Exercises for the course "Feynman integrals"
Sheet 5

Exercise 8

Consider the elliptic curve defined by the quartic polynomial $P(w,z) = 0$, where

$$P(w,z) = w^2 - z(z+4)\left[z^2 + 2(1+x)z + (1-x)^2\right].$$

x is a parameter. Determine two independent periods of the elliptic curve.

Exercise 9

At the end of the tutorial let's play a game: **Hironaka's polyhedra game**. This game is played by two players, A and B. They are given a finite set M of points $m = (m_1, ..., m_n)$ in $\mathbb{N}_+^n$, the first quadrant of $\mathbb{N}^n$. We denote by $\Delta \subset \mathbb{R}_+^n$ the positive convex hull of the set M. It is given by the convex hull of the set

$$\bigcup_{m \in M} \left(m + \mathbb{R}_+^n\right).$$

The two players compete in the following game:

1. Player A chooses a non-empty subset $S \subseteq \{1, ..., n\}$.
2. Player B chooses one element i out of this subset S.

Then, according to the choices of the players, the components of all $(m_1, ..., m_n) \in M$ are replaced by new points $(m_1', ..., m_n')$, given by:

$$\begin{aligned} m_j' &= m_j, \quad \text{if } j \neq i, \\ m_i' &= \sum_{j \in S} m_j - 1. \end{aligned}$$

This defines the set M'. One then sets $M = M'$ and goes back to step 1. Player A wins the game if, after a finite number of moves, the polyhedron Δ is of the form

$$\Delta = m + \mathbb{R}_+^n,$$

i.e. generated by one point. If this never occurs, player B has won.

You may play the game with a fellow student. Actually, one may show that player A can always win the game and you may want to think about a winning strategy for player A (this is not easy).