Combining Results

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Problem

- Want to combine results of different publications
- Set of results 1 + correlation matrix
- Set of results 2 + correlation matrix
- .....
- Parameters might not all be the same between the publications and they might be correlated
- Target might be a subset, so there are constraints or linear combinations of the source parameters
- Code available to combine, but with restricted access. Please contact us if you want to use it
- Code also available for combination with LHC experiments ATLAS, CMS, LHCb (more later)

Motivation

- Want to combine our Run 2 results with the Run 1 results
- **ResultSet**: set of parameters with their values, statistical and systematic uncertainties, and their correlation matrices
- Our ResultSet consists of 9 parameters: $\{\phi_s, \Gamma_s, \Delta\Gamma_s, |\lambda|, \Delta m_s, |A_\perp|^2, |A_0|^2, \delta_\parallel, \delta_\perp\}$
- Each parameter in the ResultSet has:
  - Fit result with statistical uncertainty
  - Multiple systematic uncertainties
  - Statistical correlations with the other parameters
  - Potential systematic correlations with the other parameters

Motivation

- Combining two variables, $x_a \pm \sigma_{a,stat} \pm \sigma_{a,syst}$ and $x_b \pm \sigma_{b,stat} \pm \sigma_{a,syst}$ means:

$$x_{av} = \frac{x_a/\sigma_a + x_b/\sigma_b}{1/\sigma_a^2 + 1/\sigma_b^2} \qquad \sigma_{av} = \sqrt{\sigma_a^2 + \sigma_b^2}$$

- BUT this is simplest case
- This is not yet including statistical or systematic correlations
- Parameters are statistically correlated with each other → covariance matrix
- Each parameter also has systematic errors → various systematic covariance matrices
- Run 1 and Run 2 have overlapping systematics → possible correlations
- Combination will become quite a challenge

Parameter Values

Just for visualisation below are the parameter values as $x \pm \sigma_{stat} \pm \sigma_{syst}$

Par.	Run 1	Run 2 (year 2015&2016)
ϕ_s	$-0.058 \pm 0.049 \pm 0.006$	$-0.083 \pm 0.041 \pm 0.006$
λ	$0.964 \pm 0.019 \pm 0.007$	$1.012 \pm 0.016 \pm 0.006$
Δm	$17.711^{+0.055}_{-0.057} \pm 0.011$	$17.703 \pm 0.059 \pm 0.018$
$\Delta\Gamma_s$	$0.0805 \pm 0.0091 \pm 0.0033$	$0.0773 \pm 0.0077 \pm 0.0026$
$\Gamma_s - \Gamma_d$	$0.0015 \pm 0.0027 \pm 0.0015$	$-0.0041 \pm 0.0024 \pm 0.0015$
$A_\perp^2$	$0.2504 \pm 0.0049 \pm 0.0036$	$0.2456 \pm 0.0040 \pm 0.0019$
A_0^2	$0.5241 \pm 0.0034 \pm 0.0067$	$0.5186 \pm 0.0029 \pm 0.0024$
$\delta_\perp$	$3.08^{+0.14}_{-0.15} \pm 0.06$	$2.64 \pm 0.13 \pm 0.10$
$\delta_\parallel$	$3.258^{+0.10}_{-0.17}{}^{+0.06}_{-0.07}$	$3.062^{+0.082}_{-0.074} \pm 0.037$

Run 1 statistical correlation matrix

	Γ_s	$\Delta\Gamma$	$A_\perp^2$	A_0^2	$\delta_\parallel$	$\delta_\perp$	ϕ_s	λ	Δm
Γ_s	1.00	-0.45	0.39	-0.31	-0.07	-0.02	0.01	-0.01	0.01
$\Delta\Gamma$	-0.45	1.00	-0.69	0.65	0.02	-0.03	-0.08	0.02	-0.03
$A_\perp^2$	0.39	-0.69	1.00	-0.59	-0.29	-0.1	0.04	-0.03	0.0
A_0^2	-0.31	0.65	-0.59	1.00	-0.02	-0.04	-0.03	0.02	-0.03
$\delta_\parallel$	-0.07	0.02	-0.29	-0.02	1.00	0.42	0.01	0.05	0.05
$\delta_\perp$	-0.02	-0.03	-0.1	-0.04	0.42	1.00	0.14	-0.17	0.67
ϕ_s	0.01	-0.08	0.04	-0.03	0.01	0.14	1.00	-0.02	0.09
λ	-0.01	0.02	-0.03	0.02	0.05	-0.17	-0.02	1.00	-0.21
Δm	0.01	-0.03	0.0	-0.03	0.05	0.67	0.09	-0.21	1.00

Run 2 statistical correlation matrix

	$\Gamma_s - \Gamma_d$	$\Delta\Gamma$	$A_\perp^2$	A_0^2	$\delta_\parallel$	$\delta_\perp$	ϕ_s	λ	Δm
$\Gamma_s - \Gamma_d$	1.0	-0.47	0.39	-0.32	0.03	0.0	-0.03	0.02	-0.01
$\Delta\Gamma$	-0.47	1.0	-0.69	0.63	-0.01	0.01	-0.01	-0.04	0.02
$A_\perp^2$	0.39	-0.69	1.0	-0.60	0.12	0.0	-0.02	0.02	-0.03
A_0^2	-0.32	0.63	-0.60	1.0	0.0	0.01	0.02	-0.02	0.01
$\delta_\parallel$	0.03	-0.01	0.12	0.0	1.0	0.20	0.01	0.0	-0.04
$\delta_\perp$	0.0	0.01	0.0	0.01	0.20	1.0	0.0	0.07	0.74
ϕ_s	-0.03	-0.01	-0.02	0.02	0.01	0.00	1.0	0.19	0.01
λ	0.02	-0.04	0.02	-0.02	0.0	0.07	0.19	1.0	0.0
Δm	-0.01	0.02	-0.03	0.01	-0.04	0.74	0.01	0.0	1.0

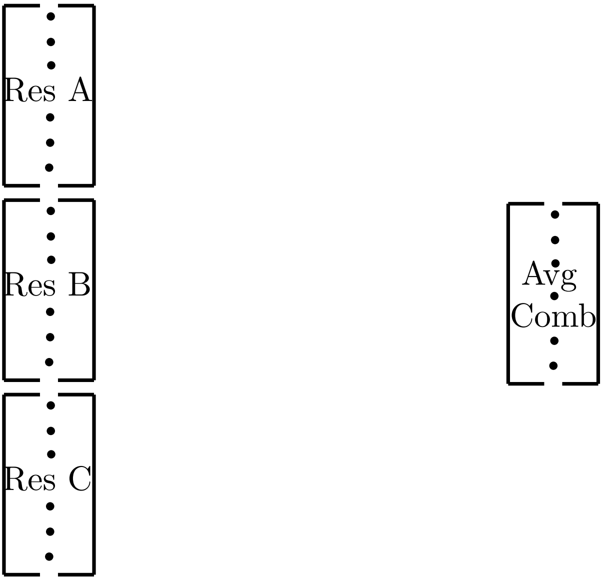
Run 1 Systematics

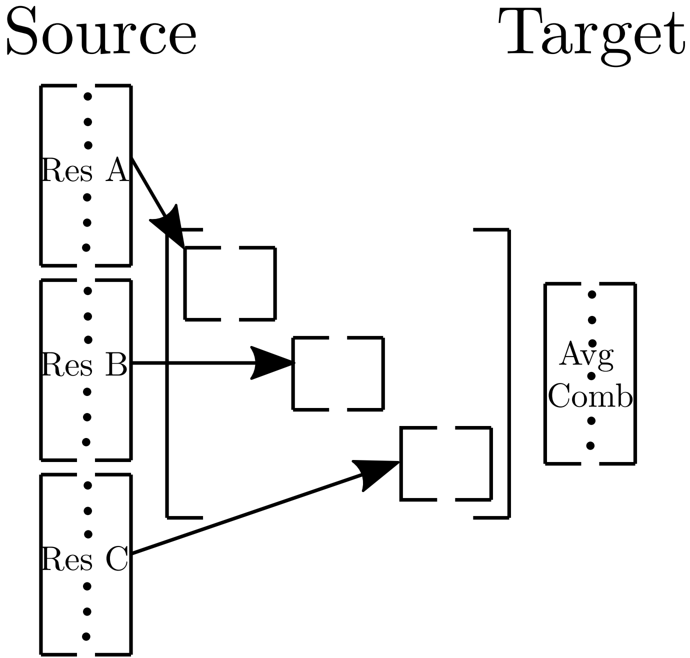
Source	Γ_s [ps^{-1}]	$\Delta\Gamma_s$ [ps^{-1}]	$\lvert A_\perp\rvert^2$	$\lvert A_0\rvert^2$	$\delta_\parallel$ [rad]	$\delta_\perp$ [rad]	ϕ_s [rad]	$\lvert\lambda\rvert$	Δm_s [ps^{-1}]
Total stat. uncertainty	0.0027	0.0091	0.0049	0.0034	$^{+0.10}_{-0.17}$	$^{+0.14}_{-0.15}$	0.049	0.019	$^{+0.055}_{-0.057}$
Mass factorisation	–	0.0007	0.0031	0.0064	0.05	0.05	0.002	0.001	0.004
Signal weights (stat.)	0.0001	0.0001	–	0.0001	–	–	–	–	–
b-hadron background	0.0001	0.0004	0.0004	0.0002	0.02	0.02	0.002	0.003	0.001
B_c^+ feed-down	0.0005	–	–	–	–	–	–	–	–
Angular resolution bias	–	–	0.0006	0.0001	$^{+0.02}_{-0.03}$	0.01	–	–	–
Ang. efficiency (reweighting)	0.0001	–	0.0011	0.0020	0.01	–	0.001	0.005	0.002
Ang. efficiency (stat.)	0.0001	0.0002	0.0011	0.0004	0.02	0.01	0.004	0.002	0.001
Decay-time resolution	–	–	–	–	–	0.01	0.002	0.001	0.005
Trigger efficiency (stat.)	0.0011	0.0009	–	–	–	–	–	–	–
Track reconstruction (simul.)	0.0007	0.0029	0.0005	0.0006	$^{+0.01}_{-0.02}$	0.002	0.001	0.001	0.006
Track reconstruction (stat.)	0.0005	0.0002	–	–	–	–	–	–	0.001
Length and momentum scales	0.0002	–	–	–	–	–	–	–	0.005
S-P coupling factors	–	–	–	–	0.01	0.01	–	0.001	0.002
Fit bias	–	–	0.0005	–	–	0.01	–	0.001	–
Quadratic sum of syst.	0.0015	0.0032	0.0036	0.0067	$^{+0.06}_{-0.07}$	0.06	0.006	0.007	0.011

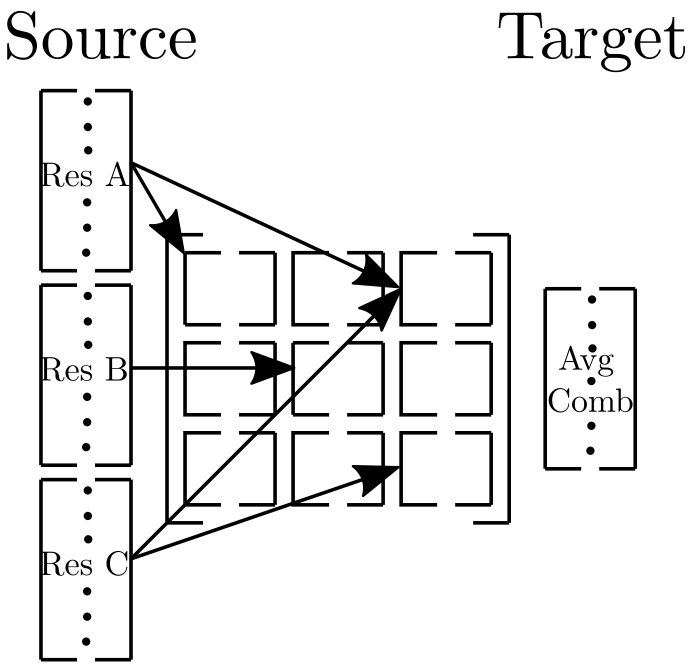
- For each systematic there is a priori a correlation matrix
- Run 2 also has systematics, some different and some the same which could be correlated with Run 1

Source

Target




Source
Target
Res A
Res B
Res C
Avg
Comb

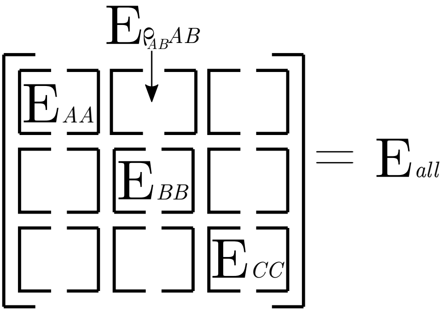

Source
Target
Res A
Res B
Res C
Avg
Comb

Formalism

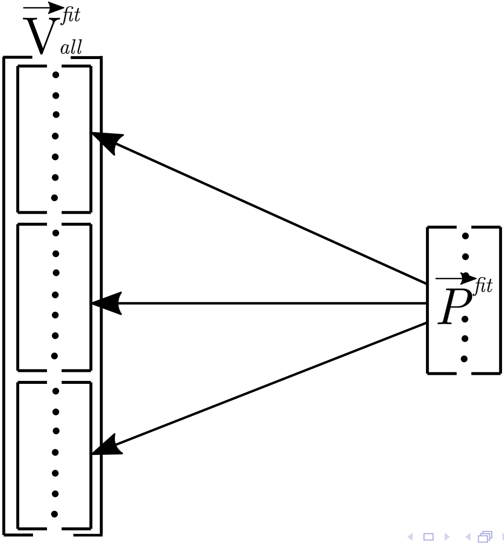
1. Create uber-vector of the measurements

$$\begin{bmatrix} \begin{bmatrix} \vdots \\ \text{Res A} \\ \vdots \end{bmatrix} \\ \begin{bmatrix} \vdots \\ \text{Res B} \\ \vdots \end{bmatrix} \\ \begin{bmatrix} \vdots \\ \text{Res C} \\ \vdots \end{bmatrix} \end{bmatrix} = \overrightarrow{\mathrm{V}}^{meas}_{all} = \begin{bmatrix} \overrightarrow{\mathrm{V}}^{meas}_{A} \\ \overrightarrow{\mathrm{V}}^{meas}_{B} \\ \overrightarrow{\mathrm{V}}^{meas}_{C} \end{bmatrix}$$

2. Create uber-covariance matrix



3. From current iteration of the fit parameters create values corresponding to each measurement



Fit procedure

1. $\vec{V}_{all}^{meas}$, uber-vector measured parameters
2. E_{all}, uber-covariance matrix
3. $\vec{P}^{fit}$, agreed combined output parameters
4. $\vec{V}_{all}^{fit}$, uber-vector current fit values

- Define difference vector:

$$\vec{\Delta} = \vec{V}_{all}^{meas} - \vec{V}_{all}^{fit}$$

- Create the χ^2 to be minimised:

$$\chi^2 = \vec{\Delta}^T E_{all} \vec{\Delta}$$

- Minuit is used to minimise the χ^2 with respect to $\vec{P}^{fit}$ to get the best combined value of all parameters, taking into account all correlations

In practice

Making up covariance matrix for one ResultSet

- To obtain covariance matrix per ResultSet
  $\mathrm{E} = \mathrm{E}_{\mathrm{stat}} + \mathrm{E}_{\mathrm{Y1}} + \mathrm{E}_{\mathrm{Y2}} + \mathrm{E}_{\mathrm{Y3}} + \ldots$ where Y represent a certain systematic error
- Systematic covariance matrices are mostly diagonal

Statistical **Systematic**

$$\begin{vmatrix} \sigma_1\sigma_1 & \sigma_1\sigma_2\rho_{12} & \ldots & \sigma_1\sigma_n\rho_{1n} \\ \sigma_2\sigma_1\rho_{21} & \sigma_2\sigma_2 & \ldots & \sigma_2\sigma_n\rho_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ \sigma_n\sigma_1\rho_{n1} & \sigma_n\sigma_2\rho_{n2} & \ldots & \sigma_n\sigma_n \end{vmatrix} + \begin{vmatrix} \sigma_1\sigma_1 & 0 & \ldots & 0 \\ 0 & \sigma_2\sigma_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \sigma_n\sigma_n \end{vmatrix} +$$

Systematic

$$\begin{vmatrix} \sigma_1\sigma_1 & \sigma_1\sigma_2\rho_{12} & \ldots & 0 \\ 0 & \sigma_2\sigma_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ \sigma_n\sigma_1\rho_{n1} & 0 & \ldots & \sigma_n\sigma_n \end{vmatrix} + \ldots$$

- Want to properly take into account off-diagonal systematic covariance terms

ResultSet


```
{
"ResultSet": [

{     "ResultSetLabel": "2016-JPsiKK",
      "Description" :
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      "Parameter":
      [
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          "Value": -0.0041,
          "Error": 0.0024
        },
        { "Name": "deltaGammas",
          "Value": 0.0773,
          "Error": 0.0077
        },
        { "Name": "AperpSq",
          "Value": 0.2456,
          "Error": 0.0040
        },
        { "Name": "AzeroSq",
          "Value": 0.5186,
          "Error": 0.0029
        },
        { "Name": "para",
          "Value": 3.062,
          "Error": 0.082
        },
        { "Name": "perp",
          "Value": 2.64,
          "Error": 0.13
        },
        { "Name": "phis",
          "Value": -0.083,
          "Error": 0.041
        },
        { "Name": "lamb",
          "Value": 1.012,
          "Error": 0.016
        },
        { "Name": "dms",
          "Value": 17.703,
          "Error": 0.059
        }
      ],
```

```
"StatisticalCorrelationMatrix": [
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  [-0.47,   1.0, -0.69,  0.63, -0.01, 0.01, -0.01, -0.04,  0.02],
  [ 0.39, -0.69,  1.0 , -0.60,  0.12, 0.00, -0.02,  0.02, -0.03],
  [-0.32,  0.63, -0.60,   1.0,   0.0, 0.01,  0.02, -0.02,  0.01],
  [ 0.03, -0.01,  0.12,   0.0,   1.0, 0.20,  0.01, -0.00, -0.04],
  [-0.00,  0.01,  0.00,  0.01,  0.20,  1.0,  0.00,  0.07,  0.74],
  [-0.03, -0.01, -0.02,  0.02,  0.01, 0.00,   1.0,  0.19,  0.01],
  [ 0.02, -0.04,  0.02, -0.02, -0.00, 0.07,  0.19,   1.0, -0.00],
  [-0.01,  0.02, -0.03,  0.01, -0.04, 0.74,  0.01, -0.00,   1.0]
],
"SystematicErrors": [
  { "Name": "AngEfficiency",
    "Values": [0.00003, 0.0, 0.00036, 0.0003, 0.0044, 0.0025, 0.0011, 0.0018, 0.0011 ],
    "SystematicCorrelationMatrix": [
      [    1.0, 0.0,   0.89,  -0.22, -0.0071, -0.028,  0.027, -0.033,   0.14],
      [    0.0, 1.0,    0.0,    0.0,     0.0,    0.0,    0.0,    0.0,    0.0],
      [   0.89, 0.0,    1.0,  -0.36,  -0.027,  -0.12, -0.058,  0.025, -0.065],
      [  -0.22, 0.0,  -0.36,    1.0,    0.14,  0.054, 0.0054, -0.040, -0.097],
      [-0.0071, 0.0, -0.027,   0.14,     1.0,   0.60,  0.029, -0.013,  -0.19],
      [ -0.028, 0.0,  -0.12,  0.054,    0.60,    1.0,   0.72,  -0.48,   0.51],
      [  0.027, 0.0, -0.058, 0.0054,   0.029,   0.72,    1.0,  -0.58,   0.82],
      [ -0.033, 0.0,  0.025, -0.040,  -0.013,  -0.48,  -0.58,    1.0,  -0.76],
      [   0.14, 0.0, -0.065, -0.097,   -0.19,   0.51,   0.82,  -0.76,    1.0]
    ]
  },
  { "Name": "DecayTimeEfficiency",
    "Values": [0.0012, 0.00083, 0.0003, 0.0002, 0.0, 0.0, 0.0, 0.0, 0.0],
    "SystematicCorrelationMatrix": [
      [1.0, -0.077, 0.060, -0.058, 0.0, 0.0, 0.0, 0.0, 0.0],
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      [0.060, -0.99, 1.0, -1.0, 0.0, 0.0, 0.0, 0.0, 0.0],
      [-0.058, 0.99, -1.0, 1.0, 0.0, 0.0, 0.0, 0.0, 0.0],
      [0.0, 0.0, 0.0, 0.0, 1.0, 0.0, 0.0, 0.0, 0.0],
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      [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.0, 0.0, 0.0],
      [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.0, 0.0],
      [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.0]
    ]
  },
```

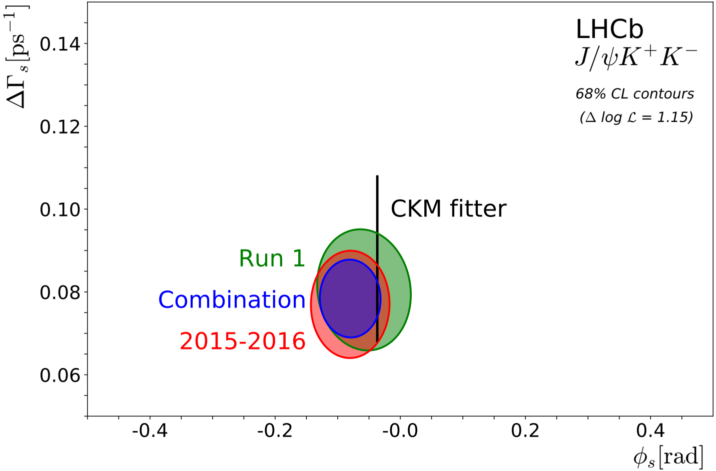
ResultList

- Are able to work with any number of ResultSets


```
{
"ResultSet":
  [
{     "ResultSetLabel": "2012-JPsiKK",
      "Description" :
      ["Contains the 2011-2012 3fb results for phis et. al. from JPsiKK"],
      "Parameter":
      [....]
}
,
{     "ResultSetLabel": "2016-JPsiKK",
      "Description" :
      ["Contains the 2015-2016 2fb results for phis et. al. from JPsiKK"],
      "Parameter":
      [....]
}
,
{     "ResultSetLabel": "2012-JPsiPiPi",
      "Description" :
      ["Contains the 2011-2012 3fb results for phis et. al. from JPsiPiPi"],
      "Parameter":
      [....]
}
,
...
  ]
}
```

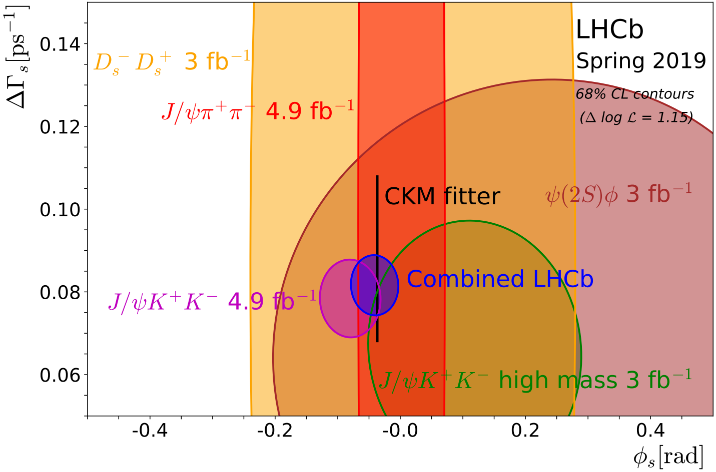

- We can also deal with correlations between the ResultSets

Results



Combination code used to combine results as in CERN-EP-2019-108

Results

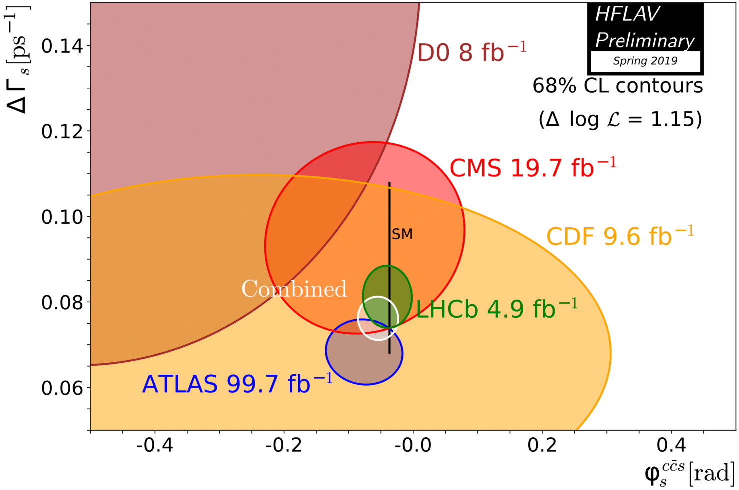


Run 1: 3 fb^{-1} 2015&2016: 1.9 fb^{-1}

Combination code used to combine results as in CERN-EP-2019-108

Next step: combine with other LHC experiments

Combination with others



LHC experiments' published results and HFLAV average

Combination with others

Experiment	Parameters
LHCb	$\phi_s, \Gamma_s - \Gamma_d, \Delta\Gamma_s, \|\lambda\|, \Delta m_s, \|A_\perp\|^2, \|A_0\|^2, \delta_\parallel, \delta_\perp$ $F_{S1}, F_{S2}, F_{S3}, F_{S4}, F_{S5}, F_{S6}, \delta_{S1}, \delta_{S2}, \delta_{S3}, \delta_{S4}, \delta_{S5}, \delta_{S6}$
ATLAS	$\phi_s, \Gamma_s, \Delta\Gamma_s, \|A_\parallel\|^2, \|A_0\|^2, \delta_\parallel, \delta_\perp$ $\|A_S\|^2, \delta_S$
CMS	$\phi_s, \Gamma_s, \Delta\Gamma_s, \|A_\perp\|^2, \|A_0\|^2, \delta_\parallel, \delta_\perp$ $\|A_S\|^2, \delta_S$

Veronika's slides from meeting with other experiments

Treatment of Δm_s

	ATLAS		CMS		LHCb	
	Run I	Run II	Run I	Run II	Run I	Run II
Δm_s [ps^{-1}]	fixed (17.77)	fixed (17.757)	Gaus. constr. (17.69 ± 0.08)	free	free	free

- We propose all experiments do their Run 1 and Run 2 combination internally
- This way they deal with Δm_s combinations as they see fit and appropriate, including any statements about correlation of their results with Δm_s
- Then, depending what these statements are, we see if we need to do a lot of work, or whether we can just use the two floated values (LHCb and CMS Run 2) in the fit

Treatment of S wave

	ATLAS		CMS		LHCb	
	Run I	Run II	Run I	Run II	Run I	Run II
$m(KK)$ range [MeV]	[1008.5,1030.5]		[1010,1030]		[0.990,1.050]	
$m(KK)$ bins	1		1		6	
S-wave model	const.	non-res.	non-res.		f_0	f_0
S-wave phase	$\delta_\perp - \delta_S$		$\delta_S - \delta_\perp$		$\delta_S - \delta_\perp$	

- LHCb bins: [990, 1008, 1016, 1020, 1024, 1032, 1050] MeV
- Four middle bins define a range [1008, 1032] close to ATLAS and CMS range

Method

1. Define either one ([990, 1050]) or three bins ([990, 1008, 1032, 1050]) and renormalize fractions and phases using knowledge of S wave
2. Do total average in one (or three) instead of six bins

Conclusion

- Code available for combination of results, please contact us if you want to use it
- Takes care of all statistical and systematic errors and correlations
- Target parameters do not have to match source parameters
- Can combine any number of ResultSets
- Has been used in CERN-EP-2019-108
- Is now used for a combination between ATLAS, CMS and LHCb

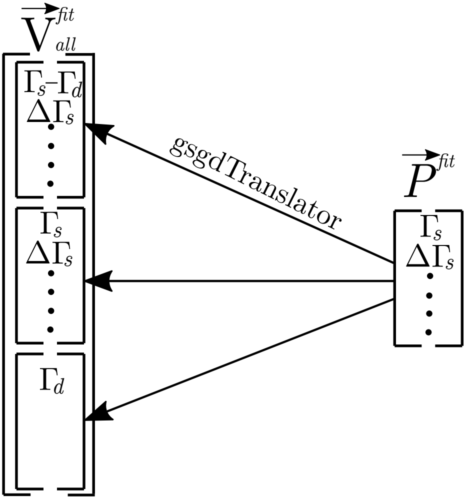
Thank you!

Translators

Parameters	Value
Γ_s Run 1	$0.6603 \pm 0.0027 \pm 0.0015$
$\Gamma_s - \Gamma_d$ Run 2	$-0.0041 \pm 0.0024 \pm 0.0015$
Γ_d WA	0.65789 ± 0.00173

- In Run 2 the average B_s^0 decay width was chosen to be published with respect to the B^0 one using its world average value
- Any relation can be included in the combination fit with a linear translator as:

```
def gsgdTranslator( fitDict ):
    #Calculates gamma in terms of the fit parameters  (in this case gammaD and gsgd)
    gd = fitDict[ 'gammaD' ]
    gamma = fitDict[ 'gamma' ]
    gsgd = gamma-gd
    return gsgd
reslist.addParameterTranslator( 'gsgd', gsgdTranslator )
```



ResultSets have different linearly dependent parameters, e.g. Γ_s in Run 1 and $\Gamma_s - \Gamma_d$ in Run 2. Can only fit for one