# Exercises for the course "Feynman integrals"
**Sheet 1**

## Exercise 1

Consider a connected graph $G$ with $n_{\mathrm{ext}}$ external edges, $n_{\mathrm{int}}$ internal edges and loop number $l$. Denote by

$$E^{\mathrm{in}} \;:\; \text{set of edges, which have a vertex of valency 1 as source,}$$
$$E^{\mathrm{out}} \;:\; \text{set of edges, which have a vertex of valency 1 as sink.}$$

The edges in $E^{\mathrm{in}}$ and $E^{\mathrm{out}}$ are necessarily external edges.

Show that momentum conservation at each vertex of valency $> 1$ implies momentum conservation of the external momenta:

$$\sum_{e_j \in E^{\mathrm{in}}} q_j = \sum_{e_j \in E^{\mathrm{out}}} q_j.$$

In other words: Momentum conservation at each internal vertex implies momentum conservation of the external momenta.

## Exercise 2

Let $U, V > 0$ be some constants and $a, \nu \in \mathbb{C}$. Show that

$$\int \frac{d^D k}{i \pi^{\frac{D}{2}}} \frac{\left(-k^2\right)^a}{\left(-U k^2 + V\right)^\nu} = \frac{\Gamma\left(\frac{D}{2}+a\right)}{\Gamma\left(\frac{D}{2}\right)} \frac{\Gamma\left(\nu - \frac{D}{2} - a\right)}{\Gamma(\nu)} \frac{U^{-\frac{D}{2}-a}}{V^{\nu-\frac{D}{2}-a}}.$$

Hint: Repeat the steps we did in the calculation of the tadpole integral.