Hadron Resonances and Scattering

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Acknowledgements

UNIVERSITY OF CAMBRIDGE

Science and Technology Facilities Council

DiRAC

Hadron Spectrum Collaboration

[*www.hadspec.org*]

hadspec

Jefferson Lab and surroundings, USA:

Raúl Briceño[1], Jozef Dudek[2], Robert Edwards, Bálint Joó, David Richards, Frank Winter, Luka Leskovec ([1] Old Dominion University, [2] William & Mary)

ODU: Andrew Jackura, Luka Leskovec; W&M: Arkaitz Rodas, *Christopher Johnson, Archana Radhakrishnan, Felipe Ortega*

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Particular thanks to Robert Edwards for some material for this talk

Hadron spectroscopy

Intriguing observations at experiments, e.g. $X(3872)$, $Y(4260)$, $Z_c^+(4430)$, $Z_c^+(3900)$, Z_b^+, $D_s(2317)$, light scalars, $\pi_1(1600)$ [$J^{PC} = 1^{-+}$], P_c , Roper, other baryon resonances

KLOE CLAS12

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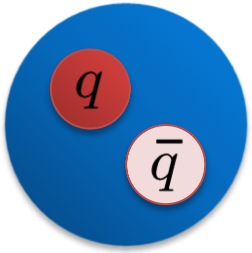

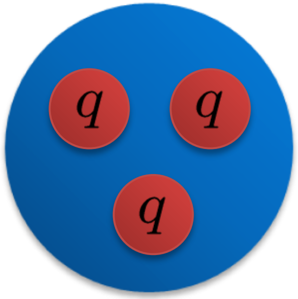

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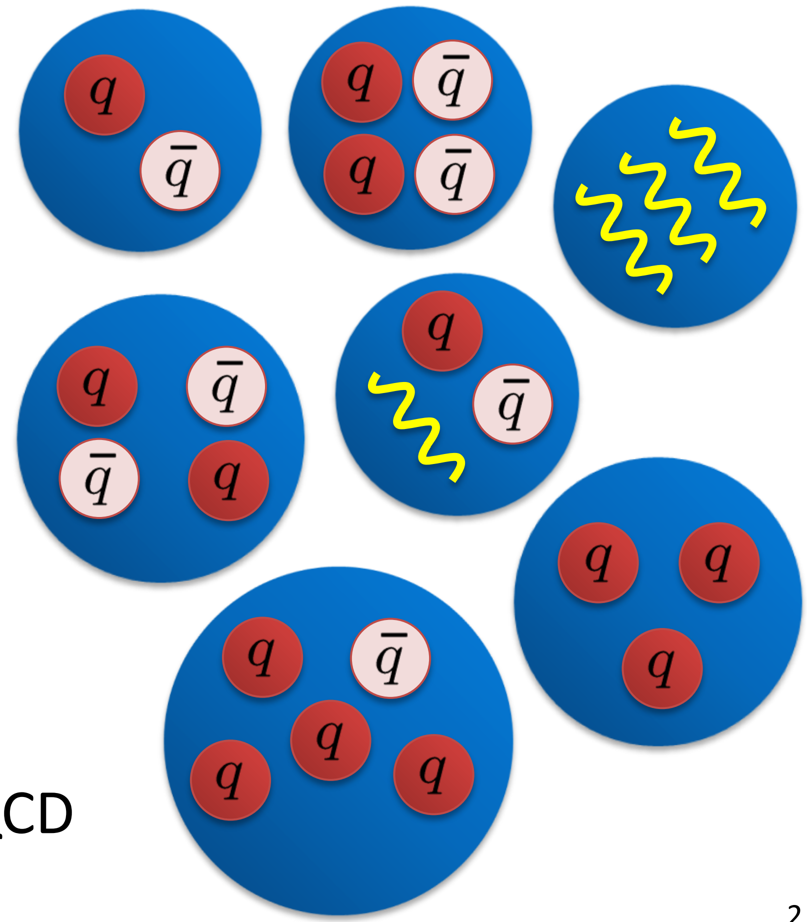
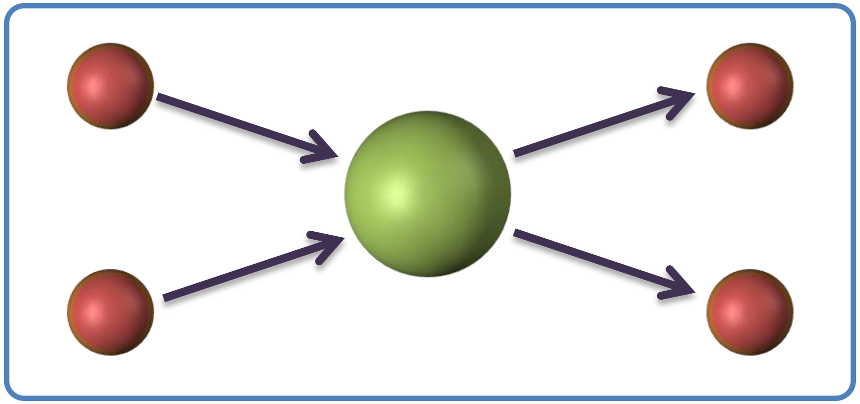
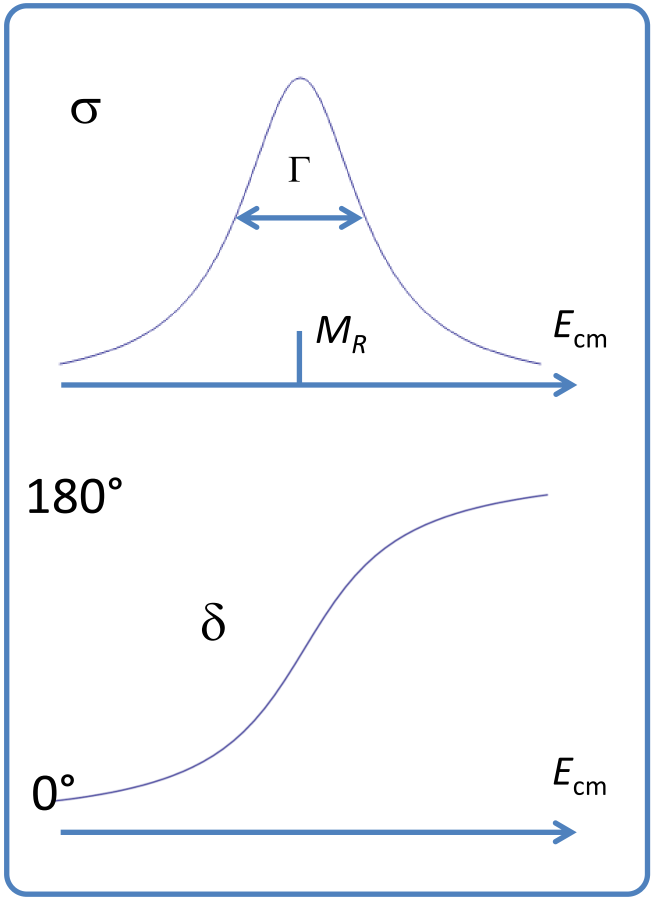


First-principles calculations → lattice QCD

Scattering and resonances

Most hadrons appear as resonances in scattering of lighter hadrons





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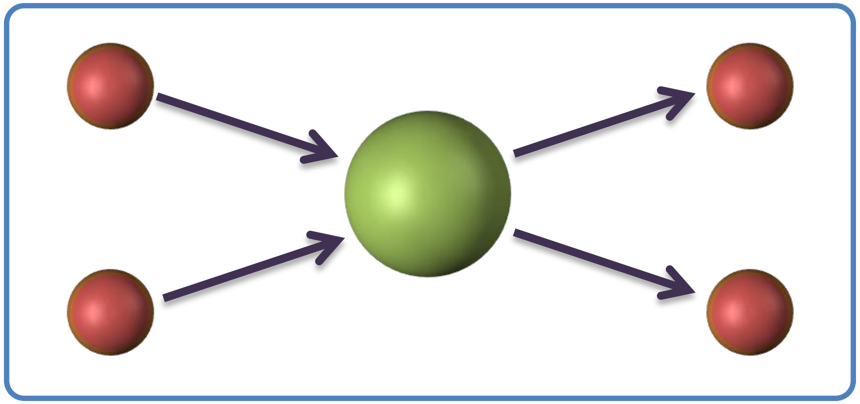
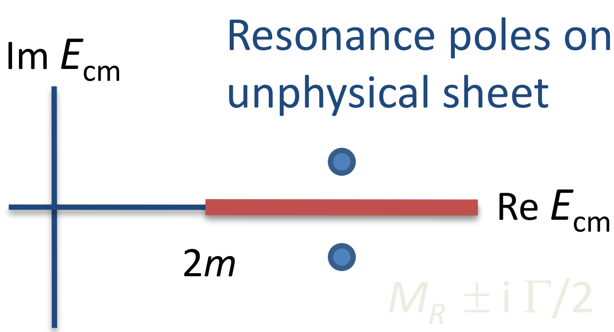
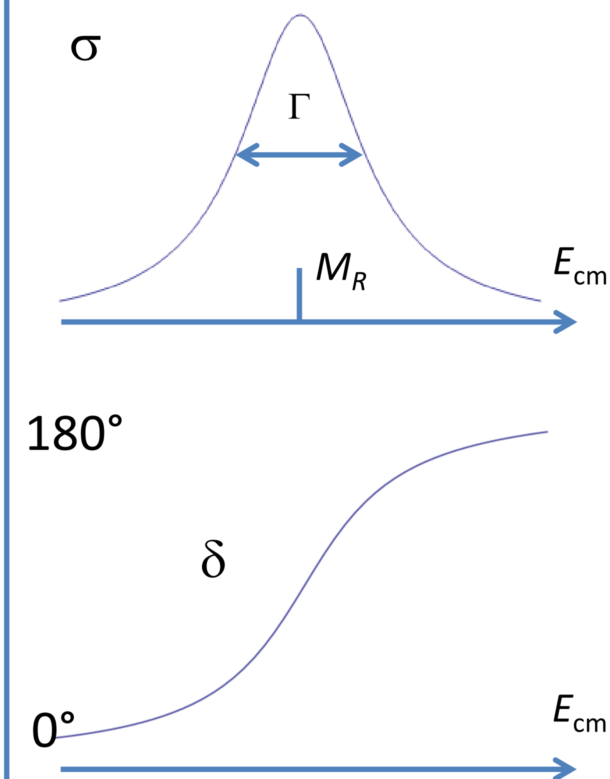


Singularity structure of scattering matrix (poles → state content)

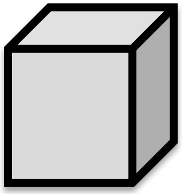




Scattering in lattice QCD

Can't directly compute scattering amplitudes in lattice QCD

Lüscher method [NP B354, 531 (1991)] (and extensions): relate discrete set of **finite-volume energy levels** $\{E_{\mathrm{cm}}\}$ to **infinite-volume scattering *t*-matrix**.



$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

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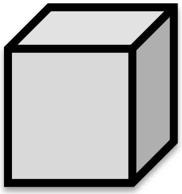

$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

$$\det\left[1 + i\,\boldsymbol{\rho}(E_{\text{cm}})\,\boldsymbol{t}(E_{\text{cm}})\,\left(1 + i\,\boldsymbol{\mathcal{M}}^{\vec{P}}(E_{\text{cm}}, L)\right)\right] = 0$$

Elastic scattering: one-to-one mapping $E_{\text{cm}} \leftrightarrow t(E_{\text{cm}})$

Coupled channels: under-constrained problem (each E_{cm} constrains t-matrix at that E_{cm})

[Complication: reduced sym. of lattice vol. → mixing of partial waves]

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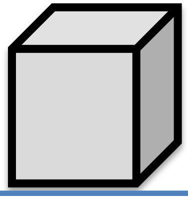

Require:

- Large set of E_{cm} in a range of channels:
  - various symmetry channels (irreps), and
  - overall non-zero momentum, different volumes, and/or twisted b.c.s
- Large enough spatial volume ($m_\pi L \gtrsim 4$)

N.B. Lüscher method is for hadron-hadron scattering (any no. of coupled channels); will return to >2 hadron scattering later

Lattice QCD spectroscopy

Finite-volume energy eigenstates from:

$$C_{ij}(t) = \left\langle 0 \left| \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \right| 0 \right\rangle$$

$$= \sum_n \frac{e^{-E_n t}}{2\ E_n} \langle 0|\mathcal{O}_i(0)|n\rangle \langle n|\mathcal{O}_j^\dagger(0)|0\rangle$$

In each symmetry channel: matrix of correlators for **large bases of interpolating operators** with appropriate variety of structures.

$$\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\ \bar{\psi}(x) \left[\Gamma \overleftrightarrow{D} \overleftrightarrow{D} \ldots \right] \psi(x)$$

$$\epsilon^{abc} \psi_a \psi_b \psi_c$$

Variational method (generalised eigenvalue problem) → $\{E_n\}$

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$$\epsilon^{abc}\psi_a\psi_b\psi_c \qquad \sum_{\vec{p}_1,\vec{p}_2,\vec{p}_3,\ldots} C(\vec{P},\vec{p}_1,\vec{p}_2,\vec{p}_3,\ldots)H(\vec{p}_1)H(\vec{p}_2)H(\vec{p}_3)\ldots$$

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Distillation

[Peardon *et al* (HadSpec), PR D80 054506 (2009)]

Smearing operator, **project onto low-lying modes**

$$\square_{xy}(t) = \sum_{k=1}^{N} v_x^{(k)}(t) v_y^{(k)\dagger}(t) = V_x(t) V_y^\dagger(t)$$

$$\psi(x) \to \tilde{\psi}(x) = \square \psi(x)$$

e.g. $-\nabla^2 v^{(k)} = \lambda^k v^{(k)}$

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$$\psi(x) \to \tilde{\psi}(x) = \Box \psi(x)$$

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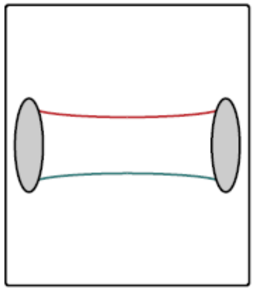
Factorizes computation of correlators

$$C_{ij}(t) = \langle \bar{\psi}\, \Box(t)\, \Gamma_t^i\, \Box(t)\, \psi(t) \cdot \bar{\psi}\, \Box(0)\, \Gamma_0^j\, \Box(0)\, \psi(0) \rangle$$
$$= \mathrm{Tr}\left[\Phi^i(t) P(t,0) \Phi^j(0) P(0,t) \right]$$



$$P(t,0) = V^\dagger(t) M^{-1}(t,0) V(0)$$ *perambulator*

$$\Phi^i(t) = V^\dagger(t) \Gamma_t^i V(t)$$ *elemental meson op* (mom, spin, derivs)

$\Big\}$ (N_s N) x (N_s N) matrices [c.f. $(N_s\, N_c\, (L/a_s)^3)^2$]

elemental baryon op is rank-3 tensor

Distillation

[Peardon *et al* (HadSpec), PR D80 054506 (2009)]

Factorization: reuse perambulators and elemental ops

Enables complicated op structures at source and sink
(with component ops projected onto definite momentum)

Quark annihilation lines $P(t, t)$

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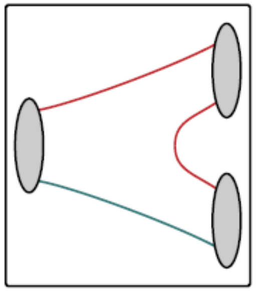
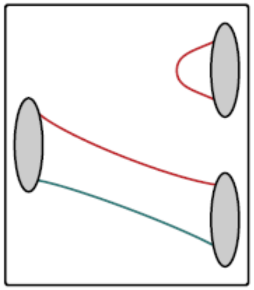


$$\mathrm{Tr}\left[\Phi^A(t)P(t,0)\Phi^B(0)P(0,0)\Phi^C(0)P(0,t)\right]$$



$$\mathrm{Tr}\left[\Phi^A(t)P(t,0)\Phi^C(0)P(0,t)\right]\,\mathrm{Tr}\left[\Phi^B(0)P(0,0)\right]$$

Workflow

Gauge cfgs. (CHROMA)

Perambulators (CHROMA)
(multigrid, GPUs/KNLs/CPUs)

Elemental ops (CHROMA)
(CPUs)

Contraction code to compute correlation functions:
construct ops (incl. flavour and irreps), work out Wick contractions, efficient organisation of operations.
Multiplying and contracting tensors, combinatoric factors
(CPUs/KNLs/GPUs)

[http://jeffersonlab.github.io/chroma/] [hep-lat/0409003, 0911.3191, 1011.0024, 1011.2775, 1005.3043, Lect. Notes Comput. Sci. 7905 (2013) 40, 1612.07873]

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Disk storage per cfg for $m_\pi \approx 239$ MeV, $32^3 \times 256$, $m_\pi L \sim 4$, $N = 384$
$\sim$ 1.7 GB for perambulator (per quark mass and t_{src}),
$\sim$ 370 GB for meson elemental ops (up to $p^2 = 6\,(2\pi/L)^2$)

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Gauge cfgs, perambs, contractions: e.g. OLCF, NERSC, TACC, JLab (USQCD)
Contractions: DiRAC/Cambridge (also some perambs) & TCD, Ireland

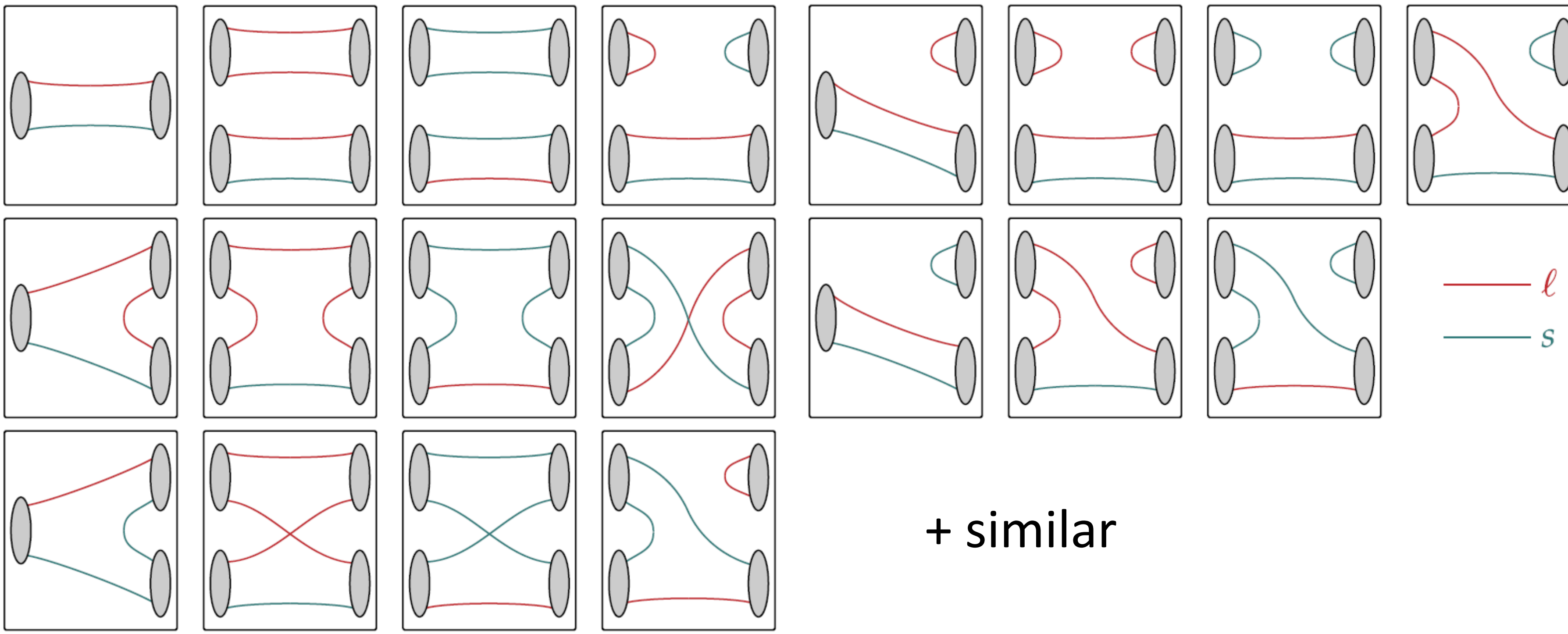
[htt
1011.2775, 1005.3043, Lect. Notes Comput. Sci. 7905 (2013) 40, 1612.07873]

Examples of recent lattice QCD calculations

πK, ηK (isospin=1/2)

[Wilson, Dudek, Edwards, CT,
PRL 113, 182001 (2014); PR D91, 054008 (2015)]

Many diagrams:



+ similar

also sums over momentum directions
(e.g. overall at rest: 6 x [0,0,1][0,0,-1], 12 x [0,1,1][0,-1,-1],
8 x [1,1,1][-1,-1,-1], ...)

πK, ηK (isospin=1/2)

[PRL 113, 182001 (2014);
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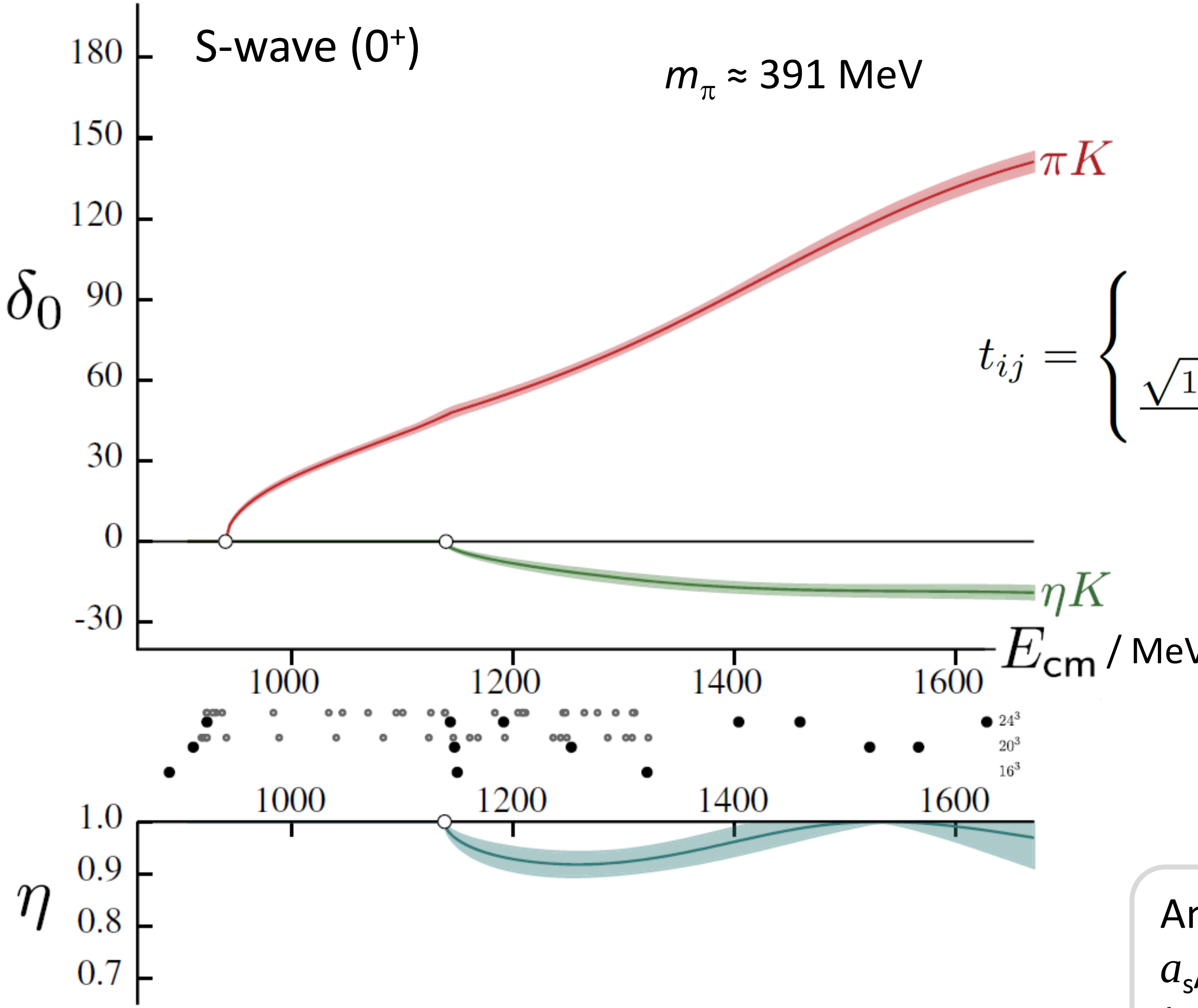

$$t_{ij} = \begin{cases} \dfrac{\eta\, e^{2i\delta_i} - 1}{2i\,\rho_i} & (i = j) \\[2ex] \dfrac{\sqrt{1-\eta^2}\; e^{i(\delta_i + \delta_j)}}{2\sqrt{\rho_i\,\rho_j}} & (i \neq j) \end{cases}$$

80 energy levels
(3 volumes)

Anisotropic clover (N_f = 2+1)
$a_s/a_t \approx 3.5$, $a_s \approx 0.12$ fm
$L \sim 2 - 3$ fm, $m_\pi L \sim 4 - 6$

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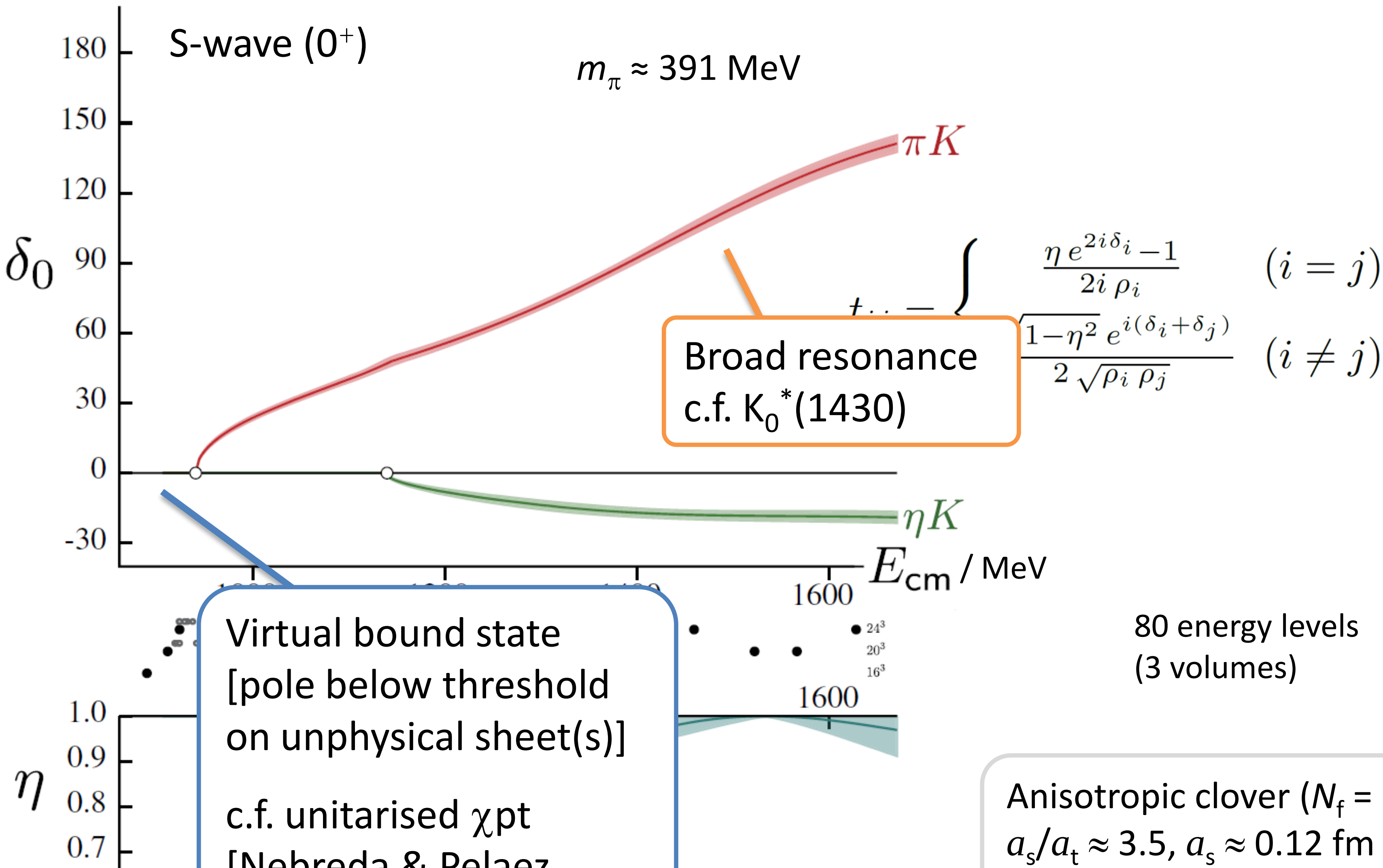

Broad resonance
c.f. $K_0^*(1430)$

Virtual bound state
[pole below threshold
on unphysical sheet(s)]

c.f. unitarised χpt
[Nebreda & Pelaez,
PR D81, 054035 (2010)]

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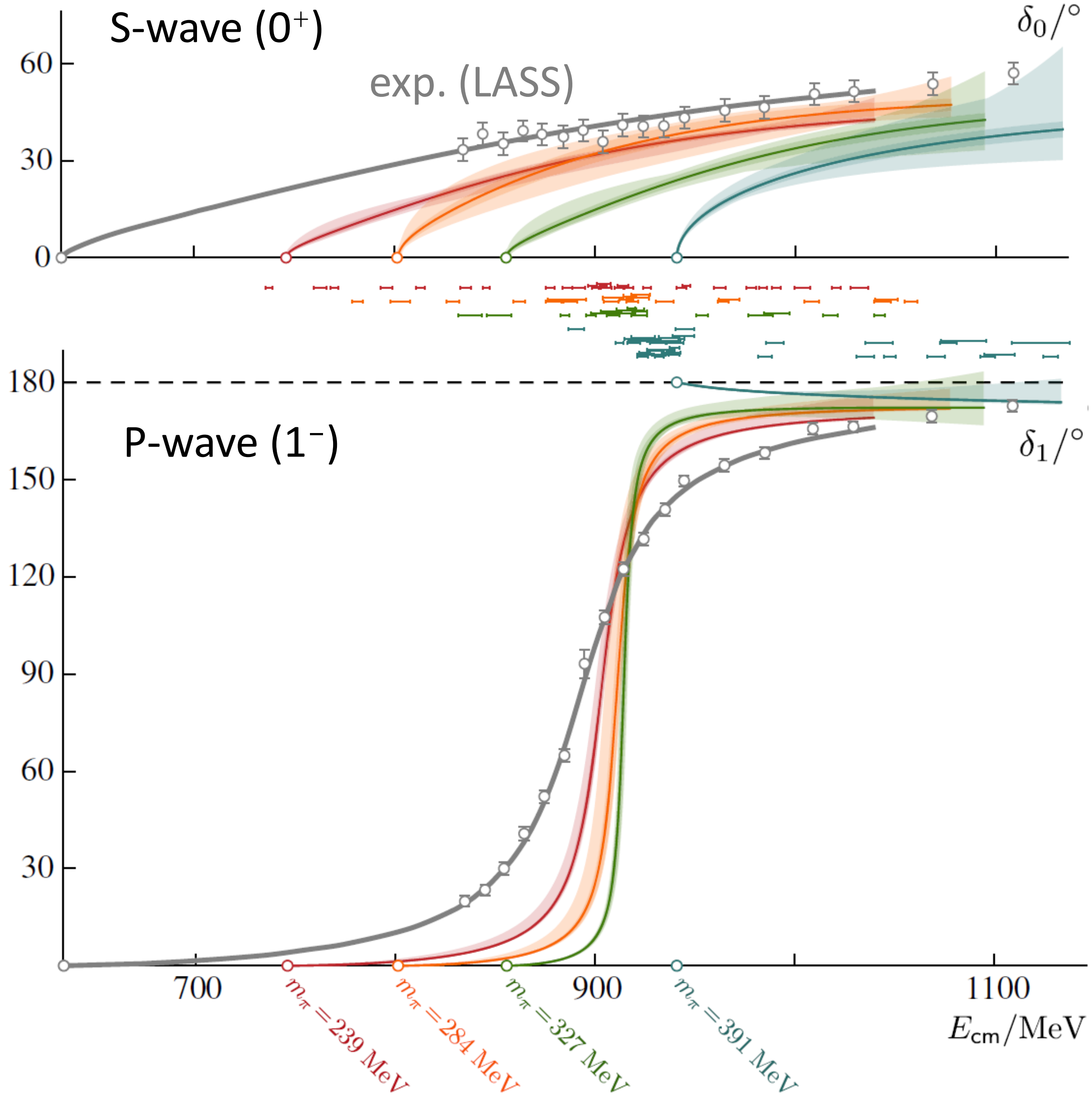
πK (isospin=1/2)

$m_\pi \approx 239, 284,$
$327, 391$ MeV
(28, 21, 18, 36 energies)



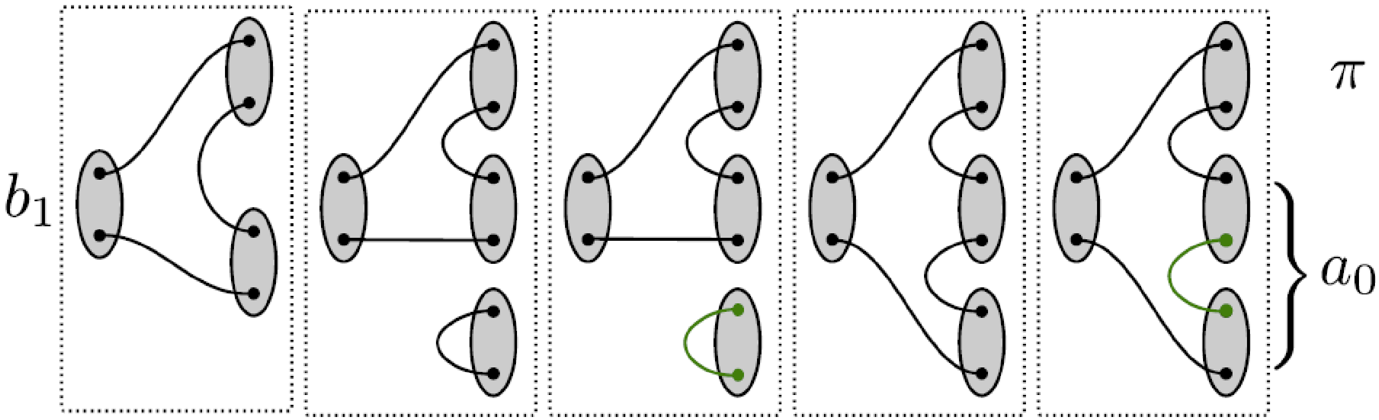
[Wilson, Briceño, Dudek, Edwards, CT, PRL 123, 042002 (2019)]

b_1 in isospin-1 $\pi\omega$, $\pi\phi$

[**Woss**, CT, Dudek, Edwards, Wilson, PRD 100, 054506 (2019)]

Coupled *S* & *D*-wave $\pi\omega$ and *S*-wave $\pi\phi$

Diagrams of similar topologies to before and also, e.g.



b_1 in isospin-1 πω, πϕ

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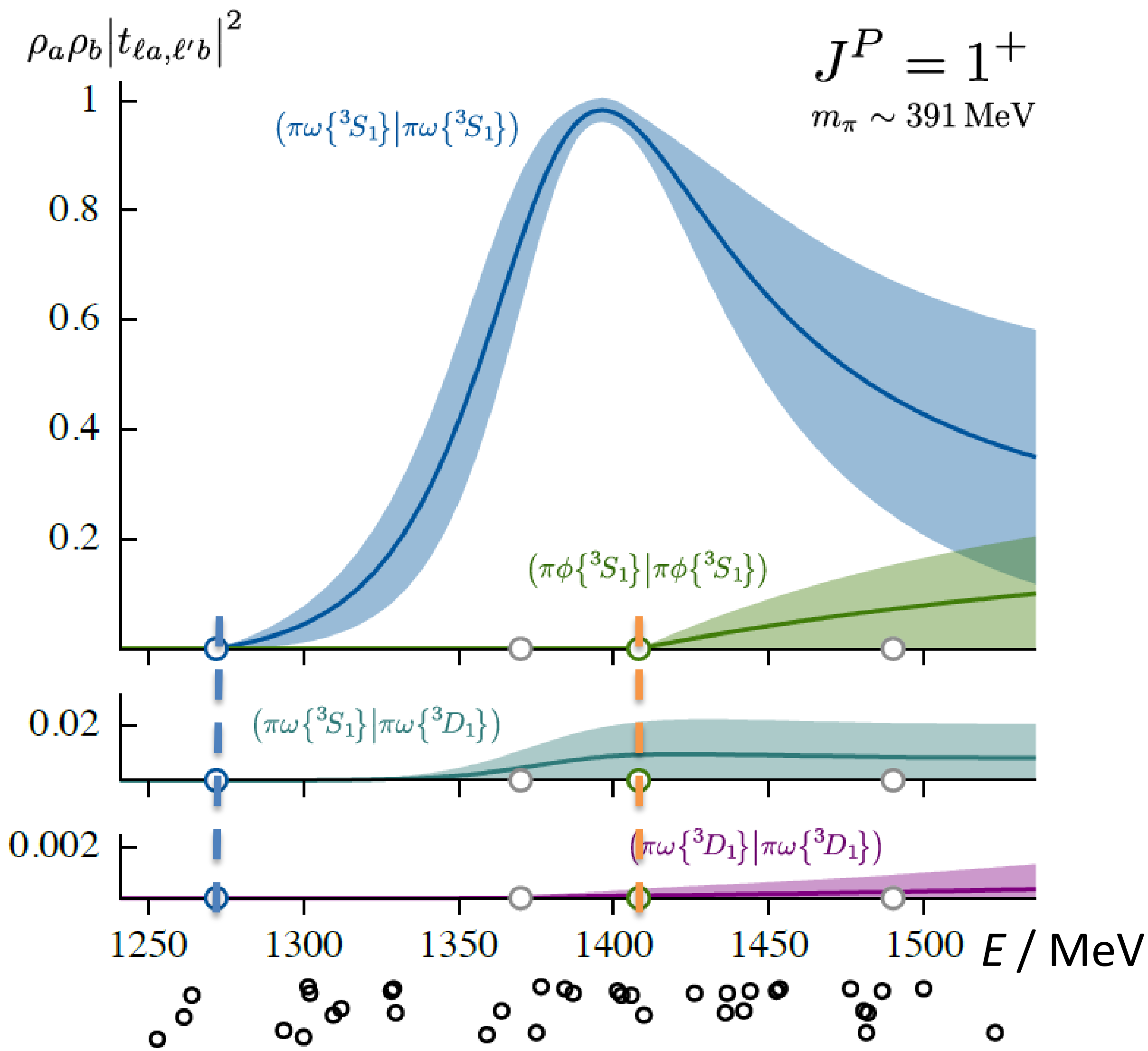

Coupled *S* & *D*-wave πω and *S*-wave πϕ

36 energy levels
[5 irreps on each of 3 vols
+4 extra irreps on 1 vol]

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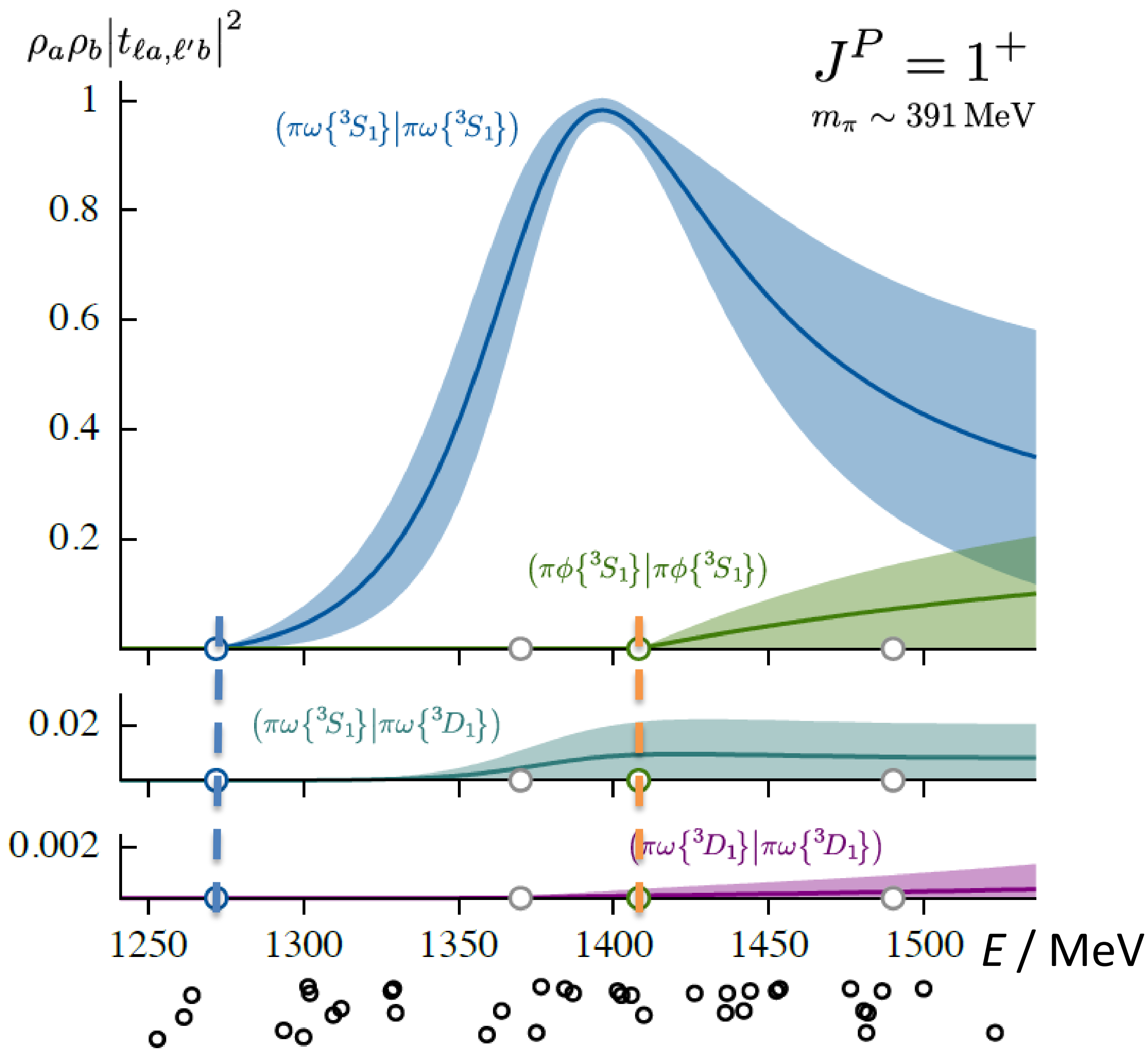

Coupled *S* & *D*-wave πω and *S*-wave πϕ

Clear enhancement in *S*-wave πω

Small *S*-wave πϕ, no evidence for Z_s analogue of Z_c

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[5 irreps on each of 3 vols
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b_1 in isospin-1 $\pi\omega$, $\pi\phi$

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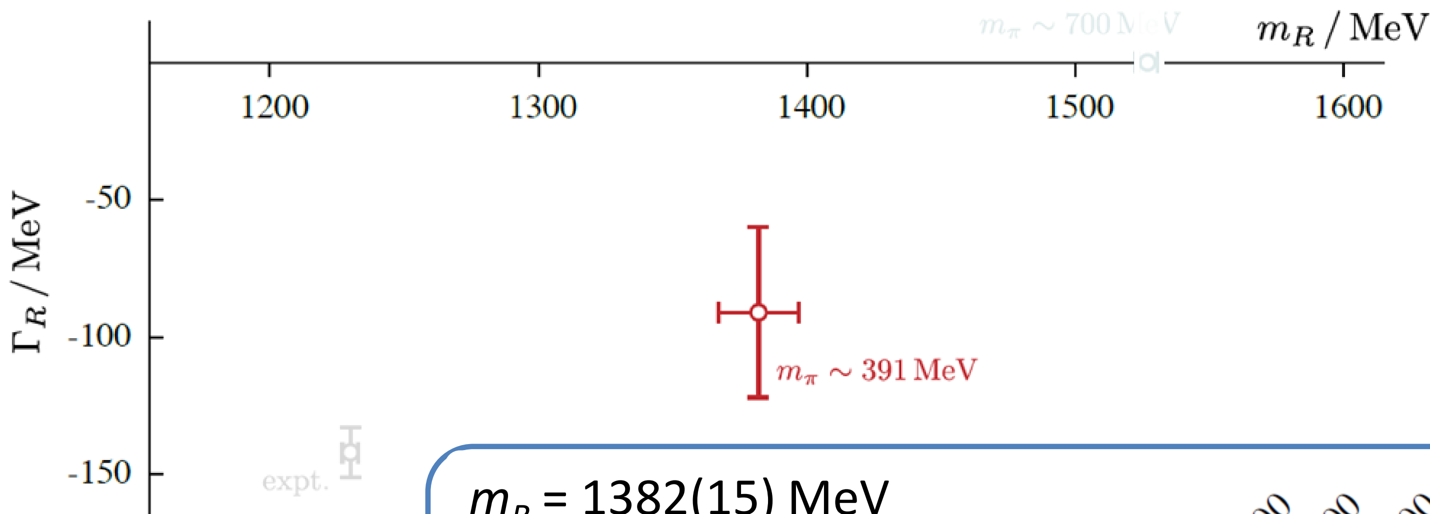
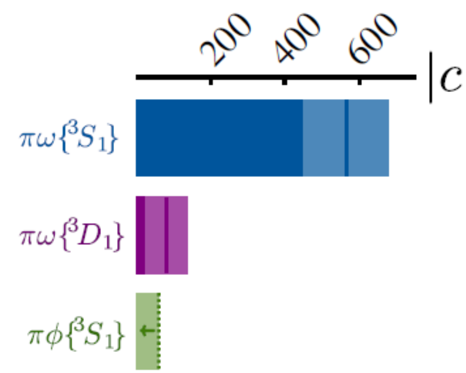


m_R = 1382(15) MeV
Γ_R = 91(31) MeV

$$\left|c_{\pi\omega\{^3S_1\}}\right| = 564(114)\ \mathrm{MeV}$$
$$\left|c_{\pi\omega\{^3D_1\}}\right| = 81(56)\ \mathrm{MeV},$$
$$\left|c_{\pi\phi\{^3S_1\}}\right| = 59(41)\ \mathrm{MeV}.$$



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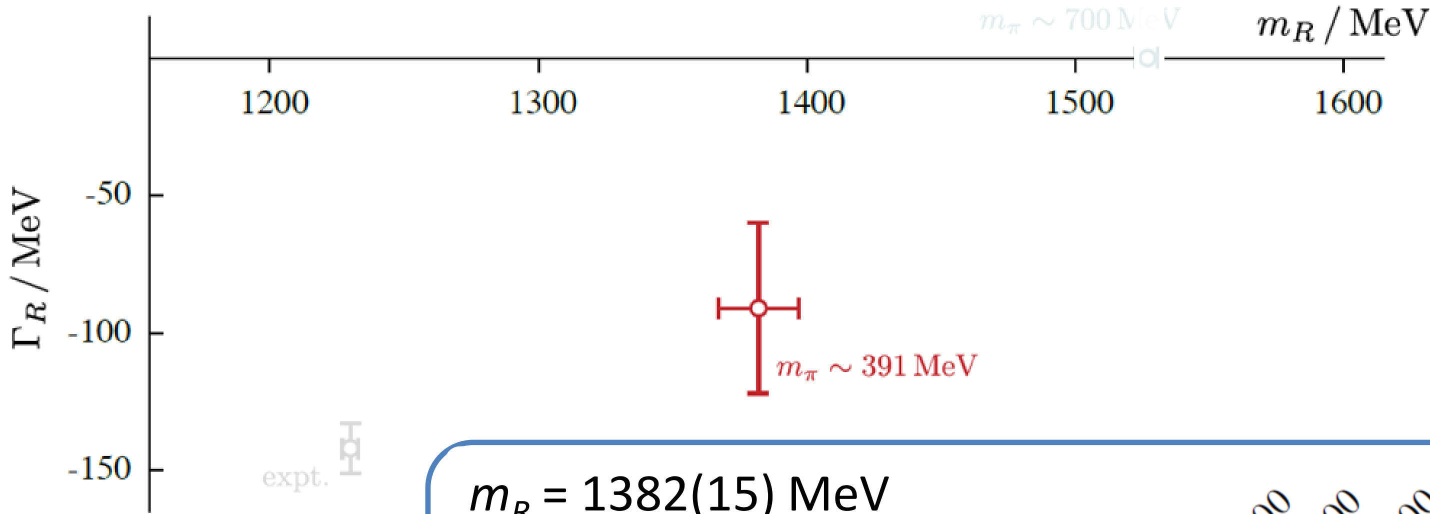

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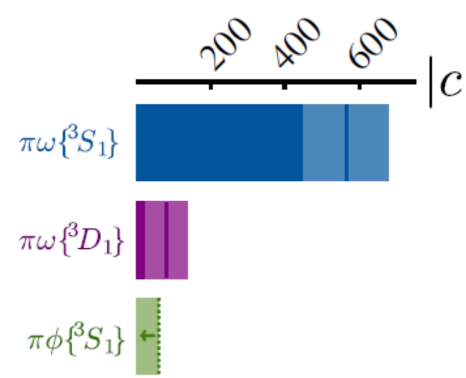


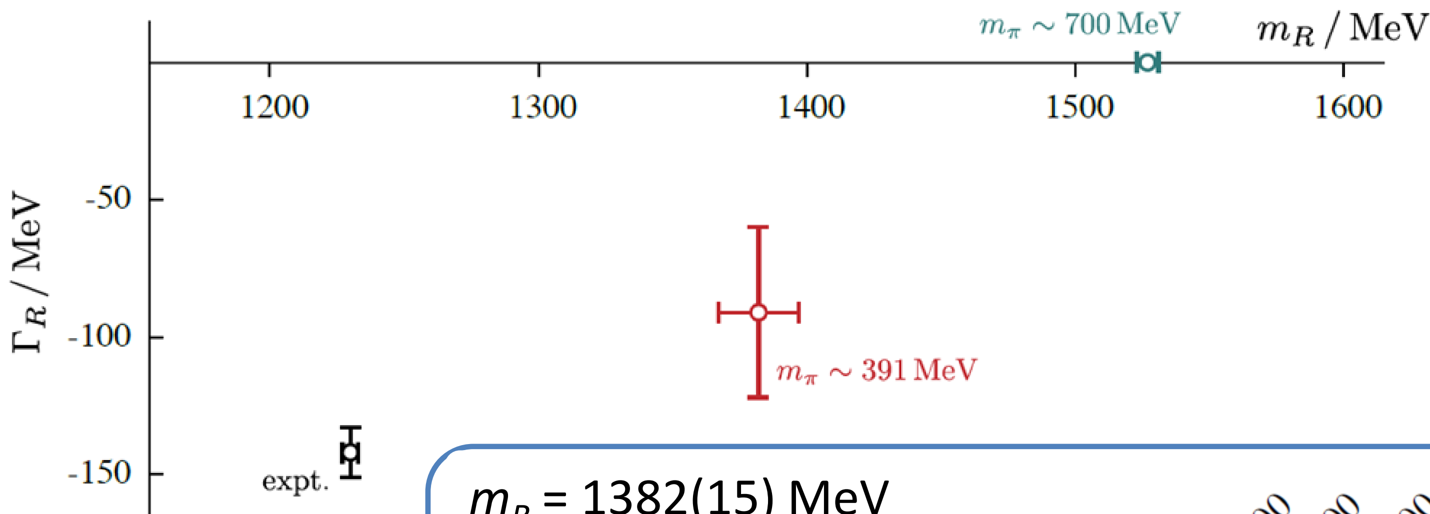
$$\mathrm{Br}\left(b_1 \to \pi\omega\{^3S_1\}\right) \sim 93\%$$

$$\mathrm{Br}\left(b_1 \to \pi\omega\{^3D_1\}\right) \sim 2\%$$

$$\mathrm{Br}\left(R \to a\{^3\ell_J\}\right) \equiv \frac{1}{\Gamma_R} \cdot \frac{|c_{\ell J a}|^2}{m_R} \rho_a(m_R).$$

b_1 in isospin-1 $\pi\omega$, $\pi\phi$

[**Woss**, CT, Dudek, Edwards, Wilson, PRD 100, 054506 (2019)]



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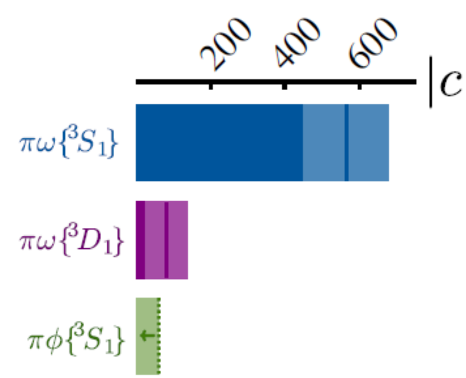

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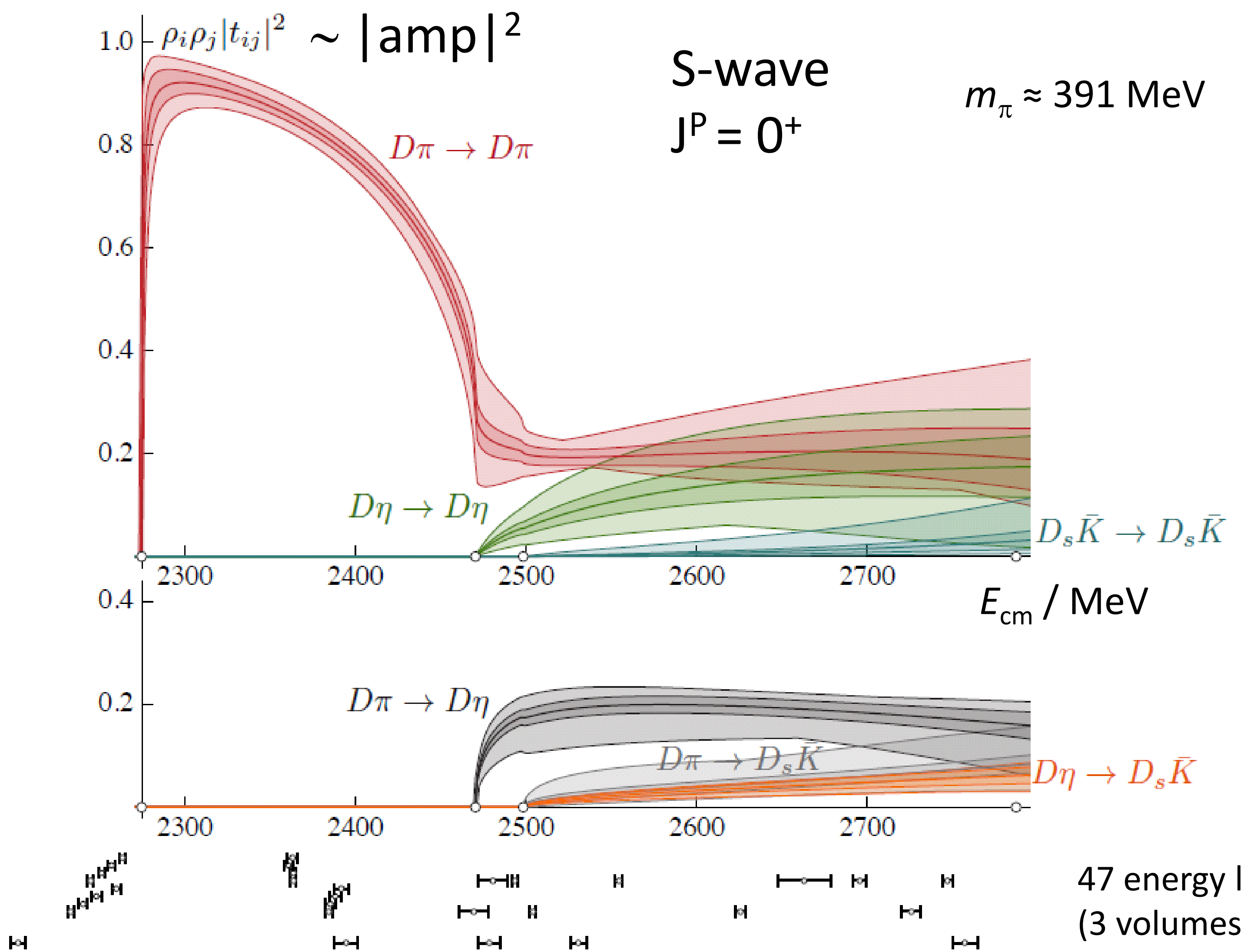
Dπ, Dη, $D_s\bar{K}$ (isospin=1/2)

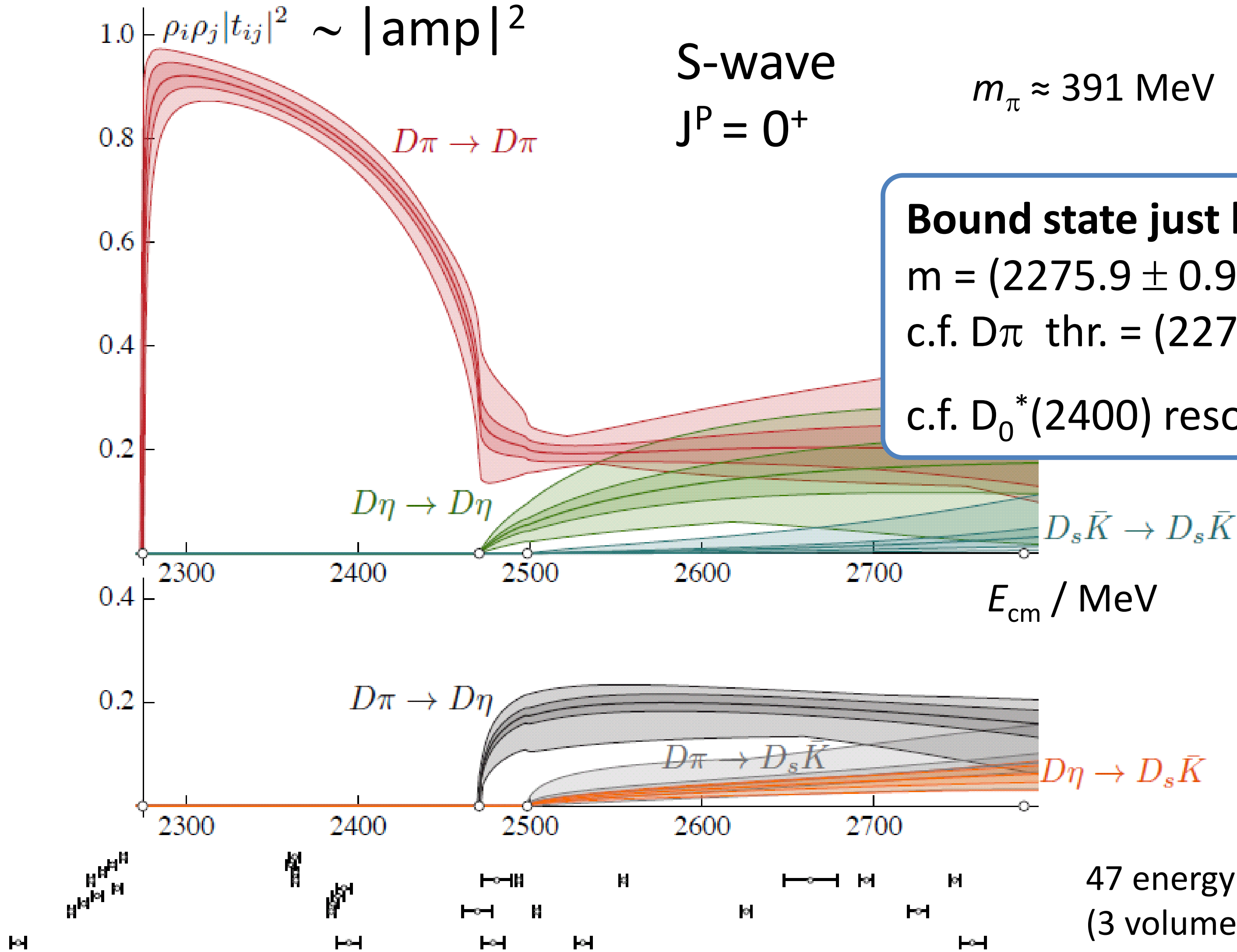
[Moir, Peardon, Ryan, CT, Wilson (HadSpec) JHEP 1610, 011 (2016)]



47 energy levels
(3 volumes)

Dπ, Dη, $D_s\bar{K}$ (isospin=1/2)

[Moir, Peardon, Ryan, CT, Wilson (HadSpec) JHEP 1610, 011 (2016)]



S-wave
$J^P = 0^+$

$m_\pi \approx 391$ MeV

Bound state just below thresh.
m = (2275.9 ± 0.9) MeV
c.f. Dπ thr. = (2276.4 ± 0.9) MeV

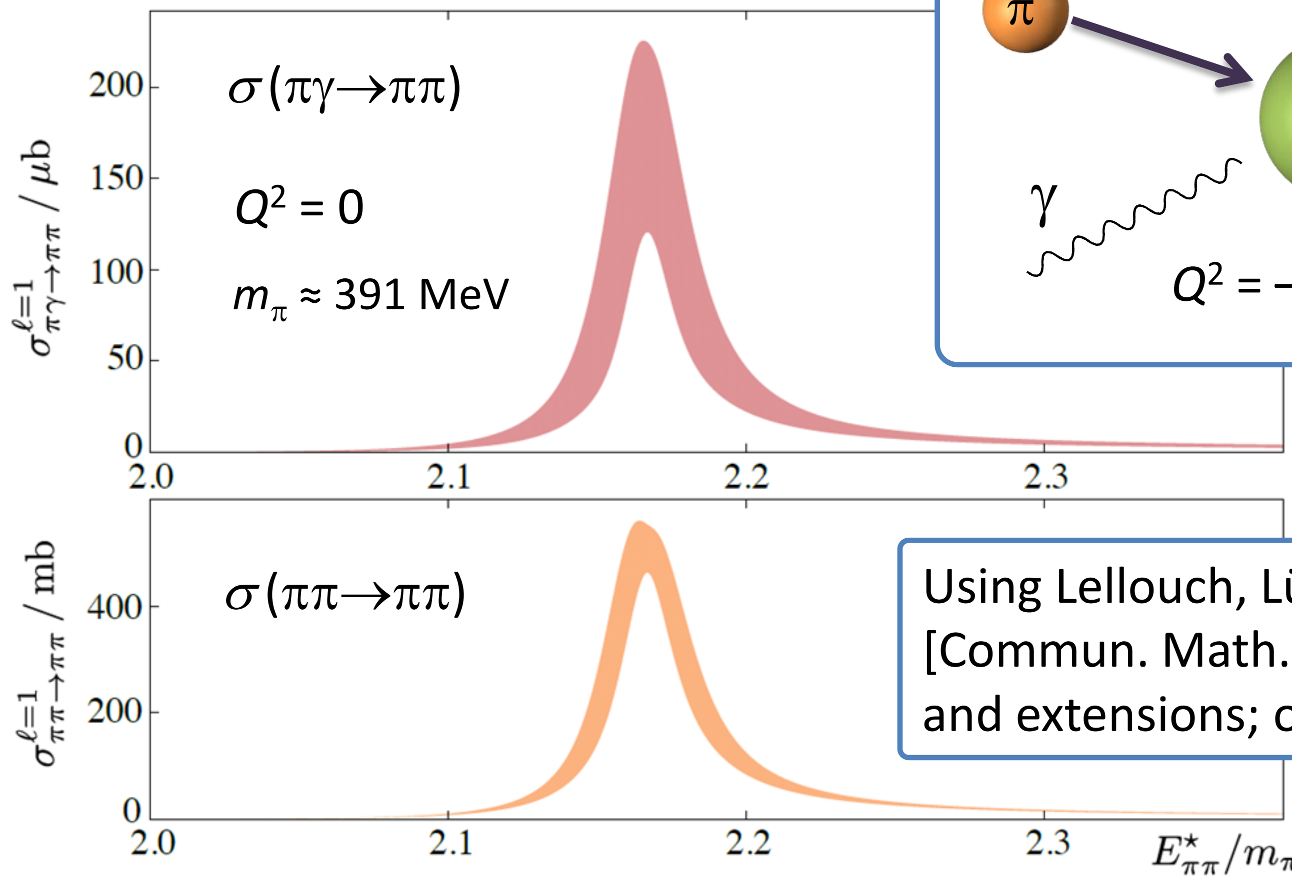
c.f. $D_0^*(2400)$ resonance

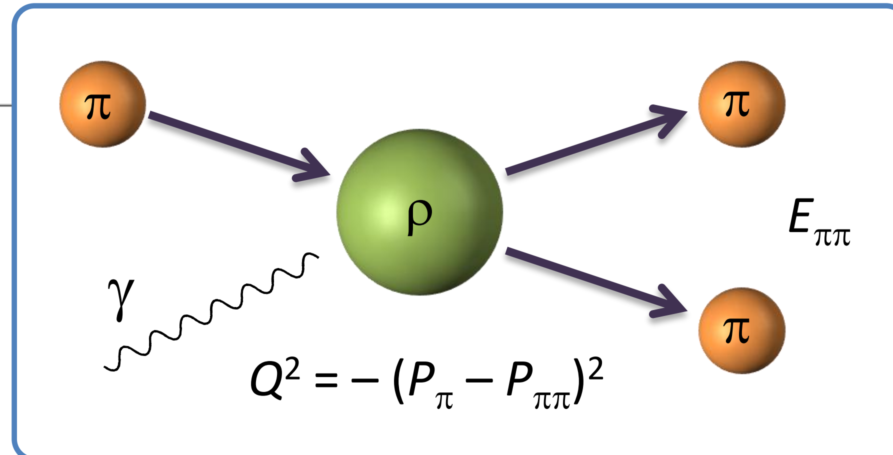
47 energy levels
(3 volumes)

Resonant $\pi^+ \gamma \to \rho \to \pi^+ \pi^0$ amplitude

[Briceño *et al* (HadSpec)
PRL 115, 242001 (2015);
PRD 93, 114508 (2016)]

Need: $\langle 0 \left| \mathcal{O}_i(t_f) \;\; \bar{\psi}(t)\gamma^\mu\psi(t) \;\; \mathcal{O}_j(t_i) \right| 0 \rangle$





Using Lellouch, Lüscher result on 2 → 1 [Commun. Math. Phys. 219, 31 (2001)] and extensions; ongoing work, e.g. 2 → 2

Some challenges

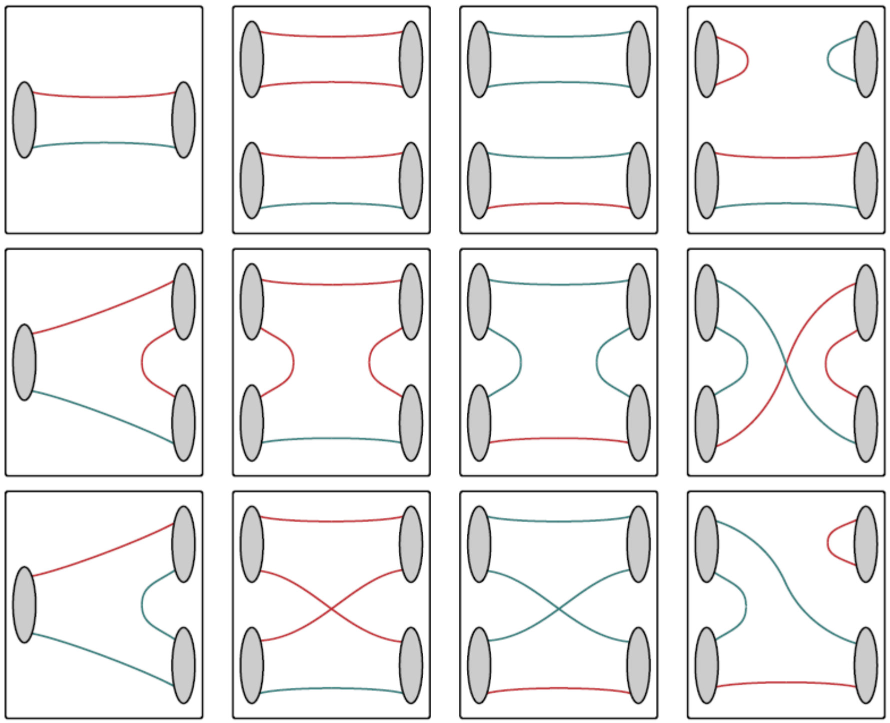
To get to physical m_π: larger volumes to keep $m_\pi L$ reasonable
$\rightarrow$ larger no. of distillation vectors ($N \propto$ volume).

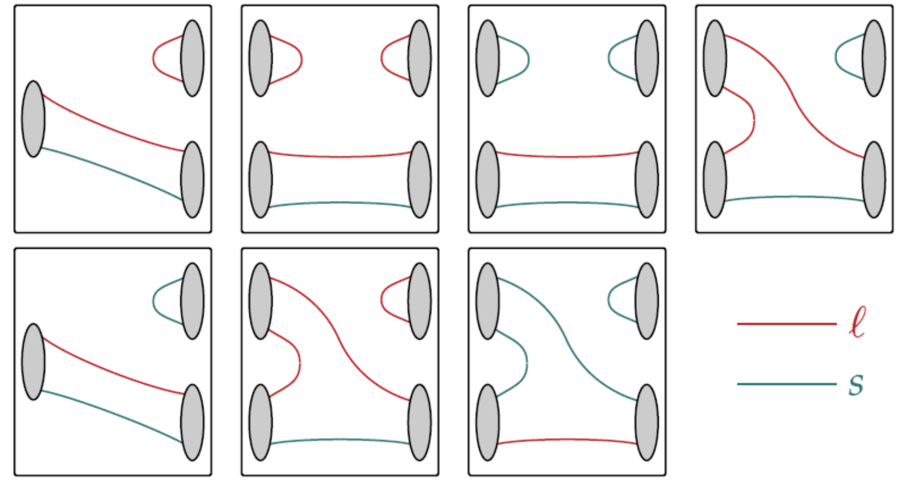
Baryons (rank-3 tensors) are more expensive

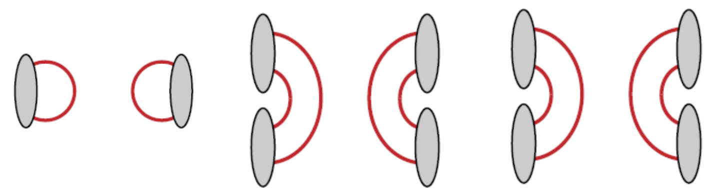
Contraction cost (multiplying and contracting tensors)
$\sim O(N^3)$ for M, MM, MMM, ...; $O(N^4)$ for BM, ...

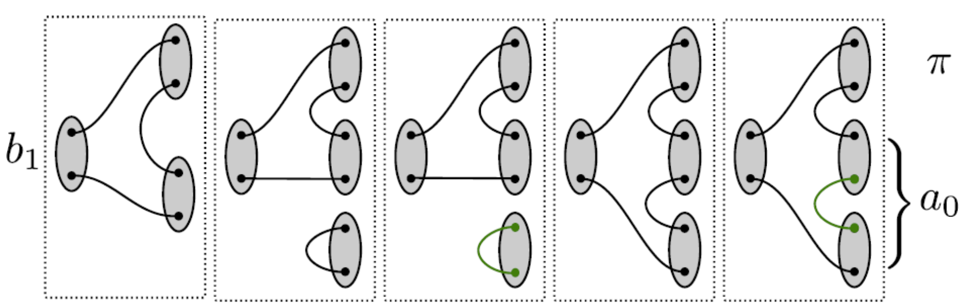
More efficient computation, stochastic distillation, another version of distillation (effectively smaller N), something else...?

Contraction code









Also sums over momentum directions (e.g. overall at rest: 6 x [0,0,1][0,0,-1], 12x [0,1,1][0,-1,-1], 8 x [1,1,1][-1,-1,-1], ...)

Lots of graphs! (~100,000's)

Many vertices for given *t*-slice

Contraction code

Efficiently organise computations and reuse common pieces (order of evaluation, temporaries) to reduce cost and memory

E.g. don't want to make larger rank tensors

Exploit hardware

Contraction code

Efficiently organise computations and reuse common pieces (order of evaluation, temporaries) to reduce cost and memory

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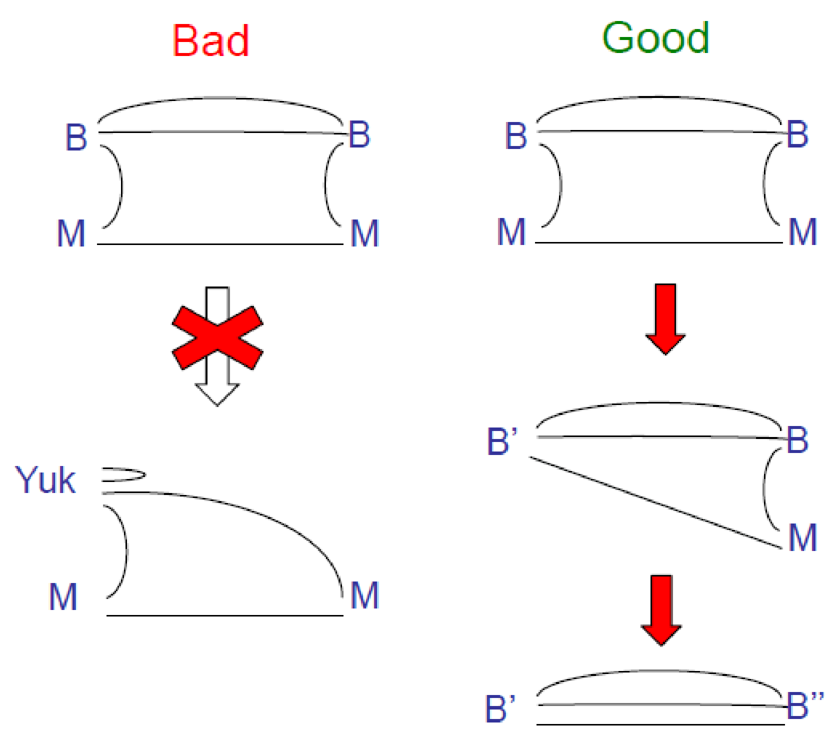
Exploit hardware



[from Robert Edwards]

Contraction code

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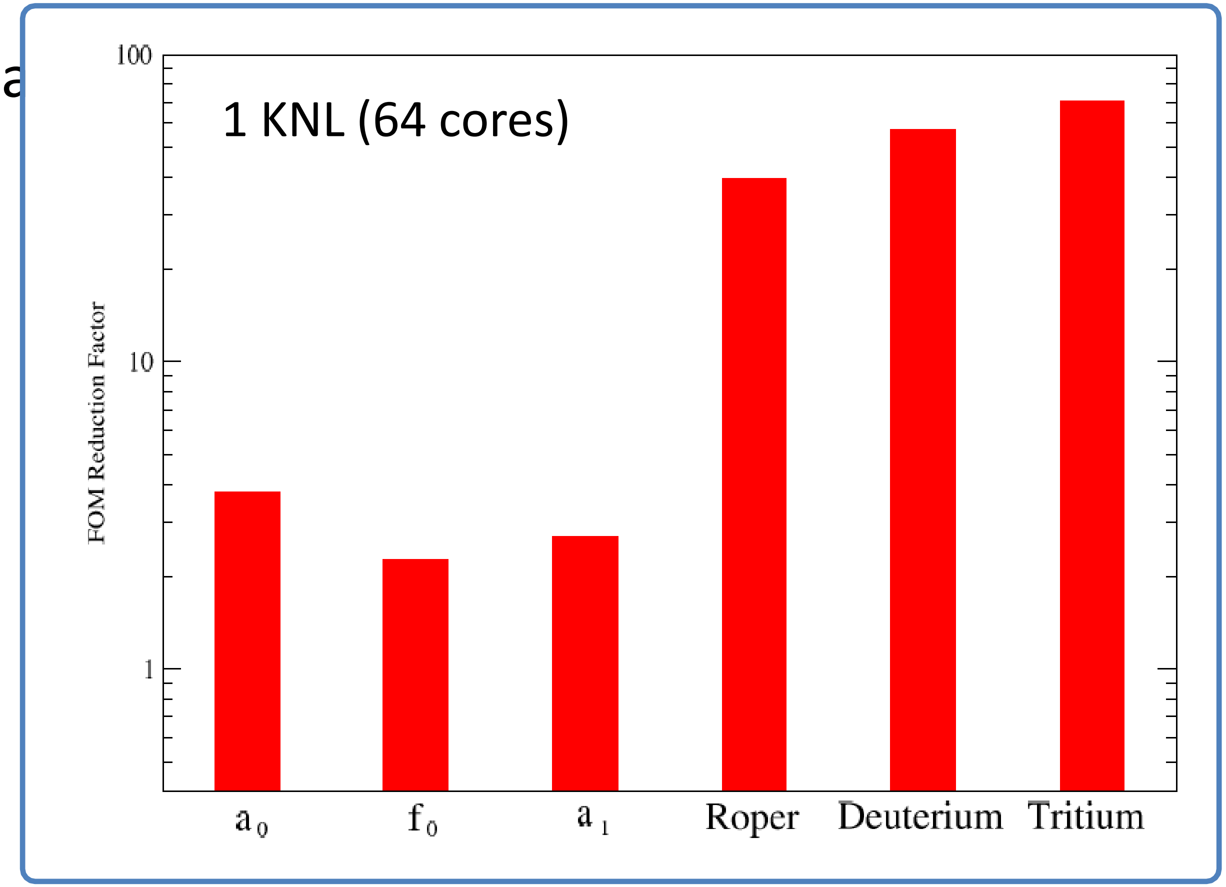
Exploit hardware



Test cases:

a_0 ($\eta\pi$, $\bar{K}K$, $\eta'\pi$) – O(N^3)

f_0 ($\pi\pi$, $\bar{K}K$, $\eta\eta$, $\eta'\eta$) – O(N^3)

a_1 ($\pi\pi\pi$) – O(N^3)

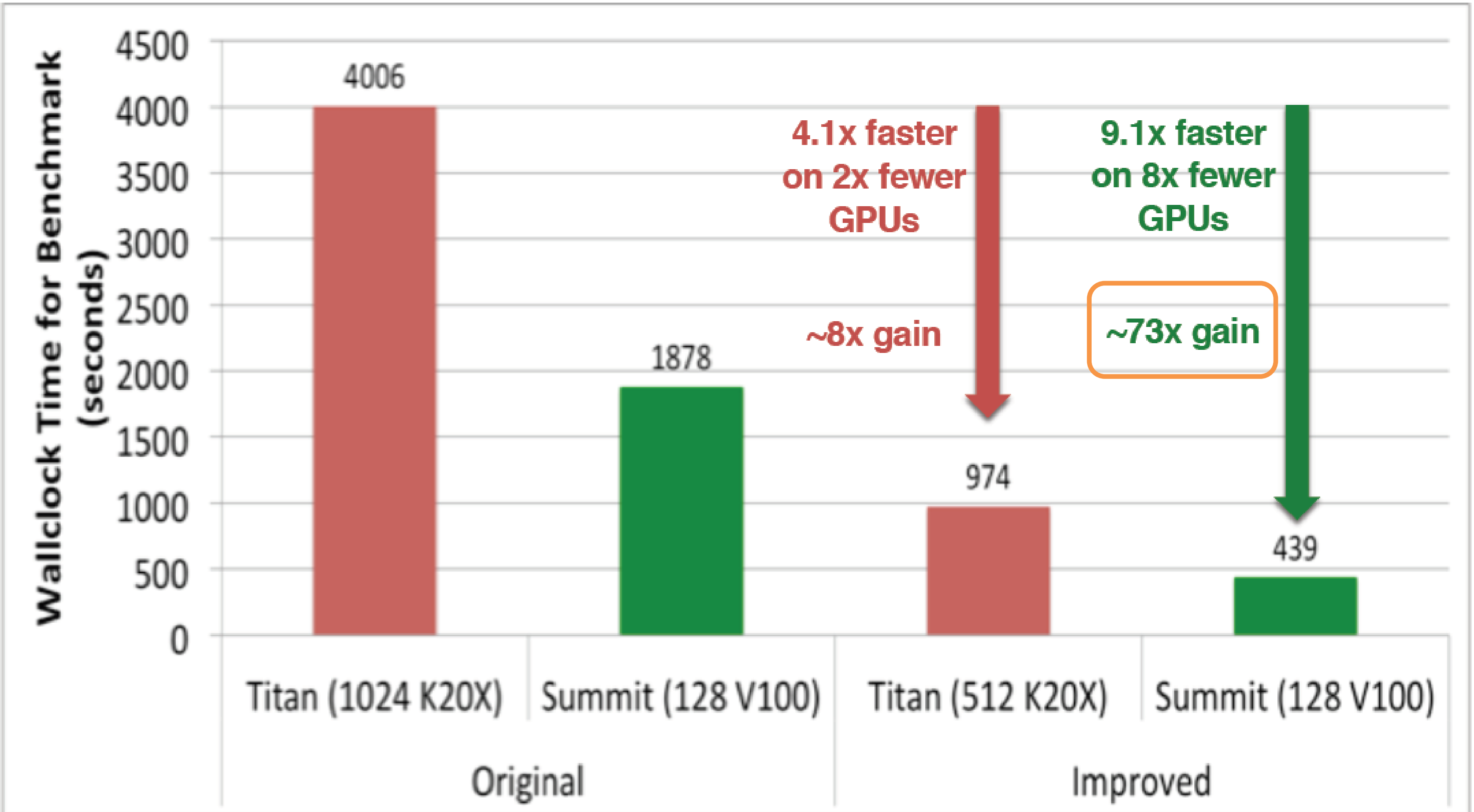
Roper ($N\pi$) – O(N^4)

Deuterium (NN) – O(N^4)

Tritium (NNN) – O(N^5)

[Jie Chen, Robert Edwards, Frank Winter (Jefferson Lab)
US DOE Exascale Computing Project]

Aside: gauge cfgs – further exploiting GPUs



[K. Clark (NVIDIA), B. Joo (JLab), B. Yoon (LANL), https://lqcdscidac4.github.io/]

Some more challenges

Suppose we could compute arbitrary set of finite-volume energies – there are still challenges...

Some more challenges

Suppose we could compute arbitrary set of finite-volume energies – there are still challenges...

- Relate energies to infinite-volume scattering amplitudes
- Analytically continue amp to complex plane (find poles) (connection with analysis of exp data)
- 3-hadron channels?

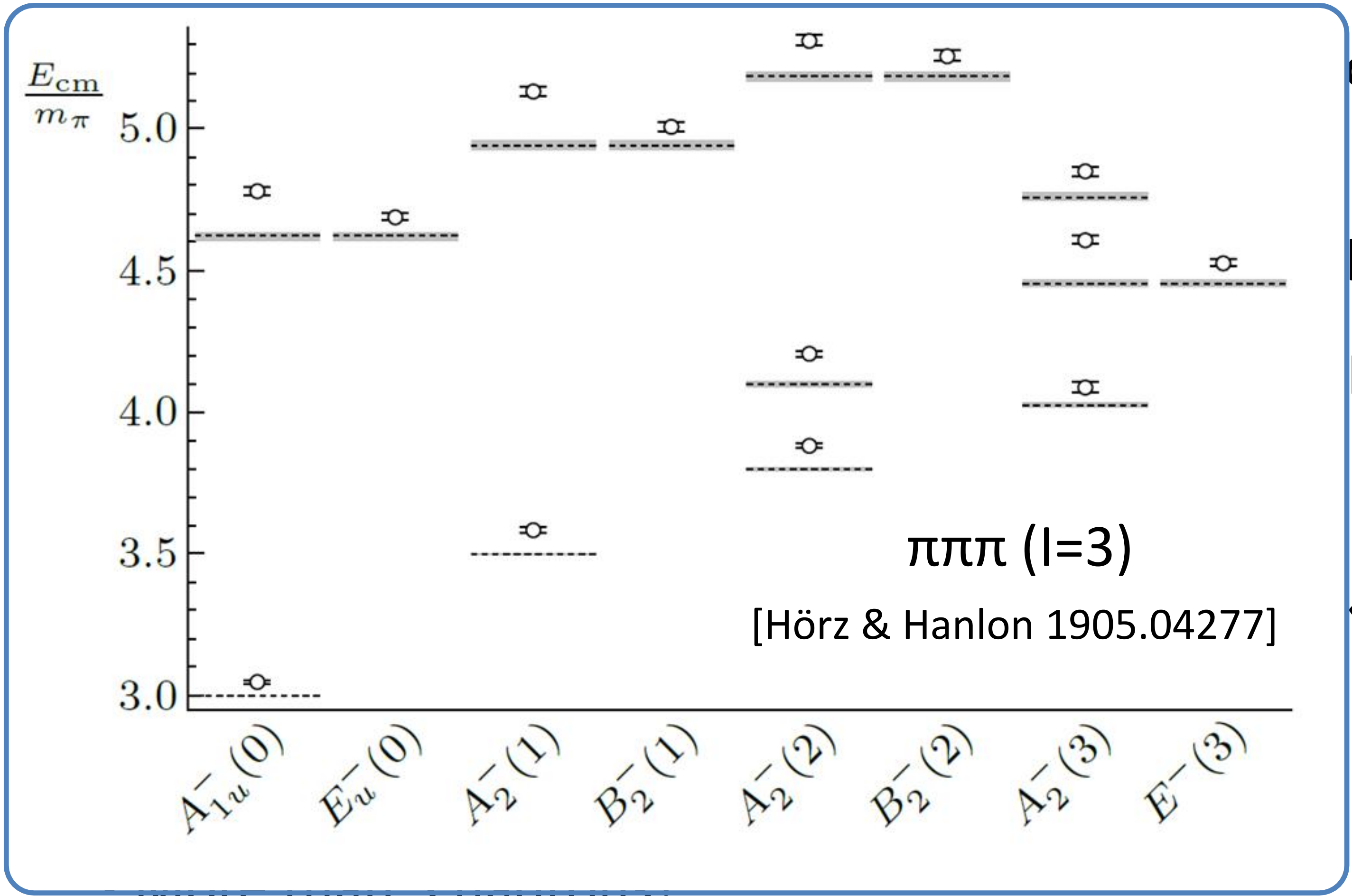
(formulation reviews: Hansen & Sharpe 1901.00483; Rusetsky 1911.01253)

Some $\pi\pi\pi$ (isospin=3) results, also see b_1 study
[1905.04277, 1909.02973, 1909.05749, 1911.09047]

- More than 3 hadrons?

There are more open channels for smaller m_π

Some more challenges



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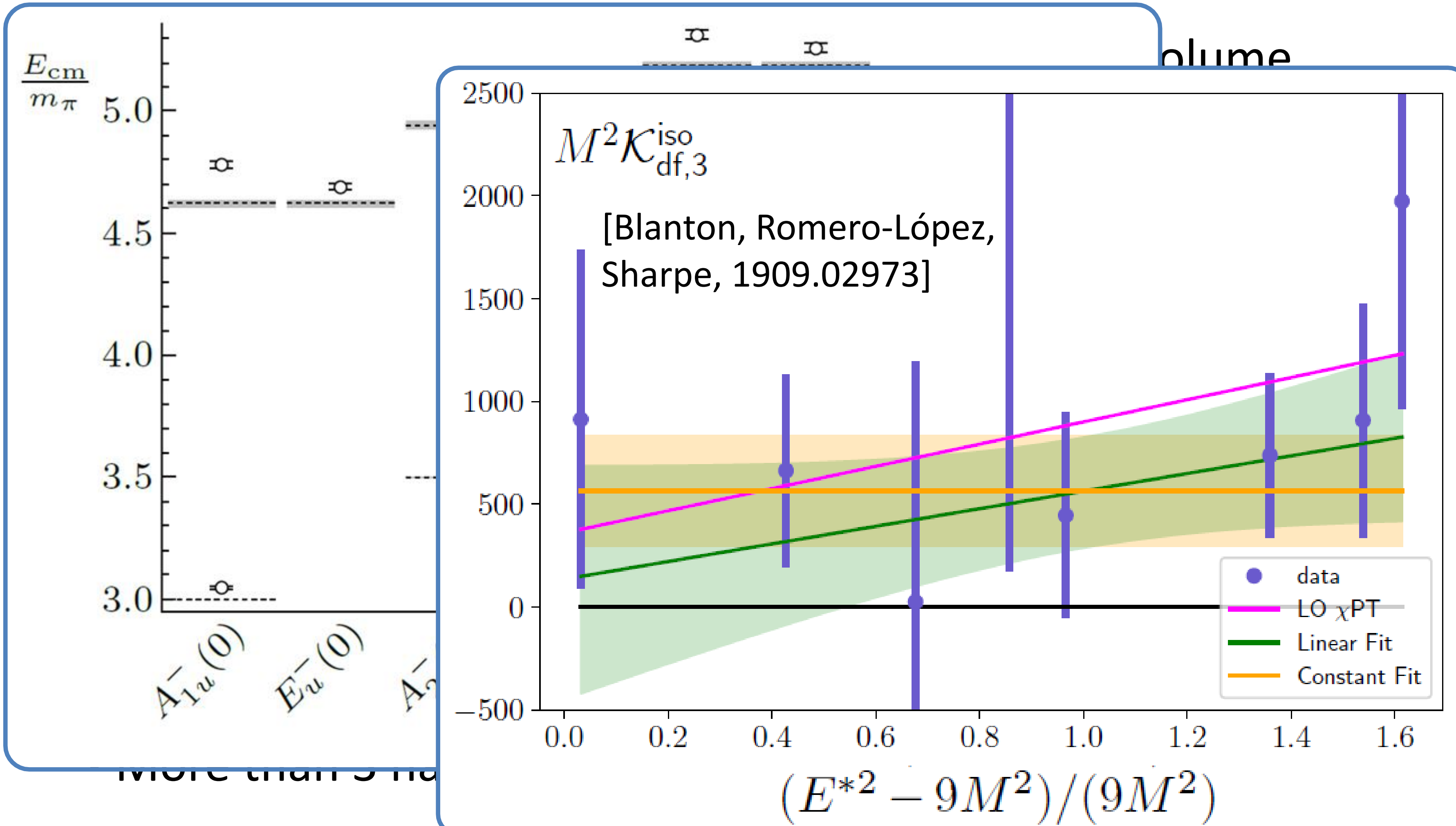
There are more open channels for smaller m_π

Some more challenges



There are more open channels for smaller m_π

Outlook

Significant progress in studying excited hadrons, resonances, etc. using lattice QCD over last 5-10 years

Advances driven by, and will continue to be driven by,

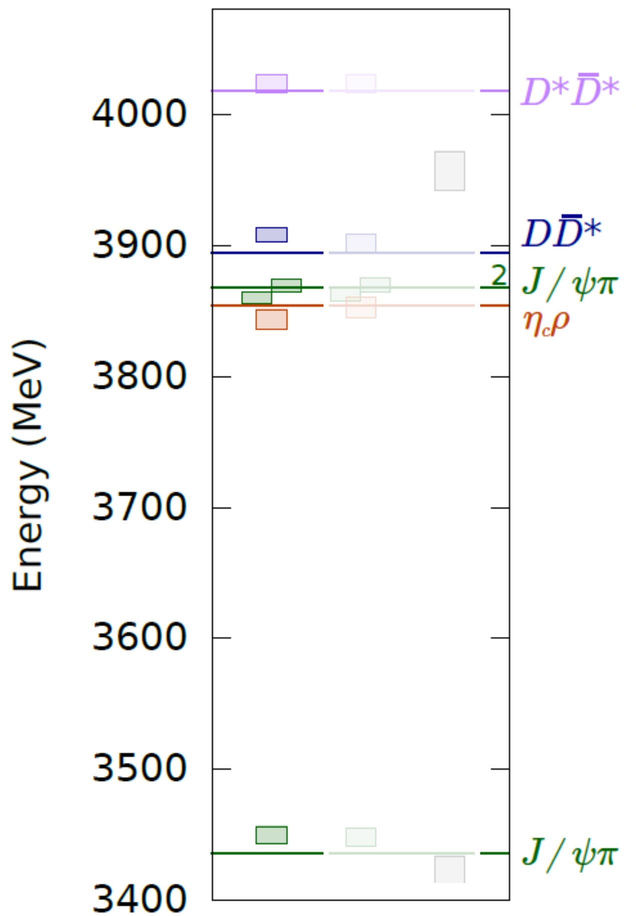
- increase in computational resources (also need storage), exploiting different architectures, computational methods
- novel lattice QCD and analysis techniques, development of theoretical formulation

Good prospects for furthering our understanding of the strong interaction, e.g. light scalars, hybrids, puzzling charmonia and bottomonia (“XYZ’s”), Roper, P_c, ...

Extra slides

Hidden-charm I=1 ($c\bar{c}l\bar{l}$)

[**Cheung**, CT, Dudek, Edwards (HadSpec), JHEP 1711, 033 (2017)]



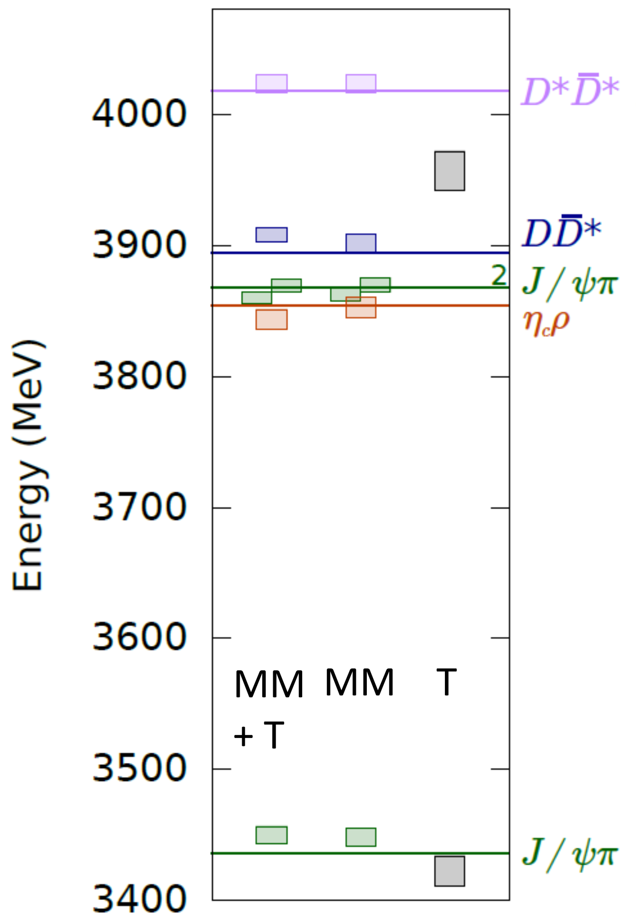
$J^P = 1^+$

G = +

$m_\pi \approx 391$ MeV

Hidden-charm I=1 ($c\bar{c}l\bar{l}$)

[**Cheung**, CT, Dudek, Edwards (HadSpec), JHEP 1711, 033 (2017)]



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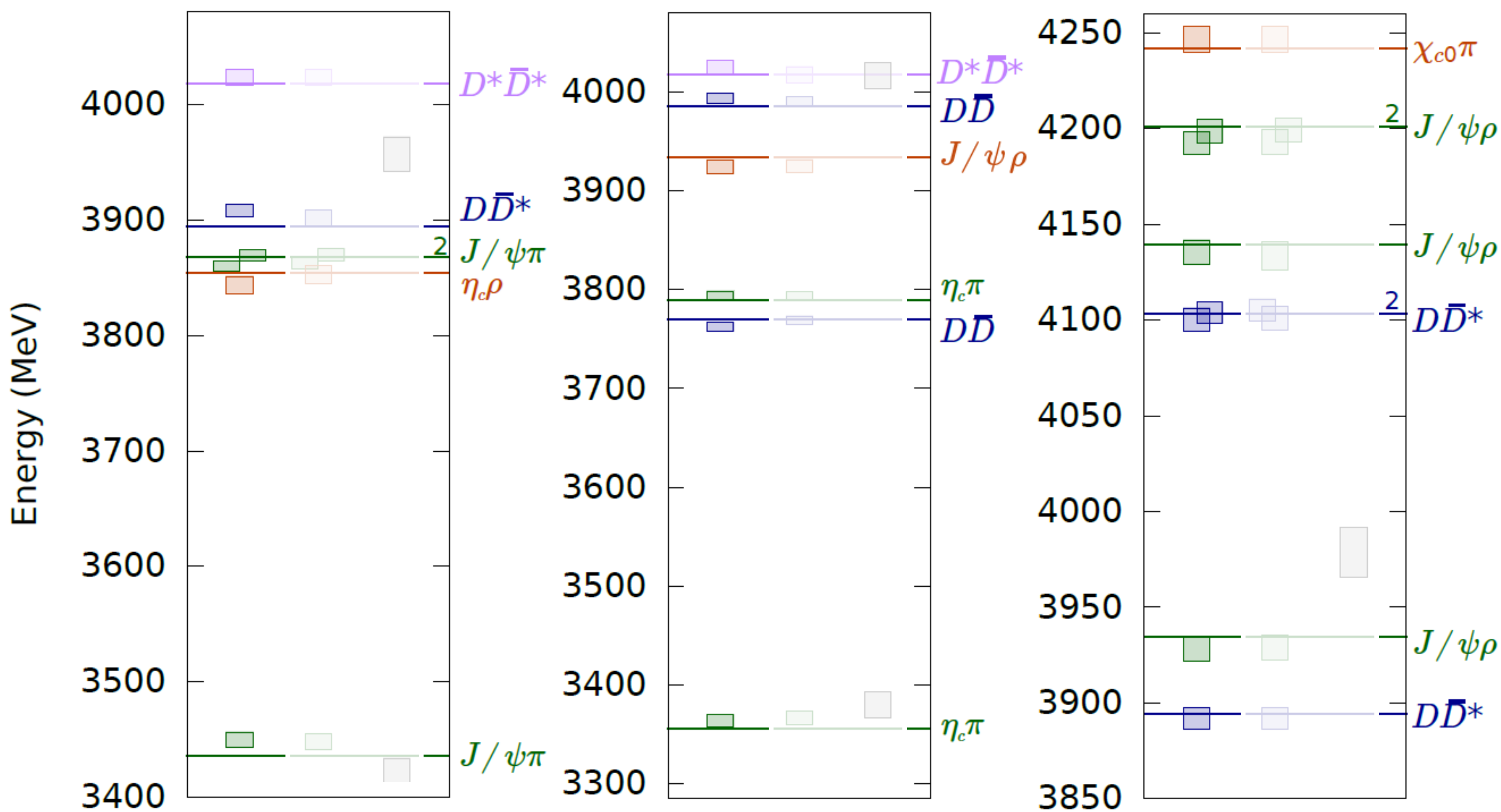


$J^P = 1^+$
G = +

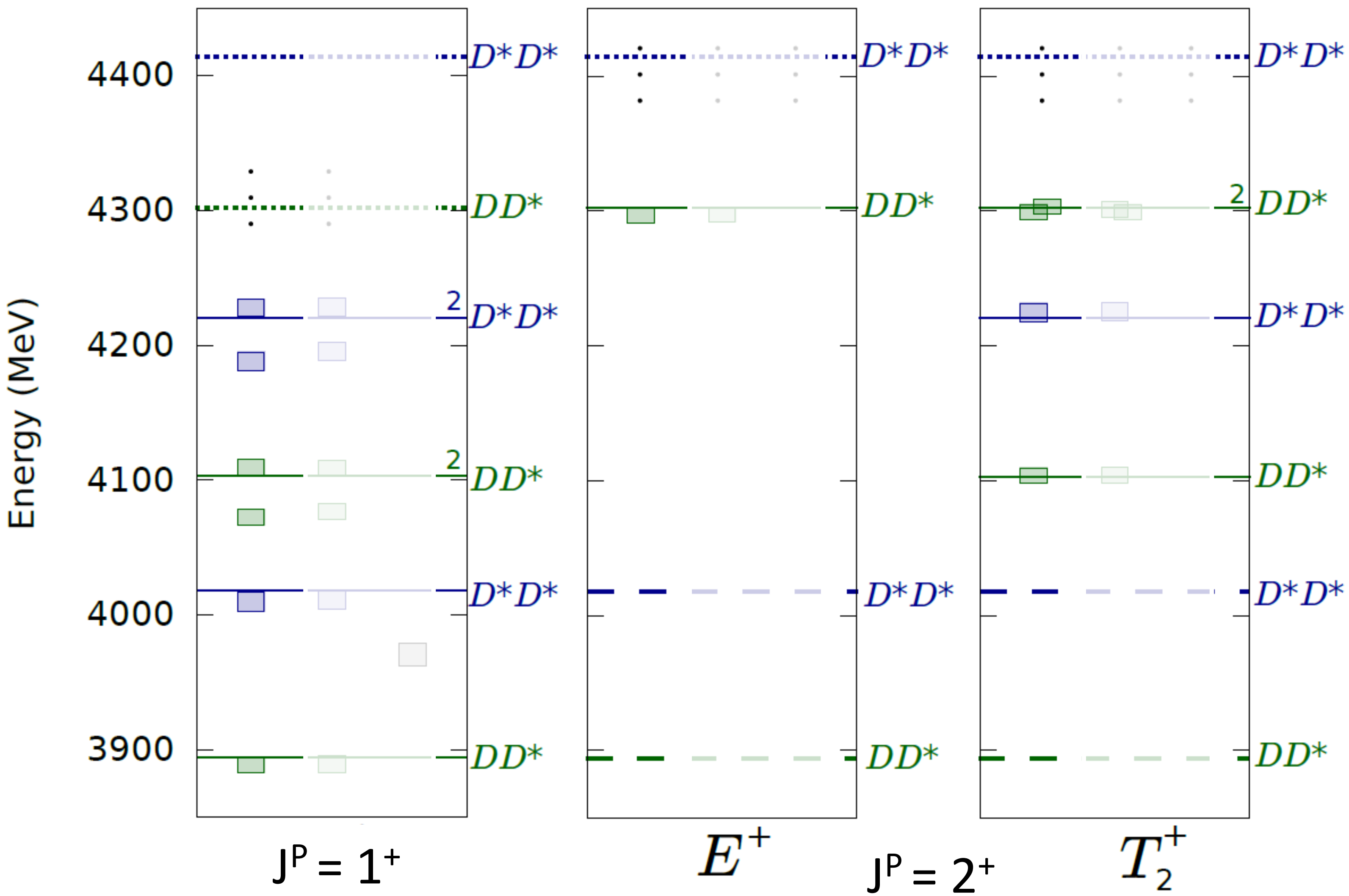
$J^P = 0^+$
G = -

$J^P = 1^+$
G = -

$m_\pi \approx 391$ MeV

Doubly-charmed I=0 ($cc\bar{l}\bar{l}$)

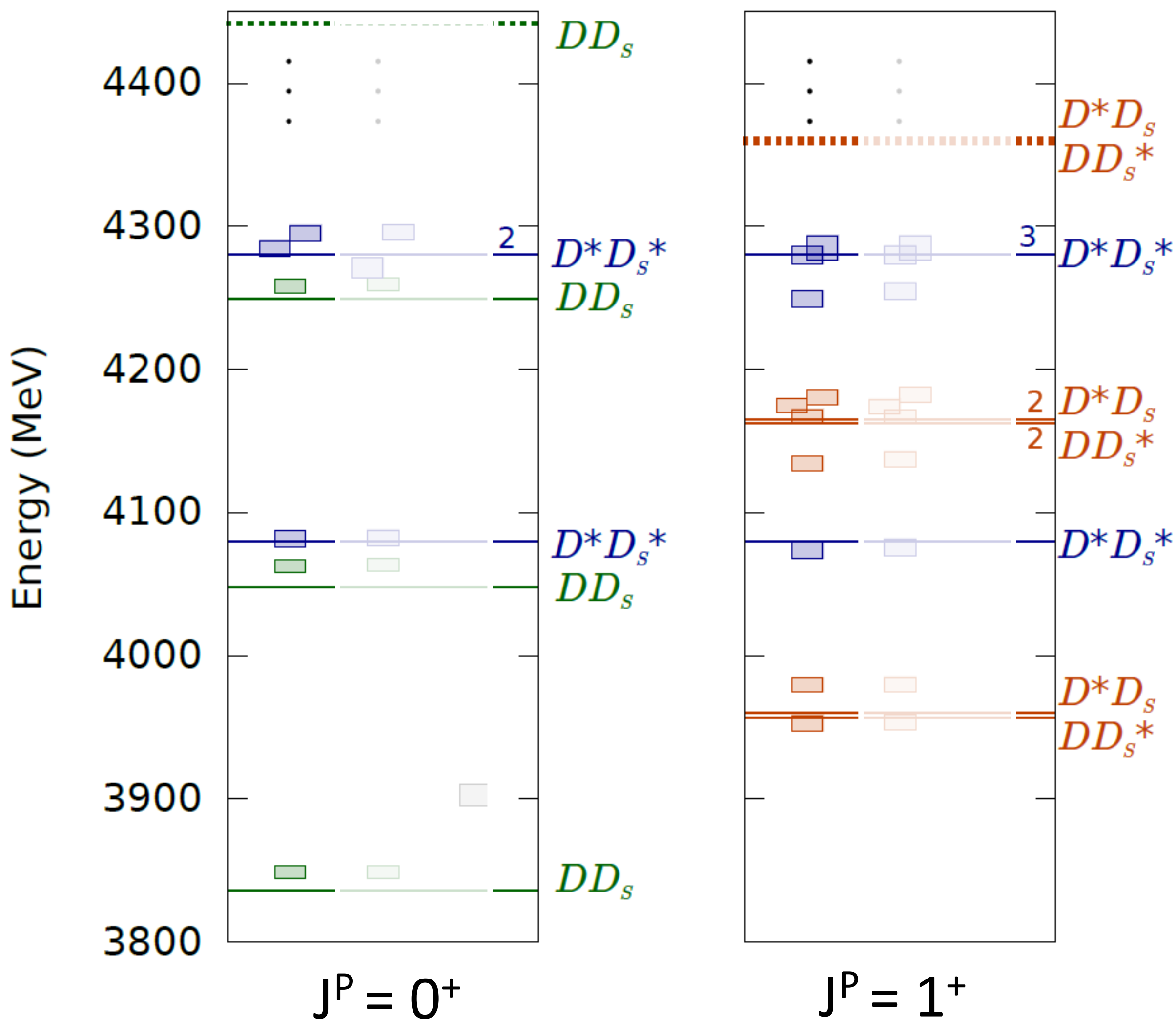
[**Cheung**, CT, Dudek, Edwards (HadSpec), JHEP 1711, 033 (2017)]



$m_\pi \approx 391$ MeV

Doubly-charmed I=½ ($cc\bar{l}\bar{s}$)

[**Cheung**, CT, Dudek, Edwards (HadSpec), JHEP 1711, 033 (2017)]



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