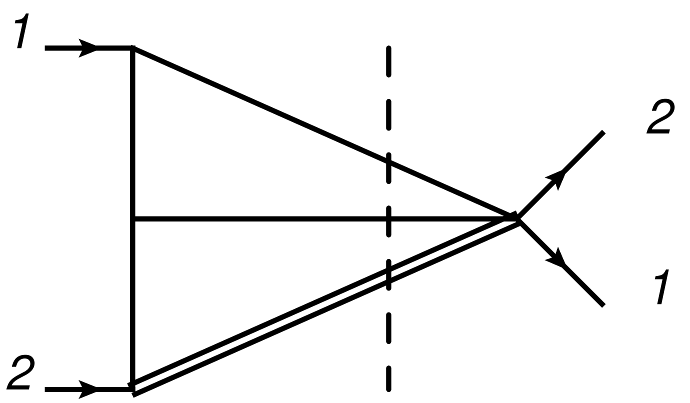


$$= w_2 x\frac{1-2\epsilon}{w_1^2\epsilon}\times$$

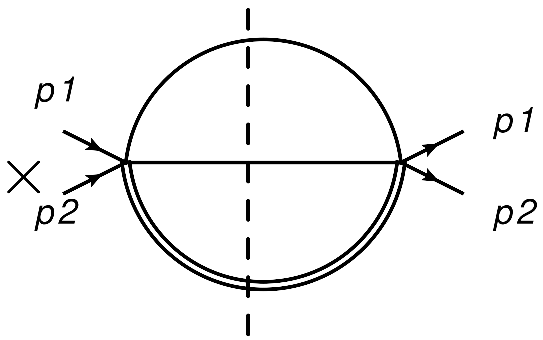


PHASE SPACE INTEGRALS

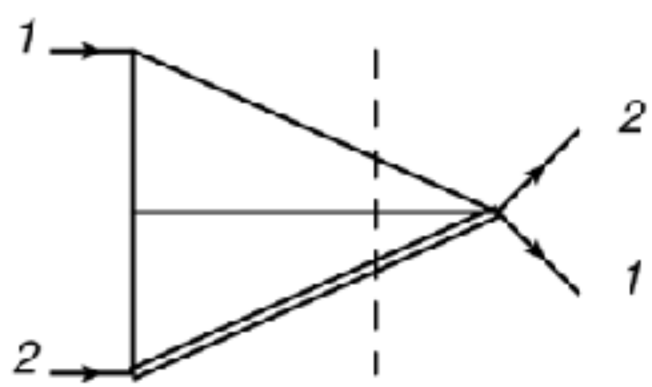
- Series expansion for our example



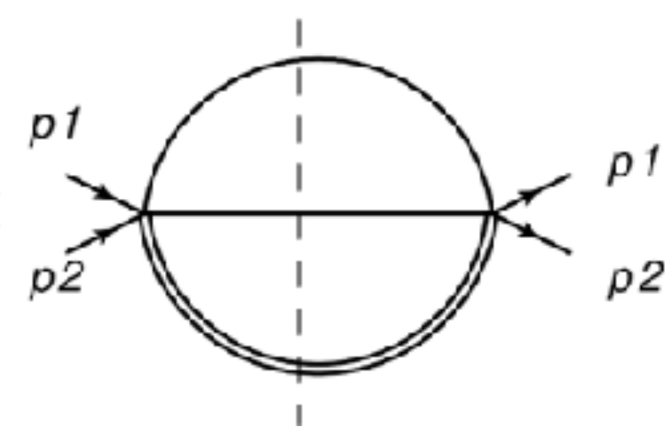
$$\rightarrow \quad -\frac{1-2\epsilon}{w_1^2\epsilon}\lambda^{1-2\epsilon} \times \left[1 + \lambda w_2 x + \mathcal{O}(\lambda^2)\right] \quad \times$$



- Indeed matches the exact result!

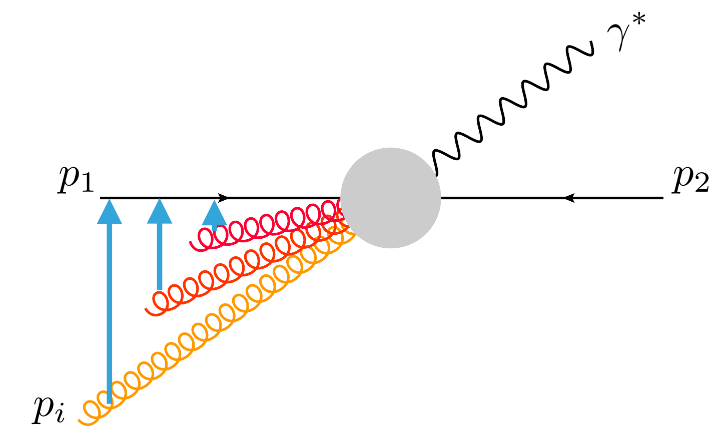


$$= \quad -\frac{1-2\epsilon}{w_1^2\epsilon}\frac{1}{1-w_2 x} \quad \times$$



- Loops: Method of Regions

SUMMARY



- Method for the expansion of cross sections around the collinear limit.
- Parametric expansion in terms of w_2
- Systematically improvable.
- Expansion at the integrand level.
- Simplification of individual terms in the expansion. (IBPs, Differential Eqs., etc.)
- Higher order terms contain the same functions as first order terms and are easily relatable to those.

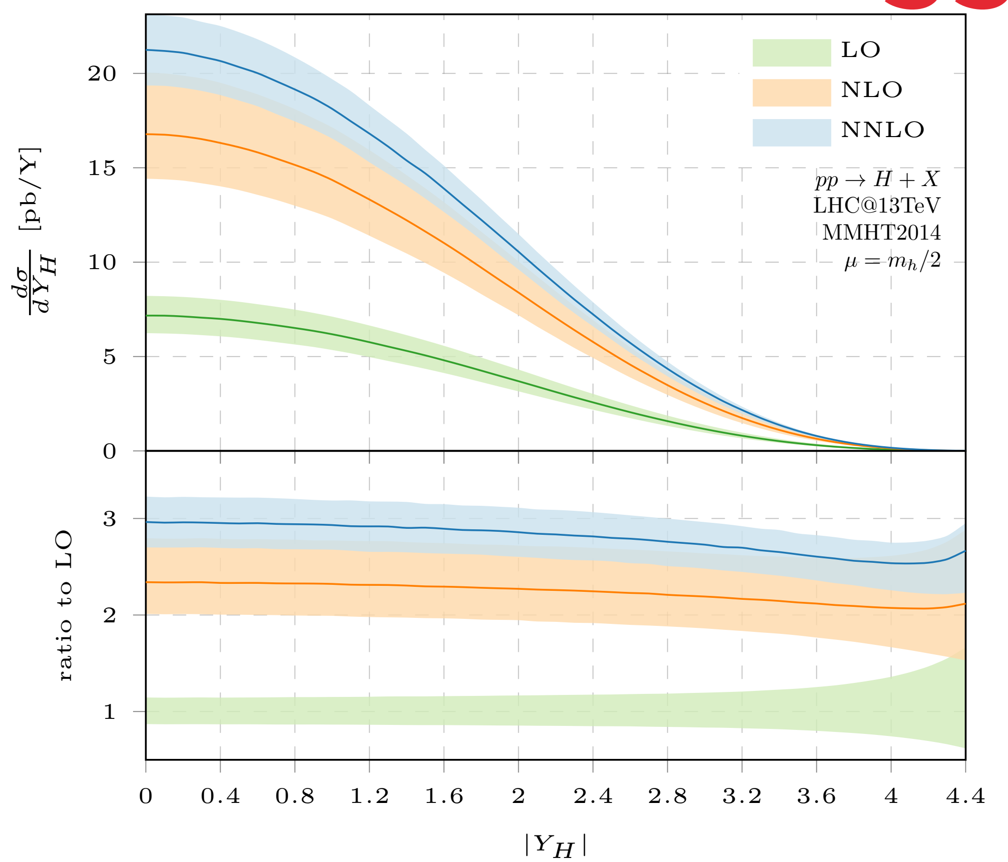
1. Motivation to look at Kinematic Limits
2. Expanding around Collinear Limits
3. Applications

RAPIDITY DISTRIBUTION

$$\frac{\partial \sigma}{\partial Y}$$

- **The** differential observable!

Gluon-Fusion: Higgs

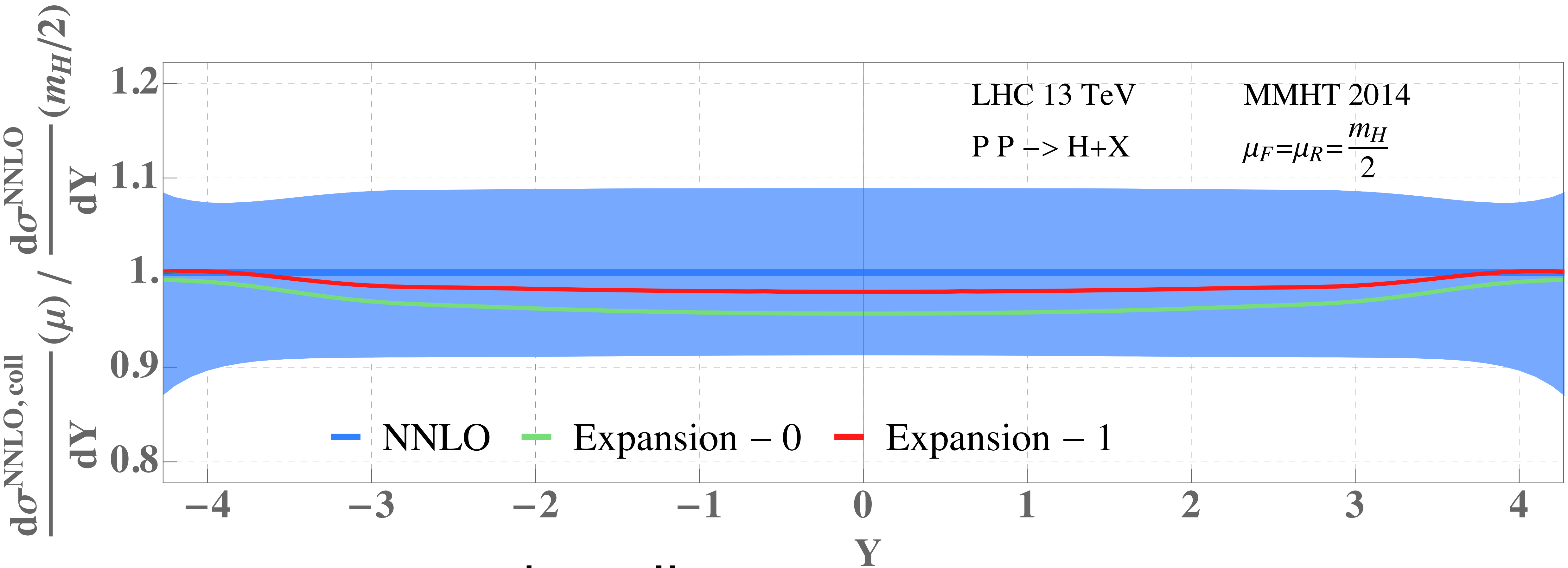


Expanded the NNLO partonic cross section to second power in collinear limit.

$$\frac{\mathrm{d}\eta_{ij}^{R,\,\mathrm{approx.}}(z_1,z_2)}{\mathrm{d}Q^2\mathrm{d}Y} = \left.\frac{\mathrm{d}\eta_{ij}^{R}}{\mathrm{d}Q^2\mathrm{d}Y}\right|_{\bar{z}_2\sim\lambda^2} + \left.\frac{\mathrm{d}\eta_{ij}^{R}}{\mathrm{d}Q^2\mathrm{d}Y}\right|_{\bar{z}_1\sim\lambda^2} - \left.\frac{\mathrm{d}\eta_{ij}^{R}}{\mathrm{d}Q^2\mathrm{d}Y}\right|_{\bar{z}_{1,2}\sim\lambda^2} + \mathcal{O}(\lambda^2)\,.$$

neglect: $\sim \mathcal{O}(w_2)$ $\sim \mathcal{O}(w_1)$ $\sim \mathcal{O}(w_1, w_2)$

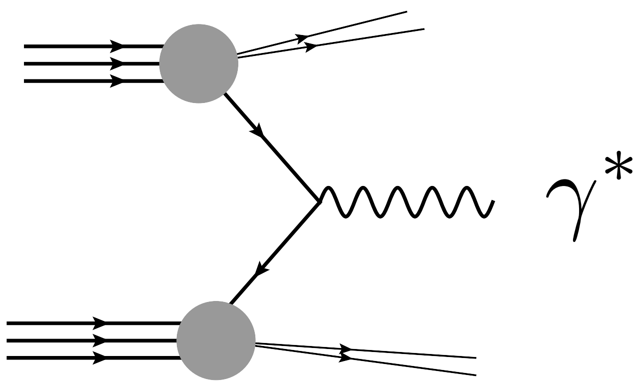
RAPIDITY DISTRIBUTION



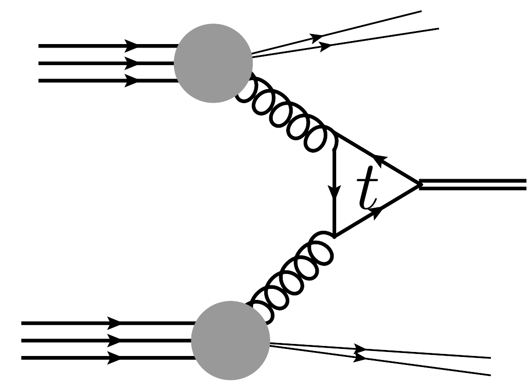
- Approximation works well!
- Systematically improvable.
- Currently, an example; N3LO in progress.

RESULTS AT N3LO

Drell - Yan



Higgs



- **Expanded partonic cross section up to N3LO to first order in collinear limit.**

- Feynman diagrams: $\mathcal{O}(10^6)$
- 492 new master integrals using differential equations
- Fully analytic results for the collinear limit of the partonic cross section $\frac{\mathrm{d}\eta_{ij}}{\mathrm{d}Q^2\mathrm{d}w_1\mathrm{d}w_2\mathrm{d}x}$

RESULTS AT N3LO

- Cross sections for infrared sensitive observables can be written in general as:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}\mathcal{T}} = \sigma_0 \sum_{i,j} H_{ij}(Q^2)\left[B_i(x_1^B,\mathcal{T})\otimes B_j(x_2^B,\mathcal{T})\otimes S(\mathcal{T})\right]\times\left[1+\mathcal{O}(\mathcal{T}/Q)\right].$$

Hard **Collinear** **Soft**

The only part that depends on the specific process.

B_i & S_i **Universal, process independent.**

Beam Functions: B_i

Since they encode collinear physics, they can naturally be extracted from our computation of the collinear limit of cross sections.

BEAM FUNCTIONS AT N3LO

- Explicit relation of beam functions to the collinear limit of colour singlet cross sections:

$$B_i(x_1^B, \mathcal{T}) = \sum_j \int_{x_1^B}^1 \frac{\mathrm{d}z_1}{z_1} f_j\left(\frac{x_1^B}{z_1}\right) \times \int_0^1 \mathrm{d}x \int_0^\infty \mathrm{d}w_1 \mathrm{d}w_2 \, \delta\left[z_1 - (1 - w_1)\right] \\ \times \lim_{\text{strict } n-\text{coll.}} \left\{ \delta\left[\mathcal{T} - \mathcal{T}(Q, Y, w_1, w_2, x)\right] \frac{\mathrm{d}\eta_{j\bar{i}}}{\mathrm{d}Q^2 \mathrm{d}w_1 \mathrm{d}w_2 \mathrm{d}x} \right\} .$$

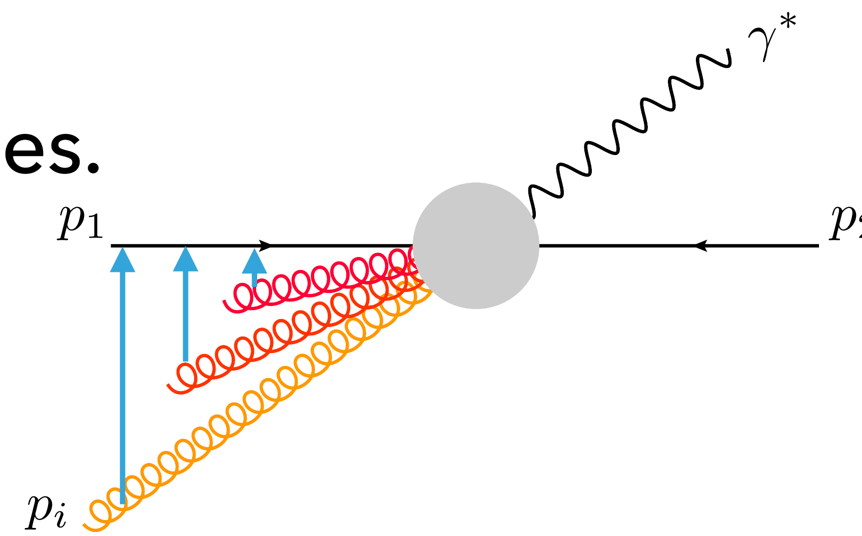
- Beam functions: Probability to find a parton in a proton with momentum fraction x_1^B and observable value $\mathcal{T}$.

- Related to standard PDFs via

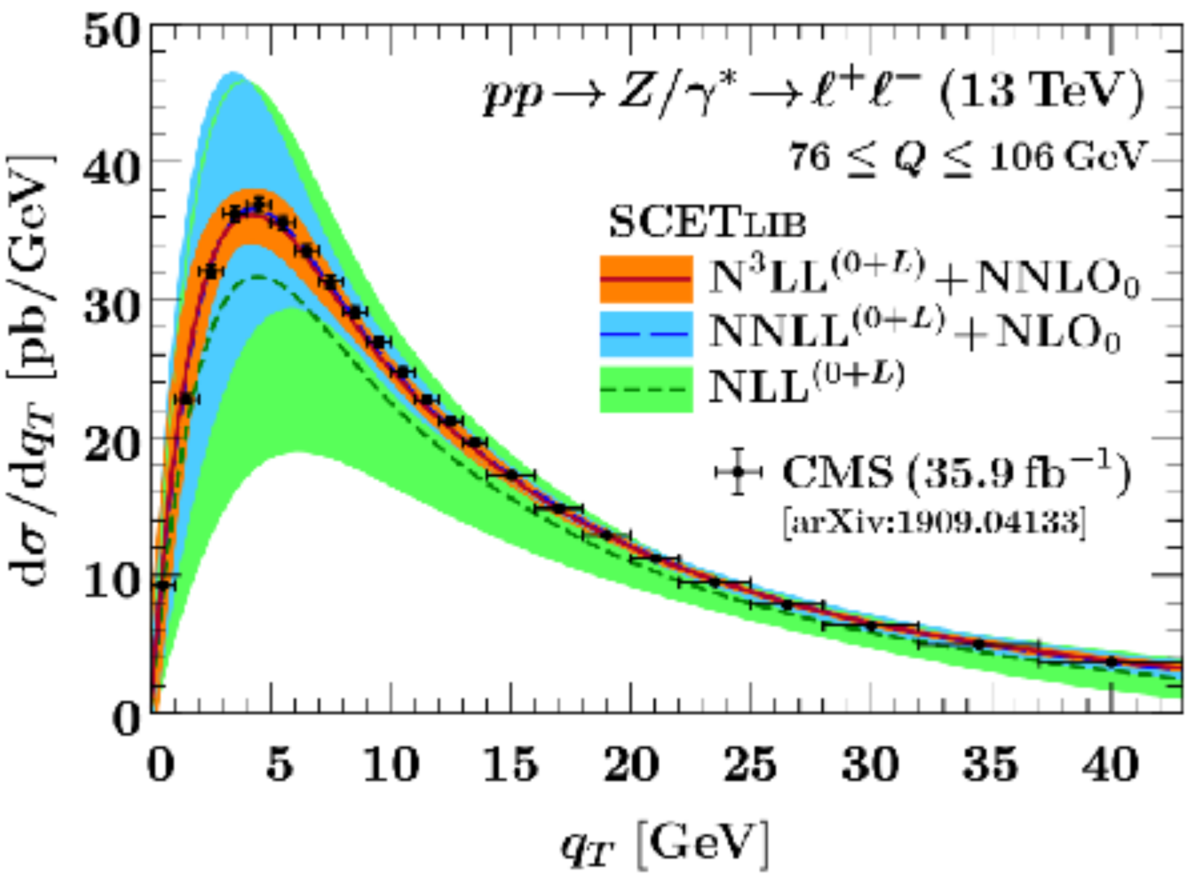
$$B_i(x, \mathcal{T}, \mu) = \sum_j \mathcal{I}_{ij}(x, \mathcal{T}, \mu) \otimes_x f_j^R(x, \mu) \times \left[1 + \mathcal{O}(\Lambda_{\mathrm{QCD}}/\mathcal{T})\right] .$$

BEAM FUNCTIONS AT N3LO: $q_\perp$

- Collinear Limit: Transverse momentum vanishes.
- Transverse momentum dependent beam functions play a role in many applications:



Resummation:



[Ebert et al., 2006.11382]

Slicing:

$$\frac{d\sigma}{d\mathcal{O}} = \int d\mathcal{T}\, \frac{d\tilde{\sigma}}{d\mathcal{T}\, d\mathcal{O}} + \int d\mathcal{T} \left[\frac{d\sigma}{d\mathcal{T}\, d\mathcal{O}} - \frac{d\tilde{\sigma}}{d\mathcal{T}\, d\mathcal{O}} \right]$$

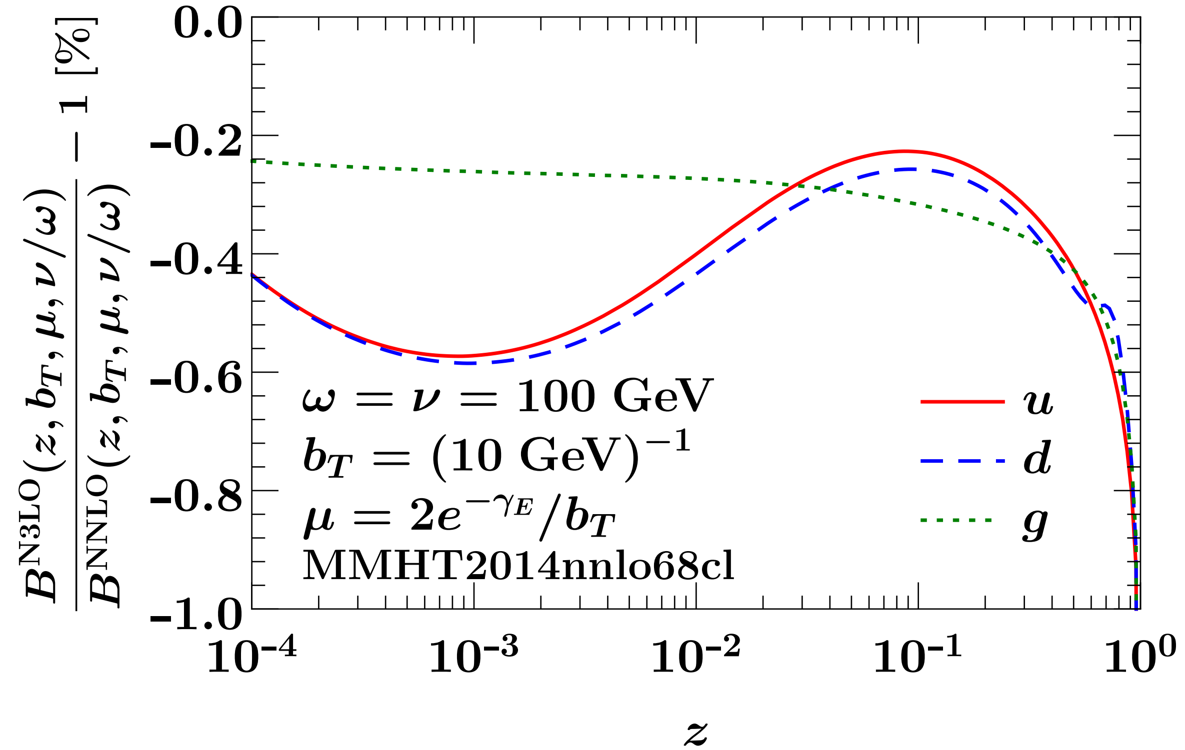
Approximate!

Lower Order!

BEAM FUNCTIONS AT N3LO: $q_\perp$

https://arxiv.org/abs/2006.05329

- Computed the N3LO corrections to the transverse momentum dependent beam functions.



- All channels, fully analytic.

See also [Luo et al.,1912.05778]

- Last missing universal ingredient for $q_\perp$ subtraction at N3LO.

- Input for fully differential predictions for arbitrary colour singlet processes at N3LO.

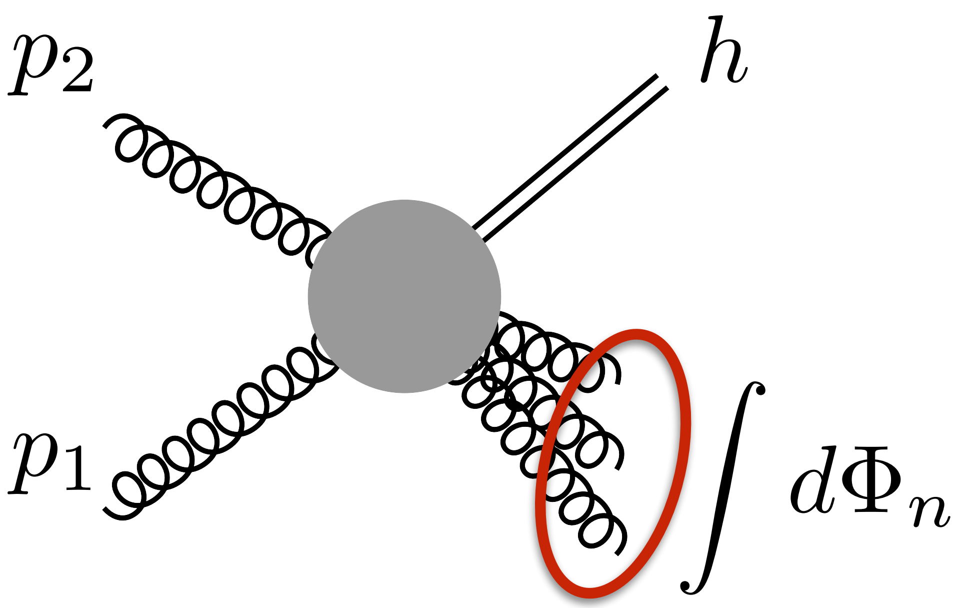
$$\gamma^* \; W \; Z \; H \quad bbH \quad H^* \; G \qquad WH \;\; ZH \quad ZZ \; WW$$

In other news: We also computed the N-Jettiness Beam Functions at N3LO.
https://arxiv.org/abs/2006.03056

CROSSING: (IN PROGRESS)

- We started with a production process:

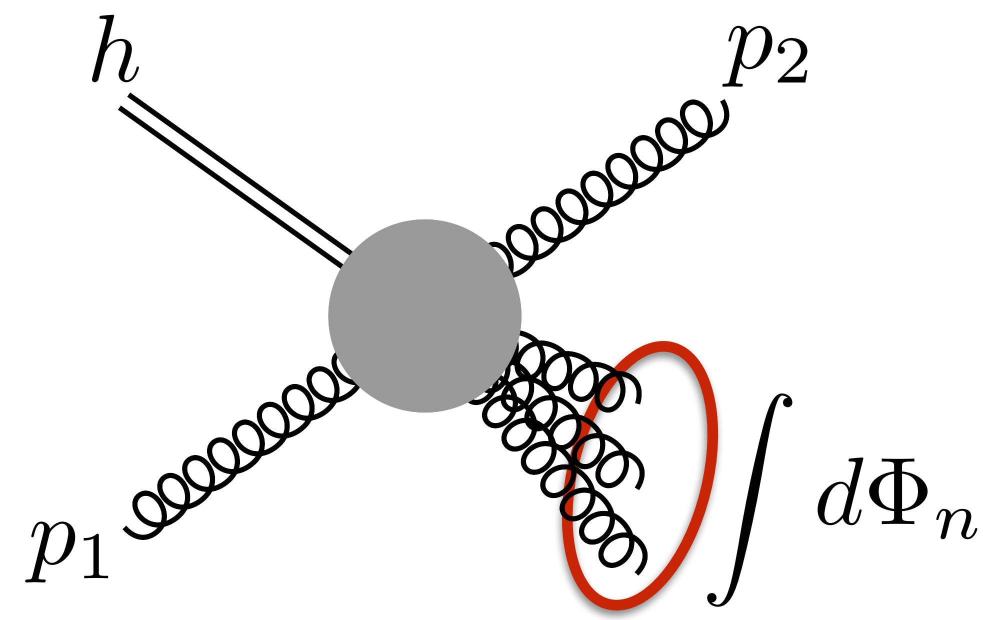
PRODUCTION



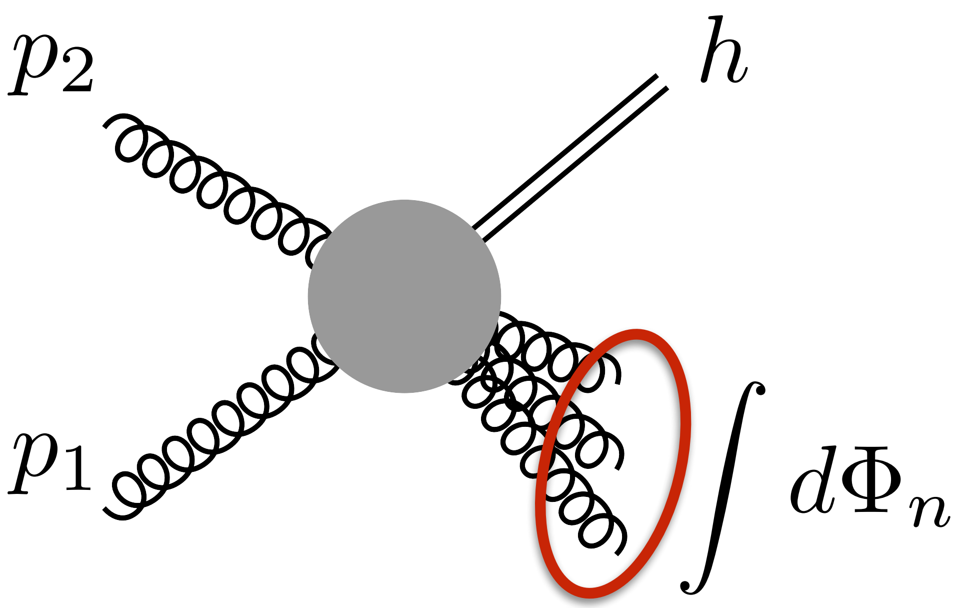
- Radiation in final state.
- Differential in the other momenta.

- We started with a production process:

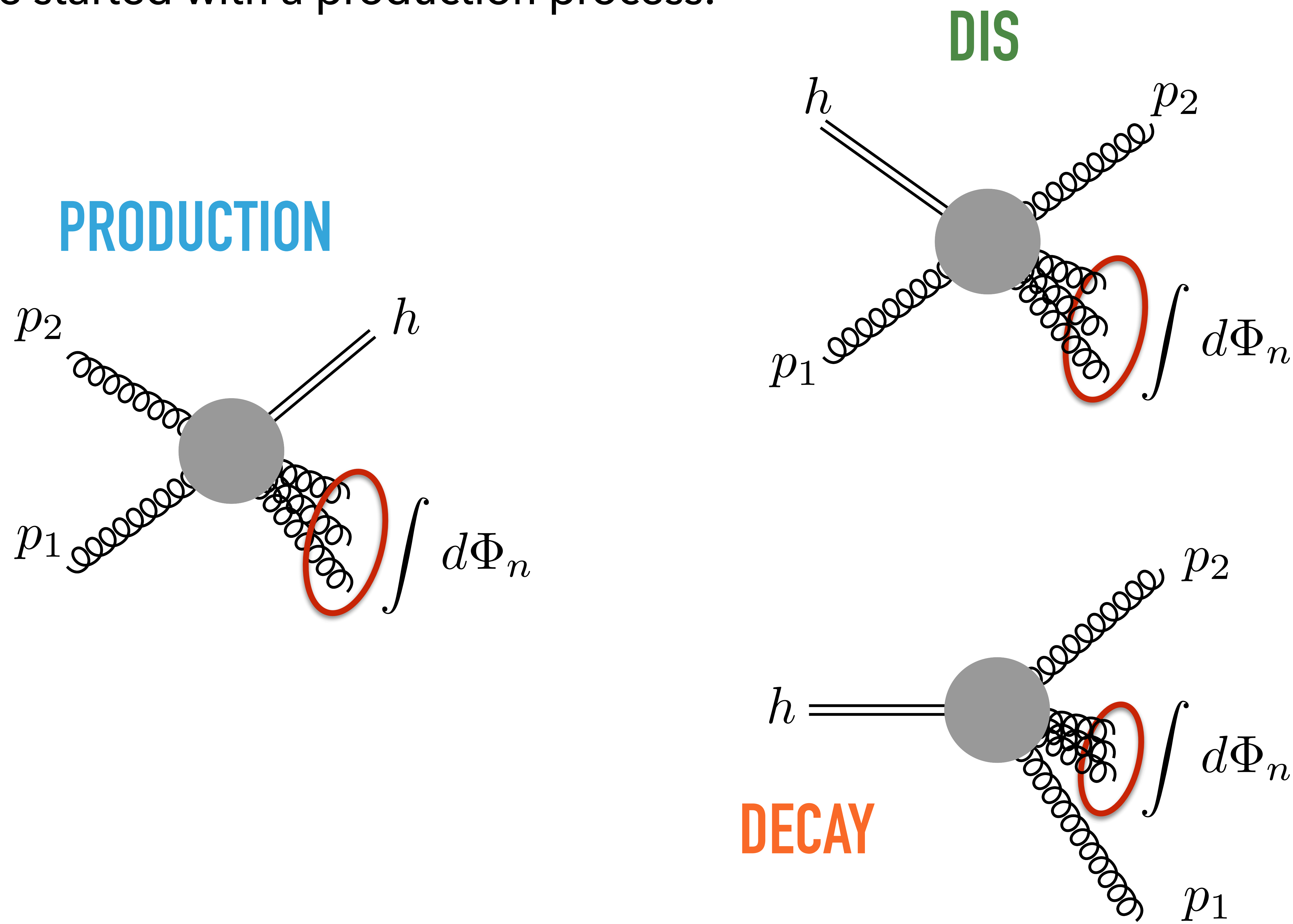
DIS



PRODUCTION

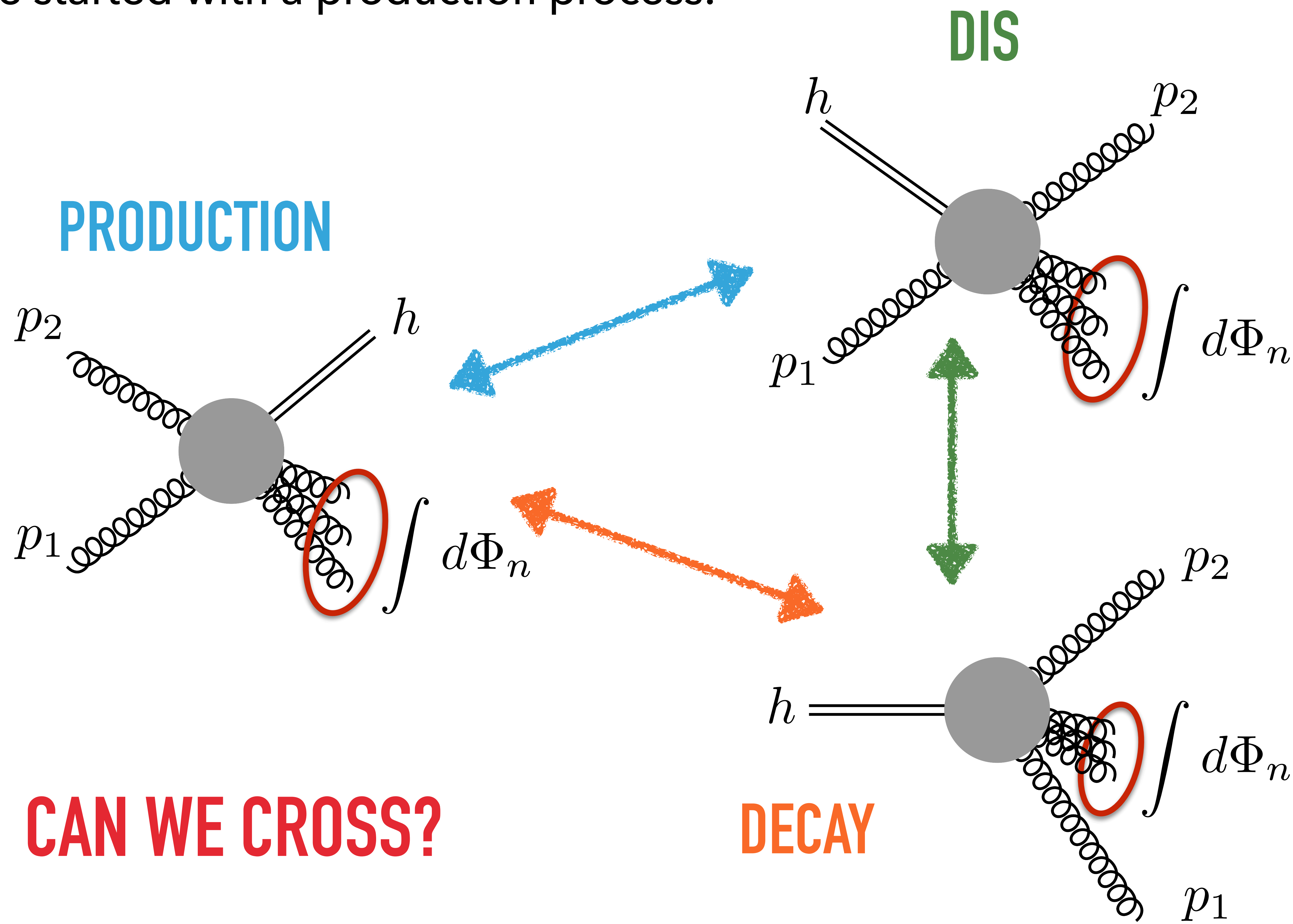


- We started with a production process:



CROSSING: (IN PROGRESS)

- We started with a production process:

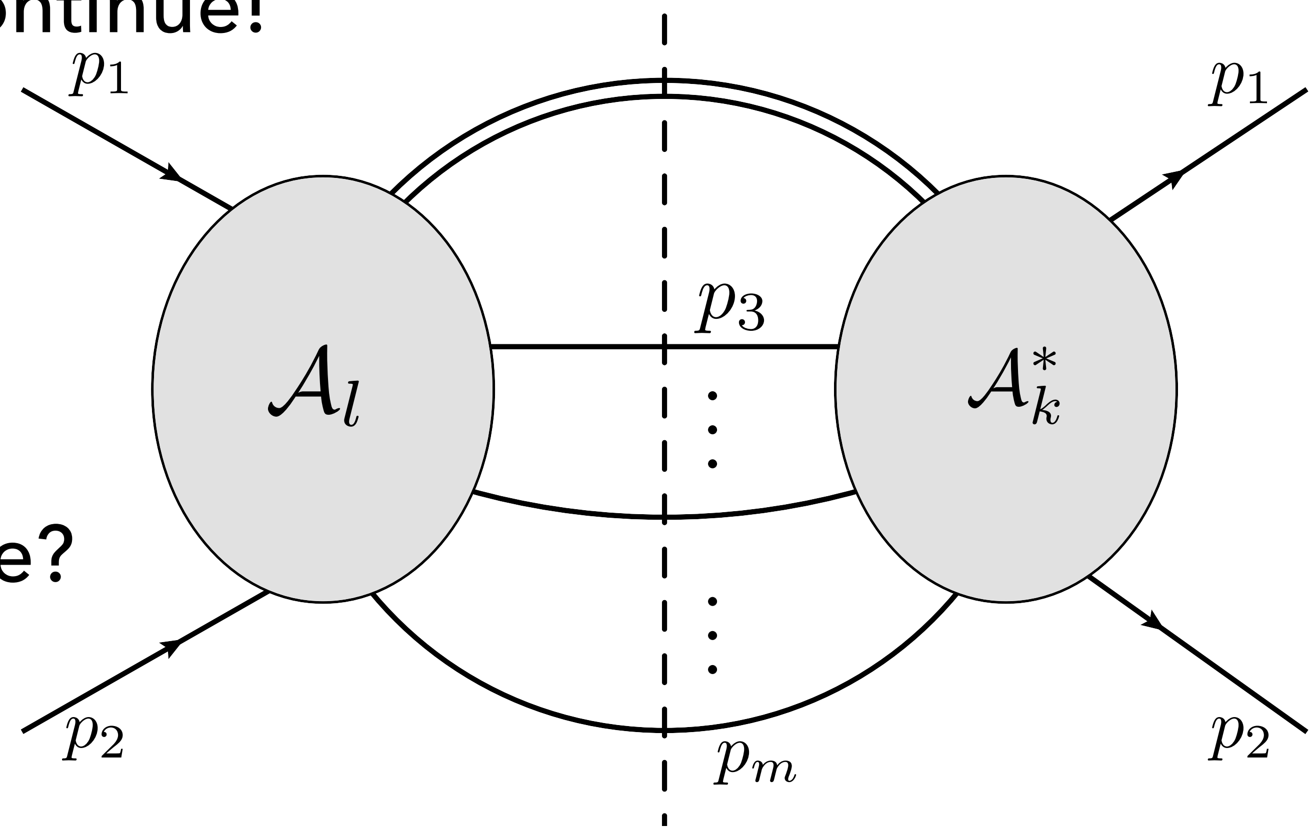


CROSSING: (IN PROGRESS)

- To cross means to analytically continue!
- Our variables change sign:

$$\{s, w_1, w_2\}$$

- What is their branch cut structure?



[cross section]

$$\frac{\mathrm{d}\eta_{ij}^{(m+l+k)}}{\mathrm{d}Q^2\mathrm{d}w_1\mathrm{d}w_2\mathrm{d}x} = (sw_1w_2)^{-m\epsilon} \times \left[\sum_{i_1,i_2=0}^{l} \sum_{j_1,j_2=0}^{k} \frac{\mathrm{d}\eta_{ij}^{(m+l+k,i_1,i_2,j_1,j_2)}}{\mathrm{d}Q^2\mathrm{d}w_1\mathrm{d}w_2\mathrm{d}x} \right.$$

$$\left. \times\ \Re\left(\left[(-s)^{(i_1+i_2-l)\epsilon}(sw_1)^{-i_1\epsilon}(sw_2)^{-i_2\epsilon}\right] \left[(-s)^{(j_1+j_2-k)\epsilon}(sw_1)^{-j_1\epsilon}(sw_2)^{-j_2\epsilon}\right]^* \right) \right].$$

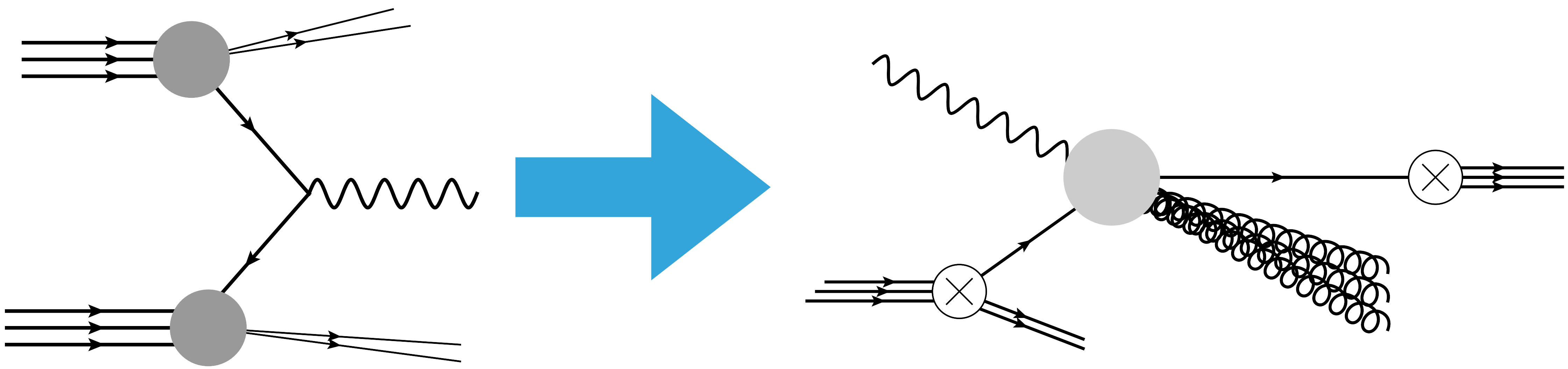
[real coefficients - complicated stuff]

[Mandelstam Invariants! - Continue!]

[See also talk by Hua Xing!]

CROSSING: FRAGMENTATION FUNCTIONS (IN PROGRESS)

Drell - Yan **Deep In-Elastic Scattering**

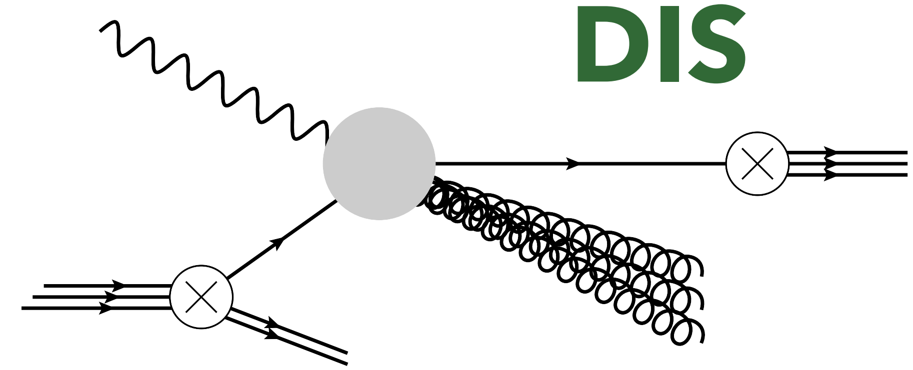


- Semi-Inclusive DIS in the small $P_{2,\perp}$ limit:

$$\frac{d\sigma_{P+h\to H+X}}{d\vec{P}_{2,\perp}^2\, d\xi} = \hat{\sigma}_0 \sum_{i,j} H_{ij} \times (Q^2,\mu)\tilde{f}_i(P_{2,\perp}) \otimes d_j(P_{2,\perp},\xi)\left[1+\mathcal{O}\left(\vec{P}_{2,\perp}^2/Q^2\right)\right]$$

CROSSING: FRAGMENTATION FUNCTIONS (IN PROGRESS)

- Semi-Inclusive DIS in the small $P_{2,\perp}$ limit:



$$\frac{d\sigma_{P+h\to H+X}}{d\vec{P}_{2,\perp}^2\, d\xi} = \hat{\sigma}_0 \sum_{i,j} H_{ij} \times (Q^2,\mu)\tilde{f}_i(P_{2,\perp}) \otimes d_j(P_{2,\perp},\xi)\left[1+\mathcal{O}\left(\vec{P}_{2,\perp}^2/Q^2\right)\right]$$

- Transverse Momentum Dependent Fragmentation Function (TMDFF)

The probability to find a Hadron inside a parton with a given longitudinal momentum fraction ξ and transverse momentum $P_{2,\perp}$

- Related to the longitudinal fragmentation function via

TMDFF

FF

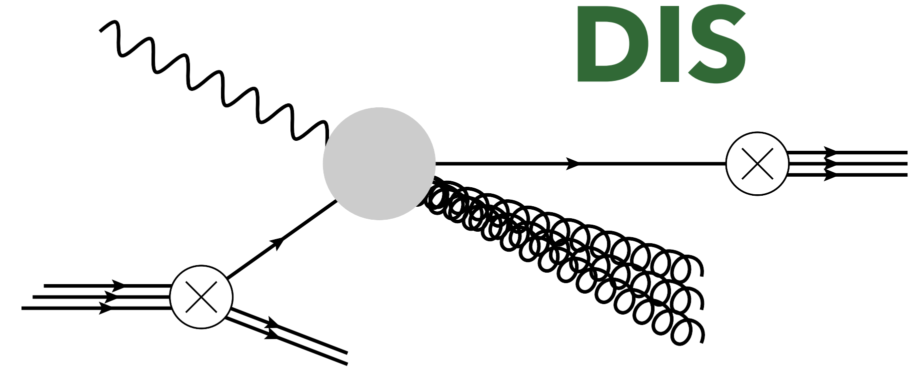
$$\tilde{d}_j\left(\xi,b_T,\mu,\tau,\omega_b\right) = \sqrt{\tilde{S}(b_T,\mu,\tau)}\sum_j \int_\xi^1 \frac{d\zeta}{\zeta}\tilde{\mathcal{I}}_{ij}^{\mathrm{FF}}\left(\zeta,b_T,\mu,\tau,\omega_b\right)D_j\left(\frac{\xi}{\zeta},\mu\right).$$

Soft Function

Matching Kernel

CROSSING: FRAGMENTATION FUNCTIONS (IN PROGRESS)

- Semi-Inclusive DIS in the small $P_{2,\perp}$ limit:



$$\frac{d\sigma_{P+h\to H+X}}{d\vec{P}^2_{2,\perp}\,d\xi} = \hat{\sigma}_0 \sum_{i,j} H_{ij} \times (Q^2,\mu)\tilde{f}_i(P_{2,\perp}) \otimes d_j(P_{2,\perp},\xi)\left[1+\mathcal{O}\left(\vec{P}^2_{2,\perp}/Q^2\right)\right]$$

- Transverse Momentum Dependent Fragmentation Function (TMDFF)

The probability to find a Hadron inside a parton with a given longitudinal momentum fraction ξ and transverse momentum $P_{2,\perp}$

- Related to the longitudinal fragmentation function via

TMDFF — FF

$$\tilde{d}_j\left(\xi, b_T, \mu, \tau, \omega_b\right) = \sqrt{\tilde{S}(b_T,\mu,\tau)}\sum_j \int_\xi^1 \frac{d\zeta}{\zeta}\tilde{\mathcal{I}}^{\rm FF}_{ij}\left(\zeta, b_T, \mu, \tau, \omega_b\right) D_j\left(\frac{\xi}{\zeta},\mu\right).$$

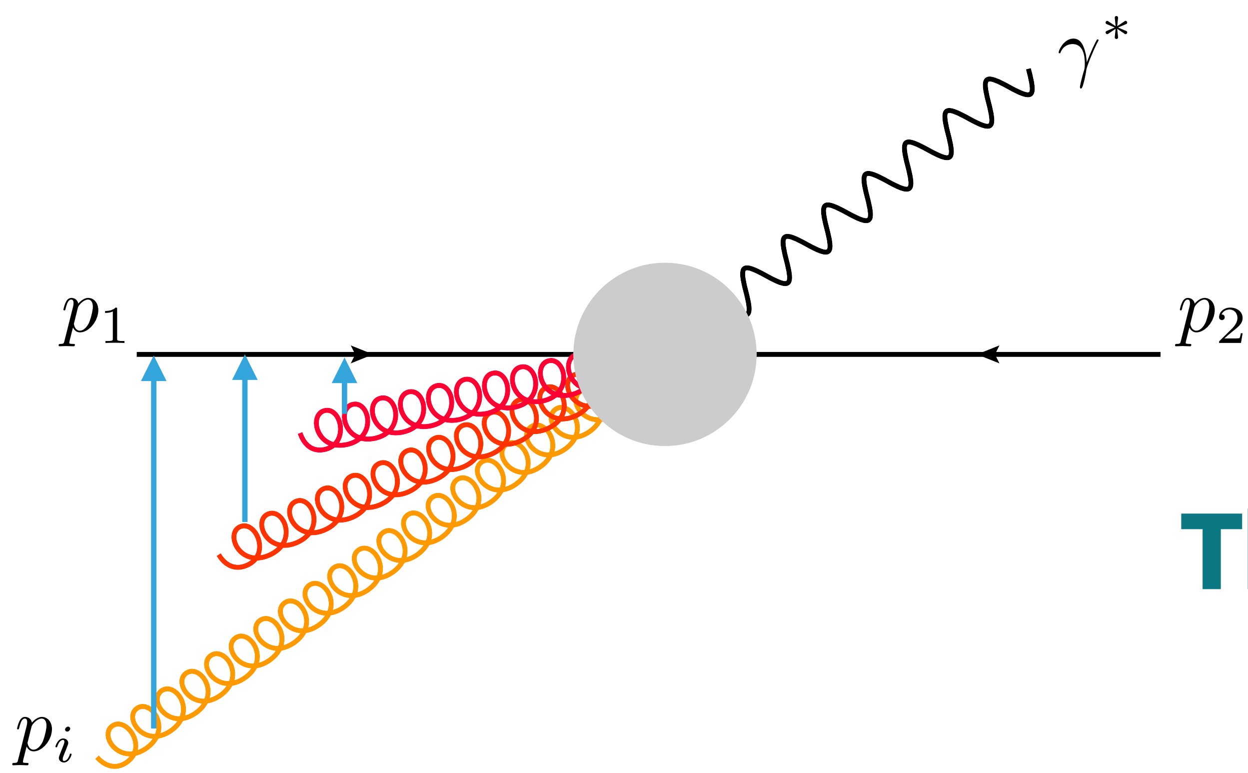
Soft Function — Matching Kernel

NEW AT N3LO! – TO APPEAR SOON

[See also talk by Hua Xing!]

CONCLUSIONS

- Collinear expansions for colour singlet processes.
- Very efficient, versatile way to compute many different quantities.
- Efficient way of performing approximations for collider cross sections. **Higgs Boson Rapidity at NNLO**
- First new results: Beam Functions at **N3LO** for $q_\perp$ & $\mathcal{T}$
  Fragmentation Functions at **N3LO** for $q_\perp$



Thank you!